

Varieties of Necessity in John Buridan

Logic and Natural Philosophy in the Late Middle Ages

Guido Alt



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Abstract

This dissertation is a study of John Buridan's (c.1300-c.1361) conception of modalities. Modal concepts - concepts of necessity, possibility, impossibility, and contingency - describe the ways in which things could and could not be otherwise. These concepts became notoriously central for philosophical discourse in the late Middle Ages. In recent years, Buridan's philosophy and modal theory have received sophisticated scholarly attention. The main contribution of the dissertation is to show new ways in which Buridan's modal theory is embedded in its contextual practical aims, as providing methods for argumentation schemes and analysis used in his natural philosophy and metaphysics.

The dissertation is divided into two parts. In Part I, I conduct a detailed analysis of Buridan's account of varieties of modality in logical contexts. In Chapter 2, I show that Buridan distinguishes between broad and restricted forms of necessity in his treatment of logical consequence. Moreover, I show how the distinction between these forms of necessity underpins his modal syllogistics. I argue in Chapter 3 that Buridan acknowledged a variety of modal concepts that are distinguished as a matter of degree. I identify the main modal concepts Buridan's theory reckons with, show how he motivates the distinctions among them, and clarify how they are logically related. Part II turns to applications of Buridan's modal analyses to natural philosophy. In Chapter 4, I address the relationship between necessity with sempiternal truth in Buridan's commentary on Aristotle's De Caelo and compare Buridan's treatment of a key passage in that commentary with the treatment by John of Jandun (c. 1285-1328), a near-contemporary master of arts at Paris. Chapter 5 focuses on Buridan's account of the relationship between power-based concepts of modality and his modal semantics. Chapter 6 describes Buridan's account of contingency in the Physics, and sets Buridan's account of the relationship between forms of contingency and chance against the background of a received debate between Avicenna's and Averroes' views on the subject. Finally, in Chapter 7, I analyse some important applications of Buridan's distinction between logical and metaphysical possibility in physical contexts. I conclude this section by showing how Buridan considered merely conceivable possibilities useful in natural philosophy, and draw further conclusions for investigating the connections between logic and natural philosophy in the later Middle Ages.

Keywords: History of Philosophy, John Buridan, Medieval Philosophy, Medieval Logic, Natural Philosophy, Nominalism, Modality, Natural Necessity, History of Logic, Philosophy in the Fourteenth Century.

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Para Helena.

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1. Introduction

1.1 Aims and Scope of the Study

Modal discourse is about the ways things might or might not have been, about what could or could not be otherwise. In the late Middle Ages, modal concepts - concepts of necessity, possibility, contingency and impossibility - have notoriously come to the center of attention in the domains of theology, logic and natural philosophy. From the perspective of the time, these concepts were central for philosophical treatments of issues thinkers of the period cared deeply about, such as scientific knowledge, freedom of action, and the investigation of the natural world.

In the intellectual world of the fourteenth-century in particular, scholastic philosophers shared the belief that the world could have been radically different from how it is. Belief on the radical contingency of the world is manifested in various forms in the culture of the period more broadly, and merely conceivable possibilities had a grip on medieval imagination. Theologians stretched divine power to embrace all logical possibilities, sculptors carved up beasts in cathedrals displaying possible uncreated animals, and jurists referred to the temporary suspension of moral laws by extraordinary powers. This generalization serves to give a first impression of the manifold reflections that the belief in the contingency of the world has had also on more technical aspects of philosophical discourse in the late middle ages.

A crucial distinction in modal philosophy is between forms of necessity and possibility. Things are said to be necessary in different ways - truths of mathematics or logic are said to be necessary in a stronger sense than truths of the natural world. For example, water boils when heated at a certain temperature

¹For the pervasive belief on the radical contingency of the world among late medieval scholastics, see Courtenay [1985], 15-16; Gelber [2004], 1-4; Knuuttila [1993], 138-155, and Normore [1985].

is necessary, but things could be otherwise were the laws of nature different. Thinking that the natural world could be otherwise than it is is an intuition that goes back to medieval philosophy. In the fourteenth-century, the distinction between logical and natural or metaphysical possibility played a manifold role in philosophical discourse. For example, metaphysical impossibilities were progressively incorporated in methods of philosophical inquiry 'according to imagination' (*secundum imaginationem*). The acknowledged usefulness of operating with impossible hypotheses in philosophical and scientific discourse had a counterpart in sophisticated theories of modality drawn from the logical textbooks of the period, which provided analytical techniques and methods for the purpose of application to a broad range of areas in philosophy, theology and natural philosophy.

John Buridan (†c.1361), a later medieval arts master active at the University of Paris, exercised an important influence in shaping philosophical discourse of the period in many areas. Scholars have often noted that through his logical works and commentaries on Aristotle, Buridan has had an important role in crystallizing the distinction between logical and natural or metaphysical possibilities in the fourteenth-century. Moreover, due to his special position as an arts master - not having produced any work on theology - a complex interaction between philosophical and theological modes of argumentation can

²Not all thought experiments 'according to imagination' (*secundum imaginationem*) concerned metaphysical impossibilities. But a great part of them which will be relevant in this thesis presuppose a distinction between metaphysical and merely conceivable possibilities. On *secundum imaginationem* procedures in medieval philosophy, see [Hugonnard-Roche] [1989], [King] [1991], [Grellard] [2011], and for a survey of imaginable impossibilities in medieval philosophy, see [Binini] [2022].

³Zupko [2003] presents the most comprehensive assessment of Buridan in this regard. Klima [2009] focuses on some key areas of Buridan's thinking which resemble modern interests.

⁴This role was foremost emphasized and brought to light by Simo Knuuttila, who devoted a series of studies on Buridan's distinction between logical and natural or metaphysical modalities. See for example, early on Knuuttila [1989], Knuuttila [1989], and also Knuuttila [1989], 155-162 for the summary of the findings of these earlier studies. Joël Biard has contributed to understanding Buridan's speaking of 'natural orders' Biard [2001], Biard [2012]. Recently, Calvin Normore and Robert Pasnau have approached Buridan's theory of varieties of modalities, Normore [2013] and Pasnau [2020]. In addition, several studies on more technical aspects of Buridan's modal theory will be cited below.

be found in Buridan which sets him apart from other scholastic philosophers.

This dissertation attempts to address some of the foundational questions concerning Buridan's modal philosophy: How did Buridan conceive necessity, possibility, and contingency? What functions his modal analyses have in his philosophical program? Buridan's treatment of modalities has received sophisticated scholarly attention in the recent decades. The contribution this dissertation aims at making is to clarify the functions of Buridan's modal analyses in natural philosophy and metaphysics. Most scholarly treatments of Buridan's theory of modality have focused on the technical aspects of his modal theory, and on his modal syllogistics in particular. As a result of the recent contributions, scholars have come to acknowledge that Buridan's modal syllogistics systematized a tradition of medieval logic, and it is widely acknowledged as presenting one of the most refined systems in the late medieval period.

One of the reasons for investigating Buridan's modal theory from logical and in non-logical sources is that Buridan often conceived his modal analyses as methodological tools, used in order to address philosophical problems of metaphysics and natural philosophy. According to the opening paragraph of Buridan's main logical textbook (the *Summulae de Dialecticae*, or *Summaries of Logic*), logic was regarded as a practical science, in the sense that it was foremost concerned with clarifying propositions, judging the validity of arguments, and with assessing the principles pertaining to any inquiry.

In order to investigate Buridan's theory of modality from the perspective of his logical and non-logical texts, I have divided the dissertation in two parts. Part I investigates Buridan's account of varieties of modality. I will be concerned

⁵See Sylla [2001].

⁶For studies on Buridan's logic as a whole, see, for example, King [1985], Klima [2009] ch.2, Read [2015c], Zupko [2003], chs. 1-9. For studies focusing on Buridan's modal theory in particular, see Knuuttila [1993], Knuuttila [2001], Lagerlund [2000], ch. 5, Normore [2013], and Pasnau [2020].

⁷See, for example, Lagerlund [2000], 136-192; King [1985]; Thom [2003], Johnston [2015a].

The opening paragraph of the *Summulae* states that logic (or 'dialectic') was meant to guide science by serving as a method to distinguish truth from falsity, and good from bad arguments (Buridan [2001b], 1-4; Buridan [2005], 7-6). For Buridan's conception of logic is practical science, see Zupko [2003], 29-32.

with the question of how Buridan distinguishes between different modal spaces, and with some of the main motivations and functions of his modal analysis. My overarching goal in Part I is to identify the main kinds of modal concepts operative in Buridan's philosophy and investigate the logical relationships which obtain between these concepts. In Part II, I will be concerned with the question of how Buridan conceptualizes modalities in his natural philosophy, that is, in his commentaries on Aristotle's writings. In this part, I will address how Buridan's distinction between varieties of modality is used in schemes of argument in natural philosophy, and attempt to situate what is distinctive to Buridan's account of natural necessity and contingency, by contrasting Buridan's commentaries on Aristotle with some of his near contemporary Parisian arts masters.

As we will see, Buridan acknowledges a variety of forms of necessity. At the highest end there are logical necessities, those which not even God could upset. These are followed by natural or metaphysical necessities, which are inviolable by natural powers, although they are logically contingent. Logical and metaphysical necessities are categories promptly recognized by contemporary philosophers. Still, medieval modal concepts are not quite the same as their contemporary counterparts. For example, as we will see, Buridan's theory also includes the necessity of the past, which is notoriously alien to contemporary standard treatments of modality.

Nonetheless, there are certainly reasons for philosophers today to care about medieval discussions of modality. The first is a historical one. The historical roots of the contemporary distinction between logical and natural or metaphysical necessity is much indebted to the medievals - even if they are to be found in unexpected places. These roots are found in the progressive use of acknowledged metaphysical impossibilities in philosophy and theology, which were thought to be possible by divine power. Medieval theologians understood that distinction primarily in the context of God's omnipotence, which was

⁹The terminology itself started in the late medieval period, with Scotus' coinage of the terms 'logical possibility' and 'metaphysical' or 'real possibility.' See King [2001b], Cross [2015].

of the past among medieval philosophers, and the rejoinder from a contemporary perspective by Ahmed [2020], 263-272. We shall see below in 3.5 and 3.6 in which ways Buridan regarded the past as necessary - but only in the weakest grade of necessity.

considered from two perspectives - in the sense of absolute power and in the sense of ordained power. It and this distinction acquired a definite shape in the writings of late fourteenth century figures such as Duns Scotus († 1308) and William of Ockham († 1357). What the notion of absolute power emphasized was the possibility of acting against or outside natural laws by extraordinary or miraculous divine action. Dut talk about absolute power was talk about absolute or logical possibility - about what is 'to repugnant to be' (*quod non repugnat esse*) in a well-known formula.

Secondly, Buridan's modal theory has features of contemporary relevance. Buridan has been foremost appreciated as a logician. His treatment of logical consequence has been interpreted a predecessor to modern accounts. ¹⁴ and his modal syllogistics systematizes a tradition of modal logic which bears some similarities to systems of quantified modal logic developed in the twentieth century. ¹⁵

Since this dissertation focuses on Buridan's modal theory both from logical and non-logical perspectives, some different aspects of Buridan's philosophy will be treated together. It will be helpful to briefly summarize some of the aspects of Buridan's modal theory that this dissertation attempts to clarify:

(I) The concepts of modality that Buridan's theory acknowledges. Buridan's theory of varieties of necessity incorporates different ways of carving out modal space, and I will discuss how he thinks about the logical relationships between different forms of necessity across his writings. For example, both

¹¹The conceptual groundwork of that distinction was already available in the twelfth century. According to Courtenay, Gilbert of Poitiers († 1154) was the first to use the term 'absolute' as a qualification of divine power, and William of Auxerre († 1231) "acknowledged God's power to have acted otherwise, *de potentia pure considerata*, which added special meaning to the natural and moral orders God actually chose to establish, *de potestate determinata*" (Courtenay) [1985], 247).

¹²See Gelber [2004], 309-324, for an up to date summary of the centrality of Scotus and Ockham in reshaping the distinction between absolute and ordained power.

¹³Frequently, the formula is translated as 'what is not contradictory to be,' but the *caveat* should be made that it is not a syntactic understanding of inconsistency, but it is a semantic concept. For example, conceptual impossibilities are in Buridan's case beyond the realm of what God can bring about. See Knuuttila [1993], Cross [2015] and Pasnau [2020].

¹⁴See, e.g. Klima [2016].

¹⁵Lagerlund [2000], 130-162 for a comprehensive account, and Read [2020a].

in the *TC* and in the *SD*, Buridan distinguishes between broad (*ample*) and restricted (*restrictive*) acceptations of modal concepts. I will argue that thid distinction informs Buridan's treatment of logical consequence, and underpins his distinction of different kinds of consequence. Moreover, as we shall see, in the treatment of modal syllogistics contianed in *TC* IV, Buridan operated with a distinction between simple (*simpliciter*) and conditional (*condicionalis*) necessity. The import on reading the modal proposition provides alternative interpretations, one which favors a broader modal space quantifying over merely possible beings, and another which endorses a more restricted modal space quantifying over only actual beings.

The significance of these distinctions in logic shows Buridan's characteristic sensitivity to context of discourse. It also sets Buridan apart from his nominalist predecessor, William of Ockham, who tends to adopt a more restrictive approach to the semantics of modal propositions. [16]

In the *SD* 8, Buridan develops a fine grained treatment of varieties of modality, which he ranks by degree of modal force. Discussion of this passage forms the core of the first part of this dissertation. We will see that Buridan acknowledges four concepts of modality (displayed at Table 3.1): logical, natural, conditional and historical. The complex relationships between these concepts will be drawn. He suggests they come in degrees (*gradus*), and the logical relationships between these different grades of being necessary is a question that will be treated in connection with other writings, in particular his *QAPr*. Chapter 3, therefore, will be an attempt to comment on the forms of necessity and their degrees of being necessary across Buridan's writings.

(II) The motivations for Buridan's modal distinctions. The distinction between absolute and ordained powers we mentioned above laid a commonly shared understanding of logical and metaphysical or natural modal spaces at the time. But Buridan had often more immediate concerns in mind when he resorts to the distinction. For example, one of the reasons Buridan had for keeping logical and metaphysical modal spaces separate comes from his epistemological concerns. Buridan draws a famous division between forms of certainty in terms

¹⁶See Pasnau [2020]. There is some debate as to whether Ockham acknowledges merely possible beings in his ontology. For an argument that he does, see Panaccio [2019].

of these modal concepts. 17 and famously forcefully argued for keeping these domains separate against skeptic concerns of the period.

I will argue also that Buridan's modal analyses have a relevant methodological dimension. The application of modal analyses in terms of logical possibilities to natural philosophical issues is an important part of Buridan's philosophical methodology. I will attempt to highlight this aspect of Buridan's philosophy by comparing his approach to natural philosophical problems with alternative approaches contained in commentaries on Aristotle around the same time, in order to identify what is distinctive to Buridan's use of schemes of argument derived from his modal semantics in non-logical domains. For example, Buridan widely used schemes of arguments based on natural impossibilities, namely, schemes of argument which consist in reasoning from naturally impossible hypotheses. Chapters 4 and 7 provide case studies of some of the ways in which Buridan thought supernatural (logical) possibilities were philosophically useful, and even required, to make sense of key natural philosophical positions of the period. [18]

(III) The logical tools used by Burdan to distinguish modal spaces. In this study I aim also to show that Buridan's way to carve out different modal spaces was not solely based on the theory of absolute and ordained powers. As we have briefly noted above, that was a standard framework of the period, and Buridan largely appropriates schemes of argument derived from accounts of omnipotence. Nonetheless, Buridan never took a position on the main tenets of the theory of omnipotence, and he conceptualizes modal space instead in a distinctive way by referring to 'supernatural' and to 'natural' modes of argumentation. [19] More

¹⁷To be sure, Buridan's epistemology will not be treated in this dissertation. For central studies which I will rely on, see Zupko [2001], Zupko [1993a], Lagerlund [2010], Grellard [2014].

¹⁸In particular, in section 7.2 we will see how Buridan applies his analysis of modal propositions to a discussion of divisibilism - the thesis that continuous entities are infinitely divisible - and section 7.3 deals with Buridan's approach to the use of impossible hypotheses in indirect arguments in his commentary on the *Physics*.

¹⁹As Joël Biard argues, Buridan's language is rather different, and suggests (Biard [2001], 64) "a naturalist version of omnipotence built around the opposition between 'supernatural and miraculous' and 'natural' modes." I am using here the term 'secularized' - known that it may have misleading implications when it comes to the Middle Ages -, to avoid the label 'naturalist.'

often, Buridan used familiar semantic tools from medieval logic to distinguish modal spaces, such as suposition theory (roughly, the medieval counterpart of the modern notion of reference), and obligations logic (a logical genre in the Middle Ages that concerned dialectical argumentation). [20]

One important such semantic tool will be treated in 3.4. In order to keep natural necessities irreducible to a logical variety of conditional necessity, Buridan rehabilitates - against the predominant nominalist tendency of the time - a semantic theory that fell out of popularity in the fourteenth century, namely the theory of natural supposition. This theory in Buridan's treatment serves to give an account of how universal affirmatives expressing natural laws can be true when their subjects are be empty. Buridan finds in the theory of natural supposition a tool to preserve the necessity and truth of universal affirmatives expressing natural regularities, given that as such they are logically contingent and can be falsified.

I will therefore throughout the thesis seek to explain what is distinctive about Buridan's theory of modality. My specific aim will be on identifying the connections between Buridan's logical writings, and its applications in non-logical domains.

1.2 Buridan as a Master of Arts

John Buridan's life and writings are situated at the first half of the fourteenth century. As with many medieval figures, many important details of his life are not well known. Buridan was born in the historical French nation of Picardy, presumably at some time before 1300, ²² and he must have died sometime before

²⁰On particular feature of this genre is the usage of schemes of argument proceeding from impossible scenarios (*positio impossibilis*). In an obligational disputation, a form of institutionalized dialectical dispute in the medieval period, a respondent given such an impossible scenario is bound to reply according to what follows from assuming an impossibility according to logical rules. For conceptions of modality in obligations logic, see Yrjönsuuri [2007], Yrjönsuuri [2015] and Binini [2022].

²¹Note that in traditional logic, affirmatives have existential import. In order for an affirmative proposition to be true, the referent of the terms occurring in an assertoric proposition must exist. This will be explained in section 3.4.

²²For a profile, see Zupko [2003]. The still authoritative biography by Bern Michael suggests he was born "at the latest 1304/5, based on the fact that Buridan's inception in

1361. Buridan traced a career path which was highly unusual for a scholastic philosopher. For one, instead of moving to the faculty of theology, he remained active as a master of arts at the University of Paris throughout his career [23] As a result, his teaching and writings concerned almost exclusively logical textbooks and commentaries on Aristotle's works, and he has not lectured nor written a work on theology.

One aspect of Buridan's unusual scholastic career will be important in this thesis. When Buridan referred to what is logically possible or to what is possible according to God's omnipotence, he did so taking care to not enter into theological domain. As is well known, due to a vow undertook at inception at the Arts Faculty at the University of Paris, arts masters in Buridan's time were forbidden to determine on theological questions. This fact shapes Buridan's use of the distinction between logical and natural possibilities in distinctive ways. In the commentaries on Aristotle we will address, Buridan's purpose in drawing on natural impossibilities is purely philosophical. In that regard, Buridan had a role in crystallizing and 'secularizing' the distinction between logical and natural possibilities.

Buridan is best known for his nominalist semantics, and for turning nominalist positions - then foremost understood as the teachings associated with William Ockham, and as a reductionist program in ontology ²⁵ - into plausible contenders in the camp of ideas about how to do logic and philosophy. ²⁶ Nominalism is a contested category in the history of medieval philosophy, and its explanatory

the corporation of Parisian *Magistri* took place by 1325, and the receipt of a benefice "cum cura animarum" which was given to candidates around 25 years old" (cf. Michael | 1985||, 400.)

²³For example most scholastic Latin philosophers had a religious affiliation to a religious order, such as the Dominicans or Franciscans, but in this regard Buridan remained unaffiliated to a religious order and retained some independence from their philosophical orientations. See Zupko [2003], xii-xiii.

²⁴See Sylla [2001], 221-222 for a famous reference Buridan makes to this vow in his commentary on the *Physics*.

²⁵For the reception of Ockham's theory at Paris, see Courtenay [2008].

²⁶For an overview of the complex history of medieval nominalism, see Normore [1992] and Normore [2017]. Gyula Klima has studied in detail the differences between realist and nominalist commitments in medieval semantic theories, and has explored their mutual influences on the metaphysics of the period. See in particular Klima [2008].

value is lively debated still today in the scholarship. What can be safely said is that Buridan's appropriation of Ockhamist ideas is not so much driven by a reductionist program in ontology, as it is by a careful approach to philosophical problems by applying logic and the analysis of language. In the period Buridan was active, the key theories associated commonly with nominalism - such as the rejection of real universals - were in fact taken as a given. As a result of this, as Jack Zupko points out, Buridan saw no need to defend a reductionist program in philosophy, as much as to apply the tools of nominalist logic to concrete cases. Buridan's Parisian milieu was comprised of many scholars of similar nominalist orientation in logic. Buridan is associated with circle of other important figures within the Parisian milieu in the philosophical landscape of the fourteenth-century such as Albert of Saxony and Marsilius of Inghen.

In this thesis, I have sought to look at Buridan's milieu from an extended perspective, and I have chosen John of Jandun's commentaries as a point of contrast. [30] Jandun was not in the same intellectual wavelength as Buridan's - the former an Averroist, the latter a nominalist philosopher -, but both were primarily masters of arts at the University of Paris around the same time. The reason for choosing Jandun's commentary as a point of contrast in these places is twofold. Firstly, Jandun endorsed the principles of a temporal understanding of modalities, which he thought to be compatible with Aristotelian positions in natural philosophy. [31] Secondly, Jandun generally regarded the use of supernatural possibilities in natural philosophy as illegitimate - the temporal

²⁷Cf. Zupko [2003]. See also Normore [1981] for the primacy of language over ontology in Buridan's approach to ontology

²⁸In general, Buridan shares with the nominalist program a repudation of any "tendency of philosophers to reify terms and categories insisting that there be some kind of isomorphism between the world and the language we use to describe it (...) the proper antidote for this is to be found in a semantics of terms - or, at a higher level, a system of rules governing the analysis of propositions and arguments - which does not have the same ontology-enriching consequences" (Zupko) [2003], 162.)

²⁹For some time, Buridan was believed to have exercised a major influence and leading role on these thinkers. Nowadays, the circle of nominalist philosophers in Buridan's milieu has been shown to comprise a network of mutual influences. See Thijssen [2004].

³⁰For example, in chapters 4 and 7.

 $^{^{31}}$ As we will see, in *De Caelo* and in his commentary on the *Physics*. See below also 1.3

model that he believed compatible with Aristotelianism was thought to have an empirical basis on the natural world, whereas supernatural (or merely logical) possibilities are regarded as elusive and of no use in natural philosophy. As we will see, in his commentaries on Aristotle, Buridan exhibited a different attitude towards merely conceivable possibilities in natural philosophy. Instead of regarding supernatural possibilities as contrary to reason and based solely on faith, Buridan instead frequently uses counterfactual posits in natural philosophy as a method of investigation of nature.

1.3 Approaches to Medieval Modalities

Throughout this thesis, I will draw on many studies on Buridan's modal theory in particular. In this brief introductory section, it will be useful to depict the background of standard scholarly treatments and interpretations of medieval modalities in general, in order to introduce some of the categories that I will use through the thesis, and to place Buridan's modal theory within the context of general studies on this fourteenth-century accounts of modality.

According to the standard view on the history of medieval modal theories, the first half of the fourteenth century witnessed a conceptual shift. Due to the pivotal work of Simo Knuuttila on this issue, most scholars tend to interpret this conceptual change as a break from premodern and Aristotelian conceptions of modality, to a modern picture in which global conceptions of possibility emerged, and modal space was broadened to include a wider space of conceivable possibilities. In Knuuttila's view, to which we will turn often in this study, Buridan thought natural necessities were intimately connected with the unchangeability and immutability of the natural domain, which is

³²The standard book in this regard is Knuuttila [1993], see also Knuuttila [2012]. Knuuttila characterized the premodern, Aristotelian model as defining modality solely in temporal terms (sometimes referred to as 'temporal-frequency' model), and characterized a new model gradually emerging in the later medieval period as based on 'synchronic alternatives,' in which possibility is understood primarily on the basis of alternative histories or states of affairs which are not required to be realized in time.

In his later writings, Knuuttila did not attribute to Aristotle himself the temporal conception of modalities, but instead held that medieval thinkers thought it to be compatible with Aristotelianism. For key interpretations of the impacts of this conceptual shift, see also Normore [196b] and Gelber [2004].

outstripped by considering the many alternative ways things could have been.^[33]

In some parts of this dissertation, I will often use these categories as interpretative tools. However, I will argue that Buridan did not fully incorporate the temporal understanding of modality, although Knuuttila was right in that Buridan regarded a temporal understanding of modalities as compatible with Aristotelianism in natural philosophy. In particular, when we contrast Buridan's views with his Parisian near contemporary John of Jandun - who seemed to have endorsed the temporal model in his commentary on *De Caelo* -, we will see that Buridan conceptualizes the distinction between logical and natural modalities in original ways, and often explicitly criticizes some main tenets of the temporal model [34]

To be sure, the extent of the conceptual change that the fourteenth-century witnessed in modal theory is not wholly consensual. Robert Pasnau has recently forcefully challenged the assumption of a "radical break with Aristotelianism," and in a series of case studies he claimed that the alleged discontinuity in medieval modal thinking, which would justify comparisons with modern possible worlds semantics is "entirely a misimpression." Instead, what characterizes premodern modal discourse in Pasnau's interpretation is its "tendency to restrict attention to the actual world" One issue this thesis will explore which was absent from Pasnau's discussion of medieval modal theories in his most recent account is the prevalent notion use of 'supernatural cases,' namely of counterfactual and naturally impossible assumptions in medieval philosophy, to which Buridan so often resorts. As we will see, in his natural philosophy Buridan often makes claims that are unpacked as claims not only about how the world is the way it is, but how it could have been otherwise.

In general, in this study I will refrain from committing to a particular view concerning Buridan's role in the landscape of fourteenth century accounts of

³³See Knuuttila [1989] and Knuuttila [2001].

³⁴In chapter 4, we will see that Buridan's criticisms are mostly based on his logic of modal propositions, as developed in the *TC* and *SD*.

³⁵See Pasnau [2020] and the replies from Ahmed [2020].

³⁶Pasnau [2020], 226: "This tendency to work within a smaller modal space can make premodern modal talk look wholly alien, if not simply confused. In fact, however, these authors prescind from our wide-open modal spaces because they seek to adapt their modal discourse to the explanatory and linguistic demands of their context."

modality. Many of the innovative aspects of Buridan's logic are already well known and established in the literature. My aim will be rather limited, and I will attempt to assess Buridan's modal theory locally from the perspective of some of its key methodological dimensions. In doing so, I hope to contribute to the interpretation of Buridan's modal theory by shedding light on underexplored connections between his modal logic and its applications to natural philosophy.

1.4 Chapter Outline

The dissertation is divided in two parts. Part I analyses Buridan's discussion of modality in primarily logical contexts. Part II turns to some applications of the modal analyses in natural philosophical contexts. The scope of the study is to highlight where connections can be found.

Starting Part I, chapter 2 discusses Buridan's modal logic. I will clarify some main distinctions between modal concepts that Buridan makes in his account of logical consequence and in his interpretation of the modal proposition in his *TC*. This chapter introduces some key features of Buridan's semantics, the applications of which we will often return to in the remainder of the thesis. In Chapter 3, I discuss Buridan's elaborate account of varieties of necessity at a central passage in his treatise of demonstrations at *SD*. The remainder of the chapter will discuss each of the modal grades Buridan distinguishes, and give a synoptic account of these concepts in Table [3.1]. I will also address the question of the logical relationships between each modal concept Buridan distinguishes.

Starting Part II of this thesis, chapter 4 addresses the relationship of necessity and eternality or omnitemporal truth. The main focus of the chapter is to compare John of Jandun's and Buridan's approaches to this relationship in their commentaries on *De Caelo*. As we shall see, this contrast brings significant differences to the fore concerning the extent to which Buridan accepted the temporal model understanding of modalities.

In Chapter 5, I will turn to Buridan's account of the relationship between powers and modal concepts. In his questions on the *Physics*, Buridan associated with Aristotle and Averroes a usage of modalities based on powers. Nevertheless, I will argue that Buridan ultimately did not think of modalities as power-based, and point to some of the discrepancies between his semantic account of

modalities and powers-based conceptions.

In chapter 6, I will approach Buridan's logical and physical descriptions of contingency, with a special focus on his treatment of the problem of chance at *QPhys*. II.11. In that context, Buridan referred to a 'famous controversy' between the views of Avicenna and Averroes on this issue, which touched on the definitions of causal necessity and contingency. The controversy concerned whether chance events are to be classified in the category contingencies that happen rarely, or in the category of 'each-way contingencies' (*ad utrumlibet*). Ultimately, I will argue that Buridan's answer, according to which chance events can be the outcome of contingent causes, shows some limitations of the temporal understanding of modalities when it comes to Buridan's applications of modal concepts in natural philosophy.

Chapter 7 turns to some key methodological applications of the distinction between supernatural and natural possibilities in Buridan's natural philosophy. In this chapter, I will explore the ways in which Buridan thought counterpossible scenarios were useful in natural philosophy. In section 7.1 I argue that Buridan operated with an implicit distinction between logically and naturally impossible objects of signification. The next two sections explore ways in which Buridan uses his modal analyses in order to articulate two natural philosophical issues. First, I will address his application of the semantics of divided modal propositions to analyse divisibilist propositions in his *QPhys.* and in his *QDGC* (7.2). In the last section, we will address Buridan's distinction of logical and natural possibilities in the context of indirect schemes of argument (arguments proceeding from naturally impossible hypothesis) in his commentary on the *Physics*.

Part I Varieties of Modality in Buridan's Logic

2. Some Aspects of Buridan's Modal Logic

This chapter revisits some aspects of Buridan's logic, and addresses the problem of how he conceptualizes necessity and possibility in his logical writings. I aim to clarify how and why Buridan thought different modal spaces were needed to explain logical concepts, such as the treatment of consequence and the treatment of modal propositions figuring in his modal syllogistics. The main conclusion the chapter draws is that Buridan's distinction between different concepts of necessity and possibility has the function of meeting the explanatory requirements of central concepts of his logical theory: first, to account for the division of simple and as-of-now consequences (section 2.1). Secondly, to provide an interpretation of modal propositions as used in modal syllogistics (section 2.2). Finally, I will turn to a neglected aspect of Buridan's logical theory, namely his account of compossibility, and I will indicate how this account connects with applications of his modal analyses to natural philosophical problems (section 2.3).

2.1 Necessities and their Consequences

Buridan's main treatment of logical consequence is contained in his *Tractatus de Consequentiis*, written around 1335. This treatise is part of a broader tendency of writing small books specially on logical consequence, as opposed to larger and more comprehensive summaries. By the first half of the fourteenth-century, similar treatises on *consequentia* started to flourish and integrated late ancient and former medieval accounts of valid argument and inference into single unified

³⁷For the probable dating of the treatise, see Hubien's remarks on his Introduction to Buridan [1976], Ed. Hubien, 9.

treatments. Several treatises under the heading of *consequentiae* began to appear, whose content is the account of semantic principles and rules (*regulae*) governing consequence.

What are consequences? The mediaeval term for consequence (*consequentia*) was often used interchangeably with the terms 'inference' (*inferentia*), and 'illation' (*illatio*), referring to the process of drawing a conclusion form a set of premises (Klima [2016], Transl. Klima, 319). Buridan starts the chapter 3 of *TC* II saying that consequences are hypothetical constructions composed of two parts, namely antecedent and consequent. However, Buridan insists that not all conditional constructions are consequences strictly speaking. Conditionals are valid in virtue of the corresponding consequences they express whenever the consequent follows from the antecedent of the conditional.

³⁸For a summary of the content of these treatises, see Dutilh Novaes [2020]. The nature of the shift that later medieval accounts of consequence introduced has been widely acknowledged, and described in different ways. For example, (Kneale & Kneale [1971], 277) speak of a "change of fashion", comparable to the move from Aristotle's presentation of syllogistics to "Boethius' presentation by means of inference schemata", and referring to the former thirteenth-century realist Aristotelian accounts of validity as based on essences, Stump [1982] sees the change as a "gradual erosion of this Aristotelianism, and an increasing concentration on the nature and the rules for consequences" (286). For a reassessment of these earlier views, which characterizes consequences in the first half of the fourteenth century as proof theory, see King [2001a].

³⁹Thus, Buridan says that broadly speaking a consequence is "is constituted from several propositions conjoined by the expression 'if', or the expression 'therefore' or something equivalent. For these expressions mean that of propositions conjoined by them one follows from the other; and they differ in that the expression 'if means that the proposition immediately following it is the antecedent and the other the consequent, but the expression 'therefore' means the converse." (Buridan [2015b], Transl. Read, 66). Cf. Buridan [1976], Ed. Hubien, 21: "[...] est propositio hypothetica; constituta enim ex pluribus propositionibus coniunctis per hanc dictionem 'si' uel per hanc dictionem 'ergo' aut aequivalentem. Dictae enim dictiones designant quod propositionum per eas coniunctarum una sequatur ad aliam; et in hoc differunt quia haec dictio 'si' designat quod propositio sequens eam immediate sit antecedens et alia sit consequens, sed haec dictio 'ergo' designat econuerso." The mediaeval denotation of 'consequentiae' is quite broad. It covers both conditional constructions, as well as valid arguments more strictly (Boh [1982], 300-301). However, see King [2001a], 119-121, who makes a definite case for the mediaeval's distinction between consequence and inference. See also Ciola [2017], 436-443.

⁴⁰Buridan [2015b], Transl. Read. 66: "[...] in this treatise I shall mean by 'consequence' a true consequence, and by 'antecedent' and 'consequent' I shall mean

a consequence only obtains (*tenet*) or is good (*bona*) when antecedent and consequent stand in a special relationship.

This relationship is defined by Buridan by a Modal Criterion. Buridan defines the consequential relation at *TC* I.3 as follows (Buridan [2015b], Transl. Read, 67):

(T2.1) the one proposition is antecedent to the other proposition if it is impossible that it be true the other not being true when they are formed together [...]

one proposition is antecedent to another, which is such that it is impossible for things to be altogether as it signifies unless they are altogether as the other signifies when they are proposed together.

The criterion for consequence thus runs on modality. In a consequence, the antecedent is related to its consequent in such a way that it is *impossible* for things to be as it signifies unless they are as the consequent signifies - alternatively, it is necessary for things to be as the consequent signifies if they are as the antecedent signifies. [42]

The usage of the modal criterion has an important history, and it is not original to Buridan. Christopher Martin has argued 43 that it was already formulated by a group of logicians in the twelfth century exploring earlier accounts of implication. A variant of this criterion is explicitly used by Buridan as well in the text above, and throughout the TC. Buridan's theory of consequence is

propositions of which one follows from the other in a true or good consequence." Cf. (Buridan [1976], Ed. Hubien, 21).

⁴¹Buridan [1976], Ed. Hubien, 21-22: "illa propositio est antecedens ad aliam propositionem quam impossibile est esse ueram illa alia non existente uera illis simul formatis [...] "illa propositio est antecedens ad aliam quae sic se habet ad illem quod impossibile est qualitercumque ipsa significat sic esse quin qualitercumque illa alia significat sic ipsis simul propositis."

⁴²Note that necessity takes the wide scope over the whole conditional (*necessitas consequentiae*), instead of the narrow scope over the consequent (*necessitas consequentiis*). In other words, it is necessary that: if things are as the antecedent signifies, they are as the consequent signifies.

⁴³Martin [1986].

⁴⁴Normore [2015] emphasizes its centrality by saying that the *Tractatus de Consenquentiis* is an articulation of this criterion and its implications.

largely focused on semantic principles that the Modal Criterion validates.⁴⁵

The Modal Criterion is therefore at the heart of the consequential relation for Buridan. It will be useful to keep keep in mind its function: if a conditional meets the Modal Criterion, then a *consequentia* is said to hold. Hence we can summarize the criterion as follows:

Modal Criterion: Q follows from P if it is impossible things to be as P signifies without also being as Q signifies.

The relevant question in this regard is what conception of necessity and impossibility underpins Buridan's characterization of consequence. As Gyula Klima noted, the necessity of a consequence for medieval thinkers was not uniformly understood in the period. In that regard, 'logical' necessity could be based on, for example, essential and causal features connecting the antecedent and consequent, and in the later medieval and early modern period, 'logical' necessity was also taken to be based on psychological features.

The best way to address the question of what Buridan's understanding of the necessity of a consequence is, is by turning to Buridan's classification of kinds of consequence. We will see that all kinds of consequence are governed by the Modal Criterion, but for different reasons. In the following, two divisions between kinds of consequence that Buridan makes will be important: (I) the division of *formal* and *material* consequence, and (II) the division between *simple* and *as-of-now* consequence.

Let us start with the first division. For Buridan, an argument may be valid by two distinguishable dimensions of consequence: namely, an argument can be valid due to a consequence holding on formal grounds (*gratia formae*) or in virtue of a consequence holding on material grounds (*gratia materiae*). The notion of formal consequence was mentioned before in mediaeval Latin logic by Simon of Faversham († 1306). It was elaborated systematically by William of

⁴⁵Although earlier accounts tended to focus on 14th century logical treatises as presenting rule-based, axiomatic systems (Stump [1982]); King [2001a]), it is nowadays accepted that Buridan's approach focuses on presenting general semantic principles instead, cf. Normore [2015]; Thom [forthcoming].

⁴⁶See Klima [2016], 233; Green-Pedersen [1984], 279.

⁴⁷See Normore [1993].

⁴⁸The earliest occurrence of the phrase, approximating the Buridanian meaning

Ockham († 1347). It is well known that the notion of formal consequence is not used uniformily accross late medieval logic, and different traditions used different criteria to define the notion of formal consequence. [49]

A formal consequence is one that preserves truth in all of its equiform instances. In order to understand this, we need to make clear how Buridan's semantics distinguishes between categorematic and syncategorematic terms and concepts. In Buridan's usage, categorematic terms are expressions that signify simple mental concepts. Those terms can serve as subjects or predicates in propositions, such as 'man' and 'white.' Syncategorematic expressions, on the other hand, do not signify anything by themselves except when adjoined by other categorematic terms, and they have the effect of modifying the signification of the proposition or the reference of the terms in the context of the proposition in which they occur. Typical examples of syncategorematic terms are propositional connectives ('and', 'or', 'therefore'), negation, and quantifier signs.

we will deal with, appears in Simon of Faversham's questions on the *Sophistical Refutations* (*Quaestiones super libro Elenchorum*), noting that the argument 'an animal is a substance; therefore a man is a substance,' does not hold in virtue of form (*ratione formae*), since it holds only "for features which are essential," that is, due to the matter of the propositions (what they are about), whereas a formal consequence should hold in all terms and in all matter. For the relevant passage, see Martin [2005], and also for its history see Dutilh Novaes [2020].

⁴⁹For example, William of Ockham claims (Ockham [1974], Eds. Boehner *et al* 589.45-54) that a formal consequence 'holds immediately by an intrinsic middle' and 'mediately by an extrinsice middle', which is related to the 'form of the proposition.' It is unclear what Ockham means by 'form' here, but the idea of a consequence holding in virtue of an extrinsic or intrinsic middle suggests that Ockham does not have a notion of *logical* form the any modern sense in mind. For example, in Ockham's account 'Socrates does not run, therefore a human does not run' is a formal consequence, holding by an extrinsic middle (namely, 'Socrates is a human'), which is related to the general conditions of the proposition (*generales condiciones propositionum*). For a comparison of Ockham's and Buridan's diverging views on formal consequence, see Crimi [2018]. On the Oxford/Paris split on semantic theory more generally, see Libera [1982].

⁵⁰Buridan [2001b], Transl. Klima, 232: "Again, some incomplex utterances are categorematic, some are syncategorematic, and some are intermediate or mixed. The purely syncategorematic ones are so called because they signify nothing besides the concepts they immediately signify, except, perhaps, the things that the terms to which they are attached signify, as in the cases of the words 'not,' 'and,' 'or,' 'therefore,' and the like. The purely categorematic ones are so called because they do not signify only their concepts, which they immediately signify, but also the things conceived by these concepts, and can be subjects or predicates in themselves, and do not include any purely syncategorematic term, e.g., 'man,' 'stone,' 'whiteness,' 'white,' and the

With that distinction in mind, Buridan claims that propositions share the same *form* when they share the same syncategorematic arrangement: that is, the same copula, the order of terms, and the quantifier signs, [51] and he claims that a consequence is "called formal if it is valid in all terms retaining a similar form. Or if you want to put it explicitly, a formal consequence is one where every proposition similar in form that might be formed would be a good consequence" (Buridan [2015b], Transl. Read, 68).[52]

The Buridanian criterion for formal consequence is often referred to in the literature as the Substitutional Criterion (Dutilh Novaes [2020]). According to it, a consequence is valid if and only if it is valid in all substitutions of its (categorematic) terms, retaining a similar form. As an example of what Buridan has in mind, consider the argument below:

C1 Every human is running
Therefore, some human is running

The gist of the idea behind Buridan's understanding of formal consequence

like." Buridan [1998], Ed. Van der Lecq, 18: "Item, vocum incomplexarum quaedam sunt categorematicae, quaedam syncategorematicae et quaedam mediae seu mixtae. Dicuntur autem pure syncategorematicae, quia praeter conceptus quos immediate significant, nihil significant, nisi forte ea quae termini quibus adiujuntur significant, ut istae dictiones 'non,' 'et,' 'vel,' 'ergo,' 'omnis,' et huiusmodi. Dicuntur autem pure categorematicae, quia non solum significant conceptus quos immediate significant, sed etiam res illis conceptibus conceptas. Et sunt per se praedicabiles vel subicibiles et nullum purum syncategorema includunt, ut 'homo,' 'lapis,' 'albedo,' 'album,' et huiusmodi."

⁵¹Buridan writes the following concerning form (Buridan [2015b]), Transl. Read, 74): "I say that when we speak of matter and form, by the matter of a proposition or consequence we mean the purely categorematic terms, namely, the subject and predicate, setting aside the syncategoremes attached to them by which they are conjoined or denied, or distributed or given a kind of supposition; we say all the rest pertains to the form." Buridan [1976], Ed. Hubien, 30: "Et dico quod in proposito, prout de materia et forma hic loquimur, per 'materiam' propositionis aut consequentiae intelligimus terminos pure categorematicos, scilicet subiecta et praedicata, circumscriptis syncategorematicis sibi appositis, per quae ipsa coniunguntur aut negantur aut distribuuntur uel ad certum modum suppositionis trahuntur; sed ad formam pertinere dicimus totum residuum."

⁵²Cf. Buridan [1976], Ed. Hubien 22-3: "Consequentia 'formalis' uocatur quae in omnibus terminis ualet retenta forma consimili. Vel si uis expresse loqui de ui sermonis, consequentia formalis est cui omnis propositio similis in forma quae formaretur esset bona consequentia [...]"See further Karger [1993].

⁵³For further discussion of Buridan's criterion, and a comparison with Ockham's understanding of formal consequence, see Crimi [2018], Dutilh Novaes [2020].

seems clear: any replacement of its categorematic terms would yield a valid argument holding in virtue of form [54] In other words, a formally valid argument is valid because any categorematic term put in place of the schematic letters would yield a material consequence.

On the other hand, material consequences fail to be formal, since they do not uphold the Substitutional Criterion, but they fulfill the Modal Criterion laid out in (T2.1). One of the example Buridan provides is the following:

While it is impossible for Socrates to be a human without being an animal, this argument has invalid equiform instances. For example, the conditional 'if Jack is a philosopher, then Jack is a musician', shares the same form as the inference [C2], but it is invalid, for it is clearly possible for things to be as the antecedent signifies without being as the consequent does, even though both inferences share the same form.

Material consequences which are *simply* valid, Buridan writes, are those that can be made into formal consequences by adding a necessary premise [55] For example:

However, note that Buridan is not making claims about propositional schemata when discussing validity. His focus is on arguments (*argumenta*), which are valid or not in virtue of the corresponding consequences they are based on. See again Karger [1993].

⁵⁵Buridan [2015b], Transl. Read, 68: "It seems to me that no material consequence is evident in inference except by its reduction to a formal one. Now it is reduced to a formal one by the addition of some necessary proposition or propositions whose addition to the given antecedent produces a formal consequence." Buridan [1976], Ed. Hubien, 23: "Et uidetur mihi quod nulla consequentia materialis est euidens in inferendo nisi per reductionem eius ad formalem. Reducitur autem ad formalem per additionem alicuius propositionis necessariae uel aliquarum propositionum necessariarum quarum appositio ad antecedens assumptum reddit consequentiam formalem."

⁵⁴We should make the *proviso* here that, in traditional logic, the universal quantifier has existential import. Therefore, any instance of following scheme is valid:

In the pair of arguments above, [C2] expresses a simple material consequence, which can be transformed into a formal consequence as in [C3] by the addition of a necessary premise, namely 'Every human being is an animal.' What is obtained by reducing a material consequence to a formal one?

Buridan claims that by reducing a material consequence into a formal consequence, all that is obtained is the evidentness of the consequence, namely, that a consequence is 'perfected' when so transformed. We can find evidence for claim in his *Quaestiones Topicorum*. When discussing the *species* of argumentation, Buridan distinguishes two senses in which an argument is said to be perfect, namely a modal and an epistemic sense. In the modal sense, perfection means that a consequent *necessarily* follows from its antecedent. In the epistemic sense, perfection means that a consequent *evidently* follows from its antecedent. Buridan writes (Buridan | 2008b|), Ed. Green-Pedersen, 66-67):

(T2.2) an argumentation is said to be perfect with respect to inference (*illationem*) in two ways: in one way, when its conclusion is inferred by necessity, in another way when its conclusion is inferred evidently. That is, one way when the consequence, which entails the conclusion is necessary, in another way when it is evident. And the second way presupposes the first, but not conversely; it may very well be, that the conclusion of some consequence is necessitated by its premises, but that is not altogether evident; but in order for it to be evident, it is required that it be necessary. [56]

This passage suggests an answer to the question of whether formal and material consequences reflect a modal distinction between formal and material necessity. [57]

The answer seems to be the negative. The distinction between formal and material consequences is first and foremost an epistemic difference for two

⁵⁶"[...] argumentatio dicitur perfecta quantum ad illationem dupliciter: uno modo quia de necessitate infert suam conclusionem, alio modo quia evidenter sua conclusionem infert. Hoc est, quod consequentia, qua infertur conclusio, est necessaria, alio modo quod cum hoc sit evidens. Et secundus modus praesupponit primum, sed non e converso; potest enim sic esse, quod aliqua consequentia conclusionis ex praemissis est necessaria, non tamen est evidens; sed ad hoc, quod sit evidens, oportet, quod sit necessaria."

⁵⁷This interpretation was put forward by Angel D'Ors (D'Ors [1993]). We will turn to his interpretation shortly below.

reasons. First, there can be cases of necessary consequences which are not evident; they are not *perfect* in the epistemic sense. Second, when Buridan speaks of reducing a material consequence to a formal one, necessitation is not obtained by the addition of the further necessary premise. Rather, as a result of adding a further necessary premise, and reducing a materially valid consequence to a formal consequence, what is obtained is that the consequence becomes evident. Therefore, the necessity of a consequence, for Buridan, does not hinge primarily on its form.

Just what sense of modality Buridan's characterization of consequence is based on has been a matter of debate between two interpretations offered by Angel D'Ors and Calvin Normore respectively. Both interpretations suggest opposing answers to the question of whether formally valid inferences are valid in the same sense of *following* as materially valid inferences. In light of our previous observation concerning (T2.2), the difference between both is ultimately an epistemic distinction. But in order to articulate this position in more detail, it is important to address this debate.

The focus of the discussion presented by D'Ors and Normore is one of Buridan's principles on the TC, namely the principle that from an impossibility anything follows (ex impossibili quodlibet, henceforth EIQ). The EIQ is presented by Buridan as an immediate corollary of the Modal Criterion. Since the latter states that a consequence holds if it is impossible for things to be as the antecedent signifies without being as the consequent does, then if the antecedent is impossible, it is impossible for things to be as the antecedent signifies without also being as the consequent does, whatever the consequent may be. [59] Throughout the TC I.8 Buridan articulates the EIQ principle and argues for it.

⁵⁸In a passage which Calvin Normore points to (Normore [2015], 266), Buridan claims that material consequences are not *strenghtened* by adding further necessary premises. The passage is in *TC* I.8 (Buridan [2015b], Transl. Read, 78): "If from any proposition with some necessity or necessities adjoined to it there follows some conclusion, the same conclusion follows from the same proposition alone without the adjoining of that necessity or those necessities." Cf. Buridan [1976], Ed. Hubien, 36: "ad quamcumque propositionem cum aliqua necessaria sibi apposita uel aliquibus necessariis sibi appositis sequitur aliqua conclusio ad eandem propositionem solam squitur eadem conclusio, sine appositione illius necessariae uel illarum necessariarum."

⁵⁹Cf. Buridan [2015b], Transl. Read, 79: "[...] it immediately follows that anything follows from such [a proposition], indeed also from anything implying a contradiction, because any such [proposition] is impossible."

In particular, Buridan sets out to prove that from "every conjunctive proposition consisting of two mutual contradictories any other proposition follows in a formal consequence" (Buridan [2015b], Transl. Read, 79). In that context, Buridan presents a formal proof of the EIQ (Buridan [2015b], Transl. Read 79):

(T2.3) So I say that from this conjunction, "Every B is A and some B is not A", anything follows and in the same way in whatever terms this consequence is formed. Proof: for example, this follows: every B is A and some B is not A, so a stick stands in the corner. For from "Every B is A and some B is not A", it follows that every B is A, since from a conjunction each of its conjuncts follows. Then from "Every B is A" it follows that either every B is A or a stick stands in the corner, since anything implies itself in disjunction with anything else. Then from this and the second part of the original antecedent let me argue like this: every B is A or a stick stands in the corner; and some B is not A; so a stick stands in the corner. This is a case of disjunctive syllogism - that from a disjunction, if either disjunct is denied, the other may be inferred. So, from first to last, from the original antecedent the stated conclusion follows, because whatever follows from the consequent follows from the antecedent. [60]

Here, Buridan effectively provides a version of the same proof that was formulated earlier by William of Soissons in the twelfth century, which is transmitted through John of Salisbury's († c. 1180) *Metalogicon*. This argument is often referred to in the literature as 'William's Machine' (Martin [1986]), since it was compared to a siege engine which, as John of Salisbury reports, was

est A et quoddam B non est A' sequitur quaelibet, et eodem modo in quibuscumque terminis talis consequentia formetur. Probatio. Pono, gratia exempli, quod sequatur 'omne B est A et quoddam B non est A; ergo baculis stat in angulo'. Quia sequitur 'Omne B est A et quoddam B non est A; ergo omne B est A', quia ad copulatiuam sequitur quaelibet eius pars. Deinde sequitur 'omne B est A; ergo omne B est a uel baculus stat in angulo', quia ad quamlibet sequitur ipsamet sub disiunctione ad quamlibet aliam. Tunc ex ista et secunda parte primi antecedentis arguam sic: 'omne B est A uel baculus stat in angulo; et quoddam B non est A; ergo baculus stat in angulo'. Et iste syllogismus tenet per locum a diuisione, quia duobus positis sub disiunctione si alterum interimatur reliquum concludetur. Ergo, de primo ad ultimum, ex primo antecedente sequebatur dicta conclusio, quia quidquid sequitur ad consequens sequitur ad antecedens."

"produced for capturing, as his friends say, the citadel of the old logic, building up unexpected links of argument, and demolishing the opinions of the ancients" (*Metalogicon* II.7, *apud* Kneale & Kneale [1971], 201). Christopher Martin (Martin [1986]) has provided a reconstruction of the argument analogous to the derivation below:

Such an argument is comparable to the one Buridan presented at the text above. Crucial here are the rules of disjunction introduction and the disjunctive syllogism that can also be seen at [T2.3]. As we have seen, Buridan in this text Buridan presents a formal proof to the effect that, as a formal consequence, in a conditional in which its antecedents are two mutual contradictories, the consequence may be any sentence, and the conditional will still be true. That is, from such an antecedent anything follows.

On the basis of Buridan's presentation of this proof, D'Ors claims that Buridan's treatment of consequence operates with a distinct concept of impossibility, namely, formal impossibility (D'Ors [1993], 204-5). D'Ors points to a passage in the *TC* IV which, referring back to the formal proof of the EIQ, Buridan uses the term (*formalis impossibilitas*). Buridan claims there that

⁶¹Virtually the same argument was presented by C.I. Lewis in the twentieth century in Lewis & Langford [1959], 250-1.

are valid because their premises form a contradictory pair, Buridan writes (Buridan [2015b], Transl. Read, 114): "Again, other consequences are also formal on account of the formal impossibility of the premise or the formal necessity of the conclusion. For since from an impossibility anything follows and what is necessary follows from anything, if a proposition is impossible on account of its form there will be a formal consequence from it to anything, and if it is necessary on account of its form there will be a formal consequence from any other [proposition] to it. Now a conjunction made up of two contradictories or contraries is impossible on account of its form, as is [a proposition] in which some finite term is affirmed of its infinite counterpart

an impossibility on formal grounds (gratia formae) is one constituted by a conjunction of a proposition and its negation, and that anything follows from it - that is, for any arbitrary B, $(A \land \neg A) \rightarrow B$. Analogously, a necessity on formal grounds is constituted by a disjunction of a proposition with its negation, and any proposition can be antecedent to it - that is, for any arbitrary $B, B \rightarrow (A \vee \neg A)$. Ultimately, what D'Ors assessment of the EIQ purports to show is that Buridan has a notion of formal necessity and impossibility which is distinct from material possibility and impossibility. It is not uncommon to think at first glance that this corresponds to the distinction between logical and metaphysical modalities. Angel D'Ors has proposed that for Buridan there are as many senses of following as there are senses of necessity. He concludes that modal notions are ultimately equivocal for Buridan, and that in the articulation of the Modal Criterion the concept of impossibility "appears in all its multiple meanings," and that this multiplicity is "precisely that upon which the multiplicity of forms of antecedence, and derivatively of consequence is founded" (D'Ors [1993], 205).

If D'Ors interpretation is correct, my suggestion above would be misleading. However, there is an alternative interpretation on which formal and material consequences are not distinct kinds of modality. Normore raised an important objection to this interpretation (Normore [2015]). He draws attention to an argument Buridan has made shortly before above quoted passage in (T2.3) in

or vice versa, and a disjunction made up of contradictories or of subcontraries is formally necessary, as is [a proposition] in which an infinite term is denied of its finite counterpart or vice versa." Cf. [Buridan] [1976], Ed. Hubien, 80-1: "Item, sunt etiam aliae consequentiae formales propter formalem impossibilitatem antecedentis uel formalem necessitatem consequentis. Cum enim ex impossibili sequatur quodlibet et quod necessarium sequatur ad quodlibet, si propositio gratia formae sit impossibilis erit consequentia formalis de ea ad quamlibet et si sit gratia formae necessaria erit consequentia formalis de omni alia ad ipsam. Impossibilis etiam gratia formae esset coputaliua ex duabus contradictoriis uel contrariis constituta, uel etiam in qua aliquis terminus finitus affimaretur de seipso infinito aut econtra, et formaliter necessaria esset disiunctiva ex contradictoriis constituta uel ex subcontrariis, uel etiam in qua terminus infinitus negaretur de seipso finito aut econtra."

⁶³An example of a contemporary book focused on logic which draws that (misguided) comparison is Burgess [2009] (p.48): "It was even later before modal logicians recognized the importance of distinguishing logical from metaphysical necessities - and this although something much like this distinction, under the labels "formal" and "material" necessity, had been made already by medieval logicians centuries earlier".

which Buridan claims that adding a necessity to the set of antecedents does not strengthen the consequence. In support of his interpretation, Normore considers the following argument (Normore [2015], 366-7):

Consider the formal proof that q follows $p \land \neg p$. Suppose now that p is impossible. Then $\neg p$ adds no strength to the argument from p to q. Since we have already granted that q follows formally from $p \land \neg p$, we must therefore grant that q follows from p alone. Since any argument from an impossible premise p to a conclusion q can thus be turned into a formal argument, the inference from an impossible premise has the same strength as that from a contradiction.

Normore's argument convincingly shows that in the case of an impossible proposition among the antecedents, the reduction of an instance of EIQ to a formal consequence does not 'equivocate' the necessity of following in question as D'Ors suggested. In particular, Normore argues that the same kind of necessity is involved in formal and material consequences. This view seems to be further supported by Buridan's remarks on the EIQ, where Buridan states from an impossible antecedent anything follows in a material consequence as well, and to a necessary proposition anything can be an antecedent in a material

⁶⁴Cf. Buridan [2015b], Transl. Read, 79: "Proof: because suppose A is a proposition from which with many necessities adjoined to it the conclusion B follows; I say that B follows from A. Because if B follows from those necessities without A, then it is necessary, so it follows from anything. But if B does not follow from those necessities without A, then either A is impossible, and then anything follow from it, or A is possible. Then either it is impossible if A obtains for B not to obtain, and then B follows from A, or it is possible if A obtains for B not to obtain. If an opponent suppose this, then, since A cannot obtain without all the necessities obtaining at the same time, it follows that it is possible if A obtains with all the necessities for B not to obtain. Then B does not follow from A along with those necessities adjoined to it, which is contrary to the hypothesis." Cf. Buridan [1976], Ed. Hubien, 36: "Probatio. Quia sit A propositio ad quam cum multis necessariis sibi appositis sequitur conclusio B; dico quod ad A sequitur B. Quia si B sequatur ad illas necessarias sine A, tunc ipsa est necessaria, ideo sequitur ad quamlibet. Si uero B non sequatur ad illas necessarias sine A, tunc uel A est impossibilis, et sic ad eam seguitur quaelibet, uel A est possibilis. Et tunc uel impossibile est A stante non stare B, et sic adhuc ad A sequitur B, uel possibile est A stante non stare B. Et si hoc ponat aduersarius, tunc, quia non potest A stare quin simul stent omnes necessariae, sequitur quod possibile est A stante cum omnibus necessariis non stare B. Ergo B non sequitur ad A cum quibusdam necessariis sibi appositis, quod est contra positum."

consequence. [55] In Normore's interpretation (Normore [2015]), Buridan uses a single, fundamental notion of logical modality, and accordingly a 'univocal' conception of consequence. [66]

If we consider what Buridan means by the 'perfection' of an argument - namely, that reducing a material consequence to a formal consequence only makes the former evident to us, but does not move between distinct concepts of necessity - Normore's interpretation is right that there is no concept of formal impossibility (*gratia formae*) to make the necessity involved equivocal. As we have noted above, Buridan ties formality to evidentness and not to necessity; formality is a matter of perfecting valid arguments in the epistemic sense. Moreover, as Stephen Read remarks, the criterion of formal consequence we have seen above provides a definition of formality, and not of validity. [67] and that point is not exclusive to Buridan or the Parisian tradition.

This leads us to the second division, namely the division between simple and as-of-now consequences. In a well known text at *TC* I.4, Buridan makes a division of material consequences into simple (*simpliciter*) and as-of-now (*ut nunc*) (Buridan [2015b], Transl. Read, 68):

(T2.4) Because I have started to speak of the distinction of consequences, I want to say further that some material consequences are called simple consequences because they are simply speaking (*simpliciter loquendo*) good consequences, since it is not possible for the antecedent to be true the consequent being false. Others, which are not

⁶⁵See, e.g. Buridan [1976], Ed. Hubien, 38.

⁶⁶Cf. Normore [2015], 362-3: "Crucial here is whether we are to understand impossibility and necessity *gratia formae* as distinct kinds of impossibility and necessity or understand them as impossibility and necessity understood generically but *shown* to be such by the form. Professor D'Ors understood them in the first way and saw them as part of a general picture in which modal notions are equivocal. I propose that we understand them in the second way and as part of a general picture in which Buridan has a single basic conception of modality and regards other uses of modals as restrictions on it."

⁶⁷(Read [2020b], 282): "the Parisian account of validity of consequence represented by Buridan and others was radically different from the English tradition. Moreover, in both cases, formal consequence was a special case of validity in general. So the Parisian definition of formal consequence as truth-preservation under substitution of all non-logical terms was a definition of 'formality', not of validity. Where truth was preserved but not formally, the inference was said to be materially valid."

simply speaking good, are called as-of-now (ut nunc) consequences because it is possible for the antecedent to be true without the consequent, but are good as-of-now, because, things being as a matter of fact as they are, it is impossible for the antecedent to be true without the consequent. Ordinary people often use these consequences, e.g., if we say "A white cardinal has been elected Pope," we infer "So a master of theology has been elected Pope," and if I say "I see him," you will then infer "therefore you certainly see a deceitful man." Now this consequence is reduced to a formal one by adding a true, but not necessary, proposition or propositions, e.g., in the examples given, because the white cardinal is a master of theology and because this man is a deceitful man. In the same way we have here a good consequence, assuming that the only men are Socrates, Plato and Robert: "Socrates is running, Plato is running and Robert is running; so every man is running," because the consequence can be made perfect by this truth: "Every man is Socrates or Plato or Robert." ⁶⁸

This central passage of the *TC* is important for understanding of the implications of the modal criterion. The thrust of the distinction of material consequences into simple and as-of-now (or as-of-then) is that whereas simple consequences are valid absolutely, as-of-now consequences are only valid in a

⁶⁸Cf. Buridan [1976], Ed. Hubien, 23-4: "Consequentiarum materialium quaedam vocantur 'consequentias simples,' quia simpliciter loquendo sunt consequentiae bonae, cum non sit possibile antecedens esse verum consequente existente falso, vel esse ita etc. Alia vocantur 'consequentia ut nunc,' quae non sunt simpliciter loquendo bonae, quia possibile est antecedens esse verum sine consequente, sed sunt bona ut nunc, quia impossibile est rebus omnino se habentibus ut nunc se habent antecedens esse verum sine consequente. Et istis consequentiis utuntur saepe vulgares, ut si dicamus 'Cardinalis Albus est electus papam,' concludemus 'ergo unus magister in theologia est electus papam' et si ergo dico 'Ego video unum talem hominem,' tu concludes 'ergo certe tu vides unum falsum hominem.' Haec autem consequentia reducitur ad formalem per additionem propositionis verae, non tamen necessariae, vel aliquarum verarum, non tamn necessariarum, ut, in exemplis positis, quia Cardinalis Albus est magister in theologia et quia talis homo est unus falsus homo".

⁶⁹This passage is also important for the dating of the *Tractatus de Consequentiis*. It has been argued that the reference Buridan made to '*Cardinalis albus*' is a reference to Jacques Fournier (†1342), an opponent of William of Ockham's ideas, elected Pope Benedict XII in 1334. Hence, the dating of the *TC* to 1335 as a *terminus a quo* is based on that reference. See Stephen Read's introduction to Buridan [2015b], 3.

more restricted sense. What this restricted sense of validity involves?

At first, the notion of as-of-now consequence may appear to resemble the modern material conditional. Early interpretations of this concept, such as that proposed by Ernest Moody have suggested the identification of the medieval usage of as-of-now consequences with the material conditional $\frac{70}{10}$ Marilyn M. Adams has convincingly argued that this is not the case. In her interpretation which focuses on William of Ockham's treatment - she shows that the former cannot be plausibly identified with the latter due to the involvement of modality. In Buridan's specification of as-of-now consequences, the usage of modal concepts is clear: p entails q in an as-of-now consequence not merely because it is not the case that p is true and q is false in an actual circumstance, but rather the additional condition that there must be a possible situation in which p is true and q is false makes the specification of as-of-now consequence hinge crucially on the notion of possibility, which means that it is not a truth-functional conditional in contrast with the material implication.

The purpose of the division between simple and as-of-now consequence is, therefore, to articulate a weakened notion of validity. This weaker form of validity is relative to how things now actually are, even though they can be possibly otherwise (in a broader sense of possibility). This is clear from the example Buridan offers. One of the examples Buridan formulates in the *SD* is the following as-of-now valid inference (Buridan [2001b], Transl. Klima, 62):

C4 Gerard is with Buridan Therefore, he is on Rue de Fouarre

This inference is valid as-of-now, since relative to the in which Buridan is actually at the Rue de Fouarre, and to the truth of premise stating that Gerard is

Moody [1953], 79: "In contrast to the mediaeval rules for 'simple' consequences, those governing consequences 'as-of-now' determine theorems which are distinctive of the modern system of material implication." Moody identifies the as-of-now consequence with the material implication $(p \to q) \leftrightarrow \neg (p \land \neg q)$.

⁷¹Arguing against Moody, Marilyn M. Adams writes Adams [1973], 11: "Whatever the other medieval logicians Moody mentions may have thought, this is not an accurate reading of Ockham's definition in the above passage. According to what Ockham says there, in order for an inference to hold good as of now, it is necessary not only (i) that it is not the case now that the antecedent is true and the consequent false; but also (ii) that it is possible that the antecedent should be true and the consequent false at some other time. The second condition is not merely truth functional, since it involves the notion of possibility."

with him, it is impossible that Gerard is not at the Rue de Fouarre. This sense of impossibility is surely a more restricted sense.

Buridan states that as-of-now valid inferences can also be transformed into formally valid inferences, similarly as was the case with simple material consequences. But in this case, the inference is transformed into a formally valid one by adding a contingently true - instead of a necessary - premise (e.g., with respect to C4, the premise that Buridan is on Rue de Fouarre). The reason why the inference, even the formal, cannot be simply valid is that things can be as the antecedent says without being as the consequent does, namely, if both leave the Rue de Fouarre together. Thus, a material consequence as-of-now can cease to obtain by some change in the circumstances.

Buridan nowhere in the TC explicitly characterizes the unrestricted and restricted senses of necessity. In his SD 1.8.5, we find Buridan distinguishing two acceptations of necessity and possibility. which he calls necessity and possibility taken broadly (ample) and restrictively (restrictive). Buridan writes Buridan [2001b], Trans. Klima, 75-6:

(T2.5) We should note, however, lest someone object, that 'possible' is sometimes taken broadly, i.e., indifferently in relation to the past and the future, and so is necessary, as when we say that everything that either is, was, or will be is possible, or even when it simply does not imply a contradiction [to say] that it is, was, or will be. [...] In another sense it is taken to be restricted to the future, so that nothing is said to be possible to be, unless it either will be or at least it does not imply a contradiction [when we say] that it is or will be, in the sense in which it is said, in bk. 1 of *On the Heavens*, that there is no possibility (*possibilitas*)⁷⁴ over the past. ⁷⁵

⁷²I am thankful to Boaz Schuman for the reference to this passage and the example.

⁷³Normore [2015] calls attention to another, but virtually equivalent, distinction made in Buridan [2001b], Transl. Klima, 733.

⁷⁴I change here Klima's text slightly, for Buridan uses here 'possibilitas' instead of 'potestas', as Klima has it. See footnote below.

⁷⁵Buridan [2005], Ed. Van der Lecq, 91: "Notandum est tamen, ne aliquis instet, quod aliquando 'possibile' accipitur ample, scilicet indifferenter ad praeteritum et futurum, et similiter 'necesse', prout diceremus omne illud possibile esse quod est vel fuit vel erit vel quod non repugnat ipsum fuisse vel fore. [...] Alio modo capitur

Here, on a broad acceptation, 'possible' and 'necessary' relate indifferently to all times; and on a restricted acceptation it reflects the modal asymmetry between the necessity of the past and future possibilities. The distinction Buridan makes is the following:

A-Possibility: *P* is broadly (*ample*) possible if it either was, is, or will be (true), or if it simply does not entail a contradiction.

R-Possibility: *P* is restrictively (*restrictive*) possible if it either is or will be (true), and it is restrictively necessary if its opposite is no longer possible, even though it might have been in the past.

In characterizing the first notion, Buridan only spells out necessity. But since he puts the absence of a contradiction as the hallmark of A-Possibility, we can plausibly assume (without extrapolating too much the text) that the corresponding notion of logical necessity would apply to a proposition or an argument which does entail an impossibility. Thus, necessity taken broadly is the notion of necessity underlying the Modal Criterion, namely, the necessity by which a consequent follows from an antecedent in simple material consequence or in a formal consequence.

The notion of necessity and impossibility involved in as-of-now consequences, and used at [T2.3] to define the sense in which an as-of-now consequence is valid is, however, a more narrow dimension of modality, based on what Buridan calls necessity taken restrictively (*restrictive*). What Buridan means by restriction involves indexing the time of the proposition to the actual one. Since it concerns only the actual situation, this narrow conception of modality is intertwined with the assymetry of time (*differentia temporum*). As we will see in the next chapter, Buridan thought this notion of modality corresponds to a historical sense, in which the past is held fixed and necessary and the future open.

I am proposing that necessity and possibility taken restrictively should be identified with a weakened Modal Criterion, corresponding to validity as-of-now. In that interpretation, we get a stronger and weaker formulation of the Modal Criterion:

stricte ad futurum, ita quod nihil dicatur possibile esse nisi quod est vel erit, vel saltem quod non repugnat ipsum fore, sicutr dicitur primo Caeli quod possibilitas non est ad praeteritum."

MODAL CRITERION*: *Q* follows from *P* if it is A-Impossible for things to be as *P* signifies, without being as *Q* signifies.

Modal Criterion**: Q follows from P if it is R-Impossible for things to be as P signifies, without being as Q signifies.

If this is correct, then Buridan adjusts modal space according to the Modal Criterion** in order to account for the validity of as-of-now consequences.

This interpretation can meet an important objection. Elsewhere, Buridan relegates restricted modality to a merely historical sense of necessity and possibility. He says that it pertains to narrative stories (*storiae narrativae*), and that it does not pertain to logical matters (Buridan [2001b], Transl. Klima, 76). Buridan's hesitation to say that historical modalities are pertinent to logic can mean that he is reluctant to accept the notion of restricted necessity and possibility as basing the concept of a valid inference.

However, some evidence that the Modal Criterion** is used by Buridan in a systematic fashion can be gleaned from his remarks on consequences where tense differences appear, namely, in consequences that are valid 'as-of-now-for-then' (*ut nunc pro tunc*). They are discussed in the first conclusion of the first book *TC*, in a remark which Buridan makes suggesting a corresponding restricted version of the EIQ.

As we have seen, the EIQ follows from the Modal Criterion alone. That means that any consequent to an impossible antecedent makes the conditional true. Buridan notes that this should be adjusted to the case of as-of-now consequence, by saying that a falsehood which is impossible 'as-of-now' entails any consequent, and a truth that is necessary as-of-now is entailed by any antecedent. Buridan writes (Buridan [2015b], Transl. Read, 75-6):

(T2.6) If the expression is in the past or future tense, then it may be called a consequence as-of-then, or however you wish to call it. For example, this follows in an as-of-now, or as-of-then, or as-of-now-for-then consequence: "if the Antichrist will not be begotten, Aristotle never was." Because, although it is simply true that the Antichrist can fail

⁷⁶See the next chapter for dicussion, especially section 3.5.

to come to be. [77] it is impossible for things being as they are going to be that they will be such that he will not be; for he will be and it is impossible that he will be and will not be. [78]

Buridan gives here the following inference as an example of a materially valid as-of-now consequence (or 'as-of-now-for-then'). The example supposes that it will be true in the future that the Antichrist will come to be, so that the antecedent of the inference above is impossible as things now are.

As Buridan explains the validity of [C5], he has in mind the supposition I mentioned above, of a situation in which the Antichrist will come to exist. Relative to that situation, the antecedent of the argument is therefore R-Impossible. Accordingly, the inference above expresses a valid consequence according to Buridan, namely a material consequence valid as-of-now (or as-of-now-for-then). The reason why C5 is valid for Buridan indicates that the Modal Criterion is modified to a restricted version, and that a kind of restricted necessity and a corresponding restricted version of the EIQ is modified in order to capture the necessity involved in as-of-now consequences.

Therefore, the senses of necessity and possibility used in the Modal Criterion and in its restricted version can be plausibly identified with the broad (*ample*) sense of possibility and necessity and with the restricted sense (*restrictive*) of possibility and necessity respectively. The latter provides the modal justification of as-of-now consequences, while the former corresponds to Buridan's modal justification of simple consequences.

⁷⁷I have modified here Stephen Read's translation of a phrase in this passage, namely 'simpliciter sit uerum quod antichristum possibile est non fore.' Read translates this phrase as "it is simply true that the Antichrist can fail to be going to be." I have modified this translation, and replaced it instead for "it is simply true that the Antichrist can fail to come to be." Although Read's translation is not wrong, I have opted to modify it with a shorter phrasing for simplicity.

⁷⁸Buridan [1976], Ed. Hubien, 32: "Et si sit sermo de praeterito uel de futuro, tunc uocetur consequentia 'ut tunc', aut qualitercumque uolueritis nominare. Verbi gratia, sequitur consequenti ut nunc, uel ut tunc, uel ut nunc pro tunc, 'si antichristus non generabitur, Aristotiles numquam fuit'. Quia, licet simpliciter sit uerum quod antichristum possibile est non fore, tamen impossibile est quod rebus se habituris sicut se habebunt ipse non erit; ipse enim erit et impossibile est quod ipse erit et non erit."

If that is correct, the distinction between broad and restricted possibility used by Buridan in the treatment of consequences presupposes metaphysical considerations that are not explicitly addressed in the *Tractatus de Consequentiis*. For example, it presupposes the modal assymetry of time, and the necessity of the past. We will postpone discussion of these difficult issues to sections 3.5 and 3.6, since they require a more careful examination of Buridan's commitments concerning modality which extrapolate the aims of the *TC*.

2.2 The Modal Proposition

Recent treatment of Buridan's logical writings has focused on his modal syllogistics. Previously existing accounts of the modal syllogism, dating back to Aristotle's *Analytica Priora*, were systematized by Buridan in his main writings on the subject. These are, mainly, *SD* 5.6-7⁸⁰, *QAPr.*, I.28-40.⁸¹ and finally *TC* IV.1-4.⁸² In particular, Buridan's account of modal propositions is regarded as innovative, and it has been widely noted that it provides for systematic account of modal validity.

What are modal propositions for Buridan? It was commonly held in the medieval period that modal propositions come in two forms, namely in composite and divided senses. Buridan's statement of this widespread distinction in medieval logic goes as follows (Buridan [2015b], Transl. Read, 95-6):

(T2.7) Now, in the second chapter, we must acknowledge that modal propositions of this sort are commonly of two types. For some are called 'composite' and others 'divided.' They are called composite when a mode is the subject and a dictum is the predicate, or vice versa. [...] For example, I call the following composite: 'That a human runs is possible,' 'It is necessary that a human is an animal.' The

⁷⁹Among the classical studies in this regard are Lagerlund [2000], 192-164, Thom [2003], King [1985], see also Read [2020a] and Parsons [2014]. Buridan's influence in post-medieval circles is notorious, but has not been yet appreciated in more detail. On the influence of Ockham's modal syllogistics in post-medieval times, see Lagerlund [2000], 210ff.

⁸⁰Buridan [2001b], Transl. Klima, 335-364.

Buridan [n.d.b], Ed. Hubien, 57-79.

⁸²Buridan [2015b], Transl. Read, 140-159.

subject of the first of these is 'that a human runs' and the predicate is 'possible;' the subject of the second is 'necessary,' and the predicate is 'that a human is an animal.' They are called 'divided' when part of the dictum is the subject and the other part the predicate. The mode attaches to the copula as a determination of it. For example, 'A human can run' or 'A human is possibly running;' similarly, 'A human is of necessity a runner' or 'A human is necessarily running,' and the like.

The basic difference between both is first characterized in grammatical terms. In a composite modal proposition, the mode typically figures as a predicate of a *dictum*. The *dictum* is a nominalized proposition which, in Latin, is formed by an infinitive construction (e.g. *hominem currere*), best rendered in English by that-clauses, such as 'that a human runs', of which the mode is then predicated (e.g., *hominem currere est possibile*). The basic scheme for construing composite modal propositions is as follows:

⁸³ Buridan [1976], Ed. Hubien, 56: "Deinde, in secundo capitulo, supponendum est quod communiter huiusmodi propositiones modales ponuntur diplices. Quaedam enim uocantur 'compositae' et aliae 'diuisae'. 'Compositae' uocantur in quibus modus subicitur et dictum praedicatur uel econuerso. [...] Et uoco 'dictum' illud totum quod in propositione ponitur praeter modum et copulam et negationes et signa aut alias determinationes modi uel copulae. Verbi gratia, haec uocatur 'composita', 'hominem currere est possibile', et haec 'necessarium, est hominem esse animal'. Et subiectum primae istarum est 'hominem currere' et praedicatum 'possibile'; subiectum autem secundae est 'necessarium' et praedicatum 'hominem esse animal'. Sed 'divisae' uocantur in quibus pars dicti subicitur et alia pars praedicatur. Modus autem se tenet ex parte copulae, tamquam eius quaedam determinatio. Verbi gratia, 'homo potest currere', uel 'hominem possibile est currere'; similiter, 'homo de necessitate est currens' uel 'hominem necesse est currere', et huiusmodi."

⁸⁴Buridan [2015b], Transl. Read, 95: "I call a 'dictum' that whole occurring in the proposition in addition to the mode and copula and negations and signs or other determinations of the mode or the copula." Cf. Buridan [1976], Ed. Hubien, 57: "Et uoco 'dictum' illud totum quod in propositione ponitur praeter modum et copulam et negationes et signa aut alias determinationes modi uel copulae." By defining the *dictum* as whole which excludes 'signs', Buridan means here 'quantifier signs', which can also appear taking wide scope over *dicta*, such as 'every that-a-human-runs is possible. I aim leaving this complication aside for now, but the semantic effect of quantifying over *dicta* will be discussed below.

It is important to note already some points concerning the semantics of composite modal propositions. In them, the *dictum* stands materially for ('supponit materialiter pro') other equiform proposition tokens, and the modal term - occurring as a predicate - states that these propositions are possibly, necessarily, impossibly, or contingently true, modifying the way in which a proposition is said to be true or false (*in essendo verae*).

Buridan notes that modal words behave similarly to quantifiers in this context. A proposition is regarded as 'universal with respect to the mode' if it is necessary, and 'particular with respect to the mode' if its possible (Buridan [2001b], Transl. Klima, 75). Furthermore, in his observations concerning the equipollences holding between composite modals, Buridan claims that they behave with respect to negations similarly as do distributive signs (i.e., quantifiers). Such relations

⁸⁵ Material supposition is one among the kinds of reference acknowledged in medieval semantics. In particular, material supposition occurs when an expression supposits for, or refers to, other linguistic or mental items, as opposed to referring to an extra-mental object. Usually, we transliterate, rather than translate, the expression '*supponit*,' in order to keep the differences between the medieval concept of supposition apart from the modern theory-laden concept of reference. On comparisons between supposition theory and reference see, e.g. [Klima [2001b]].

⁸⁶Cf. Buridan [2005], Ed. Van der Lecq, 113-116; Buridan [2001b], Transl. Klima, 97-98. Note that, in Buridan's nominalist semantics, a *propositio* is always a concrete entity - a spoken, written, or mental utterance, and propositions in this acceptation are not abstract objects. See Klima [2009], 210ff.

⁸⁷ Buridan [2001b], Transl. Klima, 89: "The third [rule] is that if we treat modes analogously with signs, namely, so that 'necessary' is treated like 'every', 'impossible' like 'no', 'possible' like 'some', and 'possible . . . not' like 'some . . . not', then a negation placed after the mode makes it equipollent to its contrary; placed before, it makes it equipollent to its contradictory; and placed both before and after, it makes it equipollent to its subaltern". Cf. Buridan [2005], Ed. Van der Lecq, 98: "Tertia regula est quod proportionando modos ad signa, scilicet quod 'necesse' sit sicut 'omnis', et

are displayed in Buridan's *magna figura*, famous the octagon of oppositions for *divided* modals. In the case of composites, they can be represented in a simplified way in the following square of oppositions, suggested by Gyula Klima: 88

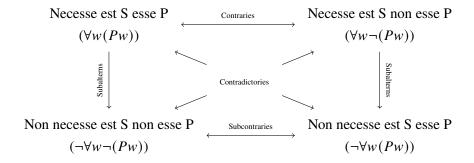


Figure 2.1: Modal Square

In this square, we take possible worlds as indices and categorical propositions as their predicates. This slightly modified version of Klima's reconstruction of the modal square captures precisely Buridan's analogy of modal words with quantifier signs (*signa distributiva*). Buridan thus has the resources to analyse modal propositions extensionally, by reference to quantification over possible situations, and this in effect approaches the modern understanding of possible worlds semantics. Buridan's analogy between quantifiers and modals might support a line of interpretation which is fairly widespread. The appeal of this

^{&#}x27;impossibile' sicut 'nullus', et 'possibile' sicut 'quidam', et 'possibile non' sicut quidam non', tunc negatio postposita modo facit aequipollere suo contrario, praeposita suo contradictorio, praeposita et postposita suo subalterno." The analogy of modals with quantifiers Buridan makes here is that in both placing negation after the quantifier yields its contrary, e.g. $\forall x \neg (Px) \rightarrow \neg \exists x (Px)$, placing it before yields its contradictory, e.g. $\neg \forall x (Px) \rightarrow \exists x \neg (Px)$, and placing it both before and after yields its subaltern, e.g. $\neg \forall x \neg (Px) \rightarrow \exists x (Px)$.

⁸⁸See the *magna figura* displayed at Buridan [2005], Ed. Van der Lecq, 100. The square that follows is based on Klima's reconstruction, in Buridan [2001b], Transl. Klima 83.

⁸⁹For example, as Gyula Klima points out, footnote 123 of his translation of the *Summulae* that Buridan [2001b], 82: "What Buridan states here is effectively the gist of the idea of modern possible-worlds semantics, which treats the intensional modal notions analogously to the extensional notions of the quantifiers, in fact treating them as quantifiers over possible worlds or situations." Knuuttila has in a number of occasions

comparison goes back to even before possible worlds talk. C.S. Peirce in 1907, referring to the 'scholastic account of modality', also held the position that to assert that A must be B is in effect to state that a class of propositions are true. Even though Peirce's remark is mistakenly formulated, it attests to the appeal of comparing modals with extensional notions on the basis of these analyses put forth by Buridan. [90]

Nonetheless, a characteristic feature of Buridan's own account of composite modals is that he regards composite modal propositions to be, for all logical purposes, equivalent to assertorics, and are not modals properly speaking (*proprie loquendo*). Why so? The main reason is that composite modals contain an assertoric copula just as in assertoric propositions. The only difference between modal composite propositions a regular assertoric proposition is that, in the former, a mode is a subject or a predicate. In modal propositions 'properly so-called' the mode is an essential part of the copula. ⁹¹ This means that in the

referred to the presence of 'extensional models' in medieval philosophy (Knuuttila | 1993|); (Knuuttila | 2008|)

⁹⁰Cf. C.S. Peirce (1901, *apud* Knuuttila [1981], ix: "The simplest account of modality is the scholastic, according to which the necessary (or impossible) proposition is a sort of universal proposition; the possible (or contingent, in the sense of not necessary) proposition, a sort of particular proposition. That is to assert 'A must be true' is to assert not only that A is true but that all propositions analogous to A are true; and to assert 'A may be true' is to assert only that some proposition analogous to A is true. If it be asked what is there meant by analogous propositions, the answer is - all those of a certain class which the conveniences of reasoning establish."

⁹¹Cf. Buridan [2001b], Transl. Klima, 70-1: "We should note that in modals properly so-called, the mode has to be placed between the subject and the predicate, and what precedes the mode and the verb is the subject or something belonging to the subject, and what follows is the predicate or something belonging to the predicate, and the aggregate of the mode and the verb is the coupla [...] we should note that if the mode is placed at the beginning of the proposition or at the end, the proposition is usually called a composite modal, as I said earlier, but more properly speaking it should instead be called an assertoric [de inesse] proposition". (Buridan [2005], Ed. Van der Lecq, 85-6: "Notandum est quod in proprie dictis modalibus modus et verbum debent situari intersubiectum et praedicatum, et quod praeecdit modum et verbum est subjectum vel se tenens ex parte subjecti, et quod sequitur est predicatum vel se tenens a parte praedicati, et congregatum ex modo et verbo est copula [...] notandum est quod si modus ponatur in principio propositionis vel in fine, propositio bene solet vocari modalis composita, ut dixi prius, sed magis proprie dicenda est de inesse".) This view, according to which composite modals are, for semantic purposes, just assertorics, is reflected in later author such as John Mair († 1550) (John Mair [1514], 89va), who were clearly influenced by Buridanian material through John Dorp's edition Summulae

Buridanian acceptation only divided modal propositions are modal propositions strictly speaking.

A second reason why Buridan thinks composites are properly speaking not modal propositions can be gleaned from his account of quantification in composite modals, namely, when a quantifier sign (*signa distributiva*) is prefixed to the composite modal proposition. His treatment of the semantic effect of quantifying over the modes reveals also the limitations of comparing Buridan's modal semantics with extensional treatments of modality. In *TC* II, Buridan discusses the quantity of composite modal propositions in cases where a distributive sign is prefixed before the mode, yielding forms such as the following (Buridan [2015b], Transl. Read, 105): [92]

- 1 Every possibility is that B is A (*omne possibile est B esse A*)
- 2 It is a possibility that B is A (possibile est B esse A)
- 3 Some possibility is that B is A (*quoddam possibile est B esse A*)
- 4 This possibility is that B is A (*hoc possibile est B esse A*)

For reasons we will see below, here what 'every possibility' (*omne possibile*) or 'some possibility' means is that every or some proposition is such that 'B is A.'⁹³ In other words, '*possibile*' in the propositional contexts mentioned above stands materially for ('supponit materialiter pro)' proposition-tokens.⁹⁴ Buridan lists these forms as gramatically universal, indefinite, particular, and singular

⁽John Dorp [1487]), annotated by Mair himself.

⁹²cf. Buridan [2015b], Transl. Read, 105: "Having clarified these matters, we now turn to composite modals. About these, it must first be said, that, as some do say, they can be universal or particular, or indefinite or singular. For example, 'Every possibility is that B is A' is universal and 'It is a possibility that B is A' is indefinite, and similarly one can speak of 'some possibility' or 'this possibility'.". Cf. Buridan [1976], Ed. Hubien, 69: "His determinatis, determinadum erit de modablibus compositis. Propter quas erit praemittendum quod illae, quidquid aliqui dicant, possunt fieri uniuersales et particulares, indefinitae et singulares. Verbi gratia, haec est uniuersalis, 'omne possibile est B esse A' et haec indefinita 'possibile est B esse A', et ita possum dicere 'quoddam possibile' uel 'hoc possibile'".

⁹³I follow Stephen Read in translating 'possibile' as 'possibility' in this context. A more accurate translation would be 'every possible [proposition] is that B is A,' for reasons we will see below. However, I avoid the complication at this stage.

⁹⁴On the concept of material supposition, see above footnote 90.

composite modal propositions respectively (cf. Buridan [2015b], Transl. Read, 105).

However, Buridan's own view is that all these forms are semantically equivalent. This presents a problem for a purely extensional interpretation of composite modals. On the one hand, adding quantifiers to composites may look as supporting an extensional interpretation of composite modal propositions, since in virtue of their grammatical surface alone it appears as if Buridan were using propositions as predicates of circumstances or possible worlds, e.g. 'every possible world is such that P' or 'some possible world is such that P'. On the other hand, once we turn to what Buridan has to say concerning their truth-conditions, the limitations of the analogy of modals with quantifiers start to surface.

Firstly, Buridan takes 'possible' in those contexts to refer to a possible proposition, not a possible circumstance. Accordingly, [1] amounts to claiming that 'every possible proposition is that B is A', [2] states that 'there is a possible proposition such that B is A', [3] that 'some possible proposition is that B is A', and finally [4] states that 'this possible proposition is that B is A'. Secondly, Buridan takes the *dicta* of composite modals to stand materially for all equiform propositions, that is to say, 'that B is A' stands for all token occurences of equiform propositions. Now, importantly, Buridan makes here the addition that, retaining all else fixed, one token signifies just the same as any other equiform proposition-token signifies. Therefore, [1]-[4] are actually all semantically equivalent, since the a *dictum* stands in material supposition for all its equiform proposition-tokens. For this reason, Buridan claims that any singular composite modal, for example, 'this possibility is that 'B is A' entails the corresponding universal composite modal 'every possibility is that 'B is A', and vice-versa. Therefore, prefixing quantifiers to composite modes leaves

⁹⁶Buridan [2015b], Transl. Read, 106: "In all composite modals in which the dictum

⁹⁵Buridan [2015b], Transl. Read, 105: "Here 'possibility' is taken not for what can be but for a possible proposition, which is said to be possible in so far as things can be altogether as it signifies. So in the examples above, saying 'Every possibility is that B is A' is the same as to say 'Every possible proposition is that B is A'." Cf. Buridan [1976], Ed. Hubien, 69: "Et capitur hic 'possibile' non quia possit esse sed pro propositione possibili, quae ex eo dicitur 'possibilis' quia qualitercumque significat ita potest esse. Unde in proposito idem ualet dicere 'omne possibile est B esse A', sicutr dicere 'omnis propositio possibilis' est B esse A'."

no trace on their truth conditions. This can also be a reason to understand why Buridan regards composites as non-modal propositions strictly speaking. This is a pervasive opinion in the later Buridanian tradition. As Geudens and Demey have argued, other authors of the post-medieval period following the footsteps of Buridan's nominalist semantics subscribed to the view stated above [97] This can also be seen as further support for the view that Buridan's account of composite modals as truly assertoric propositions is based on the reason that quantifying over *dicta* in composite modal propositions does not alter their truth-conditions, which, as we shall see, is not the case with divided modal propositions.

Let us now turn to Buridan's account of *divided* modal propositions, which as we have seen in (T2.7) are distinguished from composite ones. Quantification in modal contexts becomes semantically significant in Buridan's treatment of

is subject, from a particular there follows a universal, the rest being unchanged. For example, this follows: Some proposition 'B is A' is possible, so every proposition 'B is A' is possible, and similarly for truth and falsity, contingency and necessity. The reason is that among all the propositions 'B is A', each signifies whatever the others signify and altogether as the others signify. So if things are as one signifies, they are as any other signifies, and if not, not; so if one is true the other is true, and if false, false. Similarly for possibility, necessity and other modes. What I say concerning particular to universal, so I say concerning singular to universal. For this proposition, namely 'The proposition 'B is A' is possible' cannot be true unless 'Every proposition 'B is A' is possible' is true." Cf. Buridan [1976], Ed. Hubien, 70-1: "Nona conclusio est: in omnibus modalibus compositis in quibus dictum subicitur ad particularem sequi uniuersalem ceteris non mutatis. Verbi gratia, sequitur 'quaedam propositio B est A est possibilis; ergo 'omnis propositio B est A est possibilis', et sic de ueritate et falsitate, contingentia et necessitate. Cause est quia omnium talium propositionum 'B est A' una significat quidquid alia significat et qualitercumque alia significat. Ideo si est sicut una significat est sicut alia significat et si non non; ideo si una est uera alia est uera at si falsa falsa. Et similiter est de possibilitate, necessitate et aliis modis. Et sicut dico de particulari ad uniuersalem, ita dico de singulari ad uniuersalem. Quoniam haec propositio non potest esse uera, scilicet 'haec propositio B est A est possibilis', quian ista sit uera 'omnis propositio B est A est possibilis'.

⁹⁷Geudens & Demey [2022], 55-6: "As we have seen, the principle that dicta in composite modals are common terms that supposit materially, along with the further specification that the sets of supposita of such dicta are restricted to occurrences of the same proposition type, jointly entail quantification invariance. This principle is thus valid in the modal logics of the moderni just mentioned." Geudens and Demey quote extensively from John Fabri of Valenciennes (c. 1500) and Jeronimo Pardo († c. 1505). What they here call *quantification invariance* is the view, which Buridan definitely subscribed to, that quantifying over *dicta* leaves no trace on the truth-conditions of composite modals, since *dicta* stand materially for all its equiform proposition-tokens. This view is also held by John Mair (†1550), who was clearly influenced by Buridan.

the divided modal proposition. Buridan regards modal propositions taken in the divided sense to be modals properly so-called, since in those propositions the modal word is a part of the copula. The function of modals in divided modal propositions is that of changing the way the predicate is said of the subject. An example of divided modal proposition is 'Every human being necessarily-is an animal' (*Omnis homo necesse est esse animal*). The template below represents the combinations yielding divided modal propositions:

The main difference between divided modal propositions and normal assertoric ones in Buridan's theory is a semantic difference. In a divided modal proposition, the supposition of the subject term is ampliated to stand for not only what is actual, but also to what is not actual and only possible. Buridan writes (Buridan [2015b], Transl. Read, 97):

(T2.8) Now, in the fourth chapter, it should be realized that a divided proposition of possibility has a subject ampliated by the mode following it to supposit not only for things that exist but also for what can exist even if they do not. Accordingly, it is true that air can be made from water, although this may not be true of any air that exists. So the proposition 'B can be A' is equivalent to 'That which is or can be B can be A.'

⁹⁸Buridan [2015b], Transl. Read, 96: "In these, the subject is "human" and the predicate is "runner." The copula is the whole phrase "can be" or "is possibly" or "is of necessity" or "is necessarily.""

⁹⁹Buridan [1976], Ed. Hubien, 58: "Deinde, in quarto capitulo, supponendum est quod propositio diuisa de possibili habet subiectum ampliatum per modum sequentem ipsum ad supponendum non solum pro his quae sunt sed etiam pro his quae possunt esse quanuis non sint. Unde sic est uerum quod aër potest fieri ex aqua, licet hoc non sit uerum de aliquo aëre qui est. Et ideo haec propositio 'B potest esse A' aequivalet isti 'quod est uel potest esse B potest esse A'."

What Buridan states in this passage is that modal words have an ampliative force (*vis ampliandi*) which makes the subject terms of the propositions in which they occur refer to possible beings. In medieval logic, ampliation is a property that a term acquires in the context of propositions containing modal words, by which the reference (or the supposition) of its subject is made to supposit for things beyond the present actual referents of assertoric, present-tensed propositions. In the *TC* I.6 and II.4 Buridan develops a highly innovative account of the causes of truth for divided sense modals based on the theory of ampliation.

In order to address Buridan's semantics of divided modal propositions in more detail, we should first get clear on Buridan's notion of causes of truth (*causae veritatis*) and supposition (*suppositio*). [101]

For Buridan, a proposition is true if things are altogether as it signifies them to be, and false otherwise. Signification is broadly defined the property a term has by evoking of an understanding of some thing the mind. A term has signification by itself, as 'Socrates' or 'horse,' by evoking the concept of Socrates and the concept of horses in the mind. In a relevant sense, Buridan speaks also of signification as a relation that a term can bear to something in the

¹⁰⁰See Read [2019] for an overview of ampliation among the other properties of terms in thirteenth and fourteenth century summulists.

¹⁰¹To avoid anachronisms, although Buridan's concept of causes of truth somewhat approximates the notion of truth-conditions, it is not usually translated as such. There has been some debate over Buridan's definition of causae veritatis as rendered by Hubien's edition. The text states the following (Buridan [1976], Ed. Hubien, 19): "Et intelligo per 'causas veritatis' alicuius propositionis propositiones> quarum quaelibet sufficeret ad hoc quod propositio esset vera", which is further translated by Gyula Klima ipsis litteris in Buridan [2001b], Transl. Klima, 56 n.85 thus: "And by the 'causes of truth' of a given proposition I understand propositions which are such that any of them would suffice for the truth of the given proposition." However, Stephen Read has argued that adding *propositiones* as causes of truth is Hubien's own addition, not supported by the manuscripts (cf. his Introduction in Buridan [2015b], Transl. Read, 9), and it furthermore renders Buridan's account circular, if further propositions are what makes propositions true. He instead suggests that what makes a proposition true for Buridan is 'how things are', as signified by proposition (sicut ipsa significat). The details of this debate do not concern us here. It is worth mentioning, however, in order to add the caveat that account for causes of truth and supposition theory as an early account of truth-conditions is contentious in the literature.

¹⁰²See Spade [1982] for a general treatment of the concept of signification across medieval logic.

external world, namely as reference. This property was called 'supposition' ('suppositio'). and it was used to specify the causes of truth of propositions in the following way. For a standard assertoric, present-tensed proposition to be true, such as 'Socrates is white,' the subject must stand for (supponit pro) what it presently signifies, and the proposition is true if the predicate stands for what is the same as the subject. For example, in 'Socrates is white', the subject term signifies Socrates, but it only supposits for Socrates under the condition that he now exists, and the proposition is true if Socrates falls under the things that are now white.

By contrast, tensed or modal propositions are true if things were, will be, or possibly are as the proposition signifies them to have been, as they will be, or as they possibly are. The construal of supposition was changed in order to accompany the tenses or modals involved, and in these cases the terms were said to be 'ampliated,' namely their references are extended, in order to stand for things that may currently not exist. For example, in the proposition 'Socrates was white,' the subject again signifies Socrates, but given the past tense of the proposition in which it occurs, 'Socrates' supposits for things that either were or are actual. The latter proposition is true even if Socrates no longer exists for the subject falls under things that either were or are white, and Socrates is among things that were white.

The construal of supposition for modal cases proceeds in a similar way. As Buridan writes in (T2.7), due to the ampliative force of modal words, 'B can be A' is analysed as a proposition with a disjoint subject expressing such ampliation of the subject to supposit for possible things, namely, 'that which is *or* can be B can be A'.

In the *TC* II.4 Buridan seeks to argue for his modal analysis (Buridan [2015b], Transl. Read, 97ff.). The main feature of the analysis is that it treats all divided modal propositions as ampliated to the possible. As has been noted,

¹⁰³Buridan distinguishes between immediate and ultimate signification accordingly. See Klima [2009], 175ff.

¹⁰⁴Again, we usually transliterate this word rather than translate it, in order to avoid anachronism in comparing supposition theory with theories of reference. See again? ¹⁰⁵In that sense, supposition theory provides the basis for an identity theory of predication (as opposed to an inherence theory of predication), cf. Knuuttila [2010] and Klima [2008].

Ockham's account of the modal proposition does incorporate ampliation in that way (Priest & Read [1981]), and Ockham treats instead divided modal propositions as systematically ambiguous between composite and divided senses. For our purposes, it will be important to see how Buridan's own reading of modal propositions figures as a semantic justification for the validity of modal syllogisms by providing a few examples.

A modal syllogism is a formal consequence between modal propositions. Buridan articulates his modal syllogistics at *TC* IV The first figure syllogism below is mentioned in the fourth conclusion to book IV of *TC* (Buridan [2015b], Transl. Read, 143):

Buridan clearly explains how the validity Barbara LML is based on ampliation of the terms. Buridan states that if the major term is expressed "by a disjunction of the verb 'is' with the verb 'can', then if the minor is of possibility it will be clearly subsumed under the distribution of the major [extreme]; while if the minor is of necessity, the same is true, since that of possibility follows from that of necessity" (Buridan [2015b], 143). In other words, since the major term A refers to all possible A's, and the set of all possible C's is included in the set of all possible A's, then the all the set of possible C's are included in the set of things that are necessarily B.

Buridan reckons with ways to block or restrict ampliation, namely by forcing the subject term to refer only to what is actual. To that effect Buridan uses the relative clause 'that which is' (*quod est*) to force such a restricted reading. [108]

¹⁰⁶Cf. Ockham [1974], Eds. Boehner *et al* 273; see also Panaccio [2012], 146. See, also, for Ockham's analysis of modal propositions, Dutilh Novaes [2004]. For a comparison of Ockham's and Buridan's accounts, see Johnston [2015b].

¹⁰⁷For a complete list of the validities in Buridan's system, see the figure in Stephen Read's Introduction to Buridan [2015b], Transl. Read, table 4, and for further discussion, Lagerlund [2000], Thom [2003], and Johnston [2015a].

Buridan [2015b], Transl. Read, 144: "I call it 'restriction by that which is' when the proposition is formed as 'that which is B is necessarily A', or 'that which is B can be A,' and this is [what is meant by] the claim that in propositions of necessity or

For example, now consider the following syllogism in the first figure with a restricted major:

Although Barbara LML* has a similar formal structure as Barbara LML - it has the same order of terms, quantifier signs, and modalities - the reason why it is not valid is because the major is not ampliated, since A is restricted in the major by 'that which is' (*quod est*), it is forced to refer only to what actually is A. Therefore, the ampliated distribution of C is not subsumed under the restricted distribution of A. Buridan's counterexample to Barbara LML* is the following argument: "Everything that is shining is necessarily other than the moon and every moon can be shining; so every moon can be other than the moon" (Buridan [2015b], Transl. Read, 145). The premises can be true with the conclusion false - suppose that everything that is currently shining in the sky is a star. Then the major is true, and so is the minor, since by the ampliation of the subject, the moon is possibly in the current sky. But the conclusion is nonetheless false, since the moon is necessarily identical to the moon.

Knuuttila observed that the fact that the subject of necessity propositions is ampliated in Buridan's, but not in Ockham's interpretation, may come down to the fact that "the authors did not use the same notion of necessity" (Knuuttila [2008], 556):

One might wonder why, instead of regarding the reading with actual or possible subjects as basic for all modals, Ockham discussed the necessity propositions with restricted subject terms. This makes his modal syllogistics less systematic than those of Buridan and Pseudo-Scotus who argued that if divided possibility propositions are

of possibility the subjects are said to supposit only for things that are". Cf. Buridan [1976], Ed. Hubien, 117: "Voco 'restrictionem per "quod est"' quando propositio sic formatur quod 'quod est B necesse est esse A' uel 'quod est B potest non esse A', et hoc est dictum quod in propositionibus de necessario uel de possibili subiecta ponantur supponere solum pro his quae sunt." In Hubien's edition, the second example is different from Read's. But they do not change the main point of the text.

ampliated, the equipollences between various modals requires that necessity propositions are ampliated. It seems that the differences are related to the fact that the authors did not make use of the same notion of necessity while discussing divided modals. If the distinction between divided necessity propositions with restricted and unrestricted subject terms is not nugatory, one should admit that possible beings may have necessary properties without occurring in every possible state of affairs. This condition is fulfilled by a relative de re necessity as Buridan and Pseudo-Scotus understood it.

The observation made by Knuuttila is meant to compare Buridan's views with William of Ockham's and with another anonymous contemporary who also wrote on modal syllogistics, namely Pseudo-Scotus. However, the same distinction also is acommodated within Buridan's modal semantics. Buridan thought that different kinds of concepts of necessity require different readings of the modal proposition. Buridan writes *TC* (Buridan [2015b], Transl. Read, 141):

(T2.9) It should also be noted that although Aristotle in his examples seems to take such propositions as 'Every human is of necessity an animal', and 'Every white thing is necessarily not black' as true, nonetheless verification of such examples is not required since those propositions are simply false. For everything that can fail to be can fail to be an animal; but a human, such as Socrates or Plato, can fail to be; so a human can fail to be an animal, and this contradicts the claim that every human is of necessity an animal. [...] As to whether the proposition 'A horse is an animal' is necessary, I believe it is not, speaking simply of a necessary proposition, since God can annihilate all horses all at once, and then there would be no horse; so no horse would be an animal, and so 'A horse is an animal would be false, and so it would not be necessary. But such [propositions] can be allowed

¹⁰⁹It is called Pseudo-Scotus since it appears in the literature under Scotus' *Opera Omnia*, but the attribution to Scotus has been shown to be incorrect. For a discussion of Pseudo-Scotus' syllogistics, see Lagerlund [2000] 165-171.

to be necessary, taking conditional necessity or temporal necessity, analyzing them as saying that every human is of necessity an animal if he or she exists, and that every human is of necessity an animal when he or she exists. [110]

This passage offers the following distinction. Take Buridan's example, 'Every human being necessarily-is an animal.' As a divided modal proposition, it is ampliated to what merely can be: if there are no human beings it is still true, since merely possible humans are necessarily animals. However, Buridan suggests that according to 'simple necessity,' this proposition can be falsified by God. This suggests that Buridan is using a conception of modality in which what possible objects there are may vary from one circumstance to another. On the other hand, according to conditional-temporal necessities, the proposition is true on the condition that human beings actually exist.

Since each of these readings hinge on the possible or actual existence of their subjects, they seem to correspond to possibilist and actualist interpretations of modal logic respectively. We can summarize this analogy in the following table below:

Some evidence for the fact that Buridan was more inclined towards the conditional-temporal reading can be gleaned from his remarks at *QPr.An*.

Buridan [1976], Ed. Hubien, 112: "Notandum est etiam quod quanuis Aristotiles exemplificando uideatur ponere tales propositiones tanquam ueras 'omnis homo de necessitate est animal' et 'omne album necesse est est non esse nigrum', tamen huiusmodi exemplorum non requiritur uerificatio, quia simpliciter illae propositiones sunt falsae. Omne enim quod potest non esse potest non esse animal; sed homo, ut Sortes uel Plato, potest non esse; ergo homo potest non esse animal, et haec contradicit dicenti quod omnis homo de necessitate est animal. [...] Utrum autem haec propositio sit necessaria 'equus est animal', crederem quod non, loquendo simpliciter de propositione necessaria quia deus posset simul adnihilare omnes equos, et tunc nullus equus esset; ideo nullus equus esset animal, et sic ista esset falsa 'equus est animal', ergo ipsa non esset necessaria, quamuis tamen tales possint concedi necessiriae necessitate conditionali uel temporali, secundum tales expositiones quod omnis homo de necessitate est animal si ipse est et quod omnis homo de necessitate est animal quando ipse est".

¹¹¹See section 3.3 of Menzel [2023] for an overview of possibilism and actualism. In short, possibilism is the claim that merely possible things exist and are quantifier over by modal propositions, whereas actualism claims that only actual objects exist and quantification is rectricted to the actual world.

Mode	Divided Sense Paraphrasis	Interpretation
Simple Necessity Conditional or Temporal Necessity	Everything which is or can be A necessarily-is B Everything which is or can be A, necessarily-is B, if A exists	Possibilist Reading Actualist Reading

Table 2.1: Two Readings of Divided Sense Modals

I.26. But ultimately, which interpretation Buridan has in mind cannot be decisively established by his short remarks on these issues alone. I will close this section by pointing to two first-order logic formalizations which rely on each concept of necessity. [113]

Stephen Read has briefly suggested that the relationships of equipollence and oppositions between modal propositions can be captured by translating them to first order modal formulae. To be sure, in doing so Read does claim to formalize Buridan's modal syllogistics, since there are clear problems involved in translating regimented Latin into first order logic. Nonetheless, this

Buridan [n.d.b], Ed. Hubien,: "Having noted these matters, I say as in the preceeding question that this is true according to conditional necessity, namely 'every human being necessarily is an animal'" ("Et istis notatis, propter sequentia, dicam, sicut dixi in quaestione praecedenti, quod de necessitate condicionali haec est vera 'omnis homo de necessitate est animal'. Buridan further notes that according to conditional necessity, divided modal propositions about impossible subjects can also be true: Buridan [n.d.b], Ed. Hubien,: And in the same way this is also true, 'every void necessarily is a place', even though we posit that it is impossible for a void to exist. Since the sense [of conditional necessity] is that every human being is necessarily an animal if humans exist, and in the same way concerning the void and place" ("Et sic haec etiam est vera 'omne uacuum de necessitate est locus', quamvis ponamus quod impossibile sit uacuum esse. Quia est sensus quod omnis homo de necessitate est animal si est, et sic de uacuo et loco.") In the next chapter, we will discuss the notion of conditional necessity in more detail in connection with Buridan's *Summulae de Dialecticae*.

¹¹³There have been many comparisons of Buridan's modal syllogistics with contemporary modal formalisms. Peter King has speculated early that Buridan should be working with a version analogous to S5 (King [1985]). His arguments have been met with criticism, and the viability of providing a relational semantics for Buridan's modal theory (or even the fruitfulness of the endeavor) is also a subject of some discussion. Recently, Johnston [2015a] provides such an axiomatization of modal syllogistics which he claims presupposes only the T-Principle. Early on, Gyula Klima (Klima [1988]) has offered a semantics for ampliation which can dispense with accessibility relations, while using a possible worlds framework to specify the modal operations.

¹¹⁴See again Klima [1988], and Parsons [2014]. One of the main problems concerns the scope of quantification. In supposition theory, the scope is restricted by the subject term. Whereas using variables in place of referring terms, we get the quantifiers to

formalization offers an interesting heuristic purpose for our purpose, in order to ask whether Buridan has in mind the broader or narrow modal space in his syllogistics.

Read's formalization is originally intended to capture the logical relationships of the *magna figura*, Buridan's famous octagon of opposition. I will here use it with a heuristic purpose. The formulae he proposes can be seen in the Table 2.2 below. To take one valid form, Barbara LML can be rendered as follows:

Barbara LML
$$(\exists x) \diamondsuit Ax \land (\forall x)(\diamondsuit Ax \rightarrow \Box Bx)$$
$$(\exists x) \diamondsuit Cx \land (\forall x)(\diamondsuit Cx \rightarrow \diamondsuit Ax).$$
$$(\exists x) \diamondsuit Cx \land (\forall x)(\diamondsuit Cx \rightarrow \Box Bx) \qquad \therefore$$

Stephen Read's formalization does not make any assumptions with regard to the kind of modal semantics or the ontological commitments Buridan operates with. For example, the validity of the syllogism above follows from the non-modal Barbara. In another remark, Read states however that he sees "no reason why Buridan would not endorse merely possible beings in his semantics. The preference for a possibilist interpretation of modal syllogistics can be seen from Read's choice to express the ampliation of the subject term. In the formalization below, the quantifier in wide scope - $\Box x \diamondsuit (Ax)$ - as opposed to expressing it with the quantifier in narrow scope, as in $\Box x$

Non-modal Barbara
$$(\exists x)Ax \wedge (\forall x)(Ax \to Bx) \\ (\exists x)Cx \wedge (\forall x)(Cx \to Ax). \\ (\exists x)Cx \wedge (\forall x)(Cx \to Bx) \qquad \therefore$$

replacing the modal formulas above for the non-modal ones in the second syllogism. ¹¹⁶Read [2015c], 35-37. In that context, Read is reacting to Lagerlund's earlier interpretation in Lagerlund [1999].

¹¹⁷Under a possibilist interpretation of what quantifiers range over, namely over a domain that contains the same objects across different possible worlds, the first entails the second by the validity of the Converse Barcan Formula: CBF $\exists x \diamondsuit (Fx) \rightarrow \diamondsuit \exists x (Fx)$. See again Menzel [2023]. Whether Buridan is committed to some version of the Converse Barcan or the Barcan Formula is not a question we will pursue here. This has been a matter on some debate in the literature since Lagerlund [1999] claimed

range unrestrictedly over the domain of quantification.

¹¹⁵Since the modal formulas are inside the scope of the quantifiers, the modal syllogism above is a special case of the non-modal Barbara syllogism:

In a recent paper, Dagys et al. [2022] provided a first-order logic formalization of Buridan's syllogistics for divided modal propositions, which takes conditional-temporal necessity as paradigmatic. The formulas they used to translate Buridan's syllogistic forms can be seen in the table below 2.2 below. They argue that it seems appropriate from Buridan's remarks at (T2.9) to take the concept of conditional necessity as central for syllogistics, and propose an actualist rendition of Buridan's syllogistics.

With these two formalizations of Buridan's modal propositions in mind in the tables below (2.3, 2.2), we note that Buridan had at his disposal two modal concepts in syllogistics - one which corresponds to a broad modal space according to 'simple necessity,' and another which corresponds to a more restricted modal space according to 'conditional-temporal necessity.' I think it is unlikely that a correct answer can be given to the question of which modal space Buridan's modal syllogistics operates with. First, since Buridan's distinction between simple and conditional-temporal necessity is underedeveloped at the TC passage at hand. Buridan is more explicit about varieties of modality in his SD, and there modal space does not have to do with quantification in modal propositions, but rather primarily with whether modal discourse concerns possibilities in the wide space of 'supernaturally possible cases,' or in the narrower sense of natural or metaphysical possibility. These concepts are not addressed in the TC, and we should treat them in more depth in the following chapters. Secondly, whereas in the TC Buridan identifies the narrow reading

he was. Recently, Johnston [2017], and Read's Introduction to Buridan [2015b], Ed. Transl. 35-36 have challenged this view. Lagerlund does not pursue his arguments further in Lagerlund [2000], 145-6. The relevant texts are the conclusions concerning inferences between divided and composite modals presented in Buridan [2015b], Transl. Read, 102, 104, 119.

¹¹⁸I have corrected Dagys formalization of necessary universal negatives writing Gx insead of $\neg Gx$ [Dagys et al.] [2022].

¹¹⁹Cf Dagys et al. [2022], 272,3: "Buridan introduces a distinction between simple necessity and what he calls 'conditional or temporal necessity'. The formalizations by Read and by Hodges and Johnston effectively take propositions of simple necessity as the paradigms of necessity. Following the last sentence in Buridan's quote above, it seems quite appropriate to take propositions possessing conditional or temporal necessity as paradigm and thus to formalize the fragment of the formula that represents the predicate of Buridan's necessary modal propositions [...] as a conditional $\Box \exists y(x = y) \rightarrow Gx$, meaning that in all possible worlds whenever x is actual, x is G."

¹²⁰See below chapter 3.1, and in particular the text (T3.1).

Table 2.2: Divided Modal Propositions in FOML (Possibilist)

Modal Proposition (Quantity, Mode, Quality) Latin Regimentation	Latin Regimentation	FOL Formalization (S. Read)
Universal Necessity Affirmative Universal Possibility Afirmative Particular Necessity Affirmative Particular Possibility Affirmative	Omnis A necesse est esse B Omnis A possibile est esse B Aliquid A necesse est esse B Aliquid A possibile est esse B	$(\exists x) \diamondsuit Ax \land (\forall x)(\diamondsuit Ax \rightarrow \Box Bx)$ $(\exists x) \diamondsuit Ax \land (\forall x)(\diamondsuit Ax \rightarrow \diamondsuit Bx)$ $(\exists x)(\diamondsuit Ax \land \Box Bx)$ $(\exists x)(\diamondsuit Ax \land \triangle Bx)$
Universal Necessity Negative Universal Possibility Negative Particular Necessity Negative Particular Possibility Negative	Omnis A necesse est non esse B Omnis A possibile est non esse B Aliquid A necesse est non esse B Aliquid A possibile est non esse B	$\neg \exists x (\diamondsuit Ax \land \diamondsuit Bx) (\forall x) (\diamondsuit Ax \to \diamondsuit \neg Bx) \neg (\forall x) (\diamondsuit Ax \to \diamondsuit Bx) \lor \neg (\exists x) \diamondsuit Ax \neg (\forall x) (\diamondsuit Ax \to \Box Bx) \lor \neg (\exists x) \diamondsuit Ax$

Table 2.3: Divided Modal Propositions in FOML (actualist)

Modaf Proposition (Quantity, Mode, Quality) Latin Regimentation	Latin Regimentation	FOL Formalization (Dagyns et al)
Universal Necessity Affirmative Universal Possibility Afirmative Particular Necessity Affirmative Particular Possibility Affirmative	Omnis A necesse est esse B Omnis A possibile est esse B Aliquid A necesse est esse B Aliquid A possibile est esse B	$\Box \forall x (Ax \to \Box(\exists y (x = y) \to Bx)) \land \Diamond \exists x Ax$ $\Box \forall x Ax \to \Diamond(\exists y (y = x) \land Bx)) \land \Diamond \exists x Ax$ $\Diamond \exists x (Ax \land \Box(\exists y (x = y) \to Bx))$ $\Diamond \exists x (Ax \land \Diamond(\exists y (x = y) \land Bx))$
Universal Necessity Negative Universal Possibility Negative Particular Necessity Negative Particular Possibility Negative	Omnis A necesse est non esse B Omnis A possibile est non esse B Aliquid A necesse est non esse B Aliquid A possibile est non esse B	Omnis A necesse est non esse B $\Box \forall x (Ax \to \Box(\exists y (x = y) \to \neg Bx))$ Omnis A possibile est non esse B $\Box \forall x (Ax \to \Diamond(\exists y (x = y) \land \neg Bx))$ Aliquid A necesse est non esse B $\Diamond \exists x (Ax \land \Box(\exists y (x = y) \to \neg Bx)) \lor \Diamond \exists x Ax$ Aliquid A possibile est non esse B $\Diamond \exists x (Ax \land \Diamond(\exists y (x = y) \land \neg Bx)) \lor \Diamond \exists x Ax$

with 'temporal or conditional necessity,' in the *SD* he distinguishes sharply between conditional and temporal interpretations of modality. In his mature views, temporality interacts in complex ways with modal discourse.

Ultimately, what Buridan's distinction between both senses of necessity and possibility in his divided modal syllogistics shows is that his concerns in doing modal logic are not driven by any underlying metaphysical assumptions at the outset. But I believe that we can learn more about Buridan's modal views on these issues once we turn to its applications.

2.3 Compossibility and de re Predication

In his TC, Buridan makes almost no mention of a distinction we will see in the next chapters. That is the distinction between supernatural and natural modal domains. One issue that is touched on by the modal analysis we find connects with that distinction, is the notion of compossibility. In this section, I will argue Buridan's remarks on compossibility made in TC presuppose the distinction between these modal domains. In particular, Buridan operates with a distinction between logical and metaphysical concepts of compossibility. which although only briefly mentioned in the TC, are developed in applied contexts at the QDGC and QSPhys. III.19.

In the *TC* II, seventh conclusion (Buridan [2015b], Transl. Read, 110), Buridan declares single-premise inferences between affirmative divided modals of possibility to composite modals (i.e., assertoric propositions) to be invalid. That is, the inference below is not a valid consequence:

$$1 \frac{\text{Every A possibly is B}}{\text{Every A is B}}$$

¹²¹In what follows, I will sometimes refer to a set of propositions or to things as compossible or compatible with each other. Whenever I refer to a set of propositions or things as compossible with each other, I mean that each of its members are pairwise compossible.

¹²²Buridan [2016], Eds. Streijger and Bakker, 187ff. I will develop the implications of this concept for his applications of modal logic to the *Physics* and *De Generatione et Corruptione* in [7.2].

¹²³Cf. Buridan [2015b], Trans. Read, 110: "From no affirmative composite of possibility does there follow a divided one of possibility with the mode affirmed, or conversely."

The reason why [1] is not valid is clearly related to the semantics of ampliation in a straightforward way. For in circumstances where the premise is made true on account of the ampliation of the subject - that is, in which nothing is A, but something which can be A can be B -, the conclusion remains false. [124]

However, Buridan raises a special doubt about this inference in his *TC* it as follows (Buridan [2015b], Transl. Read, 109-110): "Now there is a doubt whether it is permissible that every divided proposition of possibility warrants an assertoric, that is to say that the doubt is whether and in what way there corresponds to every true divided proposition of possibility a possible assertoric proposition." This doubt does not concern the invalidity of [1] itself, but rather in which way an affirmative divided modal of possibility is verified by singular *de re* predications.

Buridan remarks that from a universal divided affirmative of possibility, sometimes the corresponding universal assertoric is not only false but impossible. The counterexample Buridan gives goes as follows (Buridan [2015b], Transl. Read, 110):

(T2.10) Then I say first that it is not necessary that if a divided universal of possibility is true, the corresponding universal assertoric is possible. For whereas every star existing in the zodiac is possibly shining on our hemisphere, nonetheless, "every star existing in the zodiac is shining on our hemisphere" is by the laws of nature impossible. So if the universal should be formed as an assertoric, it should be formed as an assertoric divisively (*diuisim*) for its singulars. [126]

This counterexample is meant to pinpoint that the conclusion is impossible

¹²⁴Remember that, in such a circumstance, since universal affirmatives in medieval logic have existential import, the conclusion is false.

¹²⁵Buridan [1976], Ed. Hubien, 75: "Utrum autem liceat omnem propositionem de possibili diuisam ponere inesse dubitatio est, hoc est dictum quod dubium est utrum et quomodo omni propositioni de possibili diuisae verae correspondeat propositio de inesse possibilis."

¹²⁶Buridan [1976], Ed. Hubien, 75: "Dicam ergo primo quod non oportet si uniuersalis diuisa de possibili est uera quod uniuersalis de inesse sit possibilis. Quamuis enim omnem stellam existentem in zodiaco possibile sit lucere super nostrum hemisphaerium, tamen haec est impossibilis naturaliter 'omnis stella existens in zodiaco lucet super nostrum hemisphaerium'. Talis ergo uniuersalis si debeat poni inesse debet poni inesse per suas singuaris diuisim".

because of an incompossibility holding between a set of singular *de re* predications. Namely, whereas it is possible for each of the stars of the zodiac to be shining on our hemisphere, it is impossible - and as Buridan in addition remarks, at least impossible according to the natural laws - that all of them are at the same time located on the same side of the atmosphere. More generally, the universal proposition of possibility should not be verified collectively, but disjunctively (*divisim*), since it suffices for the truth of such propositions that any of its *de re* singulars be possible, and the universality need not be verified by all of its *de re* singulars simultaneously. [127]

The same example appears, again, in Buridan's *Physics*, and it is clear that it is supposed to illustrate a natural sense of incompossibility. Buridan writes:

(T2.11) First there is a manifest counterexample in any universal propositions, or in those in which there is a distributed term, according to which this is true 'every star you can see', whereas this is impossible, 'every star you see'; I always speak naturally (*sempter dico naturaliter*). I say that the first is true by induction, since one day you can see some stars, and at another day or night you can see others; therefore you can see all stars. But it is sufficiently clear that this is impossible, namely 'every star you see,' since then you would see those stars which the earth stands now between you and them. Similarly this is true, 'every human being can be dead', that is naturally; and nonetheless speaking naturally Aristotle would not concede that this is possible, namely 'every human being is dead', since then a species which is of the perfection of the universe would be fail to exist. [128]

¹²⁷ Cf. Buridan [2010], Eds. Streijger, Bakker and Thijssen, 65: "Therefore I say that for the truth of such propositions of possibility it suffices that from none of its singular instances, when transformed into an assertoric, an impossibility follows. That is to say, from 'this star you see', no impossibility should follow, and so of others." ["Ideo dico quod ad veritatem talis propositionis de possibili sufficit quod ad nullam singularem suam positam in esse sequatur impossibile. Verbi gratia, nam ad istam 'istam stellam tu vides', adhuc non sequitur impossibile, et sic de aliis."]

¹²⁸"Primo est manifesta instantia in multis propositionibus universalibus vel in quibus est terminus distributus, quoniam haec est vera 'omnem stellam tu potest videre', et tamen haec est impossibilis 'omnem stellam tu vides'; dico semper naturaliter. Dico enim quod prima est vera per inductionem, quia una die tu potest videre aliquas stellas

The addition 'I always speak naturally' when mentioning such cases makes clear that the notion involved is that of natural or metaphysical modalities. The passage also indicates why Buridan thinks the universal affirmative possibility is true - we verify it by induction, since each of the *de re* singulars are separately possible. Furthermore, the last example Buridan gives above, 'Every human is possibly dead' is a true universal affirmative, but it cannot be the case that all human beings all perish naturally at the same time (that is, by natural powers). Since only God can create and annihilate a species, the scenario where nothing is a human being is only possible relative to supernatural possibility for Buridan.

In his *QPhys*. III.19, he explains compossibility in the following terms (Buridan [2016], Eds. Streijger and Bakker, 187):

(T2.12) Compossibles are some propositions, any of which is not only possible, but they can be simultaneously true. [129]

A set of propositions is compossible if (i) they are separately possible, and (ii) they can be simultaneously true.

LOGICAL COMPOSSIBILITY: A set of propositions is compossible if it does not contain a contradictory pair.

But compossibility can also hold among things that are not propositions; in this case, Buridan has in a mind a metaphysical notion of compossibility. In that case, Buridan says that every possibile thing posited in actuality should not entail an impossibility with respect to some of its categorical properties (Buridan [2010], Eds. Streijger, Bakker and Thijssen, 65):

et alia die sive nocte tu potest videre alias; igiur omnes tu potes videre. Tamen satis patet quod haec est impossibilis 'omnem stellam tu vides', quia tunc tu videres illas inter quas et te est terra. Similiter haec est vera 'omnis homo potest mori', etiam naturaliter; et tamen naturaliter loquendo Aristoteles non concederet istam esse possibilem 'omnis homo moritur,' quia tunc deficeret species quae est de perfectione universi."

¹²⁹"Nam propositiones vocantur compossibiles, quarum non solum quodlibet est possibilis, sed etiam quod possunt simul esse verae, et ut sic non omnes possibiles sunt compossibiles."

¹³⁰Whereas logical incompossibility applies to a pair of contradictory propositions, whether a set of predications is compossible on account of their predicates is not a matter of logic, but of the forms connoted by the predicates in question.

(T2.13) And furthermore, if we are to speak according to possibility as an attribute of things which are not propositions, it should be said that everything possible, if assumed to be actual *where*, *how*, and *when* it is possible, and retaining all circumstances similar, no impossibility should follow. For it often happens that if some two or three things are assumed actual at the same time, an impossibility follows, just as for you to be possibly sitting, and for you to be possibly standing [no impossibility follows], but if we posit both as actual, namely that you are standing and that you are sitting, an impossibility does well follow.

In this context, *where*, *how*, and *when* stand for the natural properties a thing might have as signified by the predicate.

METAPHYSICAL COMPOSSIBILITY: A set of *de re* predications is compossible when the assumption that every predication is true does not entail in impossibility.

It is this notion that Buridan has in mind when he says that universal divided affirmatives of possibility should be paraphrased disjunctively, since their predicates should be compossible with each other. Further in the *QDGC*, Buridan sets down other counterexamples (*instantatie*) to the same pattern of inference we have seen above, by paying attention to whether the corresponding singular predications are compossible. (Buridan [2010], Eds. Streijger, Bakker and Thijssen, 64). For example, in

A white thing can be black (album potest esse nigrum)

the subject term, 'white' (*album*), connotes the property of whiteness (*albedo*), and the predicate term connotes the property of blackness. These properties are metaphysically incompossible: nothing can bear both whiteness

¹³¹"Et iterum, si loquamur de possibilitate attributa rebus quae non sunt propositiones, adhuc posset dici quod, omni possibili posito in esse ubi et sicut et quando e[s]t possibile et sic de aliis circumstantiis, nihil sequitur impossibile. Tamen saepe contingit quod si aliqua duo vel tria possibilia ponerentur in esse, impossibile sequeretur. Nam te possibile est sedere et te possibile est stare, et ad positionem quod stas et sedes sequitur bene impossibile."

and blackness. Therefore, when finding a verifying instance of the modal predication, Buridan says we should look for a *de re* verifying form that demonstrates the thing for which the subject supposited for, such that the very thing which is white can be said to be possibly black without repugnance. For example, the latter sentence should have as its verifying form a singular modal proposition, namely 'it is possible that this is black,' in which the demonstrative pronoun picks out the thing that is actually white. [133]

This way of parsing modal propositions is introduced by Buridan in order to preserve the truth of the sentences with a view to the natural compossibility between the properties connoted by the predicate of the sentences. The rule alluded to was known through William of Ockham, and it was introduced by him in order to deal with the truth conditions of tensed and modal propositions (Panaccio [2012]).

In dealing first with the tensed cases, Ockham emphasized that the predicate connotes or 'appellates' a form. Medieval logicians used 'appellatio' in varying ways, and the term has a complex history within the tradition of medieval semantics. In this case, 'appellation of a form' corresponds to what the predicate signifies, as opposed what the subject does. Adjectival terms such as 'white' are said to appellate a form (in this case, the form of whiteness), instead of referring to it (for reference only concerns individual things). In his explanation of appellatio, Ockham makes a relevant point concerning modal and tensed propositions (Ockham [1974]), Eds. Boehner *et al* I.72, p.216; translation *apud* [Panaccio] [2012], 144):

¹³²Buridan explains '*verificatio*' at Buridan [2001b], 224ff. In general, the idea is that a proposition is 'verified' when its truth conditions are spelled out in terms of further singular propositions. For example 'Socrates is white' is verified by the propositions 'this is Socrates' and 'this is white.' The general purpose of the procedure is to establish the supposition of the terms occuring in the original proposition and spelling out its causes of truth. For this terminology, see above p.48-9.

¹³³Buridan [2010], Eds. Streijger, Bakker and Thijssen, 65: "Et igitur, sicut communiter dicitur, tales propositiones de possibili si reducantur ad propositiones de inesse possibiles, debet auferri a subiecto illa connotatio quae repugnat predicato, ita quod subiectum ponatur sub pronomine demonstrativo demonstrante rem pro qua subiectum supponebat, sicut si haec est vera 'algum potest esse nigrum, tunc illo albo demonstrato haec est possibilis, 'hoc est nigrum'".

¹³⁴See section 5 of Read [2019] for an overview.

(T2.14) [...] the predicate apellates its form. This should not be understood to mean that the predicate supposits for itself or for the relevant concept. The point is that where the proposition concerns the past, the assertion is that the proposition in which that very same predicate (under its proper form) is predicated of that for which the subject supposits (or of the pronoun referring to that thing) was once true. [...] If the proposition concerns the possible hassertion is that the relevant proposition is possible.

This point is relevant since, in his exposition of modal propositions, Ockham claims that they should be verified in such a way that the forms appellated by the predicate should not be incompossible with each other. For example, in the case of 'Something black can be white,' a proper exposition of the sentence should state that what is now black can be white, and not the something can be both white and black. Ockham further expands his view of truth conditions for modal propositions by claiming that the predicate should be preserved 'under its proper form' when making explicit the supposition of terms in those propositions, such that they are verified by the possibility of singular assertorics, such as 'this is black,' in which the pronoun picks out the very thing that is no longer white (since the forms appellated by 'white' and 'black' are incompatible). Thus, concerning the exposition of divided sense modal propositions, Ockham writes (Ockham) [1998], Transl. Freddoso and Schuurman, 112):

(T2.15) [...] for the truth of such propositions it is required that the predicate under its proper form belong to that for which the subject supposits, or to a pronoun referring to that for which the subject supposits. Thus it is required that the mode expressed in such a proposition be truly predicated of a non-modal proposition in which the very same predicate is predicated of a pronoun referring to that for which the subject supposits - just as it was explained in the case of past-tense

¹³⁵"[...] praedicatum apellat suam formam. Quod est sic intelligendum: non quod supponat pro se vel pro conceptu, sed quod per talem propositionem denotatur quod propositio in qua ipsummet praedicatum sub propria forma, hoc est ipsummet et non aliud, praedicatur de illo pro quo subiectum supposnit, vel de pronomine demonstrante illud praecise pro quo subiectum supponit, fuir vera, si talis propositio sit de praeterito [...] vel quod sit possibilis, si prima propositio sit de possibili."

and future-tense propositions.

Buridan does not generally adopt the same manner of speaking in his logical writings, but he is fully aware of Ockham's methodology. [136]

These remarks on compossibility will turn out relevant once we turn to Buridan's applications of modal logic to natural philosophy. In cases such as the above, Buridan is operating with a *metaphysical* interpretation of compossibility, instead of a merely *logical* conception. We will turn to these issues and the same examples in more detail in section 7.2. What we can say for now is that Buridan's application of modal analyses to natural philosophy sheds some light on a distinction not explicitly drawn in the *TC*. That Buridan had a metaphysical notion of compossibility in mind explains why he stated that a universal affirmative of possibility is verified by its *re de* particulars disjunctively, instead of collectively. This point is restated in a number of occasions in his commentaries on the *Physics* and *On Generation and Corruption*, as we shall see in the last chapter.

¹³⁶Cf. SD 4.5.3 Buridan [2001b], Transl. Klima, 296, where Buridan states that to every tensed or modal propositions there is "a corresponding proposition of the present [time], which is formed with the entire predicate remaining the same, or with a predicate that is entirely similar with respect to all its dispositions, but this need not hold of the subject. In the same way, they [namely, Ockham] say that proposition about possibility is not true unless it has a corresponding true proposition about actuality [de inesse], with totally the same predicate". This means, as we have seen, that 'a white thing can be black' has a corresponding singular proposition asserting that 'this is black,' and pointing to that which is actually white (but possibly black).

¹³⁷This distinction will be significant in order to approach Buridan's questions on infinite divisibility in chapter 7.2.

3. Varieties of Necessity

The purpose of this chapter is to identify and categorize Buridan's main modal concepts and to clarify the logical relations that obtain between the forms of necessity and possibility that Buridan distinguishes. Buridan's most developed treatment of varieties of necessity contained in *SD* 8 forms the center of the chapter. In this text, Buridan suggests that different forms of necessity are ordered by degree in a scale of modal force.

I shall start with an analysis of Buridan's treatment of varieties of necessity at *SD* 8 (3.1). In section (3.2) I will argue that one of his motivations for keeping logical and natural (or metaphysical) modal spaces separated from each other is epistemological, since that distinction underpins Buridan's analysis of certainty. The remaining sections approach how the main forms of necessity and possibility we have identified figure across Buridan's logical writings: absolute (logical) necessity (3.3), natural (metaphysical) necessity (3.4), and historical modalities (3.5). By including the latter in his analysis of modality, I end the chapter showing that Buridan remained ambivalent with regards to the well-known problem of the necessity of the past (3.6).

3.1 Grades of Necessity

The most fine-grained treatment of a variety of modal concepts throughout Buridan's logic is found in the *Treatise on Demonstrations* [SD 8.6.3] In that context, Buridan rehearses the Aristotelian stricture that all propositions apt

¹³⁸ Buridan's modal theory is often interpreted in light of this passage. Calvin Normore (Normore [2013], Normore [2015]) believes that Buridan's basic modal concept corresponds to the highest grade of modality, of which others are restricted forms. On the other hand, Pasnau [2020] reads it as evidence of Buridan's contextual selection of the appropriate modal space depending on different explanatory requirements.

to figure in valid demonstrative arguments must be necessary and *per se*. Buridan claims there are many grades (*gradus*) of necessity and 'perseity' (*SD* 8.6.3; Buridan [2001b], Transl. Klima, 733):

(T3.1) The first grade of necessity [1] occurs when it is not possible by any power to falsify the proposition while its signification remains the same, nor is it possible for things to be otherwise than it signifies. Another grade [2] occurs when it is impossible either to falsify it or for things to be otherwise by natural powers, although it is possible supernaturally or miraculously, as in 'The heavens are moving', 'The heavens are spherical,' and 'Any place is filled.' The third grade [3] occurs with the assumption of the constancy of the subject, as in 'A lunar eclipse takes place because of the interposition of the sun and the moon,' 'Socrates is a man,' and 'Socrates is risible.' These are said to be necessary in this way because it is necessary for Socrates, whenever he is, to be a risible man [....] There is yet a fourth mode [4], which involves restriction. For just as 'possible' is sometimes predicated broadly, in relation to the present, past, and future, and sometimes restrictively, in relation to the present or the future, in accordance with what is said at the end of *On the Heavens* – that no force or power can be brought to bear on the past [...] - the same goes for 'necessary' and 'impossible', which are also predicated either with restriction or broadly. 140

¹³⁹ The mediaeval term *per se* is developed from Aristotle's *Posterior Analytics* A1 and A4, 73a35-b24. Buridan's *SD* 8 is in effect a commentary on the *Posterior Analytics*. For a general reading of *SD* 8 and its place in the *Summulae de Dialecticae* at large, see Ebbesen [2009].

¹⁴⁰Cf. Buridan [2001a], Ed. De Rijk, 141-142: "Est enim primus gradus necessitatis quia per nullam potentiam possibile est propositionem falsificari, stante significatione, vel aliter se habere quam significat. Alius gradus est quia impossibile est eam falsificari vel aliter se habere per potentias naturales, licet sit possibile supernaturaliter vel miraculose, ut 'caelum movetur', 'caelum est sphaericum', 'mundus est sphaericus', 'locus est plenus'. Tertius gradus ex suppositione constantiae subiecti, ut 'lunae eclipsis est propter interpositionem terrae inter solem et lunam', 'Socrates est homo', 'Socrates est risibilis'. Hae enim dicuntur necessariae sic quia necesse est quandocumque est Socrates, ipsum esse hominem risibilem [...] Adhuc st quartus gradus, secundum restrictionem. Nam sicut 'possibile' aliquando ample dicitur, in ordine ad omne tempus praesens, praeteritum et futurum, et aliquando restrictive, in ordine ad praesens vel

In this passage of the *SD*, Buridan distinguishes several truth-concerning or alethic modalities. To the highest degree, a proposition is necessary if it cannot be falsified by any power, not even by a supernatural power (*per nullam potentiam supernaturalem*). On the other hand, natural necessities are not susceptible of falsification by any natural power (*per nullam potentiam naturalem*), although they have competing alternatives in the broader logical domain coextensive with God's omnipotence.

In speaking about modal concepts according to the first and second modal grades, Buridan is to a large extent following a traditional usage of modal notions in the medieval period. As is well known, the distinction between the two first modal concepts which Buridan makes in [T3.1] is fairly widespread in late medieval theological and philosophical thought. For example, in his *Quodlibeta*, William of Ockham says that for something to be possible is sometimes understood according to the laws ordained by God, and sometimes understood according to what is absolutely possible for God to bring about in the sense of not being contradictory. In scholastic theology, God's omnipotence is coextensive with, and in some sense limited by, the domain of logical possibilities. Therefore, modal space was essentially carved out, in medieval theology from a theory of power. According to the notion of ordained power, the necessity of the lawful connections in the natural world are seen as a byproduct

futurum, iuxta illud quod dicitur in fine primi *de Caelo* quod non est virtus sive potestas ad praeteritum [...] - ita etiam 'necesse' et 'impossibile' dicuntur secundum restrictionem vel ample'."

¹⁴¹Such modes as 'necessity' and 'possibility' are truth-concerning (or alethic) because they modify the truth of the proposition (see Buridan [2001b], Transl. Klima, 67). Contemporary modal logic emphasizes the conformity of alethic modal concepts to the T-Axiom ($\Box p \rightarrow p$), in contrast to, for example, normative or deontic concepts.

something's being possible sometimes is understood according to the laws ordained (*ordinates*) and instituted by God and those things God is said to be able to do by his *potentia ordinata*. (But sometimes) 'to be able' (*posse*) is understood in another way for 'to be able to do all that whose doing does not include a contradiction' whether God ordains this to be going to be done or not, for God can do many things which he does not (...) and these things are said to be possible by his *potentia absoluta*." ["(...) 'posse aliquid' quandoque accipitur secundum leges ordinatas et institutas a Deo, et illa dicitur Deus posse facere de potentia ordinata. Aliter accipitur 'posse' pro posse facere omne illus quod non includit contradictionem fieri, sive Deus ordinaverit se hoc facturum sive non, quia multa potest Deus facere quae non vult facere [...] et illa dicitur Deus posse de potentia absoluta."]

of divine will. 143

On the one hand, the distinction Buridan makes between logical and natural modal spaces is analogous to the one between absolute and ordained power. But contrary to the approach exemplified by Ockham, Buridan's version of this distinction is not clearly based on a theory of power. Some of the differences between these approaches can be gleaned from the terminology Buridan uses. Firstly, in most contexts, Buridan classes in the first grade of modality those propositions whose negatives are not possible in any supernaturally possible case (*casus supernaturaliter possibilies*), while in the second grade of necessity, he classes propositions whose negatives are not possible in any naturally possible case (*casus naturaliter possibiles*). Buridan also frequently marks out each modal space by using the locutions 'naturally speaking' and 'supernaturally speaking,' which, as Joel Biärd has argued, reflects a secularized version of the theory of absolute and ordained power. Secondly, Buridan usually seems to refrain from conceptualizing the necessary connections of the natural world as a byproduct of divine will.

¹⁴³ See Courtenay [1985] for a comprehensive account of the uses of *potentia absoluta* and *potentia ordinata* in the fourteenth century. In this framework, God was considered as the first or universal cause of the universe, and the operations of the secondary or particular causes - the observed causes in operation in the natural world -, were ultimately regarded as causes in relation to God's ordained power. The interference of God in the natural world - for example, in miracles - meant that whatever a secondary cause can produce, a first cause can immediately produce as well, Gelber [2004], 287: "Ockham's contemporaries did not consider God's suspension of the causal order and his direct action in lieu of secondary causes as an intrusion of God's absolute power into the world, but as one of the primary ways God produces miracles within the ordained system." The absolute and ordained power of God are therefore not distinct powers, but rather distinct modes of acting (absolutely or 'ordinate').

¹⁴⁴For the use of 'naturally speaking' versus 'supernaturally speaking' (*supernaturaliter loquendo*) language, as opposed to the more traditional distinction between the absolute and ordained powers, see Biard [2012], who traces the former language to a common Parisian terminology stemming from Albert the Great's (†1280) Aristotelian commentaries. Biard contrasts the theory of absolute and ordained power as based on normative model, in contrast to Buridan's understanding of the natural order (Biard [2001], 64): "[...] in Buridan we find a sort of naturalist version of the operational or 'juridical-political' model of omnipotence, characterized by God's ability to intervene *de facto* in the *de jure* order of nature, a naturalist version of omnipotence built around the opposition between the 'supernatural and miraculous' and 'natural' modes."

¹⁴⁵A clear statement of Buridan's attitude in this regard can be found in, for example, his *QDC* II.9, cf. Buridan [1996], Ed. Patar, 423-424: "In natural philosophy we

according to the 'common course of nature.' [146]

While these remarks show a difference in terminology between Buridan's modal conception and the account of modalities based on the absolute and ordained powers of God, do they also reflect a conceptual difference with regard to Ockham's theory?

Calvin Normore has argued that Ockham is best interpreted as a modal monist. In Normore's interpretation, despite the apparent conceptual gulf between the necessity of logical truths and, for the example, the necessity of the past - which Buridan classes as the highest and the weakest grade of necessity respectively -, these ways of being necessary are not ultimately different kinds of necessity in Ockham's theory. Normore believes that for Ockham necessity amounts to actuality and immutability, and different ways of being necessary are closely related to the time at which a proposition is necessary. Normore writes (Normore [2016], 141):

The way in which the past is necessary seems different from other ways of being necessary. Even if one thinks that a sentence like 'Ockham existed' is now as necessary as '2+2=4,' it is nonetheless reasonable to suppose that it was not always as necessary. We can then distinguish between true sentences which could have been false once but can no longer be false and true sentences which never could have been false. Actuality plus immutability characterizes the first kind but seems too weak for the second. It is central to Ockham's view as I understand it that this appearance is illusory - that at bottom necessity just is actuality plus immutability. [...] Of course there is a difference between necessity and immutability tout court and there is also a difference between being always necessary and being necessary from some time on, but if I am right

should understand the actions and connections (*dependentias*) as always proceeding in a natural way, whence God is no less of a cause of this world and its order as the world itself if it were eternal." ["Modo in naturali philosophia nos debemus actiones et dependentias accipere ac si semper procederent modo naturali, unde non minus Deus est causa huius mundi et ordinationis eius quam si iste mundus fuisset aeternus."]

¹⁴⁶The phrase 'ex communis cursus naturae' has a number of distinct functions in Buridan's metaphysics and epistemology. For a discussion of these usages in Buridan's theory of science, see King [1987].

this difference is not a difference in the kind of necessity involved. What is different is just *when* the sentences are necessary.

What Normore's interpretation suggests is that on Ockham's view these ways of necessary are not distinguished as a matter of degree - at the present moment, the proposition stating '2+2=4' is *as necessary* as the proposition that 'Ockham existed.' If Normore is correct, Buridan's view seems to differ from his nominalist predecessor in important ways.

Let us turn our attention back to [T3.1]. For Buridan, there is a scale of modal force between modal concepts - each kind of necessity is situated on a scale of modal force, from strongest to weakest. The unifying trait that is shared by all kinds of necessity in that modal scale seems to be a notion of unfalsifiability. This notion is fleshed out by Buridan as the property of not having variation of actual truth to falsity, and conversely as the state of things not becoming otherwise related to how the proposition signifies them to be, that is of not passing from 'things-being-as-it-signifies' to 'things-being-otherwise'. [148]

That Buridan emphasizes unfalsifiability as a trait shared by all forms of necessity distinguished at [T3.1] is compatible with the practical aims of *SD* 8 as treatise on demonstration. This treatise is largely concerned with demonstrative propositions, those propositions apt to figure in demonstrative arguments and productive of knowledge. The reason why they cannot pass from actual truth to falsity is, as Buridan claims, that demonstrative knowledge cannot become non-demonstrative, nor can knowledge become 'ignorance.' In that same

¹⁴⁷Normore finds evidence for his interpretation in Ockham's logical treatises, with respect to the theory of consequence *SL* III-3 (Ockham [1974], Eds. Boehner *et al.* 243-256), and also in his theological *Quodlibeta*, cf. Ockham [2004], II.5.

¹⁴⁸Hence, Buridan claims that the phrase 'cannot be otherwise' as applied to propositions should be expounded with regard to their invariance in truth value (Buridan [2001b]), Transl. Klima, 793): "And it is in this same sense that one should appropriately expound the claim that it cannot be otherwise; and likewise that it is ungenerable and that it cannot turn from truth into falsity; and that it is incorruptible, since it is not variable with respect to change from truth to falsity, or from [having the property of] things-being-in-the-way-it-signifies to [having the property of] things-being-otherwise." Cf. Buridan [2001a], Ed. De Rijk, 211: "Et ad eundem sensum convenienter debet exponi quod sit impossibilis aliter se habere; similiter quod sit ingenita, et quod sit immutabilis de veritate in falsitatem; et quod sit incorruptibilis, quia non est mutabilis de veritate in falsitatem vel de taliter se habere sicut significat in aliter se habere."

¹⁴⁹Cf. Buridan [2001b], Transl. Klima, 750: "A property [proprietas] of enduring

sense, a simple consequence cannot cease to hold by the variation of the actual truth of the conditional to its falsity. [150]

Buridan cashes out this notion of unfalsifiability in different ways, but often it is done by appeal to the concept of a counterfactual situation (*casus*) in which the falsity of a proposition is conceivable. The concept of *casus* stems from the logical genre of the *ars obligatoria*, where *casus* was used to mean a non-actual situation put forth for the sake of argument in order to see what follows from it. In an obligational disputations. Is distinguishing between logical and metaphysical possibilities was a common practice. This is very much apparent in the use of counterpossible cases (*positio impossibilis*). In the earliest

knowledge and demonstration, therefore, is that an enduring demonstration is never turned into a nondemonstrative syllogism by the passage of time or by a change in the subjected thing, nor is a piece of enduring knowledge turned into ignorance (...)" ["Est ergo proprietas scientiae mansivae et demonstrationis mansivae quod numquam demonstratio mansiva mutatur propter transitum temporis vel rei subjectae mutationem in syllogismum non-demonstrativum, nec scientia mansiva in ignorantiam vel opinionem falsam (...)"]

¹⁵⁰As we have seen above, as-of-now consequences can cease hold. In the case of as-of-now consequences, however, modalities are taken 'restrictively.' This restricted concept of modalities seem to correspond to Buridan's fourth kind of modal concept distinguished at [3.1]. We will approach this weakest form of necessity in section [3.5].

¹⁵¹Buridan himself apparently never wrote a treatise on obligations - contrary to what Bernd reports in (Michael [1985]) -, but the obligational language is apparent in Buridan's writings.

¹⁵²The *ars obligatoria* was a dialectical art, which deals with the 'logical duties' of a respondent who, given a *casus*, is obliged to respond according to what follows from it. One type of obligation was the *positio impossibilis*, in which the respondent had to reply according to what follows from an impossible proposition. Ockham describes it as follows (Ockham [1974], III-3 c.42). "Per talem enim positionem aperitur via ad sciendum quae consequentiae sunt bonae et evidentes et quae non sunt evidentes, ex quibus potest cognosci distinctio terminorum et ordo eorum, et quandoque potest per talem modum investigari aliquarum rerum distinctio."

153 See Hanke [2020], 333-4ff. for applications of the term *casus* in treatises on consequence in later authors such as Paul of Venice. Hanke maps out several variations of the terms playing a distinctive role in the period, such as *casus imaginabilis/casus secundum imaginationem*. Among even later Jesuit scholastics in the seventeenth century, such as Sebastián Izquierdo († 1681), the notion of *casus naturaliter possibilis* was used to single out physical consequences, which are truth-preserving provided no supernatural intervention occurs (Hanke [2020], 338). Although Buridan does not have a notion of *consequentia physica* as these later authors, the distinction between logical and metaphysical necessities on the basis of counterfactual situations is perfectly in line with Buridan's theory of degrees of necessity.

known treatise on obligations, the *Tractatus Emmeranus*, we find metaphysical impossibilities such as 'Socrates is a donkey' as example of an intelligible but naturally impossible *casus*. This early usage of counterpossible cases (*positio impossibilis*) suggested that at the level of understanding some circumstances are imaginable which, however, are not metaphysically possible.

In his SD, Buridan states that absolute necessities are true in virtue of signification itself - some propositions, Buridan explains, are bound by their significations to be true, just as fire by its nature is bound to be hot 155 Insofar as his necessities *simpliciter* are logical, they are nonetheless more restricted to the concepts involved than the impossible *positio*, and presuppose a certain view of conceptual or logical truths. It was costumary to draw the distinction between

154Cf. Anonymus [1974]. Thus, the inference 'if it is a man, then it is not a donkey' fails since humanity and asinity are not incompatible at the level of understanding. One kind of *impossible positio* here has to do with union. Bear in mind that the *Tractatus Emmeranus* defines consequence with the containment criterion: the understanding of the cosequent is contained (*claudat*) in the understanding of the antecedent. See Binini [2022] for a survey of *positio impossibilis* in the thirteenth century.

Buridan [2001b], transl. Klima, 727: "Just as fire is said to be *per se* hot because it is bound by its nature to be hot, and water is not said to be *per se* hot because it is not bound by its nature to be hot, so also that a proposition is said to be per se true that by its own and by its terms' signification is bound to be true; indeed, it is impossible, given this signification, for it to be false, and this is for it to be necessary. But a proposition is said to be *per accidens* that is not so bound [to be true], indeed, it is possible for it to be false, and this is for it to be contingent."

156Buridan does not seem to distinguish the level of intelligibility from the level of metaphysical possibilities, and explains away the load of *positio impossibilis* by appeal to the conventionality of language. As he further explains in sophism 'is is in our power that a man is a donkey' (Buridan [2001b], transl. Klima, 936ff), since words signify as we please, and it is within our power to create a convention in which 'man' to signifies 'whiteness' and 'donkey' signifies 'color', then since 'whiteness is a color' is true, so the *positum* also is. Further, given Aristotle's rule in the *Categories* (12.14b14ff.) that if something is true, then it is the case, namely,

$$T^{\sqcap}p^{\sqcap} \to p$$

we should grant that if in this new convention, 'man is a donkey' is true, then it is the case that some man is a donkey. Buridan's solution to the sophism states Aristotle's rule should be restricted to the presently imposed signification of terms. Referring to shifts in convention which can happen in obligational disputations, Buridan writes that (Buridan [2001b], 932): "it commonly happens in obligational disputations in schools that the master stipulates that for the duration of the disputation the term 'donkey' should signify for the disputants precisely the same as that which the term

a certain kind of consistency based on the terms from the nature itself. For Buridan, necessity *simpliciter* holds in virtue not of natures, but of concepts and their logical relationships - namely, necessities *simpliciter* are those propositions and thoughts so related to things that not even a supernatural scenario brought about by divine power would be able to falsify.

What about the second degree? When Buridan equates the natural necessity with the invariant course of nature, he often claims to be making a distinction that has escaped Aristotle, and says that since Aristotle did not have a concept of absolute possibility by a divine power, he took those invariances to be necessary absolutely (*simpliciter*). This is specially the case for the examples Buridan gives in (T3.1), but more often it concerns propositions stating metaphysical facts about natural kinds (*species*), as for example 'every horse is an animal'. Buridan says that Aristotle took them to be perpetually true and thus necessary given the perpetual existence of natural kinds, but that he calls them 'naturally necessary,' since things cannot be otherwise as they signify through natural powers, even though it could be otherwise by supernatural powers.

Are Buridanian natural necessities also metaphysically necessary? While one could take metaphysical modalities to be a higher modal kind, what Buridan calls natural necessities are undistinguished from what we call metaphysical necessities. One particularly clear example as to why natural and metaphysical

^{&#}x27;animal' signifies for us when used in accordance with its common signification; and the respondent and the others agree. Then the proposition 'A man is a donkey' is true for them and is to be conceded by them, but a proposition similar in utterance would be totally false and impossible were it propounded outside of the context of such an obligation in the church of Notre-Dame to those there present."

¹⁵⁷Binini [2022], and Yrjönsuuri [2007].

^{158°}Cf. Buridan [n.d.b] I.25: "Et tamen verum est quod Aristoteles, ponens perpetuitatem mundi et esse necessarium quod perpetuarentur generationes et corruptiones in istis inferioribus, secundum has opiniones opinabatur consequenter has esse simpliciter necessarias 'asinus est animal', 'homo est animal', et ita de consimilibus. Et ipsae possent dici 'naturaliter necessariae', ad istum sensum quod non potest aliter esse quam ipsae significant per potentiam naturalem, solum per supernaturalem". ["And it is true that Aristotle, assuming the world to be perpetual and that it is necessary for generation and corruption in the inferior beings be perpetuated, took those propositions to be absolutely necessary, namely "A donkey is an animal", and "a human being is an animal", and similar. Those can be called "naturally necessary", in the sense that things cannot be otherwise as they signify by some natural power, only by supernatural powers.]

modalities are undistinguished in mediaeval philosophy is the notion of inseparable accidents (*propria*). In his example of a the third 'grade' of necessity at [T3.1], Buridan has made use of an example of *proprium* as well, namely Socrates' risibility. The thrust of the example is that an inseparable accident, such as Socrates' capacity for humor, can only be destroyed if Socrates is also destroyed - in contrast to, say, Socrates' having a beard, since Socrates does not cease to exist when he shaves his beard.

In his analysis of Ockham's notion of inseparable accidents, André Goddu has argued that Ockham's treatment offers evidence for the close connection between natural and metaphysical necessity in the medieval approach Goddu proposed the following explanation for the modality underpinning inseparable accidents in Ockham's theory (Goddu [1984], 68ff.). Suppose the affirmative 'Every human being is able to laugh' is true at w_1 , but false at w_2 . Since according to supposition theory, an affirmative proposition is true when the subject's referents (*supposita*) fall under the predicate term, and false otherwise (in particular, false when their subjects are empty), and negative propositions are true when the the subject's referents (*supposita*) do not fall under the predicate term and false otherwise (in particular, false when their subjects are not empty), then it is true that 'Some human being is not able to laugh' at w_2 .

- 1 Every human being is able to laugh is true at w_1
- 2 Some human being is not able to laugh is true at w_2

Goddu's point is that these worlds would differ from each other in regards to which species inhabit them. Given the definition of an inseparable accidents, human beings exist at w_1 , but no human being exists at w_2 . This point suggests that relative to God's absolute power - the highest form of possibility it is *not* possible to create a world in which Socrates exists *and* Socrates is not able to laugh. The same holds true essential properties as well. God cannot bring about any world in which there are human beings which are not rational

¹⁵⁹Medieval scholastics took *risibilitas* to mean the capacity for humor - it was regarded as an inseparable accident pertaining to human beings due to their rational nature.

¹⁶⁰The main passages Goddu relies on are from *SL* I.24 (Ockham [1974], 78-81).

¹⁶¹See Kusch [1990] for a criticism of Goddu's exegesis of Ockham's point.

beings. What God can do - and, therefore, what is logically possible - is to destroy the world. Hence he can bring about by supernatural power empty circumstances which would falsify the corresponding affirmatives, but only because their subjects fail to supposit for anything in this scenario. The same point applies for Buridan's own understanding of inseparable accidents and essential properties. This suggests that it is not possible for God to create alternative natural orders in which such metaphysical facts are violated.

Buridan's intention in distinguishing logical from natural or metaphysical possibility is thus not so much to show that there are other possible natural orders, but it is, as he puts it, to 'salvage' the Aristotelian view of necessity as he understands it, which he associates both with Aristotle and with the 'White Cardinal,' most likely referring to Robert Kilwardby. What Buridan wishes to emphasize is that natural necessities could well be falsified by a supernatural case, but only because the corresponding affirmatives would fail to refer to anything. As Buridan notes in *QAPr.* I.25, this should pose no obstacle to their necessity, otherwise no geometrical proposition would be necessary either, since just as God can annihilate all human beings and thus falsify the proposition that all human beings are animals, so too he can annihilate all magnitudes, and thus falsify all propositions of geometry. Buridan's view that the propositions of natural science should be taken as necessary means that they are true in every *casus* complying to the common course of nature.

The third form of necessity Buridan presents in the modal scale at 3.1 may sound less familiar. The key idea here is that this notion of necessity

¹⁶²Cf. Buridan [2015b], 24.

¹⁶³ Cf. QAPr. I.25a (Buridan [n.d.b]), 53): Further, if it was proposed that it is not necessary, that is only insofar as God can annihilate every human being. But I argue that it should not prevent it from being necessary: since if it did, no geometrical propositional would be necessary, since God can in the same way annihilate all magnitudes as he can all human beings. And then it would further follow that geometry is not a science, which everyone regards false and inconvenient; and this consequence holds since science is only of necessities. ["Item, si poneretur quod non esset necessaria, hoc esset pro tanto quia deus posset annihilare omnem hominem: ideo nullus homo esset, et sic nullus homo esset animal. Sed probo quod hoc non debet obstare quin ipsa sit necessaria: quia si hoc obstaret, nulla propositio geometrica esset necessaria, cum deus ita possit annihilare omnes magnitudines, sicut omnes homines. Et tunc ultra sequeretur quod geometria non esset scientia, quod reputatur ab omnibus falsum et inconveniens; et patet haec consequentia per hoc quod scientia non est nisi de necessariis".]

is conditional on the existential import of its terms. Buridan's phrase for that condition, *constantia*, is usually translated as 'existential requirement.' Constancy' is used by Buridan to specify the requirement for a proposition to be true is that its terms must refer to something. This existential requirement means simply that if the supposita of the subject term exists, then the predicate applies to the same as the subject.

Why does Buridan include the third form of necessity in his modal scale? One reason may be connected to a historical debate of the period. The word 'constancy' is connected with an historical issue in logic during the later middle ages. In 1277, a number of condemned articles were issued at Oxford, possibly under the direction of Robert Kilwardby. Among the condemned propositions that Arts Masters should not propound is that 'necessity only remains with the persistence of the subject.' This thesis was again among the prohibited views in the condemnations of 1346 and 1347, targeting in the latter case Nicholas of Autrecourt's ideas.

¹⁶⁴The translation of *constantia* as 'existential requirement' is adopted by Stephen Read in his translation of the *Tractatus de Consequentiis*. Gyula Klima opts for 'constancy' in his translation of the *Summulae de Dialecticae*. In Latin, *constantia* means generally that the item in question is keep in existence (*constat*). See De Rijk's Introduction to Gerald Odonis' *Logica* for a survey of the usage of *constantia* (Giraldus Odonis) [1997], 49ff.).

terminology is explained in *SD* 6.4.4 (Buridan [2001b], 429): "But by the removal of a term in this context we should understand the nonconstancy of the term, and by the positing of a term the constancy of the term, in the way logicians talk about the constancy and nonconstancy of terms, namely, that we say that a term has constancy if it supposits for something and nonconstancy if it supposits for nothing. But we signify that a term has constancy, and that it supposits for something, when we affirm of it the verb 'is' absolutely ['est' secundum adiacens] and that it has nonconstancy when we deny of it this verb." Cf. Buridan [2013], 6.4.4: "Per remotionem termini debemus intelligere non-constantiam termini, et per positionem constantiam termini, prout logici loquuntur de constantia et non-constantia, scilicet quod terminus dicamus constare, si supponat pro aliquo, et non constare, si pro nullo supponat. Modo terminum significamus constare et pro aliquo supponere, quando de ipso affirmamus hoc verbum 'est' secundum adiacens, et non constare, quando dictum verbum de illo termino negamus".

¹⁶⁶Note that in the medieval context, a condemnation meant an "exercise of teaching authority," not exactly censorship. See Thijssen [1990].

¹⁶⁷The article of the latter condemnation states that "truth with necessity only holds so long as the subject persists" ("Item quod veritas cum necessitate tantum est cum constantia subjecti"), cf. [Thijssen [1990], p.59 n.21. As Thijssen remarks, Autrecourt's

the former case, we should note that in one of the sophisms attributed to Robert Kilwardby by Sten Ebbesen and Jan Pinborg, Kilwardby defends the thesis that the proposition 'a human being is an animal' is true even when its subject ceases to exist. The reason Kilwardby adduces to his position is that truth and necessity require primarily that the concept of the predicate is included in the concept of the predicate, instead of hinging directly on the existence of the reference of the subject. Buridan was certainly aware of these debates, and this may be one of the reasons why he distinguishes a third form of necessity in his modal scale. In the end, Buridan's view attests to the little effect these condemnations had in practice. In his semantic theory, a proposition is only true when its referents exist in some way, and he therefore believes that necessary truth presupposes existence.

Given this description of the first forms of necessity Buridan distinguished at [T3.1], what can we say about their logical relationships? The intended relationships holding between the three first grades of necessity are stated by Buridan elsewhere in the *QAPr*. I.25 in the following terms (Buridan [n.d.b], I.25):

(T3.2) Lastly, you should know that these three modes of necessity follow transmitted writings themselves do not indicate further why this condemnation was associated with him.

states that the proposition is true even when the *res* ceases to exist because what is required for truth is that the concept of the predicate is included (*intelligitur in*) in the concept of the subject (Ebbesen & Pinborg [1970] 88): "[...] what is understood by something is truly predicated of it. Hence, since in the concept of man the concept of animal is understood, 'animal' is truly predicated of 'man'. Since therefore this was proven earlier to be true, namely 'a human being is a human being', even when no human being exists, similarly this will be true, that 'a human being is an animal'. And therefore it is clear that 'a human being is an animal is true' even when no human beings exist' ([...] quod intelligitur in aliquo, vere enuntiatur de ipso. Cum igitur in homine intelligitur animal, vere enuntiabitur de eo. Si ergo ut visum est hec sit vera 'homo est homo' nullo homine existente, hec similiter erit vera 'homo est animal'. Et sic patet quod hec est vera 'homo est animal' nullo homine existente"). For Kilwardby's position, see; [Thom [2007], p.25; on the question of the impact of the 1277 condemnations on logic in particular, see [Uckelman [2010]].

¹⁶⁹Even though it may not presuppose *actual* existence. We will see below that, in his treatment of natural supposition, Buridan changes the normal truth conditions for assertoric propositions in order to preserve their truth even when their subjects are empty, by using elements of his theory of ampliation.

upon each other (*se consequuntur*) in a certain way and in a certain order. For every proposition of absolute necessity is also necessary *de quando* and conditionally necessary, but not the other way around; on the other hand, every proposition of *de quando* necessity is also conditionally necessary, but not the other way around. [170]

The three kinds of necessity mentioned by Buridan in the passage above seem to correspond to the first three concepts of necessity listed in the modal scale at **(T3.1)** even if their formulations differ slightly. In the last passage, Buridan means by *de quando* or temporal necessity the same modal concept according to which the universal affirmatives mentioned in **(T3.1)** are necessary. Namely, universal affirmatives are necessary because whenever (*quandocumque*) their subjects refer to actual things, the predicates hold true of them in the common course of nature. What the *QAPr*. I.25 passage tells us that the former passage only suggests, is that these modal concepts are logically related in a certain way and in a certain order. Buridan uses supposition theory in order to clarify the logical relationships holding between these modal concepts in the following way:

- 1 Conditional necessity: it must be the case that if the subject and predicate stand for (*supponit pro*) something, then they stand for the same.
- 2 Temporal (*de quando*) necessity: it must be the case that for any time when the subject and the predicate stand for something, then they stand for the same.
- 3 Absolute necessity: it is impossible that there is some time the subject and predicate do not stand for the same when the proposition is formed, or it is impossible for things to be otherwise in an absolute sense.

Hence, these three modes follow from each other (*consequentur*) in a certain way and in a certain order, because of the requirements imposed on the

¹⁷⁰Cf. Buridan [n.d.b] I.25: "Et debetis scire ultimo quod isti tres modi necessitatum se consequuntur quodam modo et quodam ordine. Quia omnis propositio simpliciter necessaria est etiam necessaria de quando et ex conditione, sed non convertitur; etiam omnis propositio necessaria de quando est necessaria ex condicione, sed non convertitur."

supposition of their terms differ. In absolute necessities, it is impossible that the subject and predicate do not stand for the same thing. In turn, natural or *de quando* necessities are propositions that at some time, during the course of nature, their subject and predicate stand for the same. Lastly, a proposition is conditionally necessary (*ex condicione*) when, if the subject stands for something, the predicate would stand for the same.

If we combine what Buridan says at *QAPr*. I.25 ((T3.2)) with the conception of necessity in terms of degrees developed in *SD* 8.4.6 (T3.1) we can understand Buridan's theory of varieties of modality by turning each grade of necessity into corresponding nested spheres of possibility. In this conception, every *casus* is logically possible by supernatural power, and some *casus* are only naturally possible by natural powers. Further, some propositions are only possible assuming constancy, and still others are only possible restrictively as-of-now. We may picture the intuitive model Buridan had in mind in the following diagram below (see [3.1]):

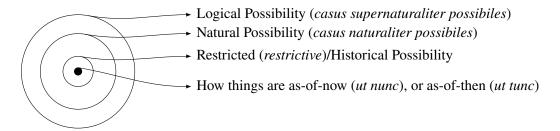


Figure 3.1: Nested Modal Spaces

Given a general notion of possibility (of being possible by a *casus* or by a power), the relationship between Buridan's modal concepts could be understood as involving modal spaces around these spheres as follows. In the actual world, Socrates does not exist any longer, and therefore cannot laugh. In a narrow sense, it is thus impossible that Socrates can laugh. In all naturally possible cases (*casus naturaliter possibiles*), so long as natural powers exist, our generalizations concerning *species* hold for all times and situations, such as 'Every fire is able to heat', or 'The heavens are moving'. Moving yet further apart to yet broader modal spaces, considering situations with divine powers that outstrip natural ones we get supernaturally possible cases (*casus supernaturaliter possibiles*).

Buridan's claim at (T3.2) that logical necessities entail natural necessities can be intuitively captured by modal operations ranging over the cases (*casus*) regarded as possible circumstances. Let L_s be a function that takes p to the set W of all possible worlds; then $L_s p$ expresses that a proposition is absolutely or simply necessary. If we take L_n to be a function taking p to a subset W_n of W instead, namely the class of naturally possible worlds, then $L_n p$ expresses a natural necessity. The class of naturally possible worlds can be understood as those complying with the common course of nature, and we can thus state that Buridan endorses $L_s p \to L_n p$ when he says that *de quando* or natural necessity follows from (*consequuntur*) absolute necessities.

Buridan's claim that conditional necessities follow from both logical and natural necessities does not fit neatly into this scheme. Let us consider again Buridan's favored example of conditional necessity: it is necessary that Socrates is able to laugh if he exists. Since risibility is an inseparable accident of Socrates, God cannot destroy Socrates' risibility without destroying Socrates himself. This is true not only of inseparable but all essential attributes as well, which are true of the thing so long as that very thing exists, and only cease to be true of the thing when that very thing ceases to exist. Since a conditional can be true with its antecedent false, God cannot falsify conditional necessities expressing such conceptual links, and since Buridan's definition of the highest degree of necessity states that these propositions are unfalsifiable by any supernatural power or case, the third form of necessity can be construed as disguised logical necessities of the form $L_s(q \to p)$, where q expresses an existence postulate, and the necessity involved is at least as strong as the first kind of necessity in the modal scale. In other words, it is unclear how the third concept of necessity distinguished at (T3.1) is supposed to express different form of necessity from the first.

One distinction that Buridan does make between natural and conditional necessities in [T3.2] is that, in natural necessities the predicate applies whenever (*quandocumque*) the subject is actual. This imposes the condition that the subject is possibly actual at some time during the actual history of the common course of nature. The same restriction does not hold of conditional necessities,

¹⁷¹I am thankful to Calvin Normore for having raised this question at conference in Stockholm on April 2022.

since Buridan is clear that there are conditional necessities about impossible objects. This is due to the fact that the proposition at stake is read as a disguised conditional which can be vacuously true, even necessarily vacuously true if the subject is logically impossible. Hence, if q is false or impossible, the whole conditional is still true. One of Buridan's examples is the following:

1 If a void exists, a void is a space not filled with matter

Buridan mentions the void as a paradigmatic example of an impossible object according to Aristotelian physics. [174] Still, the conditinal above would count for Buridan as a weaker grade of necessity than the second, since as we have seen the second grade of necessity (namely, natural necessity) requires for Buridan that the referents of the subject term are naturally possible.

Furthermore, there is an ambiguity the Buridan's claim that the modal concepts articulated above differ as a matter of degree (*gradus*), as he phrases

¹⁷²Cf. *QAPr.* I.25 (Buridan [n.d.b], 54): "Whence conditional necessity obtains by the requirement that if the subject and the predicate supposit for something, then they supposit for the same thing. And so I concede that this is necessary, 'a human being is an animal' [...] Morever in that sense this is also necessary, 'a void is a place', notwithstanding the fact that it is *simpliciter* false.[...] For to say that 'a void is a place' is conditionally necessary is nothing other than saying that this is necessary, 'if a void exists, a void is place'". ["Unde necessitas condicionalis ex hoc est quod oportet si subjectum et praedicatum pro aliquo supponant quod supponant pro eodem. Et sic concedo quod hae est necessaria 'homo est animal' [...] Immo etiam haec est necessaria 'vacuum est locus', quamvis sit simpliciter falsa [...] Unde dicere quod haec 'vacuum est locus' est necessaria ex condicione non est aliud dicere quam quod hae est necessaria: 'vacuum, si est, est locus'.]

¹⁷³E:g. Buridan [n.d.a] I.11: "Many of these propositions are necessary, and therefore, per se true, which do not inherit their necessity or perseity from the necessity or perseity of the categorical parts composing them, instead these categoricals are neither true, nor per se, nor necessary. Hence this proposition is necessary and per se true, namely, 'if a donkey flies, the donkey has wings'; even though each categorical proposition is impossible". ["Quia multae istarum sunt necessariae et, per consequens, per se uerae quae tamen non habent suam necessitatem siue suam perseitatem ex necessitate uel perseitate categoricarum eam componentium, immo quod istae categoricae nec uerae sunt per se nec necessariae. Unde haec est necessaria et per se uera 'si asinus uolat, asinus habet alas'; tamen suae categoricae sunt impossibiles."]

¹⁷⁴We will return to this issue in the sixth chapter - that Buridan thinks the void is naturally impossible, but logically possible. Buridan dedicates a set of questions on the void in his commentary on the *Physics*, IV.7-11 (Buridan [2016], 258-304). See also Sylla [2003] and Sylla [2001].

it himsefl above. If the distinct modal concepts that Buridan orders as a scale in (T3.1) seem to differ as a matter of *degree*, the most natural interpretation would be that he does not think they are different kinds of modality.

Does that mean that Buridan thought they express the same kind of necessity? This question is made difficult by the fact that Buridan only speaks of necessity in articulating the first three 'grades' of modality above, and does not explicitly articulate a corresponding notion of possibility. But a case for a negative answer to the question of whether Buridan is willing to reduce all grades of modality to a basic notion can be made in two ways. First, as we have just seen, the notion of conditional possibility cannot be properly ordered as a grade of modality.

Secondly, the notion of physical possibility cannot be reduced to the notion of logical or absolute possibility. For Buridan, being naturally possible presupposes the existence of natural powers, without which the natural operations observed in the natural world cannot be accounted for. We can infer from the fact that whatever a natural power can bring about, a supernatural power can, that a possibility in the second degree entails a possibility in the first, but not the other way around; in other words, a logical possibility is a necessary but not a sufficient condition for natural possibility. As Buridan states in his *QPhys.*, what would be sufficient for the latter is the existence of natural powers acting as the concurring causes without which the diversity in the operations observed in the natural world cannot be accounted for [175]

Buridan [2015a], 339-340: "Secondly, I say that it is necessary that God brings about diverse causes, from the diversity of which there follow observed the changes and effects apparent to us in this world, since even though God can by his infinite power and freedom of will produce and create diverse and contrary effects without other concurring causes, this is only possible supernaturally and miraculously, but according to what is possible naturally it cannot be the case that diverse and contrary effects come about from an invariable and simple cause, and that things are sometimes so and some other times not so, unless there are other concurring causes (...)." ["Secundo dico quod necesse est post ipsum Deum ponere alias causas secudum quarum diversitates consequuntur diversitates transmutationum et effectuum in hoc mundo nobis apparentium, quia licet Deus per suam infinitam potentiam et voluntatem liberam posset sine aliis causis concurrentibus producere et creare diversos effectus contrarios sive in eodem tempore sive in diversis, et hoc modo supernaturali et miraculoso, tamen modo naturali non esset possibile quod ab eodem simplici et invariabili provenirent diversi effectus contrarii, ut nunc tales et cras alii, nisi essent aliae causae concurrentes diversae (...) Tertio etiam apparet mihi quod non potest sufficienter reddi causa talis diversitatis transmutationum et effectuum ex ipso Deo et prima materia (...)] Thus, Biard (Biard [2001], 84) notes

Therefore, I believe it is implausible that Buridan operated with a single, basic form of modality in terms of which the others are derived from by restriction. This interpretation was suggested by Calvin Normore. The who seems to interpret Buridan's account of modality in similar terms as he interprets Ockham's account. I propose that reducing all forms of modality to a basic notion was not part of Buridan's concerns, and although his language of degrees may well suggest it, the textual basis for this interpretation seems to suggest Buridan did not take an explicit position on this matter.

For now, it remains to be seen what Buridan calls modalities with restriction (*cum restrictione*), namely those that involve a special relation of modes with tense. Its core feature is that possibility and necessity are intrinsically interwoven with time. A longer discussion of what a restricted mode means is in *QDGC* I.4. Buridan writes (Buridan [2010], 57):

(T3.3) Note that possibility and impossibility or necessity should be understood in two ways. In one way, they are indifferently related to any time, present, past or future. And in this way only that which always was, is and will be the case, and neither could nor will possibly fail to be the case, is genuinely called necessary. And the impossible accordingly is that which never was, is or will be the case, and neither was it able to be nor will it be able to be the case. And that is called possible which sometimes is, was or will be the case. In such a way we say that Aristotle is possible and not impossible, since in accordance with that modality, 'Aristotle exists' is possible, and not impossible. According to the second way, possiblity, impossibility and necessity are related determinately to the present or the future, excluding the past. Using this sense Aristotle claims in the first book of De Caelo that no power is for the past. After all, in that sense everything which was the case, it is now impossible that it was not the case. And in this modality, 'Aristotle exists' is naturally impossible'. 177

that Buridan claims here that the diversity of observed causes can only be salvaged 'speaking naturally' (*naturaliter loquendo*).

¹⁷⁶Normore [2013].

^{177&}quot;Nota quod possibilitas et impossibilitas aut necessitas solent capi dupliciter. Uno

In this passage Buridan makes explicit that modalities taken in a broad acceptation are related indifferently (*respiciunt indifferenter*) to the distinction of past, present, and future times. But in the restricted acceptation, possibility concerns only the present and the future, to the exclusion of the past, whereas necessity concerns only the past.

The key feature of the restricted sense is that the modal status of a proposition can change. As Buridan notes, in that sense what was once possible becomes impossible, just as 'Aristotle can walk' was possible in the past, but it became impossible now. In earlier texts, such as the *Quaestiones super Perihermeneias*, Buridan prefers to use the term 'settled' (*determinatum*) to characterize the historical necessity of the past. However, there seems to be conflicting evidence in Buridan's writings as to whether the necessity of the past is a genuine form of necessity. We will turn to this issue discussed below in section 3.4. Before addressing these questions, we are now able to have a synoptic account of Buridan's varieties of necessity in the table below (3.1):

modo prout respiciunt indifferenter omne tempus praesens, praeteritum et futurum. Et illo modo illud solum dicitur necessarium quod semper fuit, est et erit et numquam potuit vel poterit non esse. Et tunc dicitur illud impossibile quod numquam fuit, est vel erit nec umquam potuit aut poterit esse. Et illud dicitur possibile quod aliquando est, fuit vel erit. Et ita diceremus Aristotelem esse possibilem et non impossibilem, ita quod illo modo concederetur quod haec est possibilis 'Aristoteles est' et non impossibilis. Alio modo possibilitas, impossibilitas et necessitas accipiuntur prout determinate respiciunt praesens aut futurum, ita quod non praeteritum. Unde illo modo dicit Aristoteles primo *Caeli* quod potentia non est ad praeteritum. Illo modo etiam dicitur quod omne illud quod fuit, impossibile est non fuisse. Et sic diceretur quod illa est impossibilis naturaliter 'Aristoteles est'."

¹⁷⁸E.g., see Buridan [1983], 46: "Oppositum videtur de intentione Aristotilis esse ponentis quantum ad hoc differentiam inter illas de de presenti vel preterito et istas de futuro. Et arguitur ratione quia voco determinate verum quod inpossibile est de cetero non esse vel non fuisse verum, et determinate falsum quod impossibile est de cetero non esse vel non fuisse falsum."

Table 3.1: Varieties of Necessity

Modality	Description	Examples
Simple Necessity	No supernatural power or <i>casus</i> can falsify the proposition, its signification standing (<i>stante significatione</i>), nor is it possible for things to be otherwise as it signifies.	'God exists', 'Every part is a part of some whole', all abso- lutely valid arguments and demonstrative ar- guments.
Natural Necessity	No natural power or circumstance (casus) can falsify it, nor is it possible for things to be otherwise by natural powers, although it is possible miraculously.	'The universe is moving', 'Any place is filled' (i.e., no place is a void)
Conditional Necessity	It is necessary assuming the constancy of the subject, that is if the subject's significate exists.	If Socrates exists, Socrates is a human being.
Historical Necessity	It is necessary if it was true, and possible if it either is or will be true.	All as-of-now valid arguments are necessary; propositions about the present or the future are possible.

3.2 Firmness of Truth

One of the reasons why Buridan cared to keep the natural necessities irreducible to logical necessities is connected with his epistemological concerns in two ways. First, logical necessities come with some constraints about how we come to know them that natural necessities do not have. Secondly, Buridan uses the modal distinction to explain modes of certainty and firmness of truth explicitly in his writings. In order to see how Buridan's distinction between logical and

natural necessities is also motivated by his epistemological concerns, I will briefly touch on how Buridan uses the modal distinction to distinguish modes of certainty. [179]

In the *QSM* II.1 Buridan uses the distinction between logical and metaphysical modal spaces to make a distinction between modes of certitude and related modes of 'firmness of truth.' As Buridan points out, some modes of certainty are derived from evidentness that can only be ascribed to certain kinds of propositions. As there are two types of necessities, *simpliciter* and *ex suppositione*, so too there are two types of *evidentia* (*QSM* II.1; Buridan [1518], fols 8vb-9ra):

(T3.4) In a third sense, the firmness of assent comes from the evidentness, and the evidentness of a proposition is said to be *simpliciter* when from the nature of the senses or the intellect, a human being is compelled without necessity to assent to the proposition, in such a way that it cannot dissent, and this mode of evidentness pertains to the first complex principle, according to Aristotle in the fourth book the *Metaphysics*. However, in another way, evidentness is taken *secundum quid* or on a condition (*ex suppositio*), as it was said earlier, insofar as it concerns entities complying with the common course of nature; and thus it is for us evident that 'every fire is hot,' and that 'the skies are moving' - even though the opposite is possible by divine power, this kind of evidentness is sufficient for the principles and conclusions of natural science.

¹⁷⁹The literature on Buridan's epistemology is quite vast, I will only touch upon some issues on the connection between necessity and evidentness here. For studies of Buridan's epistemic positions, see Klima [2009], chs. 11 and 13; Zupko [2001], Zupko [1993a], Grellard [2014].

¹⁸⁰I use here Jack Zupko's translation of *evidentia* as evidentness. As Zupko points out, *evidentia* is not used in the broad sense of reasons which justify beliefs, but more narrowly as a quality of appearences (Zupko [1993a], 204 n.30).

¹⁸¹"Tertio modo firmitas assensus provenit ex evidentia et vocatur evidentia propositionis simpliciter quando ex natura sensus vel intellectus homo cogitur sine necessitate ad assentiendum propositioni, ita quod non potest dissentire et huiusmodi evidentia secundum Aristoteles conveniret primo principio complexo ut patet 4o huius. Sed alio modo accipitur evidentia secundum quid sive ex suppositione ut prius dicebatur, quod observeretur in entibus communis cursus naturae, et sic esset nobis evidentia quod

This well known passage distinguishes levels of certainty [182] corresponding to grades of objective security or firmness of truth. Here, the distinction between absolute and natural modal spaces plays an important role in this regard. The first mode of certainty and firmness of truth corresponds to truths that are unfalsifiable by any supernatural power, whereas the second mode of certainty corresponds to propositions whose truth is firm on the supposition of the common course of nature.

Buridan also famously distinguishes between levels of certitude on the subjective side according to epistemic possibility. In one way, we can assent to a proposition without any trepidation (*formidine ad oppositorum*) or with trepidation, namely when it is epistemically possible that the opposite is true for all the agent knows. The table below summarizes the relationship between these concepts (Cf. Table 3.2).

omnis ignis est calidus et quod celum movet, licet contrarium sit possibile per potentiam dei: et huiusmodi evidentia sufficit ad principia et conclusiones scientiae naturalis." A further and weaker grade of evidentness, required to meet normative principles and to act well (*ad bene agendum*), corresponds to moral certainty.

¹⁸²Buridan's usage of the word *certitudo* is manyfold. I am using 'certainty' to the objective state - the one that is connected to the assent the intellect makes on the basis of evidentness of propositions unfalsifiable in any natural case -, distinguishing it from 'certitutide' for a kind of firm assent that is made that lacks such evidentness. In this I am following Zupko's distinction between these kinds of '*certitudo*' remark (Zupko [2001]), 168.

¹⁸³Cf. (Buridan [1518], fol. 8vb): " (...) it should be noted that in order to assent to the truth with certainty, firmness of truth and firmness of assent are required. Firmness of truth is possible in two ways: in one way absolutely, for example in the proposition that God exists, since in no case it can be falsified. But there is also firmness of truth under the hypothesis of the common course of nature - and that way it is a firm truth that the heavens are in motion, that fire is hot, and so of other propositions and conclusions of natural science, notwithstanding the possibility that God can make things such that fire is cold and so falsify that every fire is hot; but it is clear from what has been said that firmness of truth is possible." ["Et tunc oportet notare quod ad assentiendum veritati cum certitudine requiritur firmitas veritatis et firmitas assensus. Modo firmitas veritatis est possibilis: uno modo simpliciter ut in hac propositione, deus est, quia in nullo casus falsificari potest, sed etiam est firmitas veritatis ex suppositione communis cursus naturae, et sic esset firma veritas quod celum movetur, quod ignis est calidus, et sic de aliis propositionibus et conclusionibus scientie naturalis, non obstante quod deus posset sic facere ignem frigidum et sic falsificaretur ista, omnis ignis est calidus, sic igitur patet quod firmitas veritatis est possibilis".]

¹⁸⁴I am adopting Gyula Klima's translation of '*cum/sine formidine*' as 'with/without trepidation,' that is, with or without fear that the opposite may be true.

Table 3.2: Security, Firmness, and Modal Cases

Type	Firmness of Assent	Firmness of Truth	Modal Case
Absolute (simpliciter)	•	God exists, the First Principle, Valid Con- sequences	Absolute Necessity
On assumption (ex suppositione)	With trepidation (believing the opposite may be true)	Every fire is hot, I see something before me	Natural Necessity

For Buridan, logical necessities have an epistemological constraint that natural necessities do not have in how we come to know them. As is well known, the epistemological role of this distinction is to serve as an anti-skeptical strategy against Nicholas of Autrecourt's views on certainty. Autrecourt's view is diametrically opposed to Buridan's: he claims that 'the certitude of evidentness has no degrees, and the only genuine certain principle we can know is the Principle of Non-Contradiction. This view denies that we can come to know natural necessities by demonstrative argument, and only inferences that ultimately reduce to that Principle are really evident to us. Autrecourt understands an evident inference as satisfying the condition that the consequent be 'the same in reality' (*idem realiter*), either wholly or partially, with what is signified by the antecedent.

To be sure, by introducing supernatural modalities into the conception of knowledge, Buridan acknowledges the possibility of divine deception as a possible scenario. In his *QSM* II.1, Buridan writes:

(T3.5) As usually said, the senses can be deluded, and certainly the species

¹⁸⁵Buridan was very likely familiar with Autrecourt's ideas, and mentioned some of the condemnations connected to Autrecourt in his commentary on the *Metaphysics*. See Zupko [1993a].

See Zupko [1993a].

186 This is the second collorary of his letter to Bernard, cf. Nicholas of Autrecourt [1994], 60-61.

Nicholas of Autrecourt [1994], II.10, 64-65: "The sixth corollary is this: In every evident inference, reducible to the First Principle by any number of steps your wish, the consequent is the same in reality as the antecedent, or with part of what is signified by the antecedent." ["Sextum correlarium est quod in omni consequentia evidenti, recucibili in primum principium per quotvis media, consequens est idem realiter cum antecedente, vel cum parte significati per antecedens."]

of sensible objects can be conserved in the absence of those very objects and of [the activity of] the organs of the senses, as was said in De Somno et Vigilia, and then we make judgments about what is not as if it were the case; therefore the senses lead us to error. But this difficulty is greatly magnified by things we believe on the basis of faith, since God can form in our senses the *species* of the sensible objects in the absence of those objects, and he can preserve them for a long period of time, and then we would judge as if those sensible objects were present before us. Furthermore, since God can do this and much more, and you are ignorant as to whether God wishes to do this now, you do not have certitude and evidentness that here there is a man before you whilst you are awake or if you are sleeping, since during your sleep God can make a sensible species as much vivid, and even a hundred times more vivid than the sensible objects themselves. Therefore, in such a scenario you would judge that there is a sensible thing before you just as you do now; and since you are ignorant about God's wishes, you cannot be certain of anything. [188]

While the questions raised by this passage are epistemological, without the modal distinction they would not arise. By supernatural possibility, God can detach a natural representation that would have naturally arisen by the presence of the thing, and supernaturally preserve its appearance in my intellect for a long time in the absence of its object, so that I would judge 'as if they were present' (ac si essent sensibilia presentia). And since we cannot tell whether God is actually doing this or not, then we do not know anything with certainty.

¹⁸⁸Cf. Buridan [1518], 8rb: "Item. Sensus possunt illudi ut communiter dicitur et certum est quod species sensibilium possunt conservari in absentia sensibilium et organis sensuum ut habetur de sono et vigilia, et nunc iudicamus de eo quod non est ac si esset: ideo erramus per sensum. Et difficultas augmentatur multum per ea que credimus ex fide quia deus potest in sensibus nostris formare species sensibilium sine ipsis sensibilibus et longo tempore potest eas conservare, et tunc iudicamus ac si essent sensibilia presentia. Modo ultra tu nescis cum deus hoc possit et maiora facere utrum deus facere hoc vult; immo tu non habes certitudinem et evidentiam utrum ante te sunt homines dum vigilas vel dormes quia in tua dormitione posset deus facere species sensibiles ita clara immo in centuplo clariores quam obiecta sensibilia possent facere. Ideo ita formaliter iucares quod essent res sensibiles ante te sicut nunc iudicas; immo cum nihil scias de voluntate dei tu non potes esse certus de aliquo."

Buridan's reply to these cases are well known and quite influential. Buridan's reply relies on the distinction we have seen above. For Buridan, demanding absolute necessity and evidentness of natural (and moral) principles undermines (*intermirere*) the possibility of both natural and moral knowledge, since both can be falsified in supernaturally possible circumstances. Instead, Buridan's view, as stated in (T3.4), claims that evidentness in the natural realm is evidentness enough for knowing with certainty.

The passage at (T3.4) states that evidentness in the common course of nature is evidentness enough in order to have demonstrative and causal knowledge of natural necessities. Buridan states in the *Physics* that we can know demonstratively conditionals expressing a causal relationship between distinct things. For example, it is possible to establish demonstratively a proposition of the form 'a exists, therefore b exists', which be reduced to a formally demonstrative argument, when we add a premise expressing such a causal link in a *propter quid* demonstration. For example, 'if a human being exists, a heart exists' can be demonstratively known when we demonstrate that a human being cannot live without a heart [191]

¹⁸⁹See Lagerlund [2019] for the influence of Buridan's anti-skepticism.

Buridan [1518], fol.9: "[...] immo conclusum est correlarie quod aliqui valde mali dicunt volentes intermirere scientias naturales et morales eo quod in pluribus earum principiis et conclusionibus non est evidentia simpliex, sed possunt falsificari per casus supernaturaliter possibiles quia non requiritur ad tales scientias evidentia simpliciter, sed sufficiunt predicte evidentiae secundum quid sive ex suppositione". [Therefore, the corollary is concluded that some are ill intended to undermine natural and moral sciences claiming that for the most part their principles and conclusions are not evident absolutely, as they can be falsified in supernaturally possible cases, since absolute evidentness is not required of such sciences, rather it suffices for them to have evidentness in a qualified manner, or under some hypothesis.]

¹⁹¹Cf. *Phys.* I.4 (Buridan [2015a], 41.5-15): "The fourth conclusion is that in some cases I can know something demonstratively by means of a premise of such as 'a exists', not taken in an isolated way, but with another premise, by means of which I can demonstratively know the conclusion that 'b exists', even though b is a distinct thing from a and a is a distinct thing from b. For example, it is not known to you through the senses that a heart exists, but it is known to you through the senses that a human being exists. Therefore you argue: if a human being exists, a heart exists; but a human being exists, therefore a heart exists. The minor premise is known to you through the senses. And the major premise will be known to you, when it is demonstrated that a human being cannot live without a heart. And that this is so is the reason why since the existence of movement is apparent to us we conclude that there is a mover, by virtue of that further premise that there cannot be movement without a mover, and so of many

To be sure, Buridan's distinction between modes of certainty is an epistemological distinction. It does not concern the nature of modalities considered in themselves. For example, as we have seen in section [2.1] there are many logical necessities which are not evident to us. As Buridan phrases matters, material consequences are cases of logical necessities which are not by themselves evident and known with the highest degree of certainty, and we may well fail to assent with certainty to necessary truths. My point in this section has been to show how Buridan's motivations for the modal distinctions presented in the *SD* are intimately connected with his epistemic positions.

3.3 Absolute Necessity

As we have seen, necessities in the highest grade are unfalsifiable, in the sense that no power or *casus* can bring about their falsity. What counts among such necessities for Buridan? As we have pointed out above, Buridan ranks conceptual and logical truths among the first grade necessities. Some of these examples, as Jack Zupko points out, cover the category of 'analytic' or 'definitional truths' In this section, I will focus on a special case of first grade necessity, namely the Principle of Non-Contradiction.

The text of interest in the present context are questions on the *Metaphysics* IV. [193] Aristotle provides different formulations of the PNC which bases Buridan's

similar cases". ["Quarta conclusio est quod in quibusdam per illam propositionem 'a est' non solitariam, sed cum alia praemissa, ego possum demonstrative scire illam conclusionem 'b est', licet a sit aliud quam b et b aliud quam a. Verbi gratia non est tibi notum ad sensum quod cor est, sed tibi notum est ad sensum quod homo est. Igitur tu argues: si homo est, cor est; sed homo est, igitur cor est'. Minor patet ad sensum. Et maior erit tibi nota, quando demonstratum erit quod non potest homo vivere sine corde. Et ex hoc etiam quod apparet nobis motum esse concludimus motorem esse in virtute illius alterius praemissae quod non potest esse motus sine motore. Et sic de multis aliis."]

¹⁹²Zupko [1993a], 201: "The examples Buridan gives of first-mode principles indicate that this category covers what have come to be known as analytic or definitional truths." These principles do not rely on experience, and are thus *a priori* in a contemporary sense.

¹⁹³Buridan's discussion of the Principle of Non-Contradiction is mainly present in the set of questions in Buridan [1518], IV qq.11-13 and q.15, fols. 20vb-23ra and fol. 24ra-26rb. Buridan also discusses the PNC in his *QSP* I.4 (Buridan [1983]), Ed. Van der Lecq, 41ff.).

considerations. First, that it is impossible for anything to be and not to be at the same time and in the same respect. There are two issues pertinent to Buridan's treatment of the PNC. One concerns its *ontological* status, and another concerns its *epistemic* status. Buridan takes different stances on the matter on both issues, but for the present purpose we will focus on the second set of epistemic concerns.

An important part of Buridan's considerations in *QM* IV.11 is to defend the Aristotelian view that the PNC is an absolute and non-relative principle, and evident to the highest degree. In that context, Buridan lists the following characteristics of a contradition according to the Aristotelian *desiderata*: a contradiction is an opposition between propositions which is "*repugnantissima*", that is, most inconsistent, its opposites are most distant ("*distantissima*"). [196] and 'most evident' ("*evidentissima*"), meaning that there is no more fundamental principle from which it can be demonstrated (Buridan [1518]), fol. 20vb). Buridan also explains is status of being 'most evident' by saying it is the 'most general' (*comunissime*) principle for reasons we will see below.

Importantly, Buridan rejects one version of the formulation of the PNC in his commentary at QM IV.13, according to which the PNC is formulated as 'it is impossible for the same to be and not to be at the same time in the same respect.' The first set of considerations Buridan makes is that the

¹⁹⁴cf. *Met.* IV.4 1006a1f.; Aristotle [1995], Ed. Barnes, Transl. Ross, 3419.

¹⁹⁵Regarding its ontological status, Buridan claims in his *Quaestiones super Perihermeneias*, I.9 that to settle whether a pair of propositions forms a contradiction is ontologically relative to the time one chooses to measure its truth. Thus, to establish whether a given pair of propositions is a contradictory pair depends on whether the same time is taken as a whole interval, instead of taking any particular subpart of it. Cf. Buridan [1983], Ed. Van der Lecq, 43: "The fifth conclusion is that two mutual contradictories are true in the same time. The reason is that since there are no indivisible instants, the present should be taken as a divisible time. And it is possible that in one part [of the present time] one proposition is true and in another part its contradictory proposition is true. Therefore at that time both are true and both are false." ["Quinta conclusio est quod due invicem contradictorie sunt vere in eodem tempore. Probatur quia ex quo non est dare instantia indivisibilia, oportet per presens intelligere tempus divisibile. Et possibile est quod in una eius parte una est vera in alia parte alia. Ideo in isto tempore utraque est vera et utraque falsa."].

¹⁹⁶In the sense of having no middle between them. Cf. (Buridan [1518], fol.20vb)

¹⁹⁷Buridan's question opens as follows (Buridan [1518], 22ra): "In the thirteenth question, it is asked whether this proposition is the first complex principle, namely,

PNC is understood as a categorical proposition in the formulation above. But categorical propositions cannot be absolutely evident on formal grounds (*gratia formae*), rather, only logically complex propositions can be evident on account of their form. The reason is that the truth of categorical propositions always depends on the reference of its terms, and on the contextual standards against which its truth is measured. On the other hand, the truth of logically complex propositions can be independently evident. Truth on formal grounds does not depend on the supposition of the terms, but it is instead true 'in all terms.' Buridan writes (Buridan) [1518], fols.22vb-23ra):

(T3.7) In the fourth conclusion, it is argued that a proposition with hypothetical extremes is more evident than a simple categorical proposition. That is, this is more evident: 'anything is or is not,' than this one, 'anything is.' The reason is that that proposition with hypothetical extremes has its truth, evidentness and firmness due to form, that is in all terms, e.g. 'every donkey is a donkey or is not a donkey,' 'every human being is an animal or is not an animal,' 'every Chimera is a goat-stag or is not a goat-stag,' but no simple categorical proposition, be it affirmative or negative has such evidentness and firmness due to form, rather if they have evidentness it is due to matter, that is in some terms but not in others.

Eventually, Buridan does not settle with a single, correct formulation of the PNC, but he lists several logically complex propositions which would qualify as

^{&#}x27;for the same to be and not to be simultaneously and according to the same respect and retaining all similar is impossible.'" ["Queritur decimotertio utrum ista propositio, 'idem inesse et non inesse simul eidem secundum idem et sic de aliis circumstantiis est impossibile,' sit primum principium incomplexum."]

¹⁹⁸Cf. Buridan [1518], fol.22vb-23ra: "Quarta conclusio ponitur quod propositio de hypothetico extremo est evidentior simplici cathegorica. Verbi gratia, ista est evidentior, 'quodlibet est vel non est', quam illa 'quodlibet est', et etiam ista 'quodlibet est ens vel non ens' quam ista 'quodlibet est ens'. Probatur quia ista de hypothetico extremo habet veritatem, evidentiam et firmitatem gratia formae, scilicet in omnibus terminis. Verbi gratia, 'omnis homo est asinus vel non est asinus', 'omnis homo est animal vel non est animal', 'omnis chimera est hicocervuus vel non hicocervuus', nulla autem simplex cathegorica sive affirmative siva negativa habet talem evidentiam et firmitatem gratia formae, sed si habeant evidentiam hoc est gratia materiae, scilicet in certis terminis et non in aliis."

indemonstrable and fit all the Aristotelian requirements concerning its firmness, and all of them should be instances of the form $P \vee \neg P$ Buridan claims that only as a formal principle can the firmness of the PNC be secured, for the opposite of categorical propositions can be more easily imagined as false than the opposites of logically true hypothetical propositions. What is common to all these formulations is that every equiform instance of the principle should be true.

Buridan's point is that the Principle of Non-Contradiction should be formulated in a logically complex way (*hypothetica*) in order to be absolutely evident. The passage above states that any instance of the propositional form

²⁰¹Krieger [2001] briefly remarked that Buridan's use of the expression *hypotheticae*

¹⁹⁹ In this regard, Buridan deviates from Aristotle's understanding articulated in the *Posterior Analytics*, according to which the PNC always has some content. Cf. Aristotle [1995b], Ed. J Barnes, Transl. J. Barnes: "That everything is affirmed or denied truly is assumed by demonstration *per impossibile*, and that not always universally but as far as is sufficient in so far as it bears on the genus (I say on the genus—i.e. the genus about which one is bringing the demonstrations), as has been said earlier too."

²⁰⁰Buridan [2008a], Ed. De Rijk, q. 16A p. 115: "Therefore, you see that the opposite of this categorical proposition is more easily imaginable than the opposite of the hypothetical. Therefore, these hypothetical propositions have evidentness on formal grounds, and it is so in every term and remains evident in others, for example, 'a human being exists or a human being does not exist, 'a Chimera exists or a Chimera does not exist,' 'a void exists or a void does not exist.' But categorical propositions do not have their evidentness on formal grounds, since many of its equiform propositions can be false, for example, 'a Chimera exists,' 'a void exists,' and so in many other terms. For a categorical [proposition] requires supposition for its truth, namely that its subject supposits for something. But the former hypotheticals do not require any supposition. Since the First Principle should be granted independently of any other assumption, as Aristotle wanted (Met. IV 3, 1005b12-14), it is clear that those attending to the evidentness and firmness of the First Principle should posit it as an hypothetical rather than as a categorical proposition." ["Etergo vos videtis quod facilius est ymaginari oppositan istius kathegorice esse possibilem quam oppositam illius ypotetice. Et tunc ista ypotetica habet evidentiam gratia forme, unde ita est in omnibus terminis et evidens est in aliis. Verbi gratia, 'Homo est, vel homo non est', 'Chymera est, vel chymera non est', 'Vacuum est, vel vacuum non est'. Sed kathegorica non habet evidentiam gratia forme, quia in tali forma multa essent false, scilicet 'Chymera est', 'Vacuum est', et sic de multis aliis. Et tunc kathegorica ad sui veritatem exigit suppositionem, scilicet quod subjectum pro aliquo supponat. Et ista ypotetica nullam talem suppositionem exigit. Modo principium firmussimum debet esse concessum absque suppositione, ut vult Aristotiles [Metaph. IV 3, 1005b12-14]. Ergo manifestum est quod attendentes ad evidentiam et ad firmitatem primi principii magis (debet) poni ypoteticum quam kathegoricum".]

 $P \lor \neg P$ is more 'hardly falsifiable' than any categorical proposition, since in the latter case many other instances of equiform propositions are false. It is patent from the passage that a concept of logical truth grounded on form is being used, after all, the difference between them is that only the latter have their evidentness due to its form (*gratia formae*). The appeal to the concept of impossibility, in this case, means that it is impossible to believe its opposite.

Buridan indicates that the reason why the PNC should be formulated as a a logically complex proposition is that it is impossible to deny it in any situation. In *Met.* IV.12, Buridan raises the question of whether it is possible to be mistaken concerning the Principle of Non-Contradiction (Buridan [1518]), fols. 21rbff.).

Two of the points raised in the positive part concern directly the issue of omnipotence, and whether it is possible to assent to a contradiction considering the broad sense of possibility by supernatural power. Buridan mentions a case in which the 'Old Lady' (*vetula*) - a recurrent persona in Buridan's writings — may assent to the opposite of the PNC, when presented the possibility that God could make contradictories true. The idea behind the example is that a person

in this context meant that Buridan relativizes the evidentness of the principle; thus, he interprets literally the formulation 'hypothetical' as being true under an hypothesis. However, Buridan places the principle in the first mode of firmness and certainty, which is not merely true, firm or necessary on a condition (*ex suppositio*). In that regard, Buridan follows Aristotle in considering the PNC as a non-hypothetical and non-relative principle, cf. [Wedin [2004]].

Buridan [1518], fol. 23ra: "And furthermore that proposition with hypothetical extremes, or the disjunctive hypothetical proposition should be conceded: it is an evident belief that it is evidently impossible to believe in its opposites at the same time, since the believer would need to have opposite opinions in his intellect simultaneously, which is impossible." ["Et iterum illa de hypotetico extremo vel etiam propositio hypotetica disiungens concedenda est: ita evidentis opinionis quod impossibile est ipsi opposita opinari in quocumque casu, quia oporteret opinantem habere simul in intellectu opiniones contrarias, quod est impossibile".]

²⁰³Other issues raised in the *quod sic* part are: that one can indirectly believe the PNC to be false, in the sense that 'the proposition that you are thinking is false,' and 'the proposition you are thinking is the PNC' are jointly possible. This issue, however, concerns the difference between epistemic sentences that make direct or oblique reference to the object of belief. Buridan approaches this issue in *QAnPost*. See Burge [1978] and Buridan [1518], fol. 22ra.

²⁰⁴See Grellard [2014] for a throughout analysis of the case in the context of the possibility of error concerning first principles.

²⁰⁵Cf. Buridan [1518], fol. 21va): "Item sicut dicebatur alias vetula erravit cum

may not rule out the possibility that divine power can make contradictions true, suggesting that it is possible for her to doubt the first principle.

However, the case of the *vetula* does not show that it is possible to believe contradictions. Buridan's final statement is on the matter is that "concerning the last argument, it is said that the Old Lady does not make a mistake, but is only hesitant" (Buridan [1518], fol. 22ra.) This means that state of the *vetula* is not that of doubting the first principle or assenting to its opposite, but rather instead of wondering about the extent of God's omnipotence. This is also how that passage was read by Peter of Ailly († 1420). In his commentary on the *Analytica Posteriora*, Peter claims that Buridan's *vetula* case shows only an hesitancy to subtract something from omnipotence Although the interpretation of this passage is somewhat difficult and Buridan's statements on it are short, it would be mistaken to conclude from it that it is possible to assent to the opposite of the PNC. Instead, Buridan's position is that no supernatural power can make the mind assent to the opposite of the first principle.

3.4 Natural Necessity

We have seen above that Buridan regarded natural necessities as unfalsifiable in any natural case or power. In order preserve the modal status of universal propositions of natural science, Buridan used a semantic tool which was developed earlier in the thirteenth century by logicians of a realist orientation, namely the theory of natural supposition. This element of Buridan's semantics is important to understand why he thought that the second grade of necessity is not reducible to the first grade of logical necessities. In this section, we will see the arguments Buridan presents for it against background of supposition

peterem ab ea utrum possibile esset quod ipsa curreret et non curreret, et bene respondit quod hoc non poterat esse et quando querebam. Nonne credis quod deus bene posset facere: hoc nescie, et sic dubitavit de primo principio, modo de quo dubitamus possumus errare."

²⁰⁶Peter of Ailly [1518], q.1, fol 6: "Concerning Buridan's *vetula*, I say that she did not doubted the first principle, but was hesitant to subtract something from divine omnipotence" ["De vetula Buridani dico quod non dubitabat de primo principio sed timebat subtrahere aliquid de omnipotentia dei".]

theory. 207

Natural science in the medieval acceptation typically deals with affirmative universal propositions. Buridan distinguishes two senses of universality. In one sense, a proposition is called universal "as required for a syllogistic form, if it has a common distributed subject, as for example 'Every B is A' or 'Every man runs'" (Buridan [2001b], Transl. Klima, 736; Buridan [1986]). In this regard, a universal proposition is just one with a sign of universality (*omnis*, i.e. 'Every'), which makes its subject to stand in common distributed supposition for all it signifies. In the second sense of universality, which Buridan calls '*de omni*' it is required further of a proposition that "it be true and necessary" (Buridan [2001b], 763), and a predication which holds 'of all' requires truth, necessity, and the existence of its referents for all times.

The first notion of universality is the one treated by the standard reading of supposition theory, and it corresponds for Buridan to the 'literal meaning' of universal propositions (*de virtute sermonis*). Accordingly, an affirmative universal predication such as 'Every S is P' is true if the predicate stands for all the things the subject stands for, with respect to the time connoted by the copula. The subject following a distributive sign has distributive and confused supposition - meaning that if 'Every S is P' is true, then a conjunction of its verifying forms "'this₁ is P', and 'this₂ is P', and [...]" picking out anything that is S with respect to the present time, must be true. This means that it is

²⁰⁷For the development of natural supposition, see De Rijk [1971]. On Buridan's use of natural supposition in particular, Scott [1965], King [1987] and Klima [2001b].

²⁰⁸The Latin term translates Aristotle's description of a proposition holding 'of all' in his *Analytica Posteriora* I.4 73a 27ff Aristotle [1995b], Ed. Barnes, Transl. Barnes, 272.

²⁰⁹See Buridan [2001b], Transl. Klima, 95). See also Ockham [1998], Transl. Freddoso, 95:"And it should be first noted that for the truth of such a universal proposition it is not required that the subject and predicate be in reality the same thing. Rather, it is required that the predicate supposit for all those things that the subject supposits for, so that it is truly predicated of them [...] And what is commonly said is that for the truth of such a universal proposition it is sufficient that each of its singulars be true". Cf. Ockham [1974], Eds. Bohner *et al* 260: "Est igitur primo sciendum quod ad veritatem talis propositionis universalis non requiritur quod subjectum et praedicatum sint idem realiter, sed requiritur quod praedicatum supponat pro omnibus illis pro quibus supponit subjectum, ita quod de illis verificetur [...] Et hoc est quod communiter dicitur quod ad veritatem talis propositionis universalis sufficit quod quaelibet singularis sit vera".

possible to descend from the universal form to a particular proposition of the form 'Some S is P'²¹⁰

This in a nutshell is why universal affirmatives have existential import they are false when the subject is empty. Suppose a situation where there are no human beings. Since, in the situation we assumed, it is false that some human being is an animal, then its contradictory is true, namely, that no human being is an animal. And since the universal affirmative and the universal negative are contraries and cannot both be true, then it is false that every human being is an animal. Note that this problem does not come up in standard contemporary quantification theory. In the latter, universal propositions are ordinarily formalized as conditionals such as

$$\forall x (Fx \rightarrow Gx)$$

In first order logic, universal propositions can thus be true in an empty domain, since the corresponding conditional is true if its antecedent is false. Given that the medieval approach to universal affirmatives assumes existential import, the problem arises concerning how do scientific generalizations remain true given that their subjects are contingent, and thus may fail to exist.

The main function of Buridan's theory is that propositions put forth with universal force can be true, even necessary, even when their common terms are empty - at least in the natural cases and setting miracles aside. That is to say, Buridan modifies the normal analysis of assertorics according to their 'literal meaning' (de virtute sermonis), in order to fulfill the modal requirement of the second type of universality we saw above - that in a true affirmative predication which holds of all (*de omni*), it holds of all for all times. In order to see this, recall that Buridan understands natural necessities at O. An. Pr I.25 in terms of de quando or temporal necessities, namely that they are necessary in the sense that whenever the subject stands for something the predicate stands for the same. Thus, their subjects must be actual at some time, in distinction from conditional necessities which can be vacuously true. Buridan writes (Buridan [n.d.b], Ed.

²¹⁰On the medieval theory of descent and ascent, see Spade [1988].

²¹¹See Klima [2001b], 2-3 ²¹²See King [1987]

Hubien, I.25):

(T3.8) [...] those temporal [propositions] require that their subjects sometimes supposit for something; and in that way this is not necessary, 'a void is a place', because the following is false, namely 'whenever a void is, was or will be, then it is, was or will be a place'. And it seems to me that according to this sense the propositions mentioned in the *Posterior Analytics* are said to hold 'of all' (*de omni*), where 'of all' denotes the universality and the existence of the supposita and of the times. Whence the demonstrative, natural and mathematical sciences are concerned with these necessities.

This passage states a semantic reason for keeping the second and third forms of necessity mentioned above ((T3.1) and Table (3.1) separate from each other. In contradistinction to conditional necessities, which can have impossible antecedents, temporal or natural necessities only concern possible objects.

In particular, Buridan rehabilitates the distinction between *suppositio naturalis* and *suppositio accidentalis* from an earlier tradition of summulists, where *natural* supposition meant that a common term is taken to stand for whatever is naturally suited to be shared by its supposita. Buridan rehabilitates this notion without the realist commitments to shared natures from the earlier tradition. His use of natural supposition rests on the fact that in scientific usage, propositions of natural science can fail to refer to anything in the present

²¹³"Et tales temporales requirunt quod subiecta earum aliquando supponant pro aliquo; ideo sic ista non est necessaria 'vacuum est locus', eo quod haec est falsa 'vacuum quandoque est, fuit vel erit, tunc est, fuit vel erit locus'. Et videtur mihi quod secundum istum sensum propositiones praedictae in libro Posteriorum dicerentur esse de omni, prout 'de omni' notat universalitatem et exsistentiam suppositorum et temporum. Unde de hac necessitate sunt scientiae demonstrativae, naturales et mathematicae."

²¹⁴Cf. Peter of Spain [2014], Transl. Copenhaver, Normore and Parsons, 243: "Natural supposition is taking a common term in place of all those that it is naturally suited to be shared by, as 'human' used by itself supposits of its own nature for all the humans who were, who are, and who will be." ["Suppositio naturalis est acceptio termini communis pro omnibus a quibus aptus natus est participari, ut homo per se sumptus de natura sua supponit pro omnibus hominibus qui fuerunt et qui sunt et qui erunt.", 242].

²¹⁵To be sure, even though Buridan rejects real and separated universals, he presents the theory of natural supposition as ontologically noncomittal at all. In the *QNE* VI.6,

moment, but nonetheless remain true, in the sense that they can be about past, present or future individuals falling under the common term. Buridan describes the distinction thus (Buridan [2001b], Transl. Klima, 259):

(T3.9) Common supposition is usually divided into natural and accidental supposition. Supposition is called 'natural' when a term supposits indifferently for everything for which it can supposit, present, past and future; this is the sort of supposition we use in the demonstrative sciences. Supposition is called 'accidental' when a term supposits only for present things, or only for present and past, or only for present and future things, as the verbs and predicates require, and will be explained later. [216]

addressing the question of whether what is knowable should always exist - which he posits natural supposition to account for the knowability of scientific propositions even when their referents don't exist - Buridan claims that "even if a real distinction between universals and singulars were to be posited, it would have no bearing on this question" (cf. Buridan [1637], fol. 499): "I believe that there are no universals outside of the mind and distinct from singular things, which to the present purpose I presuppose what was discussed in the seventh book of the Metaphysics [....] Furthermore it is known, as it appears to me, that if every rose were now destroyed, in such a way that they do not have any kind of existence, or if no thunder exists, nor stars in the sky or no eclipses of the Sun nor of the moon, nonetheless physicians would not lose knowledge about roses, nor would astronomers lose knowledge about eclipses, and nor would you lose the knowledge you have about thunder and stars in the sky from the books on Metereology. For you could teach me the science of these books just as well as if there were a thousand thunders. Therefore, even if a real distinction between universals and singulars were to be posited, it would have no bearing on this question." ("Sed ego puto, quod universale non sit prater animam distinctum a singularibus, quod ad praesens suppono ex septimo Metaphysicae [...] Et tamen notum est, ut mihi videtur, quod si omnes rosae nunc essent corruptae, sic quod nullo modo essent, vel modo si nulla sint tonitrua, nullae stellae comatae, aut nullae eclipses Solis aut lunae; tamen medicos ob hoc non amitteret scientiam quam habet de rosa, nec astrologus scientiam quam habet de eclipsibus; nec tu scientiam, quam habes de libro Meteorum de tonitruis et stellis comatis. Immo tu potest me docere scientiam libri Meteorum, sicut si essent mille tonitrua. Ideo talis distinctio de universali et singularli pro re si concederetur, tamen non valeret ad propositum."). For Buridan's discussion of the problem of universals, see King [2001c].

²¹⁶Buridan [1998], Ed. Van der Lecq, 45: "Suppositio communis solet dividi in suppositionem naturalem et accidentalem. Naturalis suppositio vocatur secundum quam terminus indifferenter supponit pro omnibus pro quibus potest supponere tam praesentibus quam praeteritis vel futuris. Et hac suppositione utuntur in scientiis demonstrativis. Suppositio accidentalis vocatur secundum quam terminus supponit

The gist of the distinction is that a term is said to stand in *natural* supposition when it stands for all its significates indifferently with respect to time, and a term stands in *accidental* supposition when it stands only for those significates according to the time consignified by the copula. For example, in 'a human being is white', the term 'human being' stands only for presently existing human beings. The reason is that the copula connoted the present time. By contrast, when standing in natural supposition for what it signifies, a common term stands not only for present things, but also for past, future, and possible things. In the latter case, the present-tensed copula is taken as an abbreviation (*breviloquium*) for a nominal disjunction, namely 'a human being is, was, or will be white', in a similar fashion to how the ampliation produced by modal concepts is expressed. Although Buridan thinks that in the literal sense (*de virtute sermonis*) the assertoric copula connotes only the present, and that 'many moderns' (*multi moderni*) deny natural supposition, he claims that we commonly use this sense in scientific discourse.

Buridan presents different arguments for natural supposition across his writings. In the *SD*, a set of four different arguments for natural supposition (Buridan [2001b], Transl. Klima, 259-260; Buridan [1998], Ed. Van der Lecq, 46):

- 1 The use of epistemic verbs causes ampliation of the subsequent terms (e.g., 'a man is thought of')
- 2 Propositions with an ampliated subject force the subject to supposit in relation to all times
- 3 Words such as 'always', 'perpetually', and 'eternally' confuse their subject to stand for things that either were, are or will be
- 4 The demonstrative sciences use that sort of supposition, for they do not

solum pro praesentibus, vel pro praesentibus et praeteritis, vel pro praesentibus et futuris secundum exigentiam verborum et praedicatorum, ut post dicatur."

²¹⁷Cf. SD 4.3.4, Buridan [2001b], Transl. Klima, 259. Cf. Buridan [1998], Ed. Van der Lecq, 45: "Multi moderni negant suppositionem naturalem ad istum sensum qui positus est in textu." It is unclear who Buridan refers to here, but plausible candidates are William of Ockham (Dewender [2016], 184) or even Albert of Saxony, see Fitzgerald [2006]

claim to establish their demonstrative proofs about present things only, but for all past, present or future things.

The four arguments listed in *SD* are presented in different phases of Buridan's writings. Besides the *Summulae*, Buridan addresses the [4] in his *QNE* VI.6. [218] [1] in his *QDGC* I.2. [219] and in [2] the *QAPo* I.18a. [220]

Gerhard Fitzgerald has shown that these arguments have different goals. [221] One focuses on the 'logic of nouns', and aims to establish that the significates of nouns can be apprehended by abstracting from them any temporal difference. The first argument shows that we can conceive things indifferently with regard to their temporal existence. [222] The second goal, however, relies on the 'logic of the

²¹⁸Buridan [1637], fols. 497-501

²¹⁹Buridan [2010], Eds. Streijger, Bakker and Thijssen, 43-49.

²²⁰Buridan [n.d.a], Ed. Hubien. For the respective differences between these phases, see Braakhuis [1999], 139.

²²¹Fitzgerald [2006]

²²²Cf. Buridan [n.d.a], Ed. Hubien, I.16: "Furthemore, I show the cause by which this kind of supposition should be posited. In whatever ways a thing can be understood (intelligi), in so many ways we can impose [words] to signify it, and in whatever ways concepts can be composed in the mind, in so many ways they can be designated by vocal propositions. Certainly, the intellect can apprehend together (comprehendere) every human being by a common concept, independently of whether some temporal difference is concomitant to that apprehension. And according to that common concept the name 'human being' is imposed to signify human beings; therefore we say that it signifies without time. It is immediately manifest that this term, by its proper principal signification, indifferently signifies past or future human beings just as it signifies present human beings. For according to the proper signification of terms this is true, 'Aristotle was a human being', and so is 'The Antichrist will be a human being', and 'Johannes is human being'. Since it is certain that the intellect can have general common concepts alongside more specific concepts which is indifferent to those specific differences, then just as we can have a general common concept of animals to which the name 'animal' applies (a quo sumitur), with all specific differences left aside, so we can have a general common concept in addition to the concepts of presence, pastness (praeteritionis) and futuricity (futuritionis), and abstracted away from such special temporal concepts". ["Deinde, ostendo causam per quam talis suppositio est ponenda. Quia qualiter res possunt intelligi, ita possunt imponi a significandum, et qualiter etiam conceptus in mente sunt componibiles, taliter designantur per propositiones uocales. Modo certum est quod intellectus potest comprehendere omnes homines conceptu communi et absque hoc quod concurrat in ista apprehensione aliqua differentia temporis. Et secundum istum conceptum imponitur hoc nomen 'homo' ad significandum homines; ideo dicimus quod significat sine tempore; et statim manifestum est quod iste terminus de sua propria principali significatione significat ita homines praeteritos aut futuros

copula', and its aim is to show that the copula can be taken in an *atemporal* sense. The difference is significant - while the first issue seems to be uncontentious, the second was the position associated with Buridan by his near contemporary Albert of Saxony. [223]

How does Buridan read the temporality in the case of natural supposition is relevant to the problem of temporal necessities as discussed above. Some further interpretations stress that the copula as Buridan means it by natural supposition is taken to signify all times. De Rijk took this line of interpretation, 224 and Knuuttila, on the basis of it, claimed that Buridan's reading of natural supposition exemplifies his acceptance of the 'temporal-frequency' model of necessity.

However, there is one shortcoming of this interpretation. While Buridan does take the 'mental copula' corresponding to propositions in natural supposition to be indifferent with respect to time, he clearly takes it that its meaning is expressed by a disjunction of tenses. Therefore, even such propositions have tensed truth-conditions. Furthermore, it is not the case that the proposition's being eternally true makes it necessary, in the sense that P is the case 'at every time'. Buridan explicitly rejects this view when making recourse of [3]. In stating the effect of the term 'always' on the suppositions, Buridan writes (Buridan [2001b], Transl. Klima, 260):

(T3.10) Third, the same is clear also when the term is confused by the words 'perpetually', 'eternally', 'always', and their like. For such words were imposed, as it would appear from the usage of authors, to

sicut praesentes. Quia secundum propriam significationem terminorum haec est uera 'Aristotiles fuit homo' et haec 'antichristus erit homo' sicut haec est uera 'Iohannes est homo'. Modo certum est quod intellectus potest habere praeter conceptus specificos conceptum generalem communem et indifferentem illis speciebus. Unde <sicut> possumus habere conceptum communem generalem animalium a quo sumitur hoc nomen 'animal' circumscriptis omnibus conceptibus specificis, sic praeter conceptum praesentiae, praeteritionis uel futuritionis, possumus habere conceptum communem et generalem abstractum a dictis conceptibus specialibus temporis."]

²²³See Fitzgerald [2006].

²²⁴De Rijk [1971].

Knuuttila [1993], 159: "Another more elaborate version of the statistical model in Buridan is connected with his view that the terms of sentences used in natural philosophy have natural supposition. They supposit for everything (past, present, future) they signify. [...] According to Buridan, in natural philosophy the necessity of sentences means that they are true universal affirmative sentences of this type."

distribute for present, past, and future times. So it would not appear to be correct usage of the word 'always' if we said that Socrates always runs, given that he ran only in this one hour, even if we took only that hour as the present. Therefore, in the given case the proposition 'Socrates runs at every time' could be conceded, but everybody would shrink from conceding that Socrates eternally runs, or that Socrates perpetually runs, or that Socrates always runs. Aristotle, however, conceded such propositions concerning the movement of the heavens. So it seems that 'Man is always an animal' is not equivalent to 'Man at every time is an animal' but to 'At every present, past, and future time, man is, was or will be an animal.' 226

This passage corresponds to the reason [3] for adopting natural supposition. Although this passage was not explicitly discussed in Fitzgerald's nor Knuuttila's interpretations, it makes a significant case for the fact that propositions in natural supposition do not inherit their necessary from the omnitemporal existence of their subjects. On the contrary, just as the word always, it confuses their subjects to stand for either past, present, or future (and possible) supposita. By the same token, this passage makes clear also why universal propositions taken in natural supposition do not entail particular propositions. The reason is that such copulas force their terms to stand in non-distributive confused supposition for a disjunction of times, and the descent for a conjunction of particulars is not valid.

²²⁷See Buridan's chapter on the causes of confusion in Buridan [2001b], Transl.

etiam hoc apparet ubi terminus confuderetur per istas dictiones vel consimiles, 'perpetue', 'aeternaliter', 'semper'. Tales enim dictiones impositae fuerunt, prout videtur apparere ex usu antiquorum auctorum, ad distribuendum pro omni tempore praesenti, praeterito vel futuro. Unde non apparet esse usus illius dictionis 'semper', si diceremus semper Socratem currere, ex eo quod curreret tota solum hora, licet ea sola uteremur tamquam praesente. Unde licet in dicto casu posset ista concedi, 'omni tempore Sortes currit', tamen omnes obhorrerent dicere quod 'aeternaliter Socrates currit', vel 'perpetue Sortes currit' vel 'semper Socrates currit'. Sed tales concederet Aristoteles de eo quod est caelum moveri. Unde videtur quod ista 'semper homo est animal' non aequivalet isti 'omni tempore homo est animal', sed isti 'omni tempore praesenti, praeterito vel futuro homo fuit, est vel erit animal'. Sed haec, ut mihi videtur, non possunt demonstrari, nec eorum opposita, quia dependent ex significatione ad placitum."

3.5 Historical Necessity and Possibility

The weakest modalities Buridan distinguishes in (T3.1) are those with restriction (*cum restrictione*). As we have already seen, restriction for Buridan means that a proposition along with its reference is indexed to the actual moment. Modal concepts taken restrictively are thus intertwined with the modal assymetry of time: possibility is taken 'determinately' for the present or the future, and necessity is taken 'determinately' for the past. Which logical relationships obtain between Buridan's broad and restricted reading of modalities? Referring to the *QDGC* I.4, which we have seen above (T3.3), Normore suggests the following (Normore) (2013), 397):

A claim is possible in the broad sense if it was, is or will be possible. A claim is possible in the restricted sense only if it is or will be possible. From the way Buridan distinguishes these two senses it is pretty clear that something is necessary per accidens (in the terminology of others) just in case it is possible in the broad sense, but not possible in the restricted sense (in his terminology).

Necessity *per accidens* is not a terminology Buridan explicitly uses, but it was a widespread fashion to characterize the necessity of the past, notably associated with William of Ockham. Clearly it is a *desideratum* of an account of accidental necessity that it should be compatible with possibility in the broad sense. If something is necessary in virtue of its being past, this should have nothing to do with neither causal relationships nor with logical truths. Buridan at some points qualifies that it is not possible to change the past *naturally* - that is, by no natural power. and further still some texts suggests that God cannot change what is past either, suggesting it is not possible supernaturally to overturn it.

Klima, 265ff.

²²⁸See Freddoso [1983] for an account of accidental necessity along Ockhamist lines. ²²⁹In the same text, namely *QDGC* I.4, Buridan writes (Buridan [2010]), Eds. Streijger, Bakker and Thijssen, 58): "Note that regarding the second mode of possibility and impossibility, a proposition which is in fact true and therefore possible, can be made impossible; and a false proposition can be made necessary. For example, 'Aristotle is speaking' was true at some time, but it is now impossible (*I say naturally*), since it is now impossible for Aristotle to speak. Moreover, 'Aristotle is not speaking' was was false at some time, and now it is necessary." ("Nota quod quantum ad secundum modum possibilitatis aut impossibilitatis, propositio quae de facto est vera et per consequens

Does that suggest there is a tension in Buridan's account of restricted necessity, and that our analysis of modal spaces above is misguided (3.1)? While the issue of the necessity of the past suggests there is, I believe that as Buridan has it the past is not necessary in the strongest sense. Before addressing this issue, let us turn to Buridan's motivation for adding modalities in the restricted sense in his conception of grades of necessity.

The motivation for the restricted reading comes from the intuitive observation that the past is settled in the sense that no agent can revoke the past, although neither the present nor future are settled in the same way. Buridan insists that his observation only supports an historical sense of necessity and possibility. When narrating a series of events as unfolding in time by telling stories we presuppose it. Buridan says that this notion of necessity has no place in logic or demonstrative sciences [230]

In his early texts on this issue Buridan often uses these historical senses of modality in connection with classical discussions around determinism. In his commentary to *De Interpretatione*, Buridan writes (Buridan [1983], Ed. Van der Lecq, 56.21-25):

(T3.11) Those [propositions] are called necessary which are and always will be true, and cannot be false, even though they could have been false earlier. And those are said to be impossible which neither are nor can be true, even though they could be true. And those are called 'possible' since they are now or will be able to be true. [231]

The sources of such definitions would have been known to Buridan through Boethius' commentary on *De Interpretatione*. In Boethius' commentary, he lists three theories of modality, the Philonian, Diodorean and the theory espoused by

possibilis, potest fieri impossibilis; et propositio etiam falsa potest fieri necessaria. Verbi gratia, aliquando erat vera 'Aristoteles loquitur', et tamen nunc est impossibilis (dico naturaliter), quia impossibile est Aristotelem loqui. Illa etiam aliquando fuit falsa 'Aristoteles non loquitur', et tamen modo est necessaria.") Italics are mine.

²³⁰That is not to say it does not have a use in logic at all. If 'as of now' validity is necessary restrictedly, then it does have a place in specified what makes such conditionals true and valid.

²³¹"Ista dicantur 'necessaria' que est et semper erit vera et quod non poterit esse false, licet ante fuit false. Et dicitur 'inpossibilis' que nec est nec poterit esse vera, licet potuit esse vera. Et dicitur 'possibilis' quia est vel de cetero poterit esse vera."

the Stoics (Boethius [2010], 140-145). He explains the Diodorean conception of modality in virtually the same terms as Buridan defines historical modalities (Boethius [2010], Transl. Smith, 45): "Diodorus decides that the possible is what either is or will be; the impossible, what when it is false will not be true; the necessary, what when it is true will not be false; what is not necessary, what either already is or will be false."

The Diodorean definitions are the most likely source for Buridan's own concept of historical modalities, and perhaps Buridan explicitly referred to it as such in his *Metaphysics*.

The discussion around Diodorean modalities is largely centered on their function as the bases for the famous Master Argument.

The discussion around Diodorean modalities are attributed to him, which is interpreted as an argument for a kind of logical determinism according to which what is possible either is or will be the case.

The explaint has been defined as a possible of the past as a coldentally necessary.

In the dicussion of restricted modalities contained in Buridan's commentary on *De Interpretatione*, Buridan prefers to stick with the terminology that the past is 'settled' (*determinatum*). Is the present also settled in the same way for Buridan? He seems to suggest it is not, but it can be taken to be settled only insofar as the proposition stating the necessity of the present is formulated as a

Diodorus saw this conflict and exploited the convincingness of the first two to establish the conclusion that 'Nothing which neither is nor will be true is possible'."

²³²Very little is known about Diodorus Cronus, except that he belonged to the Megarian school of Hellenistic dialectics. For an interpretation of Diodorus' views on modalities, see Bobzien [1998], 98-107.

²³³In one of the incunabula of his *Metaphysics*, Buridan makes reference to the view held by certain *mechanici* (most likely a scribal mistake for *megarici*) that nothing is possible unless it is actual (Buridan [1518], fol.57rb).

²³⁴The Master Argument is as follows according to Epictetus' rendering (Long & Sedley [1987], 230): "These seem to be the sort of starting-points from which the Master Argument is posed. The following three propositions mutually conflict:

^{1 &#}x27;Every past truth is necessary'

^{2 &#}x27;Something impossible does not follow from something possible'

^{3 &#}x27;there is something possible which neither is nor will be'

²³⁵Kneale & Kneale [1971], 117-122; Vuillemin [1996], 3-14. The most well known reconstruction of the Master Argument is found in Arthur Prior [Prior [2003]], 22-38).

disjunction in the following way (Buridan [1983], Ed. Van der Lecq, 47):

(T3.12) [...] everything that is, is determinated to be or to have been under disjunction, since it is impossible to be otherwise than for it to be or to have been. Whence of that which was, it is not required to formulate a disjunction, since the past cannot be reverted into the present. But of that which is, it is required to formulate a disjunction due to the fact that the present promptly comes to pass and becomes the past. [236]

What Buridan may have meant here by saying that one should state the necessity of the present by adding a disjunction to that past is that the present is less fixed than the past is [237]. The notion that the present is contingent is never argued for by Buridan, rather it is framed as a matter of belief that God could make things otherwise than they presently are. Buridan says that there are many possibilities that we hold on merely by faith and not demonstratively, when we hold the contingency of the present, but these are genuine possibilities nonetheless - for example, that God can create infinitely many more human beings than will ever be, and that we hold ourselves to be free with regard to actions that we never did nor will undertake [238] This suggests Buridan is able

²³⁶"Tamen omne quod est, est determinatum ad esse vel fuisse sub disiunctione, quia de cetero impossibile est quin ipsum sit vel fuerit. Unde de eo quod fuit, non oportet ponere disiunctionem quia preteritum non potest reverti presens. Sed de eo quod est, oportet apponere disiunctionem propter hoc quod presens cito fluit et efficitur preteritum"

²³⁷E.g., see Buridan [2015a], Eds. Streijger and Bakker, 23, and Buridan [1509]In an analogous way, Robert Grosseteste († 1253) puts it in his *De Veritate Propositionum* [Robert Grosseteste [1912]], Ed. Baur, 145): "Something, which in part is or is going to have been, and in part is going to be, is not necessary until the full completion of its present or past being. But once in completion, henceforth it is necessary *simpliciter* that it is or was. Between its beginning and completion, it only exists or will have existed in an accidental sense". ("Rem, quae partim est vel fuit et partim futura est, non necesse est ante complementum sui totaliter esse vel fuisse. Sed cum completum est aliquid, tunc et de cetero necesse est, ipsum simpliciter esse vel fuisse. Inter initium et complementum necesse est, ipsum secundum quid esse vel fuisse".)

²³⁸Buridan [1983], Ed. Van der Lecq, 52: "The second conclusion is that many things that will come about will not come about by necessity, and many things will not come about which nevertheless can come about. This conclusion is theological since it is taken by faith. We thus believe that God can create infinitely many more human

to take the present and the future as contingent in a sense in which the past is not [239]

Buridan does not take a definitive stance on the issue of the necessity of the present moment, nor does he discusses it at length in his later writings. Part of the reason might be that this issue was not as pressing for him as it was for other figures such as Duns Scotus. It is well known that Buridan's temporal semantics takes the notion of temporal truth as truth at an interval rather than at an instant. Since Buridan thinks that what time one takes as the present time for semantic purposes is wholly conventional, and one can set up the interval or stretch of time in which a proposition is true conventionally as one wishes (*ad placitum*), this makes the problem of the necessity of the present less pressing for him. Buridan claims that the length of time one takes as the present is not 'determined for us', but rather we conventionally fix the present when we take this year or this hour as the present.

Does the conventionality of the present put in question the modal asymmetry of time involved in historical possibility and necessity? We have noted that the

beings than he will ever create, and that he can annihilate all creatures, even though he does not. We believe furthermore that we have freedom of will, according to which we can undertake and will to undertake many actions which we neither undertake nor wish to undertake. And we can further abstain from many actions which we do undertake. Otherwise our wrongdoings could not be imputed to us [...]" ("Secunda conclusio est quod multa eveniant que non est necesse evenire, et multa non eviniant que possunt evenire. Et hec conclusio est theologica et ex fide credenda. Credimus enim quod infinitos homines Deus potest creare in futurum, non tamen infinitos creavit. Et etiam potest anichilare omnem creaturam, non tamen omnem anichilavit. Credimus etiam nos habere arbitrii libertatem secundum quam possumus agere et velle agere multa que non agemus nec volemus agere. Et a multis possumus abstinere que tamen faciemus. Aliter nostra mala facta non essent nobis inputanda[...]".

²³⁹See Lagerlund [2002] for the claim that Buridan follows Scotus in denying the necessity of the present.

²⁴⁰Famously, Scotus rejects the necessity of the present. See Knuuttila [1993], Normore [1996a].

Buridan [2001b], Transl. Klima, 942: "And I say that it is not determined for us how much time we ought to use as the present, but we may use as much as we want. For we call this year the present, and this day the present, and this hour the present, and if we use this day as the present, then the first hour is and the noon hour is and the vesper hour is, but successively. If, however, we use only the noon hour as the present, then we say that the first hour is past and is no longer, and the vesper hour is future and not yet is." For a throughout discussion of this issue, see Normore [2013], and Uckelman & Johnston [2010]

modal asymmetry of time is important to the validity of as-of-now consequences above in chapter 2.1, and Buridan's position on the conventionality of time may be in tension with our previous remarks in that section, 242 In this regard, the *QSP* and the *TC* approaches seem to differ in relevant ways.

In the *QSP*, Buridan introduces an important distinction for his temporal logic which he developed in more detail in his *Sophismata*. He claims that the terms 'past' and 'future' may be taken to supposit for times either absolutely or relatively. If taken absolutely, then no part of the present is called past or future. If taken relatively, then the first part of the present precedes the succeeding part. If taken in an absolute sense Buridan mentions, however that such inference is valid once time is defined in such a way that the past and the future are essentially distinct from the present. Buridan writes (Buridan [1983], Ed. Van der Lecq, 33-34):

(T3.13) Concerning past or future tense propositions it should be said what was said in *Physics*. Since indivisible instants should not be posited, it follows that everything which is was, and everything which is will be, understanding 'past' and 'future' relatively. Therefore, the first two conclusions hold in that sense, both for future and past tense propositions as well as for present tense propositions. But understanding 'past' and 'future' absolutely, insofar as it is distinguished from every present, then not everything is which was or will be. Therefore, these first two conclusions concerning past or future tense propositions do not hold. And more concerning this

²⁴²See the discussion above at the end of 2.1

²⁴³Buridan [2001b], Transl. Klima,: 949-50: "[...] the names 'past' and 'present', which supposit for time, are sometimes taken simply and absolutely, sometimes relatively. If they are taken absolutely, then no part of the time that we use as the present should be called past or future. For in this way no past time is, but was, and no future is, but will be. [...] In the other way 'past' and 'present' are taken relatively, so that an earlier part of the present time is past relative to a later part, and a later part is future relative to an earlier one. And this way of taking [these terms] is used for the reason that if we were to use only the earlier part as the present, then the later part would truly be called future, and conversely, if we were to use only the later part as the present, then the earlier part would be called past." For discussion a discussion of Buridan's *Sophismata*, see again Uckelman & Johnston [2010], who provide a formalization of Buridan's temporal logic within the framework of interval-based semantics.

The passage on the *QPhys*. Buridan mentions above is very likely the sixth question of the third book of his *quaestio*-commentary. Buridan explores there the distinction between relative and absolute acceptations of time in some detail. The key idea is that on the absolute acceptation, the present is singled out as the privileged actual moment, whereas on the relative acceptation no present moment is singled out as the privileged actual moment. Thus, in the *QSP* Buridan takes the modal asymmetry to depend on taking the terms 'past' and 'future' in their absolute acceptation, as distinguished from the present. This view is quite different from the one we saw above in the *TC*, where as-of-now consequences express a kind of restricted necessity.

3.6 The Necessity of the Past

Virtually all medieval thinkers thought the past to be necessary, but not in the strongest sense of necessity. Whether there were diverging views on this issue is contentious in the literature, but it is well spread enough to be almost consensus among the scholastics. Buridan seems to speak in some contexts as if the past were necessary in the strongest degree - in the sense that not even God

²⁴⁴"De propositionibus autem de preterito vel de futuro oportet dicere secundum ea que in *libro Phisicorum* dicuntur. Cum enim non sit dare instantia indivisibilia, sequitur quod omne quod est fuit et omne quod est erit, sumendo 'preteritum' et 'futurum' respective. Ideo sic valerent due prime conclusiones tam in illis de preterito et futuro quam in illis de presenti. Sed sumendo 'preteritum' et 'futurum' simpliciter prout distinguitur contra omne presens, tunc non omne quod est fuit vel erit. Ideo non valent iste due prime conclusiones de preterito vel futuro. Et horum dictorum declarationes videantur in questionibus *Phisicorum*.

²⁴⁵See Buridan [2016], Eds. Streijger and Bakker, 65: "Pro secunda obiectione notandum est quod istis nominibus 'praesens', 'praeteritum' et 'futurum' possumus uti absolute. Et tunc nullum praesens est praeteritum vel futurum. Et cum omnis pars praesentis sit praesens, nulla pars praesentis est praeterita vel futura. Immo si hac totali die uteremur tamquam praesente, hora prima esset praesens et hora meridiei esset praesens et hora completorii esset praesens; et quaelibet horarum esset praesens, licet haec prius et alia posterius. Et nulla istarum horarum est praeterita vel futura. Et esset verum dicere quod hodie pulsatur ad primam et ad vesperas."

²⁴⁶Sometimes, the early monastic writer Peter Damian († 1072) is seen as holding that the past is contingent in the fullest sense (Knuuttila [1993]; Gaskin [1995]). For a divergent view, see Bornholdt [2017], 22 n.40.

could upset facts about the past. In addressing the question of whether an infinite magnitude is possible, Buridan replies to a case where God creates and separately conserve at every past day one stone that is one foot long and thus - since the past is infinite - an infinite magnitude exists. Buridan's objection makes appeal to the necessity of the past (Buridan [2016], Eds. Streijger and Bakker, 194-5):

(T3.13) One can object by referring to what is said in the first book of *On The Heavens*, namely that there is no power over the past. Accordingly it is said that it is not possible even by divine power that what is past was not or that Aristotle did not exist; and therefore, since God did not make something yesterday, it is not possible that he would have done it yesterday. As much as it is true that God could create a stone one foot long at every past day, since he has not created it, it is not possible now that he would have created it. Therefore, it is impossible that 'At each past day God created and conserved a stone one foot long'. To that it is further objected: since God could at each day create a one foot long stone and conserve it always afterwards, wouldn't it follow that there are infinitely many stones? I answer that, if he would have done so, stones would now be infinite. But the antecedent and what follows from it in the objection are impossible, if the eternity of the world and of time are posited it was always impossible that God at every past day created a stone and conserved it afterwards, since it was always true to say that he has not done so, and since there is no power over the past, it is impossible that if God at some time has done b, he is able to not have done b. Similarly, it is impossible that if at some time he has not done b, he is then able to do b, rather it was only before doing it that he was able to do it. 247

²⁴⁷"Ad illud potest responderi per illud quod dicitur primo *Caeli*, quod non est potentia ad praeteritum. Ideo dicitur quod non est possibile etiam per potentiam Dei iam praeterita non fuisse vel Aristotelem non fuisse; et ita etiam, quod non fecit heri Deus, non est possibile quod faceret illud heri. Quamvis igitur verum sit quod Deus omni die praeterito potuit facere unum lapidem pedalem, tamen, quia non fecit, non est possibile quod fecerit. Ideo haec est impossibilis 'omni die praeterito Deus fecit unum lapidem pedalem semper conservando eum post'. Sed adhuc obicitur quia: ex quo omni

In the last part of this passage, Buridan clearly states that God cannot change the past. God cannot do what he has not done, provided he has not done so, nor undo what he has already done, provided he has already done so. One *proviso* to be made is that Buridan may be attaching necessity to the consequent - if God has done b, then necessarily he cannot undo b. This was one of the solutions to problems of theological determinism, in accordance with the widespread distinction of the 'necessity of the consequence' vs. the 'necessity of the consequent' (necessitas consequentis/consequentia) frequently used in debates surrounding omnipotence, omniscience and foreknowledge. The necessity involved here would then not absolute, insofar as absolute is distinguished from conditional necessity. However, that is not the move Buridan makes in the passage above. In the first reply to the second objection, he states that both antecedent and consequent are impossible independently (namely, it is impossible, assuming eternal duration of the world and time, both that God has created a stone one foot long each day, and that there are infinitely many stones, simply because God has not done so and there is no power over the

die Deus potuit creare unum lapidem pedalem et post semper conservare, quaeritur, si ita fecisset, quid modo esset. Nonne modo essent lapides infiniti? Respodeo quod, si ita fecisset, lapides nunc essent infiniti. Sed huius antecedens et consequens sunt impossibilia, immo posita mundi et temporis aeternitate semper fuit impossibile quod Deus omni die praeterito creavit unum lapidem semper post conservando, quia semper fuit verum dicere quia ita non fecit, et tamen, cum non sit potentia ad praeteritum, impossibile est, si Deus aliquando fecit b, ipsum non fecisse b. Et similiter impossibile est quod, si aliquando non fecit b, ipsum tunc fecisse b, licet ante fuerit impossibile quod ipsum faceret."

²⁴⁸Aquinas' discussion of foreknowledge in *De Veritate* q.2 a.12, similarly uses 'absolute' to mean the independent necessity of either an antecedent and a consequent. Thomas Aquinas [1972], 81-2: "Furthermore, in every true conditional proposition, if the antecedent is absolutely necessary, then the consequent will be absolutely necessary. But that is a true conditional proposition: 'is something is known by God, it will come to be,' therefore since the antecedent, 'this is known by God' is absolutely necessary, then the consequent will be absolutely necessary. Thus, everything that is known by God is necessary absolutely." ("Pratera, in omni vera condicionali si antecedens est necessarium absolute et consequens erit absolute; sed ista est vera, 'si aliquid est scitum a Deo, illud erit,' cum hoc ergo hoc antecedens 'hoc esse scitum a Deo' sit absolute necessarium et consequens erit absolute necessarium; ergo omne quod scitum est a Deo necesse est absolute esse.'") This is only the set-up of an ensuing argument to reconcile divine foreknowledge with contingency. Aquinas mentions four different solutions to the problem of reconciling omniscience with divine foreknowledge in Thomas Aquinas [1972], 84-6, but its details need not concern us at the present.

past). Recall that for Buridan the strongest necessities are those that not even a supernatural power can falsify, and this passage might suggest that such strong kind of necessity is involved in the necessity of the past.

On the other hand, Buridan claims that some propositions about the past are not settled in that way, namely those that depend on the future. I believe we get a better grasp of why Buridan holds the necessity of the past to be a weaker form of necessity by looking at his reply to a form of logical determinism in the *Quaestiones super de Caelo*. In that context, Buridan is addressing an opponent arguing that it is impossible for something to be possibly generated but nonetheless remain never generated - as the opponent has it, if it is possible that *A* comes to be, then *A* will come to be. The argument to that effect relies on the usage of the necessity of the past applied to propositions in the past tense. Buridan writes (Buridan) [1996], Patar 374):

(T3.16) To the other, when it is said that this conjunctive [proposition] is false, namely 'A is generable and A will not be generated', I deny it. To the contrary, I claim it is true. But you would say, 'From one conjunct there follows the opposite of the other in that proposition' - I deny it. To the proof: from 'A will not be generated', it follows if that proposition is propounded yesterday, namely 'A will not be generated', then this proposition was true then. I concede that. But in addition, you could say that then it follows that it is impossible that it was not true - but this I deny. Because is possible that it was not true but false, as is had in *Perihermeneias*, it is was not settled that it will be true nor it was settled that it will be false, since that settledness depends on a future act which is not determinated. But you claim that this rule is conceded by Aristotle, that 'for everything which was, it is impossible that it was not'. I concede that rule in the sense that it is impossible now that the proposition did not exist. But this rule is false, namely, 'everything which was true, it is impossible that it was not true'. The latter rule follow from the former, since there is a fallacy of figure of speech, moving from 'quid' to 'ad aliquid', since 'truth' is relative to something - a proposition is said to be true since things are in reality as it says

they are. 249

The argument Buridan wishes to oppose here states one of the corollaries of the Diodorean Master Argument, namely that if something is possible, it will at some time be the case. What is relevant here is that Buridan's refutation of this claim involves mitigating the necessity of the past in order to avoid the deterministic implications of the argument. The principle Buridan rejects at (T3.16) is that if a true past-tensed proposition is true, it is necessarily true-since no power ranges over the past. However, Buridan claims above that he rejects this rule (*regula*), by noting that truth is a relational predicate, namely that a proposition is true if things are as it signifies them to be. The thought that truth is a relational predicate means that a proposition can become false due to a change in one of its *relata*, namely, the way things will be.

Buridan's point with this remark is that in the case of a proposition about the past which depends on a future act (*ex actu futuro*), the proposition can turn out to be false if things will not be the way it says they will be. In one of the rare occasions where Buridan mentions prophetical speech in his *QSP*, we find the example of someone uttering the proposition that Buridan will be lecturing a thousand years before he actually is. [251] In this example, Buridan makes the

²⁴⁹Cf. Buridan [1996], Ed. Patar, 374: "Ad aliam, quando dicitur ista copulativa esta falsa: *a est generabile et a non generabitur*, nego; immo dico quod est vera. Et tu dicis: ad unam partem huius copulativae sequitur oppositum alterius partis, nego. Ad probationem. Ad istam: *a non generabitur*, sequitur quod haec propositio: *a non generabitur*, heri proposita, fuit vera, concedo; et ultra: *sequitur quod impossibile est ipsam non fuisse veram*, nego; immo dico quod adhuc possibile est ipsam non fuisse veram sed falsam, quia, sicut habetur in *Peri Hermeneias*, nondum est determinatum quod fuerit vera nec quod fuerit falsa, quoniam haec determinatio dependet ex actu futuro ad quem non est adhuc facta determinatio. Sed tu dicis quod ista est regula etiam concessa ab Aristotile, quod 'omne quod fuit, impossibile est non fuisse'. Concedo regulam, ideo concedo quod impossibile est illam propositionem non fuisse. Sed ista regula est falsa, 'omne quod fuit verum, impossibile est non fuisse verum'; nec ista regula sequitur ex alia, imo est fallacia figura dictionis, mutando 'quid' in 'ad aliquid', quia 'verum' est ad aliquid - ex eo enim dicitur propositio vera, quia taliter est in re."

²⁵¹Buridan [1983], Ed. Van der Lecq, 46: ("Deinde etiam supponitur quod aliquis a mille annis citra de me dixerit ore prophetico 'Buridanus leget'. Manifestum quod ista propositio fuit vera, quia ego legendo feci omne quod ipsa significabar fore. Et certum est quod cum ipsa fuisset vera, hoc fui quando ipsa fuit, scilicet a mille annis cintra. Et si tunc fuit vera, inpossibile fuit in posterum quod non fuisset vera, secundum predicta.

case that everything is now as the proposition then signified they would be, but states that it is still the case the future was not determinated to be that way. The reason is that the proposition about the past, namely, that it was true a thousand years before that Buridan will lecture at that day, does not inherit its necessity in virtue of being grammatically about the past alone, since it has a future oriented content.

This strategy of mitigating the necessity of the past for propositions which are grammatically about the past but semantically depend on the future is closely associated with William of Ockham. In his *Tractatus de Praedestinatione*, Ockham argues that the truth of the propositions 'Peter is predestinate' or 'Peter was predestinated' (which is clearly about the past) do not entail the inevitability of Peter's receiving grace in the future for that very reason Buridan's strategy for allowing that some propositions about the past are contingent in that way resembles closely Ockham's terminology as well, in stating that such propositions are not determined to be true since they depend on a future act (*ex actu futuro*), in that they depend on the truth of a proposition that is about the future Ockham thought that a proposition is accidentally necessary if 'it was contingent and it became necessary, even though they not always were

Igitur per equivalentias necesse fuit in posterum quod fuisset vera. Sed hoc est vel fuit determinate tale, scilicet quod necesse est vel fuit esse tale. Ideo propositio dicta a mille annis citra semper postea fuit determinate vera et non nisi quando proponebatur. Ergo tunc fuit determinate vera.")

²⁵²Ockham [1983], Transl. McCord Adams and Kretzmann, 38: "Every proposition about the present that is true at some time has [corresponding to it] a necessary proposition about the past. For example, if 'Socrates is seated' is true, 'Socrates was seated' will be necessary forever after. But suppose 'Peter is predestinate' is now true; in that case 'Peter is predestinate' will always be necessary forever after. [...] I maintain that the major premise is false; for that proposition that is about the present in such a way that it is nevertheless equivalent to one about the future and its truth depends on the truth of the one about the future does not have [corresponding to it] a necessary proposition about the past. On the contrary, the one about the past is contingent, just as its its [corresponding proposition] about the present. All propositions having to do with predestination and reprobation are of this sort, since they all are equivalently about the future even when they are verbally (*vocaliter*) or about the past. Therefore, 'Peter was Predestinate' is contingent just as is 'Peter is predestinate'."

²⁵³In the text above (T3.16), Buridan thus rejects the rule that every proposition which was true necessarily was true in analogous terms as Ockham does. See further Ockham [1983], Transl McCord Adams and Kretzmann, 46-7.

necessary ²⁵⁴. As we have seen above, Buridan's notion of restricted necessity also imples that a proposition can change its modal status over time. It is very likely that Buridan was aware of Ockham's solution to this problem and that he is following him in this point. However, as Wojciech Wciórka has shown, characterizing accidental necessity by invoking past-tensed propositions with future oriented contents was a move already found in early twelfth-century texts. ²⁵⁵

²⁵⁴In the Prologue to *Ordinatio* q,6, Ockham writes William of Ockham [1967], Eds. Brown *et al.* 178.4-7: I say concerning those propositions that they are accidentally necessary, since it was contingent that they would be necessary and it is not the case that they were always necessary. ("Quod dico propter propositiones necessarias per accidens, quia contingens fuit quod essent necessariae, nec semper fuerunt necessariae".) For a contemporary elaboration see Freddoso [1983].

²⁵⁵See Wciórka [2020] argues that one can find virtually the same terminology in authors as early as Gilbert of Poitiers († 1154)

Part II Modalities in Buridan's Natural Philosophy

4. Modal Necessity and What Always Is

The previous chapters have discussed Buridan's conception of varieties of modal concepts in his logical writings. Part II of this dissertation examines applications of Buridan's analysis of modality to problems in the philosophy of nature and metaphysics.

In this chapter, I will investigate in what ways Buridan associates modality and temporality with a view to a particular context, namely in his commentary on Aristotle's *De Caelo*. We will see that although Buridan incorporated central features of the temporal model associated by medieval figures with Aristotelian modalities, he did not fully accept it on the basis of his critical remarks to interpretations of *De Caelo* I.25. A central goal of the chapter, therefore, is to describe Buridan's assumptions concerning the connection between necessity and eternality, as addressed in the *Expositio* and *Quaestiones* on Aristotle's *De Caelo* in particular.

The former set of questions contain Buridan's take on the problem concerning the eternal duration of the world, and on the Aristotelian arguments in favor of the view that true propositions that always were and will be true cannot ever fail to be true. Aristotle's arguments at *De Caelo* I.11-12 (hereafter *DC*) were

²⁵⁶Cf. mainly Knuuttila [1993], and also Van der Lecq [1983].

²⁵⁷That is, I will be concerned mainly with the fourth treatise, chapters 2 and 3 of the *Expositio* on book I of *De Caelo* (Buridan [1996], 69-88), and the corresponding *Quaestiones* on the same problems, ranging from qq. 23-26 on the first book (Buridan [1996], 358-379). These are commentaries on the last chapters of the first book of Aristotle's *De Caelo*, namely *DC* I.11-12, where Aristotle develops a famous argument purporting to establish that the cosmos by necessity always is. Cf. [1995]

frequently interpreted in the Middle Ages²⁵⁸ and in contemporary scholarship²⁵⁹ as founded on a modal framework in which modality is essentially spelled out in temporal terms.

As we shall see, Buridan has a predominantly critical approach to Aristotle's arguments at DC I.11-12. I will argue Buridan's approach is based on the distinction between logical and natural modalities. In order to show how Buridan's approach was distinctive in the period, it will be useful to contrast Buridan's interpretation of this text with that of another Arts Master active at Paris shortly before Buridan's time, namely John of Jandun (†1328) Jandun largely endorses Aristotle's positions in the relevant passage of the DC, the contrast between his and Buridan's approaches is helpful to understand the debates concerning the legitimacy of using the distinction between supernatural and natural modalities in philosophical discourse in the period.

4.1 Modality and Temporality in Buridan's Logic

In his logical writings Buridan sometimes uses temporal expressions such as 'always' (*semper*), 'never' (*numquam*) and 'at some time' (*aliquando*) in order to spell out the meaning of modal concepts. These accounts of the meaning of modal concepts were common throughout premodern and medieval analyses of modality. This form of explanation is clearly articulated in *SD* 1.8.5 ('*On the Quantity of Modals*'), where Buridan remarks that the operation of necessity essentially has the effect of a universal quantifier, distributing the time consignified by the copula of the proposition for all times (*pro omni tempore*), and possibility has the effect of a particular quantifier, leaving the time consignified by the copula undistributed (Buridan [2001b], Transl. Klima, 75):

²⁵⁸The first Latin translations of this work were made by Gerard of Cremona (1114-1187) in the twelfth century, in the thirteenth century William of Moerbeke's (1215-1286). Aquinas' and Buridan's approach to I.11-12 are discussed by Williams [1965] and Williams [1966].

²⁵⁹For contemporary assessments, see Broadie [2009], Denyer [2000], Rosen & Malink [2012].

²⁶⁰For a description of John of Jandun's life and works, see Brenet [2020] and the references therein.

²⁶¹See in particular Knuuttila [1993], 106-128.

(T4.1) But concerning the third section we should note that the mode 'necessary' distributes the time consignified by the verb for all times. Therefore, if B has to be A, then it follows that some B always is, was and will be A, and similarly, if every B has to be A, then it follows that every B always is, was and will be A. And the same goes for 'impossible, whence 'B is unable to be A; therefore, some B never is, was or will be A' is valid, and so is 'every B is unable to be A; therefore no B is, was or will be A'. But the term 'possible' leaves the time undistributed; therefore, it follows that if a B sometimes is, was, or will be A, then that B can be A.

This passage is an explanation of the syncategorematic operations of modality over times. On the one hand, necessity has a semantic effect analogous to that of a universal quantifier and it is thus called by Buridan a 'universal mode,' and on the other hand possibility has a semantic effect analogous to that of a particular quantifier, and it is thus called 'particular mode.' In other words, in necessity propositions, the modal expression modifies the tense of the present-tensed copula from the present to consignify all times; whereas in possibility propositions, it modifies the tense of the present-tensed copula to consignify some particular time. Buridan's usage of temporal concepts to clarify modal notions in (T4.1) is stated through the following implications:

- P1 If B necessarily-is A, then B always is, was and will be A
- P2 If B sometimes is, was or will be A, then B possibly-is A.

It is important to note the direction of the implication relating modes with times. Both principles state the relationship of modality with time, but the

Buridan [2005], Ed. Van der Lecq, 90-91: "Sed de tertia clausula sciendum est quod per istum modum 'necesse' distribuitur tempus consignificatum per verbum pro omni tempore. Ideo, si B necesse est esse A, sequitur quod aliquod B semper est, fuit, et erit A. Et similiter sequitur, si omne B necesse est esse A, quod omne B semper fuit, est et erit A. Et ita etiam de impossibili, unde si B impossibile est esse A, sequitur: ergo aliquod B numquam fuit, est vel erit A. Et similiter sequitur 'omne B impossibile est esse A, ergo nullum N est, fuit vel erit A'. Sed iste terminus 'possibile' dimittit illud tempus non distributum. Ideo sequitur, si B aliquando est, fuit vel erit A, quod illud B potest esse A."

direction goes in opposite ways in [P1] and in [P2] respectively. In [P1] the implication holds from a mode-to-time direction - that is, if humans are necessarily animals, then there is, was, or there will be some time at which humans exist and in any such time they are animals. On the other hand, [P2] states the implication from time-to-mode direction.

Notice that the explanation exemplified in T4.1 does not define modal concepts in terms of temporal concepts. Both implications above go in the direction mode-to-time (in the case of necessity) and time-to-mode (in the case of possibility). In his semantic account, Buridan does not endorse an equivalence between necessity and always being. and possibility and sometimes being. The converses of [1] and [2] are never explicitly stated in Buridan's logical writings, namely:

P3 If B always is, was, or will be A, then B necessarily-is A

P4 If B possibly-is A, then B sometimes is, was or will be A

Knuuttila holds that Buridan endorses these principles in his natural philosophical writings, in what he calls the 'temporal-frequency' model of modalities. Knuuttila writes (Knuuttila [1993], 158):

In his treatises on Aristotle's works, Buridan makes use of the Aristotelian modal paradigms called above the potency model and the statistical or temporal-frequency interpretation of modal notions. Buridan delineates the first model as follows. There is a certain number of types of potencies in nature. All natural possibilities as generic tendencies are realized. Potencies cannot be

²⁶³Buridan at some points does so in his natural philosophical texts, as we will see below in the case of his commentary on *De Caelo*. Even so, when he associates necessity with eternality he does so only conditionally, on the assumption that the world is eternal.

²⁶⁴Here I part ways from Van der Lecq and Knuuttila's interpretation. Van der Lecq [1983], xl: "For him [Buridan] as well as for Aristotle necessary being was equivalent to omnitemporal being, and possible being was equivalent to sometime being. The same holds for propositions: omnitemporal truth is equivalent to necessary truth, etc. In other words, something that exists or happens always, exists or happens necessarily; the statement expressing this event or state of affairs is always true, i.e. necessarily true." See also Knuuttila [1993], 158ff.

eternally frustrated, because nothing is in vain in nature. Individual possibilities in the sense of partial potencies may remain unrealized. Buridan also reminds his readers that, according to Aristotle, what always is, is by necessity, from which it follows that what never is, is impossible, and that genuine possibilities cannot remain unrealized.

As support for this interpretation, the main passages Knuuttila mentions are QSP I.11, 265 TC^{266} However, in all these passages, Buridan is stating what he believes to be compatible with Aristotle's opinion (as he sees it, Aristotle believed that that since natural kinds are eternal beings, affirmative propositions about them are necessary), and not expressing his own views. As we will see, in Buridan the connection between eternality and necessity is significantly challenged in his QDC. At this point, we can state two further reasons why explaining modal concepts as in $\boxed{4.1}$ does not commit Buridan to conflating modality with temporality.

First, [P1] presupposes that the subject of necessity propositions pick out an

²⁶⁵Referring to affirmatives with subjects standing for something always, Buridan writes that Aristotle took affirmatives stating essential relationships between natural kinds - such as 'Every horse is an animal' to be necessary, since natural kinds are eternal beings in for Aristotle conception. Buridan writes: (Buridan [1983], 52): "Aristotle would have said that that those terms always supposit for something, if the proposition is formed. And hence he regarded these [propositions] necessary." ["Et Aristotiles diceret quod illi termini semper supponunt pro aliquo, si propositio formetur. Ideo ipse reputaret eas esse necessarias."]

²⁶⁶Buridan [2015b], 141: "It should also be said that Aristotle believed such [propositions] to be simply necessary because he thought that the eternity of the world and universal nature could not allow that at some time nothing was a horse or a dog. And it is true that it is not possible by natural means, although it is by a supernatural miracle, that at some time nothing is a horse, nothing the earth, nothing fire. So speaking only naturally such [propositions] as 'A horse is an animal,' 'Fire is hot,' should be taken as necessary, in the sense that it is not possible by nature, without a miracle, for them to be false; in what follows we will take such [propositions] to be necessary." Cf. Buridan [1976], 112: "Dicendum est etiam quod Aristotiles credidit tales esse simpliciter necessarias quia opinabatur aeternitatum mundi et natural universalem non posse permittere quod aliquando nullus esset equus uel canis. Et uerum est quod non est possibile per actiones naturales, quamuis bene sit per miraculum supernaturale, quod aliquando nullus sit equus, nulla terra, nullus ignis. Ideo naturaliter solum loquentes capiunt tales tanquam necessarias 'equus est animal,' 'ignis est calidus,' ad isum sensum quod non est possibile per naturam, circumscripto miraculo, eas falsificari; et sic in posterum utemur talibus tamquam necessariis."

actually existing thing. As we have seen in chapter 2.2 and in table 2.1. Buridan has two readings of necessity propositions at disposal. The first, according to simple modality takes possible things to be among the supposita of divided modal propositions, resembling possibilist conceptions of modal logic. The second reading, according to conditional-temporal modality, assumes that only actual things are among the supposita of necessity propositions. Thus, when Buridan states the analogy of modals with temporal notions in (T4.1), it seems he has only the second reading in mind. For on the first reading, it can be true that 'every human being is an animal,' even if nothing is or ever was a human being, but something possibly is.

Second, Buridan's formulations in his commentaries on Aristotle's natural philosophy indicate a certain care not to conflate modality with temporality. In *Quaestiones De Generatione et Corruptione*, I q.4, that we have already seen above (T3.3), according to which necessity in the broad sense is what always was/is/will be the case *and could not* ever fail to be to be the case ²⁶⁷. The last addition would not be needed if Buridan were assimilating modality with temporality, since it would be already entailed by the first part of the *definiens* of necessity.

4.2 The Argument in *De Caelo* and Jandun's Interpretation

The main tenets of the temporal interpretation applied by Knuuttila and Van der Lecq to Buridan's theory of modality appear early on in Hintikka's work on Aristotle from 1973. In this study he attributed to Aristotle the following principles connecting modality with time, namely, that: (1) every possibility is realized at some time; (2) nothing eternal is contingent; (3) that which never is, is impossible (Hintikka [1973], 102-103; see also Knuuttila [1993], 5-7).

In his later writings, Knuuttila revised his position and did not attribute this interpretation of modalities to Aristotle. However, he thought that medieval

²⁶⁷For ease of reference, I cite here part of this passage again: Buridan [2010], 57: "And in this way only that which always was, is and will be the case, and neither could nor will possibly fail to be the case, is genuinely called necessary." ("Et illo modo illud solum dicitur necessarium quod semper fuit, est et erit et numquam potuit vel poterit non esse. Et tunc dicitur illud impossibile quod numquam fuit, est vel erit nec umquam potuit aut poterit esse.")

authors regarded it to be compatible with Aristotelian conceptions in natural philosophical works. In *DC* I.11-12, Aristotle puts forward a series of arguments for the necessity of what always is the case, and in particular for the position that the eternal cosmos could not possibly fail to be. The main argument to that effect proceeds by indirect proof, by assuming the opposite of what is to be proved in order to derive a contradiction. The bulk of Aristotle's argument is at *DC* 281b20-26 (Aristotle) [1995], Transl. Legatt, 101):

(T4.2) In consequence, if something that is for an unlimited time is perishable, it would have the capacity for not-being. If, then, it is for an unlimited time, let its capacity be realised. At the same time, therefore, it will be and will not-be in actuality. The result will be false, then, because a falsity was assumed. But unless the assumption were impossible, the result would not also be impossible. Therefore, everything that always exists is absolutely imperishable.

Aristotle makes a modal claim in this passage, namely the claim that something is imperishable, is unpacked as a claim to the effect that it cannot perish. Furthermore, the meaning of 'cannot' here corresponds to the strongest sense of impossibility Aristotle had earlier distinguished in the same text (280b12-14), that is, the impossibility involved is neither conditional nor is it a merely physical sense impossibility, rather it is impossibility *tout court* [269] The argument can briefly be analyzed in the following way. The starting point of the indirect proof is the assumption that (i) something is (is F) for unlimited time and is able to cease to be (cease to be F). If, for example, a conjunction of the form 'x is eternal and x is able to perish' is true, by the first conjunct, (ii) x is (is F) for all time. But by the second conjunct, (iii) there is a possible moment of time such that x ceases to be (or to be F). Aristotle thinks that since that is a possible moment, (iv) it can be assumed to be actual without entailing any

²⁶⁸Cf., for example, Knuuttila [2012] for his later views. Knuuttila was correct on this point. As we shall see, Buridan and Jandun interpreted Aristotle as using temporal modalities in his *De Caelo* commentary.

²⁶⁹See (Aristotle [1995], 101) for the distinction between and absolute and hypothetical impossibility. Aristotle is using the first sense in the argument at (T4.2), cf. Denyer [2000], 165.

²⁷⁰See Rosen & Malink [2012], Broadie [2009].

impossibility. Since there is no time that is not comprehended in the course of things running from the actual moment, then (v) at some time something would both be and not be. Since an impossibility ensued, then the negation of the conjunction is true, so (vi) either something is for all time or it is perishable.

The majority of scholars have taken Aristotle's argument at (T4.2) to hinge on the key principle evoked at step (iv). The principle, testing a possibility by checking whether its actualization at some time does not entail any impossibility, is sometimes referred to as 'Actualisation Test' or 'Hypothesis Test,' and can be found in Aristotle's logical works. Aristotle states a version of it in his definition of possibility in *An. Pr* A 13 32a19-22: (Aristotle [2009], Transl. Striker, 17-18):

(T4.3) I use the expressions 'to be possible' and 'what is possible' in application to something if it is not necessary but nothing impossible will result if it is put as being the case (for it is only equivocally that we say that what is necessary is posssible).

In the case of DC, this definition of possibility is applied to pick out a time at which x ceases to be and assume it to be the actual time, in order to obtain a contradiction with the claim that x is eternal. In this concrete case, the problematic assumption according to many interpreters is not the stricture imposed at (T4.3) on the meaning of possibility in order to pass the Hypothesis Test - namely that a candidate possibility has be possibly actual at some time -, but the fact that the Hypothesis Test is used inside the argument without regard to changing the original assumption that x is eternal. As Lindsay Judson notes, in DC I.12 (Judson [1983], 239) "a candidate for possibility (is supposed to be actual) without regard to whether the supposition of its holding requires changes in what else is taken to be true". In other words, for 'x is eternal' and 'x is perishable' to be incompatible, Aristotle needs an additional warrant to the

²⁷¹For a careful reconstruction of Aristotle's deduction here, see Rosen & Malink [2012]. It is also addressed in Denyer [2000] on a more positive note.

²⁷²According to the majority of interpreters, this passage is not held in high esteem, and they tend to identify a logical mistake in the argument. Cf. Gaskin [1995], 104-127; Sorabji [1980], 180, Hintikka [1973], 210-13 and recently Rosen & Malink [2012]. More positive assessments of Aristotle's argument can be found in Broadie [2009] and Denyer [2000].

effect that 'x is necessary'. [273]

In the Middle Ages, the issues raised by this passage touched on a dispute over the boundaries between philosophical and theological modes of argumentation, the latter based on divine possibilities and powers. For example, in the thirteenth century, the view propounded by earlier philosophers accepting that the world by necessity is eternal was the object of nine articles in the famous condemnations of 1277 issued by Bishop Tempier at Paris [274] As we shall see, John of Jandun and John Buridan's commentaries on *De Caelo* bear witness to divergent viewpoints concerning the usage of divine possibilities in natural philosophical argumentation.

John of Jandun was active as a master of arts at the University of Paris shortly before Buridan. He probably wrote his commentary on *De Caelo* during his active time there, possibly around the 1320's or shortly before. Jandun's commentary is much indebted to Averroes' commentary, and in general he had a special interest on unveiling Aristotle's intentions through the lens of Averroes. Besides being a close contemporary to Buridan and holding a similar post as a master of arts at the University of Paris, Jandun is also relevant to our purposes for having shown a great interest in principles connecting modality with time. Among one of his shorter earlier texts is a disputed question (*quaestio disputata*) on whether possibility entails some time actuality. In

²⁷³This underlies Rosen & Malink [2012] diagnosis that the argument is circular (299). Jaakko Hintikka thought however that this needed additional warrant is imbued in the 'habit of thinking' of equating necessity with omnitemporality, and concludes that it must have been natural for Aristotle " [...] that every moment of time during any course of events that we consider possible must be equated with some moment of time during the actual course of events. This must have seemed a very natural assumption, for how could there be moments of time not identical with some moment of time in the actual history of the universe?'" (Hintikka [1973], 208).

²⁷⁴In particular, the condemnations 80-89 touched on the world's eternity (Thijssen [2018]). One of the views condemned, stated in article 89, states "That it is impossible to refute the arguments of the Philosopher concerning the eternity of the world unless we say that the will of the first being embraces incompatibles" (Klima, 2007, 195).

²⁷⁵He was then known was 'the prince of the Averroists', and alternatively (somewhat pejoratively) as 'Averroes' monkey,' due to a passage in his Question-Commentary on the *Metaphysics* in which he declares that a master of arts should take Averroes' authority as normative (Jandun [1525], fol. 84v).

²⁷⁶The *quaestio disputata*, estimated to be composed around 1318, is entitled 'Whether everything that can be generated will of necessity be generated ('*Utrum omne generabile*

his question 33 on his commentary on *De Caelo* (John of Jandun [1552], fols. 21raff.), Jandun comments and aims to reconstruct the sense in which Aristotle's proof of the necessity of the world's eternality is correct. He claims that the argument at (T4.2) is established on the basis of two principles. The first is the Hypothesis test, namely that if a possibility assumed to be actual, no impossibility follows, and the second is the principle of non contradiction. According to Jandun, the argument proceeds as follows (John of Jandun [1552], fol. 21ra):

(T4.4) This conclusion is proved through these [principles]. For it to be impossible that a thing is corrupted, means that if this thing is assumed to be corrupted in actuality (*inesse*), some impossibility would ensue. Now if we posit that a sempiternal thing (*semper ens*) is corrupted, then something impossible does follow - it would be the case that a sempiternal thing would cease to be -, for 'perpetual' means either what always is or what always is-not, and 'generated' means what comes to be from what earlier was not. Just consider an instant of time or moment in which it is possible for it [a sempiternal thing] to cease to be - since it is corruptible it ceases to be. But since it was assumed that it is a sempiternal thing, it would both be and not be in the same instant.

de necessitate generabitur'). See Lambertini [2013], 400 f.68, and the further references therein.

²⁷⁷Cf. (John of Jandun [1552], fol.21ra): "[...] This is proved by Aristotle assuming two principles, one of which says that if the possible is assumed to be actual, nothing impossible follows, although a falsity may ensue. This is naturally understood. Another principle supposes that it is impossible for one and the same to be and not to be simultaneously, since that is the first principle. From those he proves his conclusion: only that is impossible to be destroyed, which, if it were assumed that it is corrupted, some impossibility would follow. But if what always is (*semper ens*), is assumed to be corrupted, an impossibility would follow; since what always is would be corrupted." "([...] hoc probat Arisdtoteles supponendo duo principia, quorum unum est *possibili posito in esse*, *nullum sequitur impossibile*, licet possit sequi falsum, quod est naturaliter intellectum. Aliud supponid quod impossibile est unum et idem esse et non esse simul, quia prima dignitas est, non contingit idem esse et non esse simul. Ex his probatur conclusio: illud est impossibile corrumpi, quod, si poneretur corrumpi inesse, sequeretur impossibile. Sed si semper ens ponitur corrumpi, sequitur impossibile, quod semper ens corruptum esset [...]".)

²⁷⁸"Et his probatur conclusio. Illud est impossibile corrumpi, quod, si poneretur

Jandun's rendering of the argument uses temporalized versions of modal terms. It starts by defining sempiternal being as what falls under the class of what 'always is' or what 'always is-not,' and 'generated' or 'corrupted beings' as being at some time and later not being, and conversely. Furthermore, he unpacks to modal meaning of 'possibly generated' as being at some time the case. Furthermore, he mentions that in the case of something eternal any possibility for non-being should be distinguished and realized in time (*potentia ad esse et non esse non distinguitur in tempore*). In the subsequent passages of his question, he states more clearly how modal terms are spelled out in terms of temporal ones in his reading. Jandun takes for granted the oppositions that Aristotle has set out in in his text, making reference also to the *Perihermeneias* oppositions between necessity and impossibility. Consequently, he writes (John of Jandun [1552], fol 21vb):

(T4.5) Always being and always not-being are opposites, just as the impossible and the necessary are. From that I argue thus: everything which always is necessarily is; therefore, everything which always is-not is impossible. The antecedent is clear, since the terms are equivalent (*conuertuntur*). And the consequence is evident, by the rule that two opposites are so related as are the proposed terms. But always being and always not-being are opposites on one part, and the necessary and impossible opposites of the other. Hence, what is impossible cannot be generated, therefore, what never is cannot be generated. These convert in the following: nothing that can be generated never comes to be, from which it follows that everything which can be generated will come to be.

corrumpi inesse sequeretur impossibile. Sed si semper ens ponitur corrumpi, sequitur impossibile, quod semper ens corruptum esset, quia perpetuum dicitur esse semper et non esse, quia genitum est quod iam est et prius non fuit. Modo signetur illud instans vel hora in quo sit possibile non esse, et cum sit corruptibile non est, et tu ponis semper esse, ergo in eodem instanti est et non est."

²⁷⁹"Semper ens et semper non ens sunt opposita, sicut impossibile et necessarium. Ex hoc sic: omne semper ens est necessarium esse; ergo omne semper non ens est impossibile. Antecedens patet, quia illa conuertuntur. Consequentia patet: si oppositum in opposito et propositum in proposito. Sed semper ens et semper non ens opponuntur ex una parte, sicut necessarium et impossibile ex alia parte. Modo, quod impossibile est non est generabile, ergo, quod numquam est non est generabile. Modo conuertuntum in

Jandun clearly espouses a temporal understanding of modalities in this passage. This is evidenced by the notion that necessity is considered the opposite of impossibility - as are 'always being' is the opposite of 'always notbeing' - and Jandun thought that this position was in line with the metaphysical model presented in DC I.12. In order to understand why Jandun takes the concepts of 'necessity' and 'impossibility' to be opposed likewise the concepts 'always' and 'always-not,' we should turn the attention to a square of oppositions between temporal terms that was frequently drawn in commentaries on DC I.12, and which Jandun could have taken from Averroes' commentary on De Caelo In his commentary, Averroes presents the oppositions between four temporal concepts as below:

- A Always being (semper ens)
- B Always not being (semper non ens)
- C Not always not being (non semper non ens)
- D Not always being (non semper ens)

We find Averroes' rendition of a square of oppositions involving these temporal concepts below (Averroes) [1562a], fol. 39v):

istam: nullum generabile semper non est, et ex his sequitur ista, omne generabile de necessitate habebit esse, et non potest habere esse nisi per generationem. Ergo omne generabile de necessitate generabitur."

²⁸⁰Aristotle has mentioned several squares in his own *De Caelo*. Cf. Williams [1965]. Averroes' text is plausibly the closest source to Jandun in his own question-commentary, so it is useful to discuss Averroes' rendition here.

Figura,ostendês q. de quo alterius contrariorú negatio uera est, de codem & alteriusn egatio uera erit.

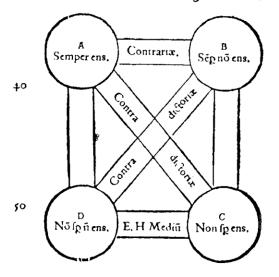


Figure 4.1: Averroes' Temporal Square

For Averroes and John of Jandun - following the Aristotelian view -, this square represents an exhaustive metaphysical classification of types of beings, based on the fact that they are either perishable or eternal in nature (not able to undergo becoming and perishing). Accordingly, the terms 'ungenerable' and 'incorruptible' apply to what always exists, and the terms 'generable' and 'corruptible' apply to what sometimes exists and sometimes does not. Each

²⁸¹Averroes explains the square thus (Averroes) [1562a], 39vb-40ra): "Cum declarauit ratione quod illud, quod habet potentiam ut sit in alia hora, et ut non sit in alia hora, non est illud, quod habet potentiam ut semper sit, neque illud, quod habet potentiam ut semper non sit, sed medium, vult declarare haec litteris et dixit. [...] quod habet potentiam ut semper sit A et eius contrarium, quod semper non sit B, et sunt quod impossibile est ut congregentur in eodem insimul. Et sit C contrarium ad A, et D contrarium ad B. Idest ponamus contrarietatem quae est in hoc sermone, verbi gratia, illud quod est ens in aliqua hora et non ens in alia hora C et D, et sit C contrarium ad A, quod posuimus semper ens. Ex C ergo intelligendum est illud, quod non est in aliquo tempore, et erit D quod est ens in aliquo tempore, contrarium ad B, quod est semper non ens. Erunt ergo tria, semper ens, scilicet A et semper non ens, scilicet B, et esse et non esse, scilicet CD. Et cum posuit hoc, quaesiuit de qua re non dicitur esse et non esse, ad demonstrandum quia, cum hoc non dicatur de extremis, necesse est cum dicatur de natura media inter duo extrema,"

corner of the square is understood on the basis of this metaphysical model, it was thought that the pairs A/B are contraries - that is they cannot be both true at the same time but can be both false, namely, they can be both false when said of things that undergo generation and corruption -, A/C and B/D are contradictory, and the pairs A/D and B/C are subcontraries. The contingent is a middle, namely that to which both being and not being apply at different times [282]

In this regard, Jandun endorses a temporal understanding of the Aristotelian modalities involved in the passage of *DC*. One of the reasons why he did so is that Jandun does not see the distinction between supernatural possibilities and natural possibilities as pertinent to assess philosophical claims. Jandun wants rather to claim that the conclusion that the world is eternal is what philosophy can establish. At the same time, he says that this does not limit belief in supernatural possibilities. In recognizing that the world is ultimately contingent - for it depends on divine power and will - he took such possibilities that are beyond nature (*super naturam*) to be outside the scope of natural philosophical argumentation. Ultimately, Jandun claims that divine possibilities are inaccessible to us, for God can make many more things than our intellect can conceive (John of Jandun) [1552], fol. 21va):

(T4.6) But that does not limit divine power and faith: since it is the truth that God can make something generable which never was, and what is sempiternal corruptible. But that is beyond nature (*super naturam*), since it cannot be demonstrated from sensible things; for if it could be demonstrated, we would not have any merit in believing them. For divine power can make more things than the intellect can conceive. And Aristotle's arguments are valid since nothing can be and not be at the same instant. We should say that it follows according to nature; but what God can do is to infer that there is no contradiction involved. But in this life we are ignorant as to how the impossibility can be avoided [283]

Averroes [1562a], 49r: "Sint ergo A et B contraria, non cadentia in eodem insimul, et sit C contrarium ad A, et D contrarium ad B. Et dicamus igitur quod illud, quod non est in A, neque in B, est in toto C et D. Sit ergo H, quod est inter A et B. Illud enim, quod non est alterum duorum contrariorum, medium est inter illa. Erit ergo H, sicut C et D, necessario"

²⁸³Cf. John of Jandun [1552], fol. 21va: "Sed istud non preiudicat divinae potentiae

This is not the only passage in his commentary on *De Caelo* where Jandun states that we remain ignorant of supernatural possibilities. The main reason why Jandun endorses the temporal-frequency model is that we can ascertain whether something is a possibility or not once it is actualized in time, and possibilities that are never actualized remain elusive to natural philosophy. As Knuuttila has shown, Jandun at some other passages regards merely logical and never actual possibilities as "mysterious and contrary to reason." ²⁸⁴

4.3 Buridan's Critique

In Buridan's *DC* we see a treatment of the arguments that is radically different from that of Averroes and Jandun. First, in Buridan's theory, the claims concerning the temporal status of assertoric propositions are not modal claims, and Buridan often accuses Aristotle of confusing modal with non-modal propositions in this treatise. In his *Expositio*, referring to *DC* I.12, Buridan writes (Buridan [1996], Ed. Patar, 86, 65/66):

(T4.6) All the arguments in this chapter, or almost all, appear to be logically flawed (*sophisticae*), insofar as they proceed from a proposition of possibility in the divided sense to [a proposition] possible in the

et fidei: quia veritas est, quod Deus potest aliquod quod numquam erit facere generabile et sempiternum corriptibile. Sed hoc est super naturam, quia ex sensibilibus non potest demonstrari; quod si demonstrari posset, tunc non haberemus meritum credendo. Unde divina potentia plus facit quam intellectums unumquoque potest concipere. Et rationes Aristoteles valent qoud idem esse in eodem instanti et non esse et cet. Dicendum quod sequitur secuncum naturam. Sed Deus potest facere hoc est deducere quod nulla erit contradictio. Sed in hac vita ignoramus quomodo evitaretur illud impossibile".

²⁸⁴Knuuttila refers to Jandun's *DC* I.34, referring to the question of whether a possibility can remain eternally unrealized. Knuuttila writes (Knuuttila [2012], 322): "Supernatural possibilities are treated as mysterious and contrary to reason here. By statements like 'one can do what one never can do', Jandun apparently means that the possibilities of faith may be naturally impossible. A great deal of the extensive discussion of divine power was more religious than philosophical, stressing the sovereignty of God, but there were also attempts to reconsider the meaning of modal concepts in this context." Certainly, Buridan falls in the latter category.

²⁸⁵Buridan [1996], 79: "It should be noted that Aristotle often mixes up the aforesaid assertoric propositions with propositions about possibility, as if he makes no distinction between them." ["Et sciendum est quod saepe Aristoteles intermiscet praedictas de inesse et de possibili, quasi non faciens differentiam inter eas."]

composite sense. Such inferences do not always hold: it does not follow, for example, that if something white can be black, then it is possible that something white is black. Nor does it follow that if I can see every star, then it is possible that I see every star. Analogously, from two possible categorical propositions to a conjunction of them, concluding that the conjunction is possible and that no impossibility ensues. But the whole procedure is wrong. In the case at hand, similarly it should be said that it is possible that a will be corrupted for an infinite time, and it is possible that a will remain incorrupted for an infinite time, but the conjunction of both is not conceded to be possible. [286]

Buridan points to two interrelated sources of logical mistakes that he attributes to Aristotle in this chapter. The first concerns the invalidity of an inference proceeding from a divided modal proposition of possibility to a composite proposition, and the second the inference from two possible composite modal propositions to the possibility of their conjunction. In what follows, we will see why Buridan thinks that way and how these remarks base his critique of the argument at *DC* I.12.

To be sure, despite the overall critical tone of Buridan's commentaries, in the positive parts of the treatise he claims that the corresponding questions of DC should be approached on a charitable way, and that the modal components of Aristotle's argument contained in DC I.12 - such as the assumption that every possibility is realized at some time - correspond to what should be said if it was assumed (*posito quod*) that the world is eternal and incorruptible. ^[287] In

²⁸⁶"Et hoc est valde dubitabile, et simpliciter falsum, secundum quem sensum vera prima facie procedunt. Et omnes rationes huius capituli, vel quasi omnes, videntur sophisticae, procedentes de possibili in sensu diviso ad possibile in sensu composito. Et non semper tenet processus: non enim sequitur, si album potest esse nigrum, quod haec est possibilis: album est nigrum, nec sequitur, si omne astrum possum videre, quod haec sit possibilis: omne astrum video. Similiter istae rationes videntur procedere de duabus categoricis possibilibus, non tamen compossibilibus ad copulativam compositam ex eis, concludendo quod illa sit possibilis et quod ad eam non debet sequi impossibile. Et hoc totum est falsum. Verbi gratia diceretur quod haec est possibilis: a infinito tempore erit corruptum, et haec etiam possibilis: a infinito erit incorruptum; et tamen copulative ex eius composita non concederetur esse possibilis. Et ideo praeiecta an sit vera vel habeat aliquod sensum verum indigent sensibili perstructione in quaestionibus."

²⁸⁷Referring to the assumption that if there is a power for opposites - such as the

his *Expositio*, Buridan says that the following principles structure Aristotle's argument for the world's eternality (Buridan [1996], Ed. Patar, 74):

- 1 If something possibly is and possibly is not, the corresponding power for each opposite should be temporally determined, each power cannot be realized for an infinite time. Otherwise, as Aristotle argues, there would be two infinite times. [288]
- 2 The false and the impossible, and the possible and the true are not equivalent 289
- 3 From an impossible posit another impossibility follows, but from a contingent falsity the impossible does not follow. 290

possibility of becoming and ceasing to be something (generation and corruption), each of these alternate possibilities should be realized at some or another time, Buridan writes: Buridan [1996], Ed. Patar, 359: "The purpose of this question should be restricted to what should be answered if it is posited, with as Aristotle believed, that the world is eternal and incorruptible, and that something cannot come to be out of nothing, but rather everything that comes to be presupposes matter, since nothing comes to be by nature in a different way." ["Sed modo questio restringatur: quomodo esset dicendum de quaesito, posito quod mundus esset aeternus et incorruptibilis secundum quod Aristoteles opinabatur, et posito quod non possit aliquid fieri ex nihilo, sed quod si necesse omne quod fit fieri ex materia praesupposita, sicut verum est quod non potest aliquid fieri aliter modo naturali."]

²⁸⁸Buridan [1996], Ed. Patar, 74: "The first assumption is that if it is possible for something to be or to fail to be, each of these powers, that is the power to be and the power to fail to be, is a power for a finite time, and neither power can be realized in infinite time. [...] Aristotle thought the reason for this assumption is that if each power were not for a determinate time, then one power would be for an infinite time and another for another infinite time; which is impossible, since there cannot be many infinite times, for one infinite time would contain the whole of time. ("Prima suppositio est quod, si es aliquid possibile esse et possibile non esse, utraque potentia, scilicet tam ad esse quam ad non esse, est in tempore determinato, ita quod neutra est in infinito tempore (...) Et Aristoteles assignat rationem illius suppositionis, quia, si non sit quaelibet in tempore terminato, tunc erit una in uno tempore infinito et alia in alio tempore infinito; quod est impossibile, quia non contingit sic esse plura tempora infinita, eo quod unum tempus infinitum contineret omne tempus." Buridan relegates criticism of this assumption to the questions.)

²⁸⁹Buridan [1996], Ed. Patar, 75: "Secunda suppositio est quod non est idem convertibiliter falsum et impossibile, neque verum et possibile, quia ego nunc sedeo de facto et ideo me stare est falsum, et tamen me stare non est impossibile, immo me stare est possibile, et tamen non est verum".

²⁹⁰Buridan [1996], 75: "Tertia suppositio est quod ad positionem impossibilis sequitur

4 Something can simultaneously have a power for opposites, but no power for opposites can be exercised simultaneously. [291]

Buridan grants to Aristotle that [1] and [4] are sound if he is speaking according to natural powers. Buridan refers to the Commentator's (i.e., Averroes') conclusions that natural powers are limited both at the level of species and at the level of individuals, for once they are actualized, their possibility of coming to be ceases. As Buridan notes, this derives from the fact that Aristotle uses a notion of potency defined as that which is not actual. The main criticisms Buridan draws concerns the applications of [2] and [3], which he sees to be logical principles underlying Aristotle's indirect proof at [4.1]. In this regard, he does not critize Aristotle's conclusions as much as the arguments themselves.

The assumption [3] is a well known principle from his treatises on logic. Its source is Aristotle's definition of the possible as we have seen above at (T4.3), and it is one of Buridan's semantic principles governing consequence in the TC. In the present context, Buridan's critique is not targeted at the principle

tale inconveniens quod non sequitur ad positionem falsi possibilis, quia ad impossibile sequitur bene impossibile; sed ad falsum possibile numquam sequitur impossibile, licet sequatur falsum; ideo si consequens est impossibile, sequitur quod antecedens non solum est falsum sed etiam impossibile"

²⁹¹Buridan [1996], Ed. Patar, 76: "Quarta suppositio est prima facie. Videtur esse quod idem habet simul potentiam ad opposita, sed non habet potentiam ad existere simul opposita, ut ego nunc sedens habeo potentiam sedendi (non enim sederem, si non possem sedere), et tamen cum hoc habeo potentiam non sedendi, quia possum surgere, sed non habeo ad hoc quod simul sedeam et non sedeam. Et haec suppositio est capiendo possibile large, tamen pro eo quod est quam pro eo quod contingit esse."

²⁹²Buridan [1996], Ed. Patar, 371-2.

Buridan [1996], Ed. Patar, 360-1: "Et potest argui suis rationibus, supponendo quod numquam sunt simul potentia ad esse rei et potentia ad non esse ipsius, quia nos loquimur hic de potentia proprie dicta, scilicet prout hoc nomen *potentia* connotat carentia actus. Ita quod non dicamus aliquem esse in potentia ad sedere quando sedet, nec ad non sedere quando non sedet; immo dicamus eum actu sedere quando sedet, et actu non sedere quando non sedet. Et ideo, quia impossibile est eum simul sedere et non sedere, ideo etiam impossibile est eum habere simul potentiam ad sedendum et ad non sedendum." See also Buridan [1996], Ed. Patar, 362: "Pro quaestione solvenda notandum est, sicut tangebatur, quod hic intendimus de potentia proprie dicta, scilicet prout *posse* vel *potentia* connotat carentiam actus. Saepe enim utimur *potentia* vel *possibili* magis communiter prout se extendit tam ad contingens quam ad necessarium, et tam ad illud quod est quam ad illud quod non est, cui tamen non repugnat esse; et de tali modo communi non indentimus ad praesens."

²⁹⁴Cf. Buridan [2015b], Transl. Read, 109.

itself, but its application. First, let us see how Buridan construes Aristotle's argument strategy at (T4.1). In his *Expositio* commentary, Buridan explains Aristotle's argument in terms similar to those used in obligational disputes. The proposition 'what always is can fail to be' is put forward by an opponent, and the respondent (referred to by Buridan in the first person) should reply in accordance with the rule corresponding to [3], namely that from a possible proposition no impossibility should follow. Buridan writes (Buridan [1996], Ed. Patar, 77):

(T4.4) It is argued in this way to the fourth conclusion. The propositions 'something which always is can be corrupted' and 'something which always is can fail to be' are equivalent; but this is impossible: 'what always is can fail to be'. Proof: let a be something which always is, which according to the opponent can fail to be; now this would be possible: a is not, even though it is false. And I claim that it is impossible, since it follows from it not only something false, but impossible, that is for the same to be and fail to be, since we have said that a is not, but it was posited that a is always, in order to conclude that what always is does not exist, and that is impossible.

As we will see below, Buridan thinks that this argument rests on a misapplication of the principle that from the possible no impossibility follows. In the construal of the argument here, since an impossibility allegedly follows from the opponent's posit, therefore something which always is cannot be destroyed is concluded. The purpose of framing the argument strategy within an obligational context is to test the compatibility between these different propositions. The original argument is construed by Buridan as follows. First it is assumed by both parties in the dispute that (i) a is a sempiternal being. Then, the respondent claims that (ii) a can fail to be, in order to see if (i) and (ii) are compatible

²⁹⁵"Arguit ergo sic ad quintam conclusionem. Idem valet dicere: *semper ens est corruptibile* et *semper ens potest non esse*; sed haec est impossibilis: *semper ens potest non esse*. Probatio: quia sit *a* semper ens quod potest non esse secundum adversarium, tunc ista esset possibilis: *a non est*, licet esse false. Et ego ostendo quod ipsa sit impossibilis, quia sequitur ad eam non solum falsum, sed impossibile scilicet idem simul esse et non esse, quia dicemus quod *a* non est, et tamen ponitur *a* semper esse, et squitur ergo semper ens non est, et hoc est impossibile."

positions. However, from (ii) it follows that it is (iii) possible that a is not - that is, at some time it is the case that a is not. But from (i) it follows that a exists for all time. Therefore, in order for both (i) and (ii) to be true there must be some time (iv) a exists and a fails to exist. But since that is impossible, something which always is cannot fail to be.

In the subsequent passages Buridan goes on to evaluate the argument, and criticizes it as failure to distinguish between divided and composite modal propositions. Namely, while (ii) is a divided modal proposition, (iii) is a composite one, and in his modal syllogistics - as we have seen in chapter 2.3 - any inference from an affirmative divided proposition of possibility to a composite one is invalid. Buridan illustrates it again with similar remarks that we have seen in the *TC* (Buridan [1996], Ed. Patar, 77):

(T4.5) And without doubt it appears to me that this way of arguing is not valid in many cases, since, we can concede this in the divided sense, 'what always is can fail to be', but deny this, 'it is possible that what always is fails to be', just as we can say that what is white can be black, while it is impossible that something black is white. We would therefore say that this is possible: *a always is* and this therefore possible: *a is not*, since from neither does any impossibility follow. But the conjunction of both is impossible, so from it the impossible follows [296]

²⁹⁶"Arguit ergo sic ad quintam conclusionem. Idem valet dicere semper ens est corruptibile et semper ens potest non esse; sed haec est impossibilis: semper ens potest non esse. Probatio: quia sit a semper ens quod potest non esse secundum adversarium; tunc ista esset possibilis: a non est, licet esse falsa. Et ego ostendo quod ipsa sit impossibilis, quia sequitur ad eam non solum falsum, sed impossibile, scilicet idem simul esse et non esse, quia dicemus quod a non est, et tamen ponitur a semper esse, et sequitur ergo semper ens non est. et hoc est impossibile. Et sine dubio videtur mihi quod iste modus arguendi non multum valet, quia, licet concederemus istam de possibili in sensu diviso: semper ens potest non esse, tamen negaremus istam: possibile est semper ens non esse, sicut album potest esse nigrum et tamen impossibile est algum esse nigrum. Diceremus etiam quod haec ese possibilis: a semper est, et haec etiam possibilis: a non est, ideo ad neutram istarum sequitur impossibile; sed copulativa consitituta ex eis est impossibilis, ideo ad eam seguitur impossibile, sicut haec est possibilis: omne currens est homo, et haec etiam possibilis: omnis equus est currens; sed copulativa ex eis est impossibilis, ideo ad illam copulativam seguitur impossibile in primo modo primae figurae, scilicet quod omnis equus est homo."

The invalid inference Buridan attributes to Aristotle is one of the steps in the indirect proof going from a divided modal of possibility to an assertoric as follows:

1 Something which always is can fail to be
1 It is possible that something which always is fails to be

As we have seen, Buridan's distinction between divided and composite modal propositions implies that divided modals have merely possible supposita and composite modals are ultimately claims about actuality. In this regard, we may say that Buridan draws a sharp line between modality and temporality. Since here there is no temporal alternative for 'a always is' - if true, then there is no time at which a is not -, then Buridan must be thinking about a modal alternative described by the premise, namely a possible circumstance which is never actual. Moreover, in the *Quaestiones* Buridan clearly argues against the position that something which never is generated or corrupted is impossible what Buridan is clear on in the *Quaestiones* is that the kind of possibility that can be eternally frustrated is only meaningful on the level of supernatural powers.

²⁹⁷Arguing against the proposition - which we have seen Jandun endorsing above - that if something is never generated (or corrupted) it is impossible for it to be generated or corrupted, Buridan objects with an example Buridan [1996], Ed. Patar, 370-1: "The opposite can be clearly shown in the following way. Vinegar can be generated from this wine, but this lies within your free power (*ex voluntate tua*), since you can conserve it and mix it with another vinegar, which would transform wine into vinegar. But it remains nonetheless possible that vinegar will never be generated from this wine, since you can drink the wine first. Therefore it is manifestly possible that something which can be generated will never be generated." ["Oppositum tamen arguitur manifeste, quia ex hoc vino potest generari acetum; et est in voluntate tua, quia tu potes ipsum servare et cum alio aceto ponere, quo facto mutaretur in acetum. Et tamen possibile est quod numquam illud acetum ex illo vino generabitur, quia statim liberere tu potes potare illud vinum. Ideo haec est valde possibilis quod aliquod generabile numquam generabitur."]

Buridan [1996], Ed. Patar, 372: "If we speak supernaturally, it is promptly conceded that something can generated substantially - be it subjectivelly (*subiective*) or terminatively (*terminative*), which happens to be never generated. Since the matter of this donkey can receive generation subjectivelly (something other can be made out of this matter, once the donkey is corrupted), but God could make it so that from this matter nothing will be ever generated, since he can annihilate it". ["Si igitur loquamur supernaturaliter, statim posset concedi quod aliquid est generabile substantialiter, sive subiective sive terminative, quod forte numquam generabitur. Quia materia huius

Buridan very often critizes Aristotle in that text for confusing (*intermiscet*) the assertoric and modal oppositions at hand in the square of oppositions we find above, in Averroes' version in figure (4.1). As we have seen in [2.2], Buridan makes a sharp distinction between composite and divided modal propositions, and he thinks that only divided modal propositions are properly about possibility, whereas composite propositions are in the end propositions about actuality (*de inesse*). Therefore, in the square that Jandun and Averroes endorsed between temporal concepts above does not involve modal propositions in Buridan's sense. In his *Expositio*, Buridan has mentioned one such square for divided modal propositions. Buridan does not elaborate on the semantic implications of this square, but it is important because it was influential later as we shall see in a moment The propositions in the modal-temporal square in the divided sense Buridan mentions in his *Expositio* are arranged as follows:

- A Always possible to be (semper possibile esse)
- B Always possible not to be (*semper possibile non esse*)
- C Not always possible not to be (non semper possibile non esse)
- D Not always possible to be (non semper possibile est esse)

A version of the modal-temporal square Buridan has mentioned can be found - in the form of a pentagon of oppositions - in Nicole Oresme's French commentary the *Livre du Ciel et du Monde*. Lorenz Demey has noted the

asini est generabilis subiective (potest enim ex ea aliud generari, corrupto asino), et tamen potest Deus facere quod numquam ex ea aliquid generabitur, quia posset ipsam annihilare."]

²⁹⁹Cf. Buridan [1996], Ed. Patar, 78-9: "Verum est quod hic Aristoteles ordinat oppositiones suas in illis de possibili, sed postea ipse hoc faciet in illis de inesse. Et ideo nunc primo videamus de illis de inesse, quia in eius sunt manifestiores huiusmodi oppositiones. Dicamus ergo quod istae se habent per modum contradictionis: semper ens et non semper ens, et similiter istae: semper non ens et non semper non ens. Et istae se habent modo contrario: semper ens et semper non ens; ideo istae sunt subcontrariae: non semper ens et non semper non ens, quae equivalent istisduabus: quandoque ens et quandoque non ens. Sed de illis de possibili dicit Aristoteles quod istae contradicunt: semper possibile esse et non semper possibile esse, et similiter istae: semper possibile non esse et non semper possibile non esse".

³⁰⁰Cf. the reference by Patar in Buridan [1996], Ed. Patar, 79.

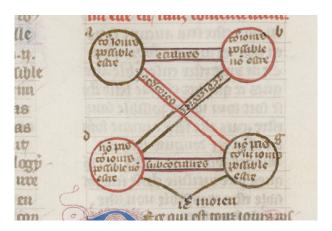


Figure 4.2: Oresme's Modal-Temporal Square

presence of this diagram in Oresme's commentary, and conjectured that it may have been influenced through Buridan's [301] Although this conjecture cannot be supported here, it would not be implausible to state Oresme's version of the square could have been influenced from Buridan's presentation in his *Expositio* commentary. Oresme's rendition of the modal-temporal square can be seen in the figure [4.2][302] Oresme's pentagon of oppositions contains the same propositional forms that Buridan has listed, with the inclusion of a middle (*le moien*), which applies to things that sometimes can be, and sometimes cannot be [303]

As we have seen, the basis for Buridan's critical remarks on *DC* I.25 is that Buridan took Aristotle to assimilate modality and temporality. In particular, Buridan claims that the argument we have seen above lacks a distinction between divided modal propositions and composite modal propositions. Since that is the case, one could expect Buridan to think that only this second kind of diagram is relevant for the modal oppositions. However, Buridan does not develop further the logical oppositions represented in the diagram in terms of his ampliation semantics. Instead, Buridan claims the reason why Aristotle conflates one type of opposition with another (namely, the oppositions between divided modal

³⁰¹Demey [2019].

³⁰²This figure is taken from BnF. Ms. fran. 1082, fol. 51r.

³⁰³The texts in Oresme's pentagon are, respectively, A: *tousjours possibile estre*, E: *tousjours possible non estre*, I: *non pas tousjours possible non estre*, and finally O: *non pas tousjours possible estre*.

propositions and between composite modal propositions) is that he believes Aristotle interprets possibility in the sense of being in potency. According to that conception, everything that is potency is opposite to what is actual. Thus, something which always can be is something that always is in pure potency to being ³⁰⁴ In the context of the *Expositio* - which is, after all, meant to be a literal commentary - his interests are in harmonizing with what he takes to be Aristotle's views.

To conclude this chapter, we should note that Buridan was not hesitant to apply his logical distinctions between divided and composite modal propositions in order to criticize earlier interpretations of Aristotle. By contrasting Jandun and Buridan's positions in this commentary, we can see how they took opposite views concerning the usage of merely logical possibilities in natural philosophy.

Buridan [1996], Ed. Patar, 79: "And it should be noted that Aristotle in this treatise often mixes up (*intermiscet*) the aforesaid propositions about actuality (*de inesse*) with propositions about possibility, almost making no distinction between them, because Aristotle means takes possibility in the sense of potency, which connotes the lack of actuality; in that sense everything which can be is not actual, and everything which can fail to be is actual, and in this regard 'always possible to be' is understood almost the same way as 'always not being', and 'always possible not to be' understood almost the same way as 'always being'."Et sciendum est quod saepe Aristoteles intermiscet praedictas de inesse et de possibili, quasi non faciens differentiam inter eas, propter hoc quod ipse intendit de possibili prout *potentia* connotat carentiam actus; ideo sic omne quod potest esse non est, et omne quod potest non esse est, propter quod *semper possibile esse* accipitur quasi *semper non esse* et *semper possibile non esse* accipitur tamquam *semper esse*."

5. Powers, Possibility, and Causal Necessity

In this chapter, I aim to shed some light on contexts where powers-based modalities are evoked in Buridan's metaphysical and natural philosophical writings. In section [5.1] I aim to clarify the distinction drawn at *QPhys*. I.22 between powers-based and logical possibility. I will argue that Buridan believed powers-based explanations of modal concepts to be more common in scientific and everyday discourse, but he also thought that they do not provide the correct basis for logical analyses of modal propositions. Section [5.2] turns to Buridan's application of the concept of historical possibility and necessity at *QDGC* I.4 and I.24. In that context, Buridan claims that the weakest degree of modality (cf. Table [3.1]), according to which no power can be over the past, is not adequate to explain the necessary connections between types of changes in the common course of nature.

5.1 The Possible Through Powers

The concept of possibility is articulated in medieval modal idioms in various ways. As we shall below, in his commentary on the *Physics* Buridan describes some of the non-logical acceptations of possibility as based on the notion of powers, capacities and abilities. Buridan's explanation os the non-logical sense of possibility goes along the following lines. Alongside the 'possible' (*possibile*) used as a predicate of propositions in logic, other usages of non-logical possibilities include 'being in potency' (*ens in potentia*), as said of possible objects relative to a power in the sense of being the end-point of their activities, and 'being able' (*potens esse*), as said of active or passive powers in nature in the sense of their capacities to bring about change (Buridan [2015a], Eds. Streijger and Bakker, 220ff.). As we have seen in section [2.2], Buridan

claims that on the basis of their proper meaning (*de virtute sermonis*), the subjects of modal propositions are ampliated to stand for what merely can be, without any connotation of powers. However, when he turns to natural philosophy and metaphysics, the comparisons Buridan draws between the logical and non-logical usages of the concept possibility poses the question of the unity of that concept.

John Duns Scotus (†1308) has notably provided an elaborate and influential account of the relationship between logical and non-logical possibilities. Scotus's account provides an important point of departure. and his influence on Buridan in this regard are usually not ruled out by interpreters on One point of innovation from Scotus which is widely agreed on his coinange of the term 'logical possibility' (possibilitas logica; also potentia logica) for the first time in the history of philosophy, distinguishing it from 'metaphysical possibility' (potentia metaphysica), which is related to the former as its real counterpart. Scotus characterizes logical possibility as a property of propositions whose terms are not incompatible (non repugnantia terminorum), and as a mode pertaining to the composition of these concepts in the intellect. Metaphysical possibility, on the other hand, is not a mode of composition of concepts in the mind, but instead a real kind of possibility, that is, it concerns possibilities which are inscribed in the structure of real beings, their capacities and powers.

These kinds of possibility are certainly related in Scotus's theory, at least in the sense that no real power can bring about a conceptual impossibility, and so

³⁰⁵The literature on Scotus's modal theory is quite vast, but representative studies are Knuuttila [1996]; Normore [1996a], King [2001b] and Cross [2015].

³⁰⁶Knuuttila's interpretation of Buridan is heavily influenced by his views on Scotus, cf. Knuuttila [1993], and recently Spencer Johnston has also suggested that in some respects the two accounts share important points of departure (Johnston [forthcoming]).

³⁰⁷Cf. Cross [2015]

³⁰⁸ Cf. *Quaestiones super libros Metaphysicorum Aristotelis*, IX qq.1-2, n.16, where he says that "the name 'potency' is adopted elsewhere to signify logical potency, as for instance in possible propositions (Scotus [1997], 514; translated by King [2001b], 6. See also Cross [2015].) By using the phrase 'the absence of a contradiction', Scotus' focus is conceptual rather than syntactical. Thus, 'A human being is a donkey' is logically impossible in that sense, even though it does not meet the syntactical criteria of a contradiction.

³⁰⁹Scotus claims that a real possibility concerns a potency which either inheres in a thing or is the object of its power. Cf. John Duns Scotus [1966] 1 d.7 q.1 n.31 and King [2001b], 4, 10-19, for an analysis of real possibility in Scotus's writings.

real or metaphysical possibility presupposes logical possibility. But they do not bear a fundamental unity. Instead, as Scotus claims, the concept of a potency is to be understood equivocally (*aequivoce*) between these two senses [310] What is Buridan's conception of non-logical possibility? An important passage to explore this issue is his *QPhys*. I.22, where Buridan provides a similar classification of the meanings of possibility as Scotus did [311] For our purposes, the distinction Buridan draws between logical and non-logical varieties of possibility is specially relevant. Buridan writes (Buridan [2015a], Eds. Streijger and Bakker, 221):

(T5.1) But in another way, as Aristotle says in the fifth book of the *Meta-physics*, possible and impossible are names predicated properly of propositions. In that basic sense, a proposition is said to be necessary from the fact that it is true whenever it is proposed, and it cannot be false. Accordingly, a proposition is said to be possible because its contradictory [proposition] is not necessary, and contingent when neither itself nor its contradictory is necessary. In that way we say that the possible is common between the necessary and the contingent, since the contradictories of those [necessary and contingent propositions] are not necessary as well.

³¹⁰ Scotus claims in his *Lectura* 1.20 q.unica n.10 that 'potentia sumitur aequivoce', cf. King [2001b], 2.

³ To be sure, the question Buridan directly addresses in *QPhys*. I.22 concerns the potency (in the sense of the passive power) of prime matter, which is a potency to receive all substantial forms (Buridan [2015a], Eds. Streijger and Bakker, 218-224). But it is in that context that Buridan fleshes out in more depth the kind of real possibility associated with the active and passive powers inscribed into the hylemorphic structure of substances, in contradistinction with logical possibility. The details of Buridan's approach to prime matter are not directly concerning us here, but see Friedman [2021] for a discussion of this issue.

³¹²"Sed iterum alio modo, sicut tangit Aristoteles quinto *Metaphysicae*, dicitur possibile ve impossibile prout haec nomina appropriate conveniunt propositionibus. Primitus enim propositio dicitur necessaria, quia quandocumque proponitur, est vera et non potest esse falsa. Deinde propositio dicitur possibilis ex eo quod sua contradictoria non est necessaria. Et dicitur contingens, quia nec ipsa nec sua contradictoria est necessaria. Et sic dicimus quod possibile est commune ad contingens et necessarium, quia neutrius horum contradictoria est necessaria. Cuius autem contradictoria est necessaria, illa est impossibilis. Et illo modo potentia vel possibilitas vel possibile, similiter necessitas vel necessaria."

In this passage, Buridan has in mind something quite similar to Scotus's notion of logical possibility. In that sense, possibility is predicated of propositions, the contradictories of which are not necessary. In other words, a proposition which is logically possible does not imply a conceptual impossibility. In the following pages, Buridan turns to the distinction between logical and non-logical acceptations of possibility. His distinction goes as follows. On the one hand, things are called possible on account of the existence of active and passive powers to produce them, namely as the objects of powers [313] On the other hand, things are also called possible insofar as they are the significates of propositions about possibility. Buridan writes (Buridan [2015a], Eds. Streijger and Bakker, 220-221):

(T5.2) However, in yet another sense, 'potency', 'possibility', 'being able' (*potens*), 'being possible' or 'being in potency' means that which can be, just as we say that the Antichrist is in potency or is possible thing, and that there is a potency to generate the Antichrist or to receive its form in a body, etc. And it seems that the Commentator [i.e., Averroes] and Aristotle have used this sense of potency or possibility according to attribution to a passive or active power. The Antichrist is said to be possible or in potency because there is an active power able to bring it about or to bring about its form in being, and because there is a subject or matter able to receive it or its form; and its first active power is God and its subject is prime matter. Alternatively, 'potency' or 'possible' can be understood according to possibility as an attribute of propositions. We thus say that the Antichrist is in potency to being or that it is a possible thing [...] because that [proposition] is possible, namely 'the Antichrist is' [...]

³¹³Active and passive powers are understood by Buridan in a standard Aristotelian way as principles of change, where the same change is considered from two different perspectives - one the one hand it involves an active principle to change something else, on the other it involves a power to be changed by something else. Cf. Buridan [1518], fol. 56ra: "Active potency is the principle of changing something else *qua* something else; and passive potency is the principle of being changed by something else *as* something else." ("Potentia activa est principium transmutandi alterum aut inquantum alterum; et potentia passiva est principium transmutandi ab altero aut inquantum altero".)

³¹⁴"Sed adhuc alio modo dicitur potentia vel possibilitas, potens vel possibile vel ens

Without using the same terminology, the passage above certainly indicates that Buridan drew the distinction between logical and real senses of possibility which we can find more explicitly stated in Scotus. In that passage, Buridan further attributes the powers-based, real kind of possibility, to the usage that Aristotle and Averroes made in the *Physics* whenever they speak about potency. As the object of passive or active powers, to call something possible - such as the Antichrist, the example of choice throughout medieval logic of a merely possible person - is to refer to the object of a power to generate that substance. Analogously, something is said to be 'in potency' ('*in potentia*') to being also on account of the passive powers of matter and the active powers of the forms that combine to engender a substance. We can represent Buridan's distinction between these various senses possibility found in (T5.1) in the following diagram below (5.1).

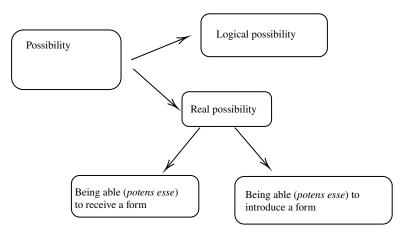


Figure 5.1: Senses of possibility

in potentia ex eo quod potest esse, sicut Antichristum diceremus esse in potentia vel possibilem et quod est in potentia ad Antichristum generari aut forma eius recipi in corpore etc. Et videtur Commentatori et forte Aristoteli quod huius modi potentia vel possibile dicitur secundum attributionem ad potentiam activam vel passiva. Ex eo enim Antichristus est possibilis vel in potentia, quia iam est principium activum potens ipsum vel formam eius producere in esse et quod est subiectum vel materia potens ipsum vel formam eius suscipere; et illud principium activum est Deus et illud subiectum est prima materia. Vel etiam potest dici quod huiusmodi potentia vel possibile dicitur secundum attributionem ad possibilitatem propositionibus attributam. Ex eo enim dicimus Antichristum esse in potentia vel ens in potentia vel possibilem vel quod est in potentia ad esse Antichristum vel ad ipsum fieri etc., quia haec est possibilis 'Antichristus est', vel 'Antichristus fit, etc''.

Given this distinction between logical and non-logical or real senses of possibility, it is relevant to ask if what Buridan says in *QPhys*. I.22 coheres with his analyses of the modal proposition we have seen in section [2.2]. To a large extent, Buridan's concern is not to account for the unity or lack of unity between these usages of possibility. Buridan instead claims that he does not intend to apply the same interpretation of modal propositions in logic and in natural philosophy. Some interpreters have held that Buridan ultimately considered modal concepts to be equivocal between both senses in a similar fashion as Scotus did. In his early work, Knuuttila described Buridan's distinction between varieties of modality as acknowledging that necessity and possibility are equivocal concepts. Recently, Spencer Johnston has argued that Buridan has changed perspectives on this issue over the course of his writings. In particular, Johnston argues that in Buridan's early logical writings we find a notion of logical modality that was based on modal alternatives, whereas in his later writings Buridan noved towards powers-based conceptions.

Buridan's modal semantics does not incorporate the powers-based reading. At the core of Buridan's modal logic, as we have seen, is a uniform treatment of divided modal propositions as ampliated to the possible. Let us recall his famous example of an ampliated proposition given in the TC II.4, "air can be made from water, although this may not be true of any air that exists" (Buridan)

³¹⁵Referring to the logical acceptation of possibility, Buridan writes in *QPhys*. I.22 (Buridan [2015a]), Eds. Streijger and Bakker, 211): "Since we are not dealing at present with such concepts of possibility, I do not intend to discuss whether what was said is generally correct or not" (Et quia de huiusmodi potentia vel possibilitate non intendimus ad praesens, ideo non amplius hic discutio utrum universaliter sit bene dictum illud quod nunc est dictum.") Buridan is referring to the powers-based concepts of possibility at [T5.1], and suggesting that is not discussing in that context whether these remarks concern possibility in other contexts (e.g., in logical contexts).

³¹⁶Knuuttila's reading assumes: "[...] Buridan's acceptance of the idea of equivocity of 'necessity' and his insistence that nomic necessities are unchangeable *ex suppositione communis cursus naturae*'" (Knuuttila [1993], 157). Although Knuuttila's point concerns necessity, he holds a similar view concerning possibility (Knuuttila [1989]). I believe Knuuttila's interpretation is heavily influenced by his reading of Scotus.

³¹⁷In raising the question of how Buridan understand the expressions 'necessary' and 'possible', Johnston argues that Johnston [forthcoming]: "The answer, we contend, is that while Buridan starts off endorsing a temporal analysis of modal propositions, Buridan ultimately settles on an analysis of modality that is grounded in the idea of what an agent can or cannot bring about."

[2015b], 18). The example is not based on the assumption of actually existing powers - namely, the passive power of air to be generated from water when water is heated to a certain temperature - for it refers to merely possible air.

In the *QM* 9.5, Buridan addresses the discrepancy between logical and real senses of possibility as a foremost contextual issue of language use. He believes that powers-based conceptions are more usual in everyday life, but according to their proper meaning (*de virtute sermonis*), modal propositions of possibility refer to merely possible beings, not referring to actual powers, capacities and abilities.

In order to indicate how logical and power-based conceptions of possibility differ, Buridan chooses as example a proposition that is true according to the ampliated reading, but counterintuitive in everyday usages of modal terms. Buridan's example is largely sophismatic - in the sense of providing a perplexing proposition for analysis -, and he chooses to discuss the proposition that "everything someone will bring about, it is possible now for that agent to bring about." It seems this proposition is true on the ampliated reading - as we have seen above in chapter 3.5, possibility broadly considered is indifferent to time, which suggests that the principle that if it will be the case that p, then it is possible that p. But in the case of powers-based reading of modal propositions, this principle is certainly counterintuitive. One of the counterexamples Buridan offers is the following proposition: "it is possible for a baby in the womb of his mother to run fast." This proposition is counterintuitive since the baby

³¹⁸cf. Buridan [2015b], Transl. Read, 18: "Now, in the fourth chapter, it should be realized that a divided proposition of possibility has a subject ampliated by the mode following it to supposit not only for things that exist but also for what can exist even if they do not. Accordingly, it is true that air can be made from water, although this may not be true of any air that exists." Buridan [1976], Ed. Hubien, 58: "Deinde, in quarto capitulo, supponendum est quod propositio diuisa de possibili habet subiectum ampliatum per modum sequentem ipsum ad supponendum non solum pro his quae sunt sed etiam pro his quae possunt esse quamuis non sint. Unde sic est uerum quod aër potest fieri ex aqua, licet hoc non sit uerum de aliquo aëre qui est."

³¹⁹Buridan [1518], 58rb: "In the fifth question, it is asked whether everything someone will bring about, it is possible for them to bring about (*ipse potest facere*), and so on for other ways of speaking about possibility, that is from the fact that something will be made it is possible for it to be made." ["Quaeritur quinto utrum omne quod aliquis faciet ipse potest facere, et sic de aliis modis loquendi quantum ad posse videlicet utrum ex quo aliquid fiet ex ipso illud potest fieri."]

³²⁰According to broad logical possibility, the sentence is true, but it certainly would

in the womb clearly lacks the power to run, and in his reply, he states that that modal discourse (*modi loquendo ad posse*) should be adapted in accordance with conventions of communication in everyday life and in scientific disputation, and appeals to the need to contextually distinguish the senses of propositions, rejecting the modal proposition in accordance with one sense, and conceding it according to another. [321]

Buridan's approach to logical and non-logical acceptations of possibility thus shows his characteristic sensitivity to conventional language. It is nonetheless important to keep in mind that according to their literal meaning (*de virtute sermonis*), modal propositions are ampliated to what merely can be, reflecting a sense of logical possibility, and that powers-based modalities are not considered by Buridan to be an appropriate basis for modal semantics. The passage of the *QSM* 9.5 cited above is the only text I could find in which Buridan appeals to context in order to disambiguate modal propositions, which is not his *ex professo* attitude in the logical writings. Furthermore, as we have seen in

be regarded by everyone as false on the basis of everyday usages of the concept of possibility. Cf. Buridan [1518], 58ra: "[..] the example is posed of whether a baby in the womb of his mother can run fast. It is argued for the affirmative: since they will at some time run, namely when they are a grown adult, and furthermore what is impossible to be done will not be done. If it is not impossible for them to run, it follows that it is possible for them to run." ["Videtur quod sic, et ponitur exemplum utrum infans qui adhuc est in ventre matris potest velociter currere. Arguitur quod sic: quia iste aliquando curret, scilicet quando erit vir perfectus, et tamen impossibile fieri non fiet: ergo non est impossibile ipsum currere, et si non est impossibile ipsum currere sequitur quod ipse potest currere."]

³²¹Cf. Buridan [1518], fol. 57rb: "And it seems to be that the sense of possibility according to proximate powers is more commonly used than possibility according to remote powers, since everyone would agree that is absurd to claim that a child has the same powers (*equipotens*) as an adult, and that she can carry the same weight as an adult; this is so because discourse (*sermones*) only has meaning by conventional imposition, and the imposition can only be known from use." ["Et videtur mihi quod homines magis communiter utuntur illo sensu qui est secundum potentiam propinquam quam illo qui est secundum remotam: quia omnes communiter reputarent absurdum quod infans esset equipotens sicut vir robustus et perfectus, et quod tantum pondus posset levare sicut vir, quia sermones non habent virtutem nisi ex impositione et impositio non potest sciri nisi ex usu."]

³²²The strategy of disambiguating senses of propositions (*propositio est distinguenda*) seems to be quite isolated in Buridan's writings, and Van der Lecq and Braakhuis (Van der Lecq & Braakhuis [1994], 30) have noted argue that Buridan does not generally adopt this strategy in his treatment of fallacies.

chapter 2.2. Buridan explicitly denied that modal propositions are ambiguous in the sense William of Ockham assumed, namely as ambiguous between an ampliated and a non-ampliated sense 323 Despite the differences between the logical analysis and account of powers-based possibility we find in the *Physics*, ultimately Buridan's main account of modality is not predicated on causal powers.

5.2 Becoming and Perishing

In his question-commentary on *De Generatione et Corruptione* Buridan at different occasions addresses, although indirectly, the problem of how powers-based conceptions of possibility function as explanations of the connection between kinds of substantial and accidental changes - namely generation, corruption, and various forms of qualitative alteration. In particular, in *QDGC* I.4 Buridan raises the question of whether the following consequence holds in the common course of nature, namely, "if it is impossible for the elements to undergo generation, it is impossible for them to undergo alteration." The Aristotelian theory of reciprocal changes claims that each of the basic elements change into another, in particular in alterations which involve contrariety; for example, when water is boiled it leads to the generation of new air, for which there must be passive and active powers to explain that change. The main problem addressed in the question concerns the necessary connection (*naturalis*

³²³On Ockham's view that modal propositions are ambiguous between two senses, one in which the subject picks out actual beings, and another in which it picks out merely possible beings, see Priest & Read [1981].

Buridan [2010], Eds. Streijger, Bakker and Thijssen, 56: "In the fourth question it is asked whether if it is impossible for the elements to be generated, it is impossible for the elements to undergo alteration." ["Quaeritur quatro utrum, si impossibile est elementa generari, impossibile est ea alterari".] This question is raised in the same terms by many of Buridan's near contemporaries, in particular by Albert of Saxony [T516]), Marsilius of Inghen (Marsilius of Inghen [T518]), and Nicole Oresme (Oresme (T996)), to which we will briefly turn below. Gensler [2006] provides interesting texts from Walter Burley's corresponding question (643). What motivated the insertion of the question in commentaries on *De Generatione et Corruptione* at that time was that qualitative alteration was thought to be a necessary condition for substantial change. For an approach to the problem of qualitative changes more broadly, see Pasnau [2021].

habitudo) between these kinds of change in the common course of nature, namely between qualititative change - changes involving the accidental properties of things - and substantial change - changes that involve a thing's coming or ceasing to be.

Buridan mentions two lemmas in addressing the goodness of the consequence stated in the question. On the one hand, if substantial generation of air from water were impossible, then the natural end of such alterations cannot be achieved, making them only apparent and not real features of the world. On the other hand, according to the restricted sense of posssibility - namely, the sense in which possibility is intrinsically interwoven with time and the past is necessary -, the same elements cannot be regenerated by natural powers, since, for example, this water cannot reoccur as numerically the same individual. Which concept of possibility is required to address the consequence? Buridan makes clear that there is no logical connection between the concepts of alteration and generation, 326 It seems the question should be evaluated by a layer of validity according to natural necessity, and the candidate descriptions thereof lead us back to the interpretation of the broad and restricted (or accidental) modalities that we have seen in section 3.5. In particular, at the QDGC question at hand, Buridan further expands on the broad and restricted modal concepts that we have seen in (T3.3) (Buridan [2010], Eds. Streijger, Bakker and Thijssen, 58). Recall that in a broad sense, possibility is indifferently related to time,

³²⁵ This holds from the *locus* that if the end is impossible, the means ordered towards the end are impossible as well. In this case, generation is the natural end of alterations involving contrariety. Cf. Buridan [2010], Eds. Streijger, Bakker and Thijssen, 60:"To the first argument, when asked from which *locus*, it is said that proceeding from generation to alteration the *locus* at hand concerns final causes to their effects, in saying that 'if it is impossible for the elements to be generated, it is impossible for them to undergo alteration.' And the maxim is that if the end is not possible, the means ordered towards the end are not possible as well." ["Ad primam rationem, quando quaerebatur unde locus, respondetur quod si procedatur de generatione ad alterationem, locus est a causa finali ad effectum suum, ut dicendo 'impossibile est elementa generari, igitur impossibile est ea alterari.' Et est maxima quod si finis non est possibilis, ordinatum in finem non est possibile."]

³²⁶Buridan [2010], Eds. Streijger, Bakker and Thijssen, 60: "Ad secundam, quando dicitur 'si unum potest esse sine altero, non oportet, si illud alterum est impossibile, quod primum sit impossibile,' ego dico quod hoc est verum, nisi illa habeant ad invicem naturalem habitudinem. Sed si habeant habitudinem ad invicem, sicut finis et ordinatum ad finem, dico quod si unum est impossibile, quod alterum est impossibile."

whereas in a restricted sense it concerns only the present or the future, while the past is necessary. It might be useful to briefly recall the distinction:³²⁷

- 1 Broad (*ample*) possibility: something is broadly possible if it either is, was, or will be the case.
- 2 Restricted (*restrictive*) possibility: something is restrictively possible if either is or will be the case.

These two senses are used to articulate the answer to the question raised at *QDGC* I.4. Buridan claims here that the necessary connection between alteration and generation holds in the common course of nature if we take the broad sense of possibility and necessity, and therefore the consequence stated in the main question is valid in that sense. On the other hand, according to the restricted sense, powers are intrinsically future-directed, and the consequence should therefore be denied, since for possibility taken in the restricted the same individual cannot reoccur as numerically the same given the necessity of the past. The relevant point of drawing this distinction in a question concerning

³²⁷For a more detailed discussion of the broad and restricted senses, see again section

Buridan [2010], Eds. Streijger, Bakker and Thijssen, 58: "Note that if we speak of possibility according to the first way, I say that the four elements are generable and corruptible, that it is possible for them to undergo generation and corruption, since according to the first way something is possible which at some time was, is, or will be, and at some time these elements were generated; therefore, their generation is possible. But if 'possibility' is understood in the second way, I say that the elements which now exist are no longer generable, and it is impossible for them to undergo generation, since what is already completely generated cannot be generated again in the future; therefore those things are impossible to generate insofar as possibility is taken determinately for the future." ["Nota. Si loquamur de possibilitate primo modo, dico quod quattuor elementa, etiam quae modo sunt de facto, sunt generabilia et corruptibilia, ita quod sunt possibilia generari et corrumpi, quia illo modo primo dicitur aliquid possibile quod aliquando fuit, est vel erit; modo aliquando haec elementa generabantur; igitur possibilis est eorum generatio, et sic vocantur generabilia. Sed si capitur 'possibile' secundo modo, dico quod illa elementa quae nunc sunt, non sunt generabilia, immo impossibilia generari, quia quod iam complete genitum est, non potest in futurum amplius generari; ideo tale dicitur impossibile generari prout possibilitas respecit determinate futurum tempus."] See also, referring to the restricted sense see DC I.26, Buridan [1996], Ed. Patar, 376)

³²⁹Buridan [2010], Eds. Streijger, Bakker and Thijssen, 58: "To the question I answer that if possibility is taken in the second way, this consequence is not good, namely

the necessary connection between types of changes is to show that in the sense of historical modalities powers are intrinsically interwoven with time, and there is no power over the past. For that reason, in Buridan's modal theory, they are not regarded as useful modal concepts for the purpose of explaining change.

These arguments, it should be noted, concern only natural possibility. Can a numerically identical individual be recreated according to the supernatural sense of possibility? In *QDGC* I.24, Buridan's answer to that question is positive. Although Buridan's positive answer places a limitation on the necessity of the past, he believes that God's power to recreate the very same individual once it is already corrupted must be assumed in order to preserve omnipotence. After all, if the past is necessary in a strong sense and God cannot make the same individuals reoccur, it seems his power would diminish as time passes.

Buridan claims in this context that arguing for the possibility of regeneration of an individual after its corruption involves the mixture of issues pertaining to theology and to natural philosophy (*miscendo theologiam et philosophiam naturalem*), and he refrains from settling the matter. Buridan states, however, that since according to the standard opinion held by the faithful (*sicut ponuntur fideles*), God knows no difference between past and future, a positive answer to

^{&#}x27;these elements cannot be generated, therefore they cannot undergo generation,' since the antecedent is true, as was previously said, but the consequent is nevertheless false; therefore, the consequence is not valid. Therefore it seems Aristotle in this context takens possibility in the first acceptation, in the sense that things which are already generated can nevertheless be called generable." ["Ad quaesitum dico quod si capitur possibilitas secundo modo, non est bona consequentia 'haec elementa non possunt generari, igitur haec elementa non possunt alterari,' quia antecedens est verum, sicut dictum est, et tamen consequens est falsum; igitur consequentia non valet. Et ideo videtur quod Aristoteles in hoc loco supponit possibilitatem primo modo, prout est verum quod illae quae sunt genita possunt vocari generabilia."]

³³⁰Buridan [2010], Eds. Streijger, Bakker and Thijssen, 180: "If [the same individual] could not be recreated as numerically the same, it would follow that God's power would diminish at each day. The consequent is false. And the consequence is proved since God could yesterday create many things that he has created, and which now have ceased to be; and if he cannot create them any longer (if they cannot be recreated as numerically the same), therefore he could do many things in the past which he now cannot; and thus his power is diminished." ["Si non posset idem numero reverti, sequeretur quod potentia Dei quolibet die diminueretur. Consequens falsum. Et consequentia probatur quia: Deus heri poterat multa creare quae creavit et iam corrupta sunt; et si non potest amplius ea creare (si non possunt reverti eadem numero), ideo multa poterat quae nunc non potest; et sic potentia sua in hoc est diminuta."]

this question should be held as the most probable. [331]

It is worthwhile to briefly compare Buridan's reflection on modal concepts at *QDGC* I.4 with other descriptions of modalities in the same context. In the corresponding questions of his near contemporaries, we find a similar classification of kinds of necessity and possibility. I will briefly discuss Nicole Oresme's own version, since Oresme presents a sixfold division of necessity identical to the ones provided by other figures in Buridan's milieu. In his question 4, book I, Oresme presents a theory of degrees of necessity based on temporal permanence of states of affairs. Oresme writes (Oresme [1996], Ed. Caroti):

(T5.4) There is a second distinction, since 'necessary' is said according to six ways. In the first way according to continuous eternity each way, and such is 'that God exists' is necessary. In that way, necessity means as it were what has unceasing being (non cessans esse). In the second way 'necessity' means continuous eternity only in some

Buridan [2010], Eds. Streijger, Bakker and Thijssen, 181-2: "Another conclusion which I posit as probable is that nothing prevents that what is absolutely corrupted can be recreated supernaturally, that is the absolute power of God, since as it was said before, if everything were now annihilated by God except himself, things would be totally the same as they were before the creation of the world, and in no respect different. And whatever God then understood, now he would understand things as altogether the same as before. Whence, as the faithful believe, God does not understand any difference between past and future. Since God acts by his intellect and will, and neither himself nor things would be different then, nothing seems to prevent that he can make everything which he has already created, ["Aliam conclusionem pono mihi probabilem quod nihil prohibet quod simpliciter corruptum possit reverti idem numero supernaturaliter, scilicet per absolutam Dei potentiam, quia, sicut prius dictum est, si omnia essent nunc annihilata praeter Deum, totaliter esset ita sicut erat ante creationem mundi et nullo modo aliter; et quaecumque Deus tunc intelligebat, ista omnia nunc ipse intelligeret et omnino similiter sicut ipsa vel ea ante intelligebat. unde, sicut ponunt fideles, non aliter intelligit Deus praeteritum quam futurum. Cum igitur agat Deus per intellectum et voluntatem et nuc ipse nec alia aliter se habent quam se tunc habebant, nihil videtur prohibere quin potest omnia facere ea quae fecit, et non solum similia, sed eadem, sic quia omnia sunt in potestate sua sicut tunc, cum non aliter se habeant, ut dictum est, et cum eodem modo intelligit omnia sicut tunc, et potestas sua non est nisi intellectus et voluntas."]

³³²The same question with virtually identical formulations (*'utrum si generatio esset impossibilis alteratio esset impossibilis'*) is also raised by Marsilius of Inghen at Marsilius of Inghen [1518], I.4, fol. 68rbff.; Albert of Saxony [1516] I.3 and Oresme [1996], Ed. Caroti, I.3, p.17ff.

direction, for example afterwards, and in that way 'that Socrates existed' is necessary, or before and that way 'that the sun will be eclipsed tomorrow' is necessary. And this kind of necessity can change into impossibility. In the third way 'necessity' means interrupted eternality, in which some singular event is inevitable, as is the eclipse of the moon; and Aristotle called those necessary. In the forth way necessities are also interrupted, but their singular events are contingent, and in that way generation is necessary, since generation is not continuous, but nonetheless it was eternally true in the past that there is generation, even though no generation happens by necessity. In the fifth way necessity means some singular event which exists only once, but which is nonetheless inevitable, as is some constellation. In the sixth way 'necessity' means what was once a future contingent, whenever is actual, it is necessary; for example that Socrates exists, at the time in which he exists, is necessary.

To be sure, Oresme's reply to the main question is not different from Buridan's. He states that the consequence affirming that if generation is impossible, alteration is also impossible, although not a formal consequence, is valid on the supposition of the common course of nature. However,

^{333&}quot;Tunc est secunda distinctio quod sex modis dicitur 'necessarium'. Primo modo dicitur de eternitate continua ab utraque parte, et sic Deum esse est necessarium; et sic dicitur 'necessarium' quasi 'non cessans esse'. Secundo modo dicitur 'necessarium' de continua eternitate tantum ab una parte, ut a parte post, et sic Sortem fuisse dicitur necessarium; aut a parte ante, et sic solem eclypsari crastina die fuit necessarium. Et tale necessarium potest bene transire in impossibile. Tertio modo di citur 'necessarium' eternaliter sed intercise, cuius quodlibet singulare inevitabiliter venit, sicut hoc quod est lunam eclypsari; et hoc etiam verum est secundum Aristotelem. Quarto modo dicitur sicut prius, scilicet intercise, sed quodlibet singulare evenit contingenter, et ita generationem esse est necessarium, quia non semper continue est generatio, et tamen ante infinitum tempus fuit necessarium generationem esse, et tamen nulla generatio fit necessario. Quinto modo est necessarium sicut aliquod singulare quod est solum una hora, et tamen inevitabiliter evenit, sicut est aliqua constellatio. Sexto modo dicitur 'necessarium' quod futurum erat contingenter, sed, quia ponitur in esse, ideo tunc, quando est, est necessarium, sicut Sortem esse, quando est, necessarium est esse."

³³⁴Oresme [1996], Ed. Caroti, 21-22: "To the first, where it is said that these changes are distinct from each other, I concede, and therefore the consequence from one to another is not a formal consequence. But the consequence is valid assuming the order of nature, in which alteration happens for the sake of generation." ["Ad primam,

the conception of modality which Oresme offers in that question differs from Buridan's in important ways. In particular, Oresme appropriates a distinction between types of necessity in terms of temporal duration, ³³⁵ In that conception, the necessary is defined as what never fails to be, understanding that necessity applies primarily to eternal states of affairs, and Oresme offers a tentative terminological explication of the term 'necessity' as 'what has unceasing being' (sic dicitur necessarium quasi 'non cessans esse'; Oresme [1996], Ed. Caroti, 19.68-69). At the highest degree of necessity are those states of affairs that always persist. Secondly, Oresme mentions forwards and backwards-looking eternities, namely as states of affairs which are past or future tensed, such as that the sun will be eclipsed. The natural regularities obtaining in the common course of nature are understood as interrupted necessities (intercisae). In his introduction to Oresme's commentary to De Generatione et Corruptione, Stefano Caroti has argued that this shift in the conception of degrees of necessity was introduced by Oresme and incorporated in the commentaries to DGC by his near contemporaries. 336

In this section, my goal was to analyse Buridan's application of modal concepts in addressing the necessary connection between different kinds of change. In brief conclusion, I would like to call attention to how Buridan's application of the modal distinctions in the *QDGC* I.4 does not make explicit appeal to temporal-frequency definitions of modal terms. By contrast, the distinction between necessities as measured by temporal duration, which we have seen in Oresme's text above as a convenient example, is explicitly based

cum dicitur quod sunt mutationes distincte etc., concedo, et ideo non est formalis consequentia de una in aliam, sed consequentia valet supposito ordine nature, et tamen alteratio fit propter generationem."]

³³⁵The same sixfold division is also given by Marsilius of Inghen, cf. Marsilius of Inghen [1518], 68rb.

³⁵⁶ See Stefano Caroti's Introduction to Oresme [1996], Ed. Caroti, p.94: "Obwohl im Kommentar Buridans das Interesse semantischer Art im Vordergrund steht, finden wir in *quaestio* I. q.4 einen Hinweis auf die Beziehung zwischen Entstehen und Vergehen ein Thema, das für Oresme, Albert und Marsilius von erstrangigem Interesse ist, wobei bei letzterem der generellen Differenzierung der Bedeutung von 'necessarium' mehr Raum gewidmet ist. Est handelt sich, wie leicht ersichtlich is, um einen Wandel, der mit dem Kommentar von Oresme einsetzt und auch von Albert und Marsilius akzeptiert ist." I cannot add here further support for Caroti's claim, but it is relevant to note that other commentaries on the *DGC* also contain a similar view on degrees of necessity.

on a temporal understanding of modalities. In this regard, the temporal modalities discussed by Oresme at (T5.4) significantly differs from Buridan's own conception of degrees of necessity, which we have seen above in Table 3.1

6. Natural Contingency and Chance

This chapter addresses Buridan's understanding of contingency in logical and physical contexts. I will compare his remarks on two types of contingency, 'eachway contingency' and the contingency of natural events, with a focus on *QPhys*. I.11 on chance. According to Buridan's logical description of contingency, treated in section [6.1], contingency properly speaking is a mode that excludes both necessity and impossibility, then called 'each-way contingency' (contingens ad utrumlibet). This description singles it out from a further classification of natural contingencies that come about for the most part (contingens ut in pluribus) or only rarely (contingens ut raro). Although absent from Buridan's logical writings, the import of natural contingencies for the logic of contingency propositions is drawn by Robert Kilwardby in his commentary on the *Prior* Analytics. This is meant to set up the background to the physical question concerning the place of chance among the above-mentioned kinds of contingency, raised by Buridan in *QPhys*. II.11, ³³⁷ Buridan's question is largely remissive to what he describes as a famous controversy stemming from Avicenna's († 1037) and Averroes' († 1198) views on the same problem. I examine Buridan's setting of the problem and his own take on it in section 6.2 against that background. 338

6.1 Each-Way and Natural Contingencies

In his logical writings, Buridan espouses a common analysis of contingency as a species of the possible that excludes necessity and impossibility. This formula defines the contingent strictly speaking, that is the sense of contingency proper

³³⁷Sylla [2015], clxx-xlxxiv.

³³⁸ Besides Sylla's treatment in the Guide to the Text in Sylla [2015], Maier [1949] and Knebel [2006] have touched on Buridan's approach to this issue.

to logical matters. In the chapters of the *Summulae de Dialecticae* dedicated to modal propositions, Buridan writes (*SD* 1.8.5; Buridan [2001b], Transl. Klima, 76):

(T6.1) [...] as far as 'contingent' is concerned, we should realize that sometimes 'contingent' is taken broadly, and then it is synonymous with 'possible', and sometimes it is taken strictly, and this we call 'each-way contingent' (*contingens ad utrumlibet*), and then it is a species of 'possible' distinguished from 'necessary.' For 'possible' is common both to 'necessary' and 'contingent,' because everything necessary is able to be and is never able not to be, and everything contingent is able to be and is also able not to be.

In that passage, contingency strictly speaking - or 'each-way contingency' (*contingens ad utrumlibet*) - is distinguished from a broad acceptation of contingency that is synonymous with possibility, frequently called 'one-way contingency' in the literature. Although terminologically the distinction may appear confusing, its thrust is that whereas one-way contingency is defined solely by negating the impossible, each-way contingency is defined by negating both necessity and impossibility. The terminological oddness reflects an historical usage. In Aristotle's writings on modal syllogistics, the same words are indistinguishably used for both possibility and contingency. In his early Latin translator Marius Victorinus, as well as in Boethius' corresponding commentaries, the Latin term *contingens* was in turn employed for both, and that usage continued well into the twelfth and thirteenth centuries in Latin scholastic logic. At 2

³³⁹ Buridan [2005], Ed. Van der Lecq, 92: "De contingente autem sciendum est quod aliqundo 'contingens' accipitur large, et tunc synonyme se habet cum 'possibile,' et aliquando accipitur stricte, quod vocamus 'contingens ad utrumlibet', et est species possibilis distincta contra necessarium. Possibile est enim commune ad necessarium et ad contingens. Omne enim necessarium est possibile esse et numquam possibile non esse, et omne contingens est possibile esse et cum hoc possibile non esse".

³⁴⁰That is to say, each-way contingency is expressed formally negating a necessity and an impossibility, as $\neg Lp \land \neg L \neg p$, which is an equivalent of $M \neg p \land Mp$. On the other hand, one-way contingency - which is in effect synonymous with possibility -, is defined solely by negating the impossible, namely as $Mp \leftrightarrow \neg L \neg p$.

³⁴¹Cf. Knuuttila [1993], 106-7.

³⁴²For some examples, see again Knuuttila [1993], 107.

In Buridan's modal syllogistics, only each-way contingency forms the basis for the analysis of contingency propositions and of modal syllogisms containing them ³⁴³ Accordingly, Buridan analyses each-way contingency by noting that a divided proposition of contingency entails both an affirmative and a negative proposition of possibility, that is 'S is contingently P' entails both 'S possibly is P' and 'S possibly is not P,' since from the definition of contingency as "what can be that way and can be another way" (Buridan [2015b], Transl. Read, 104). ³⁴⁴ it becomes clear that (Buridan [2015b], Transl. Read, 104): "[...] from every proposition of contingency there follow both an affirmative and a negative proposition of possibility. So it is well said that contingency excludes necessity and impossibility."

When it comes to the interpretation of contingency on the basis of these logical definitions, Buridan has in mind the fact that each-way contingencies are indifferently related to the positive and negative conjuncts in its *definiens*, or indifferently related to 'being and not being so.' The setup of Aristotle's original formulation of the distinction we saw above at [T6.1] adds however a further complication to this analysis. In *An. Pr.* I.13 32b4-14 it is spelled out in the following way (keeping in mind that in Aristotle's terminology, both are rendered as kinds of possibility), namely (Aristotle [2009], Transl. Striker, 19):

(T6.2) After these explanations, let us add that 'being possible' is said in two ways: in one way of what happens for the most part, when the necessity has gaps, such as that a man turns grey or grows or ages,

³⁴³As Stephen Read notes whereas Aristotle's modal syllogistics does not make a systematic distinction between both, Buridan's account does. Read [2015a], 2: "Unfortunately, even though Aristotle has two words for 'possible' (dunaton and endexesthai), he uses both in both senses, often but not always noting whether he means possible in the weaker sense (not impossible) or stronger (neither impossible nor necessary). This equivocation runs right through Aristotle's discussion of the modal syllogism. As a matter of fact, he only takes possibility premises in the stronger sense (contingency), but often considers possibility conclusions in the weaker sense."

³⁴⁴Buridan [1976], Ed. Hubien 68: "Eo enim dicitur 'contingens' quia potest sic esse et potest sic non esse."

Buridan [1976], Ed. Hubien, 68: "Manifestum est ergo quod ad omnem propositionem de contingenti habentem modum affirmatum sequitur propositio de possibili tam affimatiua quam negatiua. Ideo bene dicitur quod contingend excludit necessarium et impossibile." See futher Buridan [2015b], Transl. Read, 157, and the comments on Lagerlund [2000], 141, ft. 38 and Read [2015a].

or generally what belongs by nature. For this has no continuous necessity because a man does not exist forever, but while a man exists, it happens either of necessity or for the most part. In another way 'being possible' is said of what is indeterminate, that is, what is possible both this way and not this way, such as that an animal walks or that an earthquake happens while it walks, or, generally, what comes about by chance, for this is by nature no more this way than the opposite way.

Here, instead of being indifferently related to the positive and negative parts of its *definiens*, the concept of contingency is illustrated by using examples about things which are naturally more one way than another, in contrast to things that come about merely by chance. His example that a human being turns grey haired or ages is a contingency that happens for the most part or by nature, in the sense that it is inevitable provided they reach up to age and do not die abruptly, but it is nonetheless a contingency statement. On the other hand, that an earthquake happens while an animal walks happens merely by chance. Aristotle's statement of contingency in that passage relies on a further distinction he makes in the *Physics* elsewhere between statements concerning what happens 'by nature or for the most part' from statements about what happens 'rarely' [346]

To draw the distinction between both kinds of contingency, Gisela Striker has pointed out to the formulation of natural contingencies above as 'necessity with gaps' for what happens most often or by nature, in contrast to statements about what happens by chance ³⁴⁷ The examples provide a different conception of contingency in relation to the frequency of events, in the sense that whereas chance events are contingent because "it is no more this way than that" (32b16-19; Aristotle [2009], Transl. Striker, 19), natural contingencies are exceptionable generalities, but which are necessarily satisfied when certain conditions are fulfilled.

³⁴⁶For example, most notably in his discussion of chance in his *Physics* II.5 196b10-1 (Aristotle [1995a], Ed. Barnes, Transl. Hardie and Gaye, 742. See Judson [1991], 82ff and in particular ft. 23 for a list of these uses across Aristotle's writings).

Striker [2022], 29: "That the necessity has gaps can be explained, seemingly, by the fact that it does not extend to all specimens of the human species—some people do not grow old because they die too young. And the gaps arise because certain conditions must be satisfied that are not satisfied in all cases."

Whereas Buridan did not incorporate the conception of contingency suggested in that passage into his modal syllogistics. Robert Kilwardby († 1279) has taken Aristotle's remarks at 6.2 more seriously in his commentary on the *Prior Analytics*. On Kilwardby's interpretation of Aristotle, the examples above illustrate a further mode of contingency that is distinct from each-way contingencies. In his view, contingencies which happen by nature or for the most part, such as human beings going grey haired, are at bottom a form of interrupted necessity, since the tendency of becoming grey haired belongs properly to all human beings even though it has exceptions (Kilwardby [2015], Eds. Thom and Scott, Trasl. Thom, 395):

(T6.3) Next there is a question about this statement that when there are men, they necessarily or mostly go grey. For there is a doubt how this is true, since men only go grey in old age. And it should be said that 'the act of going grey' can be speak the process of going grey, or the end-point of the process. If it be speaks the process of going grey then when there are men they are always and of necessity going grey. For greyness comes from the incorporation of phlegm into the upper part of the head - which incorporation is caused by the diminution of natural heat, and this incorporation and diminution of heat is always going on without interruption. But if it be speaks the end-point of the completed process, then it is often the case that men go grey. For greyness comes to many if they last until old age.

³⁴⁸The only passage I could find where the same example does appear is in a list of several senses of property (*proprium*) in the *Summulae de Dialecticae*, at Buridan [2001b], Transl. Klima, 125: "[...] in the third way it [namely, a *proprium*] is that which applies to all and only [those things that are contained under a species] but not always, as to become white-haired applies to man." Cf. Buridan [1994], Ed. Bos, 42: "Terio modo quod inest omni et soli, sed non semper, ut homini canescere."

Kilwardby [2015], Eds. Thom and Scott, 394: "Consequenter queritur de hoc quod dicit quod homo cum est aut ex necessitate aut frequenter canescit. Dubium enim est quomodo hoc sit uerum cum homo non canescat ut in pluribus nisi in senectude. Et dicendum quod iste 'actus canescere' potest dicere motum in canitem uel terminum motus. Si dicat motum in canitem semper ex necessitate canescit homo cum est. Prouenit enum canities ex incorporatione fleumatis in superiori parte capitis, cuius incorporationis causa est diminutio caloris naturalis, et ista incorporatio et caloris diminutio semper fit et continue. Si autem dicat terminum motus completum sic ut frequenter canescit homo. In pluribus enim si maneante usque ad etatem prouenit

In Paul Thom's comments on this passage, he notes that the conceptual difference these examples illustrate concerns two subkinds of contingency, since Thom [2007], 40: "Among non-necessary contingent propositions, Kilwardby distinguishes indeterminate from natural contingencies; the difference is that in a natural contingency-proposition one the paired possibility-statements states not just what is possible but natural." In particular, the example involving the natural propensity of humans to become grey haired is understood as a statement of contingencies that happen by nature or for the most part. In contrast with contingencies that are indifferently related to being and non-being, the natural contingency involved in that example is "underpinned by a natural law, namely a natural movement towards greyness with age" (Thom [2007], 33).

The interpretation of this part of the *Prior Analytics* raises the problem of the relationship between modal concepts and the above-mentioned frequency judgments. What is Buridan's position on the relationship between the two? Whereas the distinction between natural and each-way contingency is absent from Buridan's modal syllogistics, in his commentary on the *Physics*, this distinction is treated in the set of questions about chance (*casus*). In particular, *QPhys*. II.11 was devoted to the issue of whether chance events belong to the category of each-way contingencies. This question concerns more directly the second mode of contingency mentioned by Aristotle - namely, those that happen by chance -, but it raises more general questions about the relationship of modal concepts with the frequency classification of events put forward by Aristotle in that context.

6.2 A Controversy over Chance

Buridan approaches the relationship between each-way and natural contingencies in his commentary on the second book of the *Physics*, question 11, 'Whether chance and fortune are found in what is equally likely to occur or not to occur (*contingentibus ad utrumlibet*), or only in contingencies that rarely occur or less

canities."

 $^{^{350}}$ I am using here 'each-way contingency' instead of 'indeterminate contingencies' for the sake of uniformity.

often than not. This text is part of a broader set of questions dedicated to the problem of chance, and in particular the problem of whether events that are described as happening by chance - such as the example we saw above in Aristotle's *Prior Analytics* in (T6.2) - fall under the category of each-way contingencies.

The question, as Buridan says, is made difficult because of a famous disagreement ('famosa controversia') between the views of Avicenna and Averroes on this issue. Its starting point is Aristotle's threefold classification of frequency judgments in the *Physics* between things which come about always and by nature, for the most part, and rarely. This threefold distinction is important in a number of issues in Aristotle's philosophy. ³⁵² but in the specific context Buridan is commenting on in *QSP* II.11 the main import of the distinction involves classifying types of causes and posing questions about the causal role of chance events. ³⁵³ To these modes of contingency the medievals added the category of each-way contingency, which was absent from Aristotle's terminology in this context. ³⁵⁴ and posed the question of whether chance events

³⁵¹Cf. Buridan [2015a], Eds. Streijger and Bakker, 322-329, 'Utrum casus et fortuna reperiantur in contingentibus ad utrumlibet vel solum in congentibus raro vel ut in paucioribus.' These correspond mainly to Aristotle's remarks on chance raised at *Physics*, starting at 195b31-6 (cf. Aristotle [1995a], Ed. Barnes, Transl. Hardie and Gayer, 740ff.). Edith Sylla translates each-way contingency as 'what is equally likely to occur or not to occur' in the question title. Cf. Sylla [2015], clxx. Her choice follows Annaeliese Maier's understanding that these questions hinge on the concept of effects that, under the appropriate conditions, are equally likely to come about or not [Maier [1949], 224). I will follow this translation at some points, but it is relevant recall that the expression originally used is 'each-way contingency' (contingens ad utrumlibet).

³⁵²See Judson [1991], 82-89 for a comprehensive analysis of this classification.

³⁵³See e.g. *Physics* 196b10-17 (Aristotle [1995a], Ed. Barnes, Transl. Hardie and Gaye,742): "First then we observe that some things always come to pass in the same way, and others for the most part. It is clearly of neither of these that chance, or the result of chance, is said to be the cause—neither of that which is by necessity and always, nor of that which is for the most part. But as there is a third class of events besides these two—events which all say are by chance—it is plain that there is such a thing as chance and spontaneity; for we know that things of this kind are due to chance and that things due to chance are of this kind." Aristotle defines chance in *Physics* 196b23 as what comes to be incidentally for the sake of something. The second condition is given since the unintentional outcome of chance events are described as good (or bad) luck in relation to a purpose, as for example accidentally encountering a debitor on the market, or finding treasure while digging for some other purpose.

³⁵⁴Knebel notes that the corresponding Greek term (τὸ ἐπ' ἴσης ἐνδεχόμενον) can

belong solely to the category of what happens rarely (as they thought was Aristotle's view), or also to the category of the each-way contingent.

To a large extent, in the *QSP* II.11 Buridan's concern is the relationship between the modal concept of contingency and these categories of frequency judgments. The view Buridan opposes, which he associates with Averroes, states that chance only belongs to the class of events that rarely happen (*contingens ut raro*) as an exception to a general rule Instead, Buridan claims that he prefers Avicenna's view, according to which chance belongs to what is equally likely to occur or not to occur, and therefore to the class of each-way contingencies.

The main issue of this debate of relevance for my present purpose concerns the descriptions of necessity and contingency that underlie the Avicennian and Averroan answers to this problem according to Buridan's interpretation. It will be important to briefly describe each of these positions on the relationship between chance, frequency judgments, and the place of the each-way contingent in this classification. For Buridan, Avicenna understood the difference between causes which always bring about their effects and causes which only sometimes do in terms of the presence or absence of competing factors hindering their effects. But according to Averroes' objection, explaining necessity and contingency by reference to those competing impediments as Avicenna did eventually collapses necessity and contingency. Buridan recites Averroes' objection in the following way (Buridan [2015a], Eds. Streijger and Bakker, 325):

(T6.4) The second conclusion [i.e., Averroes'] is that Avicenna defines the necessary and the contingent ineptly and badly, when he operates by the presence or absence of an extrinsic impediment. Since if what is called contingent is so called because it has an extrinsic impediment, it would follow that every contingent thing is naturally necessary, which is false. The consequence is evident, since a thing should be said such as it naturally is from its intrinsic nature, abstracting away from the extrinsic factors which do not belong to that nature; but abstracting from the extrinsic factors there would be no such extrinsic impediment and therefore it would be necessary according

be found in Ammonius' (c. 435/445-517/526) commentary on *De Interpretatione*, cf. Knebel [2006], 667.

³⁵⁵Cf. Averroes [1562b], fol. 66vb. See also Knebel [2006], 665; Maier [1949], 224.

This Averroan positon that Buridan recites here plays an important role in the remainder of QSP II.11. According to Catarina Belo, on "Avicenna's interpretation, an event happens always unless hindered by an external obstacle. Averroes in turn places the emphasis on the subject and its intrinsic nature" (Belo [2007], 151-2). It is clear from the passage above that Buridan understands Averroes in a similar way. In the view he associates with Averroes, what is naturally necessary or contingent for some thing is a matter of how that causal operation is compatible or not with the nature at hand, since being intrinsic to something is seen by Averroes as the very point of saying that an event comes about naturally given certain conditions. In that view, causal necessity and contingency should not be explained on the basis of anything other than the operations of the natures or substances involved, which might either contain or lack any possibility of deficiency. Averroes' original argument goes as follows (Averroes [1562b], 66vb; trans. Belo [2007], 151):

(T6.5) Therefore, actions which are not necessary, nor contingent for the most part, necessarily come to be rarely. According to this, chance is not in the equally contingent [...] You ought to know that the difference between the contingent for the most part and the necessary is not that the contingent for the most part has an impediment on rare occasions, and the necessary does not have an impediment, as Avicenna says. For that implies the view that everything is naturally (naturaliter) necessary. Rather the contingent which is for the most part is that which has in its nature the possibility that its action may fail (deficiat) on rare occasions: and therefore an external impediment is found therein. The necessary, however, because it does not have

³⁵⁶ Secunda conclusio est quod Avicenna inepte et male definit necessarium et contingens, scilicet per non habere vel habere impedimentum extrinsecum. Nam si ex hoc diceretur contingens quod habet impedimentum extrinsecum, sequeretur quod omne contingens esset naturaliter necessarium, quod est falsum. Consequentia patet, quia tale debet dici unumquodque, quale esset naturaliter ex intrinseca natura sua circumscriptis extrinsecis quae non sunt de natura sua; sed circumscriptis extrinsecis non esset impedimentum extrinsecum et sic esset necessarium secundum naturam; igitur omnia essent naturaliter necessaria."

The classification of frequency judgments was meant to subdivide types of contingency in Averroes' theory. 358 and the key point Averroes makes against Avicenna is thus that the presence or absence of extrinsic impediments should not be the defining feature of causal necessity and contingency. On the other hand, Avicenna's strategy exhibits a reversal of explanatory priorities; the difference between causes that always bring about their effects and causes which only sometimes do is explained in terms of the presence or absence of contrary or prohibitive factors (Avicenna [2007], Ed. Van Riet, 111.85-91):

(T6.6) This is the difference between what always is and what frequently (*saepe*) is, namely that which always is has no contrary impediment (whereas to that frequently is sometimes has contrary impediment). From that it follows that if contrary and prohibitive impediments are removed from that which frequently is, the same thing can be necessary. This is manifest in natural things.

Buridan likely had in mind this formulation at (T6.4). It shows that Avicenna embraces the corollary which Averroes criticized him for, namely that in some sense every contingent cause which is for the most part prevented from being actualized is possibly necessary, as we can easily isolate in imagination that

³⁵⁷Averroes [1562b] fol. 66vb): "Et debes scire quod differentia inter contingens ut in pluribus et necessarium non est quod contingens ut in pluribus habet impedimentum ut in paucioribus, et necessarium non habet impedimentum, ut dicit Avicennam; secundum hoc sequitur, ut omnia sint naturaliter necessario. Immo contingens ut in pluribus est illud, in cuius natura est possibilitas ut actio eius deficiat in minori parte, et immo invenitur illic impedimentum extrinsecum. Necessarium vero quia non habet hoc in sua natura, ideo non invenitur impedimentum illi extrinsecum."

³⁵⁸See e.g. also Averroes [1562b], 66E, transl. in Belo [2007], 125: "That which is possible is divided into that which is possible for the most part, and into that which is not possible for the most part [...] Those things which come to be for the most part are not said to come to be (*fieri*) by chance (*casus*). Therefore, if chance and the spontaneous (*ex se*) are something, they must exist in things which are not possible for the most part."

³⁵⁹"Et haec est differentia inter semper et saepe, eo quod ei quod est semper non adversatur aliquod contrarium <et ei quod st saepe adversatur aliquod contrarium>. Unde sequitur ut quod est saepe, condicione removendi contraria et prohibentia, fiat necessarium. Et hoc in <rebus> naturalibus manifestum est."

causal nature from those competing factors hindering its actualization for the most part. One of the examples Avicenna offers which concern natural causes - that is, non-voluntary agents -, is that of a stone falling and hitting someone in the head. The falling of the stone happens by an essential or intrinsic cause, namely the tendency of heavy objects to reach their natural place. However, in relation to an accidental goal, it is a cause by chance of hitting a passerby in the head.

These two views Buridan rehearses reflect different conceptions of causal necessity and contingency. Simo Knuuttila interpreted the classification of causal necessity and contingency in terms of the frequency of events as evidence for a temporal-frequency interpretation of modalities. Knuuttila holds that this modal classification of causes was prevalent in the thirteenth century, mentioning figures such as Siger of Brabant and Thomas Aquinas in support. Here, the "modal status of the cause is explicitly defined statistically" (Knuuttila [1981], 209) with regards to their possible hindrances, and each-way contingencies makes reference to things which are equally likely to come about or not (*ad utrumlibet*). [362]

In his analyses of thirteenth century notions of causal necessity and possibility, Knuuttila has rather focused on authors adopting Averroes' views such as Siger

³⁶⁰For an in depth discussion of Avicenna's position and more relevant passages, see again Belo [2007], 25-38

³⁶¹The relevant passage here, from Avicenna's *al-Samā' al-abī'ī* is translated in Belo [2007], 36: "If a falling stone breaks [someone's] head, it may stop or it may fall to its natura places. If it reaches its natural goal, this is in relation to it an essential cause and in relation to the accidental goal it is a cause by chance. If it does not reach its natural place, the stones falling is a cause by chance in relation to the accidental goal, and it is thwarted in relation to the essential goal."

Knuuttila [1981], 209: "According to Averroes, Avicenna's causal principle must be understood so that the effect is necessary when the cause is not impeded from producing the effect. And the central interest of the thirteenth century theory of causality concerned the classification of causes with regard to their possible hindrance. [...] A necessary cause is one which is never impeded from producing the effect when it is in the state of a cause. It is called *causa ut semper*. We see that the modal character of the cause is here explicitly defined statistically and the same holds for contingent causes, too. They are divided into three groups. Causa ut in pluribus produces the effect in most cases, *causa ut in paucioribus* or *ut in raro* only in a few cases and *causa ad utrumlibet* presents the special case of fifty percent incidence rate."

of Brabant († 1284) and Thomas Aquinas († 1274) [363] In the *QPhys*. II.11, Buridan however does not follow the same conception of necessity and possibility delineated by the followers of Averroes' conceptions. The main purpose of the text is rather to reject these definitions in favor of the view that chance events do not essentially belong to the category of what happens rarely, but instead belong also to the category of the each-way contingent. This position goes against the temporal-frequency definitions that Averroes seemed to sustain in defending that chance events are by definition rare occurrences. One of the examples Buridan gives concerns games of chance, the outcomes of which are random. Buridan writes:

(T6.7) [...] It is contingent each-way (*ad utrumlibet*) for the players of a dice game to win or to loose. And we say that if someone in such a game makes profit, this is by chance. Or if someone loses, this is either by chance or luck, since the definition of luck is as a whole satisfied in this case. They lose in spite of and against their intention, which was to profit, and this type of thing does not belong to what happens always or for the most part, hence we posit that it happens ad utrumlibet [364]

Buridan says that although a player of a game of dice regularly plays with the intention of winning, it is nonetheless each-way contingent whether they win or lose (Buridan [2015a], Eds. Streijger and Bakker, 322). There is a slight conceptual change in which the phrase 'ad utrumlibet' comes to mean random outcome. Buridan's example seems to be quite original, and it would be worthwhile to investigate whether there were previous usages of games of chance to illustrate that the outcome of ad utrumlibet causes and the corresponding shifts in conceptual usage of the that phrase. I am in no position to do so in the current chapter, but it is accepted that Buridan was a pivotal figure in this

³⁶³See again Knuuttila [1981], 209ff. and Knuuttila [1993], 133-137.

³⁶⁴Buridan [2015a], Eds. Streijger and Bakker, 322: "Item ludenti ad taxillos contingens est ad utrumlibet perdere vel lucrari. Et tamen dicimus quod a casu est, si quis in illo ludo lucretur. Vel si perdat, hoc est a casu vel a fortuna, quia sibi convenit tota definitio fortunae. Perfit enim praeter et contra intentionem suam, quae erat lucrandi, et extra semper et frequenter, ex quo ponimus quod hoc sit ad utrumlibet."

regard. 365

On the basis of the definitions of causal necessity and contingency that Avicenna puts forward, Buridan rejects a statement Averroes made, according to which "chance and fortune are agent causes with respect to casual and fortuitous effects, therefore their effects are not equally likely to occur or not to occur."

Buridan enlists several reasons in support of Avicenna's view instad.

Buridan further mentions in support of Avicenna's descriptions is that there is nothing inconsistent in describing something as obtaining for the most part with respect to some circumstance, but only rarely with respect to another. To the contrary, for Buridan, the same effect can be regarded as contingent relative to a preventable causal factor, but necessary with regard to an unpreventable causal factor. For example, Socrates dying at a particular time is contingent and preventable relative to a king's decree, it is however necessary and unpreventable if that circumstance includes an omnipotent agent overpowering the king's will

³⁶⁵Knebel's study, which focuses on action theory, provides some indications in this regard, cf. Knebel [2006]. Other figures who have devoted long sections of the *Physics* to study chance in relation to *ad utrumlibet* causes were mentioned by Sylla, who has placed special emphasis on Walter Burley († 1344).

³⁶⁶Cf. Buridan Buridan [2015a], Eds. Streijger and Bakker, 327: "From those it follows evidently that the argument of the Commentator is not effective, by which he wishes to claim that "chance and fortune are agent causes with respect to casual and fortuitous effects, therefore their effects are not equally likely to occur or not to occur.' This does not follow [from the definitions of chance], since chance and luck are not called such unless with regard to their non-intended and non-primary effects, but rather with regard to their accidental effects." ["Ex istis sequitur manifeste quod ineptum est argumentum Commentatoris in quo dicit: 'casus et fortuna sunt causae agentes respectu effectuum casualium et fortuitorum, igitur non sunt ad utrumlibet respectu eorum.' Non enim est bona consequentia, maxime cum casus et fortuna non dicantur casus et fortuna nisi respectu effectuum non intentorum nec primorum, sed accidentium cum intentis vel loco eorum."]

³⁶⁷Buridan [2015a], Eds. Streijger and Bakker, 327-8: "Furthermore, the Commentator is not correct when he says that the same event cannot come about for the most part with respect to one [agent] and rarely with respect to another, since profit happens for the most part with respect to dealers but rarely for gamblers, walking happens for the most part with respect to runners but rarely with respect to weavers, and winning at battles happens for the most part with regard to soldiers but rarely with regard to kinds, as so of other cases." ["Item non bene dicit Commentator quod non sit idem ut in pluribus respectu unius et paucioribus respectu alterius, quia ut in pluribus est lucrari respectu mercatorum et in paucioribus respectu taxillatorum, et etiam ut in pluribus est ambulare respectu cursorum et ut in paucioribus respectu textorum, et comes in bello vinceret ut in pluribus respectu militis et in paucioribus respectu regis et sic de aliis."]

and bringing about Socrates' death at that time as a matter of necessity. In this regard, Buridan believes that from the part of agents, the modal classification of causes acting contingently into those that do so for the most part, rarely, or *ad utrumlibet*, is not based on a description of the essences of causal agents involved as the Averroan descriptions are, but it instead takes into account the relation between causal agents and preventing or enabling circumstances.

What can be concluded from the discussion at *QPhys*. II.11? On the basis of his discussion of chance, Buridan seems to reject the temporal-frequency classifications of contingent causes. Instead of identifying chance with the class of events that happen rarely (the *contingens ut raro*), Buridan holds that chance does belong to the category of each-way contingencies in the sense of being a random outcome. Although we cannot explore this in the thesis in detail, we can observe that a conceptual shift of the phrase *ad utrumlibet*, originally having the logical meaning of 'each-way contingency,' acquires the acceptation of a random outcome in causal processes in Buridan's physical writings.

³⁶⁸Buridan [2016], Eds. Streijger, Bakker and Thijssen, 328: "Iterum de necessitatis consequentis posterioris ex priori necesse esset Socratem mori hodie respectu voluntatis Dei efficaciter hoc volentis, quia contra talem voluntatem non potest contingere impedimentum, sed hoc esset contingens et non necessarium respectu voluntatis regis, quia posset contra eum esse impedimentum. Et ita intendebat Avicenna. Non enim credebat quod illud quod est simpliciter necessarium in essendo potest non esse propter aliquam potentiam vel aliquem respectum."

7. Natural Impossibilities and Supernatural Cases

My aim in this chapter is to address the problem concerning to what extent Buridan thought an analysis of supernatural possibilities (natural impossibilities) was philosophically useful. To that end, I shall examine the functions that Buridan's distinction between logical and natural possibilities have in some natural philosophical contexts.

I will start by discussing Buridan's semantic approach to the signification of impossible objects (7.1), and I will argue that Buridan implicitly distinguishes between logically and naturally impossible objects of signification. The next section (7.2) turns to Buridan's application of the analysis of modal propositions to a natural philosophical thesis. It focuses on the articulation of the thesis that continuous magnitudes can be infinitely divisible into smaller parts at QDGC I.5 and *QPhys.* III.19. In section 7.3 I examine Buridan's view concerning the legitimacy of impossible hypotheses in natural philosophy, in particular in connection with the status of impossible assumptions in indirect demonstrations. This section focuses on the argument for a first unmoved mover in QPhys. 7.1. Knuuttila and Kukkonen have contrasted the role of Buridan's distinction of logical and natural modalities in his interpretation of the function of Aristotelian impossible hypotheses with Averroes' interpretation of this part of the *Physics* 7. My aim will be to show how John of Jandun espoused an analogous interpretation of modalities to Averroes', and that Buridan probably had in mind Jandun's interpretation as target in his critical remarks at QPhys. 7.1.

³⁶⁹See Knuuttila [2001]; Kukkonen [2005]; Knuuttila & Kukkonen [2011].

7.1 Impossible Significates

In late medieval logic, the problem of impossible significates on concerns whether there can be concepts or signs which necessarily fail to (ultimately) signify anything, although their significates can be imagined or conceived as logically possible.

In this section, I will argue that Buridan has made an implicit distinction between logically and naturally impossible significates in his semantics, by turning attention to how he treats impossible objects in semantics and natural philosophy. While Buridan did not draw the distinction between naturally and logically impossible significates in his logical writings explicitly. I will attempt to show that Buridan's discussion of some examples of impossible objects in his logic and natural philosophy presuppose that distinction.

In order to address this issue, we should first clarify some assumptions of Buridan's theory of signification. Buridan generally defines signification as the 'establishing of an understanding of a thing' (*intellectum rei constituere*). Following the long standing tradition of mental language, Buridan takes language to come in three levels: spoken, written, and mental language. In Buridan's

³⁷⁰Significate' here transliterates the Latin term '*significatum*,' namely the object of a signifying term. We will briefly describe Buridan's account of signification below.

³⁷¹For standard treatments this problem in nominalist medieval logic and semantics, and their connections with imaginability, see Hugonnard-Roche [1989], Ashworth [1977], Biard [1983], Ciola [2019] and Ciola [2020].

^{3/2}Thomas Dewender notes that Ockham also did not distinguish between them explicitly either, and argued for the stronger claim that Ockham treated logically and naturally impossible objects on a par (Dewender [2011]), 442: "The contradiction that is involved in the case of objects like a chimera or a goatstag is a logical one, assuming that incompatible properties, e.g. the essence of a goat and a stag, are united in one single object. In addition, there are impossible objects like a vacuum or an infinite, which cannot exist, but whose impossibility is not due tosome contradiction involved in forming these concepts, but becauseassuming their existence in the real world would be incompatible with the basic principles of Aristotelian natural philosophy. Ockham occasionally mentions these examples as well, but he does not make the distinction between, on the one hand, a logical, and, on the other, a 'natural' or 'physical' impossibility, instead he treats the second class of impossible objects on a par with the first class."

³⁷³See Buridan [2001b], 828 for Buridan's recitation of this phrase, which was traditionally used in medieval logic. For the theory of signification in medieval logic in general, see [Spade] [1982].

³⁷⁴For Buridan's version of the 'mental language tradition,' see Read [2015b], King

semantic theory, a spoken or written term is said to ultimately signify something when it 'supposits personally' for an object in the external world via the mediation of a mental concept, and a spoken or written term is said to non-ultimately signify when it 'supposits materially' for a concept in the mind ³⁷⁵ To say that a term or a concept has personal supposition (*suppositio personalis*) is to say, in the medieval logical vocabulary, that it bears reference to something in the external world ³⁷⁶

The distinction between immediate and ultimate signification plays a manifold role in Buridan's semantics. For our purposes, the difference between the ultimate signification and the non-ultimate (*immediate*) signification of a term is crucial here. Buridan thinks that a term is referring - namely, it has personal supposition - for something only if it can be verified of it, by predicating the term of a deictic pointing to the thing they signify. Obviously, that cannot be

^{[1985],} Panaccio [2004].

apt to supposit signify things by the mediation of their concepts, according to which concepts, or similar ones, they were imposed to signify. So in this passage we call the things conceived by those concepts 'ultimate significata', whereas the concepts we call 'immediate significata'." Cf. Buridan [1998] Ed. Van der Lecq, 39: "[...] dictiones categorematicae innatae supponere significant res aliquas mediantibus conceptibus earum, secundum quos conceptus vel similitudines impositae fierunt ad significandum. Sic ergo res illas illis conceptibus conceptas vocamus ultimata significata in proposito. Illos autem conceptus vocamus significata immediata."

³⁷⁶cf. Buridan [2001b], Transl. Klima, 252-253: "In another way supposition is divided into material and personal supposition. And supposition is called personal when the subject or the predicate of the proposition supposits for its ultimate significates or for its ultimate significate, as the term 'man' supposits for men in the proposition 'A man runs.' But supposition is said to be material when an utterance supposits for itself or for one similar to itself, or for its immediate significate, which is the concept according to which it was imposed to signify, as the term 'man' in the proposition 'Man is a species'." Buridan [1998], Ed. Van der Lecq, 38: "Et vocatur suppositio personalis quando subjectum vel praedicatum propositionis supponit pro suis ultimatis significatis vel pro suo ultimato significato, ut iste terminus 'homo' pro hominibus in ista propositione 'homo currit.' Sed suppositio materialis dicitur quando vox supponit pro se aut sibi simili aut pro suo significato immediato, quod est conceptus secundum quem imposita est ad significandum, ut iste terminus 'homo' in ista propositione 'homo est species.'"

 $^{^{377}}$ For the relevance of Buridan's distinction between the immediate significate - as the 'intramental' significate of a sign - and the ultimate significate - as its 'extramental significate,' see Klima [2009], 203-7.

Buridan [2001b], Transl. Klima, 222: "Therefore, taking signification and

the case with impossible significates - fictive terms such as the 'Chimaera' or the 'void' cannot be verified of anything, although they may well signify some mental concept by evoking a representation to the mind, as we shall see below.

The Chimera was the standard medieval example illustrating a logically impossible object, in the sense that it involves logically incompatible essences or forms [379] In one of the sophisms contained in the first chapter of Buridan's *Sophismata*, entitled "The name 'Chimaera' signifies nothing." Buridan states his negative answer to the question of whether there can be signs of impossible objects in the following way (Buridan [2001b], Transl. Klima, 828):

(T7.1) It is true that some people, trying to evade [this difficulty] rather than seek out [discernere] the truth, have wanted to reply that the utterance 'chimera' does indeed signify, but it does not signify something, so that this does not follow: 'it signifies; therefore, it signifies something, or, something it signifies' [...] Against these replies I argue as follows: first, 'to signify' is described as 'to establish an understanding of the thing;' therefore, an utterance is said to signify the thing the understanding of which it establishes for us. Therefore, if every signifying is an establishing of the understanding of a thing, then every signifying is the signifying of a thing; therefore it is the signifying of something, for 'thing' and 'something' are convertible.

supposition in this way, signification differs from supposition. For any word [dictio] that is a part of a proposition not taken materially signifies and gives rise to some concept in the person hearing it according to the signification conventionally [ad placitum] given to it. But not every such word has supposition, for only such a term is apt to supposit that, when something is pointed out by the pronoun 'this', or when some things are pointed out by the pronoun 'these', can truly be affirmed of that pronoun. Therefore the term 'chimera' cannot supposit, for whatever is pointed out, it is false to say 'This is a chimera', and whichever things are pointed out, it is false to say 'These are a chimera'." Cf. Buridan [1998], Ed. Van der Lecq, 8-9: "Dicto ergo modo apiendo 'significationem' et 'suppositionem' different significatio et suppositio, quia cuiuslibet dictionis quae non materialiter sumpta est pars propositionis, interest significare et audienti eam conceptum aliquem constituere secundum institutionem ad placitum sibi datam. Et non omnis talis dictionis est supponere, quia solus talis terminus est innatus supponere et omnis talis qui, aliquo demosntrato per hoc pronomen 'hoc' vel aliquibus demonstratis per illud pronomen 'haec,' potest vere affirmari de illo pronomine. Ideo ista terminus 'chimaera' non potest supponere,' et quibuscumque demonstratis 'haec sunt chimaera.'"

³⁷⁹For a history of this example throughout medieval logic, see Ebbesen [2008].

Again, to signify is to be a sign - but 'sign' [signum] and 'designatum' [signatum, that thing of which the sign is the sign] are correlatives - therefore, 'chimera' is not a sign, nor, consequently, does it signify, unless something is its designatum, namely, something present, past or future, or at least possible. [380]

Buridan's position as stated in this passage is that every concept or term which is significative (in the sense of having ultimate signification), must signify something at least possible. In Buridan's conception, this follows from the very definition of signification as the 'establishing of an understanding of a thing' (*intellectus rei constituere*), since it would be false to say 'I understand; but there is no thing I understand,' and concludes accordingly that "the following [propositions] are false: 'a chimera is thinkable', 'a chimera is opinable'" (Buridan [2001b], Transl. Klima, 834).

As Jack Zupko claims, these remarks illustrate the natural constraints that signification has in Buridan's theory. Zupko writes (Zupko [2003], 26):

[...] this passage makes clear that nature also imposes limits on what concepts can signify, so that from the act whereby I understand something, if I really do understand - i.e., if 'I understand [intelligo] is true - there must be something that I understand; it cannot be that I understand when there is literally nothing (no thing) for me to

Buridan [1489], fols. 67rb-va "[...] verum est quod aliqui fugere volentes voluerunt respondere quod hec vox chimera bene significat, sed non significat aliquid, ita quod non sequitur 'significat, igitur significat aliquid.' [...] Contra istos omnes arguitur primo quia significare est intellectum rei constituere, ideo vox dicitur illud significare cuius intellectum nobis constituit; ergo si omne significare est intellectum rei constituere et omne significare est rem significare, igitur est aliquid significare, quia res et aliquid convertibiliter se habent. Item significare est esse signum, signum autem et signatum dicuntur correlative. Igitur chimaera non est signum nisi alicuius sit signum. Et per consequens nihil significat nisi aliquid significet sive aliquid sit signatum scilicet vel presens vel praeteritum vel futurum vel saltem possibile."

³⁸¹Cf. Buridan [2001b], Transl. Klima, 833: "by every concept something is conceived, though this need not be only one thing but can be several things together. For it would be absurd to say that someone understands, and yet he understands nothing; or that he sees, and yet he sees nothing [...] For this reason we should hold that these are false: 'I read and I read nothing,' 'I see and I see nothing,' 'I understand and I understand nothing'; so I believe that all such propositions are false and not possible by nature [non possibiles per naturam].

understand. Non-referring significant concepts are "not possible by nature." Notice, however, that Buridan does not say "not possible absolutely [simpliciter]." This leaves open the possibility that God could, by virtue of his absolute power, operate outside the common course of nature to cause me to assent to the proposition that I understand something when there is nothing there for me to understand.

Zupko means by 'significant non-referring concepts' those terms which have immediate but do not have ultimate signification. He notes that although Buridan does not rule out the possibility that God could, by his absolute power, make us assent to propositions such as 'I understand; but there is nothing I understand,' Buridan thought nonetheless that these possibilities were not philosophically relevant, since there is no way of confirming nor disconfirming these supernatural possibilities.³⁸²

Buridan's view of impossible objects of signification is therefore quite restrictive. In a sense, it excludes metaphysically and conceptually impossible objects from his theory of signification. As is well known, this view is not unanimous in the tradition of nominalist semantics. According to the standard story, Buridan's ban on metaphysically impossible objects from signification and conceivability fell out of popularity during the post-medieval developments in nominalist semantics. Many other authors after Buridan have given a special status to metaphysically impossible objects in their semantics by positing a kind of ampliation to 'imaginable objects' (*imaginabilia*). Marsilius of Inghen is the foremost representative of this trend, and according to Graziana Ciola it has given a special kind of place in his accounts of ampliation. His treatise on ampliation includes a form of ampliation to merely imaginable beings, and

³⁸²Zupko [2003], 26: "As we shall see in chapter 12 below, Buridan does not regard such possibilities as philosophically relevant, since there is no way of confirming or disconfirming supernatural contingencies in terms that would make sense to us as empirical creatures."

³⁸³See Biard [1983] for a reference to John Mair's treatment of the issue, and also Ashworth [1977].

Biard [1983] traces the use of ampliation to imaginables to a British tradition in semantics going back to Heytesbury, and which was made popular in Paris by Marsilius of Inghen. For a detailed analysis of Marsilius' position, see Ciola [2019].

³⁸⁵Ciola [2020] and Ciola [2019].

the latter can serve as the significates of propositions such as 'a chimera is thinkable.' This approach to impossible significates is less restrictive than Buridan's, in the sense that 'Chimera' may well signify something merely imaginable, even though we are dealing with a metaphysically impossible object. [387]

However, stating that for Buridan impossible significates are not conceivable altogether would be too far stretched. Given the distinction between immediate and ultimate signification that we have mentioned above, Buridan does think that the spoken or written sign 'Chimera' evokes a concept in the mind, a concept which, however, fails to ultimately signify any extra-mental object. In other words, a term such as 'Chimera' is not meaningless altogether, as an arbitrarily construed string of symbols. Instead, the typical examples

³⁸⁶Cf. Marsilius of Inghen [1983], Ed. and Transl. Bos,102: "Secondly, in regard to the above definition it should be noted that there are three time distinctions, viz. the present, the past and the future. And they are also called tenses as we usually say this is the present tense, this is the past tense, etc. Two time-distinctions may be added, viz. the possible and the imaginable, which, although not time-distinctions in the proper sense, are in our context, for terms have supposition for them in propositions for different times, as will be discussed below." ["Secundo circa dictam descriptionem superius positam est notandum quod tres sunt differentie temporum, scilicet presens, preteritum et futurum. Et tales etiam dicuntur tempora, ut solemus dicere hoc est presens tempus, hoc est preteritum, etc. Et possunt addi duo, scilicet posse et imaginan esse, que, licet non sint proprie differentie temporum, tamen in proposito sunt differentie temporum, nam respectu eorum termini supponunt in propositionibus pro diversis temporibus, ut postea dicetur."]

³⁸⁷As Ciola points out, Albert of Saxony turns out also to be explicit in this regard in his *Quaestiones circa Logicam* q.13 (Albert of Saxony [2010]), Ed. and Transl. Fitzgerald, 206): "If you concede that the term 'opinable' can ampliate a term to stand for that which is, was, will or can be, and further also for that which can be understood, imaginated or conceived, then this [proposition] should be conceded: the Chimera is opniable." ["Si concedis istum terminum 'opinabilis' posse ampliare terminum ad standum pro eo, quod est vel fuit, vel erit vel potest esse, vel potest intelligi vel imaginari, vel concipi, haec est concedenda: Chymaera est opinabilis."]

³⁸⁸As an example for an arbitrary string of symbols which is genuinely meaningless, Buridan mentions 'ba bu' and 'baf'. Buridan [2001b], Transl. Klima, 223: "But neither does an utterance that is called 'meaningless' enter into a proposition, namely, the kind of utterance that neither naturally nor by a conventional imposition signifies something, except, perhaps, itself, or its own concept, e.g., 'bu ba'. For such an utterance is neither a noun, nor a verb, nor a syncategorematic term, so it would have no function in a proposition, unless it is taken materially. If I say: 'baf runs', this is not a proposition, for the utterance 'baf' does not have a signification, nor a mode of signifying according

of impossible significates in Buridan are subordinated to complex concepts corresponding to their nominal definitions. Thus, for example, 'Chimera' means 'a beast with the head of a dragon and the tail of lion,' and 'void' means 'a place not filled with matter.' Buridan states that the reason why these terms fail to (ultimately) signify is because their corresponding complex concepts signify many things in the external world, corresponding to the constituents of their nominal definitions, all these things signified fail to correspond to a single, unitary object. Moreover, it is because the constituents of these concepts of their nominal definitions - are signifying terms that we can form ficticious and non-referring complex concepts (*conceptus complexos fictos*) by conjoining simple referring concepts.

However, these two most frequent examples of impossible significates in Buridan's semantics seem not to be on a par. While the 'Chimera' is the traditional example of an absolutely impossible object, the concepts of the 'void' or the concept of an 'infinite body' seem to be not impossible in themselves, but

to which it could provide the subject to a verb [...]" Cf. Buridan [1998], Ed. Van der Lecq, 8-9: "Sed etiam nec illa vox quae vocatur 'non significativa' intrat propositionem, scilicet illa vox quae nec naturaliter nec secundum impositionem sibi ad placitum datam significat aliquid, nisi forte se ipsam vel conceptum sui ipsius, ut 'bu ba.' Talis enim vox nec est nomen nec verbum nec syncategorema. Ideo ad nihil deserviret in propositione nisi sumeretur materialiter. Si enim dico 'baf currit,' haec non est propositio, quia non habet significationem haec vox 'baf' nec modum significandi secundum quem posser reddere suppositum verbo [...]"

³⁹⁰Buridan [2001b], Transl. Klima, 838: "The eighth conclusion follows, namely, that some complex concept that is made the subject or the predicate of a mental proposition supposits for nothing, even though many things are conceived by means of it." In this case, the many things conceived by the complex mental concept of the Chimera are its incompossible parts - the head of a lion and the tail of a dragon forming the beast.

Buridan [2001b], Transl. Klima, 635: "A nominal definition [diffinitio explicans quid nominis] is an expression convertibly explaining what thing or things the definitum signifies or connotes, and properly speaking it is called 'interpretation.' It pertains to incomplex spoken terms to which there correspond not simple concepts in the mind but complex ones, whether these terms supposit for some thing or things or do not supposit." Cf. Buridan [2001a], Ed. De Rijk, 30-1: "Definitio dicens 'quid nominis' est oratio exprimens convertibiliter quid vel quae definitum significat aut connotat; et nomine proprio vocatur 'interpretatio.' Et convenit terminis incomplexis vocalibus quibus non correspondent in mente conceptus simplices, sed complexi, sive illi termini pro aliquo vel aliquibus sive nec pro aliquo nec pro aliquibus supponant." For an in depth treatment of Buridan's theory of definitions and its concrete applications, see

rather impossible by being inconsistent with principles of Aristotelian natural philosophy. Buridan nowhere makes an explicit distinction between absolutely and naturally impossible significates, but as we shall see below some examples suggesting there may be an implicit distinction.

There are a couple of cases in Buridan's natural philosophical writings in which he uses fictive or imaginative concepts for the purposes of addressing a question about the natural world. For example, Buridan's account of impossible significates is widely applied in the two sets of questions on the infinite (Buridan [2016], Eds. Streijger and Bakker, III.14-19), and on the void respectively (Buridan [2016], Eds. Streijger and Bakker, IV.7-11), in which Buridan makes a series of remarks about fictive concepts. One such case in the former set of questions is the concept of an infinite body. Buridan's answer is that although it may well be possible to conceive an infinite body - by attending to the complex concept signified by 'infinite body'. its existence is not possible by nature. Buridan is clear that these imaginations should not be a guide to what is naturally possible. Buridan writes Buridan [2016], Eds. Streijger and Bakker, 131:

(T7.2) To the other [argument] Aristotle says that we should not believe in the things we can imagine and in things we conceive (*intelligentiae*), that is in our intellection, since we can imagine that you grow beyond the extension that you now have even though it should not be the case that this is so. When it is said that the intellect and imagination

³⁹¹There are many understandings of the infinite - of which he mentions the infinite according to power (*vigoris*), to duration and to division -, but in this question, Buridan is interested in the infinite insofar as it signifies the unlimited extension of a magnitude. Cf. Buridan [2016], Eds. Streijger and Bakker, 124: "The infinite can be said in many ways, namely according to power (*vigorem*), according to duration, according to division, etc., but here only the sense of the infinite according to the magnitude of extension is intended, insofar as the infinite is said of an extended body without limits." ["Licet igitur multis modis dicatur infinitum, ut secundum vigorem, secundum durationem, secundum divisionem, etc., tamen hic solum intenditur de infinito secundum magnitudinis extensionem, prout infinitum diceretur corpus extensum sine terminis. Et non curamus hic de 'infinito' syntategorematice accipiendo, sed categorematice]".

³⁹²Cf. Buridan [2016], Eds. Streijger and Bakker, 129: "Et hoc etiam declarat Aristoteles per definitionem corporis infiniti, quae debet esse congregata ex definitione corporis et ex definitione infiniti; definitio autem corporis est 'quod est omniquaque distans', et definitio infiniti est 'quod est distans sine termino' [...]"

should be moved by the thing, I say that is true. For it is necessary that simple concepts correspond to some present or past thing. However, falsity or fiction can emerge in composing simple concepts [...]. But by those terms we understand true things according to the complex fictional concepts. And in the same way it could be said that these terms, 'Chimera', 'void', and 'infinite body' do not signify a Chimera nor a void nor an infinite body (I always suppose that these are impossible objects), but they signify true things according to the complex fictional concepts. And it might well be true that those vocal terms signify really existing concepts in the mind and further signify extra-mental things. That is, the term 'void' signifies the same as the expression 'a place not filled with body', and that expression signifies place and plenitude, which are really extra-mental things. However, it signifies those according to such a complexion of concepts of which the complexion itself supposits for nothing [393]

Buridan adheres in that context to the principle that if something is conceivable (*intelligibile*), it should be at least possible (Buridan [2016], Eds. Streijger and Bakker, III.14, 131). Whether Buridan has natural possibility in mind

³⁹³Ad aliam dicit Aristotels quod non oportet credere imaginationi vel intelligentiae, id est intellectioni, quia possumus imaginari te crescere ultra quantitatem quam habes et non oportet ita esse in re. Quando autem dicitur quod oportet intellectum et imaginationem moveri a re, dico quod verum est. Ideo conceptui simiplici necesse est correspondere rem aliquam vel praesentem vel praeteritam. Sed in componendo conceptus simplices potest esse falsitas vel fictio. [...] Sed per tales terminos intelligimus veras res secundum conceptus complexos fictos. Et eodem modo diceretur quod isti termini 'chimaera', 'vacuum', et 'corpus infinitum' non significant chimeram nec vacuum nec corpus infinitum (dico: semper, si impossibile sit talia esse), sed significant veras res secundum conceptus complexos fictos. Et bene verum est quod tales termini vocales bene significant conceptus realiter existentes in anima et ultra etiam significant res extra. Verbi gratia idem significat iste terminus 'vacuum' sicut hae oratio 'locus non repletus corpore', et haec oratio significat et locum et plenum, quae sunt verae res extra. Sed significat ea secundum talem complexionem conceptuum quod conceptus complexus pro nullo supponit.

 $^{^{394}}$ ["Sed etiam, quando dicitur quod est bona consequentia, 'a est intelligibile, igitur a est, fuit vel potest esse', concedo. Et ideo dico quod omnes tales sunt falsae de virtute sermonis et loequando non materialiter, sed significative: 'Deum non esse est intelligibile', 'chimaera est intelligibilis vel opinabilis', 'vacuum est imaginabile', 'corpus infinitum est imaginabile vel intelligibile'. Dico quod istae sunt falsae."]

with this remarks is not clear from the text itself, but it seems to be the most plausible interpretation, as we shall see concerning his usage of arguments from imagination and conceivability in the questions on the void in the same commentary.

As Edith Sylla has shown, in the set of questions dedicated to the problem of the void (Buridan [2016] IV.7-11), Buridan uses theological reasoning and arguments from imagination in order to show how the void is possible by supernatural possibility (Sylla [2001]). As with other impossible significates, the term 'void' has only a nominal definition, corresponding to a complex concept in the mind made out of simple concepts In his *QPhys*. IV.7, Buridan addresses the question of whether it is possible for there to be a void, and his approach to this problem in that context makes it evident that the concept the void is only naturally impossible, but supernaturally possible.

Buridan argues from imagination (*secundum imaginationem*) in order to specify how a void would be possible. He first says that place (*locus*) can be imagined in two ways. In one way, place is understood as the space that remains setting aside the magnitude of a natural body. According to this imagination, the void would then be a dimension having the longitude, latitude and profundity

³⁹⁵Cf. Buridan [n.d.a], Ed. Hubien), I 4a: "Moreover, it is impossible to have an absolute concept - which is not connotative -, unless it stands for something, either present, past or future; and that is because nothing is understood (intelligitur) unless it is will be, as will be seen later. Therefore, if some term supposits for nothing, this is because that term is connotative, or because it is composed of many simple concepts. In that way, the concept of a void supposits for nothing, since the concept of the void is composed from the concepts of place and body, in a negative way. For 'void' signifies the same as 'place not filled with body.' Thus every term which supposit for nothing is analysed with respect to its nominal definition (quid nominis) in other terms, with respect to which it is clear and known that they supposit for something; and these terms are the subjects of [propositions] stating what they are." ["Iterum, impossibile est habere conceptum simplicem purum, et non connotativum, quin ille supponat pro aliquo praesenti, praeterito vel futuro; et hoc est quia nihil intelligitur quod non sit vel fuerit, sicut postea magis videbitur. Igitur, si aliquis terminus pro nullo supponit, hoc est quia est connotativus, vel quia componuntur in se plures conceptus. Sic conceptus vacui pro nullo supponit, quia conceptus vacui componit in se conceptus loci et conceptum corporis, scilicet negative: idem enim significat 'vacuum' quod 'locus non repletus corpore'. Et ita omnes tales termini, pro nullo supponentes, resolvuntur quantum ad quid nominis in alios terminos de quibus manifestum est et praecognitum est quod pro aliquo supponunt; et illi termini sunt subiecta de quibus notum est quia est."]

of the body that would naturally occupy this place [396] In another way, place is imagined as the surface surrounding a body contained within them, the void according to this way of imagining would be the remaining figure after the contents of a body were anihilated, but it would not be any extended dimension, in distinction from the first imagination [397]

In *QPhys*. IV.15, Buridan mentions a circumstance in which God would annihilate all contents of the moon and asks what would remain inside its sphere in such a circumstance. Buridan states that it is logically contradictory to say that both nothing would remain inside the concave figure of the sphere, and yet that some void space would remain, but it is not contradictory and it is possible to claim that some void space remains inside it, for which the term 'void' would in such a scenario refer. ³⁹⁸ In other words, it does not entail any contradiction

³⁹⁶Buridan [2016], Eds. Streijger and Bakker, 261-2: "Moreover it should be noted that just as place can be imagined in two ways, so too can the void be imagined in two ways. Namely if, as many have imagined, the void is a space that remains removing the magnitudes of natural bodies which were it not removed would receive the natural body, and the space of which could be occupied by a natural body according to the parts equal to itself, this space certainly would be a place (locus). And this place would be filled (plenus) when in and along it is an adequte natural body, and void when along it and in it there is no natural body. In this way it appears that the void would be a corporal dimension having as much longitude, latitude and depth as a natural body would occupy if it were filled." ["Postea notandum est quod, sicut dupliciter potest imaginari locus, ita etiam dupliciter potest imaginari vacuum. Nam si esset spatium praeter magnitudines corporum naturalium, in quo non cedente repicerentur corpora naturalia, de quo spatio unumquodque corpus naturale occuparet partem sibi aequalem, sicut multi imaginati sunt, illud spatium sine dubio deberet poni esse locus. Et si esset locus plenus, quando in eo et cum eo esset corpus naturale aedequate, et diceretur vacuus, quando cum eo vel in eo non esset corpus naturale. Et sic apparet quod vacuum esset dimensio corporea tanta secundum longitudinem, latitudinem et profunditatem, quantum esset corpus naturale per quod illud vacuum repleretur, si poneretur in eo."]

³⁹⁷Buridan [2016], Eds. Streijger and Bakker, 262: "Alio modo secundum Aristotelem ponitur locus esse superficies corporis continentis locatum. Et tunc, si vacuum esset, deberet imaginari sic quod ex loco pleno auferretur corpus contentum vel annihilaretur loco remanente in sua figura, videlicet quod latera loci non approximarentur ad invicem, verbi gratia imaginando quod ille mundus inferior annihilaretur totaliter caelo remanente in sua magnitude et figura, sicut est nunc. Si enim sic esset, tunc superficies orbis lunae, quae modo est locus repletus isto mundo inferiori, esset locus vacuus, quia non esset in eo aliquod corpus contentum ab eo, immmo nec aliquod spatium nec aliqua dimensio, immo nihil."

³⁹⁸Buridan [2016], Eds. Steijger and Bakker, 137: "I say therefore than in the assumed case nothing would be below or inside the sphere of the moon, since it was

for Buridan to assume that the void could exist, and the term 'void' would have supposition in such a supernaturally possible scenario. Buridan goes as far as to saying that if God could miraculously conserve air between the sphere of the earth, then if two persons standing in opposite sides of the sphere communicated and uttered the term 'void' this term would have personal supposition for the annihilated content of the earth. [399]

It seems safe to say that Buridan could only have held those scenarios as possible if he thought that logically and naturally impossible significates are not on the same level. Although he did not make the distinction between possible and merely imaginable items of signification as explicit as other nominalist authors as we have seen above, Buridan's treatment of the problem of the void in his *Physics* suggest that naturally impossible referents of significant concepts could refer to something in a supernaturally possible case.

posited that everything [below and inside it] was annihilated. And thus there would be no space and no void below or inside the sphere, since it implies a contradiction to say both that nothing is inside the sphere and that void or space is inside it; for if the affirmative propositions 'void is inside the concave figure' or 'space is inside the sphere' were true, it should be the case that these terms 'space' and 'void' supposit for something; therefore it would follow 'void is inside the sphere; therefore something is inside the sphere. But that would contradict the first claim that nothing is inside it." ["Dico igitur quod isto casu posito nihil esset infra sive intra concavum orbi lunae, quia totum ponitur annihilatum. Et sic non esset aliquod spatium et etiam non esset vacuum intra vel infra huiusmodi concavum, quia implicat contradictionem quod nihil sit intra concavum et quod vacuum vel spatium sit intra concavum, quoniam cum haec propositio 'vacuum est intra concavum' vel 'spatium est intra concavum' sit affirmativa, oportet, si sit vera, quod iste terminus 'spatium' vel 'vacuum' pro aliquo supponat; ideo sequitur 'vacuum est intra concavum, igitur aliquid est intra concavum', et haec est contradictoria illius primae, quae dicebat quod nihil est intra concavum etc."]

³⁹⁹Buridan [2016], Eds. Streijger and Bakker, 269: "And furthermore I conclude a corollary, concerning what was sometimes asked, namely that it would be possible to see and to hear through a void or through its parts, since God could conserve the air in the magnitude and figure which is now occupied by earth and water, and [God could] annihilate water and earth and everything contained in this sphere, and thus that air would be void. And if God sustained that air, then two men standing on opposite sides [of the sphere] would see each other through that air and could communicate with each other, just as they do now." ["Et nunc ultra concludo corollarium, de quo aliquando quaeritur, scilicet quod possibile esset per vacuum vel partes vacui videre et audire, quia Deus posset aerem conservare in magnitudine et figura in qua nunc est circa aquam et terram, et annihilare aquam et terram et omnia in eius contenta, et sic ille aer esset vacuus. Et si Deus in illo aere sustentaret duos homines prope invicem, ipse viderent se invicem per illum aerem et possent loqui invicem, sicut faciunt nunc."]

7.2 Infinite Divisibility

In the set of questions concerning continuity and infinity, contained in commentaries on the *Physics* III.15-19⁴⁰⁰ and on *De Generatione et Corruptione* I.5. Buridan applies the analyses of modal propositions we have seen in chapter 2.3 to articulate divisibilist theses. Divisibilism labels the widespread view in the middle ages according to which indivisible entities or 'atoms' do not have real existence. According to this standard view, shared by most philosophers in the late 14th century, and going back to Aristotle's *Physics*, 402 continuous entities - surfaces, lines, shapes, magnitudes, and so on - are thus not ultimately composed of indivisible things, but instead are infinitely divisible into smaller parts. The feature of Buridan's questions of interest to the present section is his distinctively logical approach to problems raised by the thesis that continuous entities are infinitely divisible. In the text I will approach, some apparent consequences of divisibilist theses and propositions are framed by Buridan as logically perplexing. His approach seeks to dissolve this impression by referring to semantic principles, and to the analysis of the modal proposition that we have discussed earlier (2.3).

It is worth briefly rehearsing the divisibilist tenets that Buridan's commentary presupposes, since the medieval theory of the composition of continuous entities is not unitary, and comes along with a variety of distinct positions taken by late fourteenth century figures. Jack Zupko has usefully described three types of claims in order to map different positions taken on the composition of *continua* around Buridan's time (Zupko [1993b], 163-4). The first, naturally, is (1)

⁴⁰⁰Buridan [2016], Eds. Streijger and Bakker, 133-200. This set of questions is particularly long, and it involves much more material than I will cover in this section. For an overview of these questions, see Sylla's Guide to the Text in Sylla [2016], cxiv-cliii, and on Buridan's stance on infinity and continuity in general, Murdoch & Thijssen [2001].

⁴⁰¹Buridan [2010], Eds. Streijger, Bakker and Thijssen, 61-66.

⁴⁰²The main statements of Aristotle's regarding the composition of *continua* are found throughout the sixth book of the *Physics*, in particular, see *Physics* 6.1 231a21-b5 and 231b15-18 (Aristotle) [1995a], Ed. Barnes, Transl. Hardie and Gayer, 861-2).

⁴⁰³For an overview of the problem, see Murdoch [2009]; Murdoch [1998]; and Grellard [2011]. Although the position that *continua* are composed of indivisibles was a minority view in the Middle Ages, it has been progressively recognized by scholars as an important feature of later medieval debates on the composition of *continua*.

divisibilism itself, the view that a *continuum* is not composed of indivisible parts, but rather of parts divisible without end. Secondly, *non-entitism* describes the position that indivisible things (*res indivisibiles*) do not properly exist in the physical world, and even though the concept of them may be used to describe reality, they are taken as non-entities, useful products of mathematical imagination. The third type of position Zupko distinguishes will be important for my purpose, namely *infinitism* - that is, the position according to which the parts of a continuum are infinitely divisible, namely that they constitute a potentially infinite set. 404

At *QDGC* I.5, Buridan addresses the question of whether a a body is divisible into any of its assignable parts (*signa*). and raises a number of problems for a positive answer to the question which he intends to solve by applying logical tools. At the outset of the question, it is clear that Buridan's intention is not to provide a defense of divisibilism itself - which he already presupposes -, and that his intention is rather to inquire into what are the logical consequences of the proposition that every part of a body can be divided. As he phrases matters, the intention of the question is to inquire into what would follow from the possibility that any part of a natural body is divided in the sense of 'real separation' of one part from another (Buridan [2010], Eds. Streijger, Bakker

⁴⁰⁴Rega Wood provides a summary of some of divisibilist views in the late middle ages varying along these parameters (Wood [1988].) Thomas Bradwardine († 1349) is typically mentioned in this context as phrasing the standard divisibilist view of the 'moderns' which, following Aristotle and Averroes, holds that a continuum is not composed of atoms, but is rather composed of parts divisibile without end. 'Nonentitism' is a category Rega Wood proposes to classify Ockham's and Adam Wodeham's († 1358) positions, according to which terms signifying indivisibles do not pick out any real entity in the physical domain. Lastly, Walter Chatton († 1344) is mentioned as an important example of a divisibilist who rejects infinitism, since he believed that a continuum is composed of a finite number of divisibles.

⁴⁰⁵Cf. Buridan [2010], Eds. Streijger, Bakker and Thijssen, 61: "In the fifth question, it is asked whether a body is divisible according to any of its assignable parts (*signum*) and according to any point, not speaking of its terminating but its continuing points, since no one would claim that a line is divisible over its extreme points" ["Quinto quaeritur utrum corpus sit divisibile secundum quodlibet signum eius, non loquendo de punctis terminantibus, sed de punctis continuantibus, quia nullus diceret quod linea esset divisibilis super extremum punctum eius."] '*Signa*' usually means the potential result of a division in this context. For example, Nicole Oresme uses *signatio* to describe the result of the activity of dividing - without entailing a real separation - in his commentary on the *Physics* (Oresme [2013], Ed. Caroti *et al* VI.3, 671-2).

and Thijssen, 62):

(T7.3) Note that it is not relevant to the present purpose whether there are indivisible points in a line or whether nothing in a line is indivisible, since if nothing is indivisible in a line, as I take it to be the case, so that points are not indivisible things, then the sense of the question will be whether a body is divisible in all of its parts, speaking of division by discontinuation of one part from another by real separation ⁴⁰⁶

In the following, Buridan lists three scenarios in which, from the proposition that a body is divisible into all of its parts, there results something (at least naturally) impossible, for the actual result of all possible divisions of the parts of a natural body would entail either of these three situations: (1) there would remain nothing, which means that the natural body is composed from nothing; (2) there would remain extensionless points, which means that indivisible entities would have a positive ontological status as physical parts of a continuum, which goes against Aristotle's position; or (3) there would remain magnitudes having extension, which means that the body is composed of extended magnitudes which cannot be further divided, which goes against the definition of extended magnitudes as things having 'part outside of part' (pars extra partem). ⁴⁰⁷ If it is

⁴⁰⁶Buridan [2010], Eds. Streijger, Bakker and Thijssen, 62: "Nota. Non est cura ad praesens utrum in linea sint puncta indivisibilia aut quod nihil sit indivisibile in linea, quia si nihil est indivisibile in linea, sicut credo esse verum, ita quod puncta non sunt res indivisibiles, tunc sensus quaestionis erit utrum corpus sit divisibile in omnes eius partes loquendo de divisione per discontinuationem partis de parte per realem separationem".

⁴⁰⁷Buridan [2010], Eds. Streijger, Bakker and Thijssen, 61: "It is argued first that a body is not divisible into any of its continuating points. And that is stated in Democritus' arguments in the following way, since 'divisible' and 'able to be divided' mean the same, but a body is not able to be divided into any of its assignable parts, therefore etc. The minor is proved in the following way: if some possibility is assumed to be actual no impossibility should follow, as stated in the *Prior Analytics*; therefore, if a body is able to be divided into any of its assignable parts, assuming that it is divided into any of its assignable parts should not entail any impossibility. However, the impossible seemingly follows, since either nothing remains after that division, or there remain extensionless points, or there remain magnitudes having extension." ["Et arguitur primo quod corpus non sit divisibile secundum quodlibet punctum eius continuativum. Et arguitur per rationem Democriti quia: idem significant 'divisibile' et 'posse dividi'; sed corpus non potest dividi secundum quodlibet signum eius, igitur etc. Minor probatur quia:

natural possibility that Buridan has in mind, then each of these options involves unacceptable posits concerning the composition of continuous entities. 408

Buridan's reply states that the difficulty involved in this question is foremost a logical problem related to the Possibility Principle, since "it is not clear how that logical principle should be understood, namely [the principle that] if the possible is assumed to be actual nothing impossible should follow" (Buridan [2010], I.5, 62) [409] Buridan unpacks the proposition in question as a modal claim, in the sense that being 'divisible' ('divisibile') amounts to 'being able to be divided' ('posse dividi'), and in order to identify whether it is a genuine possibility, he refers to the principle - stated by Aristotle in the *Analytica Priora* A13, and repeated by Buridan in many contexts - that if something possible is assumed actual, no impossibility should result:

Possibility Principle: If something possible is assumed actual, no

possibili posito in esse nihil sequitur impossibile, ut habetur primo *priorum*; igitur si corpus posset dividi secundum quodlibet signum eius, posito quod esset divisum secundum quodlibet eius signum, nihil deberet sequi impossibile; et tamen sequeretur impossibile, quia vel nihil resultaret ex illa divisione, vel remanerent puncta, scilicet nullam extensionem habentia, vel remanerent magnitudines extensiones habentes."]

⁴⁰⁸Cf. Buridan [2010], Eds. Streijger, Bakker and Thijssen, 61-62: "If it were said that nothing would remain, that is impossible, since then the body would be composed of nothing, whence at the end it is divided into nothing. If it were said that points remained, then a magnitude would be composed out of points; which goes against the Philosopher in the sixth book of the *Physics*. If it were said that magnitudes still having extension would remain, then it is clear that in them there would be still assignable parts (signa) over which division has not yet been made; which goes against our initial assumption. Therefore, it is clear that from the posit the impossible follows. Hence it is not possible that a body can be divided over any of its assignable parts". ["Si dicatur quod nihil remaneret, hoc est impossibile, quia tunc ex nihilo fuisset illud corpus compositum, exquo finaliter in nihil divideretur. Si dicatur quod remanerent puncta, tunc magnitudo esset composita ex punctis; quod est contra Philosophum sexto Physicorum. Si dicatur quod remanerent magnitudines adhuc extensionem habentes, tunc patet quod adhuc in eis essent signa super quae non est facta divisio; quod est contra positum. Ideo patet quod ad illud positum sequitur impossibile. Igitur non est possibile quod corpus dividatur super quodlibet signum."]

⁴⁰⁹Buridan [2010], Eds. Streijger, Bakker and Thijssen, I.5, 62: "Note that the difficulty in this question comes mainly from logic, since it is not clear how that logical principle should be understood, namely [the principle that] if the possible is assumed to be actual nothing impossible should follow." ["Nota quod difficultas illius quaestionis est principaliter ex logica, quia non est bene notum quomodo illud principium logicum debet intelligi, scilicet 'possibili posito in esse nihil sequitur impossibile'."]

impossibility should result.

The logical problem at hand concerns the same pattern of inference we have seen at 2.3. The Possibility Principle is applied to inquire what follows from a universal affirmative divided proposition of possibility, if we assume it as actual. The circumstance in which every part of a body is divided is however naturally impossible, since the result of all its possible divisions would yield each of the three naturally impossible circumstance as we have seen above. [410]

In addressing the problem, Buridan uses the same tools he had used in the TC as we have seen above in section 2.3. It is helpful here to recall how Buridan's statements concerning inferences from divided to composite modal propositions A As we have seen, the following inference is invalid in Buridan's syllogistics, namely:

The counterexample that Buridan had proposed in the *TC* involves a case where each of the referents is metaphysically incompossible, and thus the conclusion is impossible while the premise is true. The same example is brought up in his *QDGC* I.5 as well, namely 'Every star possibly is shining in our hemisphere,' which has an impossible assertoric counterpart if stated in its corresponding universal form ('Every star is shining in the sky'), since all stars cannot be shining on the same hemisphere of the earth simultaneously. Recall also that in the former context Buridan suggests how the universal possibility affirmative should be transformed into a proposition about actuality. We have

⁴¹⁰Buridan [2010], Eds. Streijger, Bakker and Thijssen, 63: "Deinde omnes communiter bene concedunt istas propositiones esse impossibiles 'corpus secundum quodlibet eius signum vel punctum est divisum', vel 'corpus secundum quodlibet eius signum dividitur'. Vel etiam secndum modum loquendi proprium haec est impossibilis: 'corpus in omnes eius partes est divisum', quia, sicut arguit Democritus, quaereretur quid remaneret, utrum nihil vel puncta vel magnitudines adhuc habentes extensionem; et non potest assignari, ut prius dictum est. Similiter haec est impossibilis 'in infinitum corpus est divisum', vel 'in infinitum corpus dividitur', quoniam quantumcumque corpus sit divisum, tamen non in tot partes est divisum quod non in plures; igitur non in infinitum."

⁴¹¹Recall that composite modal propositions, for Buridan, are ultimately assertoric propositions, or propositions 'about actuality' (*de inesse*).

seen that as Buridan's analysis of universal distribution in possibility propositions had it, should the distributed subject be posited in existence (*ponere inesse*) - or, should the universally distributed possibility proposition be transformed into many singular *de re* predications about existence (*de inesse*) -, then the universal distribution of the subject should be analyzed in terms of a disjunction of *re de* singular predications, but not collectivelly - since the singular predications would be incompossible with each other. In other words: it is true of each star that it is possibly shining in our atmosphere, but taken collectively, it is impossible that every star is simultaneously located in the same side of the globe. [412]

This analysis is made taking into account the metaphysical compossibility between *de re* predications. But when Buridan turns to the problem of infinite divisibility, the case differs in important ways. For example, consider the proposition Buridan discusses at *QPhys*. III.19: 'God can separate all parts of line B.' While in the first scenario we were dealing with a natural impossibility (and incompossibility), in the latter we are dealing with a supernatural possibilities (and compossibilities). Buridan writes (Buridan [2016], 196, Eds. Strijger and Bakker):

(T7.4) But you will object that the former example does not apply to the purpose at hand, since of the example concerning the stars any (of the singular propositions) is possible, but not they are not all compossible; but in the case at hand with regard to the fourth conclusion all singulars are possible and compossible; therefore the assertoric universal should be also possible.

By the fourth conclusion mentioned in this passage, Buridan is referring here to the example where 'every part of a line B God can separate from each other and separately conserve.' In comparing both examples, Buridan says in his

⁴¹² See again 2.3.

⁴¹³"Sed tu replicabis quia est dissimile de dicto exemplo et de proposito nostro, quia licet de astris quaelibet sit possibilis, tamen non omnes sunt compossibiles; sed in proposito quantum ad quartam conclusionem omnes singulares sunt possibiles et compossibiles; igitur universalis de inesse debet esse possibilis."

⁴¹⁴Buridan [2016], Eds. Streijger and Bakker, 195: "The fourth conclusion is that every part of a line b God can separate from each other and separately conserve, since

reply that differently from the case of the stars, the *de re* singular instances of 'every part of a line B God can separate' are compossible as we shall see below.

The special problems concerning the interpretation of modal propositions with regards to infinity are addressed at *QPhys*. III.19. Buridan mentions that there are special difficulties concerning the infinite with respect to propositions about the possible (*restant difficultates de infinito quantum ad propositiones de possibili*), in particular, concerning the possibility of an infinite magnitude and that in infinite parts a line is divided (*Utrum possibile sit infinitam esse magnitudinem et in infinitas partes lineam esse divisam*). It is in that context that Buridan takes the following to be a true proposition:

1 God can separate and separately conserve all parts of line B

One part of the positive case Buridan mentions is that God can know distinctly every part of a line B, as distinct as two singular substances as Plato and Socrates. Hence, it is possible that all infinite parts of a line B are actually separated in the supernatural case. Buridan addresses the question, once more by referring to the inferences between universal divided possibility propositions and assertorics (Buridan [2016], Eds. Streijger and Bakker, 195):

(T7.5) The forth conclusion states that every part of a line B God can separate from each other and separately conserve, for example these two, and these hundred, and so of all other [parts]. To none of these singular propositions a counterexample can be formulated, unless the counterexample concerns all of them collectively; and it was said

these two, these hundred, and so of other singulars [God can divide and separately conserve]. No counterexample can be found of each singular, only of 'every' taken collectively; but it was said earlier that 'every' should not be taken in the collective sense." ["Quarta conclusio est quod omnes partes lineae b Deus potest separare ab invicem et separatim conservare, quia et istas duas et istas centum et sic de singulis. Nullae enim sunt de quibus posset dari instantia, nisi daretur de omnibus collective; et dictum est prius quod omnes non sunt capiendo 'omnes' collective."]

⁴¹⁵Buridan [2016], Eds. Streijger and Bakker, 187: "Item Deus cognoscit omnes partes lineae *b* distincte, ita distincte sicut Socratem vel Platonem. Igitur potest inter omnes discernere et omnes numerare. Ex quo squitur quod omnes actu discernit et numerat, quia non est in eo potentia intelligendi distincta ab actu etiam completo. Sed inter quaecumque ipse discernit et quaecumque numerat, possibile est quod ista dividad. Igitur etc."

before that every should not be taken as 'every' in the collective sense 416

When Buridan states that God can separate each possible part of a line, but not all at the same time, the case here seems similar to the stars in the sky we have already seen. But as Buridan says at (T7.4), both cases are different in that God can separate all parts of the line, an thus the corresponding singulars - 'this part God can separate, and that part God can separate, etc' - are all compossible by supernatural possibility.

Buridan replies by reformulating his logical rules, which were made for natural compossibility, for the supernatural scenario at hand. The corollary he concludes is that some proposition of universal possibility is impossible, even though their singular instances are compossible. He adduces the following corollary (Buridan [2016], Eds. Streijger and Bakker, 196):

(T7.6) Hence a corollary should be concluded, namely that some universal [proposition] is impossible, although all of its singulars are possible and compossible. Therefore, in order for all singulars to be possible and compossible it is not required that the corresponding universal be possible, it is rather required that all [singulars] can be simultaneously true if they are formulated, or even further that of every and each [singular] it is possible that things are in the way that each signifies simultaneously. So it is in the case at hand. And furthermore this is impossible, namely, 'in whatever way every and each [singular proposition] can signify (things be), if they are formulated, so things are [417]

 $^{^{416}}$ Quarta conclusio est quod omnes partes lineae b Deus potest separare ab invicem et separatim conservare, quia et istas duas et istas centum et sic de singulis. Nullae enim sunt de quibus posset dari instantia, nisi daretur de omnibus collective; et dictum est prius quod omnes non sunt capiendo 'omnes' collective.

Buridan [2016], Eds. Streijger and Bakker, 196-7: "Dico quod hoc non sequitur. Sed bene sequitur quod omnes singulares possunt esse simul verae; tamen impossibile est quod omnes sint simul verae. Semper enim in proposito deficit consequentia de divisa de possibili ad compositam stante universalitate. Unde corollarie concludendum est quod aliqua universalis est impossibilis, cuius tamen omnes singulares sunt possibiles et compossibiles. Ad hoc enim quod omnes singulares sint possibiles et compossibiles non requiritur quod universalis sit possibilis, sed requiritur quod omnes possunt esse simul

Is that in tension with the remark concerning metaphysical compossibility made for the natural cases - as we have seen in 2.3 -, according to which a proposition is compossible if (1) each of its singulars is possible and (2) they can be simultaneously true? Whan prevents the corresponding universal distribution to be true when they can be simultaneously the case here?

Buridan's explanation has clearly to do with the problems concerning infinity, since although God can separate all possible parts of a line, there is no simultaneous separation of all possible divisions. Buridan's statement goes as follows (Buridan) [2016], Eds. Streijger and Bakker, 199):

(T7.7) Similarly, I believe that this is true, namely 'God can separate and separately conserve all parts of line B', since every singular, with respect to the singularity corresponding to the universality of 'all parts', are possible and compossible, and can be simultaneously true, although it is not possible that all are simultaneously true. [419]

Buridan, therefore, uses the same notion of metaphysical compossibility as we have seen in 2.3, and he has not changed the concept of compossibility for the supernatural case. The key difference between both cases seems to be is that the second involves problems regarding infinity. Whereas God can separate all parts of a line or all divisions of a body, he can do so in the sense of an infinite succession, but not simultaneously.

verae, scilicet si prononantur, vel etiam quod de omnibus possibile est, qualitercumque significant, ita simul esse. Et si est in proposito. Et tamen haec est impossibilis 'qualitercumque omnes significarent, si proponerentur, ita est."

⁴¹⁸For Buridan's approach to the categorematic and syncategorematic senses of infinity, see Murdoch & Thijssen [2001].

⁴¹⁹"Ideo nullo modo sequitur propter praedicatum appellare formam quod, si ego possum videre omne astrum, quod ego possum videre omne astrum simul, sed sufficit quod ego possum videre hoc astrum et quod possum videre illud et sic de aliis, licet successive unum post alterum. Ita similiter opinor quod haec sit vera 'Deus potest separare et separatim conservare omnes partes lineae *b*', quia omnes singulares, quantum ad singularitatem correspondentem isti universalitati 'omnes partes', sunt possibiles et compossibiles et possunt esse simul verae, licet non sit possibile quod omnes sint simul verae."

7.3 Indirect Demonstration in the *Physics*

The distinction between supernatural and natural modalities has also an important role in the use of indirect demonstrations in natural philosophy - that is, chains of argument that show the impossibility of a premise by deriving a contradiction from it -, which Buridan calls demonstrations *ad impossibilem*. Buridan has elaborated on indirect demonstration in his *SD* 8.10.2 and 8.11.7 (Buridan [2001b], Transl. Klima, 784, 807), and claimed that such demonstrations are composed of several rules and syllogisms. Its unity is not that of a single valid consequence but an aggregate (*aggregatum*) of many consequences. The main consequence is a syllogism starting from a pair of premises, one of which is known to be false and the other evidently necessary, and the first premise is the contradictory of a conclusion to be shown. Through a further chain of argument, the first premise is then shown to be impossible by the rule that if one contradictory is true the other must be false. A crucial part of the procedure is that the second premise assumed along with the first one cannot be an impossibility of its own nor incompatible with any other possibility. The

⁴²⁰I aim using an analogous translation adopted by Gyula Klima, which renders *demonstratio ad impossibilem* as 'indirect proofs' (Buridan [2001b], Transl. Klima, 784ff.), but I adopt here 'demonstration' instead of 'proof,' keeping closer to the historical usage.

⁴²¹Namely, the rule numbered [III] in Buridan [2001b], Transl. Klima, 784: "But an indirect demonstration is an aggregate of four consequences. The first [I] is a syllogism that, from one premise that is the contradictory of the conclusion to be proved and another evident premise, concludes to an evidently impossible conclusion. The second [II] consequence is the one that from the impossibility of the conclusion infers that one of the premises is impossible, or at least that the premises are incompossible [incompossibiles], by means of the principle that impossible or incompatible premises entail something impossible. The third [III] consequence adds to the conclusion now inferred that the premise assumed along with the premise that is opposite to the conclusion to be proved is neither impossible nor incompatible with anything possible, for it is evidently necessary; it then concludes from the other premise by means of the locus from division that the other premise, namely, the one opposite to the conclusion to be proved, is impossible. And the fourth [IV] consequence concludes from this last conclusion by means of the locus from contradictories based on the first principle that the conclusion to be proved is necessary, this mean that which was to be demonstrated from the outset. For it is necessary that if one of two contradictories is impossible, then the other is necessary, or that if one is true, then the other is false, given that they are propounded at the same time." cf. Buridan [2001a], Ed. De Rijk, 201-2: "Demonstratio autem 'ad impossibile' congregata est ex quattuor consequentiis. Prima est syllogismus

important question in this regard concerns the modal status of the assumptions in the indirect proof. Buridan's account of logical consequence commits him to the EIQ (*ex impossibili quodlibet*) principle, as we have already seen in chapter 2.1. As Buridan uses this principle, in his view no logical impossibility can serve as a premise in indirect demonstrations - since a logical impossibility entails anything in accordance with the EIQ -, and instead the premises in indirect demonstrations can only concern what is naturally impossible.

Buridan's view that indirect proofs cannot contain a logically impossible premises is not correct from a modern standpoint. It is a common practice in logic to use logically impossible assumptions in indirect proofs. However, qualifying the premises of indirect demonstrations as naturally impossible, but logically possible in themselves, seems to be central to Buridan's uses of this scheme of argument in natural philosophy as we shall see.

Knuuttila and Kukkonen have drawn attention to one text where this is particularly clear, namely the seventh book of the *Physics*. where the interpretation of Aristotle's indirect demonstration of a first mover is at issue (Knuuttila & Kukkonen [2011], Knuuttila [1989]). The arguments presented by Aristotle in the original context of *Physics* 7 to establish a first mover are quite convoluted, and its details will not need to concern us in greater depth. As interpreted by

ex una praemissa contradictoria conclusioni probandae et alia praemissa evidenti concludentibus conclusionem evidenter impossibilem. Secunda consequentia est quae ex impossibilitate illius conclusionis infert quod una praemissarum sit impossibilis, vel saltem quod illae praemissae sint incompossibiles, per illud principium quod ex impossibilibus vel incompossibilibus sequitur impossibile. Tertia consequentia illi conclusioni nunc illatae addit quod illa praemissa assumpta cum opposita conclusionis demonstrandae nec est impossibilis nec alicui possibili incompossibilis, cum ipsa sit evidenter necessaria; et concludit per locum a divisione quod altera praemissa, scilicet conclusioni probandae opposita, est impossibilis. Et quarta consequentia ex hac ultima conclusione per locum a contradictoriis fundatum supra primum principium concludit conclusionem probandam esse necessariam."

⁴²²In modern logical practice, formal logical truths are usually proven by assuming the negation of their opposites, and deriving a contradiction from it. It is relevant to mention, however, that for Buridan 'logical impossibility' covers a wider category. In the case at hand, conceptual impossibilities such as 'a human being is donkey' would count as a logical impossibility in Buridan's sense.

⁴²³The central passages in this regard are *QPhys.* 7.1-5 in Buridan [1509].

⁴²⁴Knuuttila adopts the interpretation put forward by Robert Wardy (Wardy [1990], in particular 93-120). Since I will not be concerned here with the details of Aristotle's argument and with his concept of an unmoved mover, I will follow Knuuttila's

Knuuttila and by Wardy, the relevant part of Aristotle's argument states that an infinite chain of moved movers is impossible, in order to conclude that any chain of moved movers must have a first mover which is unmoved, and is not a part of that chain in the same way as the other movers. That conclusion is established since assuming its opposite leads to an impossibility, namely, that an infinite series of moved movers would perform an infinite movement in a finite time. [425]

In order to derive the above-mentioned impossibility, Aristotle makes use of an assumption that will be important to address Buridan's qualifications concerning the modal status of the premises in the indirect proof, namely, that the infinite chain of moved movers (which is shown to be impossible) must move simultaneously and constitute a unitary movement, not in the sense that the number of moved movers is infinite, but rather that in the sense that the members of the chain are in touch and continuous to each other. Aristotle replies to an objection stating that nothing impossible follows from assuming an infinite chain of finite moved movers, since each member of that chain performs a finite movement if the series is imagined constituting an infinite plurality of movements instead of a unitary movement. Buridan renders the

interpretation for my purpose in this section. For a contemporary appreciation of the argument in Aristotle's *Physics* 7.1, see Rosen & Malink [2012], and for the conception of the Aristotelian first mover as a final cause as developed further in other contexts such as the *Metaphysics* 12.7, see Sorabji [1988], 222-226.

⁴²⁵Knuuttila's interpretation of the indirect demonstration running through *Physics* 7 242a49-243a31 goes as follows (Knuuttila [1989], 65): "Aristotle then puts forward a longer *reductio* argument purporting to prove that any sequence of causally dependent movers must terminate and that, consequently, there must be a first moved mover in any given sequence of movers and a first mover which is not a member of the finite dependent sequence of moved movers. The reductive premise is that if there is a finite movement, say A, during a finite time and an actually infinite hierarchy of simultaneous finite movers related to A, an infinite movement is performed in a finite time. This is impossible (242a49-242b43). Someone could object that assuming an infinite number of movements is not the same as assuming an infinite movement. Aristotle answers that a proximate mover is either in touch or continuous with what it moves; since the movers constitute a unity, the motion they execute is unitary, and since the movers are infinite, the movement is infinite (242b53-243a31)." I will quote the relevant text referred to here in a moment below.

⁴²⁶Cf. Aristotle [1995a], Ed. Barnes, Transl. Hardie and Gaye, 900-1:"It might be thought that what we set out to prove has thus been shown, but our argument so far does not prove it, because it does not yet prove that anything impossible results; for in a finite time there may be an infinite motion, though not of one thing, but of many: and in the case that we are considering this is so; for each thing accomplishes its own motion, and

objection as stating that this infinite series of moved movers could be infinite 'according to multitude,' and this would not constitute a unitary movement as required to derive the impossibility for the *reductio*. As Buridan restates the

there is no impossibility in many things being in motion simultaneously. But if (as we see to be universally the case) that which primarily moves locally and corporeally must be either in contact with or continuous with that which is moved, the things moved and the movers must be continuous or in contact with one another, so that together they all form a unity: whether this unity is finite or infinite makes no difference to our present argument; for in any case since the things in motion are infinite in number the motion will be infinite, if it is possible for the motions to be either equal to or greater than one another; for we shall take as actual that which is possible. If, then, A, B, C, D form, either finite or infinite magnitude that passes through the motion EFGH in the finite time K, it follows that an infinite motion is passed through in a finite time: and whether the magnitude in question is finite or infinite this is in either case impossible. Therefore the series must come to an end, and there must be a first mover and a first moved; for the fact that this impossibility rests on an assumption is immaterial, since the case assumed is possible, and the assumption of a possible case ought not to give rise to any impossible result." For commentary on the meaning of being 'in contact' relevant to this passage, see Wardy [1990], 121ff.

⁴²⁷Buridan [1509], 95ra: "Aristotle states that the argument can be objected by someone saying that, although it was proved in the sixth book of the *Physics* that it is impossible for there to be an infinite movement occurring in finite time, this does not establish that it is impossible for there to be an infinite number of movements according to multitude ('secundum multitudinem'), any movement of which is finite, and in that case nothing impossible follows from the posit of the adversary, therefore no ensuing impossibility can be held against him. These infinite number of movers would not constitute a unitary body nor an unitary movable thing, therefore neither is their movement unitary, they would rather constitute a plurality of finite movements. Against this mistake Aristotle replies supposing that the infinite number of movers would be simultaneously touching each other without any intermediary - as was explored in another question -, in such a way that if b moves a, c moves b, and d moves c, and so on to infinity, then b is first in relation to a and in touch with it, similarly c is first in relation to b and in touch with it, and so on to infinity. After that Aristotle supposes that it is possible for all bodies which are proximate and continuous to each other to be in touch (contiguari) and so to constitute a whole movement if they are simultaneously moving. If that possibility is assumed to be actual nothing impossible should follow unless that was impossible in the first place -, but according to the former exposition of these infinite movers an impossibility does follow, namely that a unitary infinite movement occurs in finite time. Therefore, the objection posed by the adversary is impossible." ["Sed dicit Aristoteles quod contra istam rationem posset aliquis cauillare dicens quod in sexto bene probatum est quod impossibile est in temporem finito esse infinitum motum sed non erat illic probatum quod impossibile sit in tempore finito esse infinitos motus secundum mulitudidinem, quorum tamen quilibet esset finitus, et nichil plus sequitur ex positione aduersarii, ideo nihil impossibile sequitur contra eum. Illa enim infinita mouentia non essent unum corpus nec unum mobile, ideo nec esset eorum assumption made by Aristotle in reply to that objection, he needs to assume as possible that every member of that chain can constitute a unitary movement, since every body sufficiently proximate to each other can constitute a whole by being in immediate contact. That auxiliary assumption was regarded as problematic, since it contradicts Aristotle's physical principles, by requiring that different kinds of bodies constitute a whole bodily movement. The problem as Buridan found it is that Aristotle assumed as possible something which is a patent impossibility, since the celestial bodies and their laws of motion are are essentially distinct from the sublunary bodies. As Knuuttila has shown, what seemed problematic for Buridan is a logical misuse of the modal assumption in the argument - since by the EIQ an impossibility entails anything, the indirect demonstration breaks down once an impossible assumption is built into it (Knuuttila [1989]], 66).

Buridan's solution consists in distinguishing between the modal status

unus motus, sed multi quorum quilibet esset finitus. Contra hanc cauillationem replicat Aristoteles supponens quod ista infinita mouentia essent simul adinuicem tangentia se sine intermedio et hoc ostendetur in alia questio, ita quod si b mouet a, et c mouet b, et d mouet c et sic in infinitum, b est primum ipsi a et tangens et c ipsi b et sic in infinitum. Postea supponit Aristoteles quod possibile est omnia corpora sibi inuicem proxima et continua contiguari ad inuicem et sic fieri unum corpus et per consequens eorum esse unum motum si simul moveantur. Si ergo ponatur quod hoc possibile inesse nulum debet inde sequi impossibile, nisi hoc sit ex impossibile, quo accepto cum expositione infinitorum talium mouentium sequitur impossibile, scilicet unum motum infinitum esse in tempore finito, ergo posito illa aduersarii est impossibilis."]

⁴²⁸Buridan [1509], fol. 95ra: "Against this objection Aristotle replies making the supposition that these infinite movers as simultaneously touching one another without any intermediary [...] Then Aristotle supposes that it is possible for every body sufficiently proximate to one another to be continuous and thus to make up one whole, and thus its movement would be also one, and they would move simultaneously."

⁴²⁹Buridan [1509] 95ra: "But against Aristotle's argument there is an important doubt since it proceeds from an impossible premise, according to which every body sufficiently proximate to one another can constitute a continuous movement. This is impossible concerning the celestial spheres with each other, and if that is not absolutely impossible - since it is possible by divine power -, Aristotle nonetheless took it to be impossible, therefore the impossible premise is of no use." [" Sed contra istam rationem Aristotelis est fortis dubitatio quia procedit ex positione impossibili, scilicet quod omnia corpora inuicem proxima possent fieri continua. Hec enim est impossibilis de species celestibus adinuicem vel etiam ad speram ignis, et si hoc non sit simpliciter impossibilem quia est possibile per potentiam duinam, tamen hoc credidit Aristoteles esse impossibile, immo male usus est illa positione."]

of assumptions in indirect demonstrations, and by reading into Aristotle's text his own distinction between logical possibilities by supernatural power and natural possibilities by natural powers. While logical impossibilities trivialize arguments, natural impossibilities can be legitimately employed in indirect demonstrations. Importantly, Buridan advances his own solution by constant reference to Averroes' conception of indirect demonstrations. As Kukkonen has argued, Averroes understands the problematic premises in indirect demonstrations on the basis of a distinction between essential and accidental impossibilities. Essential impossibilities cannot serve as premises in indirect demonstrations, while accidental impossibilities can figure in them without violating the modal principle underlying the Aristotelian procedure. In the Averroan view as understood by Knuuttila and Kukkonen the key dimension for distinguishing essential impossibilities from accidental ones is that whereas in the former case a conceptual contradiction is involved, in the latter case something can be generally regarded as possible on the basis of an abstract

⁴³⁰Cf. in particular Buridan [1509] 95rb, which I will quote at length below.

⁴³¹Averroes construes Aristotle's procedure of reasoning from impossible assumptions in several places (see Kukkonen [2005],449-451 for a useful survey), and his general interest on Aristotle's argumentative practices involving indirect proofs is manifested by the frequent quotes of the Analytica Priora. For example, in the analogous indirect demonstration that there must be a prior eternal and circular motion which is first to all finite motions, and whose first mover must be itself unmoved, Averroes claims that Aristotle sets out to prove this by "a demonstration that it is impossible for an infinite action to proceed from a finite force" (Averroes [1991], Ed. and Transl. Goldstein, 29), and refers to the general strategy of indirect demonstration as relying on the *Prior* Analytics. Cf. Averroes [1991], Ed. and Transl. Goldstein, 29: "This impossible conclusion follows because we assumed two propositions, about the first of which, namely, that there is a finite force which causes motion for an infinite time, there is some doubt; while the second proposition, namely, that whatever finite force we take, we can assume a greater finite force, is known to be possible. The impossible conclusion must follow from one of these two propositions, but it cannot follow from the proposition known to be possible; as demonstrated in *Prior Anaytics*, an impossible falsehood cannot follow from a possible falsehood."

⁴³²Kukkonen [2005], 452: "According to Ibn Rushd, Aristotle systematically distinguished between degrees of impossibility: there are accidental as well as essential impossibilities, and the same rules apply for these as for the possibly and the impossibly false. From an accidental impossibility only an accidental impossibility follows, whereas an essentially impossible conclusion implies an essential impossibility lurking somewhere among the premises."

⁴³³Knuuttila & Kukkonen [2011].

consideration of its general features. According to this model, accidental impossibilities can be conceived by attending to the general features of natures and abstracting away their specific natures. For example, a human being *qua* human being cannot fly, but insofar as human beings are considered as belonging to the genus of animals, there is nothing incompatible in their nature considered at this level with the capacity of flying.

Similarly, in the concrete case at hand, movement *qua* movement is not incompatible with constituting a unitary whole composed of different kind of movers, although the celestial spheres cannot by their specific natures be part of such a chain. As Knuuttila claims concerning Averroes' distinction between essential and accidental impossibilities, these "impossibilities do not belong to the same level of analysis. Counter-possible possibilities refer to abstract entities, such as a body *qua* body or motion *qua* motion, which do not exist as such." (Knuuttila [2012], 320) This was a crucial part of Averroes' solution to the modal part of Aristotelian indirect proofs, and it explains Buridan's frequent references to the Commentator in the *Physics* 7.1 which we will turn to below.

ABSTRACT POSSIBILITIES: If a specific property is not incompatible with the genus of some individual thing, it is not incompatible with the the

⁴³⁴One example is in the following question from Averroes' *Questions in Physics*, corresponding to *Physics* 8.10 (Averroes) [1991], 31): "It is obvious that the proposition that Aristotle assumed in the first proofs, namely, his assumption of a body larger than the heavens, is possible with reference to a body *qua* body, but accidentally impossible with reference to the heavenly body by reason of the fact, for instance, that all other bodies are contained by the heavenly body, as Aristotle said when he gave the cause of the world's being one. It is [also] obvious that the impossibility which follows from the syllogism in which [this] premise was assumed is an absolute impossibility, namely, the impossibility of the greater force not causing motion for a longer time. Thus it is clear that this impossible conclusion does not follow from the syllogism from which it was inferred because an essentially possible, but accidentally impossible, premise was assumed in the syllogism: namely, the proposition stating that whatever body qua body [is postulated], it is possible to assume a larger body. Rather, it must follow from a premise which is impossible by necessity, seeing that the conclusion is [absolutely] impossible."

⁴³⁵This is the example John of Jandun uses, see below (T7.8).

⁴³⁶Kukkonen [2005], 453: "Adopting this line of explanation, Ibn Rushd is able to solve a number of cosmological puzzles that had vexed previous commentators. Whenever something apparently impossible is being postulated with regard to the cosmic system, this is always done in the general, not the specific sense, and the argument is meant to be handled on that level."

thing.

Kukkonen explains the abstractionist model using notions of *species* and *genera* in the following way (Kukkonen [2005], 451):

Each time the Commentator asks us to consider 'the impossible insofar as it is possible', what he is doing is in effect climbing back down Porphyry's tree. What comes into focus through this process of abstraction is the more general sort of being the creature under examination is - the species underlying the individual, the genus underlying the species, etc. The possibilities open to the wider kind can be examined independently, with the end results being brought to bear on the original question. What holds true for the genus, after all, will hold for every species subsumed under it. At the same time, the abstraction allows one to focus on just those features that matter with regard to the issue at question.

The main feature of this model is that it makes no appeal to the distinction between supernatural and natural possibilities. Within this background, Averroes' abstractionist model is spelled out by John of Jandun in the same context as Buridan's discussion on the *Physics* 7, and it is very likely that Buridan has Jandun in mind. Let us describe Jandun's own solution (which largely follows Averroes' own) to the distinctions involved in the modal status of the premises in indirect proofs. In his questions on the *Physics*, Jandun applied the same distinction between essential and accidental possibilities to the indirect demonstration of the first unmoved mover (Jandun [1488], fol. 198vb):

(T7.8) And some say that moving and moved bodies can be considered in two ways. In one way they can be considered as quantities and continuous things (*quanta et continua*). In another way, they can be considered according to their proper and determinate natures by which they are so and so (*quibus sunt talia*). In the first way it is possible for every mobile thing to be made one continuous thing, since it is not repugnant to their natures as quantities (*quanta*) that they may be or constitute a continuous thing.

^{437&}quot;Et immo alii dicunt aliter quod ista mobilia et mouentia possunt considerari

In attending to bodies *qua* bodies, things that do not actually exist, it is possible to say of them that they can make a whole continuous movement, even though that is impossible attenting to their concrete individual natures. The underlying conception of modalities behind the abstractionist model is explained by Jandun as follows (Jandun) [1488], 109vb-110ra):

(T7.9) In accordance with one way of speaking we take the contingent and possible to be that which is applied to a genus and to a species. Since everyone accepts as impossible everything that is repugnant to the *genus* or *differentia* of species out of which the concept of the species is constituted. While truly considering *genera* it is taken as possible all that which is not repugnant to the concept of the genus, even if it is repugnant to the constituting difference of a species. Just as in talking about animal in general I can take it as possible that every animal has wings, but if I descend to the consideration of human beings it would be impossible for this animal to have wings. [438]

Taking accidental impossibilities as abstracted features of concrete things, Jandun thus understands them to be absolutely possible on a general level. In the premise of the indirect proof, for Jandun Aristotle would be speaking about possibilities in an abstract way, that is of movers and mobiles in general without attending to their specific differences. [439]

dupliciter. Uno modo secundum quod sunt quanta et continua. Alio modo quantum ad proprias naturas determinatas sibi quibus sunt talia. Primo quidem modo est possibile ex omnibus mobilibus fieri unum continuum, et non repugnat eius in quantum sunt quanta quod bene sunt vel fiant unum continuum."

⁴³⁸Sed dicendum quod alio modo accipitur contingens et possibile cum demonstratur aliquid de genere et cum demonstratur de specie. Quia cum agitur de specie, omnes accipi ut impossibile omne illud cui repugnat vel genus vel differentia speciei ex quibus ratio speciei constituitur. Cum vero agitur de genere accipitur ut contingens omne illud cui non repugnat ratio generis, licet ei non repugnat differentia constituens speciem. Sicut si loqueretur de animali possem accipere ut contingens quod omne animal est alatum, et si descenderem ad considerationem hominis impossibile esset hoc animal esse alatum."

⁴³⁹Jandun [1488], 110r): "Therefore Aristotle speaks here of mobile things and movers without attending to determinate mobiles. Every continuous or contiguous thing is indifferently related to the concept of a mover and a mobile. Therefore he takes it as possible that every movable thing is made continuous with one another,

Buridan rejected the abstractionist solution, and in doing so he applies his own modal distinctions between supernatural and natural modalities. His reply is to read into Aristotle the modal distinctions he himself makes as we have seen in chapter [3.1]. I believe Buridan targeted not only Averroes, but also Jandun in his reply, when he writes that this solution to the Commentator and to many others as based on the abstract, general features (*rationes communes*) of individuals [440] However, for Buridan, considering abstract possibilities which do not have any relation to the concrete possibilities is not a good way of proceeding. Buridan writes (Buridan [1509], 95va):

(T7.10) It seems to me that this solution is not good, because from an absolute impossibility anything follows, and in the proposed case if that solution were good, then I would argue thus: Every body is at

which however is impossible if the movable things are considered according to their determinate natures." ["Quia igitur Aristoteles loquitur hic de mobilibus et moventibus nondum applicando ad determinata mobilia. Omne autem continuum vel contiguum indifferenter se habet ad rationem moventis et mobilis. Ideo accipit ut contingens quia omnia mobilia sint continua ad invicem, quod tamen impossibile est si mobilia considerentur secundum suas naturas determinatas.]

⁴⁴⁰Buridan [1509], 95ra: "To this doubt the Commentator and many others reply that although it is repugnant to the celestial bodies to constitute a continuum according to their specific natures, it is not repugnant to the general features of moving and moved bodies, therefore while Aristotle proceeds here according to the movers and the moved in general without attending to their specific natures, but attending (sistendo) to their general features, it was licit to take it as possible that which is not repugnant to the general concepts (rationes communes) of moving and moved bodies. Hence, many claim that an assumption which is not incompatible with these general concepts, with some other true co-assumed premise, nothing incompatible with the general concepts should follow. But in this case something incompatible does follow. Namely, it is incompatible with the general concepts of movements and times that infinite movements take place in a finite time, therefore it follows that the objection made by the adversary was not possible." ["Ad istam dubitationem respondent Commentator et multi alii quod libet repugnet corporibus celestibus continuari secundum suas naturals speciales tamen hoc non repugnat eius secundum rationes communes corporum motuum et mobilium, immo cum Aristoteles procederet hic de moventibus et motis non descendo ad speciales earum eationes, sed sistendo in eorum rationibus communibus, licebat ei accipere tanquam possibile quod non erat repugnans rationibus communibus motuum mouentium et corporum. Unde dicunt multi quod ex tali positione non repugnante illis rationibus communibus cum aliquo vero coassumpto non debet sequi aliquid repugnans illis rationibus communis. Sed in proposito sequitur repugnans. Repugnat enim rationibus communibus motuum et temporuum quod in tempore finito sit motus infinitos, ideo seguitur quod acceptum ab aduersario non erat possibile."]

rest and some body is moving - that is a celestial body -, therefore what is moving is at rest. The conclusion is impossible since it is repugnant to the general features (*communibus rationibus*) of moved things and moved bodies, although the second premise was true, and the first was not repugnant to the general features of movement, rest and bodies, therefore any such mode of arguing is not valid.

To be sure, Buridan's argument against the abstractionist model does not seem to be very elaborate. Its point seems to be to show that although abstracted features of things (*rationes communes*) can be possible of each individual, it does not follow that they are jointly possible of the same individual. Namely, whereas 'being at rest' and 'being in movement' are possible of everything in the genus of things that are capable of moving and being moved, nonetheless the argument shows that these possibilities cannot be jointly realized.

In the remainder of the question, Buridan reads into Aristotle's text his own solution, according to which logical possibilities are possible by divine powers and are stronger than natural possibilities possible by natural powers. Buridan writes (Buridan [1509], 95rb):

(T7.10) I reply that Aristotle, to a great extent participating in our true faith, believed that many things are impossible by natural powers which, however, he believed to be possible by a supernatural power. Therefore, in many places while speaking according to natural possibility, he simply regarded those as impossible, and in other places speaking according to absolute possibility, that is according to divine power, he regarded those same things possible. [441]

Buridan goes on to list several natural impossibilities that, as he sees it, Aristotle would have taken as possible, looking for examples of Aristotle's uses of impossible hypothesis in support of his reading. As Knuuttila remarks,

442 (Buridan [1509], 7.1 fol. 95rb): ["Et hoc notavit primo *Physicorum* ubi non voluit

^{441&}quot;Respondeo quod forte Aristoteles magnam partem participans nobiscum in vera fide nostra credidit multa esse impossibilia per potentias naturales que tamen credidit esse possibilia per potentiam supernaturalem, ideo in multis locis cum loqueretur de possibilitate naturali, simpliciter reputavit illa impossibilia, et in aliis locis loquens de possibilitate simpliciter, scilicet per potentiam divinam, reputavit illa possibilia."

these statements by Buridan are beyond doubt historically inaccurate (Knuuttila [2001]). What Buridan seems to have thought is that the only way to make sense of the use of impossible hypothesis in natural philosophy is by positing different kinds of possibility by divine or by natural powers. [443]

On the other hand, what Buridan's considerations here reveal is his distinctive attitude towards the use of natural impossibilities in natural philosophy. Buridan regarded supernatural possibilities as compatible with rational principles, presupposed in applications of logic to natural philosophy, and did not consider them to be elusive and contrary to reason.

ad arguendum quamquam possibile propter non repugnantiam secundum rationes communes aliquas nisi illud esset simpliciter possibile, et gaudeo gausus usum quod illa michi apparuerunt quorum tamen subtilioribus et sapientioribus correctionem relinquo,

et secundum hec dicta moderentur et solvantur omnia predicta."]

simpliciter dicere quod impossibile esset ex nichilo aliquid fieri, sed dixit quod omnes naturaliter vel de natura loquentes dixerunt quod impossibile est ex nichilo aliquid fieri, quasi vellet dicere quod hoc estp ossibile supernaturaliter. Si ergo Aristoteles credidit quod non posser celum quietari per potentias naturales, quod non posset dividi, quod scilicet non possent orbes celestes adinvicem continuari, quod non possent moveri velocius quam moventur aut tardius, quod nichil corruptibile posset esse subtilius igne immo quod esse dare subtilissimum quo nichil posset esse subtilius, quod non posset esse penetratio dimensionum et per sonequens quod non posset esse spacium separatum non motum cum corporibus motis non extra ea situaliter existens; et hec omnia credidit non esse possibilia secundum potentias naturales, sed alibi posuit hec omnia tamquam simpliciter possibilia. Nam in septimo huius posuit cuiuslibet mobilis partem posse quiescere, quecumque corpora posse invicem continuari, et in quarto huius posuit omne corpus in infinitum posse subtiliari et sic in infinitum, idem corpus posse velocius moveri, et in primo celi spacium quiescens cum corpore moto secundum modum penetrationis eo quod hec omnia sunt simpliciter possibilia per potentiam divinam."] ⁴⁴³Buridan [1509], 95vb: ["Et non invenitur quod unusquisque posuerit aliquid

8. Conclusions

The main goal of this dissertation was to address the question of how Buridan understands modal concepts from the perspective of his logic and natural philosophy. Whereas Buridan's logical writings on modality have received a great deal of scholarly attention, most studies have focused on Buridan's modal logic in isolation. The intended contribution of this dissertation is to address underexplored connections between his modal theory and its applications to non-logical domains, and to clarify some of the ways in which Buridan applies his analysis of modality to natural philosophical and metaphysical problems. In this concluding chapter, I will briefly summarize the main results of the individual chapters and highlight their significance within the scope of the dissertation.

8.1 Nominalist Philosopher, Modal Pluralist

As we have seen in the introduction, Buridan's nominalist philosophy is shaped by the adoption of application of logical tools to philosophy, and generally by principles of theoretical parsimony and explanatory economy. However, as is well-known, Buridan's nominalism is not primarily a reductionist program in philosophy. His modal theory provides one example of this attitude. As I hope to have shown, Buridan sees no problem in acknowledging a variety of modal concepts and positing a plurality of modal concepts.

In Part I of this thesis, we have seen that Buridan developed a sophisticated theory of varieties of modality. Throughout this part, I have sought to clarify the logical relationships that obtain between the modal notions Buridan endorses, and attempted to identify some of the main functions that Buridan's modal analyses have in his logical works.

In chapter 2, I have addressed the question of Buridan's understanding of logical modalities. We have focused on his treatment of logical consequence and

on the interpretation of modal propositions in his main writings on these issues, namely the *Tractatus de Consequentiis* and the *Summulae de Dialecticae*. As we have seen, Buridan uses a Modal Criterion in order to define logical consequence, according to which a consequence holds whenever it is impossible for things to be as the antecedent signifies without also being as the consequent does. After noting the historical background of the Modal Criterion in medieval logic, I have argued that Buridan's modification of it is informed by two conceptions of necessity and impossibility that he uses to distinguish two different kinds of consequence, namely 'simple' and 'as-of-now' consequence. Two claims were made in this section. The first is that the simple/as-of-now distinction among consequential relations correspond these two modal concepts identified above. The second claim is that Buridan's take on the formal/material distinction sees it as a foremost epistemic distinction. In conclusion, the basic sense of logical necessity for Buridan is the sense in which simple material consequences hold by the Modal Criterion.

As we have seen in section 2.2, a similar distinction between broad and restricted modalities is used in his interpretation of the modal proposition. For Buridan, modal propositions are ampliated to the possible - their subjects stand for what merely can be - and this is a crucial feature of his account of modal validity. We have seen Buridan distinguishes between 'simple' and 'conditional-temporal' necessity, yielding two interpretations of the modal syllogistics (each having affinities with 'possibilist' and 'actualist' interpretations of modal logic) Buridan's approach, therefore, is not established by a metaphysical preference to either at the outset.

Section 2.3 turned to a neglected aspect of Buridan's logic, namely his discussion of compossibility. The main claim of this section is that distinguishes between *logical* and *metaphysical* senses of compossibility. Propositions are regarded as logically compossible when they can be true together, while *de re* predications are compossible when the assumption of their truth does not entail an impossibility. Whereas that distinction is only implicit in the *TC*, it is developed and widely applied in his *QDGC* and *QPhys.*, to which I have returned in the last chapter (7.2).

Contrary to standard accounts of varieties of modality in the middle ages, we have seen that Buridan does not distinguish logical and natural or metaphysical

modal spaces on the basis of the familiar distinction between absolute and ordained powers of God. I have argued that the unifying trait of the distinct concepts of necessity he acknowledges is a notion of unfalsifiability: a proposition is unfalsifiable to the highest degree when no supernatural power or *casus* can falsify it, and it is unfalsifiable in the weakest degree when it may change its modal status, but it is now necessary (such as the necessity of the past).

Buridan had distinct motivations for distinguishing the first two grades of necessity. For Buridan, keeping supernatural and natural domains apart was crucial to his analysis of knowledge, and I have argued that one of the reasons for his anti-reductionism about modal notions comes also from the need to keep distinct modes of certainty and evidentness apart from each other, and to give an account of certainty which is able to serve the anti-skeptical purposes of his epistemic positions (3.2).

In section 3.4 we have seen that Buridan rehabilitates, against his nominalist forerunner William of Ockham, the theory of natural supposition. This has an important advantage for Buridan - it allows him to hold that universal affirmatives expressing natural necessities can be true even when their subjects are empty. In that way, Buridan is able to be clearer about the necessity of categoricals expressing natural laws, without reducing them to a relative variety of conditional necessity.

The former sections 3.5 and 3.6 have dealt with Buridan's weakest concepts of modality, those which involved the modal assymetry of time. I have argued that Buridan's sources for understanding historical modalities are the Diodorean definitions of modalities. We have seen that there is conflicting evidence as to why Buridan thought the past is necessary. On the one hand, since God cannot change it, this suggests it can be necessary to the highest degree he acknowledges. I have shown on what basis Buridan rejects this view, and we have seen that Buridan uses a strategy of mitigating the necessity of the past which many philosophers before him have adopted in connection with discussions about determinism.

8.2 Changing Conceptions of Modality

In Part II of this dissertation, my aim has been to study Buridan's understanding of modalities in natural philosophy, and as well as his application of modal analyses to his commentaries on Aristotle's writings.

The standard interpretation of medieval modalities, as we have seen in the introduction, identifies two models of modalites adopted by fourteenth-century philosophers. The first one is based on synchronic real alternatives, and a second one is based on temporal notions. Buridan has incorporated both models in his logical and natural philosophical writings. Many interpreters have seen Buridan's use of modal notions in natural philosophy as incorporating the second model, and have interpreted Buridan as assuming that temporal modalities are compatible with Aristotelian uses of modal concepts in his natural philosophical works.

In this part of the dissertation, I have argued that while it is true that Buridan took the temporal interpretation of modalities to be compatible with Aristotelian positions, Buridan did not fully accept the temporal model in his conceptions of natural necessity and contingency. I have also argued that Buridan applies his version of the distinction between logical and natural modalities to his commentaries on Aristotle, a distinction which he did not think was contained in Aristotle's texts.

Chapter 4 addressed Buridan's and John of Jandun's interpretations of the modal assumptions underpinning Aristotle's argument at *DC* I.25 to the effect that what always is, is so by necessity. As we have seen in section 4.2 Jandun accepted the main tenets of the temporal model and identified it as the basis for Aristotle's claim in *DC* that what always is must be necessary. An important evidence for this claim is Jandun's use of the square of oppositions between temporal concepts, which we can also find in Averroes' commentary (4.1). As we have seen, an important reason for Jandun's endorsement of the temporal model is that he believed non-temporal possibilities cannot be confirmed or disconfirmed empirically. Jandun took supernatural (or merely logical) possibilities to be elusive and contrary to reason, and stated that God's ability to create alternative worlds is something we hold on merely on the basis of faith.

I have argued that Buridan's attitude to supernatural possibilities in his natural philosophy differs from this position in important respects. As we have seen in section 4.3, when Buridan applied his modal analyses in his corresponding commentary, he frequently makes use of the distinction between logical and natural modalities. In contrast with Jandun's approach, I have argued that Buridan criticized the temporal model in this context. The basis of Buridan's criticism is his analysis of divided possibility propositions, which refer to merely possible and never actual circumstances. In his commentary, Buridan often criticizes Aristotle for confusing (intermiscet) modal propositions with assertoric ones. I have further suggested how Buridan gestured at a change in the relationship between temporal and modal concepts in a square of oppositions which included propositions in the divided sense. A modified version of this modified square of oppositions involving divided modal propositions is found in Nicole Oresme's French commentary 4.2 on *De Caelo*. Although Buridan's influence on Oresme on this particular point has only be conjectured in the literature, his version closely resembles the one Buridan mentioned in his Expositio commentary.

In chapter 5 we have turned to Buridan's remarks on the relationship between powers and possibility. The conclusion of the chapter, by looking at *QPhys*. and *QDGC*, is that Buridan did not base his modal theory on powers. Buridan seems to have thought that an analysis of modality in terms of powers was incompatible with his semantics of the modal proposition, and in his *QM*, Buridan claims instead that although powers-based readings of modality are more common in everyday language, nonetheless they do not correspond to the literal meaning (*de virtute sermonis*) of modal words.

The second section (5.2) turned to Buridan's use of historical modalities in the *QDGC*, in which he elaborates on the connections of powers-based possibility with time. I have argued that Buridan relates the temporal assymetry of historical modalities to the fact that no power ranges over the past, but their possibilities are instead future-oriented. In that context, Buridan was interested in asking which concept of modality is involved in stating the necessary connections between different kinds of change. I have closed the section by noting that other near contemporaries offered a reflection on modal terms, and I have chosen Nicole Oresme's representative commentary as a contrast

case. We have seen that Oresme's reflection on modal concepts contained in *QDGC* commentaries show a view of degrees of necessity based on temporal duration which is different from Buridan's, which provides further evidence that Buridan does not fully incorporate the temporal model of modalities in his natural philosophical commentaries.

Chapter 6 analysed logical and physical descriptions of contingency in Buridan's logic and natural philosophy. In section 6.1, we have seen that Buridan's concept of logical contingency corresponds to the 'each-way contingent.' I have pointed out that Robert Kiwalrdby's commentary on the *Prior Analytics* distinguished, on the basis of Aristotle's remarks, a further mode of 'natural contigency' - the contingency of events that are more naturally one way than another -, which is not incorporated by Buridan. Rather, the place where Buridan discussed the classification between for the most part and rare contingencies, as we saw in in section 6.2, is his commentary to the *Physics*. This classification was foremost associated with the modal status of causes, and in that context Buridan addresses a question associated with late medieval interpretations of a famous controversy (famosa controversia) between the views of Avicenna and Averroes, of whether chance events belong to the category of rare contingencies or to the category of the each-way contingent. I have shown how this discussion hinges on different accounts of natural contingency and necessity. We have seen that Buridan argues, against Averroes, that chance events belong to the category of the each-way contingency.

The conclusion of these chapters is that Buridan reads into Aristotle a temporal model of natural necessity and contingency, but contrary to what standard interpretations have suggested, Buridan himself did not fully incorporate this model to his own modal analyses.

8.3 Logic as a Method

A central aim of this dissertation was to investigate Buridan's theory of modality from the standpoint of its methodological dimensions, by investigating its application of the semantics of modal propositions of the distinction between supernatural and natural modalities.

In Chapter 7, three aspects of Buridan's application of his modal analyses

to natural philosophy were investigated. A central thread of this chapter is showing that Buridan regarded merely logical (or supernatural) possibilities as philosophically useful in his commentaries on Aristotle.

I have argued in section [7.1] that Buridan makes an implicit distinction between logically impossible and naturaly impossible objects of signification. Buridan's nominalist logic is known from excluding impossible significates terms that signify a concept in the mind but ultimately fail to refer to anything in the external world - from his account of significative terms. I have argued that while Buridan thought logically impossible objects - such as the 'Chimera', the paradigmatic example in medieval logic - cannot be the extra-mental referent of any concept, nonetheless other examples seem to show that terms which refer to naturally impossible objects can refer to something in supernaturally possible cases. A case in point is Buridan's discussion the void in *QPhys*. Buridan's treatment - which has received attention for involving theological and natural reasoning - shows how Buridan made an implicit distinction between kinds of modality in his theory of signification. Namely, although the term 'void' lacks a referent in the natural world, it can possibly refer to something in a supernaturally possible case.

In section 7.2, we have seen how Buridan's analysis of divided modal propositions, developed in the *TC*, is applied in order to clarify the consequences of a thesis concerning divisibilism - the thesis that *continua* can be infinitely divided. Buridan's focus on this question is largely logical. He claims that some logical perplexities that follow from divisibilism find a solution which comes mainly from logic (*principaliter ex logica*). A main conclusion of this section is that Buridan adjusts his analysis of compossibility (which we have seen in section 2.3) for the supernatural cases, in order to analyse the claim claim that God can separate and separately conserve all parts of a continuous.

The last section of this chapter (7.3) turned to another application of the distinction between supernatural and natural possibility. We have seen that Buridan believed that Aristotle's use of indirect proof in the *Physics* presupposes such a distinction. In contrast with the Averroan approach exemplified by John of Jandun, which relies on the idea that merely logical possibilities are products of abstraction, Buridan thought that they reflect a genuine division of modal space between the domain of divine power and the domain of natural

powers. As a result, this section concluded that Buridan's anachronistic claim - attributing to Aristotle a distinction between supernatural possibility and natural possibility -, should be read in light of Buridan's attitude of acknowledging naturally impossible hypotheses as having an important methodological role in natural philosophy.

8.4 Concluding Remarks

Buridan's modal logic is famously one of the most refined systems of the middle ages. Following a tradition of medieval logic, Buridan understood logic as the "art of all arts, having access to the principles of all inquiries." Logic was foremost a practical science, concerned with the clarification of principles and analysis of propositions and arguments, providing analytical tools used in natural philosophy and metaphysics. This dissertation's intended contribution was to study Buridan's modal theory from this perspective. Overall, the main conclusion of this study is that Buridan's modal analyses can offer insights into his philosophy which cannot be fully gleaned once they are studied in isolation from their methodological dimension in the broader context of his philosophy.

⁴⁴⁴ Buridan [2001b], Transl. Klima, 4. Buridan [2005], Ed. Van der Lecq.

Sammanfattning

Under 1300-talet fångade det otänkbara mångas fantasi - teologer tänjde ut den gudomliga makten för att omfatta alla logiska möjligheter, skulptörer snidade upp djur i katedraler som visade upp möjliga oskapade djur, och jurister hänvisade till att extraordinära krafter tillfälligt upphävde moraliska lagar. Hur världen skulle kunna tänkas vara beskrivs med modal begrepp som nödvändighet, möjlighet, kontingens/tillfällighet, och omöjlighet. Dessa så kallade modaliteter beskriver inte hur världen är utan hur den kan vara. Att säga att vatten kan koka är inte samma sak som att säga att vatten kokar. Att säga att vatten kan koka är att säga att det är möjligt för vatten att börja koka, inte att vatten faktiskt kokar. Inom den medeltida filosofin finner vi en mer och mer förfinad diskussion om modaliteter inom systematisk teologi, logik och naturfilosofi. De spelar stor roll i deras syn på, till exempel, vetenskaplig kunskap, handlingsfrihet och studiet av den naturliga världen.

Den här avhandlingen handlar om den nominalistiska filosofen John Buridan (c.1300-c.1361) och hans analys av modala begrepp i såväl hans logiska som icke-logiska skrifter, med tonvikt på funktionerna av hans modala analyser inom naturfilosofin och metafysiken. Buridan var en av de viktigaste filosoferna under 1300-talet. Han var aktiv under hela sin karriär vid Paris universitet, där han föreläste och skrev om logik, naturfilosofi, metafysik och etik. Hans logiska och naturfilosofiska verk har haft en betydande inverkan på det filosofiska samtalet under sin tid. En central fråga för filosofisk förståelse av modaliteter är om det finns olika sorter av modaliteter. Till exempel, en vanlig tanke inom antik och medeltida filosofi är att det förflutna är nödvändigt. Det som redan har skett kan inte vara på annat sätt. Efter att Caesar har gått över Rubicon är det nödvändigt att han har gjort det. Denna typ av nödvändighet kan vi kalla historisk nödvändighet. Denna typ av nödvändighet är skild från till exempel logisk nödvändighet. En ungkarl är med nödvändighet en ogift man. Enligt så kallade modala monister kan vi reducera alla olika typer av modaliteter till

en grundläggande typ. Jag argumenterar för i denna avhandling att Buridan förnekar detta och vi bör förstå Buridan som en modal pluralist som erkänner en mängd olika irreducerbara typer av modaliteter (historisk, logisk, metafysisk och naturlig). Även om ett genomgående tema i Buridans filosofi är att han strävar efter ontologisk sparsamhet, ser Buridan dock inga problem med att erkänna en mängd olika typer av modaliteter och att använda olika modala analyser för olika syften.

Enligt den gängse synen på den medeltida diskussionen av modaliteter skedde det ett konceptuellt skifte under första hälften av 1300-talet. Forskare har tenderat att tolka denna begreppsförändring som en brytning från den förmoderna och aristoteliska synen på modaliteter. Enligt denna standardbild anses den förmoderna, aristoteliska modellen definiera modalitet i tidsmässiga termer, medan den nya modellen baseras på möjliga alternativ i samma situation. Denna studie av Buridans modalteori belyser jag hur Buridan använder sin logiska analys av modaliteter för att förstå den naturliga världen. Varje kapitel utforskar en dimension av Buridans syn på modalitet och visar på interaktionerna mellan Buridans logik och hur han använder modala begrepp i sin övriga filosofi.

Att förstå de medeltida diskussionerna om modaliteter är även av vikt för den samtida filosofin: (1) Den samtida distinktionen mellan logisk och naturlig eller metafysisk nödvändighet har sin grund i den medeltida diskussionen. (II) Både då och nu har Buridans skrifter främst uppskattats som verk av en logiker. Hans behandling av logisk konsekvens är en föregångare till moderna redogörelser, och hans modala logiska system systematiserar en modal logisk tradition som kan jämföras med kvantifierade modala logisker som utvecklats under 1900-talet.

Avhandlingen är uppdelad i två delar. Del I undersöker Buridans redogörelse för typer av modaliteter (till exempel historiska, logiska, naturliga modaliteter). Jag fokuserar på frågan om hur Buridan skiljer mellan dessa olika modala analyser och varför han använder sig av olika modalanalyser för olika syften. Mitt övergripande syfte i del I är att identifiera de viktigaste typerna av modalitet och beskriva de logiska relationerna mellan dessa typer av modaliteter. I del II fokuserar jag på hur Buridan konceptualiserar modaliteter i sin naturfilosofi. Naturfilosofi är den antika och medeltida föregångaren till modern naturvetenskap. Under medeltiden bestod naturfilosofi till stor del av att förstå och utveckla Aristoteles naturfilosofiska skrifter. Buridan skriver så kallade kommentarer på

Aristoteles verk. Dessa är dock inte kommentarer i den samtida bemärkelsen. Buridans huvudsakliga syfte är inte att förstå vad Aristoteles faktiskt menade, utan att använda Aristoteles text som grund för att besvara de mest centrala frågorna inom naturfilosofi. Denna del fokuserar på hur Buridan använder sig av olika typer av modalitet för att förstå sig på olika naturfilosofiska problem. I dessa kapitel fastställer jag vad som är utmärkande för Buridans redogörelse för naturlig nödvändighet och tillfällighet, genom att kontrastera Buridan med samtida filosofer som också är aktiva vid Paris universitet.

Kapitel 2 behandlar modaliters roll för Buridan's analys av logisk konsekvens och hur vi ska förstå modala propositioner (till exempel att vatten kan koka). I detta kapitel introduceras några viktiga aspekter av i Buridans semantik som kommer vissa sig viktiga för avhandlingen i stort. Kapitel 3 handlar om Buridans syn nödvändighet i den utförliga diskussion av demonstrationer i Summulae de Dialectica och ger en övergripande redogörelse för de modaliteter som Buridan särskiljer. Olika typerna spelar en central roll i Buridan's filosofi som helhet. Till exempel så spelar de viktig roll för hans kunskapsteori.

I kapitel 4, som inleder del II, behandlas förhållandet mellan nödvändighet och evighet. Jag fokusera på Buridan's diskussion när olika propositioner kan vara sanna och hur vissa propositioners sanning inte beror på tidsmässiga faktorer (så kallad omnitemporal sanning). Kapitlets huvudfokus är att jämföra Janduns och Buridan synsätt. Deras analyser skiljer sig markant åt och jämförelsen visar i vilken utsträckning Buridan accepterade den temporala analysen av modaliteter.

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Kapitel 5 behandlar Buridans beskrivning av förhållandet mellan förmågor/potenser och modaliteter. Buridan är i sin kommentar på Aristoteles verk Fysiken särskilt intresserad av Aristoteles och den arabiska filosofen och kommentatorn Averroes (1126-1198) användning av modaliteter som baserade på förmågor/potenser. Buridan frångår här Aristoteles och Averroes syn och argumenterar för att modaliteter inte kan vara grundade i potenser. Jag visar på centrala skillnader mellan hans semantiska beskrivning av modaliteter och Aristoteles och Averroes potensbaserade analys.

I kapitel 6 fokuserar jag på Buridans syn på tillfälligheter. Jag fokuserar särskilt på hans diskussion av vad som kan kallas tillfällighetens problem. Buridan utgår från olika analyser av kausal nödvändighet och tillfälligheter från arabisk filosofi. De två viktigaste arabiska filosoferna, Avicenna och Averroes, hade motsatta åsikter om huruvida slumpmässiga händelser tillhör kategorin av det som sällan inträffar eller om de också tillhör kategorin av det som är lika sannolikt att inträffa som att inte inträffa. Buridans egna uppfattning är att slumpmässiga händelser måste vara effekter av så kallade utrumlibet orsaker. En sådan orsak kan ge upphov till utfall med lika stor sannolikhet som till exempel att singla slant ger upphov till klave eller krona med samma sannolikhet. Denna analys visar på ytterligare begränsningar i den tidigare förståelsen av modaliteter baserade på frekvens och tid. Dessa begränsningar är viktiga för att förstå Buridans syn på modaliteter i helhet.

Kapitel 7 fokuserar på en metodologisk tillämpning av distinktionen mellan de blotta logiska möjligheter som endast Gud kan förverkliga och vad som kan åstadkommas av icke-övernaturliga krafter. Det undersöker också hur Buridan anser att formuleringen av enbart logiskt möjliga scenarier kan vara användbar inom naturfilosofi. I avsnitt 7.1 argumenterar jag att Buridans diskussion bygger på en implicit distinktion mellan naturligt och logiskt omöjliga objekt för propositioner.I de två följande avsnitten undersöker jag hur Buridan använder sin logik för modaliteter för att besvara naturfilosofiska frågor. Exempelvis visar jag att Buridans argumentation för divisiblism – det vill säga att alla materiella ting alltid kan delas upp i allt mindre utan att man når någon minsta beståndsdel

– bygger på hans analys av modala propositioner.

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