Approximating general relativistic effects in Newtonian hydrodynamic supernova simulations

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1 Abstract

In this research, the validity of using an effective potential to approximate general relativistic effects in an otherwise Newtonian setting was investigated, by making simulations of core collapse supernovae on one hand in full general relativity, and on the other hand in said Newtonian setting. This was done for a mass range covering progenitors of $12 - 60 M_\odot$; a much wider mass range than has been used in earlier research, that also includes progenitors that form black holes. Two numerical codes were used; the general relativistic hydrodynamic code GR1D, and the Newtonian hydrodynamic code FLASH. For simplicity, spherical symmetry was assumed, and a M1 neutrino transport was employed rather than solving the full Boltzmann transport equation for neutrinos. Three different versions of the effective potential; GREP1, GREP2, and GREP3, were tested, and their results compared to a general relativistic case; GR, in an attempt to investigate possible improvements of earlier research. For all parameters investigated in this research, case GREP1 (and GREP2) yielded results that agreed very well with case GR at the time around bounce, though somewhat worse later on in the evolution. This observation is consistent with that made by Marek et al. (2006), but for a much larger set of progenitors, and therefore, the reliability of using this version of the effective potential to approximate general relativistic effects in an otherwise Newtonian setting, is not only confirmed, but extended as well. Another exceptional result not seen before was the black
hole formation times, which all three effective potentials could reproduce within \( \sim 5\% \) compared to case GR. In addition to this, case GREP3 yielded excellent results for the central density, but rather poor results for the remaining properties, and is thus not recommended to use to approximate general relativistic effects, although further investigation of this potential might give valuable clues for further improvements.

2 Introduction

2.1 Theory

Core collapse is a mechanism induced by gravity that drives massive stars (more massive than 8–10 \( M_\odot \)) towards the end of their lives. Stars generate energy through nuclear fusion, initially by fusing hydrogen into helium, and as they evolve, they fuse heavier and heavier elements. Massive stars can fuse elements all the way up to iron-56, above which nuclear fusion requires energy rather than generating it. Core collapse is triggered when the iron core, initially supported by electron degeneracy pressure, exceeds the Chandrasekhar mass, becomes unstable, and begins to collapse. During this phase, most of the protons turns into neutrons through various physical processes, creating the so called proto-neutron star (PNS). The collapse proceeds until the core reaches nuclear densities, where the collapse reverses in a “bounce” due to the nuclear force (or residual strong force) becoming repulsive, creating an outward shock wave. However, this shock wave, initially driven by the thermal pressure of the underlying material, will not instantly lead to a supernova (SN) explosion. Soon after bounce, the ram pressure of the overlying material exceeds the thermal pressure of the underlying material, reversing the shock into a quasi-static accretion shock. At this point, the PNS is accreting mass at a high rate. If the shock is revived before the PNS has accreted the maximum mass possible to support with the thermal pressure available, the object will explode in a successful core collapse supernova (CCSN), leaving behind a neutron star (NS). This can be accomplished e.g. by the favoured neutrino mechanism, which is based on the fact that neutrinos are emitted in the dense core, and then absorbed by the matter underlying the shock wave, heating it, and thus making the thermal pressure exceed the ram pressure. This mechanism will be explained in more detail later on. If the shock is not revived before the PNS has accreted the maximum mass, the object will collapse in a failed CCSN, leaving behind a black hole (BH) (O’Connor 2017). While successful CCSNe can be observationally detected thanks to the optical transient signal associated with the explosion, failed CCSNe are rather difficult to detect observationally, due to the lack of said explosion. However, there are other ways to at least indirectly prove the existence of failed CCSNe. One indirect evidence is the presence of BHs, which are the expected remnants of failed CCSNe, that can be detected through their gravitational impact on nearby visible objects or through gravitational waves from compact object merges, by LIGO (Abbott et al. 2016a,b). Another indi-
rect evidence is the existence of Type-IIP CCSNe. These CCSNe are associated with red supergiant progenitors which can be observationally detected prior to the explosion, but there seem to be a lack of these progenitors in the upper mass range, implying that these high-mass progenitors instead fail to explode and thus form BH [Smartt, 2015]. In addition to this, looking at the star formation rate, there seem to be a lack of successful CCSNe overall, indicating the presence of failed CCSNe [Horiuchi et al., 2011]. Regardless of the outcome, the mechanism of core collapse takes only seconds or less to complete, depending on various properties such as the efficiency of the neutrino mechanism, the equation of state (EOS), and the bounce compactness of the progenitor [O’Connor, 2017], properties that will be explained shortly. Exactly what determines the ultimate fate of core collapse is not known, and CCSNe have been studied for decades in attempts to predict which progenitors succeed to explode, and which fail and form black holes.

Gravity is the force driving core collapse to begin with, and the energy needed to revive the shock and thus launch an explosion comes from the tremendous amount of gravitational binding energy released when the iron core collapses. This energy is initially stored in the thermal and internal energy of the matter, and the various processes responsible for releasing this energy will be dependent on general relativistic effects, effects that must therefore be taken into account when studying the mechanism of core collapse [Marek et al., 2006]. The effects of general relativity are expected to change the hydrodynamics of the extremely dense core, as well as the neutrino mechanism mentioned earlier. Since the hydrodynamics and the neutrino mechanism are strongly coupled, hydrodynamic changes will indirectly change features in the neutrino mechanism, but in addition to this, the neutrino mechanism is expected to be directly modified by general relativistic effects such as time dilation, redshift, space-time curvature, and aberration effects [Bruenn et al., 2001]. For example, general relativity has been found to yield higher neutrino luminosities and energies compared to Newtonian gravity, which increases the neutrino heating rate and thus the efficiency of the neutrino mechanism [O’Connor and Couch, 2018b]. However, solving Einstein’s equations of motion is rather hideous work, especially in multiple dimensions, and therefore, several attempts to approximate the relativistic effects have been made, by using an effective potential $\Phi(r)_{\text{eff}}$ to resemble the most fundamental properties of a relativistic gravitational potential. However, even though this is widely used, it has only been tested in a few scenarios, and not across the range of expected progenitors. Therefore, in this research, an effective potential will be employed for a larger range of progenitors, which will be described in more detail in section 3.

As mentioned, determining which progenitors succeed to explode and which fail and instead form black holes, is not easy. However, in 2011, [O’Connor and Ott] found that the post bounce dynamics can be predicted rather well by a single parameter; the bounce compactness. The bounce compactness is a measure of mass distribution at bounce, and is given by:
$\xi_M = \frac{M/M_\odot}{R(M_{\text{bary}} = M)/1000 \text{ km}} \bigg|_{t=t_{\text{bounce}}}$

where $R$ is the radial coordinate that encloses $M$ at the time of core bounce \cite{O'Connor2011}. $M$ is chosen to be $2.5M_\odot$ in this case since this is the relevant mass scale for black hole formation. It is a dimensionless parameter that allows predictions about the post bounce dynamics and the evolution towards black hole formation to be made. In 2011, \citeauthor{O'Connor2011} predicted, assuming the neutrino mechanism to be the main cause of successful explosions, an EOS of intermediate stiffness, and simplified dimensions, that progenitors with bounce compactness $\xi_{2.5} \gtrsim 0.45$ most likely fail to explode and thus form BHs. Objects with higher compactness have higher mass accretion rates and thus accrete the maximum PNS mass earlier, and thus form BHs faster. For example, low compactness progenitors ($\xi_{2.5} < 0.1$) encloses a mass of $2.5M_\odot$ at a radial coordinate of $> 25000 \text{ km}$, while high compactness progenitors ($\xi_{2.5} > 0.5$) enclose the same mass at a radial coordinate of $< 5000 \text{ km}$. This can be understood by approximating the black hole formation time as the free fall time to the origin for a mass element located at a baryonic mass coordinate of $2.5M_\odot$. In this approximation, higher compactness correspond to shorter formation times since the mass element begins its free fall from a smaller radius. On the other hand, objects with higher compactness also have higher thermal pressure support and thus higher maximum PNS masses, and thus form BHs slower. However, the first effect out weights the second, and the net effect of higher compactness is thus faster BH formation \cite{O'Connor2011}.

Another parameter of interest when studying the post bounce dynamics and evolution towards black hole formation is the neutrino mechanism mentioned earlier, one of the various processes responsible for converting the gravitational binding energy stored in the thermal and internal energy of the matter, to energy that can drive an explosion. This mechanism is based on the fact that the hot, dense matter of the PNS emits neutrinos at a high rate, a consequence of the various processes taking place within the dense object, in which the protons transform to neutrons. The region in which the emission of neutrinos dominates is called the cooling region. The emitted neutrinos get absorbed by the material underlying the shock wave and thus heats it, increasing the thermal pressure support. The region in which the absorption of neutrinos dominates is called the heating region. If the increase of the thermal pressure support is large enough, it can exceed the ram pressure of the overlying material and thus revive the shock wave, leading to a SN explosion. Objects with higher compactness have higher mass accretion rates, and a consequence of this is that more gravitational binding energy is being released, which means that more energy is stored as thermal and internal energy that can later be radiated by neutrinos via the neutrino mechanism. This means that high compactness stars have higher neutrino luminosities and neutrino average energies, encouraging shock revival. On the other hand, another consequence of higher mass accretion rates is increased ram pressure, discouraging shock revival. In general, more compact
objects will need a more efficient neutrino mechanism to launch an explosion than less compact objects do, but the exact relation between compactness and the neutrino mechanism is not known (O’Connor, 2017).

The equation of state (EOS), relating parameters such as pressure, volume, temperature and internal energy, is yet another parameter influencing the post bounce dynamics and evolution towards black hole formation. A stiffer EOS correspond to a higher nuclear incompressibility and thus higher pressure support against collapse, leading to a longer black hole formation time, while a softer EOS leads to a shorter black hole formation time (O’Connor and Ott, 2011).

Another parameter worth mentioning when studying the post bounce dynamics and evolution towards black hole formation is the mechanism of mass loss. Massive stars discard tremendous amounts of mass in stellar winds during several phases of their lives, and the details of this mechanism strongly affects the PNS core structure, and thus also the outcome of CCSNe. For example, reduced mass loss leads to increased compactness, which as already mentioned is crucial for determining the outcome of CCSNe. Unfortunately, the understanding of this mechanism is limited, and there is a shortage of studies investigating the effects of different models of explanation. Anyhow, mass loss is most likely influenced by several properties such as mass, radius, luminosity, effective surface temperature, hydrogen and helium abundances at the surface, and metallicity, and the mass loss rate can vary remarkably during the life of a star (O’Connor and Ott, 2011).

2.2 Overview of the scientific field

In most multidimensional studies of CCSNe, an effective potential is used to approximate general relativistic effects. Some of the recent world leading 3D simulations that employs an effective potential are e.g. Lentz et al. (2015); O’Connor and Couch (2018a); Melson et al. (2015b) and Melson et al. (2015a). In 2005, Liebendoerfer studied the mechanism of CCSNe by making simulations on one hand in full general relativity (GR), and on the other hand by using an effective potential to approximate the relativistic effects, and compared the results. For the general relativistic simulations, the spherically symmetric, general relativistic hydrodynamic code AGILE-BOLTZTRAN (Liebendoerfer et al., 2004) was used, which includes full neutrino transport by solving the Boltzmann transport equations for neutrinos. For the approximate simulations, the spherically symmetric, Newtonian hydrodynamic code VERTEX (Rampp and Janka, 2002) was used, which employs an effective potential to approximate the relativistic effects, and also includes full neutrino transport. These codes turned out to produce similar results at the time around bounce, but with growing differences later on in the evolution.

In 2006, Marek et al. tried to improve these results, by trying several different adjustments of the effective potential approximating the general relativistic effects in the Newtonian simulations. In addition to using the AGILE-BOLTZTRAN code for the general relativistic simulations and the VERTEX
code for the approximate simulations, the code COCONUT (Dimmelmeier, H. et al., 2002) was used as well for both types of simulations, since this code can either do a full treatment of the general relativistic equations, or use an effective potential to approximate the relativistic effects in a Newtonian hydrodynamic setting. Using these codes and trying several different adjustments of the effective potential, they managed to improve the results of Liebendoerfer (2005).

2.3 Purpose

The aim of this research is to further determine the validity of using an effective potential to approximate general relativistic effects in CCSNe, for a wider range of progenitors than has been examined in earlier research. Establishing a robust method for approximating general relativistic effects is crucial for the development of CCSN simulations, especially in multiple dimensions in which Einstein’s equations become computationally expensive to solve. In section 3.1, the effective potentials used in this research are described, while in section 3.2, the most fundamental characteristics of the numerical codes used are specified. In section 3.3, the set of progenitors used in the simulations is introduced, and in section 3.4, a resolution test is explained. In section 4.1, the results of the resolution test is presented, followed by the resulting time evolution of several parameters in section 4.2, followed by results of the same parameters as functions of the bounce compactness in section 4.3. In section 5, the results are discussed, and in section 6, the research is summarized and conclusions drawn. Throughout this report, $G = c = M_\odot = 1$.

3 Method

3.1 Effective potential

The hydrostatic equilibrium of a general relativistic, spherically symmetric body can be described by the Tolman-Oppenheimer-Volkoff (TOV) equation (Tolman, 1934; Oppenheimer and Volkoff, 1939), and with this in mind, the general relativistic effects emerging during the mechanism of core collapse might be approximated using the TOV potential given by

$$
\Phi_{TOV}(r) = -4\pi \int_r^\infty \frac{dr'}{r'^2} \left( \frac{m_{TOV}}{4\pi} + r' \rho \right) \times \frac{1}{r'^2} \left( \frac{\rho + e + P}{\rho} \right), \quad (2)
$$

where $P$ is the gas pressure, $p_\nu$ is the neutrino pressure, $\rho$ is the rest-mass density and $e$ is the internal energy density (Marek et al., 2006). The TOV mass is given by

$$
m_{TOV}(r) = 4\pi \int_0^r dr' r' \left( \rho + e + \frac{P_F}{F} \right), \quad (3)
$$
where $E$ is the neutrino energy density, $v$ is the fluid velocity, and $F$ is the neutrino flux (Marek et al. 2006). $\Gamma$ is a metric function given by

$$\Gamma = \sqrt{1 + v^2 - \frac{2m_{TOV}}{r}},$$  \hspace{1cm} (4)

(Marek et al. 2006). However, in a Newtonian setting, the distinction between coordinate volume and local proper volume is not taken into account, and thus, the TOV mass given by the volume integral in eq. 3, is larger than the baryonic mass $m_b = 4\pi \int_0^r dr' r'^2 \rho$, and in particular larger than the mass in the general relativistic case, in which the corresponding volume integral includes the negative gravitational potential energy. This means that the TOV potential given by eq. 2 cannot accurately reproduce the effects of general relativity, endorsing adjustments to be made (Marek et al., 2006). In this research, three different modifications of the TOV potential are investigated. In all modifications, the term of $vF/\Gamma$ in the equation for the TOV mass is ignored. The fluid velocity $v$ is usually very small in CCSNe, especially in the core where the matter is not moving. In terms of the speed of light, $v \ll 0.01$. In addition to this, $F$ is very small as well, since this parameter measures the net momentum of the neutrinos, and the neutrinos in the core are very isotropic (no preferred direction). Therefore, the term of $vF/\Gamma \ll E$, and can thus be ignored without noticeable consequences. The motivation for the three modifications used lies in the observed results of previous research and the observed results throughout the course of this research, and will become clearer in section 5.

First modification, GREP1: In this modification, inspired by the one yielding the best result in the research by Marek et al. (2006), an additional factor of $\Gamma$ was added in the equation for the TOV mass, in order to reduce it (since $\Gamma < 1$), and thus weaken the potential. Ergo, for modification GREP1, the effective potential is given by

$$\Phi(r)_{TOV,1} = -4\pi \int_r^\infty \frac{dr'}{r'^2} \left( \frac{m_{TOV,1}}{4\pi} + r'^3 \left( P + p_\nu \right) \right) \times \frac{1}{\Gamma^2} \left( \frac{\rho + e + P}{\rho} \right),$$  \hspace{1cm} (5)

where the TOV mass is given by

$$m_{TOV,1} = 4\pi \int_0^r dr' r'^2 \Gamma \left( \rho + e + E \right).$$  \hspace{1cm} (6)

Second modification, GREP2: In this modification, a term of $0.1 m_{TOV}^2/r^2$ was subtracted in the square root of the equation for $\Gamma$, to reduce it a bit. Including this version, $\Gamma_2$, as a factor in the equation for the TOV mass, but using the original version, $\Gamma$, in the equation for the effective potential, weakened the effective potential further. Ergo, for modification GREP2, the effective potential is given by
\[ \Phi(r)_{TOV,2} = -4\pi \int_{r}^{\infty} \frac{dr'}{r'^2} \left( \frac{m_{TOV,2}}{4\pi} + r'^3 \left( \frac{P}{\rho} + p_\nu \right) \right) \times \frac{1}{\Gamma^2} \left( \frac{\rho + e + P}{\rho} \right), \quad (7) \]

where the TOV mass is given by
\[ m_{TOV,2} = 4\pi \int_{0}^{r} dr' r'^2 \Gamma_2 \left( \rho + e + E \right), \quad (8) \]
in which the metric function is given by
\[ \Gamma_2 = \sqrt{1 + v^2 - \frac{2m_{TOV}}{r} - \frac{0.1m_{TOV}^2}{r^2}}. \quad (9) \]

**Third modification, GREP3:** In this modification, one of the two factors of \( \Gamma \) in the equation for the effective potential was set equal to 1, so that only one factor remained. Using this version of the effective potential, with the original version of the metric function, weakened the effective potential even further. Ergo, for modification GREP3, the effective potential is given by
\[ \Phi(r)_{TOV,3} = -4\pi \int_{r}^{\infty} \frac{dr'}{r'^2} \left( \frac{m_{TOV,3}}{4\pi} + r'^3 \left( \frac{P}{\rho} + p_\nu \right) \right) \times \frac{1}{\Gamma} \left( \frac{\rho + e + P}{\rho} \right), \quad (10) \]

where the TOV mass is given by
\[ m_{TOV,3} = 4\pi \int_{0}^{r} dr' r'^2 \left( \rho + e + E \right). \quad (11) \]

### 3.2 Numerical codes

In this research, two numerical codes were used for the simulations; the spherically symmetric, general relativistic hydrodynamic code GR1D (O’Connor, 2015), and the spherically symmetric, Newtonian hydrodynamic code FLASH (Fryxell et al., 2000; O’Connor and Couch, 2018b), which employs an effective potential to approximate the general relativistic effects. These two codes employ M1 neutrino transport, which is a popular method to approximate neutrino transport, when solving the full transport equation is too computationally expensive. Aside from the inevitable differences between full general relativity and an effective potential, these two codes were made as similar as possible, in order to reduce uncertainties.

### 3.3 Progenitors

In order to extend our knowledge from previous research, a wide range of progenitors were used in the simulations, with masses ranging from 12\( M_\odot \) to 60\( M_\odot \) (Sukhbold et al., 2018). These progenitors have a reduced mass loss and
thus higher compactness than normal solar metallicity stars. The EOS used is called SFHo (Steiner et al., 2013). It has a nuclear incompressibility $K$ equal to 245.4 MeV, which describes the relative volume change due to a change in pressure, a symmetry energy at the saturation density $J$ equal to 31.57 MeV, which describes how the internal energy of nuclear matter changes as neutrinos are added or removed, a logarithmic derivative of the symmetry energy with respect to density $L$ equal to 47.10 MeV, which describes how the symmetry energy changes as density is changed, and a maximum cold NS mass $M_{T=0,\text{max}}$ equal to 2.059$M_\odot$, which constrains the mass-radius relation.

3.4 Resolution test

In addition to running simulations in full GR and with three different effective potentials (GREP1, GREP2, GREP3), the impact of resolution was tested as well for the GR case and the GREP1 case. Originally for the GR simulations, the grid zone width up to a radius of 20 km was set to 300 m, after which the grid width increases for every zone, resulting in a total of 650 zones (RES1). This original resolution was then adjusted to a grid width of 200 m up to a radius of 20 km and a total of 700 zones (RES2), and then again to a grid width of 200 m up to a radius of 20 km and a total of 900 zones (RES3). Originally for the GREP1 simulations, the grid width up to a radius of 90 km was set to 244 m, after which the grid width can jump by a factor of two in order to reduce resolution, as long as $\Delta r/r$ is kept below 0.3 degrees (RES4). This resolution was then doubled (i.e. $\Delta r = 122$ m up to a radius of 90 km and $\Delta r/r < 0.15$) (RES5). The results will be presented in the following section.

4 Results

4.1 Resolution test with 60$M_\odot$ progenitor

For case GR, increasing the resolution did not make a noticeable difference. For case GREP1, increasing the resolution made the results for the shock radius and the PNS radius slightly more consistent with case GR, while making the results for the neutrino luminosity and neutrino heating neither better nor worse. However, increasing the resolution of case GREP1 made the result for the black hole formation time worse, by shortening it with $\sim 0.01$ s. All together, the impact of increasing the resolution was more or less unnoticeable for most parameters, indicating that the original resolutions were probably good enough. The results are shown in Figure 1-5.

4.2 Versus time with 60$M_\odot$ progenitor

In this section, the resulting post bounce time evolution of several properties are presented in Figure 1-5, for a progenitor of 60$M_\odot$, the most compact progenitor in this research, with a compactness $\xi_{2.5} \approx 0.72$. The results for the remaining progenitors (12 $\leq$ 59$M_\odot$) show similar trends, but for the scope of this report,
only the 60$M_\odot$ progenitor will be presented in detail. For all the parameters discussed here, case GREP1 and GREP2 yield almost indistinguishable results for the entire range of progenitors. In addition to the results for the different cases, the results of the resolution test are presented in each figure as well. As shown and discussed above, increasing the resolution does not affect the results noticeable; the solid, dashed and dotted lines are almost indistinguishable for most parameters.

The time evolution of the shock radius, $r_s$, is presented in Figure 1. The initially outward shock wave reaches a maximum of $\sim 150$ km at $\sim 0.1$ s, where it reverses into an accretion shock. The increase in $r_s$ at $\sim 0.3$ s is due to accretion of the silicon-oxygen interface. Since silicon is twice as heavy as oxygen, but the number densities on both sides of the interface are equal, there is a drop in density and thus mass accretion rate going from accretion of silicon to accretion of oxygen. This makes the thermal pressure of the underlying material exceed the ram pressure of the overlying material, which in turn makes the shock reverse and increase a bit, before decreasing again. At $\sim 0.5$ s, the PNS collapses to a black hole. Case GREP1 and GREP2 yield results that agree very well with case GR at the time around bounce, though rather worse at later times, by overestimating the shock radius. Case GREP3 yields the worst result, especially at later times, by overestimating the shock radius even more.

The time evolution of the PNS radius, $r_{\text{PNS}}$, is presented in Figure 2. This parameter is defined as the radius where the density is equal to $10^{11}$ g/cm$^3$. It first peaks at $\sim 105$ km during the collapse phase, then decreases to $\sim 50$ km near bounce. At the time right after bounce it occurs a ringing effect, where the PNS radius oscillates, which can be seen for case GR in the zoomed in inset of
the figure. This effect cannot be seen for case GREP1, GREP2 or GREP3. This is due to differences in the codes used; in GR1D the shock is stronger than in FLASH, and thus, the effect only arises for case GR. This is a known effect in GR simulations \cite{vanRiper1988}. After this oscillation, the PNS radius increases to a second peak at \( \sim 85 \) km, before it decreases again as a consequence of energy loss. Case GREP1 and GREP2 yield results that agree extremely well with case GR at the time around bounce, though somewhat worse at later times, by overestimating the PNS radius. Case GREP3 yields the worst result, especially at later times, by overestimating the PNS radius even more.

The time evolution of the central density, \( \rho_c \), is presented in Figure 3. At bounce, the density overshoots its equilibrium due to the oscillation mentioned earlier, before it increases steadily throughout the evolution, until the point at which the curves turn vertical, which indicates the BH formation. All three cases GREP1, GREP2 and GREP3 yield indistinguishable results at the time around bounce that also agree extremely well with case GR at the time around bounce. However, at later times the three cases begin to deviate from each other as well as from case GR. Even though case GREP3 seems to underestimate the central density and thus delay the BH formation, this case seems to agree better with case GR longer into the evolution than the other two cases (GREP1 and GREP2), which seem to overestimate the central density and thus hasten the BH formation. This result is even more distinct for the lower and intermediate range of the progenitors, for which case GREP3 yields results that agree extremely well with case GR.

The time evolution of the neutrino luminosity is presented in Figure 4. This parameter depends on the mass accretion rate, which in turn is affected by the
accretion of the silicon-oxygen interface, as mentioned earlier. A steeper drop (due to the accretion of said interface) in the mass accretion rate correspond to a steeper drop in the neutrino luminosity, which is the case for the 60\(M_\odot\) progenitor. For some other progenitors, there is instead a more steady change in the mass accretion rate, and thus also a more steady change in the neutrino luminosity. All three cases GREP1, GREP2 and GREP3 yield very similar results at the time around bounce that also agree very well with case GR at the time around bounce. Case GREP1 and GREP2 continues to agree rather well with case GR throughout the evolution, while case GREP3 begins to deviate from case GR at later times by overestimating the neutrino luminosity by \(\sim 5 - 10\%\).

The time evolution of the neutrino heating is presented in Figure 5. This parameter is proportional to the neutrino luminosity (Figure 4) to first approximation, but also depends on the neutrino energies and the amount of material absorbing neutrinos in the heating region. All three cases GREP1, GREP2 and GREP3 yield very similar results at the time around bounce that also agree very well with case GR at the time around bounce. However, at later times the three cases begin to deviate from each other as well as from case GR, first by overestimating the neutrino heating up until \(\sim 0.3\) s, where instead they begin to underestimate the neutrino heating. Looking at the results for the entire range of progenitors, case GREP1 and GREP2 seem to yield the best results for the neutrino heating, although not that much better than case GREP3.
Figure 4: Time evolution of the neutrino luminosity for a progenitor of $60M_\odot$ and the different cases GR (black solid), GREP1 (red solid), GREP2 (blue solid) and GREP3 (green solid), as well as case GR with higher resolution (black dashed, black dotted) and case GREP1 with higher resolution (red dashed).

Figure 5: Time evolution of the neutrino heating for a progenitor of $60M_\odot$ and the different cases GR (black solid), GREP1 (red solid), GREP2 (blue solid) and GREP3 (green solid), as well as case GR with higher resolution (black dashed, black dotted) and case GREP1 with higher resolution (red dashed).
4.3 Versus compactness

In this section, several properties as functions of compactness are presented in Figure 6-11, for progenitors of $12 - 60 M_\odot$, corresponding to a compactness range of $\sim 0.05-0.75$. In addition to this, the deviation from case GR is presented in each figure as well. This deviation is defined as $(x_{\text{GREP}} - x_{\text{GR}})/x_{\text{GR}}$ for parameter $x$.

The peak shock radius, $r_{s,\text{peak}}$, as a function of compactness is presented in Figure 6 (top panel) together with the deviation from case GR (bottom panel). For most progenitors, the peak shock radius is reached at the moment where the shock reverses into an accretion shock. But for some progenitors, the peak shock radius is reached at the moment where the silicon-oxygen interface is accreted, making the results for these progenitors to deviate from the overall trend. For one progenitor of $M = 42 M_\odot$ and a compactness of $\xi_{2.5} \approx 0.43$, the results for case GREP1, GREP2 and GREP3 deviate, while the result for case GR follows the overall trend. The shock expansion after the silicon-oxygen interface accretes is not as large in GR1D as in FLASH, and therefore, case GR reaches the peak shock radius at the moment where the shock reverses into an accretion shock, while case GREP1, GREP2 and GREP3 reach the peak shock radius at the moment where the silicon-oxygen interface is accreted, which makes the deviation for this progenitor misleading. Overall, case GREP1 and GREP2 yield results that consistently overestimates the shock radius compared to case GR, with up to $\sim 2.5\%$ for the majority of progenitors. Case GREP3 overestimates the shock radius slightly more, with up to $\sim 5\%$ for the majority of progenitors.

The time $t_{r_{s,\text{peak}}}$ at which the peak shock radius is reached, as a function of
compactness, is presented in Figure 7 (top panel) together with the deviation from case GR (bottom panel). The progenitors for which the results deviate are those for which the peak shock radius is reached at the moment where the silicon-oxygen interface is accreted, which occurs later in the evolution, compared to the moment where the shock reverses into an accretion shock. For the progenitor $M = 42M_\odot$, the consequence of the different codes reaching the peak shock radius at different moments is even more obvious. For this reason, this specific progenitor is not included in the bottom panel. Overall, case GREP1 and GREP2 yield results that deviate from case GR with up to $\sim 5\%$ for the majority of progenitors. Case GREP3 yields slightly better results for the majority of progenitors.

The peak PNS radius, $r_{PNS,\text{peak}}$, as a function of compactness is presented in Figure 8 (top panel) together with the deviation from case GR (bottom panel). Case GREP1 and GREP2 yield results that underestimate the PNS radius with up to $\sim 10\%$ for progenitors up to a compactness of $\sim 0.3$, above which the results agree extremely well with case GR. The underestimation for the lower range is due to the ringing effect at bounce, mentioned earlier. This ringing effect could be seen as a peak for case GR, and for these low compactness progenitors, this peak exceeds the following peak. Since this effect only arises for case GR, the other cases underestimate the result for this range. In addition to this, there seems to be two “families” of progenitors in the upper range, that yields higher and lower values respectively, for the peak PNS radius. These families occur in both the GR and the GREP simulations. This is probably due to the dependence on mass accretion rate, as mentioned earlier. The “upper family” are a set of progenitors with a steep drop in the mass accretion rate at $\sim 0.3\,s$, due to
accretion of the silicon-oxygen interface, while the ”lower family” are a set of progenitors with a more steady change in the mass accretion rate. However, the lower family does have some less distinct drops in the mass accretion rate very early on in the evolution, due to accretion of other interfaces than the silicon-oxygen interface, that does not appear for the upper family. This indicates that for the progenitors in the lower family, the PNS radius might be more dependent on properties earlier in the evolution. If this is the case, defining the bounce compactness with $M = 2.5M_\odot$ might not be the best choice, while a choice of e.g. $M = 1.75M_\odot$ might reduce the appearance of these two families, as argued in O’Connor and Ott (2013). Case GREP3 yields results rather similar to, but slightly higher than case GREP1 and GREP2 for the entire range.

The peak neutrino luminosity as a function of compactness is presented in Figure 9 (top panel) together with the deviation from case GR (bottom panel). Case GREP1 and GREP2 yield indistinguishable results for the entire range, that also agree extremely well with case GR for the entire range. Case GREP3 yields results slightly higher than case GREP1 and GREP2 for the entire range, that overestimates the neutrino luminosity compared to case GR, with up to $\sim 5\%$ for the majority of progenitors. Once again, the two families of progenitors appears in the upper range, that yields higher and lower values respectively, for the peak luminosity. In this case, the ”upper family” is the set of progenitors with a more steady change in mass accretion rate and thus also neutrino luminosity, that reaches the peak neutrino luminosity later on in the evolution, while the ”lower family” are the set of progenitors with a steep drop in the mass accretion rate, and thus also a steep drop in the neutrino luminosity, as in the case for the $60M_\odot$ progenitor (Figure 4). This family of progenitors
Figure 9: Peak neutrino luminosity as a function of compactness (top panel) for progenitors of $12 - 60 M_\odot$ and the different cases GR (black), GREP1 (red), GREP2 (blue) and GREP3 (green), as well as the deviation from case GR (bottom panel).

does not reach as high values as the upper family, before the drop.

The peak neutrino heating as a function of compactness is presented in Figure 10 (top panel) together with the deviation from case GR (bottom panel). Case GREP1 and GREP2 yield indistinguishable results for the entire range, that also agree extremely well with case GR for progenitors up to a compactness of $\sim 0.4$, above which the results deviate from case GR with up to $\sim 25\%$ for the majority of progenitors. Case GREP3 yields results very similar to case GREP1 and GREP2 for progenitors up to a compactness of $\sim 0.4$, above which the results are slightly higher. The two families of progenitors in the upper range can be seen here as well, since the neutrino heating is proportional to the neutrino luminosity, to first approximation.

The BH formation time $t_{BH}$ (after bounce) as a function of compactness is presented in Figure 11 (top panel) together with the deviation from case GR (bottom panel). As found by O’Connor and Ott (2011), higher compactness leads to shorter BH formation times. All three cases GREP1, GREP2 and GREP3 yield very similar results for the entire range. Unfortunately, the simulations run in full GR did not run long enough for low compactness progenitors to form BHs, and thus, only high compactness progenitors reached BH formation, and therefore the deviation is only available for this range. Case GREP1 and GREP2 underestimate the BH formation time with up to $\sim 5\%$, while case GREP3 overestimates the BH formation time with up to $\sim 5\%$. 


Figure 10: Peak neutrino heating as a function of compactness (top panel) for progenitors of $12 - 60M_\odot$ and the different cases GR (black), GREP1 (red), GREP2 (blue) and GREP3 (green), as well as the deviation from case GR (bottom panel).

Figure 11: Black hole formation time as a function of compactness (top panel) for progenitors of $12 - 60M_\odot$ and the different cases GR (black), GREP1 (red), GREP2 (blue) and GREP3 (green), as well as the deviation from case GR (bottom panel).
5 Discussion

In this section, the results presented and explained above are further discussed. In this research, three different versions of the effective potential were tested, in an attempt to investigate possible improvements of earlier research, starting with a potential very similar to that yielding the best results in the research by Marek et al. (2006). The simulations made with this potential (GREP1), were found to produce results very similar to the simulations made in full GR at the time around bounce, but with growing differences later on in the evolution, starting at \( \sim 0.1-0.2 \, \text{s} \) for most parameters. This observation lead to the first modification of the effective potential. Choosing the central density, or specifically the black hole formation time, as the parameter to mainly improve the result of, and looking at the results of the different modifications in the research by Marek et al. (2006), an improvement seemed possible by weakening the effective potential, especially at later times, or correspondingly, when the PNS has accreted a larger gravitating mass. This was accomplished by subtracting a term of \( 0.1 m_{\text{TOV}}^2 / r^2 \) in the square root of the metric function \( \Gamma \) to reduce it, and then using this version of the metric function, \( \Gamma_2 \), in the equation for the TOV mass, but the original version, \( \Gamma \), in the equation for the effective potential. The simulations made with this modified potential (GREP2), were found to produce almost indistinguishable results for most parameters, compared to the original potential (GREP1). The difference that did occur with this modification was a slight improvement of the central density, and hence black hole formation time. Obviously, the modification was not drastic enough. This observation lead to the second modification of the effective potential. In case F in the research by Marek et al. (2006), the metric function \( \Gamma \) was set equal to 1 in the equation for the effective potential, in order to weaken it. This turned out to weaken the effective potential too drastically, and inspired by this observation, the effective potential was instead weakened by setting one of the two factors of \( \Gamma \) equal to 1 in the equation for the effective potential. The simulations made with this modified potential (GREP3), were found to produce excellent results for the central density, but worse results for the remaining parameters, so obviously, it is difficult to improve the results of one parameter without worsening another. This raises the question about for which parameters agreement with the general relativistic case (GR) is more important, and for which it is less important. One might argue, that since the efficiency of the neutrino mechanism is so fundamental for the outcome of CCSNe, and since the neutrino mechanism is so strongly dependent on general relativity, parameters describing the neutrinos, such as the neutrino luminosity and the neutrino heating, should preferably agree well with the general relativistic case (GR), while parameters such as the central density might not need to agree perfectly. However, it is interesting that case GREP3 yielded such excellent results for the central density, and determining why this came to be is probably a good place to start in future attempts to improve these results. In addition to the results discussed thus far, there is obviously a strong dependence on compactness overall, especially for the peak neutrino luminosity (Figure 9), the peak neutrino heating (Figure 10), and the black hole formation...
time (Figure 11), as expected. This is consistent with earlier suggestions that the compactness is a parameter that allows predictions about the post bounce dynamics and evolution towards black hole formation to be made.

Another interesting result was the two different families of progenitors appearing in the results for the peak PNS radius, peak neutrino luminosity, and peak neutrino heating. It is worth emphasizing the fact that it appeared two distinct families, and not just a scattered continuum of progenitors. These two families, most likely originating from differences in the mass accretion rate, indicates that the mass accretion rate is a quantity of great importance for the post bounce dynamics and evolution towards black hole formation. The appearance of these two families also demonstrates that the bounce compactness \( \xi_{2.5} \), even though valuable, is not a perfect parameter for making predictions about CCSNe. The presence of the two families in the results for the PNS radius, might originate from differences in mass accretion rate very early on in the evolution. If this is the case, it indicates that the PNS radius might be more dependent on properties early on in the evolution, and therefore, defining a compactness with a lower mass than \( 2.5M_\odot \) might be a good complement to \( \xi_{2.5} \).

Regarding uncertainties in this research, several simplifying assumptions were made, in order to cut the scope and focus on the most fundamental features of CCSNe. Among other things, spherical symmetry and a simplified neutrino transport were assumed, which of course limits the reliability of the results. In addition to this, differences in the codes used also contribute to the uncertainty of the results. For example, the fact that the shock is stronger in GR1D than in FLASH, gave rise to a discrepancy in the results for the peak PNS radius. Another example is the discrepancy for the \( 42M_\odot \) progenitor for which GR1D and FLASH reached the peak shock radius at different moments in the evolution; at the time when the shock reverses into an accretion shock, and the time when the silicon-oxygen interface is accreted, respectively. It is not immediately clear if these differences are truly a difference between GR and GREP simulations, or mainly a numerical difference in the codes used. Despite these sources of uncertainty, interesting conclusions can still be made, complementing the current knowledge of this scientific field and encouraging further research to be made.

6 Conclusions

The aim of this research was to investigate whether or not it is valid to use an effective potential, in an otherwise Newtonian setting, to approximate general relativistic effects emerging in CCSNe. This was done by making simulations of CCSNe in full GR, and comparing the results with simulations made in said Newtonian setting. By using a mass range covering progenitors of \( 12 - 60M_\odot \), also including progenitors that actually form black holes, the width of the mass range in this research exceeds the ones used in previous studies. Two numerical codes were used: the general relativistic hydrodynamic code GR1D, and the Newtonian hydrodynamic code FLASH. Spherical symmetry was assumed for the sake of simplicity, and rather than solving the full Boltzmann transport
equation for neutrinos, a M1 neutrino transport was employed. Regarding the effective potential, three different versions of it were tested; GREP1, GREP2, and GREP3. Their results were compared to a general relativistic case; GR, in order to investigate possible improvements of earlier research. The post bounce time evolution of the shock radius, PNS radius, central density, neutrino luminosity and neutrino heating was investigated, as well as the peak shock radius, the time at peak shock radius, the peak PNS radius, the peak neutrino luminosity, the peak neutrino heating and the black hole formation time. Case GREP1 (and GREP2) turned out to yield results very well consistent with case GR for all parameters mentioned, at the time around bounce, though with some growing differences later on in the evolution. This observation is consistent with that made by [Marek et al.](2006), but for a larger set of progenitors, and therefore, the reliability of using this version of the effective potential to approximate general relativistic effects in an otherwise Newtonian setting, is not only verified, but extended as well. This is probably the most valuable result of this research. The consistency between the results of GREP1 (and GREP2) and GR are even more impressive for high compactness progenitors, for which the general relativistic effects are even more prominent. Compared with the results of the research by [Marek et al.](2006), the results of this research are even more reliable, considering the fact that the codes used here are in many ways more similar to each other than the codes used by [Marek et al.](2006), which reduces uncertainties due to numerical differences. One exceptionally impressive result not seen before, is that all three effective potentials could reproduce the black hole formation times within $\sim 5\%$, compared to case GR. Another interesting result is that case GREP3 yielded excellent results for the central density, but rather poor results for the remaining properties. Since the central density is crucial for the outcome of CCSNe, further investigation of this potential might give valuable clues for further improvements. However, due to the poor results for the remaining parameters, this potential is not recommended to use to approximate general relativistic effects. Anyhow, in addition to the conclusions made thus far, the compactness $\xi_{2.5}$ is confirmed to be a valuable parameter when investigating CCSNe, since a strong dependence can be seen for most parameters, and especially for the peak neutrino luminosity, peak neutrino heating, and black hole formation time. However, the appearance of the two families in the results for the peak PNS radius, peak neutrino luminosity and peak neutrino heating, indicates that the compactness $\xi_{2.5}$ is not perfect, and might need to be complemented with some other parameter describing properties even earlier in the evolution.

References


