“Here’s the truth … if roundabouts had been invented sooner the whole of the western civilisation would be different.” In The Gap of Time by Jeanette Winterson (2015, p. 141), the fateful meeting between Oedipus and Laius in a crossroad is used as a metaphor for unnecessary confrontations in life—‘unnecessary’ meaning that they could easily have been avoided by means of simple rearrangements. Winterson uses the roundabouts metaphor to manifest smooth rearrangements that divert young men from acting in a rash manner. In this article, we describe how distance education (DE) became a ‘roundabout’ in an educational feedback situation for a young male with impulse control disorder (ICD). The setting of the situation is that of the Swedish prison and probation service, where the young male convict was following the upper secondary level mathematics program. The ICD did not cause any obstacles for instruction in general. However, when it came to the face-to-face feedback situation, where the teacher had to point to weaknesses in the student’s written solutions in the hand-in mathematics assignments, he would time and again lose his temper, raise his voice, and even leave the room while slamming the door behind him. Nevertheless, the situation changed dramatically when he was transferred to a different prison and from there continued his mathematical studies in a DE format. Not only did he manifest a change in his attitude toward the lessons and the feedback process, but there is also evidence of a change of perspective on the mathematical problem solving activities themselves and the purpose in engaging in them. Now, DE is usually regarded a necessary means for propagating education and knowledge to people who are geographically dispersed or people who have no access to educational centers. Furthermore, DE is often considered ‘next best’ to traditional teaching, where teacher and student are physically present at the same time and place. The ‘roundabout’ of providing DE described in this article is an example of the exact opposite. In particular

Distance mathematics education and emotional factors
DE has made it easier to provide mathematical instruction to communities located in remote or rural areas. In particular

Internet-based or online DE has been conceived as a way of instruction offering a myriad of possibilities for teaching mathematics, not least due to the possibility of offering open mathematics courses that can be accessed by large numbers of geographically dispersed people, and this without formally being enrolled in an educational institution or program (Engelbrecht & Harding, 2005; Borba et al., 2016). Despite the potentials offered by DE, the possibility of using it to provide mathematics education in contexts where face-to-face interactions may be challenging for learners, as in the case of ICD, has not previously been considered. More generally speaking, the emotional experiences associated with remote mathematical instruction and learning have only seldom been studied. The few investigations available have been developed in online instructional scenarios, and focus on analyzing the relationships between emotional factors and student performance in mathematics courses. For example, Cho and Heron (2015) study the extent to which the components of self-regulated learning—which includes negative emotions towards mathematics—predict student achievement in a remedial mathematics course. Similarly, Kim, Park and Cozart (2014) analyze if and how achievement-related emotions such as anxiety, anger, shame, hopelessness, boredom, enjoyment and pride, predict student achievement in online mathematics courses. This article offers a different perspective to the study of emotions in mathematics DE: we explore how characteristics of DE may be helpful to ameliorate negative emotional responses originating from the process of studying mathematics.

Swedish prison mathematics educational program
In all of Sweden’s 47 prisons, convicts are offered the possibility to study lower and/or upper secondary level mathematics. The target group is adults without an upper secondary diploma, and of special focus are young adults who have difficulties in entering the job market. All studies are planned in consultation with a study- and careers adviser on the basis of the student’s goals for education. All teaching is organized as one-on-one teaching, if there is a teacher employed at the given prison. Otherwise, teaching is organized as DE, where the teacher and student communicate through an intranet and via telephone.

In Sweden it is quite common that convicts serve their time in more than one prison. Transitions are often motivated by security concerns; for example, well-behaved convicts are moved to prisons with lower security. This may
have as a consequence that a convict who enrolls in a mathematics course with one-on-one teaching may have to complete the course through DE. For the sake of continuity, teachers ‘follow’ their students through transitions in the prison system until the desired educational level is reached. Courses can only be terminated on request from the convict, in case he or she no longer wishes to study, or if the teacher sees insurmountable obstacles for completing the course in an orderly fashion.

The case of Andreas
A fair share of the convicts choose mathematics without really wanting to study the subject, simply because mathematics is a prerequisite both for entering employment training programs provided by the government and for entering many tertiary educational programs. But even though they lack interest in mathematics as a subject, they are often highly motivated to pass their math courses in order to reach their future educational goals. This was the case with Andreas, a young male of age 25, diagnosed with ICD and at the time serving a 2-year sentence in a low-security prison. During planning with the study- and careers adviser it became evident that Andreas was in need of three courses of upper secondary mathematics to enter the tertiary program he wished to attend after serving his sentence. Andreas had completed the first of these in ordinary upper secondary school.

In March 2016, Andreas enrolled in the second math course. He initially described himself as a very intelligent and high achieving student, who due to “circumstances” had failed to complete upper secondary school. However, when his teacher (Linda Marie Ahl) tested his understanding of various mathematical concepts, she found understanding in line with curriculum goals for middle school. For that reason she included a great deal of basic mathematics from the lower secondary school curriculum and the first upper secondary mathematics course in his individual study plan. This made him annoyed and rather frustrated, since he “just wanted to pass the course” and “wasn’t interested in doing stuff that didn’t belong in the course”.

Understanding and applying quadratic functions is to some extent the core of the second course. Since it had become obvious that he had a weak conceptualization of linear functions, Linda explained to him that understanding the concept of quadratic functions required an understanding of the concepts of variable and linear function. She gave Andreas short lectures and he had to read instructions in supplementary textbooks and do carefully selected challenging assignments, all with the aim of using and applying linear functions in modeling and problem solving situations. The quality of his answers determined the next step in instruction. When he handed in his written solutions, Andreas would receive oral feedback from Linda on his strengths and weaknesses. Also, she asked him to orally account for his mathematical reasoning.

As mentioned, Andreas’ ICD would cause him act very impulsively, even aggressively, in these feedback situations. Needless to say, this made the feedback situations far more ‘exciting’ than one would normally expect. When Linda told him that something in a solution was inadequate or incorrect, Andreas immediately began to defend himself. On a regular basis, he would interrupt Linda or get very upset and extremely loud. His state of mind meant that he would generally have great difficulties listening to constructive criticism. And the feedback situations oftentimes ended with Andreas storms out of the classroom, slamming the door. Some 15 minutes later he would return remorsefully and ask for forgiveness for his behavior. This pattern continued day after day. While the relationship between the teacher and student in general was characterized by mutual respect and easy conversations, the feedback situation per se constituted a major barrier and resulted in rather limited mathematical development for Andreas. Hence, Linda decided to talk to him about cancelling the course.

Before this happened, however, Andreas was transferred to another prison, which in his case also meant having to follow the remainder of the math course as DE. The actual setup was similar, however. Andreas would get instructions on what to study in various textbooks and was given assignments, only now he had to hand them in via an asynchronous intranet forum. Linda would also provide feedback in written form via this forum, and once a week they would have an ordinary telephone conversation, i.e., no video, to discuss his work.

An asynchronous feedback situation
Surprisingly, in the DE setting all the previous communication and behavioral issues in the feedback situations ceased. In this new setting, Andreas stayed calm, and acted both interested and thoughtfully. Reflecting upon the difference between the traditional teaching setting and that of DE, it seems clear that one major change was the introduction of delays in the feedback situation. We provide an illustrative example from the intranet forum.

Linda For a system of linear equations: Determine constant A so that lines \( Ax + 5y + C = 0 \) and \( 2x + 4y + D = 0 \), lacks a solution. (2016-05-16, 13:16)

Andreas Here comes my answer. [Andreas attaches a scanned document with an incorrect answer] (14:49)

Linda Andreas, your answer is incorrect. I suggest you write the equations on the form \( y = kx + m \) and consider the properties of parallel lines. (15:15)

Andreas Yeah, parallel lines have the same \( k \)-value, so \( A = 2 \). (15:47)

Linda was then occupied with other tasks and students, meaning that Andreas had to wait for two days for feedback on his work.

Linda It is correct that they have the same \( k \)-value, but \( A \) is not 2. Do the task again and consider the values of the constants \( C \) and \( D \). (2016-05-18, 08:59)

The next day, Andreas replied:

Andreas I spent last evening thinking about the task, and I felt that there was something that wasn’t
right. Then I figured it out... felt rather stupid, actually... Here is the new answer, which I’m sure is correct. (2016-05-19, 13:09)

Andreas then provided a quite long answer containing correct reasoning, but still with some calculation errors that were discussed over the telephone later in the same day. However, the answer itself is not the point here. The point is that this kind of feedback provided in the traditional face-to-face setting would have set off Andreas’ impulsive aggressive behavior, while with the built-in delays of the asynchronous intranet forum he was thoughtful and reflective. Over time Andreas developed a deeper interest for truly understanding the mathematics in question, and his attitude changed from only wanting to pass the course to taking a genuine interest in the mathematical activities, which the following example illustrates.

An example of Andreas’ mathematical achievements
Andreas was given an assignment which included an optimization problem. Since he had not yet been introduced to differential calculus, Linda’s intention was that he should use his CAS-tool to find the maximum of the volume function. Alternatively, he could try out different integer solutions to estimate an approximate maximum, as for example \((x, y) = (4, 1144)\). No instructions or clues on how to solve the task were provided (See Figure 1).

Andreas was puzzled by the task, and he later explained that he felt as if no method was appropriate for the problem. This led him to look in the textbooks at the library for similar problems. He found what he was searching for in the mathematics textbook for the third upper secondary course. In this book, he studied the entire chapter on differentiation and was able to apply this in his solution of the task (See Figure 2).

If Andreas was puzzled by the task, Linda was even more puzzled by his answer! During a telephone conversation, she asked him to explain his solution. He told her that at first he suspected that she had given him a wrong task. But he wanted to manage to solve it anyway, so he went to the library. When he read about differentiation in the textbook, he became curious and wanted to know more. This led him to study the entire chapter and eventually solve the task.

DE, ICD and changing attitude
In connection to writing this article, Linda had the opportunity to interview Andreas briefly, over the telephone. He explained that he is by nature very proud and that he felt both stupid and humiliated in the face-to-face feedback situations—that the situation itself promoted his outbursts and caused him to lose his temper time and again.

Linda Have you thought about your emotional outbursts? When you were here.

Andreas Yeah, I like talking on the phone better [pause] and that you call me [pause] and the [intranet] forum is good too.

Linda Why is it working so much better for you, do you think?

Andreas [Getting a bit upset] It’s humiliating [pause] not to know [pause] I felt stupid [pause] do you understand?

Linda Okay. I understand. But it’s working just fine now though.

Linda cut short the interview as Andreas was becoming upset. Clearly it is not productive to have Andreas lose face in this situation, since it may have affected the now well-functioning DE teacher-student relationship.

Besides the DE setting having a positive impact on Andreas’ ICD in the situations of teaching and feedback, DE also appeared to have a positive impact on his attitude towards the mathematical activities, as mentioned above. This change may to some extent be explained by the model of Di Martino and Zan (2010), who look at the interrelations between; (1) a person’s own perceived competence (can do/cannot do); (2) the person’s vision of mathematics (mathematics is...); and (3) the person’s emotional disposition towards mathematics (like/dislike). A change in either of these may affect the two others, and in total this may change a person’s attitude towards mathematics.

Figure 1. The task reads: “From an A4 size paper you are to fold a box by cutting out a square piece of each corner. What is the largest volume of the box that can be obtained in this manner?”.

Figure 2. The unexpected part of Andreas’ solution, where he differentiates the volume function of the box. The text in the excerpt reads: “The volume of the box is...; Which gives the volume function...; This is a function of power 3 with a maximum for volume. You can find maximum by differentiating. Hence, I differentiate \(V(x)\)....”.
Firstly, besides learning about differentiation in the process of solving the optimization task, Andreas certainly also developed his *problem tackling competency* (Niss & Højgaard, 2011), where competency is thought to be one’s “well-informed readiness to act appropriately in situations involving a certain type of mathematical challenge” (p. 49, italics removed). The problem tackling competency involves the ability to detect, formulate, delimit, and specify different kinds of mathematical problems as well as the ability to solve mathematical problems that have already been formulated by oneself or by others. An important aspect of this competency is that the word ‘problem’ is relative to the person who is trying to solve a task; what to one person is a routine task may be a problem to another, and vice versa. In the case of Andreas, there is no question that the optimization task posed a ‘problem’ to him. The fact that he described himself as being “puzzled” when faced with the task underpins this. Højgaard (2010) refers to such a feeling of being “cognitively stuck” and mastering the ability to cope and overcome it as “the crux of mathematical problem solving competency” (p. 260). Højgaard uses “mathematization competency” to describe someone’s insightful readiness to solve problems defined as such by a challenge to mathematize. More loosely speaking, mathematization competency is the combination of mathematical problem solving competency and mathematization.” (p. 260, italics in original). Clearly, the optimization task invites Andreas to perform a mathematization, where he defines the volume function, \( V(x) \), and next applies differentiation as part of the solution to the problem. Hence, it seems fair to say that as a consequence of the DE setting, Andreas developed his mathematical problem tackling competency (in the area of mathematical functions and basic differential calculus) and his mathematization competency, most likely illustrating to himself in the process that he actually ‘can do’.

Secondly, somewhere in that process Andreas appears to change his initial vision of mathematics as something you merely have to pass to now wanting to pursue extra-curricular literature on his own, and solving a mathematical task in a much more advanced manner than intended as part of the course. One potential way to address his development may be through Skemp’s (1976) distinction between instrumental understanding, *i.e.*, knowing what to do, and relational understanding, *i.e.*, “knowing both what to do and why” (p. 20). On his own initiative, Andreas changed from being content with an instrumental understanding leading to correct answers to the assigned tasks, to aiming more at a relational understanding. Of course, it may be argued that Andreas was in fact chasing the ‘more correct’ answer to the optimization problem. But the process he went through in order to obtain this, as well as how pleased he was with providing his teacher with this answer, suggests a reorientation on his behalf directed towards trying to understand the ‘why’. Furthermore, according to Linda, Andreas’ work with differential calculus seemed to deepen his understanding of some of the underlying concepts, including that of linear functions. In the interview, he said: “Just think that some time ago I was one big question mark, when I saw an expression like \( y = kx + m \), and now [breaks off]” Referring to Sfard (1991), we know that lower-level mathematical objects, in this case linear functions, may become reified as a consequence of working with them as part of higher-level processes, here differentiation. But the interesting thing in Andreas’ case is that it was the setting surrounding DE, which brought with it the potential opportunity for such reification to take place. Hence, the DE setting may have provided Andreas with a ‘space’ in which he could evolve and grow mathematically.

Thirdly, due to Andreas’ ICD, his emotional disposition towards mathematics appears of course a bit more complex than merely “liking” or “disliking” the subject. Hannula (2002) argues that a negative disposition towards mathematics may function as a successful defense strategy of a positive self-concept. Indeed, this seems to fit quite well with Andreas prior to the DE setting. The later observation, during DE, of Andreas’ more positive disposition towards mathematics and his more reflective learning preference (aiming for a more relational understanding) is one that is also found by Idil, Narli and Aksoy (2016), who document the presence of a similar relationship among 702 middle school pupils in Turkey. These observations along with the change of Andreas’ emotional behavior in the feedback situations do imply a change of attitude, which is further supported by Hannula’s (2002) stressing of emotions as an indicator of change of attitude. Furthermore, the study by Hannula (2002) also shows that a student’s attitude towards mathematics may sometimes change dramatically in a relatively short time span. This too is the case of Andreas, and may possibly reflect the sudden change from a traditional teaching setting to a DE setting.

We believe, nevertheless, that it is not only the asynchronous DE environment and its built-in delays as such that favored the positive development. Both concerning Andreas’ mathematical knowledge and his attitudes toward the study of mathematics, we believe that the mode of feedback he received also has been instrumental in achieving a progress—not least that of the telephone. According to Weld (2014), feedback based on audio can have at least three advantages over written feedback, namely: (1) An audio message (or communication) adds personality and a human character to the teacher feedback. This personalization of the critique serves a dual purpose of both better capturing students’ attention and enhancing the relationship between student and instructor (p. 515). (2) It may help to clarify the reasoning underlying the teacher’s criticisms. Written feedback may signal deficiencies in the process of a mathematical solution “but rely on their audience to interpret how exactly they went wrong” (p. 515). (3) The audio feedback can be complementary and subsequent to the written feedback. This encourages the student to read the comments on his work before the feedback via telephone, and then to revisit and reengage with the corrections of his assignments when receiving the feedback over the telephone. Still, the use of audio feedback in combination with the DE setting was what appeared to work in the case of Andreas.

Finally, another part of the explanation as to why the DE environment improved Andreas’ emotional experiences in studying mathematics may lie in the fact that the expression of human emotions is affected by the modes of access to interpersonal feedback. According to Parkinson (2008),
“communication technologies also impact on our mode of contact with others, thereby influencing our emotional presentations towards those others.” (p. 1516). Interactions in face-to-face scenarios, as those experienced by Andreas initially, promote the production of emotions—many of them impulsive and unmediated—through a co-regulated process in which the emotions of individuals influence one another over time. However, such processes “are only available in certain modes of interpersonal contact, and mediated communication may disallow their operation.” (p. 1515). In other words, Andreas’ aggressive expressions towards the received feedback were impulsive and unpremeditated, and could have been influenced by Linda’s nonverbal expressions, vocal timbre and pitch. But in a remote scenario, such as the intranet forum, this type of expressions have to be manifested in a more explicit and conscious way and thus lack the immediate reaction of the interlocutor and the nonverbal cues that favor their development. In short, we believe that the characteristics of the distance communication between Andreas and Linda inhibited the free development of his emotional outbursts and impulsive behavior.

Unforeseen potentials of DE
As previously stated, DE is usually regarded as a measure taken to provide mathematics education to people who are geographically disperse or people who have no access to educational centers. However, in our case DE served as a means to provide mathematical education to a person with a special mental condition, a person who might not have “survived” (academically speaking) in the traditional educational setting. So, on the one hand, the case of Andreas tells the story of an individual who is willing to drive his own learning and comes to prefer a non face-to-face environment to do this within. On the other hand, it also tells the story of a young man, who belongs to a marginalized group with lesser opportunities to access mathematics education, not least due to his interactive challenges. It is clear that the DE provided by the Swedish prison system’s education program allowed Andreas to get access to a continuous mathematical instruction, despite the transitions between prisons that he experienced. Also, the DE context seems to have increased the learning opportunities for Andreas, not least by providing him with a ‘space’ in which he, to a larger extent, could control his ICD-related outbursts. Last but not least, thanks to the mathematical instruction received at a distance, the opportunities for Andreas to access higher levels of education within the prison educational program increased. In fact, by November, 2016, Andreas completed the third and final mathematics course, achieving the highest possible grade. Upon completing his conviction, he said he would “survived” (academically speaking) in the traditional educational setting may be successful in functioning as a mechanism for providing delays between stimuli, as it did for Andreas. And who knows, maybe DE has further unforeseen potentials, e.g., in relation to ‘math anxiety’ where it has been suggested that DE and online environments may provide support for students who exhibit anxiety and other poor attitudes toward mathematics (Taylor & Mohr, 2001; Hodges & Kim, 2013). Finally, the combination of audio feedback with DE, as discussed above, may also be a potential way of reducing the negative effect of math anxiety, which is in alignment with research studies by Nuñez-Peña, Bono and Suárez-Pellicioni (2015).

Returning to the outset of the article; Oedipus killed his father in a crossroad. Taking a more modest claim than Winston, who foresaw that manslaughter could have been avoided with roundabouts, we believe that the ‘roundabout’ experienced by Andreas describes circumstances and a potential solution that may apply to students in many educational practices. DE possesses several somewhat unforeseen potentials having to do with mathematical education of individuals, who due to their psychological and social conditions are marginalized or disadvantaged in one way or another. With this article, we hope to draw the attention of the mathematics education community towards these potentials—potentials highly relevant for those interested in providing students with a better and more just mathematical education, regardless of their social and psychological conditions.

References


