Characterising the secondary maximum in the r-band for Type Ia Supernovae: Diagnostic for the ejecta mass

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ABSTRACT

An increase in the number of studied Type Ia supernovae (SNe Ia) has demonstrated that this class of explosions has a greater diversity in its observables than was previously assumed. The reasons (e.g. the explosion mechanism, progenitor system) for such a diversity remain unknown. Here, we analyse a sample of *r*-band light curves of SNe Ia, focusing on their behaviour ~ 2 -4 weeks after maximum light, i.e. the second maximum. We characterise the second maximum by its timing (t_2) and the integrated flux ($\overline{\mathcal{F}}_{r_2}$). We find that t_2 correlates with the “colour-stretch” parameter s_{BV} , which can be used as a proxy for ^{56}Ni mass, and $\overline{\mathcal{F}}_{r_2}$ correlates with the transparency timescale, t_0 . Using $\overline{\mathcal{F}}_{r_2}$ for a sample of 199 SNe from the Palomar Transient Factory and intermediate Palomar Transient Factory, we evaluate a distribution on t_0 for a sample of SNe Ia found in an “untargeted” survey. Comparing this distribution to the predictions of t_0 ranges from models we find that the largest overlap in t_0 values between models and observations is for the sub-Chandrasekhar double detonation models. We also compare our relations between t_0 and $\overline{\mathcal{F}}_{r_2}$ with that from 1-D explosion models of Goldstein & Kasen (2018) and confirm that $\overline{\mathcal{F}}_{r_2}$ can be used as a diagnostic of the total ejecta mass.

Key words: supernovae: general

1 INTRODUCTION

Type Ia supernovae (SNe Ia) exhibit diverse observable properties (see Hillebrandt, Kromer, Röpke & Ruiter 2013; Maguire 2017, for a review). SNe Ia show a great diversity in their spectroscopic display (e.g. Blondin, et al. 2012; Folatelli, et al. 2013) and their peak luminosities differ by a factor of ~ 10 (e.g. Suntzeff 1996, 2003; Stritzinger et al. 2006; Li et al. 2011; Taubenberger 2017). The amount of ^{56}Ni (Contardo, Leibundgut & Vacca 2000) and the total ejected mass (Scalzo et al. 2014) also show a wide dispersion. The dispersion in the ^{56}Ni masses can explain the width-luminosity relation (WLR; Phillips 1993) which is used to correct the SN Ia peak luminosity and use them as distance indicators in cosmology (see Goobar & Leibundgut 2011; Leibundgut & Sullivan 2018, for a review).

Spectroscopically normal SNe Ia have optical (*B*-band) light curves showing a rise to maximum of ~ 18 days and a post-maximum decline to an exponentially declining tail. However, at redder wavelengths (*izYJHK*) filters the light

curve morphology is markedly different, showing two maxima (Elias et al. 1981; Hamuy et al. 1996; Meikle 2000). Kasen (2006) explains the emergence of the second maximum as a result of the ionisation transition in the iron-group elements (IGEs) in the ejecta from doubly to singly ionised, leading to a weakening of Fe III and Co III lines and a strengthening of Fe II and Co II lines (see also Blondin, Dessart, & Hillier 2015). The timing of the second maximum (t_2) is a function of the optical decline rate (Hamuy et al. 1996; Dhawan et al. 2015) indicating that objects with more synthesized ^{56}Ni have a later second maximum. Hence, t_2 has been used to estimate the ^{56}Ni mass (Dhawan et al. 2016) and to standardize the SNe for distance measurements in cosmology (Shariff et al. 2016).

In *r*-band, SNe show a weaker second maximum compared the redder filters, akin to a plateau starting at ~ 15 days after maximum. The aim of this study is to characterise the features of the *r*-band light curve and to quantitatively search for relations with global properties of the SN explosion, e.g. the total radioactive ^{56}Ni mass and total ejecta mass. This is important also to understand the cause of the *r*-band plateau feature, which has not been studied before.

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With this paper, we aim to fill the gap in light-curve studies with observations in r -band.

The onset of large programs to observe samples of SNe Ia at low-redshift ($z < 0.1$) has led to a large library of r -band light curves. Ongoing and recently concluded campaigns, e.g. the Carnegie Supernova Project (CSP-I) (Contreras et al. 2010; Stritzinger et al. 2011), CfA supernova program (Hicken et al. 2009), Palomar Transient Factory (Rau, et al. 2009, PTF;), Carnegie Supernova Project-II (Hsiao et al. 2018; Phillips et al. 2018), Zwicky Transient Facility (ZTF; Graham et al. in preparation), Foundation Supernova Survey (Foley, et al. 2018), have provided and will provide multi-band light curves of SNe Ia. Hence, this investigation is very timely to understand the characteristics of r -band light curve features and how they connect to physical properties of SN explosions. Moreover, studies with theoretical light curves, cross-matched with spectra, have postulated a causal mechanism for the r -band plateau. Therefore, a study of the observational properties of SNe Ia will be highly complementary to the existing theoretical work.

The structure of this paper is as follows: in Section 2 we describe the datasets used in this study and in Section 3 we detail the methodology for fitting the r -band light curves. We present our results in Section 4 and finally discuss them and conclude in Section 5 and Section 6.

2 DATA

In this investigation, we analyse correlations between well-understood decline parameters, bolometric light curve properties and r -band properties at late times (note that in this paper late times refers to epochs later than 10 days after maximum light). We use data from the Carnegie Supernova Project (CSP-I) (Contreras et al. 2010; Stritzinger et al. 2011) and CfA supernova program (Hicken et al. 2009) for analysing the correlations between the different light curve properties.

The Palomar Transient factory (PTF) and its successor the intermediate transient factory (iPTF) were large field-of-view transient surveys that discovered and obtained light curves for hundreds of SNe Ia (see Papadogiannakis et al. 2019, for details). Unlike the CSP and CfA, which were exclusively follow-up campaigns, PTF/iPTF also discovered the SNe, giving a better control on the selection effects in finding them, important for characterising properties for a population of SNe Ia (Papadogiannakis et al. 2019). The drawback of this dataset is that it mostly has photometry in one filter, which is the R -band for a large fraction of the SNe. Since we want to characterise the r -band second maximum, we only use data for SNe in the phase range +10 to +40 days (an example is shown in Figure 1).

A summary of the SNe used in this study along with their derived properties of the secondary maximum is presented in the Appendix A.

3 ANALYSIS

In this section, we describe the analysis method for evaluating the parameters in the study. We use two different methods to probe the plateau or secondary maximum in the

r -band where we determine the time of the plateau (t_2) and calculate the integrated normalised flux around the dates of the plateau ($\overline{\mathcal{F}}_{t_2}$). We use the same symbols for both the r -band and R -band used with the CSP-I, CfA and PTF and iPTF survey data respectively. In this analysis, we explore whether these properties of the r -band plateau correlate with the global properties of SNe Ia, e.g. total radioactive nickel mass, total ejecta mass. In previous studies it has been shown that the ordering parameter s_{BV} (Burns et al. 2014) correlates with the peak bolometric luminosity (Hoeftlich, et al. 2017). Arnett’s rule states that the instantaneous rate of energy deposition from radioactive decay equals the output flux at maximum (Arnett 1982). Hence, we use the s_{BV} parameter as a proxy for the ^{56}Ni mass in our analyses. In previous studies, it has been shown that the transparency timescale (t_0), i.e. the epoch at which the ejecta optical depth is unity, is directly related to the total ejecta mass (Jeffery 1999; Stritzinger et al. 2006).

Hence, we test for correlations between the properties of the plateau with the ordering parameter s_{BV} and the transparency timescale. Below, we describe the method for evaluating the properties of the plateau, t_2 and $\overline{\mathcal{F}}_{t_2}$, as well as the transparency timescale. The ordering parameter s_{BV} is calculated using the “colour model” in the SNooPy light curve fitting software (Burns et al. 2014).

3.1 Time of secondary maximum

To estimate the time of the plateau or secondary bump in r -band, t_2 , we first run Gaussian processes (GP) with a Matérn $\frac{2}{3}$ kernel to get the most likely function, the latent function ℓ , that describes the data and its uncertainty.

Gaussian processes is a non-parametric way of predicting the underlying function behind data and works well with unevenly spaced data, such as the one we have. It is also able to predict an error for each given part of the function which sets it apart from other techniques.

To accommodate the GP priors we normalise the fluxes so that the mean error is 1 and compute t_2 in flux space. To get a better understanding of the error of the t_2 estimate we then perturb the data points within their errors 100 times using Monte Carlo simulations and get 100 latent functions. We then compute the derivative of the sampled functions with respect to time and choose the t_2 as a point with zero derivative and negative second derivative that lies between day +13 and +40 with respect to maximum light. Using the results of this we can then determine the probability that we have encountered a maximum, a shoulder or neither for each SN. If there is an inflection point rather than a bump we calculate the point of inflection and call it a shoulder.

In Figure 1 an example fit is shown. We used the same Monte Carlo simulations to estimate the error of t_2 . In our final sample we require the SNe to have at least 4 data points within the times +10 and +40 days with respect to maximum light and a standard deviation of the t_{max} of less than 1.6 days. All SNe rejected by these criteria were visually inspected to make sure no good fit was rejected. This leaves 112 SNe from CfA, 70 SNe from CSP and 240 SNe from PTF and iPTF for which a t_2 measurement could be obtained.

Figure 1. In the upper panel we show an example of the Gaussian processes (GP) fit and estimation of the bump time, t_{r2} with the error estimate coming from Monte Carlo (MC) simulations. The green shaded green area shows the estimate of the $\bar{\mathcal{F}}_{r2}$. In the lower panel the curve shows the derivative of the most likely function given by GP and the dispersion of the derivative in grey lines. The green solid line shows the estimation of t_{r2} averaged over all MC runs.

3.2 Integrated flux under the secondary maximum

Another way that we quantify the *r*-band secondary maximum is using the mean normalised flux integrated in the interval +15 and +40 days with respect to maximum light ($\bar{\mathcal{F}}_{r2}$). This metric to quantify the second maximum was proposed by Krisciunas, et al. (2001) for the *i*-band, and here, we adapt it for the *r*-band. We calculate $\bar{\mathcal{F}}_{r2}$ by integrating the latent function from heteroscedastic GP of the rest-frame light-curve normalised to the peak flux and divided by the number of days of the interval. By using GP we get a data driven estimate of the error that allows a robust measurement even when there are gaps in the data. We require at least 4 data points within the integration interval. We get measurements of $\bar{\mathcal{F}}_{r2}$ of 61 SNe from CfA, 53 SNe from CSP and 199 SNe from PTF and iPTF. The two parameters, $\bar{\mathcal{F}}_{r2}$ and t_{r2} are related as seen in figure 2.

3.3 Transparency timescale

In previous studies (Jeffery 1999; Stritzinger et al. 2006), it has been shown that the transparency timescale, i.e. the epoch at which the ejecta have optical depth of unity, is a proxy for M_{ej} . The transparency timescale is evaluated from the bolometric light curve by fitting a radioactive decay energy (RDE) deposition curve to the tail of the observations. We use a standard least squares fitting methodology in this analysis. We add an error of 2 days to the error from the fit of t_0 , corresponding to the average error in inferring the rise time of an SN Ia (see Scalzo et al. 2014, for details). The bolometric light curve is created from the multi-band photometry using the method described in Dhawan, et al. (2018). We convert the observed magnitudes to de-reddened fluxes and interpolate the filters onto the same time step. The fluxes are integrated using the trapezoidal rule and then converted to absolute luminosities using the observed dis-

Figure 2. Time of the bump, t_{r2} versus $\bar{\mathcal{F}}_{r2}$ the two parameters measured at the secondary maximum of the *r*-band light-curve.

Figure 3. Time of the bump, t_{r2} versus s_{BV} for the SNe from CSP and CfA. The dashed line shows the best fit straight line with a Spearman $r = 0.7$ and a p -value $< 10^{-11}$. The gray data points show the spectroscopic outliers such as 91bg and 91T-like SNe from both surveys.

tances (e.g., see Contardo, Leibundgut & Vacca 2000). The expression for the RDE deposition curve is given as follows:

$$\begin{aligned}
 E_{\text{dep}} &= E_{\text{Ni}} + E_{\text{Co}} e^+ + [1 - \exp(-\tau_\gamma)] E_{\text{Co}} \gamma \\
 &= \lambda_{\text{Ni}} N_{\text{Ni}0} \exp(-\lambda_{\text{Ni}} t) Q_{\text{Ni}} \gamma \\
 &\quad + \lambda_{\text{Co}} N_{\text{Ni}0} \frac{\lambda_{\text{Ni}}}{\lambda_{\text{Ni}} - \lambda_{\text{Co}}} [\exp(-\lambda_{\text{Co}} t) - \exp(-\lambda_{\text{Ni}} t)] \\
 &\quad \times \{Q_{\text{Co}} e^+ + Q_{\text{Co}} \gamma [1 - \exp(-\tau_\gamma)]\}, \quad (1)
 \end{aligned}$$

where the factor $(1 - \exp(-\tau_\gamma))$ is replaced by 1 for ^{56}Ni since complete trapping of γ -rays occurs at early times, when most of the light curve is powered by ^{56}Ni . λ_{Ni} and λ_{Co} are the e-folding decay times of 8.8 days and 111.3 days for ^{56}Ni and ^{56}Co respectively. $Q_{\text{Ni}} \gamma$ (1.75 MeV) is the energy release

Figure 4. *Left:* Transparency timescale t_0 vs. bump time, t_{r_2} both in units of days. *Right:* Integrated flux, $\overline{\mathcal{F}}_{r_2}$, versus t_0 for the CSP sample, the solid line shows the best fit line and the shaded region the error of the slope. Note that the errorbars in t_0 are a systematic error of 2 days from the uncertainty of the rise-time of Type Ia SNe.

per $^{56}\text{Ni} \rightarrow ^{56}\text{Co}$ decay. $Q_{\text{Co}\gamma}$ (3.61 MeV) and $Q_{\text{Co}e^+}$ (0.12 MeV) are the γ -ray and positron energies, respectively, released per $^{56}\text{Co} \rightarrow ^{56}\text{Fe}$ decay (see Stritzinger et al. 2006). Equation 1 is only applicable in the optically thin limit, when the thermalized photons can freely escape.

τ_γ is the mean optical depth, calculated by integrating from the point of emission to the surface of the ejecta (see Jeffery 1999, for a derivation of the expression). It has a simple t^{-2} dependence, given as,

$$\tau_\gamma = \frac{t_0^2}{t^2}. \quad (2)$$

where t_0 is the transparency timescale, which by construction in Jeffery (1999) is the epoch at which the optical depth is unity.

4 RESULTS

The timing of the NIR second peak has been shown to correlate with light curve properties relating to the peak absolute brightness (e.g. decline rate, Δm_{15}) (Hamuy et al. 1996; Folatelli, et al. 2010; Biscardi, et al. 2012; Dhawan et al. 2015). Here, we investigate whether the r -band bump shows any significant trends with similar light curve parameters. Burns et al. (2014) demonstrated that the Δm_{15} light curve shape parameter does not adequately capture the diversity of SNe Ia, especially at the faint end, where the SNe transition to the exponential decline at < 15 days from maximum light. As an alternative, they propose a “colour-stretch” parameter s_{BV} , which is more accurate at ordering even the faint end of the observed distribution of SNe Ia properties. Previous studies have shown that s_{BV} is correlated strongly with bolometric properties e.g. L_{max} (Dhawan, Leibundgut, Spyromilio & Blondin 2017; Hoefflich, et al. 2017) and hence, a strong indicator of global properties like ^{56}Ni

(Arnett 1982). Thus, we use s_{BV} as a proxy for the intrinsic luminosity of the SN.

In Figure 3, we plot the resulting correlation between t_{r_2} and s_{BV} and find a strong correlation between the two quantities (Spearman $r = 0.7$ and a p -value $< 10^{-11}$). This indicates that SNe with a later t_{r_2} are intrinsically brighter, similar to the behaviour for the equivalent feature at redder wavelengths (Hamuy et al. 1996; Folatelli, et al. 2010; Dhawan et al. 2015). Hence, t_{r_2} can be used as a possible luminosity indicator. However when we look at how t_{r_2} correlates with the Hubble residuals in the Mould R -band from the PTF and iPTF sample we find no correlation suggesting that at least in the R -band, t_{r_2} is not a good predictor for luminosity.

The above correlation relates t_{r_2} features to the intrinsic luminosity in the B-band. We investigate whether t_{r_2} also correlates with observables relating to progenitor properties, e.g. the total ejected mass. Previous studies have noted that the transparency time-scale (t_0) of the bolometric light curve can be an indicator for the ejecta mass (Jeffery 1999; Stritzinger et al. 2006; Scalzo et al. 2014; Dhawan, Leibundgut, Spyromilio & Blondin 2017) with longer time-scales corresponding to higher masses. The transparency time-scale is the epoch when the optical depth of the SN ejecta is unity. We derive it from the bolometric light curve by fitting a radioactive decay energy (RDE) deposition curve to the tail (+40 to +90 days) of the light curve (see Jeffery 1999; Stritzinger et al. 2006; Scalzo et al. 2014, for details).

Due to the stringent cuts on the data to sample the peak in $u \rightarrow H$ filters, the final sample of SNe with $\overline{\mathcal{F}}_{r_2}$ and t_0 measurements is comparatively smaller and coming from the CSP sample only. We note that the small size of the dataset shows the importance of having a parameter, derived only from a single filter, that correlates with t_0 to derive t_0 values for a large sample of SNe. We find a significant correlation between the transparency time-scale, t_0 and the integrated flux as indicated in Figure 4 with a Spearman $R = 0.8$ and a p -value of $< 10^{-5}$. The best fit parameters are:

$$t_0 = 44.92(\pm 5.86) \times \overline{\mathcal{F}}_{r_2} + 15.00(\pm 2.32), \quad (3)$$

Since the PTF and iPTF data have a series of well sampled R -band light curves (Papadogiannakis et al. (2019)) from an untargeted survey, we can apply Equation 3 to a large dataset where the lack of multi-band data would have otherwise prevented us from deriving t_0 in the absence of the above relation. We get an extrapolated distribution of t_0 shown in Figure 5. The uncertainty on the derived t_0 for each object is ~ 3 days, we expect that a larger sample of SNe for deriving the best fit relation will decrease this error.

In previous works a direct comparison of the model predictions for t_0 versus other global properties (e.g. ^{56}Ni mass) has been important to suggest that multiple progenitor channels could be contributing to the observed diversity of SNe Ia (Scalzo et al. 2014; Childress, et al. 2015; Wygoda, Elbaz & Katz 2017; Dhawan, et al. 2018). Additionally, studies have compared different properties of SNe Ia (e.g. brightness, ^{56}Ni mass) with the predictions from models (Ruiter, et al. 2013; Piro, Thompson & Kochanek 2014). We present the distribution of t_0 , such that it can be compared with theoretical predictions for specific model scenarios to distinguish between the different possibilities for

Figure 5. This is the extrapolated transparency time-scale t_0 distribution of the 199 PTF and iPTF SNe based on the correlation derived from the CSP sample, the errorbars show the Poisson error in each bin. The different shaded regions represent different model prediction of t_0 .

the origin of SNe Ia. In Figure 5 we overplot the ranges of t_0 from a range of different models taken from the Heidelberg Supernova Model Archive [Kromer, et al. \(2017\)](#). The models represented are the pure detonations (det) from [Sim, et al. \(2010\)](#), “double detonation” (doubledet) from [Fink et al. \(2010\)](#), gravitationally confined detonation (gcd) from [Seitenzahl et al. \(2016\)](#) and violent merger models (merger) from [Pakmor et al. \(2010\)](#) and [Pakmor et al. \(2012\)](#). Since the uncertainty on the inferred t_0 for the SNe from PTF and iPTF is of order a few days, we do not compare individual SNe to model predictions, but rather compare the range of t_0 values observed to the predicted ranges for the different model grids. We note that no model covers the entire distribution but the “double detonation” (doubledet) model is that that covers the largest part of the distribution and the violent merger models cover the least. In Table 1 we show which percentage of the distribution is covered by each of the tested models.

While our focus here is on the r -band, the method can be applied to other filters as well. We apply the same analysis for the i -band data of CSP and CfA and find similar correlations as seen in figure 6 and 7.

We note that the dispersion in the t_{i_2} vs. s_{BV} relation is tighter in the i -band than in the r -band whereas the relation between t_0 and $\mathcal{F}_{i_2}$ has similar scatter. This reproduction of correlations seems to indicate a common origin of the r - and i -band secondary maximum as discussed in [Kasen \(2006\)](#).

In Figure 8 we show the correlation between the i -band and the r -band integrated flux, $\overline{\mathcal{F}}_{i_2}$ and $\overline{\mathcal{F}}_{r_2}$.

Model	Fraction (%)
Sub-Chandra Double Detonaton (Doubledet)	77.4
Chandrasekhar mass Delayed Detonation (ddt)	17.6
Sub-Chandra Detonation (Det)	11.6
Violent Merger (Merger)	0.5

Table 1. The percentage of SNe in the distribution from 5 for the different models tested coming from the Heidelberg Supernova Model Archive [Kromer, et al. \(2017\)](#). The gravitationally confined detonations (gcd) scenario is not listed above since it only has one model and hence, we cannot calculate a range of overlap.

5 DISCUSSION

We find that t_{r_2} are correlated with s_{BV} as well as the transparency timescale, indicating its link to fundamental properties of the SNe, e.g. luminosity, ejecta mass. We derive equivalent relations for the models and compare them to the observations.

From the transparency timescale, t_0 , that we derived earlier, we can get the total ejecta mass (M_{ej}) using the equation ([Jeffery 1999](#); [Stritzinger et al. 2006](#); [Dhawan, Leibundgut, Spyromilio & Blondin 2017](#); [Dhawan, et al. 2018](#));

Figure 6. In the left panel we show t_0 vs. t_{i2} and in the right panel t_0 vs. $\bar{\mathcal{F}}_{i2}$. We note similar correlations as seen for r-band.

Figure 7. Here we show the relation between s_{BV} and t_{i2} that is tighter than that in the r-band.

$$M_{\text{ej}} = 1.38 \cdot \left(\frac{1/3}{q}\right) \cdot \left(\frac{v_e}{3000 \text{ km s}^{-1}}\right)^2 \cdot \left(\frac{t_0}{36.80 \text{ days}}\right)^2 M_{\odot}. \quad (4)$$

for the PTF and iPTF sample. This equation describes the capture rate of γ rays in an expanding spherical volume for a given distribution of the radioactive material. Here, v_e is the e-folding velocity, which provides the scaling length for the expansion and q represents the distribution of the ^{56}Ni in the ejecta. The range of expected v_e values is between ~ 2600 and 3200 km s^{-1} with a typical value for a normal SN Ia being $\sim 3000 \text{ km s}^{-1}$. For the value of q , $1/3$ implies uniformly distributed ejecta with higher values, till a maximum of unity, implying progressively more centrally concentrated ^{56}Ni . For the comparison with the relations from the model described below we use $v_e = 3000 \text{ km s}^{-1}$ and $q = 1/3$, typical values for a normal SN Ia (see Figure 9). As in pre-

Figure 8. t_{r2} in days against $\bar{\mathcal{F}}_{r2}$ with the sub-classified SNe Ia in i-band of each survey with different symbols and colours as indicated. The solid black line shows the directly proportional equation between the two parameters.

vious studies, we assume a constant γ -ray opacity of $0.025 \text{ cm}^2 \text{ g}^{-1}$ (Swartz, Sutherland & Harkness 1995).

When comparing with radiative transfer model from Goldstein & Kasen (2018) we find a strong correlation between t_0 and $\bar{\mathcal{F}}_{r2}$ suggesting that bump features in the r -band can be directly connected to the ejecta mass M_{ej} . We analyse a grid of 4500 state-of-the-art radiative transfer models from Goldstein & Kasen (2018) which span a large range of the physical parameters of SNe Ia, e.g. kinetic energy, total mass, radioactive nickel mass. We compute synthetic photometry in the SDSS- r band and evaluate t_{r2} and $\bar{\mathcal{F}}_{r2}$ values for the models. The aim is to note whether the models *directly* show a correlation between M_{ej} (which is M_{WD} for SNe Ia since the complete white dwarf (WD) is unbound in the explosion) and t_{r2} or $\bar{\mathcal{F}}_{r2}$, and hence, confirm or refute using theoretical predictions whether the r -band bump can be used to derive physical properties for large samples of SNe Ia. We note that the models of Goldstein & Kasen (2018) are computed under the assumption of local thermodynamic equilibrium (LTE), which does not hold true for the phase of the light curve probed by the bump (see Goldstein & Kasen (2018)), however, they can be used to determine relations between the parameters.

While we find that the bump like feature in the model is more pronounced than in the data and hence, cannot be used for robust estimates of the theoretical values of t_{r2} and $\bar{\mathcal{F}}_{r2}$, we can use the inferred values for understanding correlations between the parameters. For models with $M_{\text{WD}} > 0.5 M_{\odot}$ there is a strong correlation between M_{WD} and t_{r2} . Explosion modelling has shown that carbon-oxygen WDs of masses $\gtrsim 0.7 M_{\odot}$ leads to an SN Ia, the low limit on the mass of the exploding WD. Hence, for comparing observations to models, we look at the mass range ($M_{\text{WD}} > 0.7 M_{\odot}$).

We also find that the M_{WD} value is correlated with $\bar{\mathcal{F}}_{r2}$ (Figure 9). This provides corroborating evidence $\bar{\mathcal{F}}_{r2}$ values can be used as a diagnostic of the M_{ej} for SNe Ia. This

Figure 9. The total mass vs. $\overline{F}_{r2}$ from the Goldstein & Kasen (2018) models. We limit the models shown to the ones with kinetic energy between $0.7 - 1.2 \times 10^{51}$ erg since that is the physically motivated limits (e.g. Blondin et al. 2017). For comparison we have plotted the total mass vs. $\overline{F}_{r2}$ for the data (black squares). We derived the total mass from the t_0 using typical values of v_e and q from explosion models (see text for more details). The best fit line from the data is shifted using the intercept from the models but keeping the same slope (black line). This shows that the model and the data find similar trends between the total mass and $\overline{F}_{r2}$.

in turn means that we can probe the M_{ej} for higher redshifts compared to a similar analysis in the i -band in surveys such as LSST to get an estimate of the M_{ej} for higher redshifts provided a minimal cadence of 5 restframe days in order to get a t_{r2} and $\overline{F}_{r2}$ measurement. However, it is important to note that the range of the two free parameters in equation 4 is large and when converting to M_{ej} this gives rise to such large uncertainties that it is not possible to accurately quantify the best fit values. We do see a qualitative trend for both the data and the models from Goldstein & Kasen (2018) in the same direction but not with the same values.

6 CONCLUSIONS

In this paper we present two methods of measuring the secondary maximum of light-curves in the r -band that also work for redder bands, e.g. i -band. We then used the Carnegie Supernova Project (CSP-I) (Contreras et al. 2010; Stritzinger et al. 2011) and CfA supernova program (Hicken et al. 2009) to derive correlations that we applied to the light-curve sample of PTF and iPTF (Papadogiannakis et al. 2019) to show the distribution of the transparency timescale which is a proxy for total mass. Since the distribution comes from an untargeted survey it represents more closely the true distribution of SNe Ia since the bias of finding SNe in larger galaxies is limited.

In summary our main conclusions are:

- We find a significant correlation between t_{r2} and s_{BV} which suggest brighter SN have a later bump in the r -band,

since s_{BV} correlates with the peak brightness of the SN and ^{56}Ni mass.

- There is a significant correlation between $\overline{F}_{r2}$ and the transparency timescale, t_0 which is a measure of when the SN ejecta becomes optically thin and is a probe for the total mass of the SN Ia explosion.

- These correlations suggest that SN physics can be extracted from the secondary maximum in r -band just as the secondary maximum in the NIR and IR, but with more accessible resources and less observing time.

- We also find a correlation between $\overline{F}_{r2}$ and total mass when examining the light-curves in the r -band from the Goldstein & Kasen (2018) models for masses larger than $0.3 M_{\text{solar}}$. We note that the shape of the r -band light-curves in the models do not agree with our observations and that the LTE approximations and thus it is only reasonable to compare the trend which is in agreement with our data. More accurate modelling is necessary to further investigate the correlation between $\overline{F}_{r2}$ and total mass.

- We see a correlation between the total mass from the models from Goldstein & Kasen (2018) and $\overline{F}_{r2}$ strengthening the suggestion that $\overline{F}_{r2}$ is a probe of the total mass.

- When doing the same analysis in the i -band similar correlations are found.

- We present a distribution of the transparency timescale t_0 from PTF and iPTF and compare it to models. We find that no model covers the entire range of values from the observations.

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APPENDIX A: TABLES

Here we present the tables with the data used to produced the plots in this paper.

Table A1. The parameters for the CfA SNe in the r-band.

Name	t_{r_2}	$\sigma_{t_{r_2}}$	n_{points}	sbv	σ_{sbv}	Wang	Branch	$\bar{\mathcal{F}}_{r_2}$	$\sigma_{\bar{\mathcal{F}}_{r_2}}^{lower}$	$\sigma_{\bar{\mathcal{F}}_{r_2}}^{upper}$
SN2001ep	24.4	0.21	5	0.89	0.04	BL	N	0.486	0.44	0.53
SN2005am	15.3	0.29	11	0.70	0.02	BL	N	0.372	0.33	0.41
SN2005eq	22.4	2.33	15	1.14	0.03	SS	91T	0.527	0.50	0.56
SN2005hk	37.1	6.61	5	0.87	0.04	SS	-	0.532	0.47	0.59
SN2006D	17.3	2.38	8	0.82	0.01	CN	N	0.431	0.40	0.46
SN2006N	19.3	0.24	10	0.77	0.02	CN	N	0.421	0.39	0.45
SN2006S	24.3	1.85	11	1.12	0.07	SS	N	0.553	0.52	0.59
SN2006X	17.0	1.04	12	0.96	0.01	BL	HV	0.464	0.42	0.51
SN2006ac	16.8	0.36	8	0.87	0.01	BL	HV	0.423	0.39	0.46
SN2006az	16.4	6.75	18	0.84	0.01	CN	N	0.413	0.37	0.45
SN2006le	20.4	0.3	12	1.20	0.01	CN	N	0.496	0.46	0.54
SN2007co	17.8	1.76	11	0.95	0.01	BL	N	0.473	0.39	0.56
SN2007kk	23.3	1.65	13	1.16	0.04	BL	N	0.563	0.52	0.60
SN2007le	20.7	1.38	7	1.00	0.01	BL	HV	0.457	0.41	0.50
SN2007nq	14.0	4.89	6	0.76	0.03	BL	N	0.346	0.28	0.42
SN2008Z	21.1	1.69	8	1.11	0.03	SS	N	0.483	0.45	0.52
SN2008ae	21.1	7.53	9	0.84	0.07	SS	-	0.478	0.41	0.55
SN2008ar	21.9	3.09	5	0.99	0.01	CN	N	0.511	0.45	0.58
SN2008bf	22.0	0.6	9	1.03	0.02	CN	N	0.468	0.42	0.52

Table A2: The parameters for the CSP-I SNe in the r-band.

Name	t_{r_2}	$\sigma_{t_{r_2}}$	n_{points}	sbv	σ_{sbv}	SNID	Wang	Branch	z_{Helio}	$T_{B(max)}$	$\overline{\mathcal{F}}_{r_2}$	$\sigma_{\overline{\mathcal{F}}_{r_2}}^{lower}$	$\sigma_{\overline{\mathcal{F}}_{r_2}}^{upper}$
SN2004dt											0.582	0.56	0.60
SN2004ef	15.5	0.39	16	0.81	0.00	Normal	HV	BL	0.03099	53264.0	0.386	0.35	0.42
SN2004eo	17.9	1.53	16	0.82	0.01	Normal	N	CL	0.0157	53278.2	0.417	0.37	0.46
SN2004ey	20.6	0.2	11	1.01	0.00	Normal	N	CN	0.01579	53303.9	0.439	0.40	0.48
SN2004gc	16.2	0.93	11	0.92	0.02	Normal	...	...	0.0307	53324.6	0.531	0.50	0.56
SN2004gs	16.4	5.32	17	0.69	0.00	Normal	N	CL	0.02665	53356.0	0.377	0.35	0.40
SN2005A	18.2	0.76	18	0.96	0.01	Normal	HV	BL	0.01914	53380.4	0.47	0.45	0.49
SN2005M	18.4	0.54	17	1.21	0.00	91T	91T	SS	0.02462	53405.4	0.541	0.51	0.57
SN2005W	16.6	1.57	3	0.92	0.01	Normal	BL	...	0.00889	53411.8			
SN2005ag	18.1	4.5	12	1.08	0.01	Normal	N	BL	0.0798	53414.0	0.468	0.43	0.50
SN2005al	18.9	1.69	18	0.86	0.00	Normal	...	...	0.0124	53429.9	0.375	0.35	0.40
SN2005am	14.6	0.27	21	0.73	0.00	Normal	HV	BL	0.0079	53436.6	0.346	0.33	0.36
SN2005be	19.7	0.5	10	0.75	0.02	Normal	...	...	0.03502	53460.6			
SN2005bg	21.9	0.38	5	1.00	0.03	Normal	N	SS	0.02309	53470.7	0.425	0.35	0.50
SN2005bo	nan	nan	1	0.85	0.01	Normal	N	CN	0.0139	53479.8			
SN2005el	16.2	0.27	9	0.83	0.00	Normal	N	CN	0.01491	53647.1	0.374	0.35	0.40
SN2005eq	28.5	3.8	7	1.24	0.01	91T	91T	SS	0.02898	53654.4	0.502	0.47	0.53
SN2005hc	21.8	1.36	7	1.19	0.01	Normal	N	CN	0.04594	53667.1	0.526	0.49	0.56
SN2005hj											0.524	0.48	0.57
SN2005iq	17.3	0.1	5	0.87	0.00	Normal	...	...	0.03404	53687.7	0.374	0.33	0.42
SN2005kc	20.6	1.01	3	0.90	0.01	Normal	N	CN	0.01512	53697.7			
SN2005ke	17.4	9.1	9	0.42	0.00	91bg	91bg	CL	0.00488	53698.3	0.281	0.25	0.31
SN2005ki	15.8	0.41	9	0.82	0.00	Normal	N	CN	0.01921	53705.6	0.349	0.31	0.38
SN2005ku	nan	5.51	3	1.19	0.04	Normal	HV	CN	0.04544	53698.4			
SN2005lu	24.0	0.77	6	1.13	0.03	Normal	...	...	0.03201	53711.9			
SN2005mc											0.388	0.35	0.42
SN2005na											0.447	0.42	0.47
SN2006D	14.2	0.11	12	0.81	0.00	Normal	N	CN	0.00853	53757.3	0.346	0.32	0.37
SN2006X	19.5	0.93	9	0.97	0.01	Normal	HV	BL	0.00524	53785.8	0.456	0.42	0.49
SN2006ax	20.9	3.28	7	0.98	0.00	Normal	N	CN	0.01674	53827.1	0.44	0.40	0.48
SN2006bh	16.8	0.45	10	0.80	0.00	Normal	...	...	0.01085	53833.4	0.343	0.32	0.37
SN2006br	22.7	0.84	7	0.91	0.03	Normal	HV	BL	0.02459	53851.2			
SN2006bt											0.533	0.49	0.57
SN2006ef	37.0	0.26	5	0.84	0.02	Normal	HV	BL	0.01788	53969.7	0.551	0.51	0.59
SN2006ej	18.8	0.1	5	0.82	0.02	Normal	HV	BL	0.02045	53975.6			
SN2006eq	nan	7.06	9	0.62	0.03	Normal	N	CL	0.0495	53977.1			
SN2006et											0.509	0.47	0.55
SN2006ev	13.6	1.34	7	0.84	0.02	Normal	...	...	0.02873	53989.1			
SN2006gj	35.5	4.59	5	0.66	0.01	Normal	N	CL	0.02835	53999.6	0.369	0.32	0.41
SN2006gt											0.432	0.38	0.48
SN2006hb	37.8	9.55	7	0.66	0.00	86G	91bg	...	0.01534	53998.7			
SN2006hx	17.1	0.55	4	0.90	0.02	Normal	N	SS	0.04549	54022.1			
SN2006is	22.4	0.62	5	1.14	0.03	Normal	HV	CN	0.0314	54008.4			
SN2006kf	14.7	0.29	7	0.73	0.00	Normal	N	CL	0.0213	54041.3	0.345	0.32	0.37
SN2006lu	25.9	1.21	6	1.05	0.03	Normal	...	...	0.0534	54034.1			
SN2006mr	19.7	7.67	9	0.26	0.00	91bg	91bg	CL	0.00587	54050.1	0.253	0.22	0.29
SN2006ob	15.2	0.39	5	0.74	0.01	Normal	...	...	0.05924	54063.4	0.359	0.31	0.41
SN2006os	18.5	0.54	6	0.94	0.02	Normal	N	CL	0.03281	54062.8			
SN2006ot											0.618	0.57	0.66
SN2006py	19.0	0.09	2	0.95	0.03	Normal	...	...	0.0579	54070.6			
SN2007A	19.6	0.08	2	1.00	0.01	Normal	N	CN	0.01765	54112.8			
SN2007N	38.4	2.91	6	0.30	0.01	91bg	91bg	CL	0.01288	54123.8	0.26	0.22	0.30
SN2007S	21.8	0.94	7	1.12	0.01	91T	91T	SS	0.01388	54144.6	0.514	0.47	0.56
SN2007af	18.5	0.4	9	0.93	0.00	Normal	N	BL	0.00546	54174.3	0.436	0.40	0.47
SN2007ai	22.7	1.56	6	1.25	0.01	91T	91T	SS	0.03166	54171.8	0.594	0.55	0.64
SN2007as	18.9	0.62	6	0.88	0.00	Normal	HV	BL	0.01757	54181.3	0.407	0.38	0.44
SN2007ax	33.8	0.41	4	0.36	0.01	91bg	91bg	CL	0.00686	54187.6			

SN2007ba											0.368	0.30	0.43
SN2007bc	24.6	0.12	5	0.89	0.01	Normal	N	CL	0.02077	54200.1	0.437	0.38	0.49
SN2007bd											0.369	0.31	0.42
SN2007bm	18.3	0.79	5	0.91	0.01	Normal	N	CN	0.00621	54224.6	0.425	0.39	0.46
SN2007jg	20.0	0.84	8	0.92	0.01	Normal	HV	BL	0.03713	54367.2	0.417	0.38	0.45
SN2007jh	33.4	5.59	5	0.59	0.01	86G	91bg	...	0.0408	54365.3	0.349	0.29	0.41
SN2007le	19.3	0.74	6	1.02	0.00	Normal	HV	BL	0.00672	54398.7	0.452	0.41	0.49
SN2007nq	13.6	0.12	7	0.75	0.01	Normal	HV	BL	0.04503	54398.2	0.355	0.32	0.39
SN2007on	13.2	8.2	11	0.57	0.00	Normal	N	CL	0.00649	54419.7	0.299	0.27	0.33
SN2008C	19.9	0.27	6	0.95	0.02	Normal	N	SS	0.01662	54466.3	0.465	0.43	0.50
SN2008R	15.7	0.07	8	0.60	0.01	91bg	91bg	CL	0.0135	54494.3	0.326	0.30	0.36
SN2008bc	18.2	2.59	11	1.03	0.00	Normal	N	CN	0.01509	54548.7	0.446	0.41	0.48
SN2008bq	22.4	0.33	6	1.15	0.01	Normal	N	CN	0.034	54562.9	0.499	0.46	0.53
SN2008fp	17.6	2.32	10	1.05	0.01	Normal	N	CN	0.00566	54729.7	0.455	0.43	0.48
SN2008gp	22.8	3.28	10	0.97	0.01	Normal	...	...	0.03341	54778.6	0.453	0.39	0.51
SN2008hv	16.4	1.79	11	0.85	0.00	Normal	N	CN	0.01255	54816.8	0.353	0.30	0.40
SN2008ia											0.377	0.34	0.41
SN2009F	14.0	0.05	7	0.34	0.01	91bg	91bg	CL	0.01296	54841.8	0.24	0.19	0.29
SN2009dc											0.666	0.61	0.72

Table A3: The parameters for the CfA SNe in the i-band.

Name	$\overline{\mathcal{F}}_{i_2}$	$\sigma_{\overline{\mathcal{F}}_{i_2}}^{lower}$	$\sigma_{\overline{\mathcal{F}}_{i_2}}^{upper}$	t_{i_2}	$\sigma_{t_{i_2}}$	n_{points}
SN2001V	0.726	0.63	0.82	28.8	0.41	7
SN2001ep				18.3	1.11	4
SN2002bo	0.654	0.55	0.76	23.3	0.37	6
SN2002cr				24.8	0.54	4
SN2002fk	0.54	0.50	0.58	26.2	0.48	6
SN2002ha	0.489	0.40	0.57	23.7	0.29	4
SN2002hu				27.8	0.9	4
SN2003W	0.691	0.63	0.75	25.4	0.35	5
SN2003cg	0.62	0.57	0.67	24.4	0.24	5
SN2003du				28.6	0.61	4
SN2003kf	0.515	0.48	0.55	27.4	0.61	5
SN2005am	0.489	0.45	0.53	18.5	0.12	10
SN2005el	0.438	0.42	0.46	22.2	0.15	23
SN2005eq	0.556	0.52	0.60	31.5	0.81	15
SN2005hc	0.552	0.49	0.62	30.1	1.07	10
SN2005hk	0.653	0.59	0.71	nan	1.38	5
SN2005kc	0.529	0.44	0.62	22.7	0.54	6
SN2005ke	0.467	0.39	0.54	nan	8.61	7
SN2005ki				16.2	2.24	5
SN2005lz	0.504	0.43	0.58	25.2	0.35	5
SN2005mc				31.4	8.81	4
SN2005ms	0.501	0.46	0.55	26.4	0.59	5
SN2005mz	0.351	0.28	0.42	33.8	1.05	7
SN2006D	0.399	0.37	0.43	22.4	1.4	8
SN2006N	0.555	0.52	0.59	20.7	0.39	7
SN2006S	0.571	0.53	0.61	30.7	0.3	12
SN2006X	0.591	0.54	0.64	25.7	0.38	14
SN2006ac	0.452	0.41	0.49	24.1	0.33	9
SN2006ax	0.515	0.46	0.57			
SN2006az	0.502	0.46	0.54	22.3	1.35	18
SN2006bt	0.678	0.63	0.73			
SN2006cc	0.511	0.44	0.59	29.5	0.33	6
SN2006gj	0.529	0.46	0.60	19.0	0.49	5
SN2006le	0.542	0.49	0.59	30.8	2.96	12
SN2006lf	0.413	0.35	0.47	20.5	0.32	8
SN2006ob	0.41	0.32	0.49	17.3	0.29	7
SN2006ou				nan	3.02	4
SN2007aj	0.856	0.79	0.92	24.6	0.46	10
SN2007bj	0.829	0.77	0.89	23.8	0.27	10
SN2007cb				19.5	1.88	5
SN2007cf				nan	3.3	6
SN2007cn				nan	nan	5
SN2007co	0.591	0.50	0.68	26.3	0.34	11
SN2007hg	1.042	0.91	1.17	nan	2.25	10
SN2007hj	0.462	0.42	0.50	15.9	0.32	9
SN2007hu				30.1	0.89	4
SN2007if				nan	3.31	6
SN2007ir	0.759	0.57	0.95	nan	3.93	4
SN2007jg	0.52	0.46	0.58	27.2	0.56	9
SN2007kk	0.7	0.65	0.75	27.5	0.49	13
SN2007le	0.589	0.53	0.65	27.3	0.19	7
SN2007nq	0.46	0.39	0.53	15.8	0.52	7
SN2007ob	0.613	0.53	0.70	28.1	0.76	6
SN2007ss	0.546	0.50	0.60	24.1	0.43	6
SN2007sw	0.634	0.59	0.68	26.2	0.4	11
SN2007ux	0.454	0.40	0.51			
SN2008C	0.584	0.53	0.64	24.0	0.51	5

SN2008Y				28.3	1.58	6
SN2008Z	0.468	0.43	0.51	30.6	0.27	10
SN2008ac				26.0	2.32	3
SN2008ae	0.622	0.54	0.71	30.1	1.87	8
SN2008ar	0.622	0.56	0.68	23.1	0.6	5
SN2008at	0.707	0.63	0.78			
SN2008bf	0.501	0.44	0.56	28.1	0.82	9
SN2008bi				38.7	0.91	6
SN2008bw	0.739	0.64	0.84	nan	nan	5
SN2008cd				14.2	1.95	6
SN2008cm	0.485	0.40	0.57	22.0	1.54	7
SN2008fr	0.63	0.58	0.68	29.5	0.76	10
SN2008gb	0.546	0.47	0.62	23.8	1.28	6
SN2008gl	0.513	0.45	0.58	22.1	0.5	5
SN2008hm	0.573	0.52	0.63	24.4	0.67	10
SN2008hs	0.628	0.56	0.70	16.4	0.26	8
SN2008hv				19.5	0.77	4
SN2009D	0.592	0.54	0.64	22.9	0.17	7
SN2009Y	0.555	0.52	0.59	28.7	0.41	12
SN2009ad	0.558	0.50	0.62	28.0	0.32	9
SN2009al	0.632	0.58	0.68	22.0	0.45	8
SN2009an	0.489	0.43	0.55	22.2	0.39	8
SN2009bv	0.432	0.36	0.50	16.2	5.09	6
SN2009dc	0.835	0.80	0.87	20.4	1.41	14
SN2009do	0.631	0.56	0.70	23.4	0.76	8
SN2009ds	0.452	0.37	0.54	30.8	0.46	5
SN2009ig	0.501	0.47	0.53	30.0	0.31	9
SN2009jr	0.513	0.43	0.59	31.2	0.37	8
SN2009kk	0.529	0.49	0.57	23.5	0.49	7
SN2009kq	0.564	0.52	0.60	27.0	0.23	5
SN2009lf	0.601	0.53	0.67	23.3	0.55	6
SN2010Y	0.421	0.34	0.50	16.6	0.31	8
SN2010ag	0.577	0.52	0.63	27.7	0.98	6
SN2010ai	0.42	0.27	0.57	22.8	3.5	6

Table A4: The parameters for the CSP-I SNe in the i-band.

Name	$\overline{\mathcal{F}}_{i_2}$	$\sigma_{\overline{\mathcal{F}}_{i_2}}^{lower}$	$\sigma_{\overline{\mathcal{F}}_{i_2}}^{upper}$	t_{i_2}	$\sigma_{t_{i_2}}$	n_{points}
SN2004dt	0.759	18.322	19.63			
SN2004ef	0.5	11.463	13.531	22.6	0.2	16
SN2004eo	0.548	12.364	15.027	19.1	0.91	16
SN2004ey	0.518	11.854	14.032	28.2	0.14	11
SN2004gc	0.679	16.01	17.915	19.5	1.21	10
SN2004gs	0.458	10.732	12.148	18.5	0.46	16
SN2005A	0.693	16.33	18.308	25.1	0.18	18
SN2005M	0.568	13.313	15.09	25.9	1.46	17
SN2005ag	0.569	12.974	15.485	30.2	4.72	12
SN2005al	0.493	11.507	13.141	23.8	0.14	18
SN2005am	0.487	11.474	12.888	19.1	0.14	19
SN2005be	0.737	16.616	20.245	25.1	0.25	10
SN2005bg	0.52	10.126	15.869	23.9	0.66	5
SN2005el	0.432	9.877	11.711	21.9	0.11	9
SN2005eq	0.524	12.064	14.148	32.0	0.26	7
SN2005hc	0.563	12.944	15.224	31.7	0.21	7
SN2005hj	0.583	13.268	15.878	32.7	0.11	5
SN2005iq	0.398	8.525	11.366	24.4	0.23	5
SN2005ir				26.5	0.3	4
SN2005kc				22.6	0.16	4
SN2005ke	0.369	8.38	10.055	17.1	6.87	9
SN2005ki	0.426	9.605	11.688	21.6	0.14	9
SN2005ku				28.3	1.12	3
SN2005lu	0.779	18.17	20.775	30.7	0.34	6
SN2005mc	0.51	11.792	13.7	17.8	0.23	11
SN2005na	0.547	12.67	14.682	26.1	0.37	11
SN2006D	0.381	8.949	10.078	22.8	0.2	12
SN2006X	0.598	13.929	15.947	24.7	0.13	9
SN2006ax	0.497	11.151	13.686			
SN2006bd				nan	0.06	5
SN2006bh	0.422	9.787	11.33	21.2	0.1	10
SN2006br	0.696	15.594	19.196	26.6	1.35	7
SN2006bt	0.668	15.846	17.543			
SN2006ef	0.786	18.396	20.916	18.5	0.15	5
SN2006ej	0.573	13.322	15.322	23.5	0.18	5
SN2006eq	0.531	12.069	14.474	18.1	0.58	9
SN2006et	0.584	13.672	15.529			
SN2006ev	0.488	10.689	13.719	19.5	0.26	7
SN2006gj	0.414	9.132	11.583	13.4	6.65	5
SN2006gt	0.519	11.658	14.27			
SN2006hb	0.733	17.354	19.304	18.8	4.21	7
SN2006is	0.622	14.411	16.67	32.1	0.18	5
SN2006kf	0.405	9.347	10.902	18.2	0.18	7
SN2006lu	0.657	15.086	17.751	27.9	0.91	6
SN2006mr	0.324	7.119	9.099	19.3	0.14	9
SN2006ob	0.469	10.28	13.192	18.8	0.42	5
SN2006os	0.605	13.954	16.272	23.4	0.37	6
SN2006ot	0.723	16.846	19.293			
SN2007N	0.352	7.6	10.0	34.4	5.82	6
SN2007S	0.576	13.206	15.593	30.2	0.17	7
SN2007af	0.512	11.816	13.795	25.5	0.33	9
SN2007ai	0.626	14.482	16.833	30.9	0.85	6
SN2007as	0.511	11.552	14.016	24.8	0.13	6
SN2007ba	0.48	10.313	13.691	15.0	2.88	7
SN2007bc				23.2	3.56	5
SN2007bd	0.484	10.464	13.735	22.8	0.17	5
SN2007bm	0.584	13.27	15.922	22.5	0.15	5

SN2007ca				29.5	0.32	4
SN2007if				22.5	0.68	6
SN2007jg	0.466	9.929	13.378	26.4	2.62	7
SN2007jh	0.476	10.109	13.676	14.4	4.58	5
SN2007le	0.578	13.133	15.782	27.4	0.3	6
SN2007nq	0.468	10.742	12.675	18.9	0.19	7
SN2007on	0.406	9.221	11.072	14.5	0.25	11
SN2008C	0.66	15.328	17.678	21.0	0.31	6
SN2008R	0.444	10.25	11.936	16.4	0.13	8
SN2008bc	0.505	11.511	13.751	29.3	0.56	11
SN2008bq	0.591	13.387	16.163	30.4	1.84	6
SN2008fp	0.518	12.181	13.709	27.7	0.17	10
SN2008gp	0.544	11.801	15.378	25.3	4.59	10
SN2008hv	0.44	9.412	12.602	22.5	0.52	11
SN2008ia	0.488	11.149	13.261	23.2	3.31	8
SN2009F	0.296	6.165	8.653	24.7	3.14	7
SN2009dc				20.8	0.72	4

Table A5. The t_0 values from the CSP-I SNe with sufficient multi-band data.

Name	t_0	σ_{t_0}
SN2004eo	36.6655	0.0802
SN2004ey	34.8975	0.0284
SN2005M	38.2209	0.1411
SN2005el	29.4895	0.0517
SN2005ke	31.3489	0.0548
SN2005ki	30.6891	0.0613
SN2006D	29.8909	0.0486
SN2006ax	35.8279	0.0738
SN2006et	38.3934	0.0918
SN2006kf	28.9012	0.0587
SN2006mr	26.2137	0.0234
SN2007S	40.1609	0.0925
SN2007af	34.4224	0.0766
SN2007ax	27.8355	0.0396
SN2007bc	28.7434	0.046
SN2007bd	31.5881	0.0021
SN2007le	37.5743	0.0803
SN2007on	29.0764	0.0589
SN2008bc	36.7482	0.052
SN2008hv	30.8722	0.0686
SN2008ia	29.1072	0.2377

Table A6: The parameters for the PTF and iPTF SNe in the r-band.

Name	t_{r_2}	$\sigma_{t_{r_2}}^{lower}$	$\sigma_{t_{r_2}}^{upper}$
PTF10rgn	0.454	0.377	0.532
iPTF16fmb	0.251	0.113	0.389
iPTF13aol	0.463	0.398	0.527
iPTF13dhp	0.528	0.435	0.621
iPTF16aas	0.245	0.093	0.398
PTF12eac	0.484	0.305	0.663
iPTF16gdp	0.338	0.24	0.437
iPTF13ani	0.573	0.494	0.651
PTF12fxn	0.36	0.176	0.545
PTF10hpp	0.501	0.4	0.601
PTF10tce	0.557	0.473	0.64
PTF10hdn	0.544	0.46	0.628
iPTF13ax	0.505	0.448	0.561
iPTF16eka	0.418	0.351	0.484
PTF10hdm	0.488	0.416	0.561
PTF11htb	0.49	0.402	0.578
iPTF16hun	0.293	0.089	0.498
PTF10nyt	0.522	0.412	0.631
PTF10kzf	0.582	0.504	0.661
PTF11dec	0.495	0.442	0.548
PTF12cjg	0.447	0.259	0.635
PTF10urn	0.459	0.355	0.563
PTF10uzi	0.214	0.168	0.26
PTF12gmf	0.241	0.05	0.431
iPTF13ez	0.49	0.438	0.541
PTF11blu	0.524	0.459	0.589
iPTF14yl	0.688	0.586	0.791
PTF10rpt	0.507	0.438	0.575
PTF12gmq	0.444	0.246	0.642
iPTF14aia	0.406	0.331	0.482
PTF11dzm	0.286	0.191	0.381
iPTF14apu	0.341	0.182	0.5
PTF11kml	0.478	0.367	0.588
PTF09alu	0.315	0.223	0.407
iPTF16dp	0.41	0.31	0.51
PTF10tfs	0.313	0.233	0.394
PTF10hcu	0.435	0.331	0.539
iPTF13apn	0.481	0.427	0.536
PTF11bok	0.501	0.421	0.58
PTF10pvi	0.457	0.366	0.548
PTF11rnu	0.356	0.229	0.483
PTF10hne	0.493	0.426	0.56
iPTF16grm	0.348	0.255	0.441
iPTF14axv	0.467	0.4	0.533
PTF11ivb	0.395	0.32	0.47
PTF10hrw	0.449	0.368	0.53
PTF11cji	0.48	0.367	0.592
iPTF13ckk	0.527	0.471	0.583
PTF11bas	0.482	0.42	0.543
PTF10ucj	0.424	0.308	0.539
PTF12mj	0.543	0.465	0.621
PTF10nvh	0.553	0.459	0.647
PTF11cmg	0.365	0.271	0.46
PTF11cyv	0.482	0.364	0.6
PTF11cml	0.445	0.367	0.524
iPTF13beg	0.348	0.284	0.411
PTF10aayx	0.394	0.284	0.503

iPTF14alb	0.361	0.126	0.596
PTF10qrj	0.337	0.27	0.404
iPTF13dnj	0.4	0.336	0.465
iPTF14amb	0.482	0.383	0.58
PTF10qnn	0.471	0.379	0.564
iPTF14bcl	0.467	0.42	0.513
PTF11xe	0.553	0.425	0.681
iPTF13adg	0.478	0.4	0.556
iPTF13ai	0.433	0.371	0.496
PTF12fuu	0.405	0.162	0.649
PTF10aajv	0.302	0.265	0.339
PTF12kim	0.519	0.393	0.644
PTF09dxo	0.391	0.274	0.507
iPTF13ccm	0.482	0.443	0.521
PTF10mla	0.458	0.337	0.578
PTF12dhb	0.499	0.428	0.57
PTF10tum	0.552	0.411	0.694
iPTF13s	0.504	0.448	0.559
PTF12dhk	0.611	0.543	0.68
PTF10ucl	0.413	0.301	0.526
PTF10qqw	0.514	0.447	0.58
PTF12cnl	0.531	0.455	0.607
PTF10qkf	0.47	0.397	0.542
PTF12keu	0.546	0.416	0.676
PTF10glo	0.578	0.544	0.612
PTF11qpc	0.537	0.441	0.633
PTF10iah	0.496	0.397	0.596
PTF10duy	0.471	0.412	0.53
iPTF13dkx	0.525	0.471	0.579
PTF10rhi	0.551	0.432	0.67
PTF10urj	0.418	0.312	0.524
PTF12dgy	0.381	0.302	0.459
iPTF13czs	0.47	0.417	0.523
iPTF13dkl	0.476	0.409	0.543
PTF11bof	0.487	0.429	0.545
iPTF14afv	0.441	0.367	0.515
iPTF13adw	0.528	0.476	0.58
iPTF13acz	0.586	0.532	0.641
PTF10qsc	0.549	0.46	0.638
PTF10egs	0.42	0.33	0.51
iPTF13dkj	0.392	0.318	0.467
PTF10cmj	0.497	0.351	0.644
PTF12cks	0.555	0.507	0.603
iPTF14aaf	0.518	0.449	0.587
PTF10fxp	0.398	0.308	0.488
PTF12juu	0.372	0.221	0.523
PTF10hld	0.497	0.428	0.567
PTF12hwb	0.443	0.287	0.6
iPTF14aik	0.283	0.073	0.494
iPTF16gef	0.297	0.145	0.449
PTF10hei	0.446	0.374	0.519
PTF10qqt	0.519	0.403	0.635
PTF10fym	0.398	0.32	0.476
PTF10yux	0.322	0.217	0.428
PTF10ifj	0.54	0.444	0.637
iPTF14axt	0.361	0.282	0.44
iPTF13cwq	0.465	0.395	0.535
PTF10abws	0.429	0.304	0.554
iPTF14yy	0.376	0.314	0.437
iPTF13bmn	0.39	0.276	0.503
PTF10oum	0.382	0.312	0.452
iPTF13cd	0.429	0.363	0.495

iPTF16gmw	0.344	0.22	0.468
iPTF13dbp	0.248	0.225	0.271
PTF12ibh	0.439	0.326	0.551
PTF10ivt	0.388	0.271	0.505
iPTF13dnh	0.552	0.498	0.606
iPTF13cxn	0.469	0.399	0.539
PTF11rke	0.481	0.387	0.575
PTF12sz	0.454	0.388	0.52
iPTF13anh	0.441	0.373	0.51
iPTF14aqs	0.321	0.045	0.598
iPTF13ag	0.471	0.416	0.526
PTF10cko	0.494	0.4	0.588
PTF10fxl	0.556	0.513	0.599
iPTF13bjb	0.431	0.348	0.513
PTF10kiw	0.526	0.407	0.645
iPTF13bdb	0.35	0.124	0.577
iPTF13dfa	0.294	0.228	0.36
PTF10kee	0.396	0.311	0.481
PTF10kdg	0.421	0.325	0.517
PTF10abkt	0.32	0.17	0.47
iPTF14dcd	0.474	0.433	0.515
PTF10qkv	0.469	0.419	0.519
iPTF13cyy	0.487	0.419	0.555
PTF10qly	0.527	0.437	0.617
PTF11kaw	0.491	0.399	0.583
iPTF13adv	0.463	0.411	0.515
iPTF16gua	0.281	0.188	0.374
PTF10ujl	0.414	0.259	0.569
PTF12gaz	0.308	0.06	0.557
PTF11ao	0.413	0.227	0.6
PTF10rbp	0.49	0.395	0.584
PTF11hfu	0.588	0.451	0.725
PTF10feg	0.561	0.483	0.64
PTF10goo	0.498	0.44	0.555
iPTF13cor	0.346	0.27	0.423
iPTF13bun	0.443	0.247	0.639
iPTF14anq	0.408	0.203	0.614
PTF11rrq	0.403	0.326	0.48
iPTF13caz	0.423	0.355	0.492
PTF10twd	0.587	0.504	0.67
iPTF13cow	0.477	0.39	0.563
iPTF13daw	0.339	0.268	0.411
PTF10one	0.37	0.302	0.438
iPTF16sw	0.49	0.385	0.595
iPTF13dni	0.468	0.407	0.529
PTF11ilj	0.537	0.406	0.667
PTF10qyx	0.375	0.307	0.444
iPTF13adm	0.449	0.384	0.514
PTF11qvc	0.516	0.419	0.614
iPTF16ig	0.405	0.288	0.522
PTF10aaju	0.467	0.395	0.539
iPTF16fht	0.208	0.073	0.344
PTF10xup	0.473	0.411	0.534
PTF12kta	0.402	0.306	0.499
PTF12csi	0.493	0.423	0.563
PTF10lxp	0.429	0.384	0.475
PTF10wyq	0.432	0.319	0.546
iPTF14fyt	0.805	0.624	0.987
PTF10gjsx	0.48	0.423	0.537
PTF12dxm	0.388	0.146	0.63
iPTF13dad	0.475	0.426	0.524
PTF12lgq	0.497	0.288	0.707

PTF12vr	0.474	0.388	0.56
PTF10czc	0.454	0.393	0.514
iPTF13ceq	0.46	0.32	0.6
PTF10ufj	0.47	0.39	0.551
PTF10goq	0.484	0.418	0.55
PTF10qwm	0.539	0.447	0.631
iPTF13ddg	0.462	0.414	0.51
iPTF13akl	0.487	0.388	0.586
PTF12gmu	0.337	0.174	0.5
PTF10trp	0.497	0.272	0.723
PTF11dwn	0.534	0.461	0.607
PTF12qgh	0.443	0.357	0.529
PTF10abou	0.513	0.425	0.601
PTF10gsp	0.044	0.028	0.06
iPTF14bjp	0.514	0.46	0.567
iPTF13aig	0.553	0.499	0.608
PTF12gaw	0.432	0.307	0.557
iPTF16fhz	0.274	0.16	0.388

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