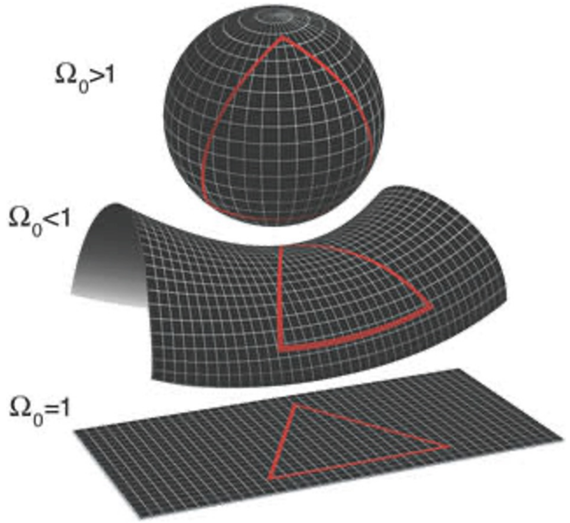
Rearranging Equations to Develop Physics Reasoning

Implications on Physics Teaching and Learning



$$\Sigma\Omega_i + \Omega_\kappa = 1$$

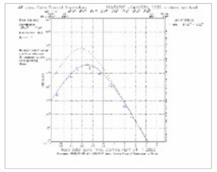
Anastasios Kapodistrias

John Airey

The project

- Semiotic Audit of Astronomy Representations
- Higher Education Courses
- Lecture notes & Textbooks
- Categorization in semiotic systems

GRAPHS
31.7%



MATHEMATICS
21.3%

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3}\,.$$

IMAGES
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DIAGRAMS
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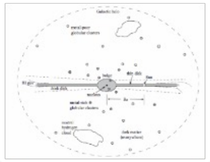

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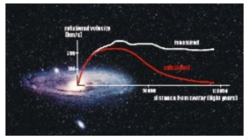

MAPS
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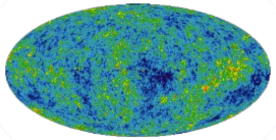

VIDEO SIMULATIONS
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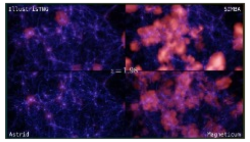

TEXT

"DENSITY
IS
DESTINY"

CODE

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- Semiotic Audit of Astronomy Representations
- Higher Education Courses
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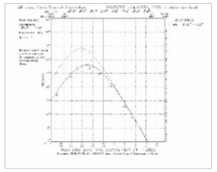

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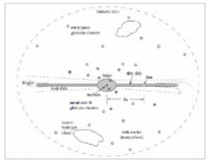

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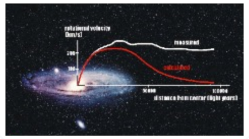

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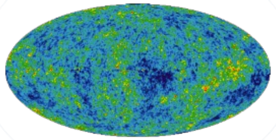

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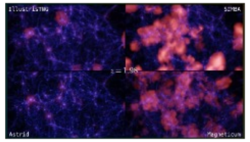

TEXT

“DENSITY
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DESTINY”

CODE

Cosmology Lectures

- Three higher education Cosmology courses.

- Friedmann equation: dynamics and curvature of the universe

The Friedmann Equation in Cosmology

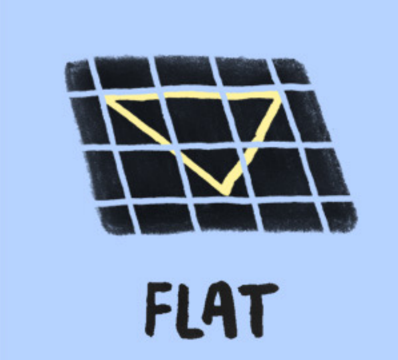
$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3c^2}\,\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\,\frac{1}{a(t)^2}$$

Cosmology Lectures

- Key relevant aspects:
    - ➢ Rate of expansion
    - ➢ Shape of the universe (curvature)
    - ➢ Contents of the universe

The Friedmann Equation in Cosmology

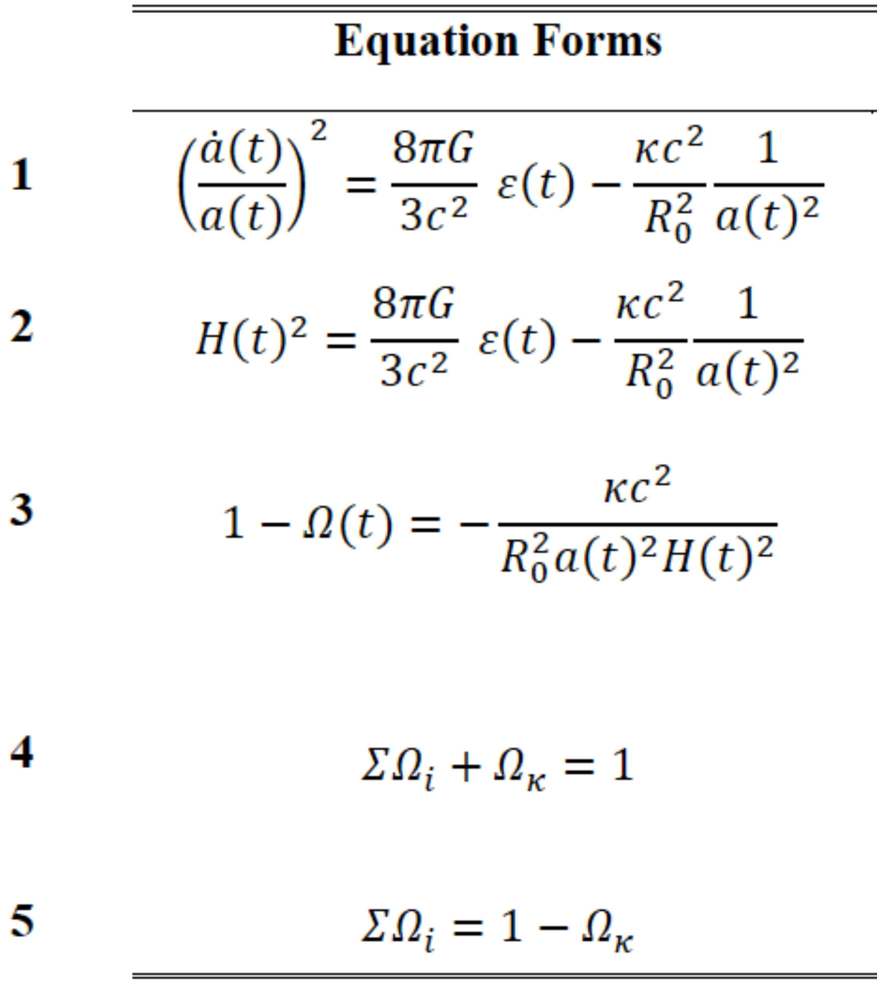
$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3c^2}\,\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$



Cosmology Lectures

- Rearrangement
- Combined forms from all three courses
- Aspects of both:
  - Mathematical knowledge
  - Physics Reasoning

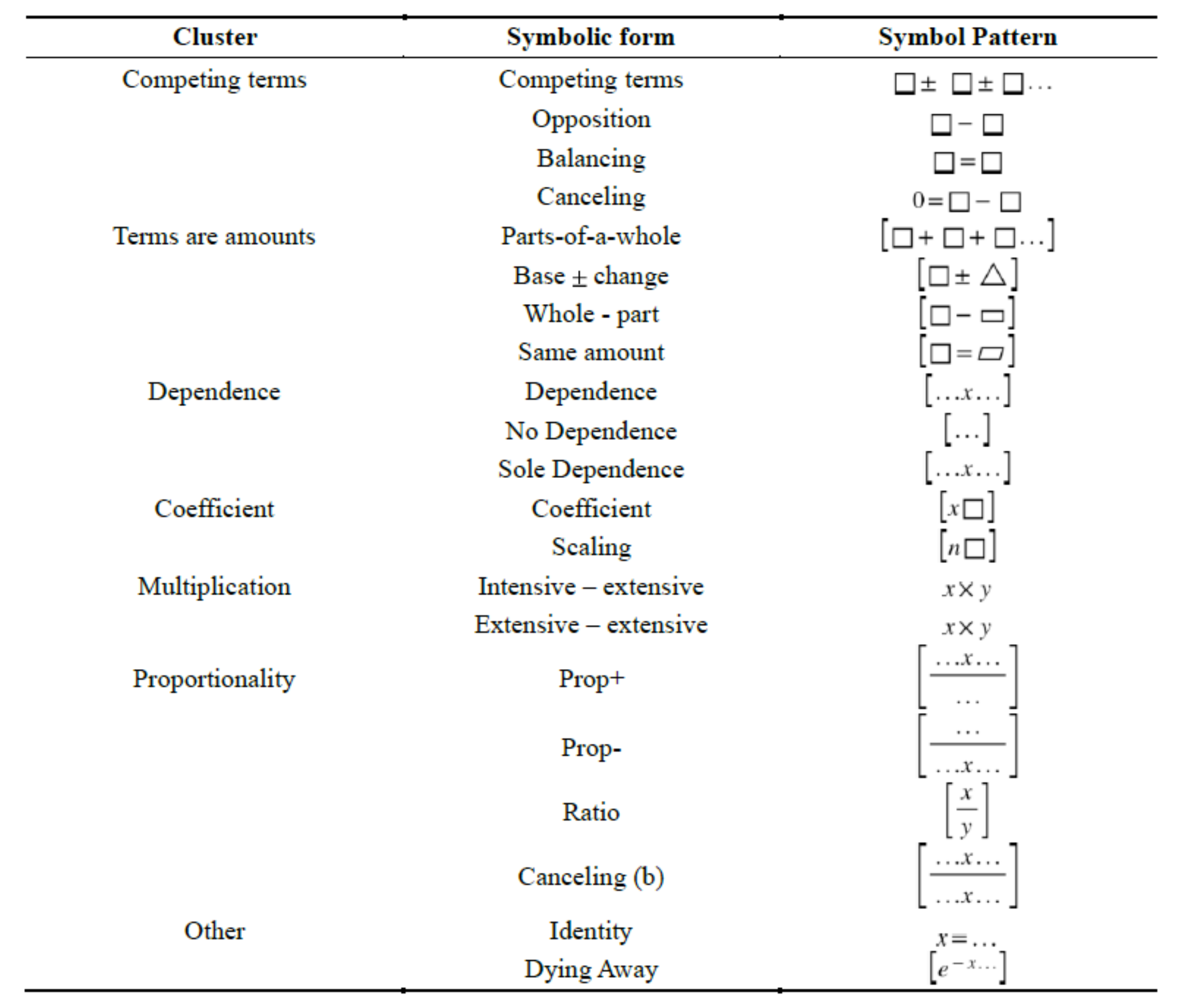
Rearrangement of the Friedmann equation



	Equation Forms
1	$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$
2	$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$
3	$1 - \Omega(t) = -\frac{\kappa c^2}{R_0^2 a(t)^2 H(t)^2}$
4	$\Sigma\Omega_i + \Omega_\kappa = 1$
5	$\Sigma\Omega_i = 1 - \Omega_\kappa$

Theoretical framework: Symbolic forms

- Symbolic Forms *(Sherin, 2001)*:
- ➢ How students understand physics equations
- ➢ Conceptual schema
- ➢ Symbol template



Cluster	Symbolic form	Symbol Pattern
Competing terms	Competing terms	$\square \pm \square \pm \square \ldots$
	Opposition	$\square - \square$
	Balancing	$\square = \square$
	Canceling	$0 = \square - \square$
Terms are amounts	Parts-of-a-whole	$[\square + \square + \square \ldots]$
	Base ± change	$[\square \pm \triangle]$
	Whole - part	$[\square - \text{▭}]$
	Same amount	$[\square = \text{▱}]$
Dependence	Dependence	$[\ldots x \ldots]$
	No Dependence	$[\ldots]$
	Sole Dependence	$[\ldots x \ldots]$
Coefficient	Coefficient	$[x \square]$
	Scaling	$[n \square]$
Multiplication	Intensive – extensive	$x \times y$
	Extensive – extensive	$x \times y$
Proportionality	Prop+	$\left[\frac{\ldots x \ldots}{\ldots}\right]$
	Prop-	$\left[\frac{\ldots}{\ldots x \ldots}\right]$
	Ratio	$\left[\frac{x}{y}\right]$
	Canceling (b)	$\left[\frac{\ldots x \ldots}{\ldots x \ldots}\right]$
Other	Identity	$x = \ldots$
	Dying Away	$[e^{-x \ldots}]$

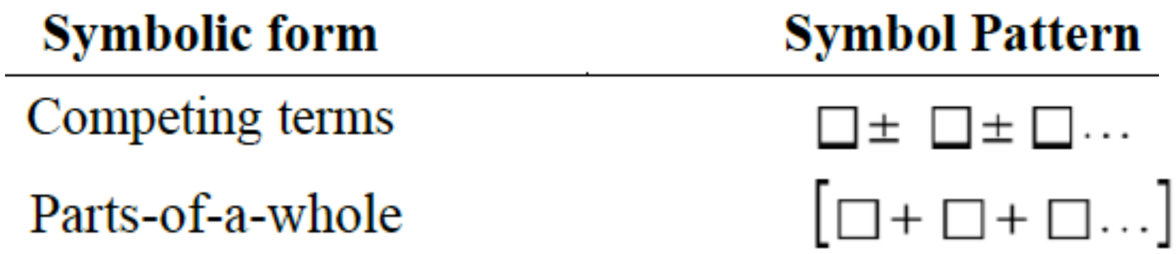
What we did

- Possible symbolic forms for each line.

Line 4

$$\Sigma\Omega_i + \Omega_\kappa = 1$$

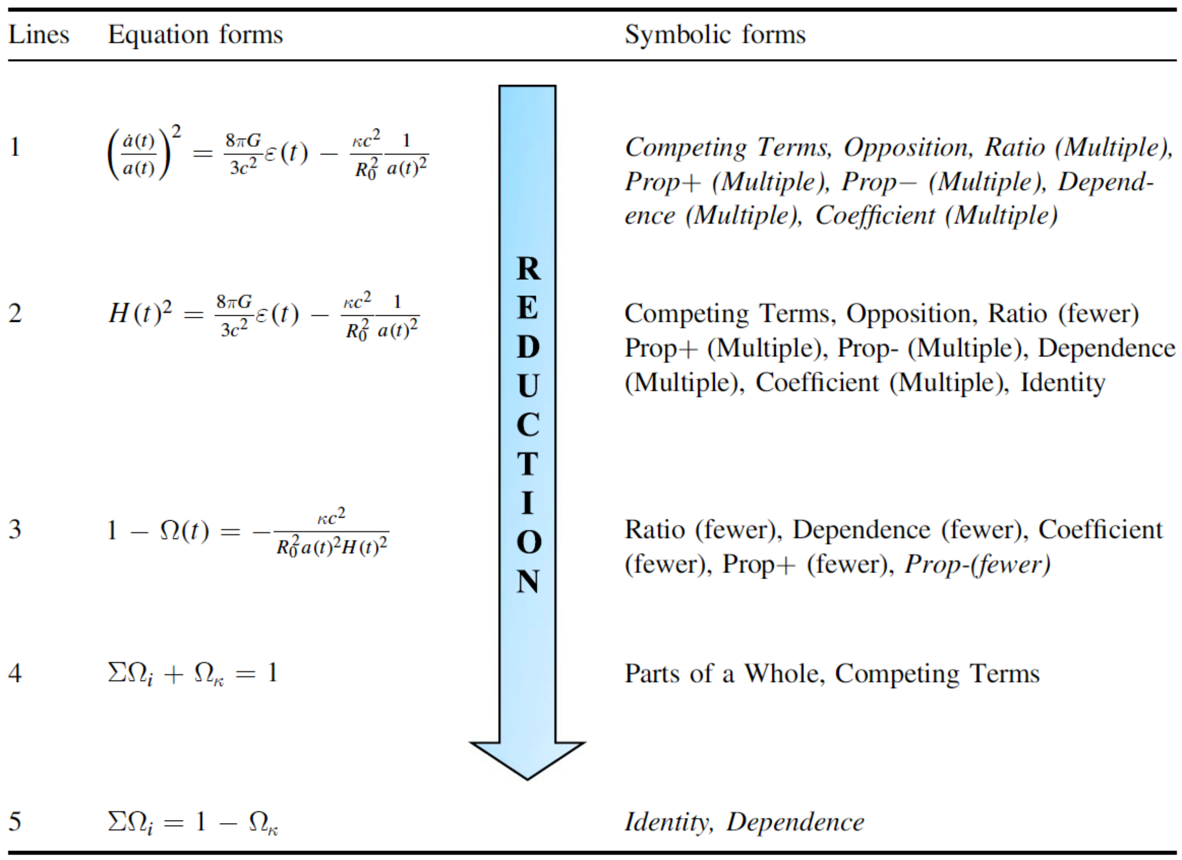
Possible symbolic forms



Symbolic form	**Symbol Pattern**
Competing terms	$\square \pm \square \pm \square \cdots$
Parts-of-a-whole	$[\square + \square + \square \ldots]$

RQ1: Rearrangement & Symbolic forms

- What aspects of physics reasoning can be identified when we use Sherin’s framework of symbolic forms to analyze the line-by-line rearrangement of the Friedmann equation?



Lines	Equation forms	Symbolic forms
1	$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$	*Competing Terms, Opposition, Ratio (Multiple), Prop+ (Multiple), Prop− (Multiple), Dependence (Multiple), Coefficient (Multiple)*
2	$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$	Competing Terms, Opposition, Ratio (fewer) Prop+ (Multiple), Prop- (Multiple), Dependence (Multiple), Coefficient (Multiple), Identity
3	$1 - \Omega(t) = -\frac{\kappa c^2}{R_0^2 a(t)^2 H(t)^2}$	Ratio (fewer), Dependence (fewer), Coefficient (fewer), Prop+ (fewer), *Prop-(fewer)*
4	$\Sigma\Omega_i + \Omega_\kappa = 1$	Parts of a Whole, Competing Terms
5	$\Sigma\Omega_i = 1 - \Omega_\kappa$	*Identity, Dependence*

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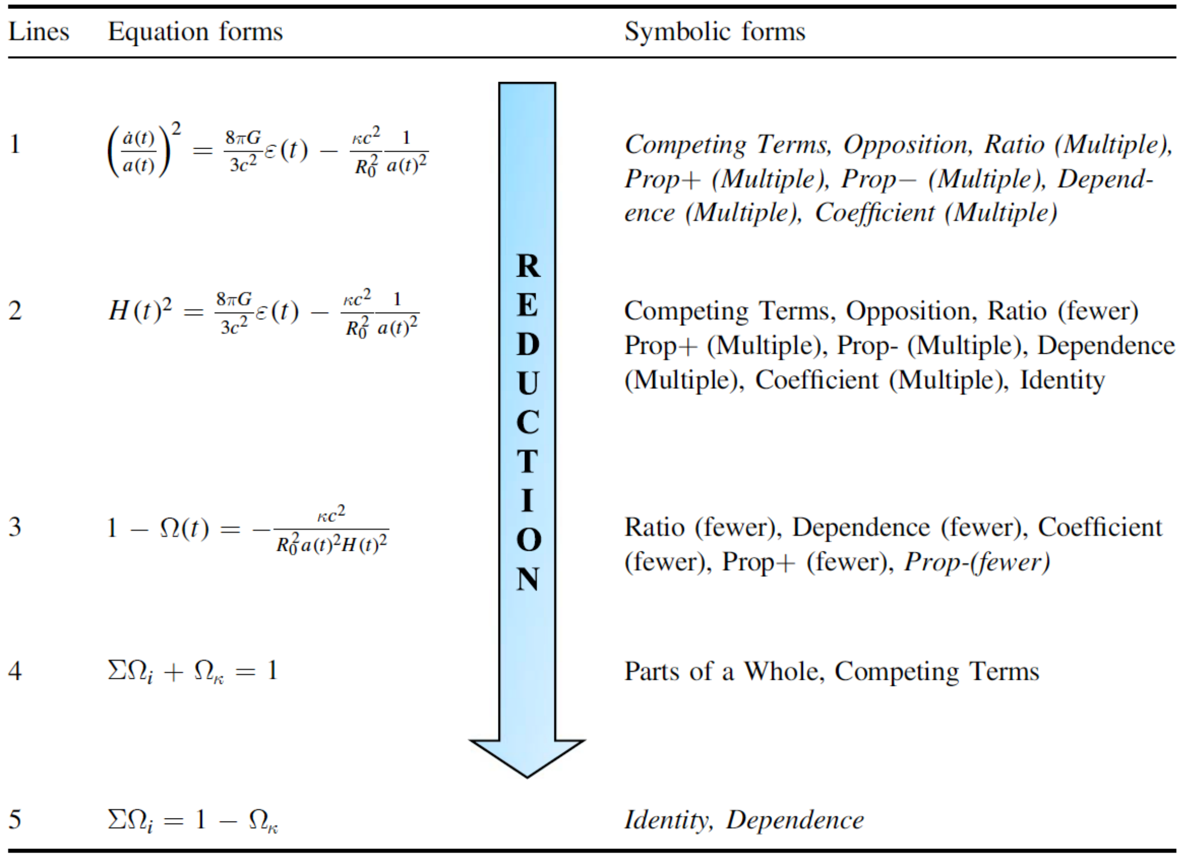
RQ1: Rearrangement & Symbolic forms

- Line to line movement:

➢ Lines 1-4: *Reduction* in the number of symbolic forms

➢ Lines 4-5: Shift in the conceptual schema



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4	$\Sigma\Omega_i + \Omega_\kappa = 1$	Parts of a Whole, Competing Terms
5	$\Sigma\Omega_i = 1 - \Omega_\kappa$	*Identity, Dependence*

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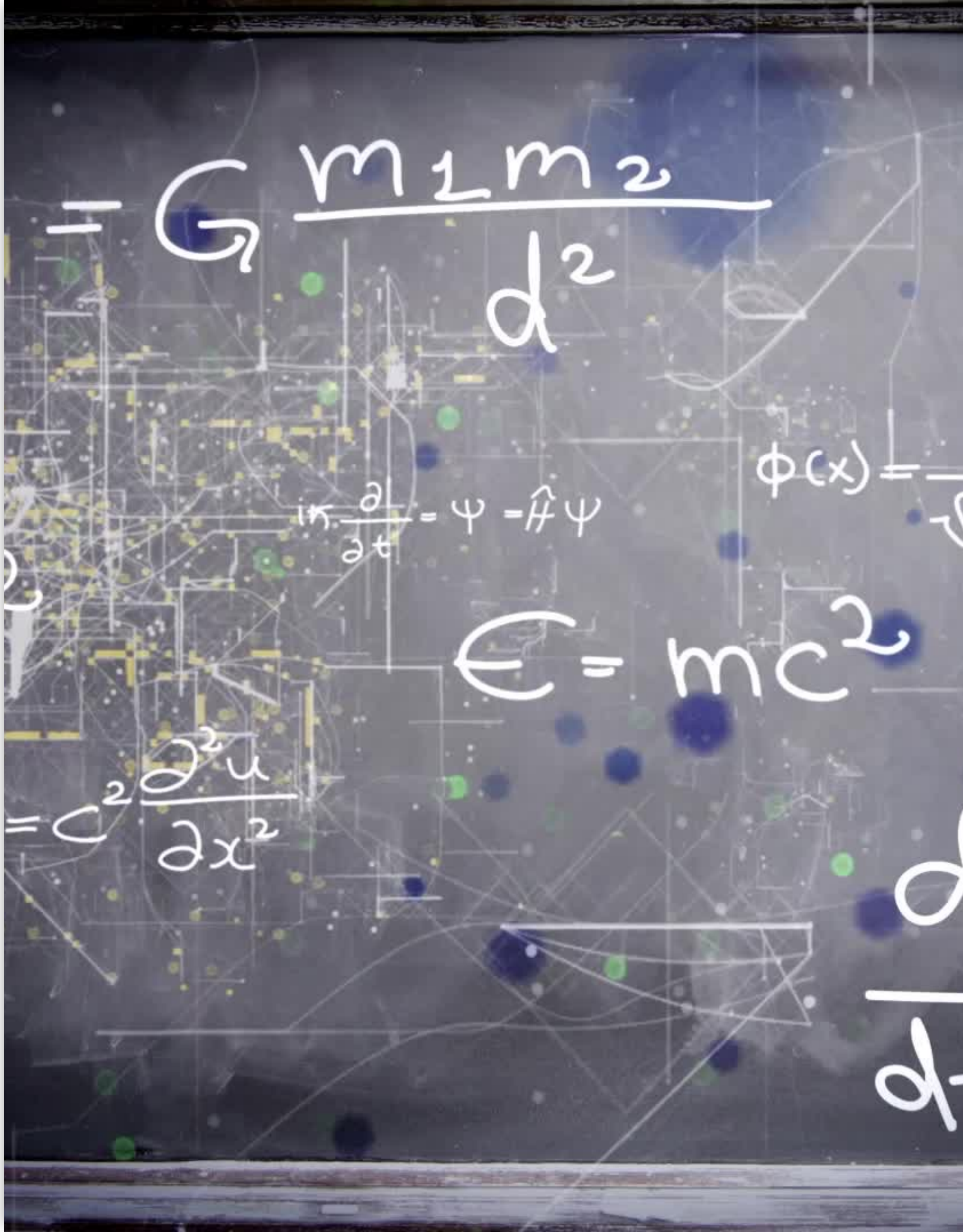
RQ2: Physics reasoning with equations

- What does this analysis suggest about the way that physicists rearrange equations in order to reason about physics?



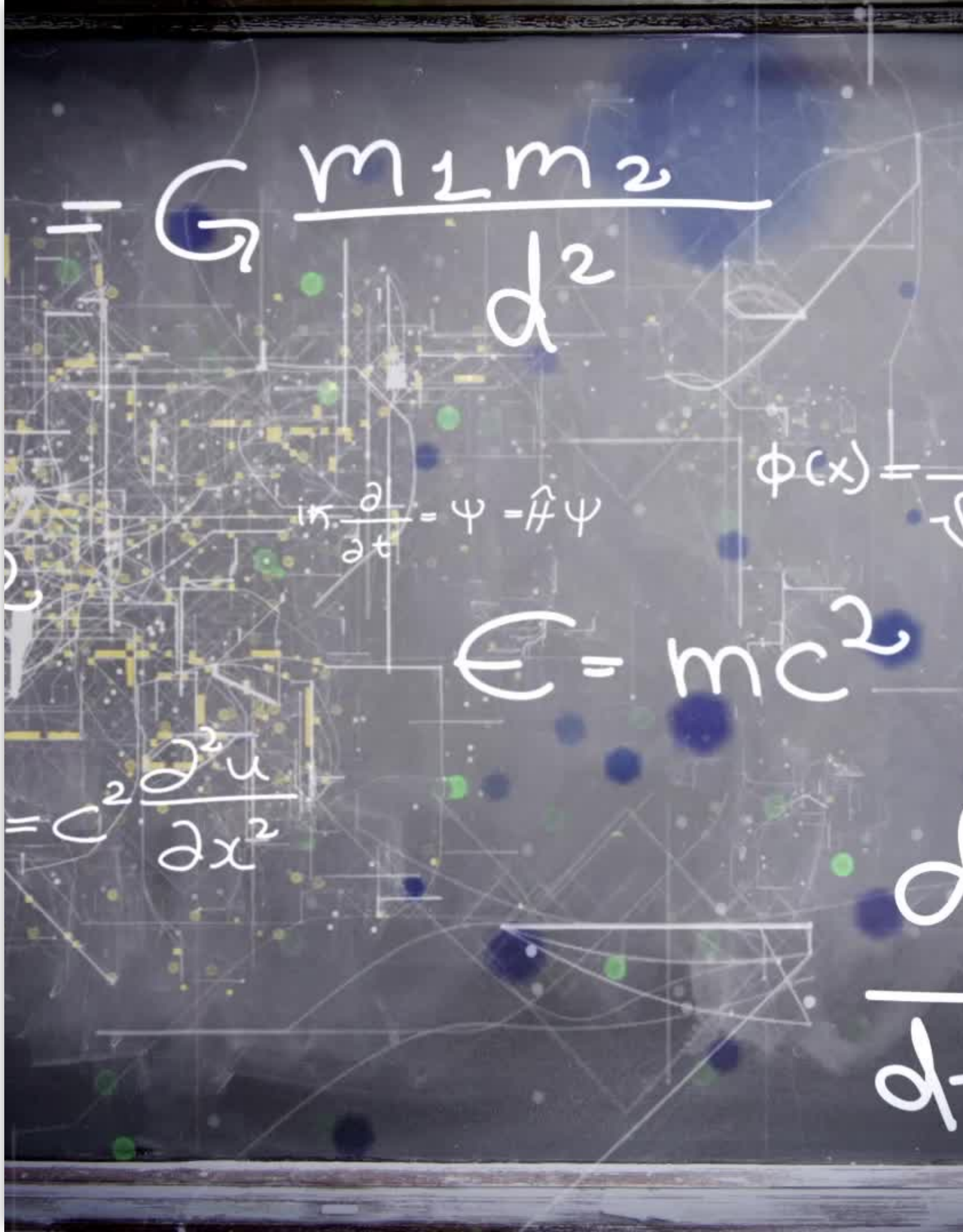
RQ2: Physics reasoning with equations

- Underlying mechanism:

➢ Narrowing down of meaning potential.

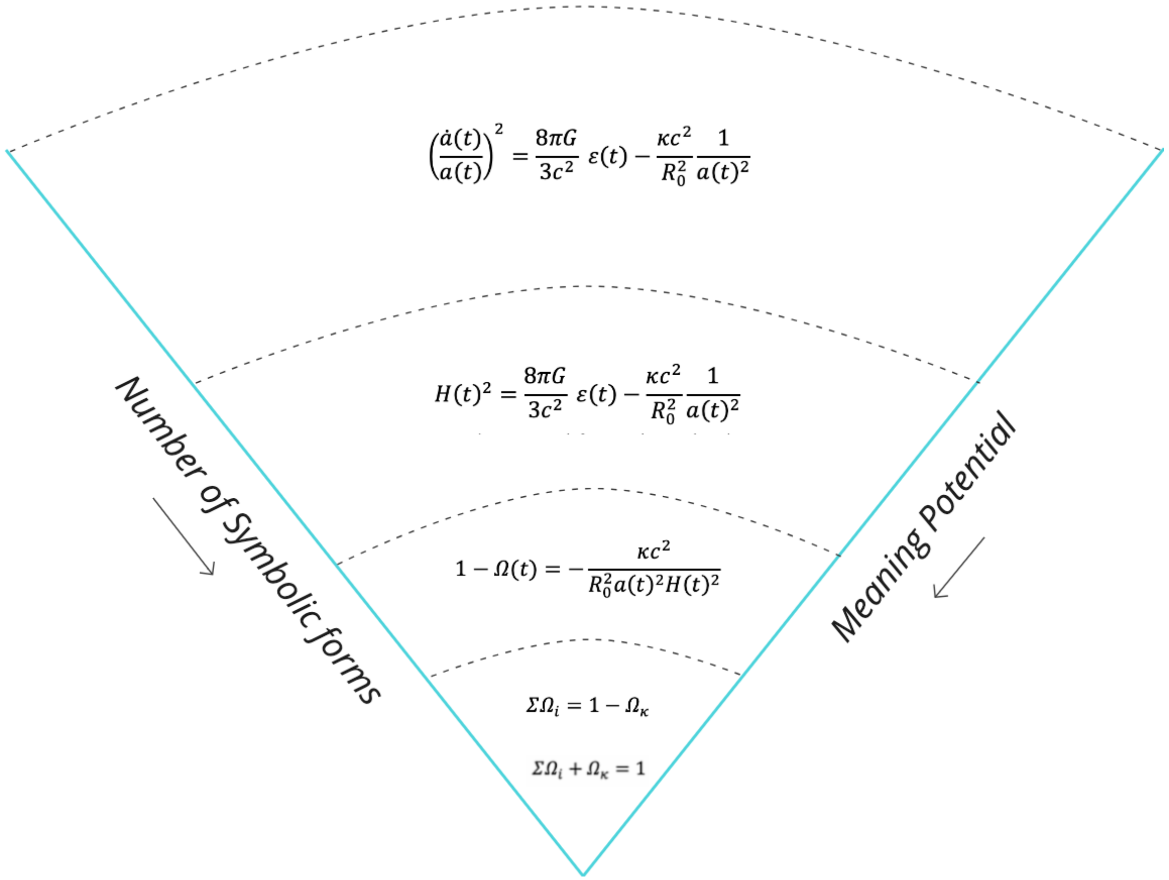
➢ Movement between foreground and background

➢ Purposefulness



Narrowing down meaning potential

- Symbolic forms ⇨ Meaning potential



$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3c^2}\,\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

$$H(t)^2 = \frac{8\pi G}{3c^2}\,\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

$$1 - \Omega(t) = -\frac{\kappa c^2}{R_0^2 a(t)^2 H(t)^2}$$

$$\Sigma\Omega_i = 1 - \Omega_\kappa$$

$$\Sigma\Omega_i + \Omega_\kappa = 1$$

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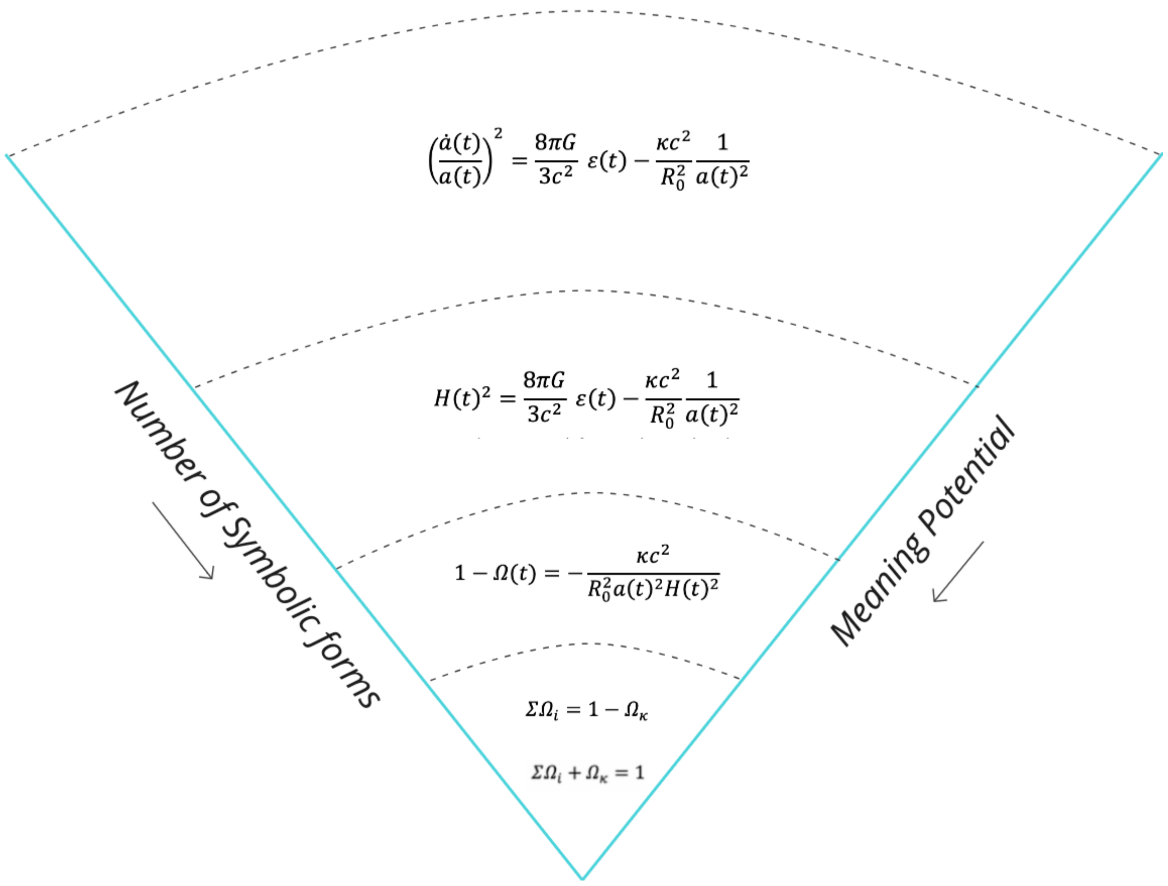
Narrowing down meaning potential

- Reduction in the number of symbolic forms

⇕

- Narrowing down meaning potential



$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

$$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

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$$\Sigma\Omega_i = 1 - \Omega_\kappa$$

$$\Sigma\Omega_i + \Omega_\kappa = 1$$

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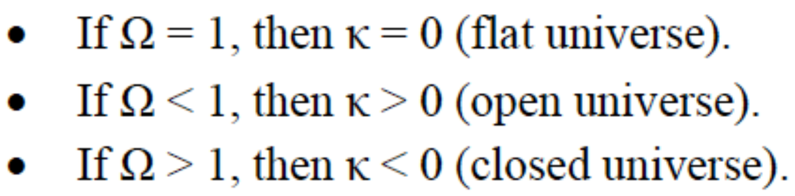
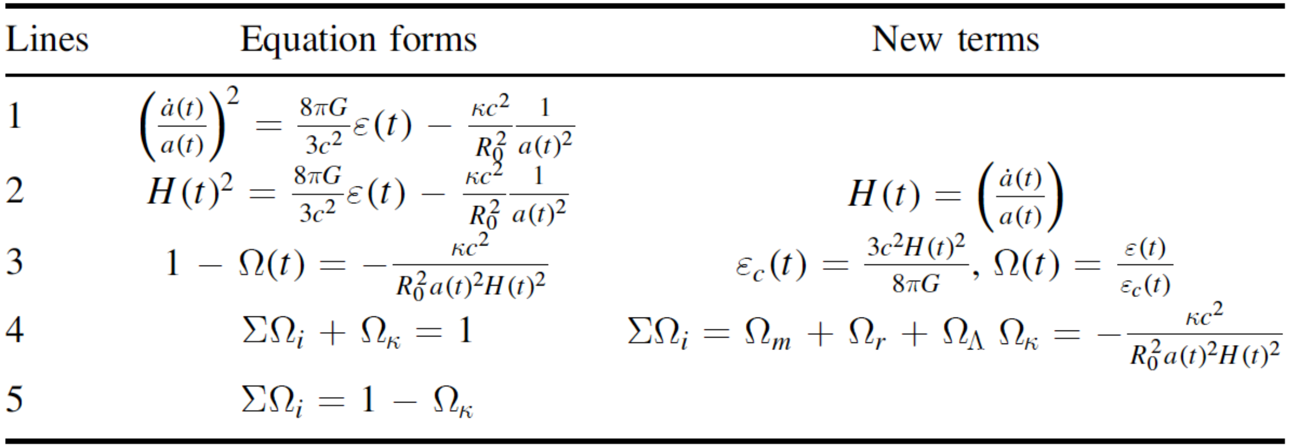
Foreground & Background

- New terms introduced in each line
- Final forms easier to reason with:



- If $\Omega = 1$, then $\kappa = 0$ (flat universe).
- If $\Omega < 1$, then $\kappa > 0$ (open universe).
- If $\Omega > 1$, then $\kappa < 0$ (closed universe).

Newly introduced terms



Lines	Equation forms	New terms
1	$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$	
2	$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$	$H(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)$
3	$1 - \Omega(t) = -\frac{\kappa c^2}{R_0^2 a(t)^2 H(t)^2}$	$\varepsilon_c(t) = \frac{3c^2 H(t)^2}{8\pi G}, \ \Omega(t) = \frac{\varepsilon(t)}{\varepsilon_c(t)}$
4	$\Sigma\Omega_i + \Omega_\kappa = 1$	$\Sigma\Omega_i = \Omega_m + \Omega_r + \Omega_\Lambda \ \Omega_\kappa = -\frac{\kappa c^2}{R_0^2 a(t)^2 H(t)^2}$
5	$\Sigma\Omega_i = 1 - \Omega_\kappa$	

Purposefulness

- Directionality

- Direction: disciplinary needs and questions of interest
- *”Ω”* parameters:
    - *Calculated from observations*
    - *Scale invariant*
    - *Question of interest: relationship between curvature and contents of the universe*

- *Purposeful*

Discussion

- *Implications on Physics Teaching & Learning:*

➢ Not being aware of the mechanism:

  ➢ Focus on the algebraic

  ➢ Not motivating the steps

➢ Awareness of the mechanism can subtly change teaching practice

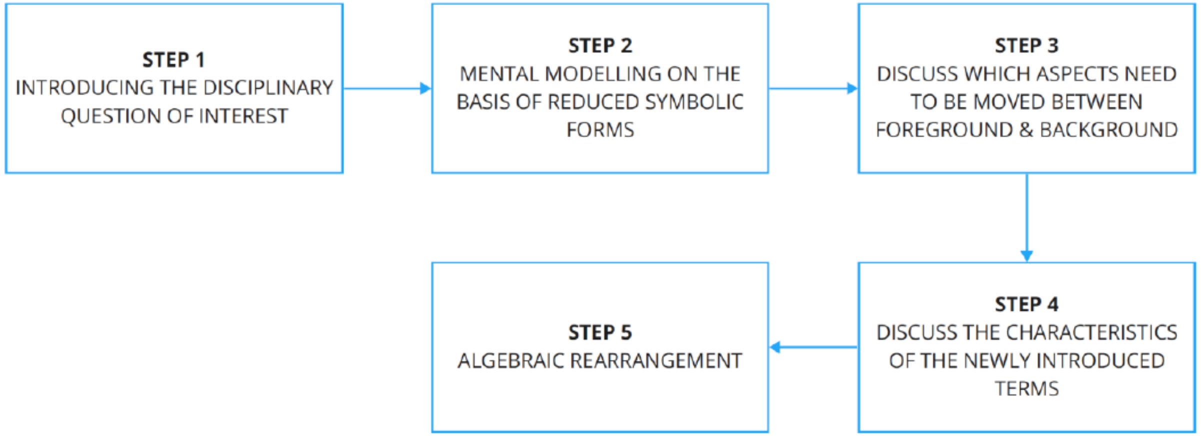
Discussion

- **Suggested teaching sequence:**

A) Leveraging the purposefulness

B) Leveraging the narrowing down of meaning potential

C) Leveraging foreground & background movement



STEP 1
INTRODUCING THE DISCIPLINARY QUESTION OF INTEREST

STEP 2
MENTAL MODELLING ON THE BASIS OF REDUCED SYMBOLIC FORMS

STEP 3
DISCUSS WHICH ASPECTS NEED TO BE MOVED BETWEEN FOREGROUND & BACKGROUND

STEP 4
DISCUSS THE CHARACTERISTICS OF THE NEWLY INTRODUCED TERMS

STEP 5
ALGEBRAIC REARRANGEMENT

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Thank you for listening!

- References

- ➢ B. L. Sherin, *How Students Understand Physics Equations*, Cogn. Instr. **19**, 479 (2001).
- ➢ A. Kapodistrias and J. Airey, *Rearranging Equations to Develop Physics Reasoning* Eur. J. Phys. 45, 3 (2024). https://doi.org/10.1088/1361-6404/ad261c