Studies of dark matter annihilation and production in the Universe

Carl Niblaeus

Still, after decades of experimental searches, the identity of dark matter remains unknown. One of the most studied hypotheses is that dark matter consists of a new form of elementary particle. In a world-spanning effort, scientists now search for traces of these particles in sensitive underground laboratories and with telescopes gazing out into space. In this thesis we study topics related to the field of indirect detection of dark matter, where one looks for the particles produced in dark matter annihilations occurring in the Universe around us. In particular, such annihilations can take place in the centre of the Sun, leading to a signature of high energy particles coming from the solar direction. We study various aspects of this scenario, looking at a background to the dark matter induced signal as well as studying refinements and modifications of the standard scenario.

We also study the production of dark matter particles. An abundance of dark matter particles surviving until today can be produced in the early Universe in a mechanism that relies on the competition between dark matter annihilations and the expansion of the Universe. We look at the effects of including higher order corrections in the calculation of the abundance of dark matter particles produced in this mechanism.
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Carl Niblaeus

Academic dissertation for the Degree of Doctor of Philosophy in Theoretical Physics at Stockholm University to be publicly defended on Friday 6 September 2019 at 13.00 in FB53, AlbaNova Universitetscentrum, Roslagstullsbacken 21.

Abstract
In this PhD thesis we investigate various aspects of particle dark matter. The proper identification of dark matter developed during the second half of the twentieth century to become one of the biggest endeavours in modern physics and astronomy. Although observations currently favour the explanation that dark matter consists of a new form of particle, no experimental search has yet provided unequivocal evidence of such a particle.

Of particular importance in this thesis is the field of indirect detection of dark matter, where one searches for the particles emerging from annihilations of dark matter particles out in the Universe. Specifically, we consider dark matter annihilations in the centre of the Sun. As the Sun moves through the galaxy, some dark matter particles scatter in the Sun and lose enough energy to become bound to the Sun. They settle in the solar core and begin to annihilate, which leads to an annihilation signal from the solar direction.

The thesis is built on novel research consisting of three papers and a monograph-type chapter. In the first paper we calculate the flux of high energy neutrinos coming from cosmic ray cascades in the solar atmosphere and investigate the role it plays as a background in solar dark matter searches. In the second paper we consider dark matter annihilating into long-lived mediators in the Sun, which leads to interesting new detection possibilities. A third paper explores more generally the fluxes of secondary particles from dark matter annihilations that are searched for in indirect detection. We look at the effects of changing the Monte Carlo event generator that generates the fluxes and of having polarized final states in the annihilations. Finally, we consider in a monograph-type chapter the production of dark matter in the early Universe through the freeze-out mechanism, looking at effects of higher order corrections in the calculation of the relic abundance in the minimal supersymmetric standard model.

Keywords: dark matter, neutrinos, astroparticle physics, indirect detection.

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Department of Physics
Stockholm University, 106 91 Stockholm
STUDIES OF DARK MATTER ANNIHILATION AND PRODUCTION IN THE UNIVERSE

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Svensk sammanfattning

Mörk materia är en typ av materia som är helt osynlig i vanliga teleskop eftersom den inte skickar ut något, alternativt en väldigt liten mängd, ljus som kan samlas upp i teleskop. Därmed kan vi i dag endast avgöra den mörka materians närvaro genom att observera hur den påverkar omgivande synlig materia. En stor mängd observationer på väldigt skilda astronomiska storleksskalor stödjer idén om att mörk materia finns närvarande i stora mängder i universum – ungefär fem gånger mer än den synliga, kända materian.

Den hypotes som fått störst fötfäste genom att testas mot observationer är att mörk materia består av en ny form av elementarpartikel, en svagt växelverkande massiv partikel som kallas wimp (förrättning av engelskans ”weakly interacting massive particle”). Sådana partiklar skulle bildas i den mängd som vi observerat i det heta plasmat som utgör det tidiga Universum.

Vi söker efter wimpar på olika sätt. En metod, så kallad indirekt detektion, är att söka efter partiklar som bildas då wimpar kolliderar med varandra. I dessa annihilationer bildas olika typer av partiklar, så som fotoner, neutriner och olika former av antipartiklar, som vi söker efter i olika experiment världen över genom att rikta teleskopen mot de platser på himlen där vi förväntar oss en hög täthet av mörk materia, och därmed många annihilationer.

Ett exempel på en typ av indirekt detektion är sökandet efter en annihilationssignal från solens inre. När solen färdas runt galaxens centrum kommer wimpar i galaxen att krocka med partiklarna inuti solen och förlora energi så att de till slut samlas upp i solens inre. När tätheten av wimpar byggs upp annihilerar de med varandra i högre grad. Av alla de partiklar som bildas i dessa annihilationer är det endast neutriner som växelverkar tillräckligt svagt för att kunna ta sig ut ur solens täta inre. Dessa oerhört lätta elementarpartiklar växelverkar endast med den svaga krafterna vilket får konsekvensen
att enorma neutrinoteleskop krävs för att detektera dem på jorden. Ett exempel på ett sådant neutrinoteleskop är IceCube, en detektor som är inbyggd i den antarktiska isen och har en total volym på mer än en kubikmiljometer.

Förutom de neutriner som skulle skapas i wimp-annihilationer i solens inre förväntar vi oss också att neutriner bildas i solens atmosfär, i de högenergetiska kaskader av partiklar som uppstå när partiklar från den kosmiska strålningen kolliderar med atomkärnor i solens yttre delar. Dessa neutriner kommer efter att de producerats att färdas genom solens inre där de växelverkar och oscillerar mellan de tre neutrinosmakerna. Efter att ha färdats genom solen kommer de oscillatora ytterligare innan de når jorden, där de kan detekteras i ett neutrinoteleskop som IceCube. I en av artiklarna som utgör denna avhandling (Paper I) har vi beräknat flödet på jorden av dessa solatmosfäriska neutriner. Motivationen för denna beräkning är delvis att bättre förstå hur den kosmiska strålningen växelverkar i den miljö som solens atmosfär utgör och hur detta kan skilja sig från liknande processer i jordens atmosfär men också det faktum att det solatmosfäriska neutrino-flödet utgör ett bakgrundsflöde för de neutriner som förväntas från wimp-annihilationer i solens inre.

I en annan av artiklarna i avhandlingen (Paper II) har vi studerat ett annorlunda scenario för mörk materia-annihilationer i solen. Vi undersöker en modell där mörk materia-partiklarna annihilerar till ett par hypotetiska partiklar som inte växelverkar med solmaterialet och har en tillräckligt lång livstid för att färdas ut ur solen innan de sönderfaller till andra partiklar. Vi beräknar och simulerar hur de experimentella förutsättningarna förändras i detta fall.

I den tredje artikeln (Paper III) har vi studerat flödet av partiklar som bildas i wimp-annihilationer med fokus på att undersöka den osäkerhet som finns i simuleringen av dessa flöden. Förutom neutriner söker man i indirekt detektion även efter gammastrålning och olika antipartiklar. Vi simulerar annihilationer av wimppar och tittar på hur simuleringsresultaten skiljer sig mellan olika program samt studerar hur polarisation av sluttillståndet i annihilationen påverkar signalen.

Slutligen beskriver vi i ett kapitel (kapitel 6) av monografi typ ett projekt som syftat till att undersöka korrektoner till beräkningen av hur mycket materia som produceras i det tidiga Universum. För
en given partikelfysikalisk modell måste parametrarna anpassas så
att denna beräkning resulterar i att samma mängd mörk materia
produceras i det tidiga Universum som den mängd vi har observerat
experimentellt. Korrektioner till denna beräkning leder till en mer
precis uppskattning av parametervärdena som krävs, samt ger en
uppskattning av osäkerheten som uppstår av att inte ta med dessa
korrektioner.
List of papers

Papers included in this thesis


Papers and proceedings not included in this thesis

### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP</td>
<td>Charge conjugation-parity</td>
</tr>
<tr>
<td>BBN</td>
<td>Big bang nucleosynthesis</td>
</tr>
<tr>
<td>CC</td>
<td>Charged current</td>
</tr>
<tr>
<td>CMB</td>
<td>Cosmic microwave background radiation</td>
</tr>
<tr>
<td>DR</td>
<td>Dimensional reduction</td>
</tr>
<tr>
<td>EA(\mu)</td>
<td>Earth atmospheric muons</td>
</tr>
<tr>
<td>EA(\nu)</td>
<td>Earth atmospheric neutrinos</td>
</tr>
<tr>
<td>FCNC</td>
<td>Flavour-changing neutral current</td>
</tr>
<tr>
<td>h.c.</td>
<td>Hermitian conjugate</td>
</tr>
<tr>
<td>IR</td>
<td>Infrared</td>
</tr>
<tr>
<td>LO</td>
<td>Leading order</td>
</tr>
<tr>
<td>LHC</td>
<td>Large hadron collider</td>
</tr>
<tr>
<td>LNP</td>
<td>Lightest new particle</td>
</tr>
<tr>
<td>LSP</td>
<td>Lightest supersymmetric particle</td>
</tr>
<tr>
<td>MS</td>
<td>Minimal subtraction</td>
</tr>
<tr>
<td>MSSM</td>
<td>Minimal supersymmetric Standard Model</td>
</tr>
<tr>
<td>MOND</td>
<td>Modified Newtonian dynamics</td>
</tr>
<tr>
<td>NC</td>
<td>Neutral current</td>
</tr>
<tr>
<td>NLO</td>
<td>Next-to-leading order (corrections)</td>
</tr>
<tr>
<td>pMSSM</td>
<td>Phenomenological minimal supersymmetric Standard Model</td>
</tr>
<tr>
<td>QCD</td>
<td>Quantum chromodynamics</td>
</tr>
<tr>
<td>SA(\nu)</td>
<td>Solar atmospheric neutrinos</td>
</tr>
<tr>
<td>SLHA</td>
<td>Supersymmetry Les Houches accord</td>
</tr>
<tr>
<td>SM</td>
<td>Standard Model (of particle physics)</td>
</tr>
<tr>
<td>SUSY</td>
<td>Supersymmetry</td>
</tr>
<tr>
<td>UV</td>
<td>Ultraviolet</td>
</tr>
<tr>
<td>WIMP</td>
<td>Weakly interacting massive particle</td>
</tr>
</tbody>
</table>
Acknowledgements

Completing a PhD thesis is a formidable task. It requires a lot of patience, but also support from others. Here I want to thank those who have helped me in my struggle to finish the thesis. First of all, I want to thank my supervisor Joakim for his constant support and enthusiasm, especially during the times when motivation has been lacking. I have enjoyed our collaboration a lot. I also want to thank my mentor Sara for the unwavering support and optimism during our meetings which has been able to infect me with new energy even during tougher periods.

In my work I have collaborated with various persons who I want to thank. Björn Herrmann and Julia Harz have been very helpful in one of the projects to explain and understand the DM@NLO code. In particular, I want to thank Björn for the hospitality during my visit to Annecy. I also want to thank my collaborators in the other projects included in the thesis—Ankit, Jonathan, Jessica and Rikard. It has been very rewarding to work with you all.

My office mates over the years, and perhaps in particular Sebastian, Axel and Jessica, also deserve thanks for all the fun and interesting discussions over the years. I am also grateful for being a part of the CoPS group and OKC, something which has been a very rewarding experience and a great environment to be a part of.

Last but certainly not least I am very grateful to my family and loved ones for all the encouragement and support over the years.
In this thesis, various aspects of particle dark matter are studied, with some emphasis on solar searches. The thesis consists of an introductory part where the necessary background for the included research papers is introduced. Apart from review material, there is one chapter in this part (Ch. 6) which includes new results and should be considered part of the new research contributions of this thesis. Part I ends with the conclusions of the thesis and a perspective on the route ahead.

The original research included consists of three publications and the aforementioned chapter. The contribution of the author to the publications and the chapter is outlined below.

**Author’s contribution**

**Chapter 6.** I have written the whole chapter as well as the majority of the code needed to run DarkSUSY together with DM@NLO, with some minor contributions from Joakim Edsjö. Julia Harz and in particular Björn Herrmann have helped with necessary discussion for the project and have been crucial in order to understand and use the DM@NLO code.

**Paper I.** I am the corresponding author of Paper I together with Jessica Elevant. I contributed in terms of modifying and running MCEq to get the neutrino fluxes at production as well as to the new code responsible for sampling events from these fluxes. I also performed validations against previous studies and wrote substantial parts of the original draft, including the parts on methodology, large parts of the background and discussion of results.
**Paper II.** I wrote the modifications of the WimpSim code needed to simulate mediator decays in the Sun and performed the simulations for the first part of the results section. I wrote about half the text of the original draft.

**Paper III.** I wrote all the code necessary in order to simulate WIMP annihilations with MadGraph, Pythia and Herwig and extract the spectra of secondary particles. Most of the simulations needed for the results were performed by me. I wrote the majority of the draft.

**Material from licentiate thesis**

Some of the material in Part I of this thesis comes from the author’s licentiate thesis in Ref. [1]. It is reused with slight modifications and updates. The following summarises chapter by chapter the material coming from Ref. [1]:

- **Chapter 1.** Sec. 1.1 and the first paragraph of Sec. 1.2.
- **Chapter 2.** Everything except Sec. 2.2.
- **Chapter 5.** Everything except Sec. 5.2.2.
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Part I

Introduction
Chapter 1

The Standard Model and astroparticle physics

In particle physics, the fundamental aim is to find and study the smallest constituents of our Universe. The twentieth century in particular represented an astonishing evolution of this field, culminating with the development of the Standard Model (SM) of particle physics which is our currently best model of the fundamental building blocks of Nature and their interactions. With the discovery of the Higgs boson in 2012 [2, 3] at the Large Hadron Collider, one of the final pieces of the SM puzzle was laid.

While particle physicists continue to look deeper and deeper into matter to determine the laws governing the smallest objects in the Universe, astrophysicists and cosmologists instead gaze outwards from Earth, out into the vast Universe. In these fields the goal is instead to determine the laws that govern the largest objects and the evolution of our Universe. From this perspective particle physics might appear distant from astrophysics and cosmology. However, there are intimate connections between the fields. After all, the elementary particles ultimately make up all the matter in the Universe and, it turns out, play a fundamental role in its evolution.

In this thesis we study the subject of dark matter, one of the most studied topics in contemporary physics and a field which explicitly connects particle physics to cosmology and astrophysics. Dark matter has been known to exist for almost a century from its gravitational effects on ordinary matter but the lack of a fundamental description was not recognised as a serious problem until the latter
half of the twentieth century. Since then, it has risen to become one of the biggest problems of modern physics. The origin of the effects attributed to dark matter remain unknown, but observations point towards a particle nature, and furthermore indicate that dark matter does not consist of any known particle but must be made up of a new form of matter. For this reason, the study of dark matter is part of astroparticle physics, a field at the border between astrophysics, cosmology and particle physics where one studies the implications of particle physics for astrophysics and cosmology and vice versa.

In this chapter, we give an overview of the content of the SM of particle physics and the ideas underlying it (for details we refer to textbooks on the subject [4–6]). We then discuss the need to go beyond and develop new models.

1.1 The Standard Model of particle physics

The Standard Model (SM) is the quantum field theory that to our best knowledge today describes the interactions of the most fundamental constituents of Nature we have discovered so far. As a quantum field theory, it describes the elementary particles as excitations of quantised fields. It is furthermore based on the incredibly successful concept of gauge invariance, where all of the physical observables are invariant under so-called gauge transformations—mathematical transformations belonging to specific groups of symmetry, one for each type of interaction. The SM includes the description of all the known fundamental forces of Nature except gravity: the strong interaction, describing the interaction of particles carrying colour charge, and the weak and electromagnetic interactions, describing interactions between particles carrying weak and electric charges respectively. The latter two are in the SM unified into a single description called the electroweak interaction.

In the SM, the elementary particles and their interactions are described by the action, which is the space-time integral of the Lagrangian density \( \mathcal{L} \), a function of the quantum fields that represent the particles and the derivatives of the fields. The Lagrangian is interpreted as the kinetic minus the potential energy of the system. The SM Lagrangian consists of all the terms allowed by the assumed symmetries, which are the gauge symmetries associated with the dif-
1.1. The Standard Model of particle physics

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Spin</th>
<th>Mass [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\mu}$</td>
<td>photon</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$W_{\mu}^+$</td>
<td>$W$-boson</td>
<td>1</td>
<td>80.3</td>
</tr>
<tr>
<td>$Z_{\mu}$</td>
<td>$Z$-boson</td>
<td>1</td>
<td>91.2</td>
</tr>
<tr>
<td>$G_{\mu}$</td>
<td>gluon</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$h$</td>
<td>Higgs boson</td>
<td>0</td>
<td>125</td>
</tr>
</tbody>
</table>

Table 1.1 – The bosonic particle content of the Standard Model. All mass values are the central values reported in [7] rounded to three significant figures.

Different fundamental forces. Thus there is one part of the full gauge symmetry belonging to the strong interaction, and one part belonging to the electroweak part. The full SM gauge symmetry $G_{SM}$ is written as

$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y.$$  \hspace{1cm} (1.1)

All observable quantities built from terms in the SM Lagrangian have to respect this gauge symmetry for the terms to be allowed. That is, a symmetry operation must leave all terms invariant\(^1\). To each gauge symmetry and therefore to each type of interaction is associated a conserved quantum number. For the strong interaction this is called colour charge, for the electroweak interaction we have weak isospin and weak hypercharge.

In terms of the particle content, the SM consists of a number of fermions interacting through the exchange of bosons. All fermions are described by spinor fields, spin 1/2 fields that represent the Fermi-Dirac nature of these particles. The interactions proceed through the exchange of gauge bosons, vector fields that describe the spin

\(^1\)Mathematically this corresponds to making sure that operations of the elements of the symmetry groups leaves all quantities invariant.

\(^2\)The quark masses deserve a special comment: since quarks are not observed as free particles their masses are ambiguous to define. Here we quote the so-called MS mass for all quarks except for the top, where the so-called “Monte Carlo” or “directly measured” mass is given, see the related reviews in [7].
1. The Standard Model and astroparticle physics

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Mass [GeV]</th>
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</thead>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st gen.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>up quark</td>
<td>$2.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>d</td>
<td>down quark</td>
<td>$4.7 \times 10^{-3}$</td>
</tr>
<tr>
<td>c</td>
<td>charm quark</td>
<td>1.28</td>
</tr>
<tr>
<td>s</td>
<td>strange quark</td>
<td>0.095</td>
</tr>
<tr>
<td>t</td>
<td>top quark</td>
<td>173</td>
</tr>
<tr>
<td>b</td>
<td>bottom quark</td>
<td>4.18</td>
</tr>
<tr>
<td>2nd gen.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ν_e</td>
<td>electron neutrino</td>
<td>$\lesssim 0.12 \times 10^{-9}$</td>
</tr>
<tr>
<td>e^-</td>
<td>electron</td>
<td>$0.511 \times 10^{-3}$</td>
</tr>
<tr>
<td>3rd gen.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ν_μ</td>
<td>muon neutrino</td>
<td>$\lesssim 0.12 \times 10^{-9}$</td>
</tr>
<tr>
<td>μ^-</td>
<td>muon</td>
<td>0.106</td>
</tr>
<tr>
<td>ν_τ</td>
<td>tau neutrino</td>
<td>$\lesssim 0.12 \times 10^{-9}$</td>
</tr>
<tr>
<td>τ^-</td>
<td>tau</td>
<td>1.78</td>
</tr>
</tbody>
</table>

Table 1.2 – The fermionic particle content of the Standard Model. All particles are fermions and have spin $\frac{1}{2}$. All mass values are the averages reported in Ref. [7] and are rounded to three significant figures when possible, otherwise two. Note that the neutrino masses are not currently known, here we quote the lowest available limit on their total sum from the Planck measurements of the Cosmic Microwave Background including observations of Baryon Acoustic Oscillations [8] (the limit depends on cosmological assumptions, see Ref. [8] for details). In the SM the neutrino masses are assumed to be zero.

Fermions are further subdivided into quarks and leptons, the former interacting both through the strong and electroweak interaction and the latter only interacting electromagnetically.
the charged leptons $e^-$, $\mu^-$ and $\tau^-$ and the corresponding neutrinos $\nu_e$, $\nu_\mu$ and $\nu_\tau$. Quarks are divided into up- and down-type quarks where the up-type are $u$, $c$ and $t$ and the down-type are $d$, $s$, and $b$.

The fermions come in three generations, each consisting of one charged lepton, the corresponding neutrino, one up- and one down-type quark. The generations have almost the same interaction properties but quite different masses. Fermions belonging to the same generation are said to have the same flavour. The fermionic content of the SM is summarised in Tab. 1.2.

The Higgs field has a vacuum expectation value $v$ that is different from zero, which means that in the lowest energy state of the theory—the vacuum state—the Higgs field has a nonzero value, unlike all other particles in the theory. Since the Higgs field behaves non-trivially under weak gauge transformations this is equivalent to saying that the SM vacuum state does not respect the electroweak part of $G_{\text{SM}}$ and since furthermore particles are excitations around the vacuum state this means that the description of the particle content of the SM (unless the energies are very large) is not symmetric under the electroweak gauge symmetry. The gauge symmetry is hidden at these lower energies, something that is often referred to as spontaneous symmetry breaking. The subpart of $G_{\text{SM}}$ that remains in the hidden symmetry phase is given by

$$G_{\text{SM}}^{\text{broken phase},v\neq0} \xrightarrow{\text{SU}(3)_c \times U(1)_{\text{em}}} SU(3)_c \times U(1)_{\text{em}}$$

where $U(1)_{\text{em}}$ is what is left of the electroweak gauge symmetry. This is the symmetry connected with electrodynamics and the conservation of electric charge. The specifics of how the electroweak symmetry is broken relates the electric charge to the weak isospin and hypercharge mentioned above.

1.2 Shortcomings of the Standard Model

Despite its tremendous success in describing practically all known phenomena involving the elementary particles and their interactions there are several reasons why the SM cannot be the final theory describing Nature. These issues are today instrumental in providing hints for a new theory beyond the SM and pushing the theoretical as well as experimental frontier.
There are several observations indicating that there are pieces missing in the SM. The two perhaps most evident ones are the lack of neutrino masses and of a description of what the dark matter in the Universe consists of.

In this thesis it is the lack of a dark matter candidate that is the main focus. It was realised in the second half of the twentieth century that the only dark matter candidate in the SM, the neutrino, was too light to be able to constitute dark matter, and ever since, particle physicists have been trying to find the identity of the dark matter particle. We discuss dark matter further in Ch. 2, 3 and 4 and present some of the most studied dark matter candidates so far in the context of their roles in solving other problems of the SM. Among these are WIMPs (Weakly Interacting Massive Particle), sterile neutrinos and axions.

1.2.1 Fine-tuning problems

One of the main drivers of ideas to extend the SM has been the hierarchy problem, originating from the fact that the Higgs field is quadratically sensitive to the energy scales of new physics. Due to this large sensitivity to higher scales, the mass parameter for the Higgs boson must be severely fine-tuned in order to produce the measured value [9].

The sensitivity to higher scales appears in the renormalisation of the Higgs mass parameter $\mu^2$ of the Higgs potential. The potential is given by

$$ V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda |\Phi^\dagger \Phi|^2, \quad (1.3) $$

where $\Phi$ is the Higgs field and $\lambda$ is the self-coupling parameter and $\mu^2 > 0$ is crucial in order to induce electroweak symmetry breaking. In the broken phase the Higgs field has a vacuum expectation value $v \neq 0$, and all fermions as well as the massive weak gauge bosons have masses proportional to $v$. Together with measurements of the weak coupling this determines the value of $v$ to be $v = 246 \text{ GeV}$. The Higgs sector parameters are related as

$$ v^2 = \mu^2 / \lambda. \quad (1.4) $$
1.2. Shortcomings of the Standard Model

When \( \mu^2 \) is renormalised, the higher order corrections beyond the leading order needs to be taken into account. The renormalised value is then

\[
\mu^2 = \mu_0^2 + \delta \mu^2. \tag{1.5}
\]

where \( \mu_0^2 \) is the bare, unrenormalised parameter, and the corrections obey

\[
\delta \mu^2 \propto \Lambda^2 \tag{1.6}
\]

for some scale \( \Lambda \) of new physics. The scale \( \Lambda \) represents some general cutoff of the theory, a scale where new physics must be accounted for, for example due to the appearance of new particles (\( \Lambda \) could e.g. be the mass of such a particle). The Higgs mass is

\[
m_H = \sqrt{2\lambda v} = 125 \text{ GeV}. \tag{1.7}
\]

which implies \( \lambda \sim 0.13 \) and \( \mu \sim 88 \text{ GeV} \). The point is now that this is the measured value and includes the corrections to all orders, notably the quadratic shift presented above in Eq. (1.5). But if we imagine the SM to be valid up to high energies, \( \Lambda \) is large and we then need to very finely tune the bare parameter in the Lagrangian in order to cancel out the large \( \Lambda^2 \)-term and end up with exactly the value of \( \mu^2 \) that is measured. The hierarchy problem is the problem of why, assuming that the SM is valid up to some high energy scale, the Higgs mass does not have a value which is much higher and of the order of the cutoff.

The hierarchy problem is thus rooted in the large sensitivity of the scalar Higgs field to higher scales. If there would be no higher energy scale where new physics come in, there would be no hierarchy problem. But there are good reasons to believe that there must be new scales where new phenomena appear, not least around the Planck scale where quantum gravity becomes relevant. To avoid fine-tuning of the Higgs mass, this new scale should then not be too large.

In the context of renormalisation group running, the fine-tuning problem for the Higgs mass can be understood as the fact that the Higgs mass at low scales is extremely sensitive to the value at high
scales. Any small perturbation can result in a vastly different mass at the electroweak scale.

Solutions to the hierarchy problem often introduce new physics at a much lower scale \( \Lambda \), to reduce the effect of the \( \Lambda^2 \)-sensitivity. One example of relevance for this thesis is supersymmetry, where one introduces supersymmetric partners to all the SM fields [10]. When including these superpartners in the loop corrections to the Higgs mass parameter, the contribution from the superpartners act to cancel out the corrections with the SM fields. The cancellation is better the lower the scale is of the superpartners’ masses. On top of solving or at least alleviating the hierarchy problem, supersymmetric theories also introduce dark matter candidates, such as the neutralino. We discuss supersymmetric dark matter in more detail in Ch. 3 and 6.

Other solutions to the hierarchy problem effectively lower the scale of new physics by considering the Higgs as a composite particle, with \( \Lambda \) representing roughly the scale where the interior can be probed [9]. Further solutions include the introduction of extra spatial dimensions [11, 12], or changing the cosmological history of the Higgs mass parameter [13] in order to explain its present-day value.

Another example of a fine-tuning problem in the SM is the so-called strong CP-problem. This is the question of why QCD does not seem to violate charge conjugation-parity (CP) symmetry. A CP-violating operator involving gluon fields, dubbed the \( \theta \)-term, since the associated coupling is usually denoted \( \theta \), can be added to the QCD Lagrangian and with \( \theta \neq 0 \), the strong sector would violate CP symmetry.

The fine-tuning appears most clearly when one considers the fact that the effective value of \( \theta \) receives another contribution from the quark mass terms in the electroweak sector, given by \( \arg \det \{ \mathcal{M} \} \) where \( \mathcal{M} \) is the quark mass matrix (the contribution is a part of the rotation necessary to make the mass matrix real and diagonal). The overall parameter can then be written

\[
\bar{\theta} = \theta + \arg \det \{ \mathcal{M} \},
\]

where the first term comes from the gluonic operator and the second term is the electroweak contribution. Somehow these two contributions (coming from two separate parts of the SM—the electroweak
and the strong sector respectively) must cancel to very high precision in order to end up with the very small value $\bar{\theta} \lesssim 1 \times 10^{-10}$ as required by the lack of observations of CP-violation in the strong sector [14].

The most popular attempt at solving this problem involves treating the $\theta$ parameter as a field that naturally relaxes towards zero in the so-called Peccei-Quinn mechanism [15, 16]. This mechanism also introduces a new field called the axion, which can act as dark matter [17, 18].

### 1.2.2 Neutrino masses

In the SM only left-handed neutrinos are included, which results in the Higgs mechanism being unable to provide mass to the neutrinos. The lack of neutrino masses is however known to be an incomplete description—many years of experimental results indicate that neutrinos oscillate between flavours [19, 20], something which is only possible if at least two of the neutrino flavours have mass.

One can extend the Higgs mechanism by introducing a right-handed neutrino field to give also neutrinos mass in a manner analogous to how the rest of the SM fermions get their masses by Yukawa couplings to the Higgs field. However, the introduction of the right-handed neutrino introduces a new problem related to the symmetry of lepton number conservation. In the SM, there is no requirement of lepton number conservation, but it still emerges as an accidental symmetry since no term violates it and has not been observed to be violated.\(^3\) The right-handed neutrino field necessary for neutrino mass generation in the Higgs mechanism is completely neutral under the SM gauge interactions—it does not interact electroweakly since it does not carry electroweak isospin or hypercharge and not through QCD since it is colour neutral. Due to this, one can write down another type of mass term called a Majorana mass term for the right-handed neutrino field, a term which however violates lepton number conservation. Thus, if neutrinos get mass through the Higgs mechanism, one needs to explain the small impact of such a

---

\(^3\)To be precise, it is the quantity $B - L$ for baryon number $B$ and lepton number $L$ which is conserved, due to some subtle non-perturbative effects in the non-abelian part of the SM.
term, and why lepton number is fully or approximately conserved.

Another issue with generating neutrino masses through the Higgs mechanism is the hierarchy of the necessary Yukawa couplings. The neutrino masses are so small that the neutrino Yukawa couplings have to be many orders of magnitude smaller than the rest of the SM Yukawa couplings. The large hierarchy between the largest and smallest Yukawa couplings in the rest of the SM is already difficult to understand—the ratio between the top quark mass and the electron mass is $O(10^6)$—and with neutrino Yukawa couplings included, the ratio between the largest and smallest Yukawa couplings becomes at least six orders of magnitude larger.

The so-called see-saw mechanism attempts to explain both the smallness of the neutrino masses and the apparent absence of lepton number violation by making the right-handed mass term very large (typically at least ten orders of magnitude larger than the electroweak scale) and thus mostly inaccessible at the energy scales of current experiments. In this mechanism, the masses of the standard neutrinos are suppressed by this large mass term which explains their smallness. One ends up with three very light neutrinos that are primarily left-handed and identified with the three known types of neutrinos, and three heavy, primarily right-handed neutrinos called sterile neutrinos.

The sterile neutrinos can also, apart from giving rise to neutrino masses, act as dark matter, since they are primarily right-handed and therefore interact very feebly with the SM [21].

### 1.2.3 Other problems

Apart from the issues mentioned above, there are other things the SM can not explain. One is the asymmetry between matter and antimatter, which in our Universe is much larger than that predicted in the SM. Another is the lack of a quantum description of gravity, which should at some point become relevant. These are both connected to the very early Universe where it is in many cases unknown how to observe and verify theoretical hypotheses. There has been a lot of research dedicated to these questions but we will not discuss them further in this thesis.
Chapter 2

The dark matter in our Universe

In this thesis the focus is on investigation of particle physics candidates for dark matter. In this chapter we review the history of dark matter discovery and the current status of dark matter searches.

Astronomers have throughout history studied the astronomical objects around us by pointing their telescopes at the sky and looking at the light emitted from these objects. In this way most of the information we have about the Universe comes from the emitted photons that can be collected in telescopes. During the 20th century, the realisation that a large fraction of the mass in the Universe does not emit or absorb light has grown from early observations by astronomers like Fritz Zwicky [22] and Knut Lundmark\(^1\) [23] in 1933 and 1930 respectively into a cornerstone of the cosmological standard model of the Universe. Although the first observations are nearly a century old today, it was not until the 1970s that the problem of dark matter was taken more seriously and it is unlikely that the early observers believed they had observed the effects of a new form of matter. The problem of *missing mass*, i.e. the fact that more matter than that observed was needed to explain the dynamics of stars and galaxies, had until then mostly been left out of focus by the scientific community, or thought to be solved by unseen ordinary matter. With new measurements by Rubin et al. of the orbital velocities of stars in

\(^1\)Lundmarks early discovery of the need for dark matter have largely gone unnoticed, possibly because Lundmark seemed to focus more on other topics of research.
galaxies that could not be explained by the visible mass alone [24], a novel interest in cosmological arguments that required more matter to be present than that visible in order to have a flat universe [25], and the understanding that no known form of matter could make up the dark matter, the general claim of large amounts of a new form of unseen mass being present in the Universe started to gain a foothold in the scientific community.

Today a multitude of observations at different cosmological scales all support the existence of dark matter, a new form of non-luminous matter that must be non-baryonic (i.e. not consisting of atoms) and cannot be explained by any known type of particle. Together with the observation that the expansion of the Universe is accelerating, which can be explained by dark energy, an energy density associated with the vacuum, the resulting perspective is one where we are currently not able to explain a vast majority of the contents of the Universe—a paradigm shift in fundamental science and an entirely new view of the Universe (see Fig. 2.1). A historical account of the development of the dark matter problem can be found in [26] and in particular the important progress during the 1970s in [25].

Figure 2.1 – The shares of the total energy density that the different matter-energy components in the Universe make up, as measured by the Planck collaboration [27]. The values are obtained in an analysis of the fluctuations in the spectrum of the Cosmic Microwave Background radiation. Image credit: ESA/Planck
2.1 Observations of dark matter

Today, there are observations supporting the existence of dark matter on scales ranging from the smallest galaxies to the very largest scales in the Universe. Some prominent examples are:

- Observations of fluctuations in the Cosmic Microwave Background [27, 28].
- The growth of structure in the early Universe [29–33].
- Dynamical observations of the motion of stars and galaxies [22, 24, 34–37].
- Gravitational lensing measurements [38–42].
- Big Bang nucleosynthesis [43].

We now go through these in more detail.

2.1.1 Fluctuations in the Cosmic Microwave Background

The Cosmic Microwave Background (CMB) is a form of relic radiation emitted in the early Universe, when the temperature had become low enough that electrons could bind to protons and form neutral hydrogen without being immediately ionised so that photons could stream freely (referred to as the time of recombination). The CMB is an almost perfect blackbody\(^2\), but there are fluctuations on small scales, coming from pressure waves in the early Universe plasma that were “frozen in” in the CMB. The fluctuations in the CMB are thought to be originating from the fluctuations of a quantum field responsible for inflation and are of great importance since they are the initial gravity wells around which matter in the Universe later grew and formed the structures we see today. The fluctuations tell us about the matter and energy density in the early universe and have by today been very well characterised by a succession of telescopes, with Planck being the most prominent one in recent years. The sizes of the fluctuations give a very precise measurement of the

\(^2\)In fact, probably the most perfect blackbody we have observed in the universe.
amount of dark matter in the Universe. The fraction of the Universe consisting of dark matter is according to Planck measurements \cite{8}^3

$$\Omega_{DM} h^2 = 0.1200 \pm 0.0012$$  \hspace{1cm} (2.1)

where $\Omega_{DM}$ is the fraction of the matter-energy in the Universe that is dark matter under the assumption of a flat Universe, and $h$ is the Hubble constant divided by 100, we have $h^2 \approx 0.5$. Measurements indicate that the Universe is to very high precision flat—this means then that about 25% of the total matter-energy density in the Universe is made up of dark matter. This is today the most precise measurement we have of the amount of dark matter in the Universe.

### 2.1.2 The growth of structure and the cosmic evolution

In the early universe plasma, the pressure of the photons kept the baryonic matter from becoming too dense but for dark matter this was not the case, due to the lack of or low degree of interactions with photons. After the time of recombination when the CMB was emitted this prohibition of density growth for baryonic matter was no longer in place and it started to fall into the already present overdense regions dominated by dark matter. Eventually this would result in the formation of stars and subsequently galaxies. Without the presence of these dark matter dominated regions in the early universe, the structures would start growing at a much later stage, in disagreement with observations of the large-scale structure of the Universe \cite{44}. The dark matter in the Universe also affected the continued evolution of galaxies and clusters. Extensive simulations of the cosmological evolution of the large scale structure of the Universe must include dark matter in order to be in agreement with observations such as presented in Refs. \cite{45, 46}.\footnote{In fact most simulations have mostly been performed with only dark matter, and only in recent years simulations including baryonic matter has been included.} Simulations range from early groundbreaking results \cite{30–32, 47} to modern state-of-the-art simulations like the \textit{Millennium} simulation \cite{33, 44}.

\footnote{There are a few different analyses of the Planck data with slight differences in the best-fit values of the cosmological parameters, here we quote the value in Eq. (23) of Ref. \cite{8}.}
2.1.3 Dynamical observations of stars and galaxies

Observations show that the orbital velocities of stars around their respective galactic centers are not decreasing with distance from the center in the way one would expect if only the visible matter was responsible for the gravitational attraction. The same principle applies to galaxies in galaxy clusters. Instead the velocities remain more or less constant with the distance from the galactic center. These types of dynamical observations were first reported already in the first half of the 20th century, notably by Fritz Zwicky regarding galactic velocity dispersions in the Coma galaxy cluster, measured to be larger than expected from the amount of visible matter [22]. In the 1970s the dark matter problem began to be taken more seriously with the arrival of more observational data supporting the existence of dark matter. Studies of stellar orbital velocities by Rubin and Ford [24] showed that the orbital velocities of stars in spiral galaxies were too high to be explained by the visible matter. Later these observations were extended the the outermost parts of galaxies, beyond the edge of visible matter, notably in studies by Einasto et al. and Ostriker and Peebles [34, 35] as well as others [36, 48]. Ostriker and Peebles had also shown that galactic disks seemed unstable without being surrounded by a massive halo of (dark) matter [49]. A recent compilation of velocity data in the Milky Way can be found in [37].

Observations of the dynamics of stars close to us in the Milky Way also seem to indicate the existence of dark matter on small scales, although the observations are not as clear as for the rotational velocities. Early efforts by Kapteyn [50] and Oort [51] pioneered this field that today is driven by observations from the GAIA telescope [52]. A review of the topic can be found in [53].

2.1.4 Gravitational lensing measurements

The light from a distant object is bent when it passes a massive object, resulting in a distorted image or duplicate images of the object. The effect is called gravitational lensing and can be used to measure the mass and mass distribution of the lens (the massive object in between us and the source that is in charge of the bending of the light) since the amount of lensing depends on the amount of mass in the lens and the way it is distributed. Since the lensing is sensitive to
the total and not just the visible mass, this can be used to infer the
dark matter distribution in the lensing objects by comparing to the
distribution of visible mass. In this way observations have shown
that clusters of galaxies are embedded in a more or less spherical
halo of dark matter with a mass density that falls of with the dis-
tance from the center of the cluster [38]. Two prominent examples
that show the presence of dark matter are the Bullet cluster [40–42]
and the cluster MACS J0025.4-1222 [39], both the result of collisions
between galaxy clusters. The observations show an offset between
the distributions of light-emitting baryonic matter and gravitating
matter in these clusters, indicating the presence of dark matter since
the baryonic matter will experience friction in the collision while the
dark matter will pass through unaffected.

2.1.5 Big Bang nucleosynthesis

When the early Universe had cooled sufficiently, neutrons and pro-
tons were able to form the first light nuclei in a process called Big
Bang nucleosynthesis (BBN). Neutrons are unstable unless they are
bound into nuclei, with a lifetime of about fifteen minutes. Further-
more the BBN process in the early Universe was slowed down by the
dilution of the proton and neutron densities due to the expansion
of the Universe. For these reasons BBN could only occur during a
very specific time interval. The proton and neutron densities are de-
termined by the amount of baryonic matter in the Universe whereas
the expansion of the Universe is determined by the total matter and
radiation density, and the relationship between the expansion and
the baryon density determines how much of each light element is
formed in the BBN process. Comparisons with observations of the
abundances of light elements in the Universe result in about 4% of
the matter-energy consisting of baryonic matter and that therefore
the rest of the matter in the Universe must consist of dark matter
[43].

2.2 Particle dark matter

In the SM, only the neutrino can act as a dark matter particle since
it is the only particle that is neutral and stable and, as we know
now, has a mass. However, during the 1970s it was realised that
SM neutrinos could not be the dark matter in the Universe when
simulations showed that neutrino dark matter would give a large
scale structure of the Universe in disagreement with observations
[47]. The problem was that in the allowed mass range for neutrinos,
they will be relativistic when they decouple from the plasma in the
early Universe and hence constitute hot dark matter, but hot dark
matter erases the smaller structures in the simulations which differs
from what is observed.

With neutrinos not able to account for the dark matter in the
Universe, one needs to look beyond the SM to find plausible candi-
dates. Today a class of particles called WIMPs (Weakly Interacting
Massive Particles) has emerged as one of the primary and most well
studied candidates [54, 55]. A WIMP is a particle that is massive
and neutral and interacts weakly with the SM. They have masses of
the order of the electroweak scale (around 100 GeV) but in principle
masses can take values in the range 1 GeV to $10^5$ GeV. We introduce
WIMPs in more detail in Ch. 3.

Other particle dark matter candidates that are not WIMPs are
for example axions, which is a very light scalar particle that appears
in what is the primary attempt at solving the strong CP problem,
and sterile neutrinos, that are a form of neutrinos with almost no
coupling to the SM at all which are introduced in mechanisms that
explain the masses of neutrinos.

2.3 Alternatives to dark matter

Since all we know about dark matter today stems from its gravi-
tational interactions it is natural to ask the question: could we get
away from postulating a new form of non-baryonic matter by instead
modifying the laws of gravity at galactic and larger scales? The idea
behind the framework of Modified Newtonian Dynamics (MOND)
[56] is to answer this question in the affirmative and postulate that
there is no dark matter but that instead the gravitational interac-
tions are different at large distance scales. This typically works well
to explain rotational velocities for stars in galaxies [57], but often has
problems when the scale is changed to that of e.g. galaxy clusters or
the CMB. Models that work well on galactic scales can not explain
the spectrum of the CMB, which includes vastly different physical mechanisms at very different cosmological scales. Often some form of dark matter is then required, conflicting with the original motivation of MOND.
Chapter 3

Dark matter candidates from particle physics

The idea that dark matter consists of a new form of particle grew successively from the 1970s when one by one, the main alternatives were disfavoured by experiments and observations, while at the same time various dark matter candidates appeared in attempts at solving problems with the SM. For example, supersymmetry gave rise to neutralinos, the Peccei-Quinn solution to the strong CP problem introduced axions and neutrino masses could be explained by introducing sterile neutrinos. There are some criteria that any candidate for a dark matter particle must fulfil. It must have survived from the early Universe until today and therefore be either completely stable or have a life-time on the order of the age of the Universe or longer. It can not be electrically charged or would otherwise not be dark, being able to emit or absorb photons. It can also not carry colour charge. In fact, if it would carry electrical or colour charge, it would condense with nuclei to form exotic isotopes in conflict with observations (see Searches for WIMPs and other particles in Ref. [7]).

In the SM only one particle fulfils all these criteria—the neutrino, which is completely neutral under electromagnetism and colour and only interacts through neutral current interactions in the SM. However, observations and numerical simulations show that dark matter should be cold (non-relativistic) in the early Universe for structures to form correctly, and since the SM neutrino is too light to constitute cold dark matter it can only at most contribute with a small, hot, component to the total dark matter abundance. Without the SM
neutrino as a viable candidate we are thus left with no other choice than to look beyond the SM for candidates for cold dark matter.

In this chapter we go through a few of the candidates for dark matter particles, focusing on WIMPs. We begin with a general historical introduction on WIMPs, inspired by Ref. [58], then give an overview of the MSSM and neutralinos, and end with a short discussion of the idea of dark matter models with long-lived mediators.

\section*{3. The WIMP scenario}

The study of the role of weakly interacting particles in cosmology and the evolution of the Universe started with neutrinos [59, 60]. In a series of papers in 1977 it was realised that a relic density of weakly interacting particles with masses above a few GeV could be produced thermally in the early Universe and survive until today [61–65]. Interest in these weakly interacting massive particles, so called WIMPs\footnote{An abbreviation which is said to originally come from Ref. [66].}, then started to gain foothold in the community.

The first studies were not motivated by the dark matter problem and in most cases made no attempt to connect the relic density calculations to the astrophysical observations of “missing mass” in galaxies and galaxy clusters. However, during the end of the 1970s and the 1980s this connection would grow sufficiently that the necessity of a new cold dark matter particle was widely accepted by the end of the 1980s [58].

If it were not for the small mass, the SM neutrino would be a good DM candidate since it is first of all known to exist and second of all is both electrically and colour neutral. However, during the 1980s, simulations of the large scale structure of the Universe showed that dark matter should be non-relativistic—cold—at the time when the first structures started to form [47]. Hot dark matter would wash out smaller structures and result in an evolution in conflict with observations, notably the CfA survey [67] which was the first three-dimensional survey of the large scale structure of the Universe. Since the SM neutrinos were constrained to be very light, they would necessarily be highly relativistic during structure formation and act as hot dark matter [68, 69]. The SM neutrino was then essentially
ruled out as a dark matter candidate.

In the typical mechanism for producing the right cold dark matter abundance, where a dark matter relic abundance is formed when dark matter particles go out of chemical equilibrium as their annihilation rate falls below the expansion rate of the Universe (see Ch. 4 for details on this calculation), a cold dark matter relic can not be too light to match the observed relic abundance. From above, the mass is bounded by unitarity constraints: requiring a unitary cross section while also producing the right dark matter abundance sets an upper bound of about 100 TeV for the WIMP mass [70]. Furthermore, the cross section implied by the matching to observations should be roughly of the same order as that of a weak interaction process. Combining these properties, one finds that a new particle interacting weakly\(^2\) with a mass in roughly the range 1 GeV to \(10^5\) GeV would be naturally produced with the right abundance in the early Universe. This, together with the fact that completely independent arguments from particle physics (such as for example attempts at solving the hierarchy problem) implies new physics at the electroweak scale, has led to the emergence of the WIMP as the most popular dark matter candidate.

### 3.1.1 Detection strategies

Due to the assumed thermal production mechanism, the hypothesis that the dark matter in our Universe consists of WIMPs is testable. In the standard thermal production mechanism one assumes that in the early Universe before decoupling, WIMPs are in equilibrium with the SM through processes of the type \(\chi \chi \leftrightarrow XY\), where \(\chi\) representing the WIMP and \(X, Y\) are SM particles. The implication of this is that WIMPs interact roughly weakly with the SM and by considering the various ways in which WIMPs can interact with the SM, one arrives at the three main strategies for detecting WIMPs:

- **Direct detection**, where one searches for the recoils of WIMPs scattering on nuclei in sensitive detectors.

\(^2\)Whether a WIMP is required to interact with the SM through specifically the electroweak interaction or whether some other interaction of similar strength is allowed is sometimes a more loose requirement than thermal production and the mentioned mass range.
3. Dark matter candidates from particle physics

Figure 3.1 – An illustration of the three main ways of searching for particle dark matter, with the blob representing the interaction that connects dark matter and the SM and the arrow direction indicating the time direction. In collider searches, one collides SM particles in the hope of producing dark matter particles, in direct detection one tries to detect the recoils of dark matter particles scattering on the nuclei in the detector and in indirect detection one searches with telescopes in astrophysical environments for signs of dark matter particles annihilating into SM particles.

- **Indirect detection**, where one searches for the annihilation products of WIMPs in astrophysical environments of high dark matter density.

- **Collider searches**, where one attempts to create WIMPs directly in collisions between SM particles.

**Direct detection**

The idea of using sensitive detectors to search for WIMPs recoiling against the detector material was developed in a paper by Goodman and Witten [71], building on previous ideas on neutrino detectors [72].

In Ref. [71], a separation was made, as is still customary, into the two cases of spin-independent and spin-dependent WIMP scattering.
on nuclei. With heavy nuclei with high mass numbers $A$, one can get a significant enhancement in spin-independent scattering due to the fact that the WIMP scatters on the nucleus as a whole, enhancing the cross section by a factor $A^2$ compared to the scattering on a single nucleon. Therefore, by choosing nuclei with large $A$, direct detection experiments are usually most sensitive to spin-independent scattering.

There are many experiments searching for WIMP recoils and with no convincing detection so far, limits on the WIMP scattering cross section are rapidly becoming more and more constraining. Over the last 20 years, the constraints on the spin-independent cross section have improved roughly five orders of magnitude for a WIMP mass of 50 GeV (see e.g. Fig. 1 of Ref. [73]). Currently the XENON1T experiment has published the strongest limits on the spin-independent cross section [74], with the minimum of a spin-independent cross section of $4.1 \times 10^{-47} \text{cm}^2$ excluded for a WIMP of 30 GeV at 90% confidence level.

Figure 3.2 – A simplified sketch, showing the idea of annual modulation. The modulation of the WIMP scattering signal is expected due to the motion of the Earth around the Sun. The WIMP wind is caused by the motion of the solar system in the galactic dark matter halo, resulting in WIMPs impinging on the Earth with a velocity of about $230 \text{km/s}$. The velocity of the Earth around the Sun either adds or subtracts to this depending on the time of the year, resulting in an oscillation in the velocity of WIMPs hitting the Earth, and therefore an annually modulated signal in direct detection since the rate of scattering depends on the relative velocity between the WIMP and the nucleon in the detector. The rate is maximised in June.
Another form of direct detection is to search for the annual modulation of the WIMP-nucleon scattering rate, expected due to the fact the solar system is not at rest with respect to the galactic dark matter halo combined with the revolution of the Earth around the Sun [75] (see Fig. 3.2 for an illustration). The movement of the solar system through the galactic dark matter halo results in a constant “wind” of dark matter particles impinging on the Earth. The rotation of the Earth around the Sun in turn means that the Earth is approximately moving towards the dark matter wind one half of the year and away from it the other half. The result is that the WIMP velocity at a point on Earth oscillates around the mean value, given by the velocity of the solar system around the galactic centre. Since the rate of WIMP scattering in a detector depends on the velocity of the WIMPs, the result of this is that the WIMP scattering rate will oscillate over the course of one year. The DAMA/LIBRA experiment has for some time reported a clear signal of annual modulation [76]. However, the implied WIMP scattering cross section and mass has by now been excluded by many other direct detection experiments, and it remains unclear how to properly interpret the DAMA/LIBRA signal.

**Indirect detection**

Of particular interest in this thesis is indirect detection, where one surveys the sky in search of dark matter annihilation products. The concept of indirect detection comes from the fact that with thermally produced dark matter, the annihilations into SM particles that keep the dark matter in equilibrium in the early Universe should give rise to an annihilation signal today. This signal will in general be expected from the galactic dark matter halo but will be stronger in regions where the dark matter density is large, such as the centres of galaxies. In indirect detection, one therefore attempts to detect an excess of SM particles over the expected background of non-dark matter origin. In Fig. 3.3 we show an illustration of some of the possible ways to search for dark matter annihilations from within our galaxy.

The idea of looking for gamma rays from the annihilations of dark matter particles was first studied in more detail in Refs. [77, 78]. This
was later extended to also other final state particles than gamma rays, such as antiprotons and positrons [79–82].

Indirect searches are often complicated by the presence of backgrounds from ordinary astrophysical processes that are difficult to estimate. One particularly interesting case is therefore the case of dwarf spheroidal galaxies, satellite galaxies to the Milky Way that are expected to have very little such backgrounds and therefore can be more constraining for the dark matter induced flux. A recent analysis of dwarf spheroidal galaxies using a combination of data from the Fermi-LAT and MAGIC experiments indeed presents some of the toughest indirect detection WIMP constraints, especially for lower WIMP masses below about 100 GeV [83].

The expected flux of SM particles in indirect detection starts with the production flux at the source, which is usually estimated for a generic WIMP with the use of Monte Carlo event generators. One simulates the annihilation of WIMPs into some final state and collects all the SM particles of the type one is interested in. In Paper III we have studied the impact on the production flux by using different event generators, as well as the impact that the polarisation of the final state in the annihilation can have. Since particle physics processes such as WIMP annihilations are in general complicated, especially in the case of coloured final states, event generators must rely on phenomenological modelling for some parts of the process. By comparing the fluxes obtained with different event generators that use different modelling one can get an estimate of the uncertainty stemming from this. Polarisation of the final state in the annihilation is expected in some concrete particle physics realisations of dark matter models. Since the final state polarisation in general changes the energy distribution of the secondary particles of interest, it is interesting to study what impact the polarisation can have in indirect searches.

The flux expected in an Earth-based telescope is different from the flux at the source, since propagation of the particles from the source to the Earth in general affects the particle fluxes. In the case of charged particles, the galactic magnetic field diffuses the source flux (which also means that charged particles do not point back to the source). Neutrinos produced in the halo will oscillate on the way from production to the Earth but are otherwise essentially unchanged by
the propagation, except at very high energies. The gamma ray signal is not affected much by the propagation but interactions cause some changes to the flux especially at high energies.

One form of indirect detection which is especially important in this thesis is the search for annihilation products from the Sun [84–86]. When the Sun travels through the galactic halo of WIMPs, some of these WIMPs scatter on the nuclei in the Sun. They lose some energy in the process and can become gravitationally bound to the Sun, eventually creating an overdensity of WIMPs in the solar core. This overdensity leads to the expectation of an annihilation signal from the centre of the Sun, which is usually only consisting of neutrinos, since all other particles are inevitably absorbed in the solar interior. The flux of neutrinos produced in the Sun is affected by oscillations but also by the interaction of neutrinos with the dense solar interior and is essentially completely absorbed when the neutrino energy is above 1 TeV. We discuss the case of WIMP annihilations in the Sun in more detail in Ch. 5. As of this writing, the best limits on WIMP annihilations in the Sun for WIMP masses above \( \sim 50 \text{ GeV} \) come from the IceCube experiment [87]. For lower masses the SuperKamiokande experiment sets more stringent limits [88].

We have in Paper I investigated an astrophysical background to the neutrino flux from WIMP annihilations, where neutrinos are produced in cosmic ray interactions on the solar surface. This flux is analogous to the neutrino flux formed by cosmic ray interactions in the Earth’s atmosphere, but differs from the Earth atmospheric flux since the environment in which the cosmic rays interact differs, and also due to the neutrino oscillations and interactions on their way from the Sun to the Earth.

In some models, the dark matter particle is \textit{secluded} from the SM, in the sense that it does not annihilate directly into SM particles, but instead to some new type of mediator particle, which subsequently can decay into SM particles. In the case that this mediator is relatively long-lived, the annihilation signal from the Sun can be vastly different, since the SM particles come from decays of the long-lived mediators away from the solar centre, and are hence less impacted by interactions with the solar material. In Paper II we have looked at the expected signal from such annihilations into long-lived mediators that decay into SM particles. We discuss such
models more in Sec. 3.3 below, and the implications for searches for WIMP annihilations in the Sun is discussed further in Ch 5.

**Figure 3.3** – A simplified illustration of various strategies for searches for dark matter annihilations within our galaxy and from galaxy clusters. The annihilation products indicated are gamma rays ($\gamma$) and neutrinos ($\nu$) from dwarf spheroidal galaxies, the galactic centre (GC) and galaxy clusters (where dwarf spheroidals are an attractive target due to small backgrounds) and charged cosmic rays such as positrons ($e^+$) and antiprotons ($\bar{p}$), whose charge cause their trajectories to be bent by the galactic magnetic field.

**Collider searches**

In dark matter collider searches, one uses particle accelerators to accelerate SM particles to high energies and collide them, hoping to see signs of dark matter in the collisions [89]. Currently, the highest energy collider is the Large Hadron Collider (LHC) at CERN, which collides protons with a centre-of-mass energy of 13 TeV.
hope is that in these proton collisions, signs of new physics, such as a WIMP, can be found. By virtue of its neutrality, a dark matter particle would not interact much in the detector and would therefore result in a signal of “missing energy”, meaning that not all energy can be accounted for when trying to reconstruct the collision. This way of searching for missing energy in collisions has been interpreted in roughly three classes of models at the LHC: complete models, in principle fully consistent, such as the MSSM; effective field theories, where new, high mass, particles are not included but the effect of them is encoded in effective low-energy constants; and simplified models, which are somewhat of a compromise between the former two, not complete particle physics models but also not effective models since they include some new particles.

Effective field theories were popular in the early days of the LHC, but have since been largely replaced by simplified model searches since there is a fundamental problem with the effective field theories in the context of the LHC WIMP searches: their validity is not always guaranteed. The effective field theories are effective models and are not valid unless the new physics appears at a high energy scale much larger than the scale probed, but if one searches for new particles at the electroweak scale new physics is expected at the probing scale, and not at a much higher energy scale [90, 91].

An important aspect of WIMP searches is complementarity. It will not be enough to find a WIMP signal in one type of search and not in others to be sure that we have found WIMP dark matter. For example, if a new WIMP-like particle would be discovered at the LHC, there is still no guarantee that this particle constitutes the astrophysical dark matter, since we can not know from collider results alone that the particle is stable on cosmological scales.

### 3.2 Supersymmetric dark matter

Of all the WIMPs studied, perhaps the most archetypical and well-studied example is the supersymmetric neutralino. During the course of the 1980s and 1990s this particle became one of the primary candidates among dark matter particles. With no signs of supersymmetry (SUSY) observed at the LHC so far, interest is starting to shift towards other candidates, but there are still large parts of the SUSY
parameter space which are not covered by the LHC searches and weak-scale SUSY remains a viable (although perhaps slightly more contrived than before) extension of the SM. In this section, we give an introduction to supersymmetric theories and the neutralino as a dark matter particle. For more details, we refer to Refs. [10, 92].

### 3.2.1 Supersymmetry

SUSY is a symmetry that relates particles with the same gauge charges but different spins: for every field there is a corresponding SUSY partner with the same gauge charges but with a spin differing by $1/2$.\(^3\) Since spin is determined by the behaviour of a particle under Poincaré symmetry, SUSY therefore introduces a connection between space-time symmetry and an internal symmetry, gauge symmetry. This extension of the Poincaré symmetry provided by SUSY is in fact the only known possible such extension. If SUSY is exact, the superpartners to the SM particles that we know of should have the same mass and since we have not found any such superpartners yet, SUSY must be broken at some energy scale.

Apart from the symmetry argument mentioned in the previous paragraph, there are several arguments motivating SUSY. One is the alleviation of the hierarchy problem in the SM. The hierarchy problem is the problem of why the Higgs mass seems to be so finely tuned in the SM: higher order corrections to the Higgs mass squared depend quadratically on the scale on new physics and so must be cancelled very precisely by the bare Higgs mass in order to end up with a physical mass at the electroweak scale, if the SM is supposed to extend to high energies. In SUSY, contributions from new particles stabilise the Higgs mass by cancelling the quadratic divergence. In exact SUSY, the cancellation is exact whereas in broken SUSY, terms of order of the SUSY breaking scale still remain.

A second motivation for SUSY is the possibility that dark matter candidates exist among the superpartners in the particle spectrum. This usually requires the assumption of a symmetry called $R$-parity, where $R = (-1)^{2s + 3B + L}$ for spin $s$, baryon number $B$ and lepton number $L$ so that $R = +1$ for all SM particles and $R = -1$.

---

\(^3\)This is strictly true for so called $\mathcal{N} = 1$ SUSY theories, which is all we will consider in this thesis. For higher $\mathcal{N}$ SUSY theories, more SUSY partners exist.
for all superpartners. With $R$ conserved the lightest SUSY particle (LSP) becomes stable and can potentially constitute the dark matter. $R$-parity violating operators can also induce proton decay which gives another motivation for its conservation.

A third motivation is the observation that SUSY can result in unification of the SM gauge couplings as they are run up to some high energy scale, in a way similar to how the electromagnetic and weak forces unify at the electroweak scale. This is due to the fact that one must include the superpartners in the diagrams that determine how the couplings run which can modify the running in such a way as to obtain unification.

3.2.2 The Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) [93] is, as the name implies, the most minimal supersymmetric model one can construct that contains all the fields of the SM along with a set of terms that parametrise the necessary breaking of SUSY. The building blocks of the MSSM Lagrangian are chiral and gauge supermultiplets, each containing two superpartner fields. A chiral supermultiplet consists of a Weyl fermion and a complex scalar field and a gauge supermultiplet consists of a gauge and fermion field. In this way, the SM fermions are associated with scalar superpartners called sfermions (with one scalar field for each fermion chirality) and the SM gauge bosons with fermionic superpartners called gauginos. In the MSSM one must include two Higgs doublets in order to avoid so called triangle anomalies, which are diagrams that can spoil the gauge invariance of the theory. Two Higgs doublets are also needed to give mass to up- and downtype particles. After electroweak symmetry breaking, one then ends up with a set of three neutral Higgs bosons, one of which is identified with the SM Higgs boson, and two charged Higgs bosons. The particle content of the MSSM is displayed in Tab. 3.1.

3.2.3 Dark matter candidates

The superpartners of the neutral gauge bosons and the Higgs bosons (the $\tilde{B}$, $\tilde{W}^3$, $\tilde{H}_1$ and $\tilde{H}_2$) will in general mix with each other. This results in four uncharged mass eigenstates $\tilde{\chi}_i^0$, $i = 1, 2, 3, 4$, called
Table 3.1 – The particle content of the MSSM. The first lines are the SM fermions and their sfermion superpartners (where $q_u = u, c, t$, $q_d = d, s, b$, $\ell = e, \mu, \tau$). Note that the sfermions are not chiral fields, the subscript $L/R$ merely denotes which SM Weyl field it is the superpartner of. We have not included a right-handed neutrino here. The line after the fermions shows the two Higgs doublets and their Higgsino superpartners and the last lines show the gauge bosons with their gaugino superpartners (where $W^a$ are the $SU(2)_L$ gauge bosons, $B$ the $U(1)_Y$ gauge boson and the $G^a$ the gluons). The mass eigenstates after electroweak symmetry breaking of the gauge and Higgs superpartners are linear combinations of the gauginos and Higgsinos. The gluino is denoted $\tilde{g}$ to avoid confusion with the gravitino (the superpartner of the graviton in SUSY models involving gravity) which is often denoted $\tilde{G}$.

 neutralinos. In the event that the lightest of the neutralinos is the LSP and this is stabilised through $R$-parity conservation, it is an excellent dark matter candidate and can be produced with the right abundance in the early Universe by the standard freeze-out mechanism. This has prompted a vast number of studies on the phenomenology of neutralinos and methods of detecting them. Some of the important early studies are Refs. [94–98].

Apart from the neutralino, there are other potential dark matter candidates in the MSSM particle spectrum. Of the particles in Tab. 3.1 also the lightest sneutrino, if it is the LSP, will be able to
act as cold dark matter. However, the sneutrino coupling to the SM is rather large which results in strong limits from direct detection experiments that essentially rule out sneutrino dark matter [99]. For the neutralino the situation is different, as the neutralino coupling to the SM contains several mixing parameters that can act to suppress the coupling.

In more general supersymmetric theories where one includes gravity and/or axionic particles, the gravitino and axino (SUSY partners of the graviton and axion respectively) are dark matter candidates. These are both however extremely weakly coupled to the SM and would be very hard to detect in a laboratory experiment. Thus, from a phenomenological point of view the neutralino is more interesting as the prospects for its detection are larger. We will not pursue the topic of gravitino or axino dark matter further in this thesis.

\subsection{The MSSM Lagrangian}

The MSSM Lagrangian can be written in terms of four parts as

\[ \mathcal{L}_{\text{MSSM}} = \mathcal{L}_W + \mathcal{L}_{gauge} + \mathcal{L}_{\text{chiral kin}} + \mathcal{L}_{\text{soft}} \]  

where the first three terms respect SUSY while \( \mathcal{L}_{\text{soft}} \) contains the terms that break SUSY, which is necessary since we know that SUSY can not be exact. \( \mathcal{L}_W \) contains interactions stemming from the superpotential \( W \) and is obtained by taking field derivatives of \( W \) (given below). Kinetic terms for fermions, scalars and gauge fields are contained in \( \mathcal{L}_{\text{kin}} \) and \( \mathcal{L}_{\text{gauge}} \). Apart from the interactions in \( \mathcal{L}_W \) there are also gauge interactions coming from \( \mathcal{L}_{\text{gauge}} \). The superpotential is given by, following the notation of Ref. [100],

\[ W = \epsilon_{ij} \left[ -\hat{e}_R^* Y_E \hat{\tilde{e}}_L H_1^j - \hat{d}_R^* Y_D \hat{\tilde{d}}_L H_1^j + \hat{\tilde{u}}_R^* Y_U \hat{\tilde{q}}_L H_2^j - \mu H_1^j H_2^j \right] \]  

where the notation is such that \( \hat{e}_R, \hat{\tilde{e}}_L, \hat{d}_R, \hat{\tilde{d}}_R, \hat{\tilde{q}}_L, \hat{\tilde{u}}_L \) are superfields—fields that contain both the fermionic SM fields and their scalar superpartner fields. The \( \epsilon_{ij} \) is the antisymmetric tensor, the \( H_{1,2}^j \) are Higgs doublets and the \( Y_{E,D,U} \) are \( 3 \times 3 \)-matrices in flavour space.
3.2. Supersymmetric dark matter

containing the Yukawa couplings of the leptons and quarks for all three generations. In this sense the first line in the superpotential represents a "supersymmetrisation" of the SM Yukawa interactions as it couples the fermions and the sfermions to the Higgs doublets. The second line introduces an additional term, a term coupling the two Higgs doublets with a mass parameter $\mu$. The $\mu$ parameter is in an unbroken MSSM the only new parameter introduced and all other MSSM parameters come from the SUSY breaking terms given below.

To get $L_W$, the contribution to $L_{\text{MSSM}}$ from the superpotential, one takes derivatives of the superpotential with respect to the fields. This gives $L_W$ as

$$L_W = -\frac{1}{2} \left[ W^{ij} \psi_i \psi_j + W^{*}_{ij} (\psi^i)^\dagger (\psi^j)^\dagger \right] - W^i W^*_i$$

(3.3)

where $\psi_i$ are fermionic fields and

$$W^i = \frac{\partial W}{\partial \phi_i}, \quad W^*_i = \frac{\partial W}{\partial \phi^*_i}, \quad W^{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}.$$ (3.4)

The remaining parts of the unbroken part of $L_{\text{MSSM}}$ are the chiral kinetic term and the gauge term. As it is not relevant in the scope of this thesis to go through the details of these terms, we will not write them down here.

Breaking SUSY is done by introducing terms such as mass terms for the superpartners in a careful way that does not cause quadratic divergences in the mass corrections and spoil the resolution of the hierarchy problem. This is called a soft breaking of SUSY and $L_{\text{soft}}$ is often written as

$$L_{\text{soft}} = -\frac{1}{2} \left[ M_1 \tilde{B} \tilde{B} + M_2 \left( \tilde{W}^3 \tilde{W}^3 + 2 \tilde{W}^+ \tilde{W}^- \right) + M_3 \tilde{g} \tilde{g} \right] - m_1^2 H_1^{*} H_1 - m_2^2 H_2^{*} H_2 - \left( \epsilon_{ij} B \mu H_1^i H_2^j + \text{h.c.} \right) - \tilde{q}_i M_2^i \tilde{q}_L - \tilde{u}_R M_2^i \tilde{d}_R - \tilde{d}_R M_2^i \tilde{u}_R - \tilde{e}_R M_2^i \tilde{e}_R - \epsilon_{ij} \left[ \tilde{e}_R A_E Y_E \tilde{L}^i H_1^i + \tilde{d}_R A_D Y_D \tilde{q}_L^i H_1^i - \tilde{u}_R A_U Y_U \tilde{q}_L^i H_2^i \right]$$

(3.5)

where we have disregarded some terms that violate CP explicitly. Here, the symbols in bold are 3-component vectors and $3 \times 3$-matrices.
in generation space and the fields with a tilde represent the sfermion parts of the superfields. The matrices $A_{E,D,U}$ give rise to trilinear couplings between the Higgs fields and the sfermions. There is also a bilinear coupling involving the Higgs fields given by the parameter $B$. We have introduced soft mass terms for the gauginos with masses $M_1$, $M_2$ and $M_3$ (for bino, wino and gluino respectively); for the Higgs fields with masses $m_1^2$ and $m_2^2$ and for the sfermions with masses given by the matrices $M_{Q}^2$, $M_{L}^2$, $M_{U}^2$, $M_{D}^2$ and $M_{E}^2$ (for the scalar superpartners of the left-handed quark doublet, the left-handed lepton doublet, the right-handed up-type quarks, the right-handed down-type quarks and the right-handed charged leptons respectively).

With $\mathcal{L}_{\text{soft}}$ included, we have accounted for SUSY breaking but at the price of introducing a very large number of new parameters. Counting all the parameters in the mass matrices and couplings that occur in $\mathcal{L}_{\text{soft}}$ results in the contribution of about 120 new parameters in the MSSM. In order to be able to make some form of predictions one typically makes theoretically motivated assumptions that reduce this number. Many of these parameters give rise to flavour-changing neutral currents (FCNCs) or large CP violation and most approaches attempt to suppress these terms, or sets them to zero by hand.

### 3.2.5 Electroweak symmetry breaking and gaugino sector

With electroweak symmetry breaking, the Higgs doublets obtain vacuum expectation values $v_1$ and $v_2$. The SM-like neutral, CP-even Higgs boson $h$ should have a vacuum expectation value $v = 246$ GeV which is given in terms of $v_1$ and $v_2$ as $v = \sqrt{v_1^2 + v_2^2}$. The ratio between $v_1$ and $v_2$ defines an angle $\beta$, given from

$$\tan \beta = \frac{v_2}{v_1}$$

The Yukawa masses of SM fermions and masses of the gauge bosons are then given in terms of $v$ and $\beta$ instead of $v_1$ and $v_2$.

The masses of neutral and charged gauginos and Higgsinos receive contributions from the soft breaking Lagrangian and the gauge part. The result is two non-diagonal mass matrices, one for the neutral and one for the charged states. Diagonalising these matrices to find the mass eigenstates results in four physical states that are mixtures
between the neutral gauginos and Higgsinos and two states that are mixtures between the charged gauginos and Higgsinos. The former are, as already mentioned, called neutralinos and are denoted \( \chi^0_i, \ i = 1, 2, 3, 4 \), while the latter are called charginos and are denoted \( \chi^\pm_i, \ i = 1, 2 \). The interplay between the lightest chargino and the lightest neutralino(s) is often important in dark matter phenomenology.

The neutralino mass terms are

\[
\mathcal{L}_{\chi^0, \text{mass}} = -\frac{1}{2}(N^0)^T M_0 N^0 + \text{h.c.} \tag{3.7}
\]

where the four-dimensional column vector \( N^0 \) is \( N^0 = (\tilde{B}, \tilde{W}^3, \tilde{H}_1, \tilde{H}_2) \). The masses of the neutralinos are then obtained by diagonalising the mass matrix as written in the above gaugino and Higgsino basis. The gaugino and Higgsino fields receive mass contributions from \( \mathcal{L}_{\text{soft}} \) and \( \mathcal{L}_{\text{gauge}} \), resulting in a mass matrix that is

\[
M_0 = \begin{pmatrix}
M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\
0 & M_2 & -m_Z c_W c_\beta & -m_Z c_W s_\beta \\
-m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\
m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0
\end{pmatrix} \tag{3.8}
\]

where we have introduced the abbreviations \( s_W = \sin \theta_W \) where \( \theta_W \) is the weak mixing angle and \( s_\beta = \sin \beta \) where \( \beta \) is the mixing angle between the Higgs doublets (with analogous definitions \( c_W = \cos \theta_W \) and \( c_\beta = \cos \beta \)). This is diagonalised by a unitary complex matrix \( Z \), leading to the four physical mass eigenstates called the neutralinos, given by

\[
\chi^0_i = Z_i1 \tilde{B} + Z_i2 \tilde{W}^3 + Z_i3 \tilde{H}_1 + Z_i4 \tilde{H}_2 \tag{3.9}
\]

where the \( Z_{ij} \) are elements of the matrix \( Z \), describing the relative contribution of the \( (\tilde{B}, \tilde{W}^3, \tilde{H}_1, \tilde{H}_2) \) fields to each of the neutralinos. The mass eigenvalues are in general rather complicated expressions.

The chargino mass terms can be written before diagonalisation in the basis \( N^\pm = (\tilde{W}^+, \tilde{H}_2^+, \tilde{W}^-, \tilde{H}_1^-) \) as

\[
\mathcal{L}_{\chi^\pm} = -\frac{1}{2}(N^\pm)^T M_\pm N^\pm + \text{h.c.} \tag{3.10}
\]
where the mass matrix is written as
\[
M_\pm = \begin{pmatrix} 0 & \mathcal{X} \\ \mathcal{X}^T & 0 \end{pmatrix}, \quad \mathcal{X} = \begin{pmatrix} M_2 \\ \sqrt{2} s_\beta m_W \\ \mu \end{pmatrix} \quad (3.11)
\]
The physical states are then obtained by writing,
\[
\chi^-_i = U_{i1} \tilde{W}^- + U_{i2} \tilde{H}_1^- \\
\chi^+_i = V_{i1} \tilde{W}^+ + V_{i2} \tilde{H}_2^+ \quad (3.12)
\]
for unitary $2 \times 2$-matrices $U$ and $V$ with elements $U_{ij}$ and $V_{ij}$ respectively, where we require
\[
U^* \mathcal{X} V^\dagger = \text{diag} \left( m_{\chi_1^\pm}, m_{\chi_2^\pm} \right). \quad (3.14)
\]
Since we now are dealing with $2 \times 2$-matrices, the expressions for the mass eigenvalues become rather compact, and they are given by
\[
m_{\chi_1^\pm}, m_{\chi_2^\pm} = \frac{1}{2} \left[ M_2^2 + |\mu|^2 + 2m_W^2 \\
\pm \left( M_2^2 + |\mu|^2 + 2m_W^2 \right)^2 - 4|\mu|M_2 - m_W^2 \sin 2\beta \right]^{1/2} \quad (3.15)
\]
where the negative (positive) sign is for the first (second) of the masses.

### 3.2.6 Ultraviolet and phenomenological models

SUSY is expected to be broken at the electroweak scale but restored at some high scale. In any ultraviolet complete SUSY model, one must specify how this breaking occurs. Often SUSY is broken in some external sector and then mediated to the MSSM. There are many explored alternatives for this such as gauge-, gravity- or anomaly-mediated SUSY breaking. In these ultraviolet models, one ends up with theoretically well motivated assumptions that reduce the dimensionality of the MSSM parameter space, and hopefully gets rid of problematic FCNC and CP violating terms. One common relation which arises from requirements of coupling unification at a high
energy scale relates the sizes of the gaugino mass parameters,

\[ M_1 = \frac{5}{3} \tan^2 \theta_W M_2 \approx 0.5M_2 \quad (3.16) \]

\[ M_2 = \frac{\alpha_{EW}}{\alpha_s \sin^2 \theta_W} M_3 \approx 0.3M_3 \quad (3.17) \]

which means that one can obtain all three \( M_1 \), \( M_2 \) and \( M_3 \) by just specifying one of them, e.g. \( M_2 \).

A historically popular example of an gravity-mediated SUSY breaking model is the constrained MSSM (CMSSM), also called minimal supergravity (mSUGRA). In the CMSSM, the number of parameters is severely reduced by assuming universality of parameters at the unification scale \( \mu_U \), specifically all scalar particles have a common mass \( m_0(\mu_U) \) and gauginos a common mass \( m_{1/2}(\mu_U) \) (so that the CMSSM contains the above condition on the gaugino masses). Furthermore, the trilinear couplings are assumed universal and equal to \( A_0(\mu_U) \) and gauge couplings assumed to unify. Requiring also that one obtains electroweak symmetry breaking in the usual way results in the reduction of all the parameters in \( L_{\text{soft}} \) down to only five:

\[ \tan \beta, \ m_0, \ m_{1/2}, \ A_0, \ \text{sign}(\mu), \]  

(3.18)

where the last parameter is a discrete parameter which gives the sign of \( \mu \). The mass parameters are then run down to the electroweak scale with the renormalisation group equations. The CMSSM is then highly predictive as the entire MSSM spectrum is obtained from just five parameters.

Another approach than the above is to remain agnostic about the specifics of SUSY breaking at the high scale and make phenomenologically motivated assumptions for the parameter values at the electroweak scale, ending up with a phenomenological MSSM (pMSSM). The hope is then that this will catch the leading characteristics of the phenomenology of the model. The number of parameters one uses is generally appended, so that pMSSM7 is a seven parameter version of the MSSM.\(^4\) Common assumptions are for example diagonal mass matrices and trilinear couplings for the sfermions, sometimes using a

\(^4\) Note that in a pMSSM model one must specify which parameters are used and which assumptions are made, as it is not clear which parameters are used just by specifying the number of them.
non-zero value for only one of the three generations. This will get rid of most or all of the problematic FCNC and CP violating terms. Another often used assumption is the unification relation in Eqs. (3.16) and (3.17) above, which relates the $M_1$, $M_2$ and $M_3$ parameters to each other.

While these approaches can reduce the dimensionality of the parameter space significantly, it can still be complicated to entangle the dependence of physical observables on the different MSSM parameters and to see the effect of experimental constraints. Therefore one usually resorts to the use of computer packages for this. Such computer packages calculate one or many observables and compare to experimental input, and can sometimes be used for scanning over the parameter space. Such scans result in a (multidimensional) volume of the parameter space being excluded by experimental observations, with some parameter values preferred, and are called *global fits*. One global fitting package of note is *GAMBIT* [101], a package that scans over the parameter space of a particle physics model, such as the MSSM, with a high-level statistical treatment. It includes the possibility to call various other codes that are tailored for calculations of specific physical observables and then compares these calculations to experimental input.

Another computer package of note in this thesis is *DarkSUSY* [102], which calculates a number of dark matter observables for a given particle physics model. It was originally developed to calculate supersymmetric dark matter observables (hence the name) but can as of the latest version handle any particle physics model. In Ch. 6 we use *DarkSUSY* together with the package *DM@NLO* [103] to compute the relic density in the MSSM. *DM@NLO* provides the necessary MSSM (co)annihilation cross sections including higher order corrections in the strong coupling. By linking *DM@NLO* to the relic density calculator in *DarkSUSY*, we can thus see the impact of higher order corrections on the relic density obtained in a given pMSSM model realisation.

### 3.2.7 Dark matter phenomenology

Since we are in this thesis primarily interested in the MSSM due to the fact that it provides a dark matter candidate in form of the
lightest neutralino, we will here devote some special attention to the phenomenology of the lightest neutralino of relevance for dark matter considerations.

As an illustrative example, we can consider the case where the soft breaking masses $M_1$, $M_2$ and the Higgsino mass parameter $\mu$ are assumed to be much larger than $m_Z$. We can then to first approximation neglect the upper right and lower left off-diagonal parts of $\mathcal{M}_0$ in Eq. (3.8), resulting in\footnote{We stress that this example is for illustrative purposes only, all results presented in this thesis use the full mass matrices.}

$$\mathcal{M}_0 \approx \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & 0 & -\mu \\ 0 & 0 & -\mu & 0 \end{pmatrix}$$

Apart from the lower right part, this matrix is diagonal. It has two eigenstates given by $(1, 0, 0, 0)$ and $(0, 1, 0, 0)$—one pure bino and one pure wino state. Given that the eigenstates of a matrix

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

are $(\pm 1, 1)$ the two remaining eigenstates are an antisymmetric and a symmetric combination of the two Higgsinos.

To be able to order the resulting neutralino states in mass to find the lightest neutralino state which is our dark matter candidate, we need to know the relative sizes of the parameters $M_1$, $M_2$ and $\mu$ (which we have here assumed to all be much larger than $m_Z$). In the case of $M_1$ being the smallest parameter of the three, $\chi_1^0$ will then be approximately bino with a mass $M_1$. If we assume the unification conditions on the $M_i$, $M_2 \approx 2M_1$ and so in that case we can never have a pure wino as the lightest neutralino. If unification conditions are not assumed, a pure wino $\chi_1^0$ is obtained when $M_2$ is the smallest out of the three mass parameters. If $\mu$ has the smallest value, we will end up with two mass degenerate Higgsino states $\chi_1^0$ and $\chi_2^0$ with mass $|\mu|$. In this case, both the two lightest neutralino states will play an important role in obtaining the right dark matter relic abundance since coannihilations, which are processes of the type
\( \chi_1^0 \chi_2^0 \rightarrow XY \) where \( X, Y \) are SM particles, will be highly relevant in the calculation of the dark matter relic abundance \[104\]. We describe the importance of coannihilations further in Ch. 4, but in general they must be included for particles close in mass to the lightest neutralino when calculating the (co)annihilation cross section.

Historically, the first studies of neutralinos as dark matter, e.g. \[94, 95\] often looked at the particular mixing where \( \chi_1^0 = \cos \theta_W \tilde{B} + \sin \theta_W \tilde{W}_3 \). Since this is the just like how the SM gauge fields \( B \) and \( W_3 \) mix to give the photon, the lightest neutralino is in this case a \emph{photino}, the superpartner of a photon.

The neutralino and chargino sectors are intimately connected, since the physical states all depend on the soft breaking parameters \( \mu \) and \( M_2 \) (with \( M_1 \) only of importance for the neutralinos). In the same approximations as discussed in the preceding paragraphs, the chargino masses in Eq. (3.15) simplify considerably. To first order, there is then one eigenstate which is approximately a (charged) Higgsino with mass \( \mu \) and one state which is approximately a (charged) wino with mass \( M_2 \). Therefore, for example in the case where \( \mu \) is the smallest mass parameter so that the lightest two neutralinos are to first order pure Higgsinos with mass \( \mu \), also the lightest chargino will then approximately have mass \( \mu \), so that chargino-neutralino coannihilations become highly relevant.

Apart from the case with mass degeneracies, chargino-neutralino coannihilations can become important when neutralino couplings are suppressed. The relevant processes for two neutralinos annihilating into SM particles are \( s \)-channel \( Z^0 \) and Higgs exchange and \( t \)-channel sfermion exchange. For a pure Higgsino, the coupling between two \( \chi_1^0 \) and a \( Z^0 \) or a Higgs boson both vanish at leading order. This also happens for a pure bino or wino. In this case, the coannihilations with the lightest chargino and the next-to-lightest neutralino can become very relevant in and even dominate the (co)annihilation cross section that gives the neutralino relic density.

Another important type of coannihilations are sfermion-neutralino coannihilations, relevant if there is one or several sfermions with mass close to that of the lightest neutralino. Here, the connection is not as direct since the masses of sfermions and neutralinos are not as directly connected as the masses of charginos and neutralinos.
3.3 Dark matter models with long-lived mediators

An alternative to the standard WIMP scenario are *secluded* models, where the dark matter particle $\chi$ resides in a dark sector that only interacts with the SM through the mediation of some new mediator particle $V$, where $V$ can have a life-time that is macroscopically large \cite{105}. In Fig. 3.4 we show a schematic view of such a model. We denote here the coupling between the mediator and the dark sector $\kappa$, and the coupling between the mediator and the SM $\epsilon$. In particular, a small $\epsilon$ then results in the seclusion of the dark sector from the SM as well as a long mediator life-time, as the life-time typically scales in inverse powers of $\epsilon$.

![Diagram](image_url)

**Figure 3.4** – A schematic view of a portal model, where the dark sector (DM) couples to the SM only through the exchange of some mediator particle. The mediator couples to the dark sector with coupling $\kappa$ and to the SM with coupling $\epsilon$. With a small $\epsilon$ one obtains a secluded dark matter model, where elastic WIMP-nucleon scattering is suppressed by $\epsilon$.

One feature that secluded models exploit is the fact that a thermal production mechanism constrains the dark matter annihilation cross section, but not the dark matter-nucleon scattering cross section. Assuming $m_\chi > m_V$ to ensure that $\chi\chi \rightarrow VV$ annihilations are kinematically allowed, secluded dark matter particles can be produced thermally in the standard freeze-out mechanism by weak scale $\chi\chi \rightarrow VV$ annihilations, thus fixing the coupling $\kappa$ to weak scale values. The coupling relevant for dark matter-nucleon scattering, $\epsilon$, is however not set by the production mechanism and can be suppressed in order to evade the strong direct detection constraints on the dark matter-nucleon scattering cross section. In Fig. 3.5 we show the rel-
evant diagrams, indicating where the couplings $\epsilon$ and $\kappa$ appear. In this way, indirect detection of the mediator decay products becomes the primary detection opportunity in secluded dark matter models.

![Diagram](image)

**Figure 3.5** – (a) The annihilation diagram responsible for dark matter production in the early universe (b) The scattering diagram relevant in direct detection ($q$ here denotes a quark from a nucleon)

In principle, the mediator life-time $\tau$ is a free parameter. One somewhat model-independent constraint on $\tau$ comes from Big Bang nucleosynthesis [106]. To ensure that mediator decays do not inject prohibitive amounts of energy into the Universe at this time, the mediators cannot have too large a life-time and should mainly decay before Big Bang nucleosynthesis. However, these constraints depend to a large extent on how big the abundance of the mediators is in the early Universe and what type of particles the mediator decays into. Other constraints come from high-luminosity fixed target collider experiments [107, 108], although this mainly constrains low masses.

One concrete example of a secluded model is the dark photon model [109–112]. In this model, dark matter particles couple to the SM through the exchange of so called dark photons, and the dark photon interaction with the SM is suppressed by a small coupling analogous to the $\epsilon$ introduced above. The dark photon consists of a new gauge boson state from the dark sector with a small admixture of the neutral SM gauge bosons which allows decays into SM pairs. Dark matter annihilations into dark photon pairs that subsequently
3.3. Dark matter models with long-lived mediators

decay into SM particles then lead to an annihilation signal which in the absence of observation has been used to put constraints on the dark photon model [113–118].

In more general terms, one can consider the case of portal models, which are simplified models (regarded as effective descriptions of full theories) where the dark sector couples to the SM through the mediation of some particle, dubbed a portal. The portal models are characterised by the spin of the mediator, resulting in vector, scalar or fermion portals. Thus, in this nomenclature, the dark photon model is a vector portal model. Numerous models with long-lived mediators and their astrophysical consequences have been studied in the context of portal models [105, 113, 116, 119–127]. One example of such a model other than the dark photon model mentioned above is the case of a pseudoscalar portal between the SM and the dark sector where SM and dark matter interactions are mediated by a pseudoscalar particle similar to an axion [105, 120, 127]. Another example is a vector portal model where one assumes a new $U(1)$ gauge symmetry that is spontaneously broken and the dark matter interactions with the SM are mediated by either the new gauge field or the new Higgs boson connected to the spontaneous $U(1)$ breaking [105, 119, 120].

In the simple model we used above, there are in principle only four relevant parameters for the new physics: the two masses $m_V$ and $m_\chi$, and the couplings $\epsilon$ and $\kappa$. This assumes that all dark sector physics is sufficiently decoupled at our scale that there are no other relevant couplings or masses that enter the phenomenology. In principle, one can then calculate all observables from these parameters, such as the mediator lifetime and the dark matter annihilation and scattering cross sections, where these will in general not be independent of each other. For example, the scattering cross section and the mediator life-time will both depend on $\epsilon$ so if one fixes $\kappa$ by requiring a thermal freeze-out production, either the life-time or the scattering cross section can be used to fix $\epsilon$.

In reality, a full model will likely include more parameters, and if different couplings dominate the annihilation and the scattering cross sections with the latter determined by a small coupling, the scattering cross section can be naturally suppressed. As an example we can consider a case where the coupling between the mediator and
the SM is of Yukawa type (we thus effectively have several different $\epsilon$ couplings). Annihilation can then occur primarily into heavy quarks whereas the scattering cross section can be suppressed since it mainly depends on the small coupling to the light quarks, which are most abundant in nucleons.

One can also take a more phenomenological approach, where one uses observables and calculated quantities as the parameters of the model rather than the fundamental couplings which give rise to them. We can then parametrise a model by the annihilation cross section, the dark matter-nucleon scattering cross section and the relevant masses. We can include the mediator decay length as an additional independent parameter, as long as we assume that there is enough freedom that it can be considered independent of the scattering and annihilation cross section (i.e. that there are at least three relevant fundamental couplings, rather than two as above). The coupling between the SM and the dark sector is thus in this approach parametrised by the dark matter annihilation cross section, the dark matter-nucleon scattering cross section and the mediator life-time. As a further simplification one can consider the case where the mediator decays with 100% branching into one specific SM final state. We end up with the following parameters:

- The dark matter-nucleon scattering cross section $\sigma_{\chi N}$
- The dark matter annihilation cross section $\sigma_{\text{ann}}$
- The mediator decay length at rest $L$
- The mass of the mediator $m_V$
- The dark matter mass $m_X$
- The SM decay channel of the mediator

In practice, we note that it is often not $L$ directly, but the boosted decay length $\gamma L$ that is the physically relevant quantity, since mediators from the dark matter annihilations are alway boosted with the same Lorentz factor $\gamma = m_{\chi}/m_V$. This is because the Lorentz factor is set by the kinematics in the process: for $\chi \chi \rightarrow VV$ with dark matter particles at rest, we will have for the Lorentz boost of
the mediator $\gamma = E_V/m_V = m_\chi/m_V$. We also note that the anni-
hilation cross section is often fixed by requiring thermal production. Constraints on the above parameters can then be translated into constraints in a full model [121].

Dark matter models with long-lived mediators lead to interesting phenomenology. Of particular importance in this thesis is the fact that the signal from WIMP annihilations in the Sun can change significantly, something which has been studied in Paper II and is also discussed further in Ch. 5.
Chapter 4

Dark matter production in the early Universe

A crucial part of a good dark matter model is to explain how the dark matter abundance in the Universe is produced. Since we have observations of dark matter at a vast range of scales in the evolution of the Universe, ranging from the CMB that probe dark matter in the early Universe to local kinematical measurements probing it today, the dark matter that we see on these various scales must be produced through some mechanism in the very early Universe, before the emission of the CMB. Furthermore, since these observations to high extent agree in their estimations of $\Omega_\chi h^2$ (that gives the fraction of dark matter in the Universe today), the number of dark matter particles must remain essentially constant since the time of the CMB (or equivalently, the dark matter density in a comoving volume should be approximately constant). In addition, observations tell us that even at the time of the CMB, dark matter is cold, or at most warm, meaning that it is non-relativistic or possibly slightly relativistic (otherwise it would not result in the large-scale structure of the Universe that we observe), and the production mechanism needs to take this into account.

Depending on the dark matter model, the mechanism to produce the right amount of cold dark matter may look very different. In this chapter we focus on a description of the chemical freeze-out mechanism, reviewing the treatment in Refs. [128, 129]. We end with some comments on alternative production mechanisms.
4.1 Overview and historical perspective

In the WIMP dark matter scenario, the present-day abundance of dark matter is typically produced in the early Universe through what is usually called the freeze-out mechanism. This mechanism is based on the fact that in the early Universe WIMPs would, through their weak interaction with the SM, be in thermal and chemical equilibrium with the SM particles, constantly annihilating and being produced in inelastic processes of the type $\chi \chi \leftrightarrow \text{SM}$, with the chemical equilibrium ensuring that annihilation and creation happen at equal rates. However, as the Universe expands and cools, eventually the interaction strength between WIMPs and the SM particles is not enough to sustain this chemical equilibrium and around the time when the expansion rate of the Universe becomes larger than the annihilation rate of WIMPs into SM particles—typically dubbed the time of (chemical) freeze-out—the number of WIMP particles becomes approximately constant so that the number density dilutes only due to the continued expansion of the Universe.

Elastic scattering on the SM background still continues after this point and due to this the WIMPs remain in thermal equilibrium with the SM background for some time after chemical freeze-out. The WIMPs stay in thermal equilibrium as long as the scattering rate is sufficiently large compared to the expansion of the Universe that the WIMPs' temperature distribution continues to reflect the temperature of the Universe. At some point in time this ceases to be true, resulting in the time of kinetic freeze-out.

The earliest studies of the freeze-out of massive particles in the early Universe can be found in Refs. [130, 131], however applied to other situations. In general, early studies of freeze-out in the early Universe did not concern WIMPs, since the idea of WIMPs accounting for the dark matter observed in e.g. the dynamics of galactic halos had not arisen yet. Instead, one often considered neutrinos, but with the motivation of investigating constraints on the neutrino mass from demanding that the neutrino abundance would not be large enough to result in an expansion rate of the Universe in conflict with observations. In Refs. [59, 60], thermodynamic arguments applied to the early Universe constrained the neutrino mass from above in this way, and there is even a mention in Ref. [60] that
massive neutrinos “may become very important in the discussion of the dynamics of clusters of galaxies and of the universe”—an early connection between the astrophysical observations of “missing mass” and ideas in particle physics.

A development of this idea came several years later, when it was realised that a heavier neutrino or some other type of massive weakly interacting particle (i.e. a WIMP) with mass above a few GeV could escape the bounds since it would be non-relativistic at freeze-out, invalidating constraints for a relativistic, light neutrino [61–65].\(^1\) The realisation that weakly interacting particles with masses above a few GeV were still cosmologically allowed (and could in fact be necessary) paved the way for the WIMP paradigm.

Today’s standard formalism where the relic abundance is obtained by solving the Boltzmann equation developed with the massive neutrino calculations in the late 1970s (although not always with the standard SM neutrino in mind) [61–65] and was further developed and refined during the 1980s and 1990s, but now applied to the case of weakly interacting dark matter particles [128, 129, 132–135]. Often these refinements were motivated by properties of supersymmetric models, although in many cases applying to most generic WIMP models. An example is the effect on the abundance that degeneracies among the lowest masses in the new physics sector have (such mass degeneracies are often expected in the MSSM), brought to general attention in e.g. Ref. [134].

Relic density calculations today are generally made with dedicated codes that solve the Boltzmann equation numerically for a given model, some examples of codes are DarkSUSY [102], MadDM [136–138] and MicrOMEGAs [139–143].

\(^1\)Actually, in the actual meaning of the word, the neutrino is a WIMP, since it is massive and weakly interacting. In this thesis however, we will not denote the SM neutrino as a WIMP and reserve that term for particles from beyond the SM.
4.2 The Boltzmann equation in the early Universe

In this section we introduce the Boltzmann equation, following the presentation in Refs. [129, 144].

The Boltzmann equation is a partial differential equation governing the time evolution of the phase space distribution \( f_i \) of some particle species \( \chi_i \). It can be written as

\[
\hat{L}[f_i] = \hat{C}[f_i] \tag{4.1}
\]

where the Liouville operator \( \hat{L} \) on the left-hand side represents the time evolution of \( f_i \) and the collision operator \( \hat{C} \) on the right-hand side describes the changes in \( f_i \) due to particle interactions and decays. In a homogeneous and isotropic Universe (as described by the Friedmann-Lemaître-Robertson-Walker metric), \( f_i \) is homogeneous and isotropic so that \( f_i = f_i(E, t) \) and the Liouville operator is in this case

\[
\hat{L}[f_i] = E \frac{\partial f_i}{\partial t} - \frac{\dot{a}}{a} |\vec{p}|^2 \frac{\partial f_i}{\partial E} \tag{4.2}
\]

where \( a(t) \) is the scale factor. We can rewrite the Boltzmann equation in terms of the number density \( n_i \) of \( \chi_i \), given by

\[
n_i(t) = g_i \int f_i(E_i, t) \frac{d^3p_i}{(2\pi)^3}. \tag{4.3}
\]

for \( g_i \) internal degrees of freedom, such as spin (for example a fermion has two spin degrees of freedom). Dividing both sides in the Boltzmann equation by \( E \) and integrating over momentum, we can then write the Boltzmann equation as

\[
\frac{dn_i}{dt} + 3 \frac{\dot{a}}{a} n_i = \frac{dn_i}{dt} + 3H n_i = g_i \int \hat{C}[f_i] \frac{d^3p_i}{E_i(2\pi)^3} \tag{4.4}
\]

where the second term on the left-hand side comes from integrating the \( \partial/\partial E \)-term in \( \hat{L}[f_i] \) by parts, using \( E \, dE = |\vec{p}| \, d|\vec{p}| \) and \( d^3p = 4\pi \, d|\vec{p}| \). We have also identified the Hubble parameter as \( H = \dot{a}/a \).

We can here note that the left-hand side can be written

\[
\frac{dn_i}{dt} + 3 \frac{\dot{a}}{a} n_i = \frac{1}{a^3 \, dt} \left( n_i a^3 \right) \tag{4.5}
\]
so that the left-hand side of the Boltzmann equation gives the time evolution of the number density in a comoving volume. This also shows that when the collision term is zero, the number density of $\chi_i$ is constant in a comoving volume and $n_i$ just dilutes with the expansion of the universe,

$$\frac{d}{dt}(n_i a^3) = 0 \quad \Rightarrow \quad n_i(t) = \text{const.} / a^3 \quad (4.6)$$

The right-hand side of Eq. (4.4) containing the collision term in general contains contributions from the various particle physics processes that affect the phase space densities, such as annihilation processes, elastic scattering and particle decays. It hence depends on the matrix elements of these processes, which one needs to obtain from the specific particle physics theory one is considering. An important aspect is that the collision term typically involves phase space distributions of all the particle species involved in a particular process. This results in a system of coupled Boltzmann equations for the phase space densities of all the particle species considered.

### 4.2.1 Chemical freeze-out of a single new particle

To gain some further understanding of the Boltzmann equation we will here present one of the simpler examples: the case of a single new, heavy particle $\chi$ in a thermal bath of SM particles. We assume that $\chi$ is equal to its antiparticle, that it is stable on the timescales involved (for example through some symmetry that stabilises the dark sector) and that it is massive enough that it can be considered non-relativistic at the temperatures we are considering. In this case, the processes of relevance in the collision term are the annihilation and creation processes $\chi \chi \leftrightarrow XY$, which in chemical equilibrium are equal in rate in both directions. As the temperature drops with the expansion of the Universe, at some point this is no longer true since the temperature will be too low for the process to proceed from right to left.

If we make the assumption that the temperature is $T \lesssim m_\chi$ and that the $\chi$ are non-relativistic so that the energies in the annihilation process are all of order $E \simeq m_\chi$, the Boltzmann equation for the
number density \( n_\chi \) can in this case be written as

\[
\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle \left[ n_\chi^2 - (n_\chi^{\text{eq}})^2 \right]
\]  

(4.7)

where the collision term has now been rewritten in terms of a thermally averaged annihilation cross section, given at temperature \( T \) by

\[
\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle = \frac{1}{(n_\chi^{\text{eq}})^2} \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p_2}{(2\pi)^3} \sigma_{\text{ann}}v_{\text{rel}} e^{-E_1/T} e^{-E_2/T}
\]  

(4.8)

where \( p_1 \) and \( p_2 \) are the momenta of the \( \chi\chi \) initial state (with energy \( E_i \) corresponding to the momentum \( p_i \) for \( i = 1, 2 \)), \( v_{\text{rel}} \) the relative velocity in the initial state and the distribution in equilibrium is

\[
n_\chi^{\text{eq}} = g_\chi \int \frac{d^3p}{(2\pi)^3} e^{-E/T}.
\]  

(4.9)

The annihilation cross section is given by

\[
\sigma_{\text{ann}} = \frac{1}{4E_1E_2v_{\text{rel}}} \int \frac{d^3p_1}{2E_1(2\pi)^3} \int \frac{d^3p_2}{2E_2(2\pi)^3} \times (2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4) \sum_{X,Y} |M_{\chi\chi \to XY}|^2
\]  

(4.10)

where the sum is over all possible final states, with final state momenta \( p_3 \) and \( p_4 \) and \( M_{\chi\chi \to XY} \) is the amplitude for the \( \chi\chi \to XY \) process.

This Boltzmann equation should now be solved to obtain the density \( n_\chi \) today for the annihilation cross section of the considered particle physics model. One can then compare and see whether the prediction matches observations. In this case of a single new particle in a thermal background of lighter particles, there is only Boltzmann equation that needs to be solved to determine the number density \( n_\chi \) after chemical freeze-out.

### 4.2.2 The Boltzmann equation for a class of new particles

With \( N \) new particle species present in the early universe alongside the SM there will be \( N \) Boltzmann equations, one for each particle
species, all coupled through the interactions between the $\chi_i$. We will in the following review the treatment in Ref. [128] of such a situation, presenting here a calculation of the dark matter relic abundance in a general way where a class of $N$ new particles $\chi_i$, $i = 1, 2, \ldots, N$, are present in the thermal bath of the SM particles. The relic abundance of DM in a given scenario is then obtained by solving a set of coupled Boltzmann equations, each tracking the number density of a particular particle species $\chi_i$ as a function of time.

We will in the following denote the masses of the new particles $m_i$ and order the particles in mass such that $m_1 \leq m_2 \leq \ldots \leq m_{N-1} \leq m_N$. Particles in the SM background are denoted $X, Y$. Specifically, we will denote the lightest of the new particles the Lightest New Particle (LNP), and in the end we want to find the present-day abundance of this particle, $n_1(\text{today})$. Similar to Sec. 4.2.1 above, we assume the temperatures to be $T \lesssim m_1$ and assume the $\chi_i$ to be non-relativistic.

When we consider all possible interactions of the $\chi_i$ with each other and the SM background as well as particle decays, the collision term of the Boltzmann equation becomes more involved than in the case of a single new particle. For the number density $n_i$ of particle species $i$, the Boltzmann equation is now given by

$$\frac{dn_i}{dt} + 3Hn_i = -\sum_{j=1}^{N} \langle \sigma_{ij}v_{ij} \rangle \left( n_i n_j - n_i^{eq} n_j^{eq} \right)$$

$$- \sum_{j \neq i}^{N} \left[ \langle \sigma'_{Xij}v_{ij} \rangle \left( n_i n_X - n_i^{eq} n_X^{eq} \right) \right.$$

$$\left. - \langle \sigma'_{Xji}v_{ij} \rangle \left( n_j n_X - n_j^{eq} n_X^{eq} \right) \right]$$

$$- \sum_{j \neq i}^{N} \left[ \Gamma_{ij} \left( n_i - n_i^{eq} \right) - \Gamma_{ji} \left( n_j - n_j^{eq} \right) \right] \quad (4.11)$$

Here, the left-hand side accounts for the evolution of $n_i$ in an expanding Universe, with the expansion governed by the (temperature-dependent) Hubble parameter $H$. The terms in the first sum on the right-hand side accounts for the change in $n_i$ due to annihilations of $\chi_i\chi_j$ into final states in the SM background for all $j$. It depends on
the thermally averaged annihilation cross section

\[
\langle \sigma_{ij} v_{ij} \rangle = \frac{\int d^3 p_i d^3 p_j \ e^{-E_i/T} e^{-E_j/T} \sigma_{ij} v_{ij}}{\int d^3 p_i d^3 p_j \ e^{-E_i/T} e^{-E_j/T}} \tag{4.12}
\]

where \( f_i \) and \( f_j \) are phase-space distributions functions of \( \chi_i \) and \( \chi_j \), the relative velocity is given by

\[
v_{ij} = \sqrt{(p_i \cdot p_j)^2 - m_i^2 m_j^2} \over E_i E_j \tag{4.13}
\]

and the annihilation cross section \( \sigma_{ij} \) is given by the cross sections of annihilations for the initial state \( \chi_i \chi_j \) summed over all final states \( XY \), \( \sigma_{ij} = \sum_{X,Y} \sigma(\chi_i \chi_j \rightarrow XY) \).

The terms in the second sum describe the conversion of \( \chi_i \) into \( \chi_j \) due to scattering on the thermal background \( X \). It depends on the the thermal average of the scattering cross sections \( \sigma'_{Xij} \), again summed over final states such that \( \sigma'_{Xij} = \sum_Y \sigma(\chi_i X \rightarrow \chi_j Y) \). The thermal averaging is performed in the same way as for \( \langle \sigma_{ij} v_{ij} \rangle \) above.

The terms in the last sum describe the loss and gain in the number density \( n_i \) due to decays \( \chi_i \rightarrow \chi_j X \) and \( \chi_j \rightarrow \chi_i X \), and this term therefore depends on the decay rates summed over final states, we have \( \Gamma_{ij} = \sum_X \Gamma(\chi_i \rightarrow \chi_j X) \) and similar for \( \Gamma_{ji} \).

Assuming that there is a symmetry that keeps the LNP stable, e.g. \( R \)-parity in the MSSM, all new particles will eventually decay to the LNP. This means that rather than solving for the LNP density \( n_1 \), we should in fact solve for the total number density \( n = \sum_i n_i \) since all densities \( n_i \) for \( i > 1 \) will eventually contribute to \( n_1 \) when the heavier particles decay (we assume here that their decay times are much smaller than the age of the Universe). Therefore it is \( n \) which in the end determines the present-day dark matter relic abundance. It can be shown [128] that the Boltzmann equation for \( n \), despite tracking all the interactions between the new particles among themselves and with the SM, is remarkably simple. The terms in the second and third sums of Eq. (4.11) cancel when taking the sum over \( i \), and we are left with

\[
\frac{dn}{dt} + 3Hn = -\langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2) \tag{4.14}
\]
4.2. The Boltzmann equation in the early Universe

where $\langle \sigma_{\text{eff}} v \rangle$ is the effective annihilation cross section, given by

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j} \langle \sigma_{ij} v_{ij} \rangle \frac{n_{eq}^i n_{eq}^j}{n_{eq}}$$

(4.15)

where the sum goes over all new particles and the relative velocity is given by Eq. (4.13).

There is an assumption made when going from the coupled Boltzmann equations for $n_i$ to the one for the sum $n$. To be able to write the Boltzmann equation for $n$ in the simple form of Eq. (4.14) one assumes that

$$\frac{n_i}{n} \approx \frac{n_{eq}^i}{n_{eq}}.$$  (4.16)

Since the equilibrium densities are known functions of temperature, this allows for the replacement of $n_i$ by a term proportional to $n$. The justification for this assumption comes from the fact that the density of the relativistic thermal background particles $X, Y$ is much higher than that of the $\chi_i$ since the non-relativistic $\chi_i$ are suppressed exponentially by the Maxwell-Boltzmann distribution. Therefore the scattering rate of $\chi_i$ particles on $X, Y$ particles is much higher than the annihilation rate of $\chi_i$ particles with each other, even if the scattering cross sections are of the same order as the annihilation cross sections. This effectively redistributes the densities $n_i$ so that the ratio $n_i/n \approx n_{eq}^i/n_{eq}$.

The most important processes contributing to $\langle \sigma_{\text{eff}} v \rangle$ are often assumed to be the LNP annihilation processes $\chi_1\chi_1 \rightarrow XY$. However, also coannihilation processes, which are processes of the type $\chi_1\chi_2 \rightarrow XY$ or $\chi_2\chi_2 \rightarrow XY$, contribute to $\langle \sigma_{\text{eff}} v \rangle$ and can be very important to determine the DM relic abundance [128, 129, 134]. Coannihilations are important when the the LNP has a mass close to other masses in the model, the coannihilation contribution depends approximately exponentially on the mass difference as we shall explain further below. Another situation where coannihilations are crucial to include is if the annihilation cross section for some reason is suppressed (due to e.g. low couplings or symmetries). In this case the coannihilation cross sections can greatly exceed the annihilation cross section and hence be of more importance in the DM abundance calculation.
As an example of a situation where coannihilations are important, consider a case where the mass difference between the LNP and the next-to-lightest of the $\chi_i$ particles is similar to or smaller than the temperature. Then $\chi_1$ is kinematically allowed to scatter on a background particle to form a $\chi_2$ in the final state even in cases where the final state mass is higher than the initial state mass. The $\chi_2$ can later decay or scatter and will in this way affect the density $n_1$. If the mass difference would be larger, the occurrence of such processes would be exponentially suppressed by the thermal distributions, but for such models with particles close in mass to the LNP, coannihilations can be very important, sometimes changing the DM relic density by orders of magnitude \cite{128, 134}.

The dependence on the particle physics model is now contained almost entirely in $\langle \sigma_{\text{eff}} v \rangle$. Some additional dependence appears in the expansion rate of the Universe given by $H(t)$ since this depends on the number of relativistic degrees of freedom at a given time.

\subsection*{4.2.3 Thermal averaging}

The thermal averaging in $\langle \sigma_{\text{eff}} v \rangle$ can be performed (a derivation can be found in Refs. \cite{128, 129}) with the result that $\langle \sigma_{\text{eff}} v \rangle$ can be expressed as a one-dimensional integration over momentum. In terms of the momentum parameter $p_{\text{eff}} = \sqrt{s - 4m_1^2}/2$ (the momentum in the centre-of-mass frame in the case of two annihilating $\chi_1$) it is given by

$$\langle \sigma_{\text{eff}} v \rangle = \frac{\int_0^\infty dp_{\text{eff}} p_{\text{eff}}^2 W_{\text{eff}}(p_{\text{eff}}) K_1(\sqrt{s}/T)}{m_1^4 T \left( \sum_i \frac{g_i}{g_1} \frac{m_i^2}{m_1^2} K_2(m_i/T) \right)^2}$$

(4.17)

where $K_k(x)$ is the modified Bessel function of the second kind of order $k$, $g_i$ the number of internal degrees of freedom of particle species $i$ and $W_{\text{eff}}$ a Lorentz-invariant effective annihilation rate, which con-

\footnote{Notably, such mass degeneracies are not unlikely in the MSSM. For example, in the case that the LNP is a pure higgsino with mass $m_\chi \gg m_Z$, it will to lowest order be mass degenerate with the lightest chargino and the next-to-lightest neutralino.}
4.2. The Boltzmann equation in the early Universe

It contains the relevant cross sections. It is defined as

\[ W_{\text{eff}} = \sum_{i,j} \frac{p_{ij}}{p_{11}} \frac{g_i g_j}{g_i^2} W_{ij} \quad (4.18) \]

where the individual annihilation rate \( W_{ij} \) for the initial state \( \chi_i \chi_j \) with energies \( E_i, E_j \) is related to the corresponding cross section, summed over final states, as

\[ W_{ij} = 4E_i E_j \sigma_{ij} v_{ij}. \quad (4.19) \]

Therefore, \( W_{\text{eff}} \) captures almost all the particle physics content of the freeze-out process, apart from that contained in the degrees of freedom parameter and the masses appearing in the denominator in Eq. (4.17). From a numerical point of view, a crucial aspect is that \( W_{\text{eff}} \) is independent of temperature. It can therefore be tabulated once for a given particle physics model, and does not have to be recalculated at each temperature step when solving the Boltzmann equation.

Plotting \( W_{\text{eff}} \) as function of momentum gives an indication of the processes that are most important in the considered particle physics model. Kinematical thresholds for final states or coannihilation processes appear at \( \sqrt{s} \) equal to the sum of the final state or coannihilating masses, and resonances of mass \( M_{\text{res}} \) appear at \( \sqrt{s} = M_{\text{res}} \).

In Fig. 4.1 we plot an example of \( W_{\text{eff}} \) from a supersymmetric model that is studied further in Ch. 6. In this model, the dark matter particle is a neutralino, denoted \( \chi^0_1 \), and there are several other particles with masses close to \( m_{\chi^0_1} \). The model is denoted a Higgs funnel model due to the important role that the Higgs resonances play in order to obtain the correct DM relic abundance.

We have in Fig. 4.1 indicated the \( p_{\text{eff}} \) values corresponding to the \( \sqrt{s} \) values where thresholds and resonances appear. We see that \( W_{\text{eff}} \) starts out at some value which is given by the effectiveness of \( \chi^0_1 \chi^0_1 \) annihilations. The first threshold appears at \( p_{\text{eff}} = 25 \text{ GeV} \) which translates to \( \sqrt{s} = m_{\chi^0_1} + m_{\chi^0_1} \), i.e. it is the threshold for coannihilations between the lightest neutralino and the lightest chargino. After the threshold, \( W_{\text{eff}} \) increases since there is now another process contributing. As \( p_{\text{eff}} \) increases further, more thresholds are passed so that more coannihilation processes contribute and \( W_{\text{eff}} \) continues.
to increase. At $p_{\text{eff}} = 116$ GeV the first resonance appears in $W_{\text{eff}}$, in fact corresponding to annihilations into both the pseudoscalar neutral Higgs boson $A^0$ with $m_{A^0} = 793$ GeV and the heavy scalar Higgs boson $H^0$ with mass $m_{H^0} = 794$ GeV. The resonance just above, at $p_{\text{eff}} = 126$ GeV, is the charged Higgs with $m_{H^\pm} = 799$ GeV.

In the end it is not $W_{\text{eff}}$, but $W_{\text{eff}}$ multiplied with the thermal factor $p_{\text{eff}}^2 K_1(\sqrt{s}/T)$ that is to be integrated to give the thermally averaged effective annihilation cross section. At low momentum, this thermal factor approaches zero. At high momentum, the Bessel function is exponentially damped. The integrand is therefore peaked at some temperature-dependent non-zero value of the momentum, meaning that the behaviour of $W_{\text{eff}}$ around this peak contributes most to $\langle \sigma_{\text{eff}} v \rangle$ and that the contributions of thresholds and resonances in $W_{\text{eff}}$ appearing at high momenta or momenta close to zero are heavily suppressed. This is the reason why coannihilations are most important for particles close in mass to the lightest state. The exponential damping of the Bessel function at high $p_{\text{eff}}$ means that the final contribution to $\langle \sigma_{\text{eff}} v \rangle$ due to a threshold at high $p_{\text{eff}}$ is
4.2. The Boltzmann equation in the early Universe

Figure 4.2 – The integrand in \( \langle \sigma_{\text{eff}} v \rangle \) at the time of freeze-out, including the thermal part that suppresses the high and low \( p \) part of \( W_{\text{eff}} \), normalised so that it integrates to one. The relic abundance is in this model largely obtained by resonant Higgs annihilation and chargino and neutralino coannihilations. The neutralino mass in this model is \( m_{\chi_1^0} = 379.25 \text{ GeV} \), which gives a freeze-out temperature \( T_f = 13.96 \text{ GeV} \).

4.2.4 Finding \( \Omega_{\chi} h^2 \)

To find the DM relic abundance \( \Omega_{\chi} h^2 \), given by

\[
\Omega_{\chi} h^2 = \frac{\rho_{\chi}(\text{today})}{\rho_{\text{crit}}} h^2 = \frac{m_1 n_1(\text{today})}{\rho_{\text{crit}}} h^2 \tag{4.20}
\]

where \( \rho_{\text{crit}} \) is the critical density resulting in a flat Universe, we need to solve the Boltzmann equation to find the number density \( n_1 \).
4. Dark matter production in the early Universe

evaluated today, with the initial condition that the number density starts out in equilibrium before freeze-out. This is today usually done with dedicated computer packages. One then ends up with a $\Omega_\chi h^2$ that is approximately inversely proportional to $\langle \sigma_{\text{eff}} v \rangle$, so that a higher $\langle \sigma_{\text{eff}} v \rangle$ results in a smaller DM relic abundance and vice versa. This is qualitatively reasonable: a high (co)annihilation cross section will mean that WIMP annihilations can compete with the Universe’s expansion longer, depleting the WIMP abundance and resulting in a smaller $\Omega_\chi h^2$.

4.2.5 Numerical packages

For a given particle physics model like the MSSM, the number of cross sections that goes into the calculation of $W_{\text{eff}}$ can become very large, especially when coannihilations are important. Furthermore there can be a large number of model parameters that go into the cross section expressions. Combined with the fact that some approximations need to be made in the calculations just from a computational perspective, e.g. restricting the number of coannihilation processes to the most important ones, the route from the input parameters of a given MSSM model to precise predictions of observables is often quite sophisticated.

For these reasons, a number of codes have been developed to calculate the DM relic density numerically, for example DarkSUSY [102], MicrOMEGAs [139–143] and MadDM [136–138] (the latter relying on the program MadGraph5_aMC@NLO [145], that generates cross sections for particle physics processes).

In the majority of cases, these computer packages perform the relic density calculations with tree level amplitudes for the calculation of $\langle \sigma_{\text{eff}} v \rangle$. Since tree level amplitudes are but the first term in the perturbative expansion of the cross section, they can only give an approximation of the true cross section. Since we now have a percent accuracy measurement of $\Omega_\chi h^2$ by the Planck collaboration [8], it is motivated to study what the impact is of using loop level cross sections in the $\langle \sigma_{\text{eff}} v \rangle$ calculation. The aim of the project described in Ch. 6 was to investigate this in the context of the MSSM and calculate the neutralino relic density using one of the Boltzmann solver codes available, DarkSUSY, interfaced to the code DM@NLO [103].
4.3. Alternatives to the freeze-out mechanism

DM@NLO provides the necessary (co)annihilation cross sections to loop level in the strong coupling constant, making it possible to study the effect of loop corrections on the obtained value for $\Omega h^2$.

4.3 Alternatives to the freeze-out mechanism

For other dark matter particle candidates, the mechanism for production can be very different from chemical freeze-out. Axions are produced through the misalignment mechanism and through the decay of cosmic strings. The former is related to the breaking of Peccei-Quinn symmetry in the early Universe and results in a relic abundance of cold axion dark matter. The contribution from the latter is more difficult to estimate. Sterile neutrinos can be produced in the early Universe through oscillations and their tiny mixing with the SM neutrinos. Very heavy dark matter particles can be produced through gravitational interactions in the very early Universe. Another idea is that dark matter is composed of extremely weakly interacting particles that come from the decay of WIMPs, where the latter can be produced in a standard chemical freeze-out process, so that the dark matter particles inherit the WIMP abundance. Since we in this thesis focus on the chemical freeze-out production mechanism for WIMPs, we will not elaborate further on these mechanisms and dark matter candidates and refer to Ref. [55] for details.
Chapter 5

Neutrinos from the Sun

The Sun is the source of most of the astrophysical neutrinos that hit the Earth. The majority of these neutrinos are lower energy neutrinos emitted in nuclear fusion process in the solar core. These solar neutrinos have been instrumental in showing that neutrinos oscillate between flavours. At higher energies, the contributions to the neutrino flux is instead expected to come from neutrinos created in cosmic ray interactions in the solar atmosphere and potentially from annihilations of dark matter particles in the core of the Sun. The former is described shortly in Sec. 5.2.3 below but is also the subject of Paper I [146]. The overall mechanism behind the latter is discussed in Sec. 5.2.1. Given that no detection has been made so far, neutrino telescopes such as the IceCube experiment at the South Pole has set limits on the cross section between dark matter particles and nuclei. This is discussed in 5.3.

5.1 The solar neutrino problem

Copious amounts of neutrinos are constantly emitted from the Sun. These are predominantly produced in nuclear fusion processes in the solar core. The flux resulting from these reactions can be calculated by taking into account all the various nuclear processes that produce neutrinos and the details of these processes. Following Ref. [147], a rough estimate can be obtained by assuming that all of the solar energy output comes from the common fusion reaction $p+p \rightarrow ^{2}\text{He}+$
$e^+ + \nu_e$, which results in a flux of about $10^{11} \text{cm}^{-2}\text{s}^{-1}$ at Earth\(^1\), meaning that a huge number of solar neutrinos pass through a human being every second. These neutrinos typically have energies in the MeV range.

Historically the measurement of this neutrino flux has been instrumental as evidence that neutrinos oscillate between flavours. The reason is that one expects only electron neutrinos to be produced in the nuclear reactions, but the measured flux of electron neutrinos is significantly lower than that expected from calculations of the fusion rate in the Sun. This apparent deficit of neutrinos compared to that expected from models of the Sun was called the *solar neutrino problem*. The solution to the solar neutrino problem with neutrino oscillations was successfully demonstrated by the Sudbury Neutrino Observatory experiment [20], which could show that the flux of muon and tau neutrinos was non-zero. Without oscillations one expects to see only electron neutrinos in the experiment, but on the way to the Earth oscillations result in non-zero fluxes of muon and tau neutrinos.

### 5.2 High energy neutrinos from the Sun

The solar neutrinos formed in the nuclear reactions have energies in the MeV range. At higher energies there are other processes that can contribute to the flux of neutrinos from the Sun. In this thesis, particular interest is awarded the high energy neutrinos that are formed in interactions of cosmic rays with the outer parts of the Sun, as well as the potential contribution from dark matter annihilations. Under the hypothesis that the dark matter in the Universe is constituted partly or completely by WIMPs (see Ch. 3), neutrinos can be emitted in the annihilations of WIMPs that have collected in the interior of the Sun.

\(^1\)Each $pp$ reaction releases about $10\text{MeV}$ of thermal energy. This gives a reaction rate of $L_\odot/10\text{MeV} \approx 10^{38}\text{s}^{-1}$, where $L_\odot$ is the solar luminosity. The result is a neutrino flux at earth of about $\phi \approx 2.5 \times 10^{38}/(4\pi d^2) \approx 10^{11} \text{cm}^{-2}\text{s}^{-1}$, where $d = 1\text{AU}$. See Ref. [147] for details.
5.2. High energy neutrinos from the Sun

5.2.1 Neutrinos from dark matter annihilations

When the Sun moves through the galaxy it sweeps through the halo of dark matter. Under the WIMP hypothesis, the dark matter particles have a non-zero cross section for scattering on normal matter, so that some of these particles will scatter on the nuclei in the Sun and become gravitationally bound in closed orbits. Given enough time, the particles will thermalise with the solar core, resulting in an overdensity of WIMPs there. This overdensity will lead to an annihilation signal from the Sun. In the annihilations all types of SM particles are typically produced, either as direct annihilation products or as secondary particles in showers from the original annihilation products. Most particles from the annihilations are quickly stopped in the dense solar interior but neutrinos can escape and propagate further to the Earth, where they can be detected in neutrino telescopes [85, 86, 148, 149].

The rate of WIMP capture by the Sun will depend on the average dark matter density along the orbit of the Sun around the galactic center as well as the sum of the scattering cross sections between the WIMPs and the different elements in the Sun, which can be factorised into the WIMP-nucleon scattering cross section $\sigma_{\chi N}$ multiplied by a sum over nucleus-dependent factors (for details, see e.g. Ref. [150]). Since the annihilation rate depends on the WIMP annihilation cross section $\sigma_{\chi \chi}$ the neutrino signal at Earth will therefore depend on both $\sigma_{\chi N}$ as well as $\sigma_{\chi \chi}$, making it a complementary approach compared to direct searches, which primarily probes $\sigma_{\chi p}$ and indirect searches, which probe $\sigma_{\chi \chi}$.

The differential equation determining the rate of change of the number of WIMPs trapped in the Sun $N(t)$ is given by

$$\frac{dN(t)}{dt} = C_C - C_A N(t)^2 - C_E N(t),$$

(5.1)

where $C_C$ is the WIMP capture rate, $C_A$ the annihilation rate which depends on the annihilation cross section $\sigma_{\chi \chi}$ times relative velocity, and $C_E$ is the evaporation rate of WIMPs. The latter describes how likely it is that a WIMP will up-scatter on a solar nucleus and obtain a velocity higher than the escape velocity. It is greatly suppressed for WIMPs with masses $m_\chi \gtrsim 5$ GeV [151, 152]. Neglecting evaporation, the collection of WIMPs in the Sun then becomes a competition
between the capture and annihilation terms with the solution for the rate of WIMP annihilations today given by

$$\Gamma(t_\odot) = \frac{1}{2} C_A N(t_\odot)^2 = \frac{1}{2} C_C \tanh^2 \left( \sqrt{C_C C_A t_\odot} \right), \quad (5.2)$$

where $t_\odot \approx 4.5 \times 10^9$ years is the life-time of the Solar System (the maximum time over which the WIMPs have been able to equilibrate with the solar interior). If $t_\odot \gg 1/\sqrt{C_C C_A}$ the solution tends to the constant value $\Gamma = C_C/2$, i.e. capture and annihilation are in equilibrium and there is a one-to-one relationship between the rate of annihilations and the capture rate. If this holds, the annihilation rate (directly related to the event rate in a neutrino telescope) only depends on the capture rate, which in turn depends on $\sigma_{\chi N}$ and the dark matter distribution in the Milky Way. The solar WIMP searches can thus be meaningfully compared to the direct detection experiments on Earth, where one attempts to detect the nuclear recoil of WIMP-nucleon scattering in a detector, although the solar WIMP signal is an integrated effect over time whereas the direct detection signal only depends on the present WIMP distribution in the galaxy.

The interaction cross section between WIMPs and nuclei of mass number $A$ is often separated into a spin-independent and a spin-dependent part, $\sigma_{\chi A,\text{SI}}$ and $\sigma_{\chi A,\text{SD}}$ respectively. Depending on what type of material the WIMPs scatter on in a particular experiment one is especially sensitive to either one or both of these. In the case of spin-independent scattering there is an additional $A^2$ enhancement of the cross section since the WIMP in this case scatters coherently on the nucleus as a whole, $\sigma_{\chi A,\text{SI}} \propto A^2 \sigma_{\chi N,\text{SI}}$ where $\sigma_{\chi N,\text{SI}}$ is the spin-independent WIMP-nucleon cross section. In reality, for a specific model, such as the MSSM with the neutralino as a WIMP, the total WIMP-nucleus scattering cross section will be a combination of spin-dependent and spin-independent parts, determined by the couplings and masses of the particles in the model. The separation into only spin-dependent or spin-independent scattering is thus a simplification, but a specific model will be somewhere in between these two extremes.

In a sense we can see the Sun as a giant direct detection experiment where we are not free to choose the material in the detector...
but are forced to consider mostly hydrogen with some abundances of heavier elements. In a direct detection experiment on Earth one can choose the material and generally chooses an element with a large number of nucleons to be able to benefit from the $A^2$ enhancement from the coherent spin-independent scattering. Stronger limits can then be put on the spin-independent WIMP-nucleon cross section $\sigma_{\chi N,\text{SI}}$ in an experiment on Earth compared to the indirect searches with the Sun, whereas the Sun often gives stronger limits for the spin-dependent WIMP-nucleon cross section $\sigma_{\chi N,\text{SD}}$, where there is no $A^2$ enhancement for heavy nuclei.

After having been produced in the WIMP annihilations in the solar core, the neutrinos must propagate to the Earth in order to be detected. In the Sun the neutrinos will interact through charged and neutral current interactions as well as undergo matter oscillations. Charged current interactions lead to loss of neutrinos while neutral current interactions result in energy losses. The neutrinos will then vacuum oscillate after exiting the Sun to the Earth [153].

The specifics of the neutrino signal expected at Earth in general depends on the model one considers and its parameter values. One therefore usually considers a generic WIMP model, with an annihilation cross section given by the value required for thermal production with the freeze-out mechanism. In a realistic model, the WIMP will annihilate into many different final states with different branching ratios, and each annihilation channel results in a different energy spectrum of the neutrinos. Annihilations into quark pairs typically peak at lower energies, below the WIMP mass, since the quarks hadronise and energy losses take place before the neutrinos are produced. Annihilations into $W^+W^-$ or $\tau^+\tau^-$ instead produce neutrinos with energies closer to the WIMP mass, since $W^\pm$ and $\tau^\pm$ decay promptly into neutrinos. See Fig. 5.1 for an example of the neutrino flux from WIMP annihilations. As benchmarks one therefore usually considers annihilations into 100% $b\bar{b}$ or 100% $W^+W^-$ or $\tau^+\tau^-$ pairs (the two latter both result in hard spectra, but $\tau^+\tau^-$ extends to lower WIMP masses since the $\tau$ mass is smaller). In an actual model the neutrino spectrum from WIMP annihilations will then be in between these extremes. In Sec. 5.3 we describe shortly the current bounds on WIMP annihilations obtained with the IceCube experiment, a km$^3$-sized neutrino telescope at the South Pole.
5. Neutrinos from the Sun

**Figure 5.1** – The flux of neutrinos per WIMP annihilation as function of neutrino energy divided by WIMP mass $m_\chi$, obtained with *WimpSim* [153, 154]. The left plot shows the $\nu_\mu$ flux directly from the WIMP annihilations, and the right plot shows the $\nu_\mu$ flux 1 AU from production (i.e. approximately at the position of the Earth), when the neutrinos have oscillated and interacted with the solar material. In both plots the blue curve is the flux for 100% annihilations into $W^+W^-$ and the red curve the flux from 100% annihilation into $b\bar{b}$. The $W^+W^-$ spectrum is much more peaked at higher energies near $m_\chi$. In the right plot one can see the neutrino oscillation patterns as wiggles in the curves. The flux is also decreased at higher energies due to interactions in the Sun.

### 5.2.2 Neutrinos from long-lived mediator decays

In Paper II we have simulated and studied the signal from dark matter annihilations in the Sun, in models where the dark matter particles do not annihilate directly into SM particles, but rather into a pair of mediators $V$. The mediators produced in an annihilation then travel some distance away from the annihilation point near centre before they decay into SM particles, with the distance between dark matter annihilation and mediator decay determined by the mediator life-time. The fact that the SM particles are injected away from the solar core means that there will be less interactions between neutrinos and the solar material on the way to the Earth, leading in general to harder neutrino spectra that extend to higher energies [155]. Data from neutrino telescopes has in this way been used to constrain secluded models [118, 156].
5.2. High energy neutrinos from the Sun

There are mainly two benefits of a harder and higher energy neutrino spectrum: firstly, the sensitivity of neutrino telescopes rises as the neutrino energy squared (one factor comes from the fact that the neutrino-nucleon cross section is proportional to the neutrino energy, the other from the muon range in the ice which is proportional to the muon energy, which in turn is proportional to the neutrino energy) so that a higher energy spectrum increases the prospects for neutrino detection in the telescope; secondly, the background, coming mainly from atmospheric neutrinos and muons, decreases quickly with the energy, increasing the ratio of signal over background. Compared to the standard neutrino telescope searches for neutrinos from WIMP annihilations in the Sun, which lose practically all sensitivity for WIMP masses above a few TeV since the Sun becomes opaque to neutrinos, searches for neutrinos from long-lived mediator decays can therefore extend to higher dark matter masses.

Furthermore, if the mediator life-time is such that it can decay outside the Sun, one can look for other particles than neutrinos, such as charged cosmic rays and gamma rays, since these are then no longer inevitably stopped by the solar material [105, 116, 119–121, 124, 126, 157, 158]. One can then complement the data from neutrino searches with observations on gamma rays and charged cosmic rays. This aspect of dark matter models with long-lived mediators has also been studied in Paper II.

5.2.3 Neutrinos from cosmic ray interactions in the solar atmosphere

When cosmic rays impinge on the Sun they collide with the nuclei in the outer parts and produce cascades of particles. Most of the particles in the cascades are hadrons in the form of pions and kaons. These propagate further in the direction of the initial cosmic ray and interact or decay. Interactions lead to energy loss in the case of elastic scattering or the loss of a particle in the case of inelastic scattering. Decays lead to the loss of the particle in question as it decays into another particle type. Depending on the energy and density that the particle is traveling in, interaction or decay will be preferred. This chain of interaction and decays continues until some stable final state is reached.
In the showers neutrinos will be produced in the decay of hadrons or leptons. The main decay channels that lead to the production of neutrinos are the decays of pions or kaons and muons, according to the following decay chain:

\[
\text{CR} + \text{nucleus} \rightarrow \pi^\pm/K^\pm + X
\]

\[
\rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)
\]

\[
\rightarrow e^\pm + \bar{\nu}_\mu(\nu_\mu) + \nu_e(\bar{\nu}_e)
\] (5.3)

The entries in brackets refer to the case of negatively charged pions/kaons and leptons. \( X \) denotes the remnant of the nucleus and the CR particle. We see that this chain produces three neutrinos and antineutrinos in the flavour ratio

\[
(\nu_e + \bar{\nu}_e) : (\nu_\mu + \bar{\nu}_\mu) : (\nu_\tau + \bar{\nu}_\tau) = 1 : 2 : 0.
\] (5.4)

The flux of neutrinos arising from the decay of charged pions or kaons in this way is called the conventional flux. Since pions and kaons are dominant in the cascades, the conventional flux is the most important contribution to the neutrino flux. However, at higher energies the interaction length of pions and kaons becomes shorter than the decay length so that there is a large probability that they interact and are lost before decaying, resulting in a decrease of the flux. Instead, decays of short-lived resonance particles containing charm quarks (for example \( D^0, D^\pm \)) become important to determine the flux. This flux component is called the prompt flux.

The neutrinos produced in the solar atmosphere will interact with the solar material on their passage through the Sun towards Earth. They will also undergo matter oscillations in the Sun and vacuum oscillations between the Sun and the Earth. This results in a different flavour ratio at Earth than in Eq. (5.4) that is approximately \( 1 : 1 : 1 \) \[146, 159\].

The Sun blocks some cosmic rays from reaching the Earth, resulting a detectable “shadow” in the cosmic ray flux at Earth \[160–163\]. These cosmic rays are precisely the ones that lead to the production of the solar atmospheric neutrinos (SA\( \nu \)) and naively the SA\( \nu \) flux could be expected to be the same as the neutrino flux from cosmic ray interaction in the Earth’s atmosphere when integrated over a solid
angle as large as that of the Sun. There are some key differences between the two however, that result in the fluxes being different: (i) the solar atmosphere is less dense than the Earth’s which leads to a larger fraction of meson decays compared to interactions, (ii) neutrino oscillations change the ratio between the different flavour fluxes on the way to Earth and (iii) the solar medium is opaque to high energy neutrinos due to interactions which leads to a large suppression of the flux for neutrino paths passing through the Sun at energies $E_\nu \gtrsim 1 \text{ TeV}$.

In Paper I we have calculated the SA$\nu$ flux at Earth. We have also investigated the impact on solar dark matter searches. Earlier studies of the SA$\nu$ flux have been performed in Refs. [146, 159, 164–170]. Out of these, Refs. [146, 164–167, 169] perform calculations of the flux$^2$, whereas Refs. [159, 168, 170] use earlier calculations. Neutrino oscillations are included completely in the calculation in Refs. [146, 169], earlier studies in Refs. [159, 168] add the effect of oscillations to the input fluxes in the solar atmosphere from Ref. [167] and propagate these fluxes with oscillations included to the Earth.

The magnetic field of the Sun will most likely affect the SA$\nu$ flux at lower energies $E_\nu \lesssim 10^2 \text{ GeV}$. In Paper I we have neglected the magnetic field and focused the analysis on higher energies, where the charged primary cosmic rays are rigid enough to be unaffected by the solar magnetic field. The magnetic field is complex in structure and our knowledge of the field in the outer parts, consisting of the chromosphere and the corona, to a large part relies on numerical simulations and modelling [171, 172]. The field varies along the solar surface and is mostly concentrated in flux tubes. The flux tubes extend out from the surface, spreading out to form a canopy-like structure higher up. Typical field strengths of the large scale solar magnetic field are of order 1 G, but in flux tubes the field strength is instead about 10 G.

The cosmic rays will be affected by the interplanetary magnetic field on the way down to the solar atmosphere, and by the atmospheric magnetic fields when they begin interacting with the solar medium. In Ref. [165] the diffusion effect of the interplanetary magnetic field is estimated to decrease the neutrino flux at neutrino en-

\footnote{Note that the calculation in Ref. [169] was performed simultaneously but independently from our analysis in Paper I.}
5. Neutrinos from the Sun

Energies $E_\nu \lesssim 200$ GeV.

From observations of the shadowing effect that the Sun has on the cosmic ray flux that reaches the Earth, we know that the solar magnetic field affects cosmic rays around the Sun, for example the cosmic ray shadow is displaced compared to the position of the Sun on the sky [160, 163, 173], and both the intensity of the shadow [160, 162] as well as the gamma ray flux has been observed to be anticorrelated with the solar activity [174]. Modelling the effect on the $\text{SA}_\nu$ flux presents a challenge however, as the magnetic field is complex, but studies with other messenger particles may help in determining the importance of the magnetic field. For example, gamma rays are produced in the decays of $\pi^0$ mesons in cascades in the solar atmosphere in a way similar to the $\text{SA}_\nu$s, and the resulting gamma ray flux will also be affected by the solar magnetic field effects. A gamma ray flux from the Sun has been observed [174–176], however the observations do not match the theoretical predictions available and are currently not completely understood [177, 178]. Further characterisation of this signal can potentially aid in determining the effect on the $\text{SA}_\nu$ flux.

5.3 Neutrino telescopes

A typical astronomical telescope observes the light emitted from astrophysical sources. This light is not restricted to be in the optical range—astronomical observatories cover the whole electromagnetic spectrum. In a neutrino telescope one is instead interested in detecting neutrinos emitted from astrophysical objects. With the IceCube experiment, situated in the Antarctic ice at the South Pole, we have for the first time observed these astrophysical neutrinos and hence entered a new era of neutrino astronomy [179, 180].

The benefit of neutrino astronomy is at the same time its drawback: the low interaction cross section of neutrinos with any type of matter encountered along the way from the emission to the detector (along with being unaffected by magnetic fields) means that the neutrinos point directly back to the source, however this also means that detecting the neutrinos in an experiment provides a significant challenge. The result is that neutrino telescopes consist of very large detectors that make up for the low detection rate with a
large detector volume. Another complicating matter is that the flux of astrophysical neutrinos is much lower than the constant background of Earth atmospheric neutrinos (EA$\nu$) produced in cosmic ray cascades in the Earth’s atmosphere.

Neutrinos can interact with the nuclei in the detector through deep inelastic charged current (CC) or neutral current (NC) interactions,

\[
\text{CC: } \nu_\ell + N \rightarrow \ell^- + X \tag{5.5} \\
\text{NC: } \nu_\ell + N \rightarrow \nu_\ell' + X \tag{5.6}
\]

where $\ell = e, \mu, \tau$, $N$ denotes a nucleus in the detector and $X$ is the remnant of the nucleus after scattering. In a CC interaction, a charged lepton is produced that gives information on the neutrino flavour since the charged leptons of different flavour have different signatures in the detector. In a NC interaction, the neutrino leaves the detector with reduced energy, leaving a hadronic remnant behind. In this case there is no flavour information. This leads to two types of signatures in the detector: cascades, from hadronic remnants or leptonic decays, and tracks from charged leptons. Cascade events typically give better energy information while tracks provide better directional accuracy.

In the searches for high energy neutrinos from the Sun it is important to have good pointing accuracy to ensure that the neutrinos come from the Sun, therefore tracks are preferred over cascades. The most useful case is then the CC interaction of a muon neutrino leading to a muon in the final state, since the muon is the most penetrating lepton type. Depending on its energy, a muon can travel several kilometers in the ice before decaying or being stopped.\(^3\) Since it is travelling faster than the speed of light in ice, it will emit Cherenkov light in a cone around the track, which can be used to determine the direction of the muon. The deposited light as well as the energy loss in the ice can be used to reconstruct the energy of the muon. The neutrino-nucleus scattering leads to an energy-dependent kinematic angle between the original neutrino and the muon. The direction of the muon will therefore not be the same as the original neutrino,

\(^3\)A 50 GeV muon has an average range of about 200 m.
leading to an uncertainty in the reconstructed neutrino direction that becomes smaller with increasing energy.

The other lepton flavours are less useful for the solar searches. Electrons are lighter than muons and will therefore be stopped in the detector much quicker, resulting in a cascade event that makes it more difficult to determine the direction of the electron and hence of the neutrino. Tau leptons also produce cascades. They quickly decay in the detector and produce secondary particles with lower energy, some of them muons.\footnote{At very high energies the tau leptons have a decay length that is sufficiently long that the cascades from the interaction vertex and decay vertex can be distinguished, called a “double bang” event. No such event has been observed to date.}

We have here focused on IceCube, other neutrino experiments include for example Super-Kamiokande \cite{181}, Antares \cite{182} and the upcoming KM3Net \cite{183}.

### 5.3.1 Dark matter searches

If WIMPs collect in the solar interior and annihilate, an excess of neutrinos is expected in the direction of the Sun. The energy range of interest is that above the solar neutrino energies, i.e. GeV energies. It is kinematically limited from above by the WIMP mass, which is typically $O(100\text{ GeV})$, as well as by neutrino interactions in the Sun that prohibit neutrinos at energies $\gtrsim 1\text{ TeV}$ from escaping the Sun’s core. To ensure that a detected excess of neutrinos really comes from WIMP annihilation one must be careful to subtract all other sources of neutrinos. At these energies, only $\text{SA}_\nu$ contribute to the neutrino flux from the Sun. Atmospheric neutrinos and muons created in similar cascades in the Earth’s atmosphere (here denoted $\text{EA}_\nu$ and $\text{EA}_\mu$ respectively) must also be taken care of.

With directional reconstruction, one can distinguish the solar neutrino fluxes from the $\text{EA}_\nu$ and $\text{EA}_\mu$ fluxes since the angular dependence on the sky will be vastly different. $\text{EA}_\mu$ are the dominating source of muons that reach the detector and must be effectively removed. The simplest way to do this is to require that the muons in recorded events are upgoing, i.e. that they enter the detector from below. Only neutrinos are able to propagate through the
Earth without being absorbed, so vertically upgoing muons must be neutrino-induced and not atmospheric (they can still come from EA$\nu$ however). The EA$\nu$ flux will reach the detector from all directions, with a magnitude dependent on the zenith angle (the angle from the vertical direction from the Earth’s surface). The EA$\nu$ flux in the direction of the Sun will therefore take values in between the horizontal and vertical flux, since the Sun will move on the sky. One possibility to subtract the EA$\nu$ flux is estimate the flux in regions of the sky where the Sun is not present. Early IceCube solar dark matter analyses used this together with a selection of only upgoing muons [184].

In later IceCube solar dark matter searches [87, 185–187], the background is instead estimated from the actual data by utilising the fact that the Sun is small on the sky and that the total background from EA$\nu$ and EA$\mu$ is approximately the same over the whole sky and much larger than any signal. The background is then taken from data randomised over the whole sky, which removes the systematic errors that can occur when instead simulating the background.

With no excess seen so far in the IceCube searches, limits can be put on the WIMP-nucleon cross section. This comes from the fact that the expected event rate is proportional to the present-day WIMP annihilation rate in the Sun, which in equilibrium is directly proportional to the WIMP capture rate in the Sun, which in turn is proportional to the WIMP-nucleon scattering cross section. By simulating such WIMP annihilation signals and comparing with data one can then get a handle on how large the cross section can be without being detected in IceCube. The latest limits come from Ref. [87], which obtains a limit on the spin-dependent WIMP-proton cross section down to $\sigma_{\chi p, SD} \sim 2 \times 10^{-41}$ cm$^2$ for WIMP masses of the order of 100 GeV. For the spin-independent cross section, the IceCube limit is as expected not as constraining as direct detection experiments. The limit goes down to $\sigma_{\chi p, SI} \sim 2 \times 10^{-44}$ cm$^2$ for WIMP masses of the order of 100 GeV, orders of magnitude above the best direct detection limits.

In the case of spin-dependent scattering neutrino telescope limits can compete with the direct detection experiments. For WIMP masses above about 100 GeV, the IceCube limits are the most stringent to date (for lower WIMP masses SuperKamiokande [88] and the
PICO experiment [188] have stronger limits). This is because the Sun to a large part consists of hydrogen, which has spin. In the case of spin-independent cross sections, the IceCube limits can not compete with the Earth-based direct detection experiments, since one in the case of the Sun does not benefit from the coherent $A^2$ enhancement one obtains with heavy nuclei in detectors on Earth. These limits are still a complement to the direct detection limits however, since the solar annihilation signal is an effect that is sensitive to the dark matter distribution during the whole lifetime of the Solar System and not just to the present day dark matter density.

5.3.2 Solar atmospheric neutrinos as a background

Since the SA$\nu$s are also emitted from the Sun they provide a different type of background for the solar dark matter searches as compared to the EA$\nu$s and EA$\mu$s. In principle, the SA$\nu$ flux can also be distinguished from a WIMP signal since the dependence on the angle from the centre of the solar disk and energy spectrum are both different. In practice however, the angular resolution of IceCube is $\sim 6^\circ$ for 100 GeV neutrinos [87] which is much larger than the angle $\sim 0.5^\circ$ that the Sun occupies on the sky. Therefore both the SA$\nu$ and WIMP annihilations act as neutrino point sources in IceCube. The differences in the energy spectra, with the SA$\nu$ flux having a power-law dependence and the WIMP signal consisting of a bump below the WIMP mass, can then potentially be used to distinguish the two. This however requires reasonably large statistics, which is unlikely given expected event rates, and a sufficient energy resolution.
Chapter 6

Neutralino relic abundance including NLO-QCD corrections

The relic abundance of cold dark matter is by now measured to percent-level accuracy by the Planck telescope\textsuperscript{1} \cite{planck},

\[ \Omega_{\chi} h^2 = 0.1200 \pm 0.0012, \]  

which means that the experimental uncertainty on \( \Omega_{\chi} h^2 \) in the standard \( \Lambda \)CDM scenario is now smaller than the theoretical uncertainty. There are several sources of uncertainty in the relic abundance prediction for a given particle physics model. On the cosmological side, for example a different expansion history than the standard one can result in differences in the treatment of the Boltzmann equation that are not accounted for in the standard calculation. There are also numerical uncertainties coming from the differences between different codes in how the relic abundance is calculated. Fast calculations, which are absolutely necessary to be able to perform scans in the parameter space of a model, can lead to compromises in numerical accuracy, and in general the differences between codes lead to different predictions of \( \Omega_{\chi} h^2 \). From the particle physics side the main source of uncertainty are the uncertainties in the cross sections

\textsuperscript{1}The exact best-fit value of \( \Omega_{\chi} h^2 \) depends on the specifics of the analysis. We quote here the 68% confidence level Planck result denoted “TT, TE, EE+lowE+lensing” in Ref. \cite{planck}, which includes the analysis of temperature and polarisation spectra.
that enter into the effective annihilation cross section \( \langle \sigma_{\text{eff}} v \rangle \). Such uncertainties can for example stem from the lack of higher order corrections in the cross sections. There can also be uncertainties in the calculation of the mass spectrum for a given set of input parameters, which in particular affects models where coannihilations between the lightest particle and other particles close in mass to it are important. Uncertainty can also come from the residual dependence on the scale at which the calculation takes place, which is inevitably present in any perturbatively calculated cross section but should decrease with the inclusion of higher order corrections.

As noted in Ch. 4, the dependence on the particle physics model in the dark matter relic density calculation is contained almost entirely in the effective annihilation cross section, presented in Eq. (4.15) and repeated here for convenience:

\[
\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j} \langle \sigma_{ij} v_{ij} \rangle \frac{n_{\text{eq}}^i}{n_{\text{eq}}} \frac{n_{\text{eq}}^j}{n_{\text{eq}}} \tag{4.15}
\]

where \( n_{\text{eq}}^i \) (\( n_{\text{eq}}^j \)) is the equilibrium number density of particle \( i \) (\( j \)), \( n_{\text{eq}} \) the sum over all such number densities for the non-SM particles and \( \langle \sigma_{ij} v_{ij} \rangle \) is the thermally averaged cross section for initial state \( ij \), summed over final states. To calculate \( \langle \sigma_{\text{eff}} v \rangle \) and later the relic abundance \( \Omega_{\chi h^2} \), we thus need the cross sections for all relevant coannihilation processes. Typically, in the computer packages that calculate the relic abundance for a given particle physics model, these cross sections are used at leading order in the relevant coupling constants, with a few higher order effects included. In this chapter, we present results obtained for the neutralino relic abundance using cross sections at next-to-leading order in the strong coupling \( \alpha_s \) (NLO-QCD), as provided by the DM@NLO package [103]. We use three benchmark models that are the best-fit points in a scan over the parameter space of a seven-parameter phenomenological MSSM model [189] made by the GAMBIT collaboration [101].

Out of the above mentioned sources of uncertainty in \( \Omega_{\chi h^2} \), we thus focus on the reduction in uncertainty resulting from using the more exact NLO-QCD cross sections in the calculation of the relic abundance. It is well-known that loop corrections can affect cross sections significantly and especially so for QCD processes since \( \alpha_s \)
6.1 Benchmark models

is large. Concerning neutralino annihilation in the MSSM, the performed studies of electroweak \[190–192\] and QCD \[193–201\] corrections indicate that the corresponding uncertainty in the relic abundance is larger than the experimental uncertainty in the Planck measurement. Apart from giving a more accurate prediction, studying the higher order corrections to $\Omega h^2$ quantifies the uncertainty in a leading order prediction of the relic abundance in the sense that it tells us how much the leading order prediction can shift when such corrections are included.

The difficulty of comparing a specific particle physics model to today’s plethora of experimental constraints in a globally consistent way was one of the reasons for the development of the GAMBIT code. In GAMBIT, one can start from a particle physics model, and perform parameter scans in a consistent way. One can choose which DM observables to include in the scan, as well as which code to use for which observable. One long-term goal of the project presented here is to include the possibility to use DM@NLO cross sections in GAMBIT, to be able to use NLO cross sections for the $\Omega h^2$ calculation in MSSM parameter scans.

6.1 Benchmark models

The most general version of the MSSM that conserves CP introduces a large number of new (continuous) parameters, where all but one (the $\mu$ parameter) come from the soft SUSY breaking terms (see Eq. (3.5) for the soft breaking Lagrangian). In Ref. [189] these parameters have been reduced down to seven by making the following phenomenological assumptions:

- All elements in the trilinear coupling matrices $A_E$, $A_D$, $A_U$ are assumed to be zero except for the third generation elements $(A_D)_{33} \equiv A_{d3}$ and $(A_U)_{33} \equiv A_{u3}$.

- All off-diagonal elements in the sfermion mass matrices $M^2_Q$, $M^2_L$, $M^2_U$, $M^2_D$ and $M^2_E$ are assumed to be zero to prevent flavour-changing neutral currents. The remaining diagonal elements are set to a universal sfermion mass $m^2_{\tilde{f}}$. 
• The GUT-inspired relation between the gaugino soft mass parameters is assumed to hold: \( \frac{1}{3} \cos^2 \theta_W M_1 = \sin^2 \theta_W M_2 = \frac{\alpha}{\alpha_s} M_3 \) at some scale \( Q \).

• A positive sign of \( \mu \).

In addition one needs to specify the energy scale where these parameters are input, this scale is for most parameters set to 1 TeV. The Higgs sector is specified by the doublet masses \( m^2_{H_u} \) and \( m^2_{H_d} \) together with ratio between their vacuum expectation values \( \tan \beta \).

With the above assumptions in place is it possible to completely specify this MSSM model with the seven parameters

\[
M_2, A_{u_3}, A_{d_3}, m^2_{f_1}, m^2_{H_u}, m^2_{H_d}, \tan \beta
\]

so that this defines a pMSSM7 model. In addition, \( R \)-parity is assumed to be conserved so that the LSP is stable and furthermore only parameter combinations that result in an LSP which is a neutralino are considered.

As stated in Ref. [189], the effective annihilation cross section is in large parts of the MSSM parameter space small, which leads to a prohibitively large value of \( \Omega_\chi h^2 \) and rules out such regions. To obtain the right value of the relic abundance (or a smaller value) one needs a mechanism for depleting the neutralino relic abundance. Typical such mechanisms are either resonant annihilation through a neutral gauge or Higgs boson (denoted a “funnel”) or coannihilation with another SUSY particle type, such as the lightest chargino \( \chi^\pm_1 \) or the lightest sboson. The benchmark models used here consist of one chargino coannihilation scenario, one heavy Higgs funnel scenario and one light Higgs or \( Z^0 \) funnel scenario, where “heavy Higgs” here refers to the \( A^0 \) or \( H^0 \) bosons (the CP-odd and the heavier of the CP-even Higgs bosons respectively) and the “light Higgs” is the lighter CP-even Higgs boson \( h^0 \). The regions are defined in the following way:

• **chargino coannihilation:** \( \chi^0_1 \) is \( \geq 50\% \) Higgsino.

• **\( A^0/H^0 \) funnel:** \( 1.6m_{\chi^0_1} \leq m_i \leq 2.4m_{\chi^0_1} \).

• **\( h^0/Z^0 \) funnel:** \( 1.6m_{\chi^0_1} \leq m_j \leq 2.4m_{\chi^0_1} \).
6.1. Benchmark models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scenario I</th>
<th>Scenario II</th>
<th>Scenario III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{d_3}$</td>
<td>9376.461</td>
<td>1639.611</td>
<td>9582.567</td>
</tr>
<tr>
<td>$A_{u_3}$</td>
<td>2923.877</td>
<td>3660.585</td>
<td>-9389.783</td>
</tr>
<tr>
<td>$M_2$</td>
<td>2469.296</td>
<td>2032.136</td>
<td>3768.368</td>
</tr>
<tr>
<td>$\tan \beta$</td>
<td>46.632</td>
<td>19.058</td>
<td>7.133</td>
</tr>
<tr>
<td>$m_{H_u}^2$</td>
<td>$-7.830 \times 10^5$</td>
<td>$-6.077 \times 10^5$</td>
<td>$-1.271 \times 10^7$</td>
</tr>
<tr>
<td>$m_{H_d}^2$</td>
<td>$2.729 \times 10^7$</td>
<td>$3.189 \times 10^6$</td>
<td>$3.748 \times 10^5$</td>
</tr>
<tr>
<td>$m_f^2$</td>
<td>$1.352 \times 10^7$</td>
<td>$9.574 \times 10^6$</td>
<td>$9.680 \times 10^7$</td>
</tr>
</tbody>
</table>

Table 6.1 – The values of the input parameters for the three benchmark models used. These models are the best-fit points in the scan over the parameter space of a pMSSM7 model presented in Ref. [189]. All parameters are defined at a scale of 1 TeV, except $\tan \beta$ which is defined at the scale $m_Z$. Mass parameters are given in units of GeV.

The reason for the first definition is the fact that if the lightest neutralino $\chi_1^0$ is close to a pure Higgsino, it will inevitably be approximately mass degenerate with the lightest chargino $\chi_1^{\pm}$ (as well as $\chi_2^0$), making coannihilations between these particles highly relevant for the determination of the relic abundance. Note that the designation of particular parameter combination to a region is not exclusive, so that a particular parameter combination can belong to several regions simultaneously. Note also that the model classification is purely based on kinematical conditions, it is not necessarily the case that a specific type of model only receives contributions from the processes it is classified as. We also note that the contribution of a specific process to $\langle \sigma v \rangle$ is a temperature-dependent statement. Different processes may be important at low and high temperatures respectively.

In the scan over the seven input parameters presented in Ref. [189], numerical values are assigned to the input parameters, making it possible to make numerical predictions for all the observables included
### 6. Neutralino relic abundance including NLO-QCD corrections

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scenario I</th>
<th>Scenario II</th>
<th>Scenario III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\chi_1^0}$</td>
<td>258.94</td>
<td>69.22</td>
<td>379.25</td>
</tr>
<tr>
<td>$(f_b, f_h)$</td>
<td>(0, 1.00)</td>
<td>(0, 1.00)</td>
<td>(0, 1.00)</td>
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<tr>
<td>$m_{\chi_2^0}$</td>
<td>262.75</td>
<td>73.65</td>
<td>381.94</td>
</tr>
<tr>
<td>$(f_b, f_h)$</td>
<td>(0, 1.00)</td>
<td>(0, 1.00)</td>
<td>(0, 1.00)</td>
</tr>
<tr>
<td>$m_{\chi_1^\pm}$</td>
<td>261.18</td>
<td>71.60</td>
<td>380.87</td>
</tr>
<tr>
<td>$(f_w, f_h)$</td>
<td>(0, 1.00)</td>
<td>(0, 1.00)</td>
<td>(0, 1.00)</td>
</tr>
<tr>
<td>$m_A$</td>
<td>5348.47</td>
<td>1804.89</td>
<td>793.44</td>
</tr>
<tr>
<td>$m_H$</td>
<td>5348.47</td>
<td>1804.92</td>
<td>793.97</td>
</tr>
<tr>
<td>$m_{H^\pm}$</td>
<td>5379.24</td>
<td>1805.87</td>
<td>798.95</td>
</tr>
<tr>
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<td>2132.46</td>
<td>9013.00</td>
</tr>
<tr>
<td>$m_{\tilde{t}_1}$</td>
<td>3698.87</td>
<td>3097.13</td>
<td>9845.05</td>
</tr>
<tr>
<td>$\Omega_{\chi^0 h^2}</td>
<td>_{\text{GAMBIT}}$</td>
<td>$8.027 \times 10^{-3}$</td>
<td>$8.382 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

**Table 6.2** – Values of physical parameters in units of GeV in the three benchmark models used, rounded to two decimal points. These models are the best-fit points in the scan over the parameter space of a pMSSM7 model presented in Ref. [189]. For neutralinos and charginos we list the bino or wino fraction ($f_b$ and $f_w$ respectively) and the higgsino fraction ($f_h$). The bino or wino fraction of $\chi_1^0$, $\chi_2^0$ and $\chi_1^\pm$ are all on the permille level or smaller. The values of the relic abundance are quoted from Ref. [189] and are there calculated in GAMBIT using MicrOMEGAs. Since they are below the measured dark matter relic abundance, they show that in addition to the neutralino, another dark matter component is also needed for these models.

in the scan. The predictions are then compared to experimental constraints to determine whether this particular parameter combination is ruled out or not. For the specific case of the relic abundance, the value of $\Omega_{\chi^0 h^2}$ is in the scan constrained to be equal to or below the measured Planck value, i.e. the surviving models do not need
to provide all of the dark matter in the Universe. Those parameter combinations that survive all experimental constraints then define a multiparameter volume in the space spanned by the parameters scanned over, with each point in the volume assigned a likelihood value that depends on how plausible it is given the experimental data.

The surviving volume is subdivided into the above mentioned regions depending on how the right relic abundance is obtained. We consider here the best-fit points in this volume (i.e. the points that maximise the profile likelihood) in the chargino coannihilation region and the light and heavy Higgs funnel regions. We will refer to these model realisations as Scenario I ($\chi^\pm_1\,$ coannihilation), II ($h^0/Z^0$ funnel) and III ($H^0/A^0$ funnel). We show the values of the model input parameters in Tab. 6.1.

In Tab. 6.2 we show the most relevant physical parameters in the three models. We can see that in all three models, the two lightest neutralinos and the lightest chargino are dominantly Higgsino and close in mass—a clear indication that coannihilations between these particles will be important in the determination of the relic abundance. We can also see that the sfermion masses are large compared to the lightest neutralino mass and hence, sfermion coannihilations will be unimportant (note that the sfermion masses not displayed in Tab. 6.2 are similar in magnitude or larger since the soft sfermion mass parameter used as input is the same for all sfermions). In one model, scenario III, the Higgs masses are within reach in the coannihilation processes so that annihilation through a Higgs resonance can play a role.

To get an idea of the dominating phenomenology in scenarios I, II and III we show in Fig. 6.1 the effective annihilation rate $W_{\text{eff}}$ for scenarios I, II and III as determined with DarkSUSY, with the standard cross sections. The horizontal axis is spanned by the momentum parameter

$$p_{\text{eff}} = \frac{\sqrt{s - 4m^2_{\chi_1^0}}}{2} \iff \sqrt{s} = 2\sqrt{m^2_{\chi_1^0} + p^2_{\text{eff}}},$$  \hspace{1cm} (6.3)$$
corresponding to the momentum in the centre-of-mass frame in a $\chi_1^0\chi_1^0$ collision. In $W_{\text{eff}}$, resonances then show up as peaks at points
Figure 6.1 – Effective annihilation rates in Scenario I, II and III evaluated with the standard DarkSUSY cross sections. Kinematical thresholds for initial states and resonances are indicated with the arrows, some of them are barely visible. One final state threshold also appears in scenario II, the $W^+W^-$ threshold at $p_{\text{eff}} = 40\text{ GeV}$
Figure 6.2 – The integrand in $\langle \sigma_{\text{eff}} v \rangle$ in Scenario I, II and III evaluated at the freeze-out temperature for each model respectively, using the standard DarkSUSY cross sections, normalised so that it integrates to one. Kinematical thresholds and resonances are indicated with the arrows (with several of them barely visible). The modified Bessel function in Eq. (4.17) gives an exponential suppression at high momenta.
where $\sqrt{s}$ is equal to the resonance particle mass. The thresholds indicate that $\sqrt{s}$ is large enough to access new initial or final states and appear at points on the horizontal axis where $\sqrt{s}$ is equal to the sum of the masses of the new initial or final state that can be accessed. Note that both initial and final states appear as thresholds in $W_{\text{eff}}$ since it is defined as a sum over both initial and final states.

We also show in Fig. 6.2 the temperature-dependent integrand,

$$p_{\text{eff}}^2 W_{\text{eff}}(p_{\text{eff}}) K_1(\sqrt{s}/T_f),$$  \hspace{1cm} (6.4)

of the $p_{\text{eff}}$ integral in the numerator of Eq. (4.17). We evaluate the integrand at the at the freeze-out temperature$^2$ $T_f$ and normalise it such that the area under the curve in the plot is equal to one. The area under the curve is therefore proportional to the integral in the numerator of Eq. (4.17) and the curves in Fig. 6.2 thus indicate what parts of $W_{\text{eff}}$ that will be most important in the determination of $\langle \sigma_{\text{eff}} v \rangle$.

We can see that in all three scenarios, coannihilation processes and especially $\chi_1^0 \chi_1^\pm$ contribute significantly to $\langle \sigma_{\text{eff}} v \rangle$. In scenario III, we see the heavy Higgs resonances as expected. We can see that for scenario II, although denoted a $Z^0/h^0$ funnel, there is no clear sign of these light resonances and in fact the spectrum is such that both the $Z^0$ and $h^0$ resonances are below the lower kinematical threshold for $\chi_1^0 \chi_1^0$ annihilations given by $\sqrt{s} = 2m_{\chi_1^0}$. This may also be due to the fact that the $\chi_1^0 h^0$ coupling is close to zero at tree level for a nearly pure Higgsino, making such an $s$-channel annihilation process highly suppressed off resonance.

### 6.2 DM@NLO

DM@NLO is a fortran package containing routines for the calculation of supersymmetric cross sections relevant for the calculation of the neutralino relic density. As opposed to earlier studies of electroweak corrections [190–192], the focus is in DM@NLO on NLO-QCD corrections, thus the corrections affect only processes with color charged

\footnote{The freeze-out temperature is in DarkSUSY defined as the temperature when the ratio between the value of the comoving abundance is twice the equilibrium comoving abundance, where the comoving abundance at temperature $T$ is defined as $Y = n_\chi/s$ for neutralino number density $n_\chi$ and entropy density $s$.}
particles. In a number of publications, the DM@NLO group has investigated the effect that the loop corrections has on the annihilation cross section relevant for the calculation of the relic density [193–201], showing that in the cases studied, the uncertainty on the neutralino relic abundance associated with higher order corrections is around 10% and thus larger than the experimental uncertainty. The relic density has in these studies been calculated with the package MicrOMEGAs together with DM@NLO cross sections, and due to the above mentioned possible differences between different Boltzmann solvers, it is of interest to also study the relic density obtained with DarkSUSY interfaced to the DM@NLO code. Apart from that, global scans in parameter space including NLO corrections will in the future be necessary in order to assess the global implications of corrections for the MSSM as a whole. In a framework like that of GAMBIT, if functioning interfaces to DM@NLO are included in both Boltzmann solvers, one can in a future study investigate consistently the effect of NLO corrections as well as the use of different Boltzmann solvers on scans in parameter space.

### 6.2.1 Interfacing DM@NLO to DarkSUSY

In order to be able to use the DM@NLO cross sections in the calculation of $\Omega \chi h^2$ we have made some modifications of DarkSUSY. We have linked the code to the DM@NLO code and included new routines that allow the user to initialise DarkSUSY with the option to use the cross sections calculated with DM@NLO instead of the ones included in DarkSUSY in the calculation of $\Omega \chi h^2$. The cross sections included in DarkSUSY have been calculated analytically and are hardcoded into routines included in the MSSM module in the code, so that one can replace these with the DM@NLO cross sections by calling the DM@NLO cross section routines instead of the DarkSUSY ones.

We have included the following processes in the interface:

\[
\chi_i^0 \chi_j^0 \rightarrow q\bar{q}, \quad i, j = 1, 2, 3, 4, \tag{6.5}
\]

\[
\chi_i^0 \chi_k^\pm \rightarrow q\bar{q}'t, \quad i = 1, 2, 3, 4, \quad k = 1, 2, \tag{6.6}
\]

\[
\chi_k^\pm \chi_l^\mp \rightarrow q\bar{q}, \quad k, l = 1, 2, \tag{6.7}
\]

for $q, q' = u, d, c, s, t, b$. The tree-level Feynman diagrams for these processes are shown in Fig. 6.3. In the interface, the DM@NLO cross
sections for the processes in Eqs. (6.5), (6.6) and (6.7) thus replace the DarkSUSY cross sections in the sum over initial and final states to obtain $\langle \sigma_{\text{eff}} v \rangle$. Any other cross sections needed are taken to be the MSSM cross sections available in DarkSUSY.

In models where chargino or neutralino coannihilation processes are most important (and e.g. sfermion coannihilation is not relevant), restricting QCD corrections to the above processes will likely capture most of the NLO-QCD effects since the remaining SM final states (leptons and gauge bosons) do not receive QCD corrections. The three benchmark models we use are examples of such models, since coannihilations between the lightest chargino and neutralino are important in all these models and furthermore the sfermions are all sufficiently heavy that neutralino-sfermion coannihilations will be unimportant, as evidenced by e.g. the large $m_{\tilde{f}}$ values (see Tab 6.1).

A technical issue regarding the interface is whether the integration of the differential cross sections takes place before or after the summation over final states. Since this ordering is different in DM@NLO compared to default DarkSUSY, it had to be changed in DarkSUSY for the interface to work. This made the cross section calculations about a factor of two slower. This may however be possible to optimise further in future work.

Another technicality has to do with low momentum values, where the NLO calculations can become unstable and/or unphysical due to complications with loop functions at low momenta. Since the low $p$ values are not very important\(^3\) in the calculation of $\Omega h^2$, we have elected to always use DarkSUSY cross section below momenta of 5 GeV.

The model parameters are read into DarkSUSY and then passed over from DarkSUSY to DM@NLO in a routine that ensures that the same parameter values are used in both codes. There are however some exceptions to this, having to do with the renormalisation scheme in DM@NLO, which we explain in Sec. 6.3.1 below. We have here read the model parameters for the three models into DarkSUSY using files in the SUSY Les Houches Accord (SLHA) format [202, 203], that have been made publicly available [204]. To be clear, this means that we do not define the models using the input pa-

---

\(^3\)We note however, that in some other applications, such as in indirect detection, the cross section values in the $p \to 0$ limit are crucial.
rameters in 6.1, but rather read the full model spectra directly into DarkSUSY.

6.3 Renormalisation and higher order corrections

Scattering amplitudes in quantum field theory are typically calculated in a perturbative expansion in terms of the relevant couplings constants. The rationale behind the perturbative expansion is that for small couplings, successive terms in the expansion will be smaller than the previous terms. We denote the lowest order term in the expansion the leading order (LO) contribution with the next order denoted next-to-leading order (NLO) and so on. Since NLO corrections typically involve diagrams with loops, these higher order corrections are often referred to as loop corrections. However, we note that some NLO corrections, like gluon radiation, are not composed of loop diagrams.

In the NLO calculations one typically encounters divergences, both at high momenta, referred to as ultraviolet (UV) divergences, and at low momenta, referred to as infrared (IR) divergences. These divergences are regulated to be able to make sensible predictions by renormalising the theory. When going beyond the LO contribution, the Lagrangian bare parameters no longer correspond to physical parameters. Renormalisation is a systematic procedure where the bare parameters are redefined (i.e. renormalised) into renormalised parameters that again have a correspondence with the physical parameters.

A renormalisation scale is introduced in this procedure, such that renormalised parameters depend on on the scale where they are renormalised. In any physical observable, all such scale dependence should cancel out when considering the scale dependence of all parameters that contribute to the observable. However, at any order, a residual scale dependence remains, caused by the fact that the scale dependence in general only vanishes completely when including corrections to all orders. Thus, at any finite order in perturbation theory, there will be a renormalisation scale dependence, which becomes smaller when higher order corrections are included.
A renormalisation scheme contains the specific choice of which parameters to renormalise and what renormalisation conditions to use for them. The renormalisation scheme specifies the definition of the renormalised parameters and gives the relationship between physical observables and renormalised parameters. Common renormalisation schemes include the modified minimal subtraction ($\overline{\text{MS}}$) scheme and the on-shell scheme. In the former one removes only the divergent parts of the loop divergences (and some additional ubiquitous finite parameters) whereas in the latter one fixes the renormalised quantities to be equal to the physical quantities (so that the renormalised mass is equal to the physical pole mass).

A common technique for regulating divergences is dimensional regularisation, where one analytically continues the number of space-time dimensions away from four. This explicitly breaks SUSY since there is no longer an equal number of bosonic and fermionic degrees of freedom when the vector fields are no longer four-dimensional. One can then not use renormalisation schemes that rely on dimensional regularisation, such as the $\overline{\text{MS}}$ scheme. A solution to this problem is to use the dimensional reduction (DR) renormalisation scheme, or the modified DR scheme ($\overline{\text{DR}}$) [205]. This scheme is similar to $\overline{\text{MS}}$ but the dimension of vector fields is also continued away from four with the resulting difference in degrees of freedom accounted for by auxiliary scalar fields. The $\overline{\text{DR}}$ scheme has been extensively used, despite inconsistencies in the original formulation at higher orders [206], which have however since been resolved [207]. Nevertheless, as emphasised in e.g. the Supersymmetry Parameter Analysis (SPA) project [208], a further understanding of possible inconsistencies of $\overline{\text{DR}}$ is still necessary.

In addition to the scale dependence, there can also be a dependence on the scheme, since values of renormalised parameters generally differ between schemes, and also a residual scheme dependence will remain at a finite order. The dependence of scale and scheme is attributed to the higher orders not included and should decrease when one includes higher orders.
6.3.1 Renormalisation in DM@NLO

The default renormalisation scheme in DM@NLO is a hybrid scheme where some parameters are defined with conditions from the on-shell scheme and some are defined with DR conditions. The on-shell conditions are applied to the top mass and the masses of squarks whereas remaining quantities are renormalised with DR conditions. Note that only QCD interactions are affected by the renormalisation, so that relevant quantities like for example the neutralino masses are not renormalised as they do not receive QCD corrections.

There is also the option in DM@NLO to use strictly the DR scheme for all parameters. This provides an opportunity to investigate the impact that the renormalisation scheme has on physical quantities, as e.g. cross sections. The scale and scheme dependence and the resulting theoretical uncertainty in DM@NLO calculations has been investigated in Ref. [200], showing that in the studied cases, the theoretical uncertainty can be about six times larger than the experimental one.

We will below compare the default renormalisation scheme with the pure DR scheme. Since mass parameters are defined differently depending on the renormalisation scheme we here summarise how the mass parameters are defined in the two schemes:

**Pure DR scheme.** The four lightest quark masses (up, down, charm, strange) are the same as the input (from DarkSUSY). Top and bottom masses are set to the DR masses evaluated at 1 TeV. All squark masses are then redefined by diagonalising the squark mass matrices using the DR masses for top and bottom.

**Default scheme.** The light quark masses are the same as the input. The pole mass is used for the top quark. For the bottom quark, the DR mass evaluated at 1 TeV is used. The masses passed over from DarkSUSY are used for first and second generation squarks as well as for three of the four third generation squarks, with the fourth one determined from the stop and sbottom mass matrices by rediagonalising using the top pole mass and the bottom DR mass in the matrix elements (this also redefines the third generation mixing).
6.4 Results

In this section we present results of comparisons between DarkSUSY and DM@NLO, first looking at the case where the tree level expressions in DM@NLO are used, and then showing the impact of using the full NLO expressions in DM@NLO. As mentioned above, the models are defined in DarkSUSY by reading the model files in SLHA (SUSY Les Houches Accord) format for the three benchmark scenarios into DarkSUSY. This defines all masses and mixings as well as the SM input parameters.

We do not use the Yukawa matrices in the SLHA files, since these are given at the scale $M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, where $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ are the two stop masses, and the scale $M_{\text{SUSY}}$ is for the models in questions much higher than the relevant scales for the relic density calculation for the high sfermion masses considered. We instead set the Yukawa couplings from the masses, using the running mass apart from some special cases. This will only impact the Yukawa coupling in DarkSUSY however—in DM@NLO the Yukawa couplings are set in different ways depending on the renormalisation scheme.

In DM@NLO, a diagonal CKM matrix is assumed, which is not the case in DarkSUSY. Therefore we set the CKM matrix diagonal also in DarkSUSY in all our comparisons below so that the differences we observe can be attributed primarily to differences between the codes, such as the higher order effects. A non-diagonal CKM matrix can change the affected cross sections with a factor of about 5% relative to the diagonal case.

6.4.1 Tree level comparisons

In this section we present some results on comparisons between DarkSUSY and DM@NLO, where we use the tree level cross sections from DM@NLO. Such a comparison is a useful calibration procedure which is also absolutely necessary to be able to make sure that the interface is working correctly and to understand the impact of adding NLO corrections. In Fig. 6.3 we show the tree-level Feynman diagrams for all the processes where we use DM@NLO cross sections.

Naively we might expect the DarkSUSY and DM@NLO tree level results to be equal, since DarkSUSY uses tree level cross sections.
6.4. Results

Figure 6.3 – Tree-level Feynman diagrams for the processes that we include from DM@NLO.
However, what one means by tree level is not unambiguous. Often, effects of higher orders are included with the use of running couplings (which include some corrections to all orders by the resummation of certain diagrams) and sometimes other higher order effects are included. Because of this, we can not generally expect that the results will agree when we compare DarkSUSY results to results with DM@NLO tree level cross sections. However, when we turn off these partial higher order effects, the codes should agree to reasonable accuracy.

Although we do not include NLO corrections in the DM@NLO results shown in this section, the renormalisation scheme used still plays a role. This is because the masses are redefined in different ways depending on the scheme, as mentioned above, so that different masses are used in the matrix elements in DM@NLO depending on the renormalisation scheme used.

In DarkSUSY, running couplings and masses are included, using the \( \overline{\text{MS}} \) scheme expressions with SM particles contributing to the running. All running is evaluated at a scale \( \mu_{\text{DS}} = 2m_{\chi^0_1} \), which means that the same scale is used regardless of the properties of the process in question, such as e.g. \( \sqrt{s} \). Hence, SUSY particles are not accounted for in the running parameters. This is different from DM@NLO, where the running quantities can include effects from the full MSSM spectrum, however also at a single energy scale of 1 TeV. In DM@NLO, the bottom and top masses also depend on the renormalisation scheme used. In the default renormalisation scheme, the \( \overline{\text{DR}} \) mass at 1 TeV is used for the bottom mass whereas the pole mass is used for the top mass.

Both codes run the strong coupling constant. In DarkSUSY, running only takes into account SM particles whereas in DM@NLO the running \( \alpha_s \) is defined in \( \overline{\text{DR}} \) with all SUSY particles contributing [209, 210]. However, this will not cause any difference between the codes at tree level, since \( \alpha_s \) does not enter at tree level in any of the neutralino and chargino coannihilation processes where we use DM@NLO.

The Yukawa couplings for quarks are treated differently in the codes and this can lead to rather large differences at tree level. In DarkSUSY, the default is to define the Yukawa couplings through the running masses, evaluated at \( \mu_{\text{DS}} \). In DM@NLO, Yukawa cou-
plings are instead always defined with the mass parameters, whatever they are set to be. In particular, the pole mass will be used for the top and the DR mass at 1 TeV for the bottom with the default renormalisation scheme. The difference between the masses can be rather large, for example the ratio between the top masses is \( m_t^{\text{pole}} / m_t^{\overline{\text{MS}}} (2 m_{\chi_1^0}) \sim 1.06 - 1.18 \) for the three scenarios considered.

With the above considerations taken into account, we expect agreement between DarkSUSY and DM@NLO tree level cross sections when we turn off all running and do not use the renormalisation scheme in DM@NLO so that the same set of parameter values are used in both codes.

**Relevant cross sections**

In Figs. 6.4 and 6.5 we show the annihilation rates for some selected processes, where the annihilation rate \( W_{ij \rightarrow kl} \) is defined as

\[
W_{ij \rightarrow kl} = 4 E_i E_j \sigma_{ij \rightarrow kl} v_{ij}
\]  

(6.8)

for a process \( ij \rightarrow kl \), where the initial states have energies \( E_i \) and \( E_j \) and relative velocity \( v_{ij} \). We show \( W_{ij} \) for some of the more important processes that contribute to \( W_{\text{eff}} \) for the three scenarios considered. We have picked processes where we have the possibility to use DM@NLO cross sections which are relevant for the determination of \( \langle \sigma v \rangle \) in these models. We show in the figures also the thermal distribution—by which we mean the expression shown in Eq. (6.4) without the \( W_{\text{eff}} \)—normalised such that it has a maximum value of 0.06 on the vertical axis. This gives an indication of which parts of the annihilation rate are most important in the determination of the thermal average.

We have compared the annihilation rates for two different cases. In the first case running is turned off in DarkSUSY and the renormalisation scheme in DM@NLO is not used, so that the same parameter values are used in both codes. In the second case the default running is used in DarkSUSY and the renormalisation scheme used in DM@NLO, so that some parameters are changed with respect to the value provided from DarkSUSY. A superscript of zero means that all running has been turned off, using the pole masses for top and bottom both for masses in kinematical factors and in Yukawa couplings.
Fig. 6.4 shows the rates for this case. We can see that, as expected, the rates agree very well in this case, with a ratio that is unity up to less than one percent.

We show in Fig. 6.5 the same processes in the second case considered: with running turned on in DarkSUSY and DM@NLO, using the default renormalisation scheme in DM@NLO. The difference compared to the case with running turned off is that now running masses are used for charm, bottom and top quarks in the Yukawa couplings in DarkSUSY. In DM@NLO the DR mass evaluated at 1 TeV is used for the bottom quark mass and the pole mass for the top quark mass.

When running is turned on, the codes give slightly more different results for the processes shown in Fig. 6.5, although the differences are still on the level of a few percent, with the exception of scenario III just at the charged Higgs resonance. The additional differences compared to no running are likely caused by the different treatment of the Yukawa couplings and masses for the quarks. The Yukawa couplings appear in in the s-channel Higgs exchange diagrams and in the t-channel squark exchange diagrams through the Higgsino part of the neutralino- or chargino-quark-squark coupling. Since there are differences in the treatment of Yukawa couplings it is expected that differences will arise, in particular at the position of the Higgs resonance where the Yukawa coupling is most important.

We have also checked more generally the agreement between the codes for all the processes where the initial state is a combination of $\chi^0_1$, $\chi^0_2$ or $\chi^0_1$ for all quark final states. These will be the most relevant processes where we use DM@NLO since the masses of $\chi^0_1$, $\chi^0_2$ and $\chi^0_1$ are close in all models so that $\langle \sigma_v \rangle$ will be dominated by coannihilations between these. The heavier initial states will barely contribute to $\langle \sigma_v \rangle$ as they are exponentially suppressed by their masses.

We find then that for all relevant processes the codes agree to very good accuracy (below one percent) when running is turned off and the renormalisation scheme in DM@NLO is not used. When running in DarkSUSY is turned on and either the default or the pure DR scheme is used in DM@NLO, there are some notable differences apart from the processes shown in Fig. 6.5. For example, in scenario III, the $\chi^0_1 \chi^0_1 \rightarrow c\bar{c}$ process (not shown in the plots) differs by a factor $\sim 10$. This can be explained by the fact that DarkSUSY uses the running
6.4. Results

(a) Scenario I, the chargino coannihilation model.

(b) Scenario II, the $Z^0/h^0$ funnel model.

(c) Scenario III, the $A^0/H^0$ funnel model.

Figure 6.4 – A comparison of the annihilation rates obtained with DarkSUSY and DM@NLO for some of the more important processes in the three scenarios with all running effects turned off. The processes agree to less than one percent.
(a) Scenario I, the chargino coannihilation model.

(b) Scenario II, the $Z^0/h^0$ funnel model.

(c) Scenario III, the $A^0/H^0$ funnel model.

Figure 6.5 – A comparison of the annihilation rates obtained with DarkSUSY and DM@NLO for some of the more important processes in the three scenarios, with running of masses and couplings turned on.
charm mass in the Yukawa coupling, whereas the pole mass is used in DM@NLO for the charm Yukawa coupling. In the model considered, the ratio between the two masses at the scale $2m_{\chi^0_1}$ used in DarkSUSY is $m_c(2m_{\chi^0_1})/m_c^{\text{pole}} \sim 3.5$. Therefore the annihilation rates, which depend on the Yukawa couplings squared, can be expected to differ by a ratio up to $\sim 3.5^2 \approx 11.9$, when the rate is dominated by the Yukawa contribution. However, we note that the contribution of this particular process to the total annihilation rate is small, so this rather large difference does not to any significant extent impact the obtained value for the relic abundance. The other differences that show up are at most a factor of about two and as expected, they all relate to the final states containing the heavy quarks (charm, bottom, top) where the differences in the quark masses used in the two codes are most significant.

### Relic abundance

We show in Tab. 6.3 the calculated values for the relic abundance for a number of different cases. The two uppermost values are the values calculated when running is turned off and the DM@NLO renormalisation scheme not used (so that the same parameter values are used in both codes). These values should be very nearly equal, and therefore provide a cross-check that everything is working as expected. Indeed we see that the values agree very well when using only DarkSUSY and DarkSUSY together with DM@NLO respectively with the differences below the percent level.

The three lowermost values in Tab. 6.3 show values obtained with running turned on in DarkSUSY and the renormalisation scheme used in DM@NLO. The differences are then slightly larger, but still at most on the percent level. In fact, the difference could be expected to be somewhat larger, given that many of the individual annihilation rates $W_{ij}$ differ by several percent or more in this case. It seems that these differences either relate to processes that do not contribute much to the determination of $\Omega h^2$, or that the differences cancel out when all processes are combined in the calculation of $\langle \sigma v \rangle$.

We note however, that the obtained values of $\Omega h^2$ differs from the values obtained in the GAMBIT study, as shown in Tab. 6.2. The difference is 4-5% in all three scenarios. One reason for this can be
that the model parameters are not exactly the same in the two cases. In the GAMBIT study the input parameters at the scale 1 TeV in Tab. 6.1 are used to obtain a mass spectrum at 1 TeV, whereas here we use the values in the SLHA files that are run to the scale $M_{\text{SUSY}}$, something that will affect the masses and mixings that enter into the amplitudes. Another difference that can lead to different values of $\Omega h^2$ is that the GAMBIT study calculates $\Omega_\chi h^2$ with MicrOMEGAs rather than DarkSUSY. Differences between these codes in the numerical implementation of the solution of the Boltzmann equation can result in different values of $\Omega_\chi h^2$ even for the same model, especially for models where the $\Omega h^2$ calculation is more involved, such as those dominated by resonant annihilation and/or coannihilations [211].

<table>
<thead>
<tr>
<th>Case</th>
<th>Scenario I ($\chi_1^\pm$ coann.)</th>
<th>Scenario II ($h^0/Z^0$ funnel)</th>
<th>Scenario III ($H^0/A^0$ funnel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega h^2_{\text{DS}}$</td>
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<td>$7.987 \times 10^{-4}$</td>
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<tr>
<td>$\Omega h^2_{\text{DM@NLO}}$</td>
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<td>$7.967 \times 10^{-4}$</td>
<td>$1.465 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\Omega h^2_{\text{DS}}$</td>
<td>$7.702 \times 10^{-3}$</td>
<td>$7.986 \times 10^{-4}$</td>
<td>$1.472 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\Omega h^2_{\text{DM@NLO}}$</td>
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<td>$7.966 \times 10^{-4}$</td>
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<tr>
<td>$\Omega h^2_{\text{DM@NLO}}$</td>
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<td>$7.965 \times 10^{-4}$</td>
<td>$1.465 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 6.3 – The relic density, using only DarkSUSY or DarkSUSY with DM@NLO tree level cross sections replacing the processes in Eqs. (6.5)-(6.7). From top to bottom, the rows are $\Omega h^2$ calculated using: only DarkSUSY cross sections without running; DarkSUSY and DM@NLO cross sections without running; the standard DarkSUSY calculation including running; the DarkSUSY and DM@NLO calculation, using the default renormalisation scheme in DM@NLO, and finally the DarkSUSY and DM@NLO calculation, using the pure DR scheme in DM@NLO.
6.4. Results

6.4.2 NLO corrections

In this section we present results for scenarios I, II and III obtained when using the NLO cross sections in DM@NLO. These cross sections include all QCD corrections to the diagrams shown in Fig. 6.3.

We expect the results in this section to depend to a larger extent on the renormalisation scheme used in DM@NLO. This is because now the renormalisation scheme does not only affect a few of the mass values used in DM@NLO, but also the actual renormalisation of all the various diagrams included (i.e. how to get rid of the divergences in quantities like self-energies). Therefore it is more difficult to estimate how big impact the renormalisation scheme can have on the results. A general principle, however, is that if the difference between schemes is large, we expect even higher order corrections that are not included to be relevant for a given cross section.

Relevant annihilation rates

In Fig. 6.6 we show the individual annihilation rates for the same processes as shown in Figs. 6.4 and 6.5, using the NLO cross sections in DM@NLO and the standard DarkSUSY ones (including the standard treatment of running masses and couplings). We can see that the DM@NLO NLO annihilation rates are larger by a factor between about three to eight percent relative to the DarkSUSY rates for all the shown processes.

Relic abundance

We show in Tab. 6.4 the relic abundance as calculated with the NLO cross sections from DM@NLO replacing the processes in Eqs. (6.5)-(6.7), with DarkSUSY providing the remaining cross sections that go into the calculation of $\Omega h^2$. We show results using both the default renormalisation scheme in DM@NLO and the pure $\overline{\text{DR}}$ renormalisation scheme. Given that the relative differences between DarkSUSY and DM@NLO in the processes shown in Fig. 6.6 is at most about eight percent, we do not expect the relative difference between the values of $\Omega h^2$ to be larger than that. We expect $\Omega h^2$ to be smaller when using DM@NLO cross sections, since $\Omega h^2$ depends inversely on the sum of the cross sections. Indeed, we see in Tab. 6.4 that the values
6. Neutralino relic abundance including NLO-QCD corrections

(a) Scenario I, the chargino coannihilation model.

(b) Scenario II, the $Z^0/h^0$ funnel model.

(c) Scenario III, the $A^0/H^0$ funnel model.

Figure 6.6 – A comparison of the annihilation rates obtained with DarkSUSY and DM@NLO for some of the more important processes in the three scenarios, with running of masses and couplings turned on, using the NLO cross sections in DM@NLO with the default renormalisation scheme.
of $\Omega h^2$ when including DM@NLO cross sections is lower by a few percent than the result using only DarkSUSY.

We can also see that the difference between the two renormalisation schemes in DM@NLO is very small, which signals a stability in the NLO results, and indicates that for the models studied, higher order corrections that are not included are likely to be less important. However, one also needs to look at how the results change when varying the renormalisation scale in order to assess this aspect in full.

We show also the approximate time taken in seconds for the relic abundance calculations in Tab. 6.5. In order to perform scans at sufficiently many points in the MSSM parameter space with a tool like GAMBIT, it is important to be able to make the $\Omega h^2$ calculations relatively efficiently. Ideally the code should not spend more than about half a minute on each parameter point. We see in the table that it takes about two to three minutes for the NLO calculation of $\Omega h^2$. This likely needs to be optimised in a future implementation in a parameter scan framework.

<table>
<thead>
<tr>
<th>Case</th>
<th>Scenario I</th>
<th>Scenario II</th>
<th>Scenario III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega h^2_{\text{DS}}$</td>
<td>$7.702 \times 10^{-3}$</td>
<td>$7.986 \times 10^{-4}$</td>
<td>$1.472 \times 10^{-2}$</td>
</tr>
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<td>$\Omega h^2_{\text{NLO}}$</td>
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<td>$7.847 \times 10^{-4}$</td>
<td>$1.434 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 6.4 – The relic density, using only DarkSUSY or DarkSUSY with DM@NLO NLO cross sections replacing the processes in Eqs. (6.5)-(6.7). From top to bottom, the rows are $\Omega h^2$ calculated using: the standard DarkSUSY calculation including running; the DarkSUSY and DM@NLO calculation, using the default renormalisation scheme in DM@NLO; and finally the DarkSUSY and DM@NLO calculation, using the pure $\text{DR}$ scheme in DM@NLO. The relative difference between the DM@NLO and DarkSUSY results is about two percent in all cases.

\footnote{The calculations have been timed on a laptop.}
### Table 6.5

<table>
<thead>
<tr>
<th>Case</th>
<th>Scenario I</th>
<th>Scenario II</th>
<th>Scenario III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($\chi_1^\pm$ coann.)</td>
<td>($h^0/Z^0$ funnel)</td>
<td>($H^0/A^0$ funnel)</td>
</tr>
<tr>
<td>$\Omega h^2</td>
<td>_{DS}$</td>
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</tr>
<tr>
<td>$\Omega h^2</td>
<td>_{\text{DM@NLO}}^{\text{NLO}}$</td>
<td>175</td>
<td>147</td>
</tr>
<tr>
<td>$\Omega h^2</td>
<td>_{\text{NLO, DR}}^{\text{DR}}$</td>
<td>176</td>
<td>159</td>
</tr>
</tbody>
</table>

Table 6.5 – Approximate time in seconds needed to compute the relic abundance with a laptop for the various calculations described in the caption to Tab. 6.4.

### 6.5 Summary and outlook

In this chapter we have presented a study of the impact of NLO-QCD corrections on the calculation of the dark matter relic abundance $\Omega h^2$. We have looked at three MSSM scenarios that are best-fit points in a recent scan over the MSSM parameter space by the GAMBIT collaboration. The results indicate that for these models, the impact of NLO corrections is about two percent on the value of $\Omega h^2$. For individual annihilation rates (or equivalently cross sections) $W_{ij}$ the relative difference can be somewhat larger, extending to almost ten percent in some cases. The differences in the $\Omega h^2$ values for these models are several times smaller than for the models presented in e.g. Ref. [197], which looks at NLO corrections to similar processes to those we have considered here.

Given that the uncertainty in the $\Omega h^2$ value measured by the Planck experiment is on the percent level, we find here that the NLO corrections are slightly larger than the experimental uncertainty on the best observational estimate of $\Omega h^2$. Therefore a calculation using only tree level cross sections, as is the default in most codes that calculate $\Omega h^2$, can be considered to have a systematic “uncertainty”...
that is larger than the experimental one. We write here uncertainty within quotation marks since it is not an uncertainty in the usual sense: the NLO corrected value of an observable should in general be considered more close to the true value, so the NLO results are rather an indication of how much the result can change compared to the tree-level only results when doing a more complete calculation.

As there is no detection of supersymmetric particles as of yet, the main impact of including NLO corrections in the calculation of $\Omega h^2$ in a more general scan over the MSSM parameter space will be to slightly shift the allowed volumes in the parameter space compared to scans made with tree level cross sections. This is because the parameter values needed to reproduce the correct $\Omega h^2$ will differ slightly in the NLO calculation.

In a more general perspective, the fact that the experimental uncertainty on $\Omega h^2$ is now so small means that the sources of systematic uncertainties from the theoretical side have become more relevant to investigate. Interestingly, we have found in this study that the values of $\Omega h^2$ obtained in the GAMBIT study (where the code MicrOMEGAs was used) are about 5% different compared to the values obtained with DarkSUSY. One source of difference between the calculations is that the parameter values used for each model are slightly different as they are evaluated at different scales. Still, we emphasise that the difference between the GAMBIT study and this one is in fact larger than the difference between the LO and NLO calculations reported here.

One of the main motivations for this study was the aim to include NLO corrections in the relic density calculations in MSSM parameter scans, such as those performed by the GAMBIT collaboration. It is therefore of interest to know how global the results presented here are in the perspective of the whole MSSM parameter space. We find however, that it is not entirely straightforward to generalise the results presented here into a broader view on the MSSM parameter space. Although the NLO cross sections are all well-behaved and the impact of the renormalisation scheme is small for the models we have looked at here, we have seen during the study that some other benchmark models we have looked at show unphysical behaviour such as negative cross sections for some processes, and the presence of such a behaviour can depend on the renormalisation scheme. For
this reason it is a nontrivial task to extend the study into a general scan in the MSSM parameter space. A systematic method to analyse if pathologies are present for a given parameter point is then needed.

When it comes to scans in parameter space, another aspect is the computational expense of a calculation. In order to be viable from a computational perspective, a code must provide a value for the relic abundance for a point in parameter space in sufficiently short time. If each parameter point takes too long time, say over a minute, this limits the possible parameter space volume that can be scanned. For the three models we have looked at here, the NLO calculations of $\Omega h^2$ have taken around two to three minutes per model while the calculations using DarkSUSY have taken around one second per model. In an extension to more general parameter space scans, it is therefore crucial to investigate whether the NLO calculations can be optimised in any way.

Regardless of the practicalities of how NLO calculations can be included, it is important to continue pushing for the inclusion of NLO corrections in future dark matter studies and parameter scans, given that the effect of the corrections is generally larger than the experimental uncertainty in the Planck measurement. In the case that a dark matter particle is observed experimentally, NLO corrections are crucial, since they can have a significant impact on the values of the model parameters that are inferred from the measurements.
Chapter 7

Outlook and discussion

7.1 Summary and discussion

In this thesis we have investigated aspects of particle dark matter. We have provided an extensive overview of the field of dark matter, including a presentation of some important particle candidates and their production in the early Universe.

The attached papers make contributions to the field of dark matter indirect detection, and a monograph-type chapter contributes on the topic of dark matter production in the early Universe. Specifically, in Paper I we consider a background for the neutrino telescopes searching for neutrinos from dark matter annihilations in the Sun, consisting of neutrinos formed in cosmic ray cascades in the solar atmosphere. Compared to the important background coming from cosmic ray cascades in the Earth’s atmosphere, the solar atmospheric flux is problematic since it comes from the Sun itself. With the current angular and energy resolution available, a distinction between a solar atmospheric neutrino flux and a neutrino flux from dark matter annihilations may prove difficult.

In Paper II we study a modification of the standard scenario for dark matter annihilation in the Sun, where dark matter particles in the solar core annihilate into a new type of long-lived mediator particle instead of Standard Model particles. The mediators then propagate away from the annihilation point and decay into Standard Model particles. This reduces the impact of neutrino interactions with the solar material and generally results in a more energetic neutrino flux.
Another important aspect is that the long-lived mediator scenario opens up for new experimental prospects—if mediators decay outside the Sun, gamma rays and charged particles are not stopped and can propagate to the Earth where they can be discovered in telescopes. We make important contributions in terms of a comparison of the prospects of gamma ray observations compared to neutrino observations, and see that gamma ray experiments are in general very competitive for the long-lived mediator scenario, even when mediator decays happen primarily inside the Sun.

In indirect searches, one needs to know the yields of the secondary particles of interest, such as neutrinos, gamma rays or charged antiparticles, at the source where the dark matter annihilations take place. Typically, particle physics codes are utilised to simulate the dark matter annihilations since the particle physics is generally complex to model, and some parts of the simulations rely on phenomenological modelling of the underlying physics. In Paper III, we study the effect on the secondary particle yields of changing the simulation code, in order to get an estimate of the impact of this phenomenological modelling.

We also study another aspect of the annihilations in Paper III, namely spin polarisation of the final state particles in the annihilation, which is expected in some dark matter scenarios. With polarised final states, the shape of the secondary particle energy spectra can change significantly. We investigate how much difference the final state polarisation has on the spectra and specifically look at the effect on neutrino fluxes from dark matter annihilations in the Sun.

The canonical mechanism to explain the dark matter abundance today is the thermal freeze-out mechanism. This explains the dark matter abundance as a competition in the early Universe between dark matter annihilations and the expansion of the Universe. At some point, the expansion rate overcomes the annihilation rate and the annihilations stop, with a remaining dark matter abundance, set by the size of the dark matter annihilation cross section.

We have discussed the production of dark matter in the early Universe through the freeze-out process in several chapters in this thesis, and make new contributions to the topic in Ch. 6. In this chapter we examine the influence of higher order corrections to the cross sections relevant in the freeze-out calculation, in a supersym-
metric model with the neutralino as the dark matter particle. With the experimental measurements of the abundance from the Planck satellite reaching percent levels [8], better accuracy in the theoretical prediction of the abundance is highly relevant. Apart from the annihilations between neutralinos, so-called coannihilations involving other particles in the model, play an important role. We find that in the specific supersymmetric model realisations studied, the higher order corrections change the abundance by a few percent. In a future implementation in a scanning framework, where one studies many more model realisations in the supersymmetric parameter space, optimisations of the calculations are most likely needed.

7.2 Outlook

There are several interesting and relevant opportunities for future studies related to the research contained in this thesis. When it comes to the solar dark matter searches, a better understanding of the background and in particular the solar gamma ray flux is necessary in order to pin down the effect of the solar magnetic field, which affects also the solar atmospheric neutrino flux. There are aspects of the observed gamma ray flux which are not understood and to be able to progress, more input, both in the form of experimental data as well as theoretical modelling, is required. The solar magnetic field affects the cosmic ray cascades as they propagate in the outer parts of the Sun, but the complexities of the magnetic field make it difficult to model this effect in detail. Given that the fluxes of gamma rays and neutrinos formed in such cascades are relevant backgrounds in solar dark matter searches (in the case of gamma rays, only in a scenario where dark matter annihilations can result in gamma rays, such as the long-lived mediator scenario), the full characterisation of the magnetic effect is crucial in order to disentangle a potential future dark matter signal from the background.

A natural extension of the analysis of Paper II is to study also charged particles from the mediator decays, such as positrons or antiprotons. In this case, one has to take into account the heliomagnetic effects on the propagation of these particles from the mediator decays to an Earth-based telescope.

Given that different particle physics simulation codes often use
similar data to tune their phenomenological parameters, the difference between predictions from different codes is not necessarily a complete estimate of the uncertainty in the modelling. A future study could consider the effect of changing the tuning of the parameters. Moreover, we have in Paper III studied a subset of the possibilities for particle type and polarisation of the final state, chosen to highlight the polarisation effects and the differences caused by the phenomenological modelling in the event generator. In a future analysis, one can consider a more complete set of particle type, spin and polarisation of the final states, to fully map out the possibilities for this. This could contribute to creating a user-friendly tool that allows for the calculation of yields from an arbitrary set of parameters, by interpolating from tables of simulation data (i.e. an update of what is currently implemented in e.g. DarkSUSY). Providing the difference between event generators can then give the user an idea of the uncertainties in the yields.

When it comes to higher order corrections in relic abundance calculations, an important future goal is to study more globally the effect of higher order corrections in the MSSM parameter space as a whole. As well as looking at the actual effect of the higher order corrections, it is important to make sure that the higher order correction calculations are theoretically sound, by for example considering different renormalisation schemes and scales. Any regions of parameter space where e.g. the renormalisation scheme makes a big difference should be mapped out and isolated. However, given the computational intensity of parameter scans, the higher order calculations most likely need to be optimised in order to be possible to use in any larger scan.

On a more general note, the search for particle dark matter continues. Although the WIMP hypothesis, and perhaps in particular the supersymmetric neutralino WIMP, is under some strain due to continued non-discovery, it is important to remember that the WIMP hypothesis remains fully viable to this day. It is necessary to complete and follow through with the WIMP searches in order to make sure that one has fully explored at the very least the thermal WIMP hypothesis (where the WIMPs are produced in a thermal freeze-out process), something which will most likely not be completely accomplished by current experiments. However, given that the ob-
servations of dark matter do not tell us much about the specifics of a dark matter particle physics model, it is important to also search broadly for other types of dark matter than WIMPs.

The dark matter problem remains today as one of the largest unsolved problems in physics. Experiments are now starting to probe primarily the WIMP hypothesis, but also other hypotheses, in more detail. There remains much to be understood in this interesting interplay between the fundamental physics of elementary particles and the cosmological and astrophysical aspects of our Universe.
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Part II

Papers