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Connecting two laboratory tasks under an umbrella of uncertainty: Hooke's law and simple harmonic motion

Jakob Gyllenpalm[1], Ulf Christiansson[2] and Patrik Friggebo[2]

[1] Department of Mathematics and Science Education, Stockholm University, Stockholm, Sweden
[2] Viktor Rydberg Gymnasium Odenplan, Stockholm, Sweden

E-mail: jakob.gyllenpalm@mnd.su.se

Abstract
Laboratory work in physics has traditionally focused on the verification of facts, theories and laws. In contrast, this article describes how laboratory tasks can be used to promote students understanding about the nature of science and scientific inquiry. In the project reported here, students learn about measurement uncertainties and a simplified graphical method for propagating errors. By using this knowledge to compare the precision of two common methods to determine the spring constant, Hooke's Law and simple harmonic motion, students learn about the nature of experimentation in physics. From this specific example, comparisons can then be made with authentic research to highlight more general aspects of the nature of science and scientific inquiry.

1. Introduction

Two common laboratory tasks in high school physics are to determine the value of a spring constant using Hooke's law and simple harmonic motion (SHM). Here we describe how students can learn about scientific inquiry by comparing the precision of these two methods. The aim was to make scientific inquiry and the nature of science more explicit in laboratory work in physics by gradually modifying laboratory tasks traditionally used at our school.

Our starting point was that students do not generally learn about scientific inquiry and the nature of science just by *doing* laboratory work [1, 2]. This is especially so if tasks are highly structured to arrive at a known result, i.e. expository labs with little or no degrees of freedom [3]. Therefore, we wanted to make learning about the nature of scientific inquiry explicit by focusing on one aspect at a time [4]. One such aspect stated in the new Swedish curriculum (from 2011) is the requirement that students should learn about evaluating and comparing methods of inquiry, and with respect to measurement uncertainties.

If the only purpose of laboratory tasks is to confirm known theory, deviations from the expected result are often dismissed by loose arguments about 'sources of error'. But with a goal to teach students about scientific inquiry the role of measurement uncertainties can acquire new meanings. A quantified uncertainty makes it possible to investigate the effect of improvements in

the method and may also lead to a better understanding of the physics involved. This can help students understand aspects of the nature of science such as: the role of creativity in research, the difference between data and evidence, and that there is not one scientific method all researchers use as an algorithm [2, 5].

A challenge for us was that there is no tradition in Sweden of handling measurement uncertainties *quantitatively* in high school physics. It was not something we had done with students before. However, after having come across a simplified method for error analysis we saw a new possibility for teaching about scientific inquiry by comparing methods. This graphical method of error analysis is described by Hamper [6] and, earlier by Tawney [7]. It is based on using a linear regression of data points to find a constant directly or indirectly given by the lines gradient. The idea is to plot data points with error bars and then find the error margins for the gradient by determining the upper and lower bounds by graphically introducing two more lines allowed within the error bars and determining their gradients.

2. The experimental error project

In *the Experimental Error Project,* we asked students in grade 12 to investigate which of two methods to determine the spring constant of a mechanical spring was more precise; Hooke's law or SHM. In anticipation of this project, students had gained some familiarity in previous laboratory work with estimating measurement uncertainties. They were already familiar with linear regressions and how to interpret these in relation to a research question. They were also familiar with estimating error margins from measurements with uncertainties using only maximum and minimum values, and that the precision of a measurement is represented by the significant numbers used. Students had also been introduced to the principles of a controlled experiment and had practiced identifying dependent and independent variables.

To help students grasp the role of experimental errors, the principles of a mechanical scale containing a spring was used together with the idea that the precision of the measured spring constant could be related to the precision of the scale. This may not be totally realistic but

served to make the idea of quantifying *uncertainties* in experimental work more concrete. To further illustrate this idea a brief introduction to the investigation of gravitational waves and the enormous degree of accuracy needed in such research was given (a current topic at the time due to the Nobel Prize for the discovery of gravitational waves). The overarching question students were faced with then was: *Which one of two methods to determine the spring constant is more precise and how large is the difference in precision?* To investigate and answer this question, students were given five classroom periods (or parts of) of 60–90 min each in the following sequence:

1. Introduction to the project and the graphical method of propagating measurement uncertainties to determine a constant with error margins.
2. Lab work to determine the value of a spring constant with error margins using Hooke's law, for which students' hand in a short report.
3. Peer feedback on the reports with a focus on ways to estimate and minimize measurement uncertainties.
4. Lab work to determine the value of the same spring constant with error margins using SHM. Students hand in a short report and formative assessment is given by the teacher.
5. Concluding lesson to discuss the overarching research question by comparing the two methods and extracting what this can teach us about scientific inquiry; in these methods, and in general.

The instructions for the two labs were altered from their original cookbook style to emphasize aspects of scientific inquiry. First, by combining them under one overarching question, and by explicitly stating that the two different methods were to address the same research question: *What is the value of the spring constant*? This may seem like a slight change, but was done to emphasize, (a) that all scientific inquiries proceed from a question, (b) that the particular methods of inquiry follow from the questions asked, and (c) that the same question can be investigated by different methods—all important understandings about the nature of scientific inquiry we want our students to learn [2]. Secondly, by not providing any details about the experimental setup or data

collection the task was given two degrees of freedom: the value for the spring constant, and the details of how to plan and conduct the data collection; in particular, how to estimate and minimize measurement uncertainties. This also served to exemplify the role of creativity in scientific inquiry and hopefully give some experience of the non-linear nature of more authentic inquiry. The theoretical subject matter was already familiar to the students so that they could focus their attention on the process of inquiry.

We pointed out that the general method was a *controlled experiment*, a concept they had used in previous laboratory tasks but which was still not clear to all. One reason for this difficulty may be the common conflation of *methods of inquiry* and *methods of teaching* in school science in Sweden [8]. Laboratory work is often described as 'experiments' whether or not it actually is an experiment. Most traditional laboratory tasks in physics are of course experiments but this makes the point even more important and difficult to see, namely that not all research in physics adhere to the logic of the controlled experiment (e.g. in astronomy). Therefore, students were asked to specify the *independent*, and *dependent* variables they had used, as well as an actual *answer* to the research question. To connect the research *question* with an explicit *answer* is an important detail somehow often forgotten in laboratory work in physics. As mentioned the students had already become somewhat used to this in previous lab work, but an improvement in hindsight would be to also ask students to list *controlled* variables and specify how these may relate to measurement uncertainties. Obviously, students also include raw and processed data together with the linear diagrams for calculating error margins. Special attention was given to the description of how they had estimated and minimized the measurements' uncertainties.

2.1. Hooke's law

Extending a spring with weights to determine the spring constant of Hooke's law ($F = -kx$) is a common task in high school physics, but perhaps not with a quantitative estimation of errors. Estimating the uncertainty in length is a matter of estimating how well one can determine the extension of a spring, in this case using a ruler.

This was not as easy as expected for students to do and we observed several strategies initially. One group held a short plastic ruler in the air next to the spring. Another group did the same but also photographed the spring and ruler for an easier and more precise measure. One group let a plastic ruler hang from the same arm holding the spring in which case both dangled to some extent. After such initial attempts, most but not all groups, decided that the best way was to use meter-long wooden rulers which could be firmly planted to the desk. Eventually groups came up with estimated uncertainties in the magnitude between ±2–8 mm. Students used 5–8 data points, and the spread in relative uncertainties was 1%–12%, with a typical value for the spring constant being $k = 21 \pm 1 \, \text{N m}^{-1}$ (5%). Although there is also an uncertainty in the mass we asked students to omit this to simplify the calculations.

2.2. Simple harmonic motion (SHM)

Another way to determine the spring constant is to set the system in motion creating a pendulum. The mathematical model of SHM can then be used to determine k by measuring the period for different masses. The analysis is simplified by writing the equation as $4\pi^2 m = kT^2$ and then plotting $4\pi^2 m$ versus T^2 to find k (a trick we left for the students to figure out by themselves by suggesting them to identify appropriate quantities to plot). The most straight forward way is then to use a stopwatch and measure the time for a number of periods. Estimating the uncertainty in the period is a matter of estimating the uncertainty for the total time measured divided by the number of periods. To calculate the uncertainty in T^2 students used the 'rule' that relative uncertainties are added when values are multiplied. This gives each data point a different error bar due to the increase in the period for larger masses.

In one class the idea spread to film the motion and then analyze the video to find the time for the period. For this several types of software were used: Tracker, Quicktime, Logger Pro, and iMovie. However, most groups then measured the time for only one period, forgetting the idea of a longer sampling time. Reasoning in slightly diverse ways the measurement uncertainty in the period was estimated to be between 17 to 80 ms. This was similar to the estimation by groups using

only a stopwatch (17–50 ms), however no group used more than 20 periods to collect data. Groups using the stopwatch had more consistent results with error margins for k between 13%–24%, while groups filming calculated error margins raging wildly between approximately 1%–50%. One group creatively came up with the idea to film the pendulum in slow motion with a large digital stop watch on a lap top in the background, resulting in one of the more precise measurements (4,6% relative error). Only one group used both video analysis and measured over 10 periods resulting in the lowest relative error of only 1,5%.

As in the case with Hooke's law students encountered many problems which needed to be resolved during their lab work. The fact that the amplitude does not enter the equation was forgotten by several groups leading to discussions about its role in the investigation. What constituted dependent and independent variables had to be clarified by many, and the significance (if any) of the pendulum wobbling from side to side. For those groups filming the motion discussions arose about what constituted their primary *data* (in contrast to *evidence*); if it was the movie itself or perhaps frames per second derived from analyzing the film. For groups using a stopwatch reaction time was discussed and sometimes investigated by e.g. having one student start and stop the watch by a random command from another student. This led, in discussion with the teacher, to talks about rhythm and timing and comparisons with sport and music as in hitting a tennis ball or dancing. The value of measuring the time over several periods over allowing the pendulum to develop a sideways wobble had to be negotiated within some groups.

3. Summarizing learnings about inquiry

In the concluding lesson students wrote their results on the white board to collectively answer the overarching question concerning the precision of methods. It was found to be comparable for the two methods for most groups; even though some came up with rather large error margins, especially for SHM. A question this raised was whether the value for the spring constant lies within the error margins of either method, and it was found to do so for 10 out of 11 groups.

Hooke's law generally gave somewhat smaller error margins which seemed to answer the overarching question for the project. However, the potential for improvements in SHM method is larger, as students realized in discussion with the teacher. One group, for example, found the value for k to be $21 \pm 1 \, \mathrm{N \, m^{-1}}$ (5%) with Hooke's law and $21 \pm 3 \, \mathrm{N \, m^{-1}}$ (14%) with SHM. But by increasing the number of periods measured from 20 to 30, the measurement uncertainty in each period can be reduced from 0, 02 s to about 0, 013 s, resulting in the same error margin for k as with Hooke's law. This evidence supports the conclusion that the 'true' value of the spring constant is in fact $21 \, \mathrm{N \, m^{-1}}$. But how certain can we be? This serves to illustrate to students that in authentic research there is no correct answer in the back of the book. All we must rely on is how well we trust our methods. This means to what degree we can say with confidence that we have been able to estimate and keep track of measurement uncertainties, and how these affect the final error margin.

Included in the project description was also a list of what we expected students to learn. This was inspired by Hart *et al* [4], who emphasize distinguishing between—and letting students know— the *educational purpose* and the *aim* of a laboratory task. The aim in this case was: (1) to determine the spring constant and to quantify the precision with two methods, and (2) to compare the precision of two methods. The educational purpose was formulated as a set of questions we wanted students to be able to answer after the project:

1. What is meant by *measurement uncertainties* in experimental investigations?
2. What can be done to estimate and minimize measurement uncertainties?
3. What is meant by *error margins* in experimental investigations?
4. How can error margins for an experimentally derived value be estimated?
5. Why is it important to know about measurement uncertainties and error margins in experimental investigations?

These questions were the focal point for the discussion in the concluding lesson. It resulted in several suggestions for how to improve these experimental investigations (table 1). From this list of particular improvements some general points were distilled to keep in mind in future

Table 1. In discussion with the teacher students summarized what they had learnt about ways to minimize the measurement uncertainty and increase the precision of the method.

Hooke's law	SHM
Positioning the ruler in a more stable way	Stable set up
Photographing the spring and ruler to provide a more stable reading	Waiting for a clear rhythm to establish before beginning to measure the period
Measure the distance from the table to the weight and deduce the extension of the spring (i.e. using a more stable reference point)	Getting a 'feeling' for the rhythm when measuring with a stop watch
Using heavier weights to get a longer extension easier to measure (without damaging the spring)	Having more than one time-keeper in the group
Using weights so that data points are more spread out	Measuring the time over a larger number of periods
	Filming the motion of the pendulum and count frames to determine the period
	Filming the motion in slow-motion together with a digital stopwatch to determine the period

investigations: (a) getting a stable and aligned experimental set up, (b) considering the precision of measuring devices, (c) considering reaction time, (d) choosing a reasonable number of data points, (e) expanding the measurement interval and considering its limits given by the set-up, (f) measuring more than once, (g) keeping all relevant variables controlled, (h) being able to reproduce a phenomenon and measurements, and finally (i) considering random and systematic uncertainties. A crucial point, more difficult than anticipated for many students, was the difference between *measurement uncertainties* resulting from the data collection process, and the resulting *error margins* for the determined constant—a consequence *both* of how data was collected, and later mathematically processed and analyzed.

4. Conclusions

In *the Experimental Error Project* two common and relatively simple laboratory tasks are connected by asking students to compare the precision of methods quantitatively. Students learn about scientific inquiry through a focus on the relationship between measurement uncertainties and error margins and are introduced to a simplified method of error analysis. As students' progress through the project a healthy source of frustration is the lack of simple algorithms to estimate measurement uncertainties, leading to deeper discussions about of the physical systems studied, the experimental setup, as well as some creative inventions to measure more

accurately. The quantitative error margins make it possible to ask more informed questions about the work of each group, both how they compare across methods and also across groups to collectively deduce insights about the physics involved and the processes of inquiry used. By relating these experiences to the functioning of a mechanical scale and an example of authentic research in physics (gravitational waves), a link between student's experiences in school, everyday life, and authentic scientific inquiry is established. These examples alone cannot teach students all they need to know about scientific inquiry but provide a concrete experience to refer to in future teaching situations.

In future versions of this project we will try to be even more explicit in connecting this project with authentic research. The possibilities for taking advantage of teachable moments about the nature of science such as the role of subjectivity in estimating uncertainties and the difference between data and evidence can be further developed [9]. Finally, we want to develop better assessments of student's learning about scientific inquiry so that this content is perceived as equally valued as the traditional theoretical content. Our impression is that the increased focus on methods of inquiry have not resulted in less opportunity for students to also learn about Hooke's law and SHM, but rather the opposite.

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Ulf Christiansson holds a Master's of Education for the Upper Secondary School in Physics and Mathematics and a Master's of applied Physics. He is currently working as a Physics teacher in upper secondary school at Victor Rydberg's Gymnasium Odenplan in Stockholm.

Patrik Friggebo holds a Master's of Education for the Upper Secondary School in Physics and Mathematics. He is currently the head of the physics department at Victor Rydberg's Gymnasium Odenplan in Stockholm, sharing his time between teaching and school development projects.

Jakob Gyllenpalm holds a Master's in Physics, a Master's in Education for the Upper Secondary School in Physics and Mathematics, and a PhD in Science Education. He has taught physics in the International Baccalaureate program and is currently a lecturer in Science Education at Stockholm University with a special research interest in inquiry and laboratory work.