To halve a fraction: An issue for second language learners

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To halve a fraction: An issue for second language learners

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ABSTRACT
The present study investigated test responses from 259 immigrant and non-immigrant school year 9 students in Sweden with the focus on how they solved two problems on fractions, one of them halving a fraction, in a test. The authors report three observations. Newly arrived second language immigrants seemed less likely to have the word 'half' in their Swedish mathematical vocabulary. Moreover, second language learners with longer experience of the new language connected the word 'half' with a division by two, but showed mathematical difficulties in correctly applying it to a fraction. A third finding was that the longer the experiences with Swedish school mathematics, the more likely both first and second language learners were to erroneously omit the percentage symbol, when choosing to use percentage representation of the fraction given in the test problem. The authors suggest seeing newly and early arrived second language immigrants as meeting different challenges. The newly arrived second language immigrants may know some mathematical concepts better and Swedish language less. In contrast the opposite seems to hold for second language learners with longer experience of the language of instruction.

KEYWORDS
Learning mathematics in a second language; mathematics; rational numbers; secondary level; student assessment

Introduction

Sweden today is a multilingual society. About one fifth of the students in the nine years compulsory school are either born abroad or have both their parents born abroad and of these many have a first language other than Swedish. Immigrant students’ achievement in mathematics has been reported in international surveys. For example OECD (2006) reported that in several countries, immigrant students underachieve in mathematics compared to native students. Moreover, Sweden showed one of the largest achievement gaps between immigrant students and native students where both parents immigrated (which PISA denotes second generation immigrants), with the advantage to the latter (OECD 2006). Sweden showed a similar gap in PISA 2012 achievement (Skolverket, [The Swedish National Agency for Education] 2013b). However, besides Parszyk (1999) and previous work by one of the authors Petersson (2012) we found no Swedish studies focusing on second language learners’ achievement when solving test problems in specific mathematical topics. Research on multilingual and multicultural
issues in mathematics education is a recent phenomenon in the Nordic countries (Abreu, César, Gorgorió, & Valero, 2005). Therefore, the present study, on which this article reports, was designed to explore this phenomenon.

Eurostat and Swedish agencies highlights that Sweden has by far the largest number of asylum applicants per capita among EU-countries (Bitoulas, 2013; Swedish Migration Agency, 2016b) and the majority of these are non-European. About 16% of the Swedish population were born abroad (Statistics Sweden, 2014). In compulsory school, 9% of the students were born abroad and another 11% had both parents born abroad. The largest proportion of the immigrant students was born in Iraq (17%). A prognosis made by the Swedish Migration Agency (2016a) is uncertain, but point to an increase of newly arrived students in Sweden within the coming four years, due to the refugee situation in Syria. Together, these aspects make it interesting to study immigrant students’ school achievement in Sweden. One explanation for the achievement gap between first and second language learners is the challenge to learn a school subject in a second language (Moore, Redd, Burkhauser, Mbwana, & Collins, 2002). In Sweden, newly arrived second language immigrants are supposed to have access to academic support in their mother tongue. Thus, there could be possibilities for second language learners to show their knowledge in mathematics in their mother tongue, without having to use Swedish. But tests in mathematics are in Swedish, and there is a severe shortage of both teachers of Swedish as a second language (Skolverket, 2015) and of mother tongue teachers. We decided to look in particular at achievement in fractions as a topic because it is a topic that is taught from first to last grade in compulsory school, and fractions are also reported to be one of the most complex topics to learn at primary school (Lamon, 2007). In the next section we provide some background to the focus of the project. Firstly, we refer to studies on second language learners in mathematics classrooms. Secondly we refer to studies on fractions, the mathematical topic relevant for this article.

**Second language learners**

A common misunderstanding in Sweden is that mathematics is easy to start studying in a second language because it is not as language intense as other school subjects. In contrast to this common misunderstanding, mathematics learning requires language skills in the language of instruction that second language learners may not yet have mastered (Cuevas, 1984; Parszyk, 1999). According to Cuevas (1984) second language learners require considerable language proficiency in both of their languages if they are going to draw on the range of linguistic activities that are required for the learning of mathematics. Some of the academic language used in mathematics may be especially difficult for second language learners to follow (Cuevas, 1984). Second language learners’ lack of fluency in the language of instruction is often seen as creating obstruction in learning mathematics; however, this claim can be seen as drawing upon a notion of deficiency of the immigrant student and their families with respect to language. Lange (2009), Stentoft (2009) and Norén (2010) challenged the view on multilingual students as deficient in the Nordic classrooms. Lange explored how young immigrant children experienced mathematics education practices. In his study, the teachers were not interested in their students’ background. As an example of system of ‘violence’ Lange
(2009) report on teachers’ “non-recognition of ethnic background” (p. 174). In Stentoft’s (2009) study the teachers’ practice of deficiency views on students, lowered the expectations on students’ performances in mathematics. The low expectations often placed the students in “predefined identities” (Stentoft, 2009, p. 1597), and the predefined identities usually determined the students’ performances in school and higher education and predicted barriers on the way. The third study mentioned (Norén, 2010) took into account the importance of student’s mother tongue as a resource for learning. In bilingual teaching of mathematics, the immigrant students were thriving. If the multilingual dimension of mathematics classrooms is neglected, it may under-privilege mathematics learning for students whose first language is not the language of instruction (Meyer, Prediger, César, & Norén, 2016).

Other results indicate that bilingual students may succeed under certain conditions, for example when code switching as Moschkovich (2007) described. Yet, Parvanehezehad and Clarkson (2008) found that code switching among bilingual students was trigged by the challenge to comprehend and interpret a mathematical task. Examples of this are specific mathematical symbols such as numbers and arithmetic operation signs, and by a bilingual environment that supported the code-switching. A high level of language proficiency contributes to educational achievement, and the impact of students’ various proficiencies in each of their languages is central for their academic achievement (Cummins, 1991, 2000). Some studies have shown that if test problems were translated, Mexican-American did better on Spanish versions than English versions of the test (Holland, 1960; Meeker & Meeker, 1973; Mycue, 1968). A similar result was found in Norén’s (2010) study where the language of testing and assessment (Swedish) was one reason for second language learners’ lower results in their mathematics achievement; and the second language learners improved when mathematics test items with text were translated into their mother tongue.

In the research literature, it is well known that second language learning students often achieve less well in mathematics than first language speakers. Cuevas (1984) and Moschkovich (2007) raise the complexity of learning mathematics in a second language. One aspect is that linguistic complexity on test items may cause achievement disadvantages for second language learners and test items with high linguistic demands may measure language skills instead of mathematics skills (Martiniello, 2008). “If students are tested in a language that is not their native language, the scores might reflect not only their competencies in the measured content area (e.g., mathematics), but also their mastery of the language” (Haag, Heppt, Stanat, Kuhl, & Pant, 2013, p. 24). Some research on second language learners in relation to areas of mathematics has had a focus on text or word problems (Gerofsky, 1996; Barwell, 2009). In order to explore the relationship between academic language features of the test items and differences in achievement of first and second language learners, Haag et al. (2013) used data from a state-wide test in mathematics. An example of academic language features are descriptors such as: word count, lexical, such as counts of academic words, and grammatical such as the grammatical construction of phrases/phrase construction. They found that these features in academic language, contributed to the achievement differences between first and second language learners. The complex learning and teaching situations of students, who are second language learners of Swedish and share neither the language of instruction nor the dominant
culture, was the focus of Parszyk’s (1999) study. Drawing on national examinations in mathematics, students solved mathematical text problems in school year 2 (six various test problems), school years 4 and 5 (six various test problems) and school year 9 (three various test problems). The result showed that students in different school years tried to use various strategies to overcome language obstacles when solving the mathematical text problems. Their strategies were not always successful and the students lost motivation and stated that they did not understand mathematics. Abedi and Lord (2001) tested English second language learners on mathematical word problems released from the National Assessment of Educational Progress, along with parallel items modified to reduce linguistic complexity. English second language learners benefited from the modified test version.

One model for understanding mathematics classrooms with language diversity is the one by Prediger, Clarkson, Bose and Vazquez (2016) in Figure 1.

In the model in Figure 1, there is a hierarchy in abstraction in the four semiotic representation levels. The technical register denotes subject specific words while the school register corresponds to Cummins’ (2008) academic register. Prediger et al. (2016, p. 201) mentioned that the school register is sometimes viewed as a condition for learning rather than a goal for learning. An early immigrated second language learner may master the language of instruction in the everyday register but not in the school register and thus does not meet the learning condition. For this student, the alternative is to participate in the classroom activities via the everyday register, which may lead to superficial knowledge in all representation levels in Figure 1. For a newly immigrated second language learner, the situation may be different. The student may not even master representation in words in any register in the language of instruction. On the other hand the newly arrived student may have participated in previous schooling in the student’s first language and through this teaching have learnt to master the other three representation levels in all registers, in for example fractions.

### Fractions in school mathematics

Besides second language learning, what mathematics is taught and how mathematics is taught, is related to culture. According to Abreu and Cline (2003), Bishop (1991), Ladson-Billings (1997) and Nasir (2002) mathematics education is based on cultural knowledge. Thus, what is taught in mathematics and what is valued as knowledge, competency and good achievement may vary between countries. Curriculums and focal topics are not the same in mathematics teaching around world. The Swedish curriculum introduces representation of fractions as parts of a whole and parts of an integer in school years 1-3 (Skolverket, 2011). In school years 4-6, the concept of
representing rational numbers is widened to encompass written fractions, percentage and decimal numbers. For school years 7-9, the curriculum includes arithmetic with rational numbers written as fractions. From an immigrant perspective, there may be differences between countries in how fraction sub-constructs are presented in curricula (Son & Senk, 2010) and in textbooks (Alajmi 2012; Charalambous, Delaney, Hsu, & Mesa, 2010). Also teaching styles can vary between countries (Andrews, 2009; Ma, 1999; Stigler & Hiebert, 1999).

Moreover research literature states that fractions are one of the most complex concepts starting in preschool and is taught throughout all school years (Behr, Lesh, Post, & Silver, 1983; Charalambous & Pitta-Pantazi, 2007; Lamon, 2007). The main reason for this is that fractions are multifaceted and have been shown to consist of five sub-constructs (Kieren, 1976; Behr, Lesh, Post, & Silver, 1983; Pantziara & Philippou, 2012). The calculation procedures used are not neutral to the representation of the numbers, as Duval (2006) emphasized. For instance, half of a fifth can be calculated in different ways as a multiplication or division and represented as a decimal or fraction number.

Half of a fifth as $0.5 \times 0.2 = \frac{0.2}{2} = \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$

These four calculation procedures require different algorithms, which in Prediger’s et al. model in Figure 1 are Symbolic-numerical representations in the technical register. Moreover, they illustrate the importance of students’ ability to change representation of concepts such as number representation and fraction constructs (cf. Duval, 2006; Pantziara & Philippou, 2012). In addition fractions are important not only for their own sake. For example, Post, Behr and Lesh (1988) see mathematical reasoning with fractions and proportions as prerequisites for success in algebra learning.

Students, who arrive in Sweden in later school years, may possess knowledge in mathematical topics not expected by their teachers. That knowledge can be important starting points for these students in their ongoing mathematical learning in the Swedish school. On the basis of the above, we draw the conclusion that there is a need for studying how students with Swedish as a second language, and specifically newly arrived students, achieve when solving problems in specific topics of mathematics.

**Research question**

To learn mathematics through a second language is complex. The same holds when taking a mathematics test in a second language (Abedi & Lord, 2001; Campbell, Davis, & Adams, 2007; Martiniello, 2008). There is a risk that a second language learner’s score on a mathematics test may also mirror the student’s knowledge in the language of the test (Haag et al., 2013). Moreover, early and newly arrived second language immigrants may master the registers and representation levels in Figure 1 at different degree, as discussed earlier. Therefore the following research question is posed:

Given Swedish first and second language learners with different length of experience in the language of instruction, the present study asks the following. What differences, if any, are
reflected in test results between these student categories? The focus for this research question is familiarity with common terminology such as ‘half of’ and usage of different number representations such as fraction, decimal or percentage?

The student categories are the following four: newly arrived second language immigrants, early arrived second language immigrants, other second language learners and first language learners. These student categories are defined in the method section.

Method

To answer the research question, the first author designed a mathematics test and defined the student categories. The students were categorized in groups according to their assumed competence in Swedish as a second language, based on the consideration that a student’s age at immigration has impact on the time needed for being able to study school subjects on the second language. This, since length of time for learning the second language is critical (cf. Cuevas, 1984; Cummins, 2008). Second language learners reach conversational aspects of proficiency, at peer-appropriate levels, usually within about two years of acquaintance with a second language, but a period of 5-7 years can be required for immigrant students to come close to grade norms in academic aspects of a second language (Cummins, 2008). Cummins writes: “Academic language proficiency can thus be defined as the extent to which an individual has access to and command of the oral and written academic registers of schooling” (Cummins, 2000, p. 67). Our assumption is that a second language immigrant student arriving before school start will have time to learn Swedish to an academic proficiency level over nine school years, while a newly arrived student will not. Accordingly many students who are not academically proficient in Swedish may experience unfair assessment on tests.

Test instrument

We have taken into account that second language learners who are taught mathematics in their second language may be in a disadvantaged position when taking a test in mathematics, especially if the test contains substantial amounts of texts in their second language. Although Barwell (2009) questioned the possibility of separating mathematical and linguistic effects, the test items for the present study were constructed with an aim of reducing their linguistic complexity (see Abedi & Lord, 2001). Taking this into account, the test construction, containing a minimum of written text in Swedish, was intended not to obscure the mathematical meaning of the test problems for second language learners (c.f. Cuevas, 1984; Daroczy, Wolska, Meurers, & Nuerk, 2015; Haag et al., 2013; Lager, 2006; Monaghan, 2009). Explicitly, word count and academic word count was kept low, and grammatical construction was kept simple following Haag et al. (2013).

We also have taken into account that some immigrant students have experienced schooling in other countries. They may have been taught in their first language and may have followed a syllabus different from that of the Swedish school. Texts in mathematics, e.g. word problems, are heavily culturally contextualised (Campbell, Davis, &
Adams, 2007; Gutstein, Lipman, Hernandez, & de Los Reyes, 1997). Thus, the author constructed the test instrument as neutrally contextualised as possible.

The mathematics test for the students was constructed with test items mainly on number sense but also algebra and statistics with the purpose to offer a mix of mathematical content similar to the compulsory national test, which the students would undertake a few weeks later. Calculators were not allowed on the test and test time was approximately 40 minutes. In line with Abedi and Lord (2001), the test items were essentially modified national test items. In this article we report on and analyse how two test items on fractions were responded to:

**Problem A.** What is half of $\frac{1}{5}$? (Original Swedish formulation was “Vad är hälften av $\frac{1}{5}$?”)

**Problem B.** What is $\frac{2}{3}$ of 60? (Original Swedish formulation was “Vad är $\frac{2}{3}$ av 60?”)

The students’ responses to these two problems were analysed with the model in Figure 1 (Prediger et al., 2016). Problem A may be suitably solved in different number representations, for example decimal as half of 0.2 with answer 0.1; percentage as half of 20% with answer 10%; fraction as half of $\frac{1}{5}$ with answer $\frac{1}{10}$; or with a graphic illustration of a box or a circle divided in five equal parts halved into 10 equal parts. Problem A thus allows the students to show flexibility in number representation (Duval, 2006; Pantziara & Philippou, 2012). As exemplified above, the students’ responses may contain symbolic-numerical or graphical representations, but problem A also demands a mastering of word representations. In problem A, the Swedish noun ‘hälften’ (English: the half) is the irregular definite declension of ‘halv’ (English: half). A regular declension in Swedish would have been ‘halven’. The irregular declension might be a challenge for second language learners (Hulstijn and Laufer, 2001). A similar problem “Vad är hälften av $1\frac{1}{2}$?” (in English “What is the half of . . .”) was given in the national test 2006 (test problem 7, part B1). For that test problem those students achieved 70% correct. However the students in the national test 2006 were not reported as having the test languages as their first or second language.

Problem B can be solved in at least the following three ways: $\left(\frac{40}{3}\right) \times 2 = 40$; $\left(\frac{2}{3}\right) \times 60 \approx 40$ and $\left(\frac{2 \times 60}{3}\right) = 40$. All of these three calculations involve handling a fraction involving a division by the number three via whole number or decimal number arithmetic. Both problems A and B have the same mathematical structure, namely: What is fraction of number? One linguistic difference is that in problem A the fraction is represented in words (the half), while in problem B the fraction has a symbolic-numerical representation $\left(\frac{2}{3}\right)$. The purpose of problem B is to contrast problem A with a different semiotic representation of the fraction.

**Student sample**

Five schools with a total of twelve entire classes of a possible thirteen agreed to participate in the study. The schools were chosen because they had a high proportion of second language immigrant students. The students (15-16 years old) came from the last year of the 9 years compulsory Swedish school since these students
could be expected to have the largest span of experiences from the Swedish school system and Swedish language instruction. In Swedish schools the students are assigned to follow either ‘Swedish’ or ‘Swedish as a second language’ courses. This decision is made by language experts and governed by the school act (Grundskoleförordning, 1994; Skolförordning, 2011), which gives two criteria for being assigned to ‘Swedish as a second language’ course. The criteria are that the student has a mother tongue other than Swedish (which may include both immigrants and Swedish native children of immigrants) and is considered to need support in Swedish language development.

Cummins’ (2008) empirical and linguistic base separates students on the grounds of whether they had conversational proficiency or academic proficiency in their second language. Cummins indicated an approximate time span of reaching conversational proficiency in about two years and academic proficiency, which is necessary for being able to learn advanced academic content in school, in five to seven years. Following Cummins’ (2008) time spans, the students were categorised in four groups based on time in Sweden, since this correlates with proficiency in the second language. In the present study, the students are in the same grade, grade 9 being the last year of Swedish compulsory school, and thus have a similar age (about 16 years old). Thus, this categorisation coincides with grade placement for those who immigrated after school start. The students following the course ‘Swedish as a second language’ were divided into three subgroups and the fourth group were students following the course ‘Swedish’. The latter are abbreviated Swe1L (Swedish as first language) in the tables and in this group there were no students that had immigrated during compulsory school age. The three categories of second language learners were Newly arrived immigrants, who immigrated during school years 8-9; and Early arrived immigrants, who immigrated during school years 1-7; and Other second language learners (in tables denoted Other2L). Students in the latter student category have either immigrated before school start or are born in Sweden with immigrant parents. All students were taught mathematics in Swedish. For the three second language learner categories, this means that they were taught in their second language, which they had developed to different degrees, the newly arrived immigrants the least.

**External validity**

When using purposive sampling in order to oversample a specific student category, there is a risk of losing external validity in the sense that the sample might have other properties than a random sample would have (Cohen & Manion, 1994; Kruuse, 1998). To control for this, the students in the present study were compared with students in a random sample of students from all over Sweden with respect to their achievements on the annual national test in mathematics. The random sample was collected by the Swedish National Agency for Education and is a part of the evaluation of the annual national test (Skolverket, 2013a). The first author received data from the national random evaluation sample from the National Test Team in Mathematics. The national random evaluation sample only categorizes students according to which of the courses in ‘Swedish’ or ‘Swedish as a second language’ they followed. The National Test Team in Mathematics has no information about the students’ school year of immigration.
each participant in the present study, the national test results were collected with kind permission from the participant schools.

**Data collection**

All students’ written responses to the test problems were compiled into a database. The student responses in this database were aggregated into larger response categories, from which are reported in the present paper.

**Results**

In the test items on fractions, the achievement and the spectrum of both number representation and correct and incorrect calculation procedures varied between different student categories.

**The students’ background**

The collected background variables of the students were their immigrant status, their school year 9 leaving grade in Swedish language and their results on the Swedish national test part B1. These are summarised in Table 1.

Table 1 shows that while nearly all first language learners had at least passed grade in Swedish first language, only about half of the newly arrived immigrants had passed the equivalent course in Swedish second language. The mathematics achievement given in Table 1 is the percentage of correct answers on part B1 of the national test in mathematics. The early arrived immigrants achieved slightly lower than the newly arrived immigrants and the other second language learners in mathematics. These differences were not statistically significant when compared using t-test, but they followed the pattern of the students’ achievements for problems A and B. Of the three categories of second language learners, 52% had Arabic or dialects of the Syrian/Assyrian language as mother tongue. The other students had various language backgrounds such as languages of Eastern Europe, Latin America, South Asia, Africa and other languages in the Middle East, usually with several languages represented in each class. The newly immigrated students are small in number, but their proportion should be set in relation to that they consist of immigrants with a maximum of two years’ experience of Swedish school out of the nine years of Swedish compulsory school.

Together, all second language learners in Table 1 achieved 46% on the mathematics national test part B1, which is identical to the achievement of the second language learners in the national random sample. The first language learners in the random sample achieved 60%, which should be compared with the figure 56% in Table 1 for the first language learners in the present study. Together this shows that the students in the

<table>
<thead>
<tr>
<th>Students’ background</th>
<th>Newly</th>
<th>Early</th>
<th>Other2L</th>
<th>Swe1L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>23</td>
<td>67</td>
<td>56</td>
<td>113</td>
</tr>
<tr>
<td>Swedish grade ≥ passed</td>
<td>52%</td>
<td>78%</td>
<td>86%</td>
<td>97%</td>
</tr>
<tr>
<td>Mathematics achievement</td>
<td>49%</td>
<td>43%</td>
<td>48%</td>
<td>56%</td>
</tr>
</tbody>
</table>
present achieved similarly to the students in the national random evaluation sample when split into first and second language categories. Moreover, the achievements on the national test for the participating schools varied between 45% and 50%.

**Responses for problem A, half of a fraction**

Students’ correct responses to problem A, as in Table 2, were classified into three categories of responses depending on how they represented numbers in their responses. The first category consists of responses containing only fraction representation in calculations and answer. The correct answer in this category is the fraction $\frac{1}{10}$. An incomplete answer in this category was $\frac{2}{10}$ without completing the solution to the problem though this response was infrequent. An incorrect answer in this category consists of incorrect halving responses such as calculation of denominator/2, calculation of numerator/2 and halving in both denominator and numerator $\frac{0.5}{1.5}$ without further calculation. Incorrect halving was more frequent among early arrived immigrants and other second language learners than among the other student categories as seen in Table 2.

The second response category in Table 2 consists of responses using decimal representation but not percentage representation, for example starting with converting to decimal as in $\frac{1}{5} = 0.2$. Correct response in this category is 0.1. Among early and especially among newly immigrants it was common to give an incomplete response by stopping with the answer 0.2. Incorrect responses in this category were to re-representations the fraction by stating that a fifth is 0.25 and similar followed by further calculations, though incorrect decimal responses were infrequent.

For the third category, the students used a percentage representation somewhere in the response, usually by starting with the conversion $\frac{1}{5} = 20\%$ (of some imagined whole). Correct response in this category was 10%. An incorrect response in this category was to omit the percentage symbol in the answer and sometimes also in the calculations. The detailed student responses indicated that this error was due to an omitted percentage symbol and not due to an omitted decimal symbol. For example some responses were the following: $\frac{1}{5} = 20\% = 10$; $\frac{1}{5} = 20\%$ and $\frac{\frac{20}{2}}{2} = 10 = 10\% = \frac{1}{10}$; $\frac{20}{2} = 10 = 10\% = \frac{1}{10}$; and $\frac{100}{2} = 20$. Among other second language learners and students having Swedish as first language, a larger proportion used the percentage representation incorrectly than correctly.

**Table 2. Categories of correct and incorrect responses for problem A.**

<table>
<thead>
<tr>
<th>Response category</th>
<th>Newly</th>
<th>Early</th>
<th>Other2L</th>
<th>Swe1L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction correct</td>
<td>6 (26%)</td>
<td>8 (12%)</td>
<td>10 (18%)</td>
<td>43 (38%)</td>
</tr>
<tr>
<td>Fraction incomplete (stop at 2/10)</td>
<td>0</td>
<td>0%</td>
<td>1</td>
<td>2%</td>
</tr>
<tr>
<td>Incorrect halving</td>
<td>2</td>
<td>9%</td>
<td>11</td>
<td>16%</td>
</tr>
<tr>
<td>Decimal correct</td>
<td>6</td>
<td>(26%)</td>
<td>9</td>
<td>(13%)</td>
</tr>
<tr>
<td>Decimal incomplete (stop at 0.2)</td>
<td>4</td>
<td>17%</td>
<td>6</td>
<td>9%</td>
</tr>
<tr>
<td>Decimal incorrect (0.25 etc.)</td>
<td>0</td>
<td>0%</td>
<td>2</td>
<td>3%</td>
</tr>
<tr>
<td>Percentage correct</td>
<td>0</td>
<td>(0%)</td>
<td>9</td>
<td>(13%)</td>
</tr>
<tr>
<td>Percentage symbol omitted</td>
<td>0</td>
<td>(0%)</td>
<td>4</td>
<td>(6%)</td>
</tr>
<tr>
<td>Other or no response</td>
<td>5</td>
<td>(22%)</td>
<td>17</td>
<td>(25%)</td>
</tr>
<tr>
<td>Total correct</td>
<td>12</td>
<td>(52%)</td>
<td>26</td>
<td>(39%)</td>
</tr>
<tr>
<td>Total incorrect</td>
<td>11</td>
<td>(48%)</td>
<td>41</td>
<td>(61%)</td>
</tr>
</tbody>
</table>
A fourth category in Table 2 consists of twelve students who gave other responses and twenty-eight students who gave no response to test item A.

Table 2 indicates three points of interest. First, the newly and early arrived immigrants had a higher proportion than other student categories, of the response stopping at 0.2. Of the fourteen students that responded 0.2 on problem A, all except one newly and two early arrived immigrants managed to calculate a correct numerical division in problem B. Of the five students that responded \( \frac{2}{10} \) on problem A, all except one first language learner and one other second language learner responded correctly on problem B. The large proportions of incomplete decimal response among newly and early immigrants is accompanied by large proportions of correct decimal responses among newly arrived immigrants. Second, there was a higher proportion of omission of the percentage symbol among the student categories that had spent all their schooling in Swedish school (first language learners and other second language learners) than among the other student categories. Third, there was a higher proportion of incorrect halving among early arrived immigrants and other second language learners than in the other student categories. This pattern is accompanied by a reversed pattern of correct fraction responses among all the four student categories. The frequencies in Table 2 of these observations can be tested with a \( \chi^2 \)-analysis. Since many cells in Table 2 have small frequencies, some rows had to be aggregated, as in Table 3, in order to make the \( \chi^2 \)-analysis reliable.

A \( \chi^2 \)-analysis of Table 3 showed a statistically significant result (p=0.00017). The major contribution to the \( \chi^2 \)-result was from the following cases:

- Incomplete responses: there were large proportions among newly immigrated and small proportions among other second language learners and this was accompanied by a smaller contribution from correct decimal representation among newly arrived immigrants.
- Use of percentage representation: there were none among newly immigrants and large proportions among other second language learner.
- Correct use of fraction responses: there were small proportions among early immigrants and large proportions among Swedish as first language learners.
- Other responses: there were the large proportions of other responses in Table 3 among the early immigrants and in this category are also the incorrect fraction responses.

### Table 3. Aggregated Table 2 (for problem A).

<table>
<thead>
<tr>
<th>Response category</th>
<th>Newly</th>
<th>Early</th>
<th>Other2L</th>
<th>Swe1L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction correct</td>
<td>6 (26%)</td>
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<tr>
<td>Decimal correct</td>
<td>6 (26%)</td>
<td>9 (13%)</td>
<td>9 (16%)</td>
<td>17 (15%)</td>
</tr>
<tr>
<td>Incomplete (0.2 or ( \frac{2}{10} ))</td>
<td>4 (17%)</td>
<td>7 (10%)</td>
<td>2 (4%)</td>
<td>6 (5%)</td>
</tr>
<tr>
<td>Percentage (correct &amp; symbol omitted)</td>
<td>0 (0%)</td>
<td>13 (19%)</td>
<td>18 (32%)</td>
<td>25 (22%)</td>
</tr>
<tr>
<td>Other responses</td>
<td>7 (30%)</td>
<td>30 (45%)</td>
<td>17 (30%)</td>
<td>22 (19%)</td>
</tr>
</tbody>
</table>

Table 4 shows that responses in decimal representation were less common among students with Swedish as first language than among second language learners with
especially newly arrived immigrants having high proportions of decimal number representation. Of the 37 incorrect whole number responses, ten responded with the number 20. Of the 10 incorrect decimal number responses, six contained calculation errors but else showed no conceptual errors.

Of those who displayed detailed calculations in their correct (40) response, 74 had calculated the answer as \( \frac{60}{3} \times 2 = 40 \) and 9 had calculated the answer as \( \frac{2 \times 60}{3} = 40 \). Of these latter there were four newly and four early arrived immigrants and only one had Swedish as first language. Though this difference between the student categories in displayed the calculation method is notable, it is not appropriate to draw any conclusions about this difference, since some of those who responded with only the answer may as well have used the latter calculation method by mental calculations.

**Discussion**

The research question for the present study was about possible differences between the student categories’ familiarity with common terminology and number representation. The discussion will focus on the three concerns identified from Tables 2 and 3: The responses stopping at ‘0.2’; The omission of the percentage symbol; and error in halving. A background to the discussion is that newly and early arrived immigrants and other second language learners showed no major differences in achievement on the national test (Table 1) and problem B (Table 4). Nevertheless there is a consistent pattern of newly arrived students achieving higher than early arrived students in both problems A and B in the present study and in the national test.

**Incomplete halving of a fraction**

Of the three observations in Table 3 for test problem A (determining the half of a fifth) one was: Why did such large proportions of the newly and early arrived immigrants stop at 0.2 and did not divide by two? An interpretation is that the newly and early immigrants in this study may not have recognised the word ‘hälften’ (English: the half) as meaning ‘the half’. Arguments for this statement were found in both the students’ knowledge in Swedish language and mathematical knowledge shown in the different student categories.

From a language knowledge point of view, the following can be argued. First, the newly arrived immigrants clearly had lower grades in Swedish language than the other student categories, as shown in Table 1. This coincides with them having less time to learn the second language. Second, in Table 2, there is a decrease in the response category ‘Stop at 0.2.’ the longer the students has stayed in Sweden. Especially for newly arrived immigrants, this was one of the major contributions to the significant result in
the χ2-analysis of Table 3. Third, irregular declensions in the language of instruction are difficult to recognize for second language learners (Hulstijn & Laufer, 2001). It is known from previous studies that second language learners might find some mathematical terms unfamiliar (Lager, 2006; Monaghan, 2009).

From a mathematics knowledge point of view, the following can be argued. First, Table 1 shows that all three categories of second language learners achieved similarly on the national test, indicating that these three groups had similar general mathematical knowledge though newly arrived students achieved a bit better than early arrived students. Second, the results stated that seven, of the ten early and newly arrived immigrants in Table 2 that responded 0.2 in problem A, managed to calculate a correct numerical division in problem B. This indicate that they had the algorithmic knowledge for completing the task of halving 0.2 in problem A but for some reason did not do so. Third, a close look at the result on students that stopped at 0.2 or \(\frac{2}{10}\) in Table 2, shows that in all four student categories less than 3% responded \(\frac{2}{10}\). For those who responded 0.2 the proportions were the same for the student categories first and other second language learners. Together this gives the impression that only a small proportion of students in all categories may have forgotten to complete the calculation by dividing by two after having re-written the original fraction as 0.2 or \(\frac{2}{10}\). One question is why several students responded with the decimal 0.2 while few responded with the equally increased fraction \(\frac{2}{10}\). It is a common test problem to reduce a fraction, but a fifth cannot be reduced. It is also common to convert the number representation from fraction to decimal or back, but to equally increase a fraction into \(\frac{2}{10}\) would rarely occur as a test problem. Fourth, Table 2 shows that the newly arrived immigrants had better conceptual knowledge in working in fraction representation than early arrived immigrants and yet the newly arrived immigrants had larger proportions of stopping at 0.2 than the early immigrants. Together these arguments of knowledge in Swedish language and mathematics indicate that the reason for their incomplete calculation procedures were not algorithmic (calculation) difficulties but linguistic. A conclusion is that if these newly and early immigrants had known the word ‘hälften’ (the half), it is likely that most of them would have given a correct response to problem A. In other words, these students gave the impression of mastering the symbolic-numerical representation of fractions in the technical register in Figure 1, but not the representation level in words in the same register (see Prediger et al., 2016).

**Correct and incorrect notation of percentage**

A second observation is students who omitted the percentage symbol. Table 3 shows that this was more common among students who have spent their whole school time in Sweden than among those who had immigrated during school age. Tables 2 and 3 taken together show that among the newly arrived immigrants in the present study there were no percentage representations at all in the responses. This is in contrast to the other three student categories, in which percentage representations were common in the present study. Moreover, omission of the percentage symbol was relatively common among the two student categories other second language learners and first language learners. In that sense, both the use of percentage representation and omission of the
percentage symbol seemed to increase with the length of experience with Swedish mathematics teaching. In detail, Tables 2 and 3 show that of the students that gave a response using percentage representation, more than half of the first language learners and other second language learners omitted the percentage symbol. Of the early arrived immigrants less than half committed this error. Collectively this gives the impression that longer experiences with Swedish school mathematics may enable familiarity with using percentages. In that aspect students who converted the fraction into a percentage, explicitly showed flexibility in number representation (cf. Duval, 2006; Pantziara & Philippou, 2012). But with this seems to have followed an increased risk of omitting the percentage symbol where it is needed. So, the students omitting the percentage symbol can be said to master fractions as percentages (a symbolic-numerical representation), but in an everyday register in Figure 1.

**Error in halving of a fraction**

A last observation as evidenced in Table 3 relates to the students who responded in the category ‘incorrect halving’. These students divided not the fraction but the number in the denominator or numerator, or both, by the number two. By doing so, these students showed two things. First, they showed that they understand ‘half’, represented as a word in Figure 1, as an instruction to divide by the number two. Second, they showed that they did not know how to correctly apply the instruction to determine the half of the fraction in problem A during the test. A conclusion from this is that the responses ‘incorrect halving’ were due to mathematical difficulties and not due to linguistic difficulties. From previous research we know that second language learner may be underprivileged when it comes to learning from instruction in their second language (Lange, 2009; Meyer et al., 2016; Norén, 2010; Stentoft, 2009). One reason for this is that second language learners may have a less well developed academic proficiency than first language learners in the language of instruction (Cummins, 2008), which in the model in Figure 1 is the school register.

We know that the students in the two student categories of newly arrived immigrants and other second language learners have had a large part or even all their mathematics teaching in their second language. This is not the case for the newly arrived immigrants, most or all of them are likely to have had a majority of their mathematics and fractions teaching in their first language, just like the first language learners in the present study. Moreover, Tables 2 and 3 point out early arrived immigrants and other second languages learners as having slightly smaller proportions of correct responses and slightly larger proportions of incorrect responses ‘incorrect halving’ than newly arrived immigrants and first language learners. An assumption we make is that these differences in Tables 2 and 3 are at least to some extent related to the difference in time of having been taught mathematics in a second language.

**Conclusions**

In the present study we have reported on three observations from students’ responses to two mathematical test problems involving fractions and with very few words. We have used a theoretical model by Prediger et al. (2016), illustrated in Figure 1, to interpret the result.
The first observation is the proportion of students who did not associate the wording ‘hälften av’ (being an irregular declension of the wording ‘the half of’) in the test language with dividing by two, decreased with the length of time since the student immigrated. Second, the conversion from fraction representation to percentage representation increased with the time experiencing Swedish mathematics teaching. So did the risk of omitting the percentage symbol for both first and second language learners. These students had responded ‘10’ instead of ‘10%’. Third, higher proportions of early arrived immigrants and other second language learners than newly arrived immigrants and first language learners had problems in correctly applying ‘half of’ to a fraction. Greater proportions of these students responded by dividing numerator, denominator or both by two.

The second observation of omitting the percentage symbol may be a local Swedish result. In contrast, we believe that the first and third observations are applicable not only to the students in this study, but to immigrant students in general during mathematics tests. This holds since newly arrived immigrants, despite previous long learning experiences in their first language and potentially solid mathematical backgrounds, will have to struggle with learning new mathematical terminology and grammatical issues (Lager, 2006). For early arrived immigrants the challenges are different. For them, the terminology may be the smaller problem while the mathematical knowledge may suffer due to learning in a second language as also noted by Moschkovich (2007). They may know the terminology, but have experienced most of their mathematical learning in a second language. Fraction teaching starts in early school years and in line with Moschkovich we assume that the second language learners in the present study might have had more opportunities for learning and would have achieved more if their first language had been used and accounted for as a resource in their learning of mathematics through their school years. As Cummins (2000) wrote, it takes five to seven years to develop academic proficiency in a second language, and a well-developed language is required for successful learning of school mathematics. Beside these theoretical arguments, an argument for validity of the student sample is that the second language learners in the present study achieved in the mathematics national test on a similar level as did the second language learners in a national random sample. Moreover there was a small variation between the schools in this respect.

An outcome of the present study is that it seems necessary to see newly and early arrived immigrants as two separate groups having different challenges when being taught and tested respectively on different mathematical topics in their second language Swedish, despite the problems in the present study not being word problems. The present study gives a picture of newly and early arrived immigrants mastering the same levels and registers in of the model in Figure 1 in different degrees. This outcome is in line with research underlining the time needed for developing proficiency in the language of mathematics instruction (Cuevas, 1984; Cummins, 2008; Meyer et al., 2016; Parszyk, 1999). Together, the responses to the fraction test item in Table 2, suggests that a large proportion of early arrived immigrants and other second language learners may need significant support in fraction concept development.

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