<http://www.diva-portal.org>

Postprint

This is the accepted version of a paper presented at *15th European Conference, EUMAS 2017, and 5th International Conference, AT 2017, Evry, France, December 14-15, 2017*.

Citation for the original published paper:

Goranko, V., Kuusisto, A., Rönholm, R. (2018)
Rational Coordination in Games with Enriched Representations
In: Francesco Belardinelli, Estefanía Argente (ed.), *Multi-Agent Systems and Agreement Technologies: Revised Selected Papers* (pp. 323-338). Springer
Lecture Notes in Computer Science
https://doi.org/10.1007/978-3-030-01713-2_23

N.B. When citing this work, cite the original published paper.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:su:diva-164749>

Rational coordination in games with enriched representations

Valentin Goranko¹ ✉, Antti Kuusisto², and Raine Rönholm³ ✉

¹Stockholm University, Sweden; valentin.goranko@philosophy.su.se

¹University of Johannesburg, South Africa (visiting professorship)

²University of Bremen, Germany; kuusisto@uni-bremen.de

³University of Tampere, Finland; raine.ronholm@uta.fi

Abstract. We consider pure win-lose coordination games where the representation of the game structure has additional features that are commonly known to the players, such as colouring, naming, or ordering of the available choices or of the players. We study how the information provided by such enriched representations affects the solvability of these games by means of principles of rational reasoning in coordination scenarios with no prior communication or conventions.

1 Introduction

Pure win-lose coordination games (WLC games) ([2]) are strategic form games in which all players receive the same payoffs, either 1 (*win*) or 0 (*lose*), and thus all players have the same preference, viz. to coordinate on any winning outcome.

This paper is a sequel to [2] (see also the extended version [3]), where we identified a hierarchy of principles of reasoning that rational players may apply in WLC games when they cannot use preplay communication and do not share any previously agreed-upon conventions. Additionally, [2] also provides a comparative analysis of the classes of WLC games that can be solved by applying such rational principles.

Here we consider WLC games with representation models enriched with additional relations on the choices or players. The additional structure is assumed to be commonly known by the players. For example, unary relations over the choices—interpreted as a labelling or colouring—can be used by the players to select choices. Also, an ordering (partial or total) is a natural example of a binary relation over the choices of players that can provide information which the players can use for their common benefit. We also consider representations with orderings of the *players* which intuitively correspond to priority orders in hierarchical systems of agents. The main research question of this study is to analyse how the additional information provided by such enriched representations affects the solvability of these games by means of rational reasoning only.

As in [2], we assume that the players are rational and commonly believe to pursue the same goal (to coordinate). Furthermore, we assume that the full structure of the game is represented to all the players and is commonly known

to them. As we will show in Section 4.5, the extra information provided by the additional relations can be either useful or detrimental for the players, and this depends not only on the content of that information but also on the reasoning principles which we assume the players to follow.

Typically, the additional representation structure can be useful to the players by creating *salient features* of the game and *focal points* amongst the winning profiles, or it can gradually establish *conventions* amongst the players. Thus, the present work is related, at least in spirit, to previous studies on focal points and conventions, originating from Schelling [6] and Lewis [4] and further developed in the context of coordination games in, e.g., [7], [5], [8]. However, the relation with that (and other) previous work on coordination and conventions is mainly conceptual, whereas both the technical framework introduced in [2] and expanded here, and the study of the effect of enriched representations of coordination games in that framework are, to the best of our knowledge, our original contributions. Some related considerations regarding symmetries in coordination games, in a technically different framework have been investigated in [1].

In the present work we only consider single-round coordination games and do not assume any preplay communication. Repeated coordination games are studied in a follow-up work. We do not discuss the use of *conventions*, either, which is a topic of another follow-up work. We only note that, in the context of single-round coordination games studied here, we regard conventions as principles of coordination that are not likely to be adopted by all players through their individual rational reasoning only, but can be explicitly agreed upon in preplay communication. Lewis [4] is a seminal study of a somewhat different concept of conventions that are gradually emerging principles of coordination adopted by the players in the process of repeated coordination attempts.

2 Preliminaries

2.1 Basic definitions and notation

We begin with the definition of win-lose coordination games. As in [2] we define them as *relational structures*, which is technically convenient for our study.

Definition 1 ([2]). *An n -player **win-lose coordination game (WLC game)** is a relational structure $G = (A, C_1, \dots, C_n, W_G)$ where A is a finite domain of **choices**, each $C_i \neq \emptyset$ is a unary predicate, representing the choices of player i , $C_1 \cup \dots \cup C_n = A$, and W_G is an n -ary relation such that $W_G \subseteq C_1 \times \dots \times C_n$. Here we also assume, for technical convenience, that the players have pairwise disjoint choice sets, i.e., $C_i \cap C_j = \emptyset$ for every $i, j \leq n$ such that $i \neq j$. A tuple $\sigma \in C_1 \times \dots \times C_n$ is called a **choice profile** for G and the choice profiles in W_G are called **winning profiles**.*

We use the following terminology for a WLC game $G = (A, C_1, \dots, C_n, W_G)$.

- For every choice $c \in C_i$ of a player i , the **winning extension of c in G** is the set $W_G^i(c)$ of all tuples $\tau \in C_1 \times \dots \times C_{i-1} \times C_{i+1} \times \dots \times C_n$ such that the choice profile obtained from τ by adding c to the i -th position is winning.

- A choice $c \in C_i$ of a player i is **(surely) winning** /respectively **(surely) losing**/ if it is guaranteed to produce a winning /respectively losing/ choice profile regardless of what choices the other player(s) make.

The n -ary winning relation W_G of an n -player WLC game G defines a *hypergraph* on the set of all choices. We give visual presentations of hypergraphs corresponding to WLC games as follows. The choices of each player are displayed as columns of nodes, starting from the choices of player 1 on the left and ending with the column with choices of player n . The winning relation consists of lines that represent the winning profiles. This kind of graphical presentation of a WLC game G will be called a *game graph (drawing)* of G .

Example 1 ([2]). Here are two examples of WLC games: a 2-player game G_Σ with 2 choices for player 1 (left) and 3 choices for player 2 (right), and a total of 4 winning profiles; and a 3-player WLC game G^* also with 4 winning profiles, each represented as a triple of choices connected by (solid or dotted) lines.

We now introduce a uniform notation for certain classes of WLC games. See the picture below for examples (and [2] for more of them). Let $k_1, \dots, k_n \in \mathbb{N}$.

- $G(k_1 \times \dots \times k_n)$ is the n -player WLC game where the player i has k_i choices and the winning relation is the *universal relation* $C_1 \times \dots \times C_n$.
- Suppose that $G(A)$ and $G(B)$ have been defined and both have the same number of players. Then $G(A + B)$ is the *disjoint union* of $G(A)$ and $G(B)$, i.e., the game obtained by assigning to each player a disjoint union of her/his choices in $G(A)$ and $G(B)$, and where the winning relation for $G(A + B)$ is the union of the winning relations in $G(A)$ and $G(B)$.
- Let $m \in \mathbb{N}$. Then $G(mA) := G(A + \dots + A)$ (m times).

2.2 Symmetries of WLC games and structural principles

As discussed in [2], in the case of no preplay communication and no conventions the choices of rational players should be independent of the names of the choices in the game and of any ordering (or naming) of the players. That is, rational principles of reasoning and choice in that setup should only take the ‘structural properties’ of the game into account. For defining this more precisely, we introduce the following notions:

- A **choice renaming** between G and G' is an isomorphism $G \rightarrow G'$.
- A **player renaming** between G and G' is any permutation β of the players names (indices) such that G' is obtained by applying β to the components of the winning relation of G .
- A **full renaming** between G and G' is a combination of choice renaming and player renaming. We say that G and G' are **structurally equivalent** if there is a full renaming between them.

For formal definition of these notions, see [2]. However, the following example should suffice for understanding the intuition behind these notions.

Example 2. The following WLC games G and G' are structurally equivalent. Indeed, the game G' is obtained from G by renaming (swapping) the players and then permuting the two choices of the player on the left.

A **protocol** is a mapping Σ that assigns to every pair (G, i) , where G is a WLC game and i is a player in G , a nonempty set $\Sigma(G, i) \subseteq C_i$ of choices of i . Thus, a protocol gives a global nondeterministic strategy for playing any WLC game in the role of any player. Intuitively, a protocol represents a global mode of acting in any situation that involves playing WLC games. Hence, protocols can be informally regarded as global “reasoning styles” or “behaviour modes”. We say that a protocol Σ is **structural** if it is “indifferent with respect to full renamings” (see [2] for the formal definition). Clearly, structural protocols must be generally non-deterministic in order to treat symmetric choices equally.

A **(structural) principle** of reasoning in WLC games is a set of (structural) protocols and therefore principles can be seen as properties of protocols. We say that a player **follows a principle P** if she uses a protocol from P. A WLC game G is **solved by the principle P (P-solvable)** if, whenever all players follow P (with possibly different protocols) when playing G , they are guaranteed to win.

Remark. The WLC games defined in [2] can be assumed to be presented to each player as abstract structures, i.e., up to re-arrangement (renaming) of the players and their choices. Thus, the names and the possible ordering of the choices and players are only known and used by each player privately, but are not shared amongst the players, so common knowledge of the names (or ordering) cannot be used in the players’ reasoning. One main objective of the present work is to investigate the effect of having some of these features commonly known to the players.

3 WLC games with enriched representations

Now we will consider WLC games with representations enriched with additional relations on choices and players.

3.1 Enriched WLC games with choice colouring and ordering

Definition 2. An *enriched WLC game* is any WLC game G (defined as in Definition 1) expanded with interpretations of some additional relational symbols in the domain of choices of G . More precisely, if G is a WLC game and $\Phi = \{R_1, \dots, R_k\}$ is a set of relation symbols, a Φ -**enrichment** of G is any enriched WLC game G^Φ , defined exactly as G except that all symbols in Φ are assigned interpretations (relations with matching arities) in the domain of choices of G . Formally, if $G = (A, C_1, \dots, C_n, W_G)$, then G^Φ is of the type $(A, C_1, \dots, C_n, R_1^A, \dots, R_k^A, W_G)$, where $R_1^A, \dots, R_k^A$ are relations on A assigned as respective interpretations of the symbols $R_1, \dots, R_k$.

Note that the enrichment of a WLC game affects only the (commonly given) representation of the game, not the underlying structure involving the winning relation and the choice sets C_i .

The notions of renaming from the previous section can be defined likewise for enriched WLC games. The additional structure given by the relations interpreting Φ adds extra requirements for the isomorphisms (in particular, automorphisms) on enriched WLC games, thus making these games ‘more rigid’ and therefore eliminates some choice and player renamings. This could be seen as an advantage for the players, since it enables more structural principles which they can use. However, as we will show further, the enrichment of the game model can also be disadvantageous for the players depending, inter alia, on the (rational) principles of reasoning they are following.

A unary relation $R \in \Phi$ can be regarded as a **colouring** on the set of choices of each player, thus splitting the choice of the game into *R-coloured choices* and *non-R-coloured choices*. WLC games enriched with one or more colours will be called **coloured WLC games**¹. As a simple example, consider a scenario where each of several people is going to buy a piece of clothing as a present for a common friend (say, who just lost all her clothes in a fire). Suppose they do not know what the others will buy and cannot communicate on that before they make their choices. Then, it would be natural that each person chooses an item in a neutral colour, say black or white, which is more likely to match the choices of the others. On the other hand, if they would have had the knowledge that their friend has a clear colour preference, say for purple clothes, then each of them would naturally choose something in purple – which could have a good or bad overall effect, depending on the concrete choices, but would at least ensure colour matching of the presents.

A special case of a colouring is **naming** of choices, when (formally) the interpretation of the unary relation R intersects each player’s choice set in at most a singleton, and thus R serves as a name of that single choice (if any) of

¹ Note that the notion of ‘colour’ is used metaphorically, rather than literally here, because we assume that a choice may have several colours, or none at all. Thus, the notion of colour used here is more in line with its traditional use in logic, not in graph theory. In particular, every choice of every player is associated with a (possibly empty) set of colours, called further ‘colour type’, rather than a single colour.

each player. We assume the names are commonly known and can be used by the players for coordination. For instance, consider the coordination problem of several people who are to meet somewhere in a completely unknown to them city without being able to communicate before the meeting, but each of them is given a map (the same for all) on which several possible meeting places are indicated, but only one of them – say the central square – is named on the map. That would immediately create a unique *focal point* that would naturally be chosen by all as the expected meeting place. Or, consider the common situation of two cars approaching each other on a narrow road. Using a simple convention or preplay communication, they can easily coordinate by each using an action choice with a commonly shared name, like ‘(swerve to the) Right’ or ‘(swerve to the) Left’. On the other hand, if all players have to coordinate by choosing the same choice but they cannot refer by (commonly known) names to their choices, and if some players have two or more commonly available choices that create a ‘bad symmetry’, then the players will not be able to reach a guaranteed coordination, even with preplay communication.

An important particular case is when all players have the same number of choices, which are named by the same set of names (unary symbols). Then one can think that all players share the same set of choices². The class of so enriched WLC games will be called **WLC games with shared choices**. Many ‘real life’ coordination games fall in this class. A typical example is when two or more persons are to meet somewhere in the city, and they have several commonly known to them choices of places but have not been able to communicate and decide on any of them. A variant of that is the coordination problem of meeting of all persons at a given place but at a time (hour or day) which has not been agreed upon in advance.

Another natural enrichment of WLC games is provided by a binary relation $R \in \Phi$ that defines a partial (pre-)order on the set of choices of each player. Such games are called **WLC games with partial choice (pre-)ordering**; when the (pre-)order is total (linear), we call them **WLC games with (total) choice (pre-)ordering**. Note, that WLC games with choice ordering and the same number of choices for each player could also be interpreted as games with shared choices, as in this case the ordinal numbers of each player’s choices in the ordering can serve as shared names of these choices (‘1st choice’, ‘2nd choice’, etc.).

When considering ‘real life’ coordination scenarios, there are several ways how a natural ordering of choices can arise, for instance:

- (1) Certain choices can be easier to execute than others (taking more time or effort) or otherwise preferable by the players.
- (2) An ordering can arise from spacial or otherwise comparable properties of different choices, e.g. by them being displayed in an order from left to right, or by physical size, weight, distance, etc.

² Even though the sets of players’ choices are formally pairwise disjoint by definition, common names for all choices would establish a natural 1-1 correspondence between the choices in the different players’ sets.

In a setting similar to scenario (1) above, it is quite natural to assume that players would prefer the first choices in the ordering when trying to coordinate. However, in a setting similar to (2), it is not so clear whether players would prefer the first or last choices in the ordering. In reality that may depend, for instance, on whether the agent’s native language uses left-to-right or right-to-left writing. We will get back to this point in the next section where we consider rational principles in enriched WLC games.

We will use a graphical presentation for game graphs of enriched WLC games with colouring(s) and/or an ordering of choices as follows.

- Each unary relation symbol is associated with a colour (or a pattern). The nodes in the game graph are then displayed with the corresponding colours.
- When there is an ordering of choices, each player’s choices are displayed in ascending order from top to bottom. We may also display a numbering on the side of the game graph to indicate that the choices in the game are ordered.

3.2 WLC games with ordering of players

In many real agents’ groups there is a natural hierarchy or priority order amongst the agents, which gives higher priority to the choices of the ‘superior’ agents over those of her ‘inferior’ ones. A typical example is a coordination problem in a military or other hierarchical organisation. When solving such coordination problems the agents are naturally assumed to respect that hierarchy in their considerations and decisions. For instance, if an employee and his boss are supposed to meet at a given time, but the place is not specified in advance, then the boss’ office would be the natural common choice.

Formally, WLC games can be enriched with a commonly known (linear) ordering of the players, which can be used by them for coordination. The enrichments of WLC games with ordering of the players will be called **WLC games with player ordering**. This could be further generalized to partial orderings or pre-orderings of players, but we leave the analysis of these for a future work.

Remark. The ordering of the players is, in fact, implicitly included in the definition of standard WLC games in [2], but that is only for the sake of the formal definition. In that basic setup we do not assume this ordering to be common knowledge amongst the players and thus they cannot use it in their reasoning. Thus, in the setup of [2], when players are using structural protocols they cannot use that ordering, and that is why these protocols must prescribe sets of choices that are invariant under player renamings.

Now, an enrichment of a WLC game G with players ordering can be practically obtained simply by making the players ordering from its definition commonly accessible to the players, and therefore by relaxing the condition prescribing that structural protocols must be invariant under player renamings. More precisely, let P be a structural principle (cf. [2]). If we want to consider P in a setting where the ordering of the players is commonly known and can be used by the players, we can do that technically as follows: we consider a principle P' which is defined

exactly as P , with the only difference being that P' may contain protocols which are not invariant under player renamings.

4 Principles of rational choice in enriched WLC games

In [2] we define and study a hierarchy of rational principles of reasoning in WLC games, some of which we discuss and list briefly here for the reader's convenience (see the precise definitions in [2]). First, some principles of basic rationality:

Non-losing principle (NL): Never play a losing choice, if possible.

Sure winning principle (SW): Always play a winning choice, if possible.

Individually rational choices (IRC): Never play a weakly dominated choice.

Collective rational choices (CRC): Assume that everyone follows IRC.

In [2] we have also defined *symmetry based principles* which use the renamings defined in Section 2.2. for lack of space, we only give here the intuition behind these principles via examples. First, consider the game $G(2(1 \times 2) + (1 \times 1))$. Here the choices with out-degree 2 are automorphic, so Player 1 should be 'indifferent' between them. Likewise, Player 2 should be indifferent between all of her choices in the subgames $G(1 \times 2)$. If players select their choices from these subgames, winning is not guaranteed and we say that these choices generate a *bad choice symmetry*. The principle of **Elimination of bad choice symmetries (ECS)** prescribes such choices to be avoided, if possible. Hence, following ECS, both players select from within the subgame $G(1 \times 1)$ and thus successfully coordinate.

Now, consider the game $G(1 \times 2 + 2 \times 1 + 1 \times 1)$. Here there is a full renaming of the game which relates the choices of players between $G(1 \times 2)$ and $G(2 \times 1)$. Therefore, if Player 1 has some (rational) reason to select a choice with out-degree 2, then Player 2 should have the same reason to select the choice with in-degree 2. Since they would fail to coordinate this way, we say that these choices generate a *bad player symmetry*. The same holds for the two other choices in these subgames. The principle of **Elimination of bad player symmetries (EPS)** prescribes to the players to avoid such choices, if possible. The principle of **Elimination of bad symmetries (ES)** combines ECS and EPS.

We now look at the applications of those principles in enriched WLC games, beginning with the following observation which follows directly from definitions.

Proposition 1. *Every rationality principle P presented in [2], except for the symmetry principles (ECS, EPS and ES), has the same strength with respect to enriched WLC games. That is, any WLC game G is solved by P iff any enriched game G^Φ is solved by P , for any Φ . Furthermore, adding the ordering of players to G^Φ does not affect the strength of these principles.*

In particular, the rationality principle from standard game theory, prescribing iterated elimination of dominated choices (using CRC), does not give anything more—or less—for enriched games, unless combined with other principles which we discuss further. Let us now consider the extensions of the symmetry principles. For any $P \in \{\text{ECS}, \text{EPS}, \text{ES}\}$ we make the following observations:

- P can be used in enriched WLC games, just like in the standard WLC games, but the actual definition of P depends on the definitions of renamings.
- Some P-unsolvable WLC games become P-solvable when the game is suitably enriched. For example, the game $G(3(1 \times 1))$ becomes P-solvable when a same color is given for a single pair of choices that are winning. (Recall the example with the map of possible meeting places, of which only one is named.) This is because then the uncoloured subgame $G(2(1 \times 1))$ becomes eliminated due to a bad symmetry (so, the players do not consider anymore playing there).
- For any nontrivial (e.g., not solvable by NL) WLC game G that is P-solvable, there is an enriched game G^Φ which is not P-solvable, obtained by adding colouring in a such way as to eliminate any non-trivial renamings.
- We also note that the ordering of players makes EPS completely unusable, but does not effect ECS, so the principle ES becomes equivalent with ECS.

4.1 Principles of rational choice in coloured WLC games

Enrichments of WLC games give rise to a range of new principles, with varying degrees of rationality. We give here a representative selection.

Let G^Φ be a coloured WLC game and let $\Psi \subseteq \Phi$ be the set of all *unary* predicate symbols (‘colours’) in Φ . Now every subset of Ψ (incl. $\emptyset$) forms a **colour type** in G^Φ and two choices c and c' are said to have the same colour type if the same predicates from Ψ hold of each of them. Thus, colour types form a partition on the choices in G^Φ , generating a respective equivalence relation there (and play the same role as vertex colours in graph theory). If $|\Psi| = n$, there are *at most* 2^n different colour types, and corresponding equivalence classes, in G^Φ .

The following principle can be naturally applied to coloured WLC games.

Colour matching principle (CM): If there is a unique colour type whose choices guarantee a win, choose from that colour type.

To ‘guarantee a win’ here means that if all players pick, no matter how, their choices from that colour type, they will win.

For example, *structurally unsolvable*³ WLC games like $G(3(1 \times 1 \times 1))$, G_O and $G(2(2 \times 2))$ can become CM-solvable by adding a suitable colourings using a single colour, ‘white’ (the non-coloured choices are indicated in black and the superscript ¹ indicates an enrichment with one colour). See these coloured versions on Figure 1.

Note that *almost every* WLC game with a non-empty winning relation can become CM-solvable by adding just one colour and colouring a suitable single winning tuple. However, this does not work with the game $G(2(1 \times 1))$, because any such colouring here creates a complete symmetry between the coloured and the un-coloured winning pair. (But one could argue that a single colour would still suffice, as colouring just 1 (or 3) of the 4 choices would break this symmetry. See further an extension of the principle CM that covers this case.)

³ Intuitively, structurally unsolvable WLC games are those that cannot be solved by *any* structural principle. For a precise definition, see [2] where we give a complete characterisation of such games.

Fig. 1. Some examples of CM-solvable coloured WLC games.

The principle CM is a particular case of a more general type of rational reasoning, as follows. Given a coloured WLC game G , consider each colour – or, more generally, each colour type C – as defining a subgame, denoted $G|_C$, of G , obtained as a restriction of G to the choices having the colour type C . Thus, a family of at most 2^n such subgames, hereafter called for short **monochrome subgames**, arises in a coloured WLC game with n colours. Now, the players can naturally consider each of these monochrome subgames on their own and try to coordinate on it. If they can coordinate in *exactly one* of these subgames by applying some (rational) principle P (e.g. the ones described in [2]), then they could focus on that subgame and use that solution for the entire game. Thus, a natural generalisation of CM (in which exactly one monochrome subgame has a complete winning relation), parameterised with a given underlying (rational) principle P , can be formulated as follows.

Generalised colour matching principle (GCM^P):

If there is a unique colour type C in the game G such that the monochrome subgame $G|_C$ is P -solvable, then select a choice according to the principle P applied to $G|_C$.

What if the players can coordinate by applying P in *more than one* of the monochrome subgames of G ? It is then possible (but not necessary) that the choices prescribed by P in these subgames of G can be combined in a surely winning way. This leads to a further generalisation of GCM^P formulated as follows. Let G^P be the union of all monochromatic games which P solves. If P also solves G^P , then the players make their choices by applying P in G^P .

For example, consider the coloured games on Figure 2. (For technical reasons, we represent the different colour types here by white, black, and different shades of grey.) In the game G_1^1 , only the ‘white’ monochrome game is solvable by applying NL and thus G_1^1 is GCM^{NL}-solvable. Likewise G_2^1 is GCM^{SW}-solvable as only the ‘black’ monochrome game is SW-solvable. In the 2-coloured game G_3^2 , the only (rationally) solvable monochrome subgame, again by applying SW, is the one indicated in dark grey. In G_4^2 , by applying the Basic individual rationality principle BIR (the combination of SW and NL), there are two solvable monochrome subgames, the ‘black’ one and the ‘grey’ one, so the principle GCM^{BIR} does not solve the game. But note that there is a solution by applying BIR for the generalised colour type combining both, i.e. in the subgame $G_4^2|_{bg}$ restricted to the black and grey colour types, pictured to the right of G_4^2 . Thus, one can argue that G_4^2 is solvable by the generalised version of GCM^{BIR}, as described above. Note also that the colourless versions of G_1^1 , G_2^1 , G_3^2 and G_4^2 are not solvable by any of the principles defined in [2].

Fig. 2. Some examples of GCM^P -solvable coloured WLC games.

In the game G_5^2 (Fig. 2), each of the ‘black’, ‘white’ and ‘dark grey’ subgames are all SW-solvable. But since their solutions are not pairwise compatible, it is not clear which of them should be preferred by the players in order to coordinate. There is also a different type of coordination problem related to the principle GCM^P : Since GCM^P is defined with respect to some (rational) principle P , it is possible that a WLC game G is solvable with both GCM^{P_1} and GCM^{P_2} , but these solutions are not compatible. This creates a *higher-order coordination* problem about making the choice between the principles P_1 and P_2 . (The same problem arises even in WLC games with plain representation, studied in [2].)

The coordination principles based on colourings presented so far can be extended even further. Consider, e.g., coloured versions of the game $G(3(1 \times 1))$ with the following colourings of the pairs of winning choices:

- (1) *(red, red)*, *(red, blue)*, *(blue, blue)*;
- (2) *(red, blue)*, *(green, green)*, *(green, green)*.

Neither of these games can be solved with the principles presented in this subsection (4.1), but they clearly suggest new principles that consider not just monochrome subgames, but also subgames with different colouring patterns (like, the ‘red-blue’ subgame). Since such new principles are generally not compatible with the ones we have presented here, further higher-order coordination problems amongst them arise. These issues we will be consider further in an extended version of this paper.

4.2 Principles of rational choice in WLC games with choice ordering

When considering WLC games with choice ordering, one can talk about **order-matching choice profiles**, each consisting of choices that have the same ordinal position in each player’s ordering. Thus, the matching choice profiles can naturally be ordered with orderings of the choices of each player (the first one being the choice profile which has the first choice in the ordering of each player).

The following principle naturally arises in WLC games with a choice ordering.

Choice order-matching principle (COM):

Play the least order-matching choice profile that is winning, if there is any.

For example, consider the WLC games with choice ordering on Figure 3.

- The WLC game G_1 , enriching the structurally unsolvable game $G(3(1 \times 1 \times 1))$ with choice ordering, is COM-solvable and the least winning choice is 1 because $(1,1,1)$ is a winning choice profile.
- The WLC game with choice ordering G_2 is COM-solvable, prescribing to both players the least winning choice 3, because $(3,3)$ is a winning choice profile, while $(1,1)$ and $(2,2)$ are not.
- The WLC game with choice ordering G_3 is not COM-solvable, though it is solvable by the principle of Collective Rational Choices (CRC).

Fig. 3. Some examples of COM-solvable WLC games with choice ordering.

As we have noted at the end of Section 3.1, the ordering of choices can arise in different ways. In the coordination scenario of type (1), described there, COM seems a rather rational principle to follow. If the ordering presents players' preferences then, by following COM, players are looking for a winning choice profile which is "equally preferable to all players" and, under this condition, "as preferable for everyone as it can be".

However, in a scenario of type (2) (in Section 3.1) it is harder to justify why the first choices in the ordering should be preferred over the last ones in the ordering. If, for instance, the ordering arises from a linear ordering of choices from left to right, then (assuming no conventions) there is no clear reason to prefer either the leftmost or the rightmost choice. And, in a situation where choosing the first or the last choice in the ordering could be considered *equally rational*, one could even argue that the most rational compromise would be to select the middle choice in the ordering, if there is a winning middle choice.

4.3 Principles of rational choice in WLC games with player ordering

If a WLC game is enriched with a commonly known ordering of players, then players can use this order in their reasoning. One possible interpretation here is that players select their choices as if they play *not simultaneously, but consecutively*, following that order (hierarchy), but *without the choices being announced* to the other players. Therefore, if the players follow a (rational) principle P, they may apply the *iterated reasoning* of P according to their hierarchical order. We call this **Hierarchical reasoning with respect to P** (HR^P).

We present here two examples where a structurally unsolvable game G becomes solvable with HR^P when we add player ordering to G . Consider first the game $G(1 \times 2 + 2 \times 1)$ and the principle of **Probabilistically optimal reasoning (PR)** (see [2]). By following PR, a player simply selects a choice with the largest winning extension. Clearly if players follow PR in $G(1 \times 2 + 2 \times 1)$,

then they both will select a choice with out-degree of 2 and lose. But if Player 2 assumes Player 1 to follow PR, then Player 2 can coordinate with her/him. Therefore this game is HR^{PR} -solvable with the player ordering $(1, 2)$.

Fig. 4. Some examples of solvable WLC games with players ordering.

Consider then the game G^{**} in Fig. 4, where the winning triples are indicated by either solid or dashed line segments. Here the choices a_1 and c_1 of Player 1 generate (cf. ECS) a bad choice symmetry. Thus if Players 2 and 3 assume Player 1 to follow ECS, then they can coordinate with her by selecting their choices on a winning choice profile (b_1, b_2, a_3) or (b_1, b_2, c_3) . Note that the choice b_1 generates (cf. ECS) a bad player symmetry and thus it is easy to see that this game is structurally unsolvable without the player ordering. However, the ordering of the players breaks this symmetry, and thus EPS arguably does not apply.

4.4 Principles combining orderings of choices and players

If a WLC game is enriched with both choice-ordering and players-ordering, then the players can naturally use the *lexicographic order* of all choice profiles determined by both the ordering of players and the ordering of choices. So, here is a natural alternative to COM:

Least Lexicographic Order principle (LLO): Play the least winning choice profile in the lexicographic order of all choice profiles, if there is one.

Clearly, *every* WLC game with commonly known orderings of choices and players and non-empty winning relation can be solved by applying (LLO). For illustration, consider the application of LLO in a scenario where the orderings of choices are made according to the players' preferences and there is a hierarchical order of the players. Now, following LLO can be interpreted as follows:

“Players select a winning choice profile in such a way as if all players get to choose consecutively, following the hierarchy from top to bottom, and each of the players makes their most preferred choice that can coordinate (win) with the choices already made by all previously choosing (i.e., superior) players.”

For example, consider the game G^* from Example 1. Suppose an ordering of choices such that Player 1 “prefers” a_1 to b_1 , Player 2 prefers b_2 to a_2 and Player 3 prefers a_3 to b_3 . By following the player ordering $(1, 2, 3)$, the LLO-prescribed solution would now be (a_1, b_2, b_3) .

4.5 On the compatibility of the new and old principles

Both CM and COM are incompatible with most of the rationality principles presented in [2]. Thus, one may ask how rational players would (and should) behave in a game which is solvable by some (purely) rational principle, but some additional feature in the game creates an alternative focal point conflicting with the prescribed choices of that principle. For instance, consider the game $G^1(2(2 \times 2) + (1 \times 1))$ in Figure 5. On the one hand, the symmetry principle ECS applied to the plain game $G(2(2 \times 2) + (1 \times 1))$ prescribes playing the pair in the subgame $G(1 \times 1)$, whereas the principle CM applied to the coloured version $G^1(2(2 \times 2) + (1 \times 1))$ prescribes playing in the ‘black’ subgame, which is readily solved. (Note that there are no bad choice symmetries in $G^1(2(2 \times 2) + (1 \times 1))$, as the added colouring breaks the corresponding automorphisms.)

Fig. 5. Examples on incompatibility of some principles.

Now, consider the 1-coloured game G_Z^1 on the right, also enriched with choice ordering. Note first that its plain version G_Z is solvable by the principle of Collective Rational Choices (CRC), which prescribes playing the middle choice profile (3,2). On the other hand, G_Z with the given colouring is solvable by CM, which prescribes playing either of the choice profiles (4,3) or (4,4). Clearly, if the two players apply different principles amongst these, they lose. Note also that G_Z with the ordering of choices is solvable with COM which prescribes playing the choice profile (1,1). So the game G_Z^1 has three different incompatible solutions, depending on if players follow CRC, CM or COM.

Furthermore, the generalised principle GCM^P applied to colourless WLC games obviously coincides with the respective underlying principle P applied to them. However, as the examples above show, on coloured games GCM^P may become incompatible with some rationality principles for plain WLC games with which P itself is compatible, or even with P itself! Indeed, the coloured game G_Z^1 is not solvable by GCM^{CRC} because each of the two monochrome subgames is solvable by CRC—respectively with any of the choice profiles (4,3) or (4,4) for the ‘white’ subgame, and any of the choice profiles (2,1) or (2,2) for the ‘black’ subgame. But these solutions are not mutually compatible and thus the principle GCM^{CRC} fails on the entire game. Hence we also see that GCM^P may turn out to be disadvantageous as compared to CM, too.

Lastly, a few words on further enrichments. Besides total (linear) orderings of choices, WLC games can also be enriched with *partial orderings*, *preorderings*,

matchings and other natural binary relations. For such games, one could define natural variants of the principles formulated above. For instance, note that a (partial) preorder on choices can also be seen as a (partial) ordering of colours. Thus, COM can be naturally generalised and can also be combined with GCM^P. For lack of space, we leave out the details here. Formal definitions and further study of these principles will be included in an extended version of this paper.

5 Concluding remarks

In this work we have studied how additional features of the representation of coordination scenarios (games), that are commonly known amongst the players, can be used by rational players in order to achieve coordination. We have shown that in the enriched games one cannot achieve much more with the principles presented in [2], but a variety of new principles emerges which still seem rational, and certainly reasonable. However, like in [2], it seems very difficult to determine which of the reasoning principles stated here can be qualified as (purely) rational and which not.

There are many natural further extensions of this work, such as adding imperfect information or considering repeated coordination games or dis-coordination games. Also, some more technical issues—such as precise characterisations of the solving powers of various principles, and computational complexities of solving coordination games with them—could be pursued further. Lastly, it would be very interesting to see how real agents (people) actually behave in coordination scenarios studied here, by setting up concrete (live or web-based) experiments.

Acknowledgements

The work of Valentin Goranko was partly supported by a research grant 2015-04388 of the Swedish Research Council. The work of Antti Kuusisto was supported by the ERC grant 647289 “CODA”. We thank the anonymous referees for valuable comments.

References

1. Alós-Ferrer, C., Kuzmics, C.: Hidden symmetries and focal points. *Journal of Economic Theory* 148(1), 226–258 (2013)
2. Goranko, V., Kuusisto, A., Rönholm, R.: Rational coordination with no communication or conventions. In: *Proc. of LORI VI. LNCS*, vol. 10455, pp. 33–48. Springer (2017)
3. Goranko, V., Kuusisto, A., Rönholm, R.: Rational coordination with no communication or conventions. Tech. rep. (2017), arXiv:1706.07412
4. Lewis, D.: *Convention, A Philosophical Study*. Harvard University Press (1969)
5. Mehta, J., Starmer, C., Sugden, R.: Focal points in pure coordination games: An experimental investigation. *Theory and Decision* 36(2), 163–185 (1994)
6. Schelling, T.C.: *The strategy of conflict*. Harvard University Press (1960)
7. Sugden, R.: Spontaneous order. *J. of Economic Perspectives* 3(4), 85–97 (1989)
8. Sugden, R.: A theory of focal points. *The Economic Journal* pp. 533–550 (1995)