Light from Dark Matter
– Hidden Dimensions, Supersymmetry, and Inert Higgs

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Recent observational achievements within cosmology and astrophysics have lead to a concordance model in which the energy content in our Universe is dominated by presumably fundamentally new and exotic ingredients – dark energy and dark matter. To reveal the nature of these ingredients is one of the greatest challenges in physics.

The detection of a signal in gamma rays from dark matter annihilation would significantly contribute to revealing the nature of dark matter. This thesis presents derived imprints in gamma-ray spectra that could be expected from dark matter annihilation. In particular, dark matter particle candidates emerging in models with extra space dimensions, extending the standard model to be supersymmetric, and introducing an inert Higgs doublet are investigated. In all these scenarios dark matter annihilation induces sizeable and distinct signatures in their gamma-ray spectra. The predicted signals are in the form of monochromatic gamma-ray lines or a pronounced spectrum with a sharp cutoff at the dark matter particle’s mass. These signatures have no counterparts in the expected astrophysical background and are therefore well suited for dark matter searches.

Furthermore, numerical simulations of galaxies are studied to learn how baryons, that is, stars and gas, affect the expected dark matter distribution inside disk galaxies such as the Milky Way. From regions of increased dark matter concentrations, annihilation signals are expected to be the strongest. Estimations of dark matter induced gamma-ray fluxes from such regions are presented.

The types of dark matter signals presented in this thesis will be searched for with existing and future gamma-ray telescopes.

Finally, a claimed detection of dark matter annihilation into gamma rays is discussed and found to be unconvincing.
**List of Accompanying Papers**

**Paper I**  
**Cosmological Evolution of Universal Extra Dimensions**  
T. Bringmann, M. Eriksson and M. Gustafsson  

**Paper II**  
**Gamma Rays from Kaluza-Klein Dark Matter**  
L. Bergström, T. Bringmann, M. Eriksson and M. Gustafsson  

**Paper III**  
**Two Photon Annihilation of Kaluza-Klein Dark Matter**  
L. Bergström, T. Bringmann, M. Eriksson and M. Gustafsson  
JCAP **0504**, 004 (2005)

**Paper IV**  
**Gamma Rays from Heavy Neutralino Dark Matter**  
L. Bergström, T. Bringmann, M. Eriksson and M. Gustafsson  

**Paper V**  
**Is the Dark Matter Interpretation of the EGRET Gamma Excess Compatible with Antiproton Measurements?**  
L. Bergström, J. Edsjö, M. Gustafsson and P. Salati  

**Paper VI**  
**Baryonic Pinching of Galactic Dark Matter Haloes**  
M. Gustafsson, M. Fairbairn and J. Sommer-Larsen  

**Paper VII**  
**Significant Gamma Lines from Inert Higgs Dark Matter**  
M. Gustafsson, E. Lundström, L. Bergström and J. Edsjö  

**Published Proceedings Not Accompanying:**

**Paper A**  
**Stability of Homogeneous Extra Dimensions**  
T. Bringmann, M. Eriksson and M. Gustafsson  

**Paper B**  
**Gamma-Ray Signatures for Kaluza-Klein Dark Matter**  
L. Bergström, T. Bringmann, M. Eriksson and M. Gustafsson  
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This is my doctoral thesis in Theoretical Physics. During my years as a Ph.D. student, I have been working with phenomenology. This means I live in the land between pure theorists and real experimentalists – trying to bridge the gap between them. Taking elegant theories from the theorist and making firm predictions that the experimentalist can detect is the aim. My research area has mainly been dark matter searches through gamma-ray signals. The ultimate aim in this field is to learn more about our Universe by revealing the nature of the dark matter. This work consists of quite diverse fields: From Einstein’s general relativity, and the concordance model of cosmology, to quantum field theory, upon which the standard model of particle physics is built, as well as building bridges that enable comparison of theory with experimental data. I can therefore honestly say that there are many subjects only touched upon in this thesis that in themselves deserve much more attention.

An Outline of the Thesis

The thesis is composed of two parts. The first introduces my research field and reviews the models and results found in the second part. The second part consists of my published scientific papers.

The organization for part one is as follows: Chapter 1 introduces the essence of modern cosmology and discusses the concept of dark energy and dark matter. Chapter 2 contains a general discussion of the dark matter distribution properties (containing the results of Paper VI), and its relevance for dark matter annihilation signals. Why there is a need to go beyond the standard model of particle physics is then discussed in Chapter 3. This is followed by a description of general aspects of higher-dimensional theories, and the universal extra dimension (UED) model is introduced. In Chapter 4, a toy model for studying cosmology in a multidimensional universe is briefly considered, and the discussion in Paper I is expanded. Chapter 5 then focuses on a detailed description of the field content in the UED model, which simultaneously gives the particle structure of the standard model. After a general discussion of the Kaluza-Klein dark matter candidate from the UED model, special attention is placed on the results from Papers II-III in Chapter 6. This is then followed by a brief introduction to supersymmetry and the results of Paper IV in Chapter 7. The inert Higgs model, its dark matter candidate, and the signal found in Paper VII are then discussed in Chapter 8. Chapter 9 reviews Paper V and a claimed potential detection of a dark matter annihilation signal, before Chapter 10 summarizes this thesis.
For a short layman’s introduction to this thesis, one can read Sections 1.1 and 1.6 on cosmology (including Table 1.1), together with Section 3.1 and large parts of Section 3.2–3.3 on physics beyond the standard model. This can be complemented by reading the preamble to each of the chapters and the summary in Chapter 10 – to comprise the main ideas of the research results in the accompanying papers.

My Contribution to the Accompanying Papers

As obligated, let me say some words on my contribution to the accompanying scientific papers.

During my work on PAPERS I-IV, I had the privilege of closely collaborating with Torsten Bringmann and Martin Eriksson. This was a most democratic collaboration in the sense that all of us were involved in all parts of the research. Therefore, it is in practice impossible to separate my work from theirs. This is also reflected in the strict alphabetic ordering of author names for these papers. If one should make one distinction, in PAPER III I was more involved in the numerical calculations than in the analytical (although many discussions and crosschecks were made between the two approaches). In PAPER V, we scrutinized the claim of a potential dark matter detection by de Boer et al. [1]. Joakim Edsjö and I independently implemented the dark matter model under study, both into DarkSUSY and other utilized softwares. I did the first preliminary calculations of the correlation between gamma-ray and antiproton fluxes in this model, which is our main result in the paper. I was also directly involved in most of the other steps on the way to the final publication and wrote significant parts of the paper. For PAPER VI Malcolm Fairbairn and I had similar ideas on how we could use Jesper-Sommer Larsen’s galaxy simulation to study the dark matter distribution. I wrote parts of the paper, although not the majority. Instead I did many of the final calculations, had many of the ideas for the paper, and produced all the figures (except Figure 4) for the paper. Regarding PAPER VII, I got involved through discussions concerning technical problems that appeared in implementing the so-called inert Higgs model into FeynArts. I found the simplicity of the inert Higgs model very intriguing, and contributed many new ideas on how to proceed with the paper, performed a majority of the calculations, and wrote the main part of the manuscript.

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Special thanks go to my supervisor Professor Lars Bergström, who over the last years has shown generous support, not only financial but also for his sharing of fruitful research ideas.
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I am also very grateful to my dear childhood friend Andreas Hegert for helping me to produce the cover illustration.

All those not mentioned by name here, you should know who you are and how important you have been. I want you all to know that I am extremely grateful for having had you around and for your support in all ways during all times. I love you deeply.

Michael Gustafsson

Stockholm, February 2008
Notations and Conventions

A timelike signature (+, −, −, ···) is used for the metric, except in Chapters 1 and 4 where a spacelike signature (−, +, +, ···) is used. This reflects my choice of following the convention of Misner, Thorne and Wheeler [2] for discussions regarding General Relativity, and Peskin and Schroeder [3] for Quantum Field Theory discussions.

In a spacetime with \(d = 4 + n\) dimensions, the spacetime coordinates are denoted by \(\hat{x}\) with capital Latin indices \(M, N, \ldots \in \{0, 1, \ldots, d - 1\}\) if it is a higher dimensional spacetime with \(n > 0\). Four-dimensional coordinates are given by a lower-case \(x\) with Greek indices \(\mu, \nu, \ldots\) (or lower-case Latin letters \(i, j, \ldots\) for spacelike indices). Extra-dimensional coordinates are denoted by \(y^p\), with \(p = 1, 2, \ldots, n\). That is:

\[
\begin{align*}
  x^\mu &\equiv \hat{x}^M \quad (\mu = M = 0, 1, 2, 3), \\
  x^i &\equiv \hat{x}^M \quad (i = M = 1, 2, 3), \\
  y^p &\equiv \hat{x}^M \quad (p = M - 3 = 1, 2, \ldots, n).
\end{align*}
\]

In the case of one extra dimension, the notation is slightly changed, so that spacetime indices take the value \(\{0, 1, 2, 3, 5\}\) and the coordinate for the extra dimension is denoted \(y (\equiv y^4 \equiv \hat{x}^5)\). Higher-dimensional quantities such as coordinates, coupling constants and Lagrangians will frequently be denoted with a ‘hat’ (as in \(\hat{x}, \hat{\lambda}, \hat{\mathcal{L}}\)) to distinguish them from their four-dimensional analogs \((x, \lambda, \mathcal{L})\).

Einstein’s summation convention is always implicitly understood in expressions, i.e., one sums over any two repeated indices.

The notation \(\ln\) is reserved for the natural logarithm \((\log_e)\), whereas \(\log\) is intended for the base-10 logarithm \((\log_{10})\).

Natural units, where \(c = \hbar = k_B = 1\), are used throughout this thesis, except occasionally where \(\hbar\) and \(c\) appear for clarity.

Useful Conversion Factors \((c = \hbar = k_B = 1)\)

\[
\begin{align*}
  1 \text{ GeV}^{-1} &= 6.5822 \cdot 10^{-25} \text{ s} = 1.9733 \cdot 10^{-14} \text{ cm} \\
  1 \text{ GeV} &= 1.6022 \cdot 10^{-3} \text{ erg} = 1.7827 \cdot 10^{-24} \text{ g} = 1.1605 \cdot 10^{13} \text{ K} \\
  1 \text{ barn (1 b)} &= 10^{12} \text{ pb} = 10^{-24} \text{ cm}^2 \\
  1 \text{ parsec (1 pc)} &= 3.2615 \text{ light yr} = 2.0626 \cdot 10^5 \text{ AU} = 3.0856 \cdot 10^{18} \text{ cm}
\end{align*}
\]
Useful Constants and Parameters

- Speed of light: \( c \equiv 2.99792458 \times 10^{10} \text{ cm s}^{-1} \)
- Planck’s constant: \( \hbar = h/2\pi = 6.5821 \times 10^{-26} \text{ GeV s} \)
- Boltzmann’s constant: \( k_B = 8.1674 \times 10^{-14} \text{ GeV K}^{-1} \)
- Newton’s constant: \( G = 6.6726 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2} \)
- Planck mass: \( M_{\text{pl}} \equiv (\hbar c/G)^{1/2} = 1.2211 \times 10^{19} \text{ GeV c}^{-2} = 2.177 \times 10^{-5} \text{ g} \)
- Electron mass: \( m_e = 5.1100 \times 10^{-4} \text{ GeV c}^{-2} \)
- Proton mass: \( m_p = 9.3827 \times 10^{-1} \text{ GeV c}^{-2} \)
- Earth mass: \( M_{\oplus} = 3.352 \times 10^{24} \text{ GeV c}^{-2} = 5.974 \times 10^{30} \text{ g} \)
- Solar mass: \( M_{\odot} = 1.116 \times 10^{57} \text{ GeV c}^{-2} = 1.989 \times 10^{33} \text{ g} \)
- Hubble constant: \( H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1} (h \sim 0.7) \)
- Critical density: \( \rho_c \equiv 3H_0^2/8\pi G \)
  \[= 1.0540 h^2 \cdot 10^{-5} \text{ GeV c}^{-2} \text{ cm}^{-3} \]
  \[= 1.8791 h^2 \cdot 10^{-29} \text{ g cm}^{-3} \]
  \[= 2.7746 h^2 \cdot 10^{-7} M_{\odot} \text{ pc}^{-3} \]

Acronyms Used in This Thesis

- BBN: Big Bang Nucleosynthesis
- CDM: Cold Dark Matter
- CERN: Conseil Européen pour la Recherche Nucléaire (European Council for Nuclear Research)
- CMB: Cosmic Microwave Background
- DM: Dark Matter
- EGRET: Energetic Gamma Ray Experiment Telescope
- EWPT: ElectroWeak Precision Tests
- FCNC: Flavor Changing Neutral Current
- FLRW: Friedmann Lemaître Robertson Walker
- GLAST: Gamma-ray Large Area Space Telescope
- IDM: Inert Doublet Model
- KK: Kaluza-Klein
- LEP: Large Electron-Positron Collider
- LHC: Large Hadron Collider
- LIP: Lightest Inert Particle
- LKP: Lightest Kaluza-Klein Particle
- MSSM: Minimal Supersymmetric Standard Model(s)
- NFW: Navarro Frenk White
- Ph.D.: Doctor of Philosophy
- SM: Standard Model (of particle physics)
- UED: Universal Extra Dimension
- WIMP: Weakly Interacting Massive Particle
- WMAP: Wilkinson Microwave Anisotropy Probe
Part I

Background Material and Results
The Universe is a big place, filled with phenomena far beyond everyday experience. The scientific study of the properties and evolution of our Universe as a whole is called cosmology. This chapter’s aim is to give a primary outline of modern cosmology, present basic tools and notions, and introduce the dark side of our Universe: the concepts of dark energy and dark matter.

1.1 Our Place in the Universe

For a long time, Earth was believed to be in the center of the Universe. Later it was realized that the motion of the Sun, planets, and stars in the night sky is more simply explained by having Earth and the planets revolving around the Sun instead. The Sun, in turn, is just one among about 100 billion other stars that orbit their mutual mass center and thereby form our own Milky Way Galaxy. In a clear night sky, almost all of the shining objects we can see by the naked eye are stars in our own Galaxy, but with current telescopes it has been inferred that our observable Universe also contains the stars in hundreds of billions of other galaxies.

The range of sizes and distances to different astronomical objects is huge. Starting with our closest star, the Sun, from which it takes the light about eight minutes to reach us here at Earth. This distance can be compared to the distance around Earth, that takes mere one-tenth of a second to travel at the speed of light. Yet these distances are tiny compared to the size of our galactic disk – 100 000 light-years across – and the distance to our nearest (large) neighbor, the Andromeda galaxy – 2 million light-years away. Still, this is nothing compared to cosmological distances. Our own Milky Way belongs to a small group of some tens of galaxies, the Local Group, which in turn

* The astronomer Nicolaus Copernicus (1473-1543) was the first to formulate the heliocentric view of the solar system in a modern way.
belongs to a supercluster, the *Virgo supercluster*, including about one hundred of such groups of clusters. The superclusters are the biggest gravitationally bound systems and reach sizes up to some hundred million light-years. No clusters of superclusters are known, but the existence of structures larger than superclusters is observed in the form of *filaments* of galaxy concentrations, thread-like structures, with a typical length scale of up to several hundred million light-years, which form the boundaries between seemingly large voids in the Universe.

This vast diversity of structures would make cosmology a completely intractable subject if no simplifying characteristic could be used. Such a desired, simplifying feature is found by considering even larger scales, at which the Universe is observed to be *homogeneous* and *isotropic*. That is, the Universe looks the same at every point and in every direction. Of course this is not true in detail, but only if we view the Universe without resolving the smallest scales and ‘smears out’ and averages over cells of $10^8$ light-years, or more, across. The hypothesis that the Universe is spatially isotropic and homogeneous at every point is called the *cosmological principle*, and is one of the fundamental pillars of standard cosmology. A more compact way to express the cosmological principle is to say that the Universe is spatially isotropic at every point, as this automatically implies homogeneity [4]. The cosmological principle combined with Einstein’s general theory of relativity is the foundation of modern cosmology.

### 1.2 Spacetime and Gravity

Since the study of the evolutionary history of our Universe is based on Einstein’s *general theory of relativity*, let us briefly go through the basic concepts used in this theory and in cosmology. As the name suggests, *general relativity* is a generalization of another theory, namely *special* relativity. The special theory of relativity unifies *space and time* into a flat *spacetime*, and the general theory of relativity in turn unifies special relativity with Newton’s theory of gravity.

#### Special Relativity

What does it mean to unify space and time into a four-dimensional spacetime theory? Obviously, already Newtonian mechanics involved three spatial dimensions and a time parameter, so why not already call this a theory of a four-dimensional spacetime? The answer lies in which dimensions can be ‘mixed’ in a meaningful way. For example, in Newtonian mechanics and in a Cartesian coordinate system, defined by perpendicular directed $x$, $y$ and $z$ axes, the Euclidian distance $ds$ between two points is given by Pythagoras’ theorem

$$ds^2 = dx^2 + dy^2 + dz^2.$$  \(1.1\)
However, a rotation or translation into other Cartesian coordinate systems \((x', y', z')\) could equally well be used and the distance would of course be unaltered,

\[ ds^2 = dx'^2 + dy'^2 + dz'^2 \]  

(1.2)

This invariance illustrates that the choice of axes and labels is not important in expressing physical distances. The coordinate transformations that keep Euclidian distances intact are the same that keep Newton’s laws of physics intact, and they are called the \textit{Galileo transformations}. The reference frames where the laws of physics take the same form as in a frame at rest are called \textit{inertial} frames. In the Newtonian language, these are the frames where there are no external forces, and particles remain at rest or in steady, rectilinear motion.

The Galileo transformations do not allow for any transformations that mix space and time; on the contrary, there is an absolute time that is independent of spatial coordinate choice. This, however, is not the case in another classical theory – electrodynamics. The equations of electrodynamics are not form-invariant under Galileo transformations. Instead, there is another class of coordinate transformations that mix space and time and keep the laws of electrodynamics intact. This new class of transformations, called Lorentz transformations, leaves another interval \(d\tau\) between two spacetime points invariant. This spacetime interval is given by

\[ d\tau^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2, \]  

(1.3)

where \(c^2\) is a constant conversion factor between three-dimensional Cartesian space and time distances. That is, Lorentz transformations unify space and time into a four-dimensional spacetime \((t, x, y, z)\), where space and time can be mixed as long as the interval \(d\tau\) in Eq. (1.3) is left invariant. Taking this as a fundamental property, and say that all laws of physics must be invariant with respect to transformations that leave \(d\tau\) invariant, is the lesson of special relativity.

Let me set up the notation that will be used in this thesis: \(x^0 = ct, x^1 = x, x^2 = y,\) and \(x^3 = z\). The convention will also be that Greek indices run from 0 to 3 so that \textit{four-vectors} typically look like

\[(dx)^\alpha = (c dt, dx, dy, dz)\]  

(1.4)

Defining the so-called \textit{Minkowski metric},

\[ \eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \]  

(1.5)
allows for a very compact form for the interval $d\tau$:

$$d\tau^2 = \sum_{\alpha,\beta=0}^{3} \eta_{\alpha\beta} dx^\alpha dx^\beta = \eta_{\alpha\beta} dx^\alpha dx^\beta.$$  

(1.6)

In the last step, the *Einstein summation convention* was used: Repeated indices appearing both as subscripts and superscripts are summed over. There is one important comment to be made regarding the sign convention on $\eta_{\alpha\beta}$ used in this thesis: In Chapters 1 and 4, the sign convention of Eq. (1.5) is adopted (as is the most common convention in the general relativity community), whereas in all other chapters $\eta_{\alpha\beta}$ will be defined to have the opposite overall sign (as is the most common convention within the particle physics community).

In general, the allowed infinitesimal transformations in special relativity are rotations, boosts, and translations. These form a ten-parameter non-abelian group called the *Poincaré group*.

The invariant interval (1.4), and thus special relativity, can be deduced from the following two postulates [5]:

1. **Postulate of relativity**: The laws of physics have the same form in all inertial frames.

2. **Postulate of a universal limiting speed**: In every inertial frame, there is a finite universal limiting speed $c$ for all physical entities.

Experimentally, and in agreement with electrodynamics being the theory of light, the limiting speed $c$ is equal to the speed of light in a vacuum. Today $c$ is defined to be equal to $2.99792458 \times 10^8$ m/s. Another way to formulate the second postulate is to say that the speed of light is finite and independent of the motion of its source.

**General Relativity**

In special relativity, nothing can propagate faster than the speed of light, so Newton’s description of gravity, as an instant force acting between masses, was problematic. Einstein’s way of solving this problem is very elegant. From the observation that different bodies falling in the same gravitational field acquire the same acceleration, he postulated:

**The equivalence principle**: There is no difference between gravitational and inertial masses (this is called the *weak* equivalence principle). Hence, in a frame in free fall no local *gravitational* force phenomena can be detected, and the situation is the same as if no gravitational field was present. Elevate this to include *all* physical phenomena; the results of all local experiments are consistent with the special theory of relativity (this is called the *strong* equivalence principle).
From this postulate, you can derive many fundamental results of general relativity. For example, that time goes slower in the presence of a gravitational field and that light-rays are bent by gravitating bodies. Due to the equivalence between gravitational and inertial masses, an elegant, purely geometrical formulation of general relativity is possible: All bodies in a gravitational field move on straight lines, called *geodesics*, but the spacetime itself is curved and no gravitational forces exist.

We saw above that intervals in a flat spacetime are expressed by means of the Minkowski metric (Eq. 1.5). In a similar way, intervals in a curved spacetime can be express by using a generalized metric $g_{\mu\nu}(x)$ that describes the spacetime geometry. The geometrical curvature of spacetime can be condensed into what is called the Riemann tensor, which is constructed from the metric $g_{\mu\nu}(x)$ as follows:

$$R^\alpha_{\beta\mu\nu} \equiv \partial_\mu \Gamma^\alpha_{\beta\nu} - \partial_\nu \Gamma^\alpha_{\beta\mu} + \Gamma^\alpha_{\sigma\mu} \Gamma^\sigma_{\beta\nu} - \Gamma^\alpha_{\sigma\nu} \Gamma^\sigma_{\beta\mu}, \tag{1.7}$$

where

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} (\partial_\nu g_{\beta\mu} + \partial_\mu g_{\beta\nu} - \partial_\beta g_{\mu\nu}), \tag{1.8}$$

and $g^{\mu\nu}(x)$ is the inverse of the metric $g_{\mu\nu}(x)$, *i.e.*, $g^{\mu\alpha}(x)g_{\alpha\sigma}(x) = \delta^\mu_\nu$.

Having decided upon a description of gravity that is based on the idea of a curved spacetime, we need a prescription for determining the metric in the presence of a gravitational source. What is sought for is a differential equation in analogy with Newton’s law for the gravitational potential:

$$\nabla \phi = 4\pi G \rho, \tag{1.9}$$

where $\rho$ is the mass density and $G$ Newton’s constant. If we want to keep matter and energy as the gravitational source, and avoid introducing any preferred reference frame, the natural source term is the energy-momentum tensor $T_{\mu\nu}$ (where the $T_{00}$ component is Newton’s mass density $\rho$). A second-order differential operator on the metric, set to be proportional to $T_{\mu\nu}$, can be constructed from the Riemann curvature tensor:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} \mathcal{R} g_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{1.10}$$

where $\mathcal{R}_{\mu\nu} \equiv \mathcal{R}^\alpha_{\mu\alpha\nu}$ and $\mathcal{R} \equiv g^{\alpha\beta} \mathcal{R}_{\alpha\beta}$. These are Einstein’s equations of general relativity, including a cosmological constant $\Lambda$-term.\footnote{Independently, and in the same year (1915), David Hilbert derived the same field equations from the action principle (see Eq. (4.4)) \cite{6}.} The left hand-side of Eq. (1.10) is in fact uniquely determined if it is restricted to be divergence free (*i.e.*, local source conservation $\nabla_\mu T^{\mu\nu} = 0$), be linear in the second derivatives of the metric and free of higher derivatives, and vanish in a flat spacetime \cite{2}. The value of the proportionality constant $8\pi G$ in Eq. (1.10) is
obtained from the requirement that Einstein’s equations should reduce to the
Newtonian Eq. (1.9) in the weak gravitational field limit.

In summary, within general relativity, matter in free fall moves on straight
lines (geodesics) in a curved spacetime. In this sense, it is the spacetime that
tells matter how to move. Matter (i.e., energy and pressure), in turn, is the
source of curvature – it tells spacetime how to curve.

1.3 The Standard Model of Cosmology

We now want to find the spacetime geometry of our Universe as a whole, i.e.,
a metric solution $g_{\mu\nu}(x^\alpha)$ to Einstein’s equations. Following the cosmological
principle, demanding a homogenous and isotropic solution, you can show that
the metric solution has to take the so-called Friedmann Lemaître Robertson
Walker (FLRW) form. In spherical coordinates $r, \theta, \phi, t$ this metric is given
by:

$$d\tau^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1-kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (1.11)$$

where $a(t)$ is an unconstrained time-dependent function, called the scale fac-
tor, and $k = -1, +1, 0$ depending on whether space is negatively curved,
positively curved, or flat, respectively. Note that natural units, where $c$ is
equal to 1, have now been adopted.

To present an explicit solution for $a(t)$ we need to further specify the
energy-momentum tensor $T_{\mu\nu}$ in Eq. (1.10). With the metric (1.11), the
energy-momentum tensor must take the form of a perfect fluid. In a comoving
frame, i.e., the rest frame of the fluid, the Universe looks perfectly iso-
tropic, and the energy-momentum tensor has the form:

$$T^\mu_{\nu} = \text{diag}(\rho, p, p, p), \quad (1.12)$$

where $\rho(t)$ represents the comoving energy density and $p(t)$ the pressure
of the fluid. Einstein’s equations (1.10) can now be summarized in the so-called
Friedmann equation:

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G \rho + \Lambda}{3} - \frac{k}{a^2}, \quad (1.13)$$

and

$$\frac{d}{dt}(\rho a^3) = -p \frac{d}{dt} a^3. \quad (1.14)$$

The latter equation should be compared to the standard thermodynamical
equation, expressing that the energy change in a volume $V = a^3$ is equal to
the pressure-induced work that causes the volume change. Given an equation
of state $p = p(\rho)$, Eq. (1.14) determines $\rho$ as a function of $a$. Knowing
$\rho(a)$, a solution $a(t)$ to the Friedmann equation (1.13) can then be completely
specified once boundary conditions are given. This \( a(t) \) sets the dynamical evolution of the Universe.

By expressing all energy densities in units of the critical density

\[
\rho_c \equiv \frac{3H^2}{8\pi G},
\]

the Friedmann equation can be brought into the form

\[
1 = \Omega + \Omega_\Lambda + \Omega_k,
\]

where \( \Omega \equiv \frac{\rho}{\rho_c}, \Omega_\Lambda \equiv \frac{\Lambda}{8\pi G\rho_c} \) and \( \Omega_k \equiv \frac{k}{a^2} \). The energy density fraction \( \Omega \) is often further split into the contributions from baryonic matter \( \Omega_b \) (i.e., ordinary matter), cold dark matter \( \Omega_{CDM} \), radiation/relativistic matter \( \Omega_r \), and potentially other forms of energy. For these components, the equation of state is specified by a proportionality constant \( w \), such that \( p = w\rho \). Specifically, \( w \approx 0 \) for (non-relativistic) matter, \( w = 1/3 \) for radiation, and if one so prefers, the cosmological constant \( \Lambda \) can be interpreted as an energy density with an equation of state \( w = -1 \). We can explicitly see how the energy density of each component\(^\dagger\) depends on the scale factor by integrating Eq. (1.14), which gives:

\[
\rho_i \propto a^{-3(1+w_i)}.
\]

1.4 Evolving Universe

In 1929 Edwin Hubble presented observation that showed that the redshift in light from distant galaxies is proportional to their distance [7]. Redshift, denoted by \( z \), is defined by

\[
1 + z \equiv \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}},
\]

where \( \lambda_{\text{emit}} \) is the wavelength of light at emission and \( \lambda_{\text{obs}} \) the wavelength at observation, respectively. In a static spacetime, this redshift would presumably be interpreted as a Doppler shift effect: light emitted from an object moving away from you is shifted to longer wavelengths. However, in agreement with the cosmological principle the interpretation should rather be that the space itself is expanding. As the intergalactic space is stretched, so is the wavelength of the light traveling between distant objects. For a FLRW metric, the following relationship between the redshift and the scale factor holds:

\[
1 + z = \frac{a(t_{\text{obs}})}{a(t_{\text{emit}})}.
\]

The interpretation of Hubble’s observation is therefore that our Universe is expanding.

\(^\dagger\) Assuming that each energy component separately obeys local ‘energy conservation’.
Proper distance is the distance we would measure with a measuring tape between two space points at given cosmological time \(i.e. \int d\tau\). In practice, this is not a measurable quantity, and instead there are different indirect ways of measuring distances. The angular distance \(d_A\) is based on the flat spacetime notion that an object of known size \(D\), which subtends a small angle \(\delta \theta\), is at a distance \(d_A \equiv D/\delta \theta\). The luminosity distance \(d_L\) instead makes use of the fact that a light source appears weaker the further away it is, and is defined by

\[
d_L \equiv \sqrt{\frac{S}{4\pi L}},
\]

where \(S\) is the intrinsic luminosity of the source and \(L\) the observed luminosity. In flat Minkowski spacetime, these measures would give the same result, but in an expanding universe they are instead related by \(d_L = d_A(1 + z)^2\). For an object at a given redshift \(z\), the luminosity distance for the FLRW metric is given by

\[
d_L = a_0(1 + z)f \left( \frac{1}{a_0} \int_0^z \frac{dz'}{H(z')} \right),
\]

\[
f(x) \equiv \begin{cases} 
\sinh(x), & \text{if } k = -1 \\
x, & \text{if } k = 0 \\
\sin(x), & \text{if } k = +1 
\end{cases}
\]

Here \(a_0\) represents the value of the scale factor today, and \(H(z)\) is the Hubble expansion at redshift \(z\):

\[
H(z) = H_0 \sqrt{\sum_i \Omega_i^0(1 + z)^{-3(1 + w_i)}},
\]

where \(H_0\) is the Hubble constant, and \(\Omega_i^0\) are the energy fraction in different energy components today.

It is often convenient to define the comoving distance, the distance between two points as it would be measured at the present time. This means that the actual expansion is factored out, and the comoving distance stays constant, even though the Universe expands. A physical distance \(d\) at redshift \(z\) corresponds to the comoving distance \((1 + z) \cdot d\).

By measuring the energy content of the Universe at a given cosmological time, \(e.g.,\) today, we can, by using Eq. (1.17), derive the energy densities at other redshifts. By naively extrapolating backwards in time we would eventually reach a singularity, when the scale factor \(a = 0\). This point is sometimes popularly referred to as the Big Bang. It should, however, be kept in mind that any trustworthy extrapolation breaks down before this singularity is reached – densities and temperatures will become so high that we do not have any adequately developed theories to proceed with the extrapolation. A better (and the usual) way to use the term Big Bang is instead to let it denote the
early stage of a very hot, dense, and rapidly expanding Universe. A brief timeline for our Universe is given in Table 1.1.

This Big Bang theory shows remarkably good agreement with cosmological observations. The most prominent observational support of the standard cosmological model comes from the agreement with the predicted abundance of light elements formed during Big Bang nucleosynthesis (BBN), and the existence of the cosmic microwave background radiation. In the early Universe, numerous photons, which were continuously absorbed, re-emitted, and interacting, constituted a hot thermal background bath for other particles. This was the case until the temperature eventually fell below about 3000 K. At this temperature, electrons and protons combine to form neutral hydrogen (the so-called recombination), which then allows the photons to decouple from the primordial plasma. These photons have since then streamed freely through space and constitute the so-called cosmic microwave background (CMB) radiation. The CMB photons provides us today with a snapshot of the Universe at an age of about 400 000 years or, equivalently, how the Universe looked 13.7 billion years ago.

1.5 Initial Conditions

The set of initial conditions required for this remarkable agreement between observation and predictions in the cosmological standard model is however slightly puzzling. The most well-known puzzles are the flatness and horizon problems.

If the Universe did not start out exactly spatial flat, the curvature tends to become more and more prominent. That means that already a very tiny deviation from flatness in the early Universe would be incompatible with the close to flatness observed today. This seemingly extreme initial fine-tuning is what is called the flatness problem.

The horizon problem is related to how far information can have traveled at different epochs in the history of our Universe. There is a maximal distance that any particle or piece of information can have propagated since the Big Bang at any given comoving time. This defines what is called the particle horizon

$$d_H(t) = \int_0^t \frac{dt'}{a(t')} = a(t) \int_0^{r(t)} \frac{dr'}{\sqrt{1 - kr^2}}.$$  \hfill (1.23)

That is, in the past a much smaller fraction of the Universe was causally connected than today. For example, assuming traditional Big Bang cosmology, the full-sky CMB radiation covers about $10^5$ patches that have never been in causal contact. Despite this, the temperature is the same across the whole

\footnote{There is also the notion of event horizon in cosmology, which is the largest comoving distance from which light can ever reach the observer at any time in the future.}
Table 1.1: The History of the Universe.

<table>
<thead>
<tr>
<th>Time</th>
<th>Size</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-43}$ s</td>
<td>$10^{-60}\times$ today</td>
<td>$10^{32}$ K</td>
</tr>
<tr>
<td><strong>The Planck era:</strong></td>
<td>Quantum gravity is important; current theories are inadequate, and we cannot go any further back in time.</td>
<td></td>
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<thead>
<tr>
<th>Time</th>
<th>Size</th>
<th>Temperature</th>
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</thead>
<tbody>
<tr>
<td>$10^{-35}$ s</td>
<td>$10^{-54}\rightarrow 26\times$ today</td>
<td>$10^{26}\rightarrow 0\rightarrow 26$ K</td>
</tr>
<tr>
<td><strong>Inflation:</strong></td>
<td>A conjectured period of accelerating expansion; an inflaton field causes the Universe to inflate and then decays into SM particles.</td>
<td></td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Time</th>
<th>Size</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-12}$ s</td>
<td>$10^{-15}\times$ today</td>
<td>$10^{15}$ K</td>
</tr>
<tr>
<td><strong>Electroweak phase transition:</strong></td>
<td>Electromagnet and weak interactions become distinctive interactions below this temperature.</td>
<td></td>
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<thead>
<tr>
<th>Time</th>
<th>Size</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-6}$ s</td>
<td>$10^{-12}\times$ today</td>
<td>$10^{12}$ K</td>
</tr>
<tr>
<td><strong>Quark-gluon phase transition:</strong></td>
<td>Quarks and gluons become bound into protons and neutrons. All SM particles are in thermal equilibrium.</td>
<td></td>
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<tr>
<th>Time</th>
<th>Size</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 s</td>
<td>$10^{-8}\times$ today</td>
<td>$10^{9}$ K</td>
</tr>
<tr>
<td><strong>Primordial nucleosynthesis:</strong></td>
<td>The Universe is cold enough for protons and neutrons to combine and form light atomic nuclei, such as He, D and Li.</td>
<td></td>
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</tbody>
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<thead>
<tr>
<th>Time</th>
<th>Size</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{12}$ s</td>
<td>$3 \cdot 10^{-4}\times$ today</td>
<td>$10^{4}$ K</td>
</tr>
<tr>
<td><strong>Matter-radiation equality:</strong></td>
<td>Pressureless matter starts to dominate.</td>
<td></td>
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<thead>
<tr>
<th>Time</th>
<th>Size</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4 \times 10^5$ yrs</td>
<td>$10^{-3}\times$ today</td>
<td>$3 \times 10^3$ K</td>
</tr>
<tr>
<td><strong>Recombination:</strong></td>
<td>Electrons combine with nuclei and form electrically neutral atoms, and the Universe becomes transparent to photons. The cosmic microwave background is a snapshot of photons from this epoch.</td>
<td></td>
</tr>
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<thead>
<tr>
<th>Time</th>
<th>Size</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^8$ yrs</td>
<td>$0.1\times$ today</td>
<td>$30$ K</td>
</tr>
<tr>
<td><strong>The dark ages:</strong></td>
<td>Small ripples in the density of matter gradually assemble into stars and galaxies.</td>
<td></td>
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<thead>
<tr>
<th>Time</th>
<th>Size</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{10}$ yrs</td>
<td>$0.5\times$ today</td>
<td>$6$ K</td>
</tr>
<tr>
<td><strong>Dark energy:</strong></td>
<td>The expansion of the Universe starts to accelerate. A second generation of stars, the Sun and Earth, are formed.</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Size</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$13.7 \times 10^9$ yrs</td>
<td>$1\times$ today</td>
<td>$2.7$ K</td>
</tr>
<tr>
<td><strong>Today:</strong></td>
<td>$\Omega_A \sim 74%$, $\Omega_{CDM} \sim 22%$, $\Omega_{baryons} = 4%$, $\Omega_r \sim 0.005%$, $\Omega_k \sim 0$.</td>
<td></td>
</tr>
</tbody>
</table>
sky to a precision of about $10^{-5}$. This high homogeneity between casually disconnected regions is the horizon problem.

An attractive, but still not established, potential solution to these initial condition problems was proposed in the beginning of the 1980’s [8–10]. By letting the Universe go through a phase of accelerating expansion, the particle horizon can grow exponentially and thereby bring all observable regions into causal contact. At the same time, such an inflating Universe will automatically flatten itself out. The current paradigm is basically that such an inflating phase is caused by a scalar field $\Phi$ dominating the energy content by its potential $V(\Phi)$. If this inflaton field is slowly rolling in its potential, i.e., $\frac{1}{2} \dot{\phi} \ll V(\Phi)$, the equation of state is $p_{\Phi} \approx -V(\Phi) \approx -\rho_{\Phi}$. If $V(\Phi)$ stays fairly constant for a sufficiently long time, it would mimic a cosmological constant domination. From Eq. (1.13) it follows that $H^2 \approx \text{constant}$ and thus that the scale factor grows as $a(t) \propto e^{Ht}$. This will cause all normal matter fields ($w > -1/3$) to dilute away†. During this epoch, the temperature drops drastically, and the Universe super-cools due to the extensive space expansion. Once the inflaton field rolls down in the presumed minimum of its potential, it will start to oscillate, and the heavy inflaton particles will decay into standard model particles. This reheats the Universe, and it evolves as in the ordinary hot Big Bang theory with the initial conditions naturally‡ tuned by inflation. During inflation, quantum fluctuations of the inflaton field will be stretched and transformed into effectively classical fluctuations (see, e.g., [13]). When the inflation field later decays, these fluctuations will be transformed to the primordial power spectrum of matter density fluctuations. These seeds of fluctuations will then eventually grow to become the large-scale structures, such as galaxies etc, that we observe today. Today, the observed spectrum of density fluctuations is considered to be the strongest argument for inflation.

1.6 The Dark Side of the Universe

What we can observe of our Universe are the various types of signals that reach us – light of different wavelengths, neutrinos, and other cosmic rays. This reveals the distribution of ‘visible’ matter. But how would we know if there is more substance in the Universe, not seen by any of the above means?

The answer lies in that all forms of energy produce gravitational fields (or in other words, curve the surrounding spacetime), which affect both their local surroundings and the Universe as a whole. Perhaps surprisingly, such gravitational effects indicate that there seems to be much more out there in

† This would also automatically explain the absence of magnetic monopoles, which could be expected to be copiously produced during Grand Unification symmetry breaking at some high energy scale.

‡ A word of caution: Reheating after inflation drastically increases the entropy, and a very low entropy state must have existed before inflation, see, e.g., [11,12] and references therein.
our Universe than can be seen directly. It turns out that this ‘invisible stuff’ can be divided into two categories: dark energy and dark matter. Introducing only these two types of additional energy components seems to be enough to explain a huge range of otherwise unexplained cosmological and astrophysical observations.

**Dark Energy**

In 1998 both the Supernova Cosmology Project and the High-z Supernova Search Team presented for the first time data showing an accelerating expansion of the Universe [14, 15]. To accomplish this result, redshifts and luminosity distances to Type Ia supernovae were measured. The redshift dependence of the expansion rate $H(z)$ can then be deduced from Eq. (1.21). The Type Ia supernovae data showed a late-time acceleration of the expansion of our Universe ($\ddot{a}/a > 0$). This conclusion relies on Type Ia supernovae being *standard candles*, i.e., objects with known intrinsic luminosities, which are motivated both on empirical as well as theoretical grounds.

These first supernova results have been confirmed by more recent observations (*e.g.*, [16, 17]). The interpretation of a late-time accelerated expansion of the Universe also fits well into other independent observations, such as data from the CMB [18] and gravitational lensing (see, *e.g.*, [19]).

These observations indicate that the Universe is dominated by an energy form that i) has a negative pressure that today has an equation of state $w \approx -1$, ii) is homogeneously distributed throughout the Universe with an energy density $\rho_\Lambda \approx 10^{-29}$ g/cm$^3$, and iii) has no significant interactions other than gravitational. An energy source with mentioned properties could also be referred to as *vacuum energy*, as it can be interpreted as the energy density of empty space itself. However, within quantum field theory, actual estimates of the vacuum energy are of the order of $10^{120}$ times larger than the observed value.

The exact nature of dark energy is a matter of speculation. A currently viable possibility is that it is the cosmological constant $\Lambda$. That is, the $\Lambda$ term in Einstein’s equation is a fundamental constant that has to be determined by observations. If the dark energy really is an energy density that is constant in time, then the period when the dark energy and matter energy densities are similar, $\rho_\Lambda \sim \rho_m$, is extremely short on cosmological scales (*i.e.*, in redshift

---

5 To translate between $z$ and $t$, one can use $H(z) = \frac{d}{dt} \ln \left( \frac{a}{a_0} \right) = \frac{d}{dt} \ln \left( \frac{1}{1+z} \right) = -\frac{1}{1+z} \frac{dz}{dt}$.

6 A Type Ia supernova is believed to be the explosion of a white dwarf star that has gained mass from a companion star until reaching the so-called Chandrasekhar mass limit $\sim 1.4M_\odot$ (where $M_\odot$ is the mass of the Sun). At this point, the white dwarf becomes gravitationally instable, collapses, and explodes as a supernova.

7 Inclusion of *broken* supersymmetry could decrease this disagreement to some $10^{60}$ orders of magnitude.
range). We could wonder why we happen to be around to observe the Universe just at the moment when $\rho_\Lambda \sim \rho_m$?

Another proposed scenario for dark energy is to introduce a new scalar field, with properties similar to the inflaton field. This type of scalar fields is often dubbed quintessence [20] or k-essence [21] fields. These models differ from the pure cosmological constant in that such fields can vary in time (and space). However, the fine-tuning, or other problems, still seems to be present in all suggested models, and no satisfactory explanation of dark energy is currently available.

**Dark Matter**

The mystery of missing dark matter (in the modern sense) goes back to at least the 1930s when Zwicky [22] pointed out that the movements of galaxies in the Coma cluster, also known as Abell 1656, indicated a mass-to-light ratio of around 400 solar masses per solar luminosity, which is two orders of magnitude higher than in our solar neighborhood. The mass of clusters can also be measured by other methods, for example by studying gravitational lensing effects (see, e.g., [23] for an illuminating example) and by tracing the distribution of hot gas through its X-ray emission (e.g., [24]). Most observations on cluster scales are consistent with a matter density of $\Omega_{\text{matter}} \sim 0.2 - 0.3$ [25]. At the same time the amount of ordinary (baryonic) matter in clusters can be measured by the so-called Sunayaev-Zel’dovich effect [26], by which the CMB gets spectrally distorted through Compton scattering on hot electrons in the clusters. This, as well as X-ray observations, shows that only about 10% of the total mass in clusters is visible baryonic matter, the rest is attributed to dark matter.

At galactic scales, determination of rotation curves, i.e., the orbital velocities of stars and gas as a function of their distance from the galactic center, can be efficiently used to determine the amount of mass inside these orbits. At these low velocities and weak gravitational fields, the full machinery of general relativity is not necessary, and circular velocities should be in accordance with Newtonian dynamics:

$$v(r) = \sqrt{\frac{GM(r)}{r}},$$

(1.24)

where $M(r)$ is the total mass within radius $r$ (and spherical symmetry has been assumed). If there were no matter apart from the visible galactic disk, the circular velocities of stars and gas should be falling off as $1/\sqrt{r}$. Observations say otherwise: The velocities $v(r)$ stay approximately constant outside the bulk of the visible galaxy. This indicates the existence of a dark (invisible) halo with $M(r) \propto r$, and thus $\rho_{DM} \sim 1/r^2$ (see, e.g., [27]).

On cosmological scales, the observed CMB anisotropies combined with other measurements are a powerful tool in determining the amount of dark
matter. In fact, without dark matter, the cosmological standard model would fail dramatically to explain the CMB observations [18]. Simultaneously, the baryon fraction is determined to be about only 4%, which is in good agreement with the value inferred, independently, from BBN to explain the abundance of light elements in our Universe.

Other strong support for a large amount of dark matter comes from surveys of the large-scale structures [28] and the so-called baryon acoustic peak in the power spectrum of matter fluctuations [29]. These observations show how tiny baryon density fluctuations, deduced from the CMB radiation, in the presence of larger dark matter fluctuations have grown to form the large scale structure of galaxies. The structures observed today would not even have had time to form from these tiny baryon density fluctuations, if no extra gravitational structures (such as dark matter) were present.

Finally, recent developments in weak lensing techniques have made it possible to produce rough maps of the dark matter distribution in parts of the Universe [30].

Models that instead of the existence of dark matter suggest modifications of Newton’s dynamics (MOND) [31, 32] have, in general, problems explaining the full range of existing data. For example, the so-called ‘bullet cluster’ observation [33, 34] rules out the simplest alternative scenarios. The bullet cluster shows a snapshot of what is interpreted as a galaxy cluster ‘shot’ through another cluster (hence the name bullet) – and is an example where the gravitational sources are not concentrated around most of the visible matter. The interpretation is that the dark matter (and stars) in the two colliding clusters can pass through each other frictionless, whereas the major part of the baryons, i.e., gas, will interact during the passage and therefore be halted in the center. This explains both the centrally observed concentration of X-ray-emitting hot gas, and the two separate concentrations of a large amount of gravitational mass observed by lensing.

In contrast to dark energy, dark matter is definitely not homogeneously distributed at all scales throughout the Universe. Dark matter is instead condensed around, e.g., galaxies and galaxy clusters, forming extended halos. To be able to condense, in agreement with observations, dark matter should be almost pressureless and non-relativistic during structure formation. This type of non-relativistic dark matter is referred to as cold dark matter.

The concordance model that has emerged from observations is a Universe where about 4% is in the form of ordinary matter (mostly baryons in the form of gas, \( w \approx 0 \)) and about 0.005% is in visible radiation energy (mostly the CMB photons, \( w = 1/3 \)). The remaining part of our Universe’s total energy budget is dark and of an unknown nature. Of the total energy roughly 74% is dark energy (\( w \sim -1 \)), and 22% is dark matter (\( w = 0 \)). Most of the dark matter is cold (non-relativistic) matter, but there is definitely also some hot dark matter in the form of neutrinos. However, the hot dark matter can at most make up a few percent [35, 36]. Some fraction of warm dark matter, i.e.,
Section 1.6. The Dark Side of the Universe

Dark Energy 74%

Dark Matter 22%

Baryonic Matter 4%

2% Luminous (Gas & Stars)
0.005% Radiation (CMB)
2% Dark Baryons (Gas)

Figure 1.1: The energy budget of our Universe today. Ordinary matter (luminous and dark baryonic matter) only contributes some percent, while the dark matter and the dark energy make up the dominant part of the energy content in the Universe. The relative precisions of the quoted energy fractions are roughly ten percent in a ΛCDM model. The figure is constructed from the data in [18, 37–39].

particles with almost relativistic velocities during structure formation, could also be present. This concordance scenario is often denoted the cosmological constant Λ Cold Dark Matter (ΛCDM) model. Figure 1.1 shows this energy composition of the Universe (at redshift $z = 0$).

Note that the pie chart in Fig. 1.1 do change with redshift (determined by how different energy components evolve, see Eq. (1.17)). For example, at the time of the release of the CMB radiation the dark energy part was negligible. At that time the radiation contribution and the matter components were of comparable size, and together made up more or less all the energy in the Universe.

The wide range of observations presents very convincing evidence for the existence of cold dark matter, and it points towards new, yet unknown exotic physics. A large part of this thesis contain our predictions, within different scenarios, that could start to reveal the nature of this dark matter.

All Those WIMPs – Particle Dark Matter

Contrary to dark energy, there are many proposed candidates for the dark matter. The most studied hypothesis is dark matter in the form of some

** The background picture in the dark energy pie chart shows the WMAP satellite image of the CMB radiation [40]. The background picture in the dark matter pie chart is a photograph of the Bullet Cluster showing the inferred dark matter distribution (in blue) and the measured hot gas distributions (in red) [41].
yet undiscovered species of fundamental particle. To have avoided detection, they should only interact weakly with ordinary matter. Furthermore, these particles should be stable, *i.e.*, have a lifetime that is at least comparable to cosmological time scales, so that they can have been around in the early Universe and still be around today.

One of the most attractive classes of models is that of so-called Weakly Interacting Massive Particles – WIMPs. One reason for the popularity of these dark matter candidates is the ‘WIMP miracle’. In the very early Universe, particles with electroweak interactions are coupled to the thermal bath of standard model particles, but at some point their interaction rate falls below the expansion rate of the Universe. At this point, the WIMPs decouple, and their number density freezes in, thereby leaving a relic abundance consistent with the dark matter density today. Although the complete analysis can be complicated for specific models, it is usually a good estimate that the relic density is given by [42]

$$\Omega_{\text{WIMP}} h^2 \approx 3 \cdot 10^{-27} \text{cm}^{-3}\text{s}^{-1},$$

where $h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$ ($h$ is today observed to be $0.72 \pm 0.03$ [18]) and $\langle \sigma v \rangle$ is the thermally averaged interaction rate (cross section times relative velocity of the annihilating WIMPs). This equation holds almost independently of the WIMP mass, as long the WIMPs are non-relativistic at freeze-out. The ‘WIMP miracle’ that occurs is that the cross section needed, $\langle \sigma v \rangle \sim 10^{-26}$ cm$^3$ s$^{-1}$, is roughly what is expected for particle masses at the electroweak scale. Typically $\sigma v \sim \frac{\alpha^2}{M_{\text{WIMP}}^2} \sim 10^{-26}$ cm$^{-3}$s$^{-1}$, where $\alpha$ is the fine structure constant and the WIMP mass $M_{\text{WIMP}}$ is taken to be about 100 GeV.

There are other cold dark matter candidates that do not fall into the WIMP dark matter category. Examples are the gravitino and the axion. For a discussion of these and other types of candidates, see for example [25] and references therein.
Without specifying the true nature of dark matter, one can still make general predictions of its distribution based on existing observations, general model building, and numerical simulations. Specifically, this chapter concentrates on discussing the expected dark matter halos around galaxies like our own Milky Way. For dark matter in the form of self-annihilating particles, the actual distribution of its number density plays an extremely important role for the prospects of future indirect detection of these dark matter candidates. An effective pinching and reshaping of dark matter halos caused by the central baryons in the galaxy, or surviving small dark matter clumps, can give an enormously increased potential for indirect dark matter detection.

2.1 Structure Formation History

During the history of our Universe, the mass distribution has changed drastically. The tiny $10^{-5}$ temperature fluctuations at the time of the CMB radiation reflects a Universe that was almost perfectly homogeneous in baryon density. Since then, baryons and dark matter have, by the influence of gravity, built up structures like galaxies and clusters of galaxies that we can observe today. To best describe this transition, the dark matter particles should be non-relativistic (‘cold’) and experience at most very weak interactions with ordinary matter. This ensures that the dark matter was pressureless and separated from the thermal equilibrium of the baryons and the photons well before recombination, and could start evolving from small structure seeds – these first seeds could presumably originating from quantum fluctuations in an even earlier inflationary epoch.

Perturbations at the smallest length scales – entering the horizon prior to radiation-matter equality – will not be able to grow, but are washed out due to the inability of the energy-dominating radiation to cluster. Later, when larger scales enter the horizon during matter domination, dark matter
density fluctuations will grow in amplitude due to the absence of counter-balancing radiative pressure. This difference in structure growth, before and after matter-radiation equality, is today imprinted in the matter power spectrum as a suppression in density fluctuations at comoving scales smaller than roughly 1 Gpc, whereas on larger scales the density power spectrum is scale invariant (in agreement with many inflation models). The baryons are, however, tightly coupled to the relativistic photons also after radiation-matter equality and cannot start forming structures until after recombination. Once released from the photon pressure, the baryons can then start to form structures rapidly in the already present gravitational wells from the dark matter. Without these pre-formed potential wells, the baryons would not have the time to form the structures we can observe today. This is a strong support for the actual existence of cold dark matter.

As long as the density fluctuations in matter stay small, linearized analytical calculations are possible, whereas once the density contrast becomes close to unity one has to resort to numerical simulations to get reliable results on the structure formation. The current paradigm is that structure is formed in a hierarchal way; smaller congregations form first and then merge into larger and larger structures. These very chaotic merging processes result in so-called violent relaxation, in which the time-varying gravitational potential randomizes the particle velocities. The radius within which the particles have a fairly isotropic distribution of velocities is commonly called the virial radius. Within this radius, virial equilibrium should approximately hold, \( 2E_k \approx E_p \), where \( E_k \) and \( E_p \) are the averaged kinetic energy and gravitational potential, respectively.

By different techniques, such as those mentioned in Chapter 1, it is possible to get some observational information on the dark matter density distribution. These observations are often very crude, and therefore it is common to use halo profiles predicted from numerical simulations rather than deduced from observations. In the regimes where simulations and observations can be compared, they show reasonable agreement, although some tension might persists [25].

### 2.2 Halo Models from Dark Matter Simulations

Numerical \( N \)-body simulations of structure formation can today contain up to about \( 10^{10} \) particles (as, \( e.g. \), in the ‘Millennium simulation’ [43]) that evolve under their mutual gravitational interactions in an expanding universe. Such simulations are still far from resolving the smallest structures in larger halos. Furthermore, partly due to the lack of computer power, many of these high-resolution simulations include only gravitational interactions, \( i.e. \), dark matter. These simulations suggest that radial density profiles of halos ranging from masses of \( 10^{-6} \) [44] to several \( 10^{15} \) [45] solar masses have an almost
Table 2.1: Parameters for some widely used dark matter density profile models (see equation 2.1). The values of $r_s$ are for a typical Milky-Way-sized halo of mass $M_{200} \sim 10^{12} M_\odot$ at redshift $z = 0$.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$r_s$ [kpc]</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFW</td>
<td>1.0</td>
<td>3.0</td>
<td>1.0</td>
<td>20</td>
</tr>
<tr>
<td>Moore</td>
<td>1.5</td>
<td>3.0</td>
<td>1.5</td>
<td>30</td>
</tr>
<tr>
<td>Kra</td>
<td>2.0</td>
<td>3.0</td>
<td>0.4</td>
<td>20</td>
</tr>
<tr>
<td>Iso</td>
<td>2.0</td>
<td>2.0</td>
<td>0.0</td>
<td>4</td>
</tr>
</tbody>
</table>

universal form

A suitable parametrization for the dark matter density $\rho$ is to have two different asymptotic radial power law behaviors, i.e., $r^{-\gamma}$ at the smallest radii and $r^{-\beta}$ at the largest radii, with a transition rate $\alpha$ by which the profile interpolates between these two asymptotic powers around the radius $r_s$:

$$\rho(r) = \frac{\rho_0}{(r/r_s)^\gamma [1 + (r/r_s)^\alpha]^{\frac{-\beta}{\alpha}}}.$$  \hspace{1cm} (2.1)

It is often convenient to define a radius $r_{200}$, sometimes also referred to as the virial radius, inside which the mean density is 200 times the critical density $\rho_c$. The total mass enclosed is thus:

$$M_{200} = 200 \frac{4\pi r_{200}^3}{3} \rho_c.$$  \hspace{1cm} (2.2)

For a given set of $(\alpha, \beta, \gamma)$ the density profile in Eq. (2.1) is completely specified by only two parameters, e.g., the halo mass $M_{200}$ and the scale radius $r_s$. The two parameters $M_{200}$ and $r_s$ could in principle be independent, but numerical simulations indicate that they are correlated. In that sense, it is sometimes enough to specify only $M_{200}$ for a halo (see, e.g., the appendix of [48]). Instead of $r_s$, the concentration parameter $c_{200} = r_{200}/r_s$ is also often introduced. Although less dependent on halo size than $r_s$, $c_{200}$ also varies with a tendency to increase for smaller halo size and larger redshifts (see, e.g., [49] and [47]).

Some of the most common values of parametrization parameters $(\alpha, \beta, \gamma)$ found for dark matter halos are given in Table 2.1. From top to bottom, the table gives the values for the Navarro, Frenk and White (NFW, [50]), Moore et al. (Moore, [51]), and the Kravtsov et al. (Kra, [52]) profile. The modified isothermal sphere profile (Iso, e.g., [53, 54]), with its constant density core, is also included.

The most recent numerical simulations appear to agree on a slightly new paradigm for the dark matter density. They suggest that the logarithmic slope, defined as

$$\gamma(r) \equiv \frac{d \ln(\rho)}{d \ln(r)},$$  \hspace{1cm} (2.3)

* The density profiles are actually not found to be fully universal as the density slope in the center of smaller halos is in general steeper than in larger halos [46, 47].
decreases continuously towards the center of the halos. In accordance with this, the following, so-called Einasto density profile is suggested [55, 56]:

$$\rho(r) = \rho_{-2} \exp \left[ -\frac{2}{\alpha} \left( \left( \frac{r}{r_{-2}} \right)^{\alpha} - 1 \right) \right].$$  \hspace{1cm} (2.4)

In this profile, $\rho_{-2}$ and $r_{-2}$ correspond to the density and radius where $\rho \propto r^{-2}$. (Note that the logarithmic slope converges to zero when $r = 0$.) Typically $\alpha$ is found to be of the order of $\sim 0.2$ [56].

The dark matter profile are sometimes referred to as cored, cuspy, or spiked depending on whether the density in the center scales roughly as $r^{-\gamma}$ with $\gamma \approx 0$, $\gamma \gtrsim 0$ or $\gamma \gtrsim 1.5$, respectively.

All the above results stem from studies of dark matter dominated systems. This should in many respects be adequate, as the dark matter makes up $\sim 80\%$ [18] of all the matter and therefore usually dominates the gravitationally induced structure formation. However, in the inner parts of e.g., galaxy halos, the baryons, i.e., gas and stars, dominate the gravitational potential and should be of importance also for dark matter distribution.

### 2.3 Adiabatic Contraction

The main difference between dark and baryonic matter is that the latter will frequently interact and cool by dissipating energy. This will cause the baryons, unless disturbed by major mergers, to both form disk structures and contract considerably in the centers. This behavior is indeed observed both in simulations containing baryons and also in nature, where the baryons form a disk and/or bulge at the center of apparently much more extended dark matter halos. It has long been realized that this ability of baryons to sink to the center of galaxies would create an enhanced gravitational potential well within which dark matter could congregate, increasing the central dark matter density. This effect is commonly modeled by the use of adiabatic invariants [53, 54, 57–66].

#### A Simple Model for Adiabatic Contraction

The most commonly used model, suggested by Blumenthal \textit{et al.} [60], assumes a spherically symmetric density distribution and circular orbits of the dark matter particles. From angular momentum conservation $p_i r_i = p_f r_f$ and gravitational-centripetal force balance $G\frac{M(r)}{r^2} = \frac{v^2}{m_i r}$, we obtain the adiabatic invariant

$$r_f M_f(r_f) = r_i M_i(r_i),$$  \hspace{1cm} (2.5)

where $M(r)$ is the total mass inside a radius $r$ and the lower indices $i$ and $f$ indicate if a quantity is initial or final, respectively. Splitting up the final
mass distribution $M_f(r)$ into a baryonic part $M_b(r)$ and an unknown dark matter part $M_{DM}(r)$, we have $M_f(r) = M_b(r) + M_{DM}(r)$. Eq. (2.5) gives

$$r_f = \frac{r_i M_i(r_i)}{M_b(r_f) + M_{DM}(r_f)}. \quad (2.6)$$

From mass conservation, the non-crossing of circular orbits during contraction, and a mass fraction $f$ of the initial matter distribution in baryons, we get

$$M_{DM}(r_f) = M_i(r_i)(1 - f), \quad (2.7)$$

This means: From an initial mass distribution $M_i$, of which a fraction $f$ (i.e., the baryons) forms a new distribution $M_b$, the remaining particles (i.e., the dark matter) would respond in such a way that orbits with initial radius $r_i$ end up at a new orbital radius $r_f$. These new radii are given by Eq. (2.6), and the dark matter mass inside these new radii is given by Eq. (2.7).

**Modified Analytical Model**

In reality, the process of forming the baryonic structure inside an extended halo is neither a fully adiabatic process, nor spherically symmetric. Instead, it is well established that typical orbits of dark matter particles inside simulated halos are rather elliptical (see, e.g., [67]). This means that $M(r_{\text{orbit}})$ changes around the orbit, and $M(r)r$ in Eq. (2.5) is no longer an adiabatic invariant. It has therefore been pointed out by Gnedin et al. [62] that Eq. (2.5) could be modified to try to take this into account. In particular, they argue that using the value of the mass within the average radius of a given orbit, $\bar{r}$, should give better results.

The average radius $\bar{r}$ for a particle is given by

$$\bar{r} = \frac{2}{T} \int_{r_p}^{r_a} \frac{r}{v_r} \, dr \quad (2.8)$$

where $v_r$ is the radial velocity, $r_a$ ($r_p$) is the aphelion (perihelion) radius, and $T$ is the radial period. The ratio between $r$ and $\bar{r}$ will change throughout the halo, but a suitable parametrization of $\langle \bar{r} \rangle$ (i.e., $\bar{r}$ averaged over the population of orbits at a given radius $r$) is a power law with two free parameters [62]

$$\langle \bar{r} \rangle = r_{200} A \left( \frac{r}{r_{200}} \right)^w \quad (2.9)$$

The numerical simulations in [62] result in $A = 0.85 \pm 0.05$ and $w = 0.8 \pm 0.02$.

† In [62] they used $r_{180}$ instead of $r_{200}$ as used here, but the difference in $A$ is very small.

In general, we have $A_{180} = A_{200} (r_{200}/r_{180})^{1-w}$, which for a singular isothermal sphere ($\rho \propto 1/r^2$) implies that $A_{180} = A_{200} (180/200)^{(1-w)/2} \approx 0.99 A_{200}$.

‡ Another, almost identical, parametrization was used in [68]: $\langle \bar{r} \rangle = 1.72 y^{0.82}/(1 + 5y)^{0.085}$, where $y = r/r_s$. 
The modified adiabatic contraction model is given by

\[ r_f = \frac{r_i M_i(\langle \bar{r}_i \rangle)}{M_b(\langle \bar{r}_f \rangle) + M_{DM}(\langle \bar{r}_f \rangle)}, \]  

(2.10)

whereas the equation for the conservation of mass is unchanged

\[ M_{DM}(r_f) = M_i(r_i)(1 - f). \]  

(2.11)

These two equations can now be solved for any given \( A \) and \( w \) value. Thus the model predicts the final dark matter distribution \( M_{DM}(r) \) if one knows the initial mass distribution together with the final baryonic distribution in a galaxy (or some other similar system, like a cluster). How well these analytical models work can now be tested by running numerical simulations including baryons.

### 2.4 Simulation Setups

In Paper VI, we aimed at investigating the dark matter halos as realistically as possible by using numerical simulations that included both dark matter and baryons. These simulations were known from previous studies to produce overall realistic gas and star structures for spiral galaxies [69–71]. Although the numerical resolution is still far from being able to resolve many of the small-scale features observed in real galaxies, the most important dynamical properties such as the creation of stable disk and bulge structures both for the gas and star components are accomplished.

Four sets of simulated galaxies were studied in Paper VI. The simulations were performed by the Hydra code [72] and an improved version of the Smoothed Particle Hydrodynamics code TREESPH [73]. In accordance with the observational data, the simulations were run in a \( \Lambda \)CDM cosmology with: \( \Omega_M = 0.3, \Omega_{\Lambda} = 0.7, H_0 = 100h \text{ km s}^{-1}\text{Mpc}^{-1} = 65 \text{ km s}^{-1} \text{ Mpc}^{-1} \), a matter power spectrum normalized such that the present linear root mean square amplitude of mass fluctuations inside \( 8h^{-1} \text{ Mpc} \) is \( \sigma_8 = 1.0 \) and a baryonic fraction \( f \) set to 0.15. By comparing simulations with different resolutions, we could infer that the results are robust down to an inner radius \( r_{\text{min}} \) of about 1 kpc. The simulations were run once including only dark matter, then rerun with the improved TREESPH code, incorporating star formation, stellar feedback processes, radiative cooling and heating, etc. The final results in the simulations including baryons are qualitatively similar to observed disk and elliptical galaxies at redshift \( z = 0 \), a result that is mainly possible by overcoming the angular momentum problem by an early epoch of

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5 Hydra is a particle-particle, particle-mesh code that calculates the potential among \( N \) point masses, and TREESPH is for simulating fluid flows both with and without collisionless matter. Each of the simulations in Paper VI took about 1 month of CPU time on an Itanium II 1 GHz processor.
strong, stellar energy feedback in the form of SNII energy being fed back to the interstellar medium (see PAPER VI and [69–71] for further details on the numerical simulations).

In the following we will focus on the generic results in PAPER VI. Although four galaxies were studied, I concentrate here on only one of them to exemplify the generic results. [All examples will be from simulation S1 and its accompanying simulation DM1 found in PAPER VI.] The simulated galaxy resembles in many respects our own Milky Way, and some of its main properties are found in Table 2.2.

### 2.5 Pinching of the Dark Matter Halo

With the baryonic disks and bulges formed fully dynamically, the surrounding dark matter halo response should also be realistically predicted. Figure 2.1 shows the comparison of the simulation that includes the correct fraction of baryons to the otherwise identical simulation with all the baryons replaced by dark matter particles. It is clear how the effect of baryons – forming a central galaxy – is to pinch the halo and produce a much higher dark matter density in the central part.

For simulations including only dark matter, the density profile in Eq. (2.4), with a continuously decreasing slope, turns out to be a good functional form. The best fit values for the two free parameters in this profile are given in Table 2.3. The simulation that includes baryons produces a dark matter cusp

Table 2.2: The main properties (at redshift $z=0$) of the benchmark galaxy and its dark matter halo.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>DM+galaxy</th>
<th>DM only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virial radius $r_{200}$ [kpc]</td>
<td>209</td>
<td>211</td>
</tr>
<tr>
<td>Total mass $M_{200}$ [$10^{11}M_\odot$]</td>
<td>8.9</td>
<td>9.3</td>
</tr>
<tr>
<td>Number of particles $N_{200}$ [$\times10^5$]</td>
<td>3.6</td>
<td>1.2</td>
</tr>
<tr>
<td>DM particle mass $m_{DM}$ [$10^6M_\odot$]</td>
<td>6.5</td>
<td>7.6</td>
</tr>
<tr>
<td>SPH particle mass $m_{baryon}$ [$10^6M_\odot$]</td>
<td>1.1</td>
<td>...</td>
</tr>
<tr>
<td>Baryonic disk + bulge mass [$10^{10}M_\odot$]</td>
<td>7.17</td>
<td>...</td>
</tr>
<tr>
<td>Baryonic bulge-to-disk mass ratio</td>
<td>0.19</td>
<td>...</td>
</tr>
</tbody>
</table>

Table 2.3: Best fit parameters to Eq. (2.4) for the spherical symmetrized dark matter halo in the simulation with only dark matter.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$r_{-2}$ [kpc]</th>
<th>$\chi^2$/dof$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.247</td>
<td>18.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

$^a$ This $\chi^2$ fit was done with 50 bins, i.e., 48 degrees of freedom (dof).
Figure 2.1: Dark matter density for a galaxy simulation including baryons (solid line) compared to an identical simulation including dark matter only (dashed line). A clear steepening in the dark matter density of the central part has arisen due to the presence of a baryonic galaxy. The curves’ parameterizations are given in Table 2.4 and Table 2.3 for the solid curve and the dashed curve, respectively. The data points, shown as solid and open circles, are binned data directly from the simulations. The arrows at the bottom indicate, respectively, the lower resolution limit ($r_{\text{min}} = 2 \text{ kpc}$) and the virial radius ($r_{200} \approx 200 \text{ kpc}$). These arrows also indicate the range within which the curves have been fit to the data.

Table 2.4: Best fit parameters to Eq. (2.1) for the spherical symmetrized dark matter halos in the simulation including baryons.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$r_s$ [kpc]</th>
<th>$\chi^2$/dof$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.76</td>
<td>3.31</td>
<td>1.83</td>
<td>44.9</td>
<td>1.4</td>
</tr>
</tbody>
</table>

$^a$ This $\chi^2$ fit was done with 50 bins, i.e., 46 degrees of freedom (dof).

and was therefore better fitted with Eq. (2.1), which allows for a steeper logarithmic slope $\gamma$ in the center. The best-fit parameter values are found in Table 2.4. It should be realized that with four free parameters in the profile (2.1), there are degeneracies in the inferred parameter values (see, e.g., [74]). Although the numbers given in Table 2.4 give a good parametrization, they

$^*$ See, e.g., [75] for a nice introduction to statistical data analysis.
do not necessarily represent a profile that could be extrapolated to smaller radii with confidence.

Let me summarize the result of all the dark matter halos studied in Paper VI. Simulations without baryons have a density slope continuously decreasing towards the center, with a density $\rho_{\text{DM}} \sim r^{-1.3\pm0.2}$, at about 1% of $r_{200}$. This is a result that lies between the NFW and the Moore profile given in Table 2.1. The central dark matter cusps in the simulations that also contain baryons become significantly steeper, with $\rho_{\text{DM}} \sim r^{-1.9\pm0.2}$, with an indication of the inner logarithmic slope converging to roughly this value.

### 2.6 Testing Adiabatic Contraction Models

The proposed adiabatic contraction models would, if they included all the relevant physics, be able to foresee the true dark matter density profiles from simulations that include only dark matter and known (i.e., observed) baryonic distributions. Having in disposal simulations with identical initial conditions except that in one case baryons are included and in the other not, we could test how well these adiabatic contraction models work.

It turns out that the simpler contraction model by Blumenthal et al. [60] significantly overestimates the contraction in the inner 10% of the virial radius as compared to our numerical simulations; see Fig. 2.3. To continue and test the modified adiabatic contraction proposed by Gnedin et al. [62] we in addition need to first find out the averaged orbital eccentricity for the dark matter (i.e., determine $A$ and $w$ in Eq. (2.9) for the pure dark matter simulation). For our typical example model, we found that the averaged orbital structure is well described by $A = 0.74$ and $w = 0.69$, as seen in Fig. 2.2 (similar values were found for all our simulated dark matter halos).

From these $A$ and $w$ values, and the baryonic distribution in our corresponding (baryonic) galaxy simulation, the final dark matter distribution is deduced from Eq. (2.10) and (2.11). Comparing the result from these two equations with the dark matter density profile found in the actual simulation including baryons showed that the Gnedin et al. model is a considerable improvement compared to the Blumenthal et al. model. However, this model’s prediction also differed somewhat from the N-body simulation result that included baryons. To quantify this, $A$ and $w$ were taken as free parameters, and a scan over different values was performed. With optimally chosen values of $A$ and $w$ (no longer necessarily describing the orbital eccentricity structure of the dark matter), it was always possible to obtain a good reconstruction of the dark matter density profile.

Figure 2.3 shows the region in the $(A,w)$-plane that provides a good reconstruction of the dark matter halo for our illustrative benchmark simulation. From these contour plots, it follows that the fits for $(A,w) = (1,1)$ – which corresponds to circular orbits and therefore the original model of Blumenthal et al. – are significantly worse than the fits for the optimal values ($A \sim 0.5$ and
Figure 2.2: $\langle \bar{r} \rangle$ versus $r$ for the halo simulation including only dark matter. The best fit (solid line), corresponding to $(A, w)=(0.74, 0.69)$ in Eq. (2.9), shows that the power law assumption is an excellent representation of the data. The large crosses represent the binned data, and the smaller horizontal lines indicate the variance for $\langle \bar{r} \rangle$ in each data point. The smaller sub-figure shows in black the 1σ (68%) confidence region whereas the lighter gray area is the 3σ (99.7%) confidence region in the $(A, w)$ plane. Figure from PAPER VI.

Typically, $w \sim 0.6$). We also see that although the Gnedin et al. model (marked by a cross in Fig. 2.3) is a significant improvement it is not at all perfect.

All of our four simulations in PAPER VI showed more or less significant deviations from the model predictions. By changing the stellar feedback strength, we could also find that this had an impact on the actual best fit values of $(A, w)$ (see PAPER VI for more details). This difference between $(A, w)$ obtained directly from the relationship between $\langle \bar{r} \rangle$ and $r$ in Eq. (2.9) and from the best fit values suggests (not surprisingly) that there is more physics at work than can be described by a simple analysis of the dark matter orbital structure.

### 2.7 Nonsphericity

We have just seen how the centrally concentrated baryons pinch the dark matter. Since the dark matter particles have very elliptical orbits and the baryons dominate the gravitational potential in the inner few kpc, it would be interesting to see how the presence of the baryonic galactic disk influences the triaxial properties of the dark matter halo. This was studied in PAPER VI.
by relaxing the spherical symmetry assumption in the profile fitting, and instead studying the halos’ triaxial properties. With an ellipsoidal assumption, and studying the momentum of inertia tensor $I_{ij}$, we determined the three principal axes $a$, $b$, and $c$ at different radii scales (see PAPER VI for more details). That is, we find how much we would need to stretch out the matter distribution in three different directions to get a spherically symmetric density profile.

**Axis Ratios**

Let the principal axes be ordered such that $a \geq b \geq c$ and introduce the parameters $e = 1 - b/a$ (ellipticity) and $f = 1 - c/a$ (flatness). Figure 2.4 shows how these quantities vary with radius. We can clearly see that ellipticity and flatness differ between the simulation with only dark matter (left panel) and the simulation including the formation of a baryonic disk galaxy (right panel). The radius $R$ on the horizontal axis gives the size of the elliptical shell — that is semiaxes $R$, $(b/a) \cdot R$ and $(c/a)R$ — inside which particles have been used to calculate $e$ and $f$.

Having obtained the semiaxes, we can determine whether a halo is prolate, in other words, shaped like a rugby ball, or oblate, i.e., flattened like a Frisbee,
Figure 2.4: The triaxial parameters $e = 1 - b/a$ (dashed line) and $f = 1 - c/a$ (solid line) of the dark matter halos in the simulation with only dark matter (left panel) and the simulation including baryons (right panel).

Table 2.5: Values of the oblate/prolate-parameter $T$ inside $R = 10$ kpc for the dark matter halo.

<table>
<thead>
<tr>
<th>Simulation:</th>
<th>including baryons</th>
<th>only dark matter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ value</td>
<td>0.076</td>
<td>0.74</td>
</tr>
</tbody>
</table>

by the measure

$$T = \frac{a^2 - b^2}{a^2 - c^2}. \quad (2.12)$$

If the halo is oblate, that is $a$ and $b$ are of similar size and larger than $c$, and the measure is $T < 0.5$, whereas if $T > 0.5$ the halo is prolate. The $T$ value for the dark matter halo with and without baryons are listed in Table 2.5.

The general result, from all our four studied simulated halos, is that the inclusion of the baryons causes the dark matter halo to change its shape from being prolate in the pure dark matter simulations into a more spherical and oblate form in simulations that include the formation of a central disk galaxy. This result agrees and compliment the studies in [76–78].

Alignments

Given these results of nonsphericity, the obvious thing to check is whether the principal axes of the dark matter and the baryon distributions are aligned. Figure 2.5 shows this alignment between the stellar disk, the gaseous disk, and the dark matter ‘disk’. The parameter $\Delta \theta$ is the angle between each of these vectors and a reference direction, defined to correspond to the orientation vector of the gaseous disk with radius $R = 10$ kpc. The figure shows that the orientation of the minor axes of the gas, stars, and dark matter is strongly
Some Comments on Observations

The amount of triaxiality of dark matter halos seems to be a fairly generic prediction in the hierarchical, cold dark matter model of structure formation, and observational probes of halo shapes are therefore a fundamental test of this model. Unfortunately, observational determination of halo shapes is a difficult task, and only coarse constraints exist. Probes of the Milky Way halo indicate that it should be rather spherical with $f \lesssim 0.2$ and that an oblate...
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structure of \( f \sim 0.2 \) might be preferable (see, \textit{e.g.}, [79] and references therein). Milky-Way-sized halos formed in dissipationless simulations are usually predicted to be considerably more triaxial and prolate, although a large scatter is expected [80–86]. Including dissipational baryons in the numerical simulation, and thereby converting the halo prolateness into a slightly oblate and more spherical halo, might turn out to be essential to produce good agreement with observations [77].

Having determined the ellipsoidal triaxiality of the dark matter distribution, we can include this information in the profile fits. Including triaxiality to the radial density profile fits would not change any results (see PAPER VI). This should not be surprising since the flattening of the dark matter halo is very weak. The oblate structure of the dark matter would have some minor effects on the expected indirect dark matter signal [87]. However, the baryonic effects found here have no indication of producing such highly disk-concentrated dark matter halo profiles as used in, \textit{e.g.}, [1] to explain the excess of diffuse gamma-rays in the EGRET data by WIMP annihilation (see Chapter 9 for more details).

Observations of presumably dark-matter-dominated systems, such as low surface brightness dwarf galaxies, indicate that dark matter halos have constant density cores instead of the steep cusps found in numerical simulations (see, \textit{e.g.}, [88–93]). This could definitely be a challenge for the standard cold dark matter scenario. Even if baryons are included in the \( N \)-body simulations, and very explosive feedback injections are enforced, it seems unlikely that it could resolve the cusp-core problem (see, \textit{e.g.}, [94] and references therein).

However, several studies also demonstrate that the cusp-core discrepancy not necessarily implies a conflict. Observational and data processing techniques in deriving the rotation curves (see, \textit{e.g.}, [89–91]), and the neglected complex effects on the gas dynamics due to the halos’ triaxiality properties [95], indicate that there might not even be a discrepancy between observation and theory. One should also note that the story is actually different for galaxy halos where the baryons dominate the gravitational mass in the inner parts (as the halos studied in PAPER VI). Here the problem of separating the dark matter component from the dominant baryonic component allows the dark matter profile to be more cuspy without any conflict with observation. As adiabatic contraction increases the central dark matter density in such a way that the dark matter density only tends to track the higher density baryonic component, strong adiabatic contraction of the dark matter halo in these systems should not be excluded.

2.9 Tracing Dark Matter Annihilation

Improved knowledge about the dark matter distribution is essential for reliable predictions of the detection prospects for many dark matter signals. For any self-annihilating dark matter particle, the number of annihilations per unit
time and volume element is given by

\[ \frac{dn}{dt} = \frac{1}{2} \langle \sigma v \rangle \frac{\rho_{DM}^2(r)}{m_{DM}^2}, \]  

(2.13)

where \( v \) is the relative velocity of the two annihilating particles, \( \sigma \) is the total cross section for annihilation, and \( \rho_{DM}(r) \) the dark matter mass density at the position \( r \) where the annihilation take place.

Indirect detection of dark matter would be to detect particles produced in dark matter annihilation processes, e.g., to find an excess in the amount of antimatter, gamma rays, and/or neutrinos arriving at Earth [25]. Since the expected amplitude of any expected signal depends quadratically on \( \rho_{DM} \), it seems most promising to look for regions of expected high dark matter concentrations. Unfortunately, charged particles will be significantly bent by the magnetic fields in our Galaxy and will no longer point back to their source. On the other hand, this is not the case for neutrinos and gamma rays as these particles propagate more or less unaffected through our Galaxy.

When looking for gamma rays towards a region of enhanced dark matter density, the expected differential photon flux along the line of sight (l.o.s.) in a given direction \( \psi \) is given by

\[ \frac{d\Phi_{\gamma}(\psi)}{dE_{\gamma}} = \frac{\langle \sigma v \rangle}{8\pi m_{DM}^2} \frac{dN_{\gamma}^{\text{eff}}}{dE_{\gamma}} \int_{\text{l.o.s.}} d\ell(\psi) \rho_{DM}^2(\ell), \]  

(2.14)

where \( dN_{\gamma}^{\text{eff}} / dE_{\gamma} \) is the energy differential number of photons produced per dark matter pair annihilation.

Expected dark matter induced fluxes are still very uncertain. Any attempt to accurately predict such fluxes is still greatly hampered by both theoretical uncertainties and lack of detailed observational data on the dark matter distribution. It can be handy to separate astrophysical quantities (\( \rho_{DM} \)) from particle physics properties (\( m_{DM}, \langle \sigma v \rangle \) and \( dN_{\gamma} / dE_{\gamma} \)). It is therefore convenient to define the dimensionless quantity [96]

\[ \langle J \rangle_{\Delta \Omega}(\psi) \equiv \frac{1}{8.5 \text{ kpc}} \left( \frac{1}{0.3 \text{ GeV cm}^{-3}} \right)^2 \frac{1}{\Delta \Omega} \int_{\Delta \Omega} d\Omega \int_{\text{l.o.s.}} d\ell(\psi) \rho_{DM}^2(\ell), \]  

(2.15)

which embraces all the astrophysical uncertainties. The normalization values 8.5 kpc and 0.3 GeV/cm\(^3\) are chosen to correspond to commonly adopted values for the Sun’s galactocentric distance and the local dark matter density, respectively. For a detector of angular acceptance \( \Delta \Omega \), the flux thus becomes

\[ \frac{d\Phi_{\gamma}}{dE_{\gamma}} = 9.4 \cdot 10^{-13} \frac{dN_{\gamma}^{\text{eff}}}{dE_{\gamma}} \left( \frac{\langle \sigma v \rangle_{\text{tot}}}{10^{-26} \text{ cm}^3 \text{s}^{-1}} \right) \left( \frac{1 \text{ TeV}}{m_{DM}} \right)^2 \cdot \Delta \Omega \langle J \rangle_{\Delta \Omega} \text{ cm}^{-2} \text{s}^{-1} \text{TeV}^{-1}. \]  

(2.16)
For example, looking towards the galactic center with an angular acceptance of $\Delta\Omega = 10^{-5}$ sr (which is comparable to the angular resolution of, e.g., the H.E.S.S. or GLAST telescope) it is convenient to write

$$\Delta\Omega \langle J \rangle_{\Delta\Omega}(0) = 0.13\ b\ sr.$$  

(2.17)

where $b = 1$ if the dark matter distribution follows a NFW profile as given in Table 2.1[97]. On the other hand, we have just seen that taking into account the effect of baryonic compression due to the dense stellar cluster observed to exist very near the galactic center, could very well enhance the dark matter density significantly. A simplifying and effective way to take into account such an increase of the dark matter density is to simply allow the so-called boost factor $b$ to take much higher values. Exactly how high this boost factor can be in the direction of the galactic center, or in other directions, is still not well understood, and we will discuss this in some more detail in what follows.

**Indirect Dark Matter Detection**

It is today impossible for galaxy simulations to get anywhere near the length resolution corresponding to the very center of the galaxy. Despite this, different profile shapes from numerical simulations have frequently been extrapolated into the galactic center. This enables, at least naïvely, to predict expected fluxes from dark matter annihilation in the galactic center. In the spirit of comparing with the existing literature we performed the baryonic contraction with our best fit values on $A$ and $w$. For typical values of the Milky Way baryon density (see PAPER VI for details) and an initial Einasto dark matter profile (given in Table 2.3) it is straightforward to apply the contraction model. The local dark matter density is here normalized to be $\rho_{DM}(r = 8.5\ kpc) \sim 0.3\ GeV\ cm^{-3}$. Figure 2.6 shows both results if a $2.6 \times 10^{6}\ M_{\odot}$ central supermassive black hole is included in the baryonic profile and if it is not.

No attempt is made to model the complicated dynamics at subparsec scales of the galaxy, other than trying to take into account the maximum density due to self-annihilation. In other words, a galactic dark matter halo unperturbed by major mergers or collisions for a time scale $\tau_{gal}$ cannot contain stable regions with dark matter densities larger than $\rho_{max} \sim m_{DM}/\langle \sigma v \rangle_{\tau_{gal}}$. In Fig. 2.6 and in Table 2.6 it is assumed that $\tau_{gal} = 5 \times 10^9$ years and for the WIMP property a dark matter mass of $m_{DM} = 1\ TeV$, with an annihilation cross section of $\langle \sigma v \rangle = 3 \times 10^{-26}\ cm^3s^{-1}$, is adopted.

Table 2.6 shows the energy flux obtained with the values of $(A, w)$ found in the previous section, as well the Blumenthal et al. estimate. This is the total energy luminosity, not in some specific particle species, and is hence the flux given in Eq. (2.13) multiplied by 2 times the dark matter mass. The Blumenthal et al. adiabatic contraction model gives fluxes far in excess of the modified contraction model. Even with the modified contraction model, and
Section 2.9. Tracing Dark Matter Annihilation

Figure 2.6: Diagram showing the contraction of a Einasto dark matter density profile, given in Table 2.3 (dot-dashed line) by a baryon profile (dotted line) as described in PAPER VI. The resulting dark matter profiles (dashed lines) are plotted for \((A, w) = (1, 1)\) and \((0.51, 0.6)\), each splitting into two at low radii, the denser corresponding to the density profile achieved from a baryon profile that includes a central black hole. The extrapolated density profile from Table 2.4 from our simulation is shown for comparison (solid line). Remember the numerical simulation is only robust into \(r_{\text{min}} \approx 1\) kpc. Also shown are the maximum density line and the radius corresponding to the lowest stable orbit around the central black hole. Figure adapted from PAPER VI.

Table 2.6: Luminosity in erg s\(^{-1}\) from dark matter annihilation for different contraction model parameters \((A, w)\). The initial dark matter profiles are an NFW profile, given in Table 2.1 or the Einasto profile, given in Table 2.3. The baryon profile includes a super massive black hole, as described in the text. Quoted values are for the flux from the inner 10 pc and 100 pc. Note that this is the total luminosity, and not of some specific particle species. A dark matter particle mass of 1 TeV is assumed.

<table>
<thead>
<tr>
<th>(A)</th>
<th>(w)</th>
<th>NFW (Table 2.1)</th>
<th>Einasto (Table 2.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(L_{10\text{pc}})</td>
<td>(L_{100\text{pc}})</td>
</tr>
<tr>
<td>Initial profile (\rightarrow)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3.9 (\times) 10(^{33})</td>
<td>3.9 (\times) 10(^{34})</td>
</tr>
<tr>
<td>0.51</td>
<td>0.6</td>
<td>2.8 (\times) 10(^{37})</td>
<td>3.5 (\times) 10(^{37})</td>
</tr>
<tr>
<td>7.9 (\times) 10(^{35})</td>
<td>3.5 (\times) 10(^{36})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
the values of \((A, w)\) found in Paper VI, we find a large flux enhancement compared to the traditional NFW profile (as given in Table 2.1). This stays true also if the initial profile is the Einasto profile, which does not initially possess a cusp at all. From Table 2.6, the boost of the luminosity compared to the standard NFW profile takes values of about \(10^2 \, \text{–} \, 10^4\) in the direction of the galactic center. This extrapolation to very small radii neglects many potential effects, such as the scattering of dark matter particles on stars or the effect from a supermassive black hole not exactly in the galactic center. Case studies of the dark matter in the galactic center [68, 98–103] show that compared to an NFW density profile the expected flux from self-annihilation can be boosted as much as 10^7, but also that the opposite effect might be possible leading to a relative depletion of an initial dark matter cusp.

2.10 Halo Substructure

So far the dark matter density has been described as a smooth halo profile with a peak concentration in the center. Presumably, the halos contain additional structure. Numerical simulations find a large number of local dark matter concentrations (clumps) within each halo (see, e.g., [47, 104–112]). Also substructures within substructures etc. are found. This should not be surprising as the hierarchal structure formation paradigm predicts the first formed structures to be numerous small dark matter halos. These first formed (WIMP) halos could still be around today as clumps of about the Earth mass and with sizes similar the solar system [44, 113–115]. In the subsequent processes, accretion to form larger structures by the merging of smaller progenitors is not always complete, the cores of subhalos could survive as gravitationally bound subhalos orbiting within a larger host system. More than \(10^{15}\) of this first generation of dark halo objects could potentially be within the halo of the Milky Way [111], but gravitational disruption during the accretion process as well as late tidal disruption from stellar encounters can significantly decrease this number [116].

This additional substructure could be highly relevant for indirect detection of dark matter. As the dark matter annihilation rate, into for example gamma rays, increases quadratically with the dark matter density, the internal substructure may enhance not only the total diffuse gamma-ray flux compared to the smooth halo, but also individual clumps of dark matter could be detectable with, e.g., gamma-ray telescopes such as GLAST. The prospects for detecting these subhalos, however, depend strongly on the assumptions (see, e.g., [117]). With surviving microhalos down to masses of \(10^{-6} \, M_\odot\), the dark matter fraction in a galaxy could be as large as about 50% of the total mass [117]. Translated into an enhancement of the total dark matter annihilation rate for whole galaxies, this gives a boost factor of a few, up to perhaps some hundred [112, 117, 118]. The local annihilation boost, compared to the smooth background halo profile, due to subhalos is,
however, expected to strongly depend on the galactocentric distance. In the outer regions, the clumps can boost the rates by orders of magnitude. On the other hand, in the inner regions the increase in annihilation rates due to clumps might be negligible. This is both because expected tidal disruption could have destroyed many clumps in the center and that the smooth component already give larger annihilation rates. Spatial variations of the local annihilation boost mean that different dark matter signals are expected to depend on both species and energy [119] of the annihilation products. For example, positrons are most sensitive to the local boost factors. Positrons are strongly affected by magnetic fields – they quickly lose energy and directional information – and become located within some kpc to their source before diffusing outside the galactic disk and escape from the Galaxy. On the contrary, gamma rays propagate almost freely in our Galaxy, and are therefore affected also by more distant dark matter density boosts. The intermediate case are antiproton signals, which, like the positrons, are sensitive to clumps in all sky directions, but due to their much higher mass they are less deflected by magnetic fields and can therefore travel longer distances in the disk (see, e.g., [119]).
Why is there a need to go beyond today’s standard model of particle physics? This chapter presents motivations and introduces possible extensions, which will be discussed further in this thesis. Special focus is here put on the possibility that hidden extra space dimensions could exist. General features expected in theories with extra space dimensions are discussed, together with a short historical review. The chapter concludes by giving motivations to study the specific model of so-called universal extra dimensions, as this will be the model with which we will start our discussions on dark matter particle phenomenology in the following chapters.

3.1 The Need to Go Beyond the Standard Model

Quantum field theory is the framework for today’s standard model (SM) of particle physics – a tool box for how to combine three major themes in modern physics: quantum theory, the field concept, and special relativity. Included in the SM is a description of the strong, weak, and electromagnetic forces as well as all known fundamental particles. The theoretical description has been a great success and agrees, to a tremendous precision, with practically all experimental results up to the highest energies reached (i.e., some hundred GeV). However, it is known that the SM is not a complete theory as it stands today. Perhaps the most fundamental drawback is that it does not include a quantum description of gravitational interactions. There are also a number of reasons directly related to particle physics for why the SM needs to be extended. For example, the SM does not include neutrino masses (neutrinos masses are by now a well-accepted interpretation of the observed neutrino
oscillations [120–125]), and extreme fine-tuning is required in the Higgs sector if no new divergence canceling physics appears at TeV energies [with no new physics between the electroweak scale (10^2 GeV) and the Planck scale (10^{19} GeV) is usually called hierarchy problem]. Physics beyond the SM is also very attractive for cosmology, where new fields in the form of scalar fields driving inflation are discussed, and since the SM is incapable of explaining the observed matter-antimatter asymmetry and the amount of dark energy. The strongest reason for new fundamental particle physics, however, comes perhaps from the need for a viable dark matter candidate.

Despite the necessity of replacing (or extending) the SM, its great success hints also that new fundamental physics could be closely tied to some of its basic principles, such as its quantization and symmetry principles. As the SM is a quantum field theory with an SU(3) × SU(2) × U(1) gauge symmetry and an SO(1,3) Lorentz symmetry, it is tempting to investigate extensions of these symmetries.

This is the idea behind grand unified theories (GUTs), where at high energies (typically of the order of 10^{16} GeV) all gauge couplings have the same strength and all the force fields are fused into a unified field. This is the case for the SU(5) GUT theory. This larger symmetry group is then thought to be spontaneously broken at the grand unification scale, down to the SU(3) × SU(2) × U(1) gauge group of the SM that we observe at today’s testable energies. However, the simplest SU(5) theory predicts a too-short proton lifetime, and is nowadays excluded.

Another type of symmetry extension (with generators that anticommute) is a spacetime symmetry that mixes bosons and fermions. These are the supersymmetric extensions of the SM, where every fundamental fermion has a bosonic superpartner of equal mass and vice versa, that every fundamental boson has a fermionic superpartner. This symmetry must, if it exists, be broken in nature today, so as to give all superpartners high enough masses to explain why they have evaded detection. This possibility is further discussed in Chapter 7.

Yet another possibility is a Lagrangian with extended Lorentz symmetry, achieved by including extra dimensions. The most obvious such extension is to let the SM have the Lorentz symmetry SO(1,3 + n), with n ≥ 1 an integer. This implies that all SM particles propagate in n extra spatial dimensions endowed with a flat metric. These are called universal extra dimensions (UEDs) [126] Other extra-dimensional scenarios also exist where all or part of the matter and SM gauge fields are confined to a (3+1)-dimensional brane on which we are assumed to live. The aim of the many variations of extra-dimensional models is usually to propose different solutions, or new perspectives, to known problems in modern physics. Most such scenarios are therefore

* Along similar lines, studies of possible effects of extra dimensions felt by SM particles were also done earlier in [127–129].
of a more phenomenological nature, but a notable exception is string theory. String theory (see, e.g., [130] for a modern introduction) certainly aims to be a fundamental theory, and by replacing particles with extended strings, it offers a consistent quantum theory description of gravity (how much this theory is related to reality is, however, still an open question). The requirement that the theory should be anomaly free leads canonically to a critical value of the spacetime dimensionality. In the case of supersymmetric strings, the number of dimensions must be \( d = 10 \) (or \( d = 11 \) for M-theory) [130]. Performing a fully consistent extra dimensional compactification within string theory that leads to firm observational predictions at accessible energies is at the moment very challenging. Therefore, fundamental string theory is not yet ready for making unique (or well constrained) phenomenological predictions in a very rigorous way. However, as string theory is perhaps the most promising candidate for a more fundamental theory today, it is of interest to try to anyway investigate its different aspects, such as extra space dimensions, from a more phenomenological perspective.

Finally, another approach to go beyond the SM could be to try to extend it as minimally as possible to incorporate only new physics that can address specific known drawbacks. One such approach, which extends only the scalar sector of the SM, is discussed in Chapter 8. There it is shown that such an extension, besides other advantages, gives rise to an interesting scalar dark matter particle candidate with striking observational consequences.

\section{General Features of Extra-Dimensional Scenarios}

Although we are used to thinking of our world as having three spatial dimensions, there is an intriguing possibility that space might have more dimensions. This might at first sound like science fiction, and seemingly ruled out by observations, but in the beginning of the 20th century Nordström [131], and more prominently Kaluza [132] and Klein [133] asked whether extra dimensions could say something fundamental about physics. To allow for an extra dimension, without violating the apparent observation of only three space dimensions, it was realized that the extra dimension could be curled up on such a small length scale that we have not yet been able to resolve the extra dimension. As an analogy, imagine you are looking at a thin hose from a long distance. The hose then seems to be just a one-dimensional line, but as you get closer you are able to resolve the thickness of the hose, and you realize it has an extended two-dimensional surface.

In the original idea by Kaluza [132], and rediscovered by Klein, the starting point was a five-dimensional spacetime with the dynamics governed by the Einstein-Hilbert action (i.e. general relativity). After averaging over the (assumed static) extra dimension, and retaining an ordinary four-dimensional effective theory, the result was an action containing both Einstein's general relativity and Maxwell's action for electro-magnetism. Although it at first
seemed to be a very ‘magic’ unification, it can be traced back to the fact that
the compactification of one extra dimension on a circle automatically gives a
$U(1)$ symmetry, which is exactly the same key symmetry as in the electromag-
netic (abelian) gauge theory. There is, however, a flaw in the five dimensional
Kaluza-Klein (KK) theory, even before trying to include the weak and the
strong interactions. Once matter fields are introduced, and with the $U(1)$
symmetry identified with the usual electromagnetism, the electric charge and
mass of a particle must be related and quantized. With the quantum of charge
being the charge of an electron, all charged particles must have masses on the
Planck scale $M_{pl} \sim 10^{19}$ GeV. This is not what is observed – all familiar
charged particles have very much smaller masses.

Today much of the phenomenological studies of extra dimensions concern
the generic features that can be expected. To illustrate some of these fea-
tures, let us take the spacetime to be a direct product of the ordinary (four-
dimensional) Minkowski spacetime and $n$ curled up, flat extra dimensions.
We can then show that in the emerging effective four-dimensional theory:

1. a tower of new massive particles appears
2. Newton’s $1/r$ law is affected at short distances
3. fundamental coupling ‘constants’ will vary with the volume spanned by
   the extra dimensions

Feature (1) can qualitatively be understood quite easily. Imagine a particle
moving in the direction of one of the extra dimensions. Even if the particle’s
movement cannot be directly observed, the extra kinetic energy will still con-
tribute to its total energy. For an observer, not aware of the extra dimensions,
this additional kinetic energy will be interpreted as a higher mass ($E = mc^2$)
for that particle compared to an identical particle that is not moving in the
extra dimensions. To do this more formally, let us denote local coordinates
by

$$\{\hat{x}^M\} = \{x^\mu, y^p\},$$

where $M = 0, 1, \ldots, 3 + n$, $\mu = 0, 1, 2, 3$ and $p = 1, 2, \ldots, n$, and consider
a scalar field $\hat{\Phi}(\hat{x})$ with mass $m$ in five dimensions. Its dynamics in a flat
spacetime are described by the Klein-Gordon equation:

$$\left(\hat{\Box}^{(5)} + m^2\right)\hat{\Phi}(x^\mu, y) = \left(\partial_t^2 - \nabla^2 - \partial_y^2 + m^2\right)\hat{\Phi}(x^\mu, y) = 0.$$  (3.2)

With the fifth dimension compactified on a circle with circumference $2\pi R$,
i.e.,

$$y \sim y + 2\pi R,$$  (3.3)

any function of $y$ can be Fourier series expanded, and the scalar field is de-
composed as

$$\hat{\Phi}(x^\mu, y) = \sum_n \Phi^{(n)}(x^\mu) e^{-i\frac{2\pi}{R}y}.$$  (3.4)
Each Fourier component $\Phi^{(n)}(x)$ thus separately fulfills the four-dimensional Klein-Gordon equation:

$$\left(\Box + m_n^2\right) \Phi^{(n)}(x^\mu) = \left(\partial^2_t - \nabla^2 + m_n^2\right) \Phi^{(n)}(x^\mu) = 0, \quad (3.5)$$

where the masses are

$$m_n^2 = m^2 + \left(\frac{n}{R}\right)^2. \quad (3.6)$$

That is, a single higher-dimensional field appears in the four-dimensional description as an (infinite) tower of more massive KK states $\Phi^{(n)}$. This fits well with the expectation that very small extra dimensions should not affect low energy physics (or large distances), as it would take high energies to produce such new heavy states. In general, the exact structure of the KK tower will depend on the geometry of the internal dimensions.

Feature (2), that the gravitational $1/r^2$ potential will be affected at small distances, is straightforward to realize. At small distances $r \ll R$, the compactification scale is of no relevance, and the space should be fully $(3+n)$-dimensional rotation invariant. At a small distance $r_n$ from a mass $m$ the gravitational potential is thus determined by the Laplace equation $\hat{\nabla}^2 V(r) = 0$, where $\hat{\nabla}^2 = \partial^2_{x^1} + \partial^2_{x^2} + \ldots + \partial^2_{x^{n+3}}$. The solution is

$$V(r) \sim -\hat{G}_{4+n} \frac{m}{r_{n+1}} \quad r_n \ll R, \quad (3.7)$$

where $\hat{G}_{4+n}$ is the fundamental $(4+n)$-dimensional gravitational constant and $r_n^2 = r^2 + y_1^2 + y_2^2 + \ldots + y_{n+3}^2$ is the radial distance. The potential will thus qualitatively behave as in Eq. (3.7) out to the compactification radius $R$, where the potential becomes $\sim \hat{G}_{4+n} \frac{m}{R^{n+1}}$. At larger distances, space is effectively three-dimensional and Newton’s usual $1/r^2$-law is retained:

$$V(r) \sim -\hat{G}_{4+n} \frac{m}{R^n} \quad r \gg R. \quad (3.8)$$

At small distances the presence of the extra dimensions thus steepens the gravitational potential. Such a deviation from Newton’s law is conventionally parameterized as [134]:

$$V(r) \propto \frac{1}{r} \left(1 + \alpha e^{-r/\lambda}\right). \quad (3.9)$$

Experiments have today tested gravity down to sub-millimeter ranges and have set upper limits in the $(\lambda, \alpha)$-plane. At 100 $\mu$m the deviation from Newton’s law cannot be larger than about 10%, i.e., $(\lambda, |\alpha|) \lesssim (100 \mu$m, 0.1) [135].

By comparing Eq. (3.8) with Newton’s law $V(r) = -G \frac{m}{r}$ we find that the ordinary Newton’s constant scales with the inverse of the extra-dimensional volume according to

$$G \equiv G_4 \propto \hat{G}_{4+n} \frac{1}{R^n}. \quad (3.10)$$
This is exactly feature (3) for Newton’s constant $G$ - fundamental constants vary with the inverse of the volume of the internal space $\sim R^{-n}$.

Another instructive way of seeing the origin of the features (1)–(3) is to work directly with the Lagrangian \cite{136}. Take for simplicity a $\phi^4$-theory in a $(4+n)$-dimensional Minkowski spacetime

$$S = \int d^{4+n} \hat{x} \sqrt{-g} \left[ \frac{1}{2} \partial_{\mu} \hat{\phi} \partial^{\mu} \hat{\phi} - \frac{m^2}{2} \hat{\phi}^2 - \frac{\Lambda_{4+n}}{4!} \hat{\phi}^4 \right]. \quad (3.11)$$

With the extra $n$ dimensions compactified on an orthogonal torus, with all radii equal to $R$, the higher-dimensional real scalar field $\hat{\phi}$ can be Fourier expanded in the compactified directions as

$$\hat{\phi}(x,y) = \frac{1}{\sqrt{V_n}} \sum_{\vec{n}} \phi(\vec{n})(x) \exp \left\{ i \frac{\vec{n} \cdot \vec{y}}{R} \right\}. \quad (3.12)$$

Here $V_n = (2\pi R)^n$ is the volume of the torus, and $\vec{n} = \{n_1, n_2, \ldots, n_n\}$ is a vector of integers $n_i$. The coefficients $\phi(\vec{n})(x)$ are the KK modes, which in the effective four-dimensional theory constitute the tower of more massive particle fields. Substituting the KK mode expansion into the action (3.11) and integrating over the internal space, we get

$$S = \int d^4 x \sqrt{-g} \left\{ \frac{1}{2} \left( \partial_{\mu} \phi^{(0)} \right)^2 - \frac{m^2}{2} \left( \phi^{(0)} \right)^2 \\
+ \sum_{\vec{n} > 0} \left[ \left( \partial_{\mu} \phi^{(\vec{n})} \right) \left( \partial^{\mu} \phi^{(\vec{n})} \right)^* - m_{\vec{n}}^2 \phi^{(\vec{n})} \phi^{(\vec{n})*} \right] \\
- \frac{\lambda_4}{4!} \left( \phi^{(0)} \right)^4 - \frac{\lambda_4}{4} \left( \phi^{(0)} \right)^2 \sum_{\vec{n} > 0} \phi^{(\vec{n})} \phi^{(\vec{n})*} + \ldots \right\}, \quad (3.13)$$

where the dots stand for the additional terms that do not contain any zero modes $(\phi^{(0)})$ of the scalar field. The masses of the modes are given by

$$m_{\vec{n}}^2 = m^2 + \frac{\vec{n}^2 \cdot R^2}{R^2}. \quad (3.14)$$

The coupling constant $\lambda_4$ of the four-dimensional theory is identified to the coupling constant $\hat{\lambda}_5$ of the initial multidimensional theory by the formula

$$\lambda_4 = \frac{\hat{\lambda}_{4+n}}{V_n}. \quad (3.15)$$

We thus again find that the four-dimensional coupling constant is inversely proportional to the volume $V_n$ spanned by the internal dimensions. The same is true for any coupling constant connected to fields in higher dimensions.
3.3 Modern Extra-Dimensional Scenarios

The original attempt by Kaluza and Klein of unifying general relativity and electromagnetism, the only known forces at the time, by introducing a fifth dimension was intriguing. After the discovery of the weak [137–139] and strong forces [140, 141] as gauge fields, it was investigated whether these two forces could be fit into the same scheme. It was found that with more extra dimensions these new forces could be incorporated [142, 143]. This developed into a branch of supergravity in the 1970s, which combined supersymmetry and general relativity into an 11 dimensional theory. The 11-dimensional spacetime was shown to be the unique number of dimensions to be able to contain the gauge groups of the SM [143,144]. The initial excitement over the 11-dimensional supergravity waned as various shortcomings were discovered. For example, there was no natural way to get chiral fermions as needed in the SM, nor did supergravity seem to be a renormalizable theory.

For some time, the ideas of extra dimensions then fell into slumber, before the rise of string theory in the 1980s. Due to consistency reasons, all string theories predict the existence of new degrees of freedom that are usually taken to be extra dimensions. The reason for the popularity of string theory is its potential to be the correct long time searched for quantum theory for gravity. The basic entities in string theory are one-dimensional strings, instead of the usual zero-dimensional particles in quantum field theory, and different oscillation modes of the strings correspond to different particles. One advantage of having extended objects, instead of point-like particles, is that ultra-violet divergences, associated with the limit of zero distances, get smeared out over the length of the string. This could solve the problem of unifying quantum field theory and general relativity into a firm physical theory. For superstring theories, it was shown that the number of dimensions must be 10 in order for the theory to be self-consistent, and in M-theory the spacetime is 11 dimensional. The extra dimensions beyond the four observed, which have to be made unobservable, and are commonly compactified on what is called a Calabi-Yau manifold. It might also be possible that non-perturbative lower dimensional objects called branes can host the four-dimensional world that we experience.

With the hope that string theory will eventually turn out to be a more fundamental description of our world, many string-inspired phenomenological scenarios have been developed. For example, the concept of branes in string theory gave room for addressing the strong hierarchy problem from a new geometrical perspective. Branes are membranes in the higher dimensional spacetime to which open strings, describing fermions and vector gauge fields, are attached, but closed strings, describing gravitons, are not. In 1998 Arkani-Hamed, Dimopoulos, and Dvali (ADD) [145] proposed a string-inspired model,

\[ \text{† Today, many techniques exist to embed the SM gauge group in supergravity in any number of dimensions, by, e.g., the introduction of D-branes [130].} \]
where all the SM particle fields are confined to a four-dimensional brane in a higher dimensional flat spacetime. Only gravity is diluted into the additional extra dimensions, and therefore the gravitational force is weakened compared to the other known forces. With the extra dimensions spanning a large enough volume (see Eq. (3.10)), the gravitational scale could be brought down to the electroweak scale – explaining the strong hierarchy of forces. For two extra dimensions spanning a volume $V_{n=2} \sim (1 \mu m)^2$ the fundamental energy scale for gravity $\hat{M}_{pl}$ is brought down to the electroweak scale, i.e., $\hat{M}_{pl} \sim (M_{pl}^2 \cdot V_n)^{1/(2+n)} \sim (1 \text{ TeV})^2$.

Even with small extra dimensions, the strong hierarchy problem can be addressed in a geometrical way. In 1999 Randall and Sundrum proposed a model with one extra dimension that ends at a positive and a negative tension brane (of which the latter is assumed to contain our visible SM). This model is often referred to as the Randall and Sundrum I model, or RS I model [146]. The five-dimensional metric is not separable in this scenario, but has a warp factor $e^{-w|y|}$ connecting the fifth dimension to the four others:

$$ds^2 = e^{-w|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \quad (3.16)$$

The bulk is a slice of an anti de Sitter space ($\text{AdS}_5$), i.e., a slice of a space-time with constant negative curvature. After integrating out the extra dimension in this model, the connection between the four-dimensional and the five-dimensional fundamental Planck mass (that is the relationship between the scale for gravity at the brane and in the bulk) is found to be

$$M_{pl}^2 = \frac{\hat{M}_{pl}^3}{w} \left( 1 - e^{-2wL} \right) \quad (3.17)$$

where $L$ is the separation between the branes. Here $M_{pl}$ depends only weakly on the size $L$ of the extra dimension (at least in the large $L$ limit); this is a completely different relation than found in the ADD model. To address the strong hierarchy problem in the RS I model, they looked at how the mass parameters on the visible brane are related to the physical higher-dimensional masses. In general, the mass parameter $m_0$ in the higher-dimensional theory will correspond to a mass

$$m = e^{-wL} m_0 \quad (3.18)$$

when interpreted with the metric on our visible brane (see [146]). This means that with $wL$ of about 50 it is possible to have all fundamental mass parameters of the order of the Planck mass, and by the warp factor still produce masses of the electroweak scale. Instead of having large flat extra dimensions, the large hierarchy between the electroweak and the Planck energy scales is here induced by the large curvature of the extra dimension, i.e., the warp factor $e^{-wL}$. In a follow-up paper [147], Randall and Sundrum demonstrated that the metric (3.16) could also allow for a non-compact extra dimension. This possibility is partly seen already in Eq. (3.17), where it is no problem to
take the indefinitely large $L$ limit. If the curvature scale of the AdS space is smaller than a millimeter, then Newton’s gravitational law is retained within experimental uncertainties [147,148]. The reason why the extra dimension can be non-compact is that the curved AdS background supports a localization of the higher dimensional gravitons in the extra dimensions. In this so-called RS II case, the hierarchy problem is not addressed.

Another more recent extra-dimensional scenario is the model by Dvali, Gabadadze, and Porrati (DGP) [149] where gravity gets modified at large distances. The action introduced is one with two gravity scales: one five-dimensional bulk and one four-dimensional brane gravity scale. This model could be used to discuss an alternative scenario for the cosmological problem of a late-time acceleration of our expanding Universe. However, problems such as violation of causality and locality make it theoretically less attractive. For a recent review of braneworld cosmology, see [150].

As mentioned above, another approach for extra-dimensional phenomenology is to look at models where all SM particles can propagate in a higher dimensional space. This is the case in the UED model [126]. The UED model will be of special interest in this thesis, as this model can give rise to a new dark matter candidate. This dark matter candidate will not only be discussed in detail, but the UED model will also serve as the starting point to go through multidimensional cosmology, the particle SM structure, dark matter properties, and dark matter searches in general.

### 3.4 Motivations for Universal Extra Dimensions

Even though the idea of the UED model is a conceptually simple extension of the SM – basically just add extra dimensions – it provides a framework to discuss a number of open questions in modern physics. Theoretical and practical motivations to study the UED model include:

- **Simplicity**: only 2 new parameters in its minimal version ($R$ and $\Lambda_{\text{cut}}$).
- **A possibility to achieve electroweak symmetry breaking without any need to add an explicit Higgs field** [151].
- **Proton stability** can be achieved even with new physics coming in at low-energy scales. With the SM applicable up to an energy scale $\Lambda_{\text{SM}}$, the proton would in general only be expected to have a lifetime of

$$\tau_p \sim 10^{-30} \text{ years} \left(\frac{\Lambda_{\text{SM}}}{m_p}\right)^4,$$

where $m_p$ is the proton mass [152]. In [153] it was shown that global symmetries within UED instead can lead to a proton lifetime of

$$\tau_p \sim 10^{35} \text{ years} \left(\frac{1/R}{500 \text{ GeV}}\right)^{12} \left(\frac{\Lambda_{\text{cut}}R}{5}\right)^{12} ,$$

(3.20)
where $\Lambda_{\text{cut}}$ is the cut-off energy scale of the UED model. This illustrates that the UEDs model can (for relevant $R$, $\Lambda_{\text{cut}}$ values) fulfill the constraint on proton stability of $\tau_p \gtrsim 10^{33}$ years [152,154].

- It has addressed the (unanswered) question of why we observe three particle generations. In order to cancel gauge anomalies that appear in an even number of UEDs, it has been shown, in the case of two extra dimensions that the number of generations must be three [155].

- Unlike most other extra-dimensional scenarios, single KK states cannot be produced, but must come in pairs. This means that indirect constraints, such as those coming from electroweak precision observables, are not particularly strong. As a result, KK states of SM particles can be much lighter than naively expected. Such ‘light’ new massive particles should, if they exist, be produced as soon as the upcoming Large Hadron Collider (LHC) operates. This is especially true in the region of parameter space favored by having the dark matter in the form of KK particles.

- Studies of the UED could lead to insights about supersymmetry, which today is the prime candidate for new physics at the TeV scale. There are many similarities between supersymmetry and the UED model, which have even led some people to dub the UED scenario ‘bosonic supersymmetry’ [156]. With two such similar models in hand, it gives a great possibility to study how to experimentally distinguish different, but similar, models. In particular, such studies have triggered work on how to measure spin at LHC. By studying the gamma-ray spectrum from annihilating dark matter particles in the UED scenario, we have learned more about similar phenomena within supersymmetry (see, e.g., Paper II versus Paper IV).

- The UED model naturally encompasses a dark matter particle candidate. Although this was not the original motivation for the model, this is perhaps one of the most attractive reasons to study it.

The following three chapters will be devoted to discussions of cosmological aspects of UEDs and its dark matter properties in detail.

---

‡ A gauge anomaly occurs when a quantum effect, such as loop diagrams, invalidates the classical gauge symmetry of the theory.
For every physical theory, it is crucial that it is consistent with observational constraints. A generic prediction in extra-dimensional models is that at least some of the fundamental coupling constants vary with the volume of the extra-dimensional space. Due to the tight observational constraints on the potential variability of fundamental coupling ‘constants’, it is necessary that the size of the extra dimensions stays close to perfectly static during the cosmological history of our Universe. If the extension of the extra dimensions is considerably larger than the Planck scale, it is reasonable that their dynamics should be governed by classical general relativity. However, in general relativity it is nontrivial to obtain static extra dimensions in an expanding universe. Therefore, the evolution of the full spacetime in a multidimensional universe must be scrutinized in order to see if general relativity can provide solutions that are consistent with current observational constraints. The results presented in this chapter, coming partly from PAPER I, show that a homogeneous multidimensional universe only can have exactly static extra dimensions if the equations of state in the internal and external space are simultaneously fine-tuned. For example, in the case of the UED model, it is not expected that the extra dimensions stay static, unless some stabilization mechanism is included. A brief discussion of the requirements of such stabilization mechanisms concludes this chapter.

4.1 Why Constants Can Vary

With coupling constants defined in a higher-dimensional theory, the effective four-dimensional coupling ‘constants’ will vary with the size of the extra-dimensional volume. In a UED scenario, where all particles can propagate in the bulk, all force strengths (determined by their coupling constants) pick up
a dependence on the internal volume. This is not the case in general, since some or all of the force carrying bosons might be confined to a membrane and therefore insensitive to the full bulk. However, since gravity is associated with spacetime itself the gravitational coupling constant (i.e., Newton’s constant) will inevitably depend on the size of the internal space.

Consider specifically \((4+n)\)-dimensional Einstein gravity in a separable spacetime \(M_4(x) \times K_n(\hat{x})\), with the internal space \(K_n\) compactified to form a \(n\)-dimensional torus with equal radii \(R\). If the matter part is confined to our four dimensions, then the extra spatial volume affects only the gravitational part of the action:

\[
S_E = \frac{1}{16\pi \hat{G}_{4+n}} \int d^{4+n} \hat{x} \sqrt{-\hat{g} \hat{R}},
\]

where \(\hat{R}\) is the Ricci scalar (calculated from the higher dimensional metric \(\hat{g}_{MN}\)) and \(\hat{G}_{4+n}\) is the higher dimensional gravitational coupling constant. By Fourier expanding the metric in KK modes, with the zero (i.e., \(y\)-independent) mode denoted by \(g^{(0)}_{MN}\), the Ricci scalar can be expanded as \(\hat{R}[\hat{g}_{MN}] = R[g^{(0)}_{\mu\nu}] + \ldots\). The missing terms, represented by the dots, are the nonzero KK modes, and in Section 4.6 it is shown that they correspond to new scalar fields, so-called radion fields, appearing in the effective, four-dimensional theory. After integrating over the internal dimensions in Eq. (4.1), the four-dimensional action takes the form

\[
S_E = \int d^4 x \sqrt{-g} \left\{ \frac{1}{16\pi G} \hat{R}[g^{(0)}_{\mu\nu}] + \ldots \right\}.
\]

In analogy with Eq. (3.10), the four-dimensional Newton’s constant \(G\) is given by

\[
G = \frac{\hat{G}_{4+n}}{V_n}
\]

and, as before, \(V_n = \int d^4 x \sqrt{-g^{(n)}} \propto (2\pi R)^n\) is the volume of the internal space and \(g^{(n)}\) is the determinant of the metric on the internal manifold. Also in the previous chapter we saw, in the example of a \(\phi^4\)-theory, how the volume \(V_n\) of the extra dimensions rescale higher dimensional coupling constants \(\lambda_{4+n}\) into the dynamical (i.e. \(V_n\) dependent) four-dimensional coupling constant

\[
\lambda = \frac{\hat{\lambda}_{4+n}}{V_n}.
\]

Similar relations hold also for other types of multidimensional theories.

### 4.2 How Constant Are Constants?

Numerous experimental and observational bounds exist on the allowed time variation of fundamental constants, and thus on the size variation of extra
dimensions. Some of these constraints are summarized below (for a more complete review see, e.g., \cite{157,158}).

The constancy of Newton’s constant $G$ has been tested in the range from laboratory experiments to solar system and cosmological observations. Laboratory experiments have mainly focused on testing the validity of Newton’s $1/r^2$ force law down to sub-millimeter distances, but so far no spatial (or temporal) variation has been detected \cite{161}. In the solar system, monitoring of orbiting bodies, such as the Moon, Mercury and Venus, sets an upper limit of $|\Delta G/G| \lesssim 10^{-11}$ during the last decades of observations \cite{157}. On cosmological scales, the best limit comes perhaps from BBN, which puts a constraint of $|\Delta G/G| \lesssim 0.2$ between today and almost 14 billion years ago \cite{i.e., z_{BBN} \sim 10^8 - 10^{10}} \cite{160}. The limit from BBN is derived from the effect a change in Newton’s constant has on the expansion rate of the Universe, and accordingly on the freeze-out temperature, which would affect the abundance of light elements observed today. It is worth noting that some multidimensional models might retain the same expansion rate as in conventional cosmology, despite an evolving gravitational constant $G$, and therefore some stated constraints on $G$ might not be directly applicable \cite{162}.

The constraints on the possible variation of the electromagnetic coupling constant, or rather the fine structure constant $\alpha_{EM}$, are both tight and cover much of the cosmological history. One fascinating terrestrial constraint comes from studies of the isotopic abundances in the Oklo uranium mine, a prehistorical natural fission reactor in central Africa that operated for a short time, about $2 \times 10^9$ yr ago. From the $\alpha_{EM}$ dependence on the capture rate of neutrons of, e.g., $^{149}$Sm, an upper limit of $|\Delta \alpha_{EM}/\alpha_{EM}| \lesssim 10^{-7}$ has been derived \cite{163,164}. Another very suitable way of testing the constancy of the fine structure constant is by analyzing light from astrophysical objects, since the atomic spectra encode the atomic energy levels. Analyses of different astrophysical sources has put limits of $|\Delta \alpha_{EM}/\alpha_{EM}| \lesssim 10^{-3}$ up to a redshift $z \sim 4$ \cite{157}. Worth noticing is that in the literature there have even been claims of an observed variation in $\alpha_{EM}$. Webb \textit{et al.} \cite{165,166}, and also later by Murphy \textit{et al.} \cite{167}, studied relative positions of absorption lines in spectra from distant quasars, and concluded a variation $\Delta \alpha_{EM}/\alpha_{EM} = (-5.4 \pm 1.2) \times 10^{-6}$ at redshift $0.2 < z < 3.7$. However, this is inconsistent with other analyses of quasar spectra. For example, Chand \textit{et al.} \cite{168} and Srianand \textit{et al.} \cite{169} get $\Delta \alpha_{EM}/\alpha_{EM} = (-0.6 \pm 0.6) \times 10^{-6}$ at redshift $0.4 < z < 2.3$. In cosmology, CMB \cite{170,171} and BBN \cite{160,172} set the constraint $|\Delta \alpha_{EM}/\alpha_{EM}| \lesssim 10^{-2}$ at redshift $z_{CMB} \sim 1000$ and $z_{BBN} \sim 10^{10}$, respectively.

* Strictly speaking, it makes no sense to consider variations of dimensionful constants, such as $G$. We should therefore give limits only on dimensionless quantities, like the gravitational coupling strength between protons $Gm_p^2/(\hbar c)$, or specify which other coupling constants that are assumed to be truly constant. The underlying reason is that experiments in principle can count only number of events or compare quantities with the same dimensionality \cite{157,159,160}.
Less work has been put into constraining the weak ($\alpha_w$) and the strong ($\alpha_s$) coupling constants. This is partly due to the more complex modeling; for example, in the weak sector there are often degeneracies between the Yukawa couplings and the Higgs vacuum expectation value, and for strong interactions there is the strong energy dependence on $\alpha_s$. Nonetheless, existing studies of the BBN – where a change in the weak and the strong interactions should have observable effects – indicate no changes in $\alpha_w$ or $\alpha_s$ [157].

Of course, we should keep in mind that there are always some underlying assumptions in deriving constraints, and that the entire cosmological history has not been accessible for observations (accordingly, much less is known at times between epochs of observations). Despite possible caveats, it is still fair to say that there exists no firm observational indication of any variation of any fundamental constant ranging from the earliest times of our Universe until today. These constraints are directly translated into the allowed variation in size of any extra dimension which coupling constants depend on. Therefore, in conclusion, observations restrict the volume of extra spatial dimensions to be stabilized, and not vary by more than a few percentages throughout the history of our observable Universe.

### 4.3 Higher-Dimensional Friedmann Equations

We now turn to the equations of motion that describe the evolution of space-time in a multidimensional universe. Standard cosmology is well described by an isotropic and homogeneous (four-dimensional) FLRW model. Any scenario with internal spatial dimensions must therefore mimic this four-dimensional FLRW model and at the same time be in agreement with the above constraints on the size stability of the extra dimensions. The gravitational dynamics of a multidimensional cosmology will be assumed to be governed by the ordinary Einstein-Hilbert action with $n$ extra dimensions

$$S = \frac{1}{\kappa^2} \int d^{4+n}x \sqrt{-\hat{g}} \left( \hat{\mathcal{R}} + 2\hat{\Lambda} + 2\kappa^2 \hat{\mathcal{L}}_{\text{matter}} \right),$$

where the notation $\kappa^2 = 8\pi \hat{G}$ has been introduced. By varying the action with respect to the metric we derive Einstein’s field equations in $d = 3 + n + 1$ dimensions:

$$\hat{\mathcal{R}}_{AB} - \frac{1}{2} \hat{\mathcal{R}} \hat{g}_{AB} = \kappa^2 \hat{T}^\text{matter}_{AB} + \hat{\Lambda} \hat{g}_{AB} \equiv \kappa^2 T_{AB}.$$  

The higher dimensional cosmological constant $\hat{\Lambda}$ is here taken to be a part of the energy momentum tensor $T_{AB}$.

To address the question whether it is possible to retain ordinary cosmology together with static extra dimensions, let us consider a toy-model\footnote{Some similar studies of multidimensional cosmologies can be found in, e.g., [162, 173–181].}.
where the multidimensional metric is spatially homogeneous, but has two
time-dependent scale factors $a(t)$ and $b(t)$:

$$
\hat{g}_{MN} \, dx^M dx^N = g_{\mu\nu}(x) \, dx^\mu dx^\nu + b^2(x) \hat{g}_{pq}(y) \, dy^p dy^q
$$

(4.6)

$$
dt^2 - a^2(t) \gamma_{ij} dx^i dx^j - b^2(t) \tilde{\gamma}_{pq} dy^p dy^q.
$$

Here $\gamma_{ij}$ is the usual spatial part of the FLRW metric (1.11) for the ordinary,
large dimensions and $\tilde{\gamma}_{pq}$ is a similar maximally symmetric metric for the internal
extra-dimensional space. The most general form of the energy-momentum
tensor, consistent with the metric, is, in its rest frame:

$$
T_{00} = \hat{\rho}, \quad T_{ij} = -\hat{p}_a a^2 \gamma_{ij}, \quad T_{3+p3+q} = -\hat{p}_b b^2 \tilde{\gamma}_{pq}.
$$

(4.7)

This describes a homogeneous, but in general anisotropic, perfect fluid with
a 3D pressure $p_a$ and a common pressure $p_b$ in the $n$ directions of the extra
dimensions.

With the above ansatz, the nonzero components of the higher-dimensional
Friedmann equations (4.5) can be written as:

$$
3 \left( \frac{\ddot{a}}{a} \right)^2 + \frac{k_a}{a^2} + 3n \frac{\dot{a}}{a} \frac{\dot{b}}{b} + \frac{n(n-1)}{2} \left[ \frac{(\dot{b})^2}{b^2} + \frac{k_b}{b^2} \right] = \kappa^2 \hat{\rho}
$$

(4.8a)

$$
2 \frac{\ddot{b}}{b} + \frac{(\dot{a})^2}{a^2} + \frac{k_a}{a^2} + 2n \frac{\dot{a}}{a} \frac{\dot{b}}{b} + \frac{n(n-1)}{2} \left[ \frac{(\dot{b})^2}{b^2} + \frac{k_b}{b^2} \right] = \kappa^2 \hat{p}_a
$$

(4.8b)

$$
\frac{\ddot{b}}{b} + 3 \frac{\dot{a}}{a} \frac{\dot{b}}{b} + (n-1) \left[ \frac{(\dot{b})^2}{b^2} + \frac{k_b}{b^2} \right] = \frac{\kappa^2}{n+2} \left[ \hat{\rho} - 3\hat{p}_a + 2\hat{p}_b \right]
$$

(4.8c)

where dots (as in $\dot{a}$) denote differentiation with respect to the cosmic time $t$
and, as usual, the curvature scalars $k_{a,b}$ are $+1, 0, -1$ depending on whether
the ordinary/internal spatial space is positively, flat, or negatively curved.

With the extra dimensions exactly static ($\dot{b} \equiv 0$) the first two equations
(4.8a, 4.8b) reduce to the ordinary Friedmann equation (1.11), with an effective
vacuum energy due to the internal curvature $k_b$. If also the third equation
(4.8c) can be simultaneously satisfied, this seems to be what was looked for –
a solution to Einstein’s field equations that has static extra dimensions and
recovers standard cosmology.

### 4.4 Static Extra Dimensions

Since the internal curvature parameter $k_b$ is just a constant in Eq. (4.8c),
exactly static extra dimensions ($\dot{b} \equiv 0$) are only admitted if also

$$
\hat{\rho} - 3\hat{p}_a + 2\hat{p}_b \equiv C \left[ = (n+2)(n-1) \frac{k_b}{\kappa^2 b^2} = \text{constant} \right].
$$

(4.9)
stays constant. This equation will severely restrict the possible solutions if we do not allow the internal pressure to be a freely adjustable parameter.

Similarly to standard cosmology, the energy content will be taken to be a multicomponent perfect fluid. Each matter type will be specified by a constant equation of state parameter \( w^{(i)} \), such that

\[
\dot{\rho}^{(i)} = w^{(i)} \dot{\rho}^{(i)} \quad \text{and} \quad \dot{p}^{(i)} = w^{(i)} \rho^{(i)}
\] (4.10)

This permits the equation \( T_{0,0} = 0 \) to be integrated to

\[
\dot{\rho}^{(i)} \propto a^{-3(1+w^{(i)})} b^{-n(1+w^{(i)})}.
\] (4.11)

The total energy \( \dot{\rho} \) and pressures \( \dot{p}_a \) and \( \dot{p}_b \) are the sum of the individual matter components, i.e., \( \dot{\rho} = \sum_i \dot{\rho}^{(i)} \) and \( \dot{p}_{a,b} = \sum_i w^{(i,a,b)} \dot{\rho}^{(i)} \). With a multicomponent fluid allowed in Eq. (4.9), [that is \( C = \sum_i C^{(i)} \), where \( C^{(i)} \equiv \dot{\rho}^{(i)} - 3\dot{p}^{(i)} + 2\dot{p}^{(i)} \)], we could in principle imagine a cancellation of the time dependency of individual \( C^{(i)} \) such that \( C^{(i)} \) still is time independent. However, in the case of static extra dimensions this requires that canceling terms have the same \( w^{(i,a)} \), and therefore it is convenient to instead define this as one matter component, with the given \( w^{(i,a)} \), and then adopt an effective \( w^{(i,b)} \).

With this nomenclature, each matter component \( (i) \) must separately fulfill Eq. (4.9) to admit static extra dimensions:

\[
\dot{\rho}^{(i)} \left( 1 - 3w^{(i,a)} + 2w^{(i,b)} \right) \equiv C^{(i)} = \text{constant},
\] (4.12)

From Eq. (4.11) and (4.12), it is clear that static internal dimensions in an evolving universe \( (\dot{a} \neq 0) \) requires that either \( w^{(i,a)} = -1 \), so that \( \dot{\rho}^{(i)} \) is constant, or that the equations of state fulfill \( (1 - 3w^{(i,a)} + 2w^{(i,b)}) = 0 \). Thus, in summary:

**In a homogeneous, non-empty, and evolving multidimensional cosmology with \( n \) exactly static extra dimensions, the equation of state for each perfect fluid must fulfill either**

\[
\text{I.} \quad w^{(i,a)} = -1 \quad \text{or} \quad \text{II.} \quad w^{(i,b)} = \frac{3w^{(i,a)} - 1}{2}.
\] (4.13)

Note that a matter component that fulfills case I. (and not II.) implies that the internal space must be curved, since it implies \( C \neq 0 \) in Eq. (4.9). Similarly, it is not possible to have a multidimensional cosmological constant \( (w^{(i,a)} = w^{(i,b)} = -1) \) together with flat extra dimensions \( (k_b = 0) \).

To actually determine the expected equations of state in a multidimensional scenario, we have to further specify the model. In the next section, we take a closer look at the scenario of UED and KK states as the dark matter.
4.5 Evolution of Universal Extra Dimensions

In the UED model, introduced in the previous chapter, all the SM particles are allowed to propagate in the extra dimensions. Momentum in the direction of the compactified dimensions gives rise to massive KK states in the effective four-dimensional theory. Furthermore, if the compactification scale is in the TeV range, then the lightest KK particle (LKP) of the photon turns out to be a good dark matter candidate. The exact particle field content of the UED model and the properties of the dark matter candidate are not important at this point, and further discussion on these matters will be postponed to the following chapters. However, from the mere fact that the KK dark matter particles gain their effective four-dimensional masses from their own momentum in the extra dimensions, it is possible to predict the pressure.

The classical pressure, in a direction $\hat{x}^A$, is defined as the momentum flux through hypersurfaces of constant $\hat{x}^A$. In case of isotropy in the 3 ordinary dimensions and also – but separately – in the $n$ extra dimensions, we find that (see Paper I):

$$3\hat{p}_a + n\hat{p}_b = \hat{\rho} - \left\langle \frac{m^2}{E} \right\rangle. \quad (4.14)$$

For the SM particles, with no momentum in the extra dimensions, there is no contribution to the pressure in the direction of the extra dimensions ($p_b = 0$), whereas for KK states, with momentum in the extra dimensions, we can always ignore any SM mass $m$ compared to their total energy $E \sim$ TeV (i.e. momentum in the extra dimensions). Thus, with KK states being the dark matter, the energy in the universe will not only be dominated by relativistic matter ($m^2/E \ll \hat{\rho}$) during the early epoch of radiation domination but also during matter domination in the form of LKPs. Thus, Eq. (4.14) gives the equation of states:

$$w_a = \frac{1}{3}, \quad w_b = 0 \quad (4D \text{ Radiation dominated}) \quad (4.15a)$$

$$w_a = 0, \quad w_b = \frac{1}{n} \quad (4D \text{ Matter dominated}) \quad (4.15b)$$

During four-dimensional radiation domination, the requirement II. in (4.13) for static extra dimensions is actually satisfied, whereas this is clearly not the case during what looks like matter domination from a four-dimensional point of view.

With exactly static extra dimensions being ruled out, a numerical evolution of the field equations (4.8) was performed in Paper I, to test if nearly static extra dimensions could be found. The freeze-out of the LKPs takes place during four-dimensional radiation domination, and a tiny amount of the energy density in the universe is deposited in the form of the LKPs (which much later will dominate the energy during matter domination). Consequently, the initial condition for the numerical evolution is a tiny amount of LKPs with
stable extra dimensions in the regime of radiation domination. However, not surprisingly, as soon as the relative amount of energy in dark matter becomes significant ($\sim 10\%$) the size of the extra dimensions start escalating, as is shown in Fig. 4.1. Such an evolution would severely violate the constraints on both $\alpha_{EM}$ and $G$ presented in Section 4.2. In conclusion, not even approximately static extra dimensions can be found in this setup, and therefore some extra mechanism is needed in order to stabilize the extra dimensions and reproduce standard cosmology.

### 4.6 Dimensional Reduction

To study dynamical stabilization mechanisms, it is more practical to consider the equations of motion after dimensional reduction of the action. In this section, such a dimensional reduction is performed before we return to the stabilization of internal spaces in the next section.

Integrating over the internal dimensions in Eq. (4.4), with the metric
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ansatz (4.6), gives (see, e.g., [182, 183]):

\[ S = \frac{1}{2\bar{\kappa}^2} \int d^4x \sqrt{-\bar{g}} \bar{b}^n \left[ \bar{\mathcal{R}} + b^{-2}\bar{\mathcal{R}} + n(n-1)b^{-2}\partial_\mu b\partial^\mu b + 2\Lambda + 2\bar{\kappa}^2\mathcal{L}_{\text{matter}} \right] , \quad (4.16) \]

where \( \bar{\kappa}^2 \equiv \hat{\kappa}^2 / \int d^n y \sqrt{-\bar{g}} \), and \( \mathcal{R} \) and \( \bar{\mathcal{R}} \) are the Ricci scalars constructed from \( g_{\mu\nu} \) and \( \bar{g}_{pq} \), respectively. With the assumed metric, the internal Ricci scalar can also be written as \( \bar{\mathcal{R}} = n(n-1)\bar{b} \). A conformal transformation to the new metric \( \bar{g}_{\mu\nu} = b^n g_{\mu\nu} \) takes the action to the standard Einstein-Hilbert form (so-called Einstein frame), i.e., the four-dimensional Ricci scalar appears with no multiplicative scalar field [182]:

\[ S = \int d^4x \sqrt{-\bar{g}} \left( \frac{1}{2\bar{\kappa}^2} \bar{\mathcal{R}}[\bar{g}_{\mu\nu}] - \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V_{\text{eff}}(\Phi) \right) . \quad (4.17) \]

This is our dimensionally reduced action. It gives ordinary four-dimensional general relativity coupled to a new scalar field (the radion field):

\[ \Phi \equiv \sqrt{\frac{n(n+2)}{2\bar{\kappa}^2}} \ln b , \quad (4.18) \]

with an effective potential

\[ V_{\text{eff}}(\Phi) = -\frac{\bar{\mathcal{R}}}{2\bar{\kappa}^2} e^{-\sqrt{\frac{2n+2}{n}}\bar{\kappa}\Phi} + \frac{1}{\bar{\kappa}^2} \left( \Lambda - \bar{\kappa}^2\mathcal{L}_{\text{matter}} \right) e^{-\sqrt{\frac{n}{n+2}}\bar{\kappa}\Phi} . \quad (4.19) \]

Although the equations of motion for the metric \( \bar{g}_{\mu\nu} \) and \( \Phi \), derived from this new action, can be directly translated to the higher-dimensional Friedmann equations (4.8), the equation of motion for the radion field,

\[ \Box \Phi = -\frac{\partial}{\partial \Phi} V_{\text{eff}} \quad (4.20) \]

where \( \Box = \frac{1}{\sqrt{-g}} \partial^\mu \sqrt{-g} \partial_\mu \), now has a much more intuitive interpretation regarding when stable extra dimensions are expected. As the internal scale factor depends only on \( t \), we have \( \Box \Phi = \ddot{\Phi} + 3H \dot{\Phi} \). We have thus found that the scalar field \( \Phi \) has the same equation of motion as a classical particle moving in a potential \( V_{\text{eff}} \), with a friction term given by three times the Hubble expansion rate \( H \). If the effective potential \( V_{\text{eff}}(\Phi) \) has a stationary minimum, then the radion field, and thus the extra dimension, has a static and stable solution.

Before we continue the discussion in the next section on how to obtain stable extra dimensions, it is interesting to note a subtlety regarding conformal transformations. From the higher-dimensional perspective the naïve
guess would be that the four-dimensional metric part $g_{\mu\nu}$ with the scale factor $a(t)$ describes the effective four-dimensional space. However, after the conformal transformation into the Einstein frame (i.e., the action written as in Eq. (4.17)) suggests that it should rather be the effective four-dimensional metric $\bar{g}_{\mu\nu}$ with the scale factor $\bar{a}(\bar{t}) = a(t)b^{n/2}(t)$ that describes the physical four-dimensional space. The problem of which of the conformally related frames should be regarded as the physical one, or if they both are physically equivalent, is to some extent still debated [184–186]. However, for the discussion in this chapter the distinction between $a(t)$ and $\bar{a}(\bar{t})$ is not of any importance, because the tight observational constraints on the variation of coupling constants imply that the extra dimensions can always be assumed to be almost static. In that case the different frames are in practice identical.

### 4.7 Stabilization Mechanism

Let us briefly investigate what is needed to stabilize the extra dimensions. The effective potential for $\Phi$ in Eq. (4.19) can be rewritten as

$$V_{\text{eff}}(b(\Phi)) = -\frac{\bar{R}^2}{2}\bar{\kappa}^2 b^{-(n+2)} + \sum_i \rho_0^{(i)} \bar{a}^{-3(1+w_a^{(i)})} b^{-\frac{3}{2}(1-3w_a^{(i)}+2w_b^{(i)})}. \quad (4.21)$$

This expression follows from the definition in Eq. (4.18), that $\hat{\rho}_{\text{matter}} = -\hat{\rho} = -\sum_i \hat{\rho}^{(i)}$, and that each fluid component dependency on $\bar{a} = ab^{-n/2}$ and $\Phi$ is given by

$$\hat{\rho}^{(i)} = \rho_0^{(i)} a^{-3(1+w_a^{(i)})} b^{-n(1+w_b^{(i)})} = \frac{\bar{R}^2}{\bar{\kappa}^2} \rho_0^{(i)} \bar{a}^{-3(1+w_a^{(i)})} b^{n/2(1+3w_a^{(i)}-2w_b^{(i)})},$$

where the four-dimensional densities $\rho_0^{(i)}$ are defined by $\rho_0^{(i)} \equiv \hat{\rho}_0^{(i)} V_n \equiv \hat{\rho}_0^{(i)} \bar{\kappa}^2/\bar{\kappa}^2$.

We can now easily verify that $\partial_\mu \Phi = 0$ in Eq. (4.20) implies the same condition (4.9) as obtained earlier for exactly static extra dimensions. However, in the perspective of the radion field, in its effective potential, it is also obvious that even the static extra-dimensional solution is only stable if the potential has a stationary minimum. In fact, the static solution found earlier (for a radiation-dominated universe with $w_a = 1/3$ and $w_b = 0$) is not really a stable minimum, since the effective potential is totally flat (when $k_b = 0$). During matter domination in the UED model, with $w_a = 0$ and $w_b = 1/n$, the radion field is of course not in a stationary minimum either. In the case of a higher dimensional cosmological constant, $w_a = w_b = -1$, there could in principle be a stationary minimum if $\rho_\Lambda(\bar{a}) < 0$ and $\bar{R} < 0$, but then the effective four-dimensional vacuum energy from the cosmological constant and the internal curvature contribution will sum up to be negative. This would give an anti de Sitter space, which is not what is observed.
At this point let us, ad hoc, introduce a stabilization mechanism in the form of a background potential \( V_{bg}(\Phi) \) for the radion field. If this potential has a minimum, say at \( \Phi_0 \), the contribution to the potential is to a first-order expansion around this minimum given by:

\[
V_{bg}(\Phi) \simeq V_{bg}(\Phi_0) + \frac{m^2}{2}(\Phi - \Phi_0)^2
\]

where \( m^2 \equiv \frac{\partial^2 V_{bg}}{\partial^2 \Phi} \bigg|_{\Phi_0} \). (4.22)

The minima of the total potential are found from \( V'_\text{tot} = V'_\text{eff}(\Phi) + V'_\text{bg}(\Phi) = 0 \), and can implicitly be written as:

\[
\Phi_{\text{min}} \simeq \Phi_0 + \left(1 - 3w_a + 2w_b\right) \frac{\sqrt{n\kappa^2}}{m^2} \rho(\bar{a}, b_{\text{min}}) - \frac{\tilde{R}}{m^2} \sqrt{\frac{2 + n}{2n\kappa^2}} b_{\text{min}}^{-(n+2)}.
\] (4.23)

Here the sum over different fluid components has been suppressed for brevity, and only one (dominant) contributor \( \rho(\bar{a}, b_{\text{min}}) \) is included. Due to the coupling to the matter density \( \rho(\bar{a}, b) \) the minimum \( \Phi_{\text{min}} \) will thus in general be time dependent. For small shifts of the minimum, it can be expressed as

\[
\Delta\Phi_{\text{min}} \simeq \left(1 - 3w_a + 2w_b\right) \frac{\sqrt{n\kappa^2}}{m^2} \sqrt{\frac{2 + n}{2n\kappa^2}} \Delta \rho(\bar{a}, b_{\text{min}}),
\] (4.24)

Thus, a change in \( b(\Phi) \) of less than 1% between today and BBN, corresponding to \( \Delta \rho \sim 10^{19} \text{ eV}^4 \), would only require the mass in the stabilization potential to be \( m \gtrsim 10^{-16} \text{ eV} \) – a very small mass indeed.

In fact, since a light mass radion field would mediate a new long-range (fifth) force with the strength of about that of gravity, sub-millimeter tests of Newton’s law impose a lower mass bound of \( m \gtrsim 10^{-3} \text{ eV} \) [187] (see also [188] for more cosmological aspects on radions in a UED scenario).

Still, only a fairly shallow stabilization potential is needed to achieve approximately static extra dimensions for a radion field that tracks its potential minimum. The stabilization mechanism could be a combination of different perfect fluids with tuned \( w_a^{(i)} \) and \( w_b^{(i)} \) to produce a potential minimum, or one of many proposed mechanisms. Examples are the Freund-Rubin mechanism, with gauge-fields wrapped around the extra dimensions [189]; the Goldberger-Wise mechanism, with bulk fields interacting with branes [190]; the Casimir effect in the extra dimensions [191]; quantum corrections to the effective potential [192]; or potentially something string theory related (like the Kachru-Kallosh-Linde-Trivedi (KKLT) model with nonperturbative effects and fluxes that stabilizes a warped geometry [193]). Whichever the mechanism might be, it seems necessary to introduce an additional ingredient to stabilize extra dimensions. In the following discussions of the UED model, it will be implicitly assumed that the extra dimensions are stabilized, and that the mechanism itself does not induce further consequences at the level of accuracy considered.

\[ \text{‡} \] In general, a small \( \Delta b/b \) shift requires \( m \gtrsim \left[ \sum_i \sqrt{2(1 - 3w_a^{(i)} + 2w_b^{(i)}) \kappa^2 \Delta \rho^{(i)}} / (2 + n) \right]^{1/2} \]
We now turn to the theoretical framework for the UED model. Proposed by Appelquist, Cheng, and Dobrescu [126] in 2001, the UED model consists of the standard model in a higher dimensional spacetime. As all standard model particles are allowed to propagate into the higher-dimensional bulk in this model, this means that, when it is reduced to a four-dimensional effective theory, every particle will be accompanied by a Kaluza-Klein tower of identical but increasingly more massive copies. Conservation of momentum in the direction of the extra space dimensions imply that heavier Kaluza-Klein states can only be produced in pairs. This chapter develops the effective four-dimensional framework, and will simultaneously give an overview of the field structure of the standard model of particle physics.

5.1 Compactification

The simplest reasonable extension of spacetime is to add one extra flat dimension, compactified on a circle $S^1$ (or a flat torus in more extra dimensions) with radius $R$. However, such a compactification of a higher-dimensional version of the SM would not only give new massive KK particles but also unwanted light scalar degrees of freedom and fermions with the wrong chirality.

The extra scalar degrees of freedom that appear at low energies for a four-dimensional observer are simply the fifth component of any higher dimensional vector fields, which transform as scalars under four-dimensional Lorentz transformations. Such massless scalar fields (interacting with usual gauge strengths) are not observed and are strongly ruled out by the extra ‘fifth’ force interaction they would give rise to [194].

In the SM, the fermions are chiral, meaning that the fermions in the $SU(2)$ doublet are left-handed, whereas the singlets are right-handed (and the oppo-
site handedness for anti-fermions). With one extra dimension, fermions can be represented by four-component spinors, but the zero modes will consist of both left-handed and right-handed fermions. Technically, the reason behind this is that the four-dimensional chiral operator is now a part of the higher dimensional Dirac algebra, and higher-dimensional Lorentz transformations will in general mix spinors of different chirality.

If the extra dimensions instead form an orbifold, then the above problems can be avoided. With one extra dimension the simplest example is an \( S^1 / \mathbb{Z}_2 \) orbifold, where \( \mathbb{Z}_2 \) is the reflection symmetry \( y \rightarrow -y \) (\( y \) being the coordinate of the extra dimension). Fields can be set to be even or odd under this \( \mathbb{Z}_2 \) symmetry, which allows us to remove unwanted scalar and fermionic degrees of freedom, and thereby reproduce the particle content of the SM. From a quantum field theory point of view, the orbifold can be viewed as projecting the circular extra dimension onto a line segment of length \( \pi R \) stretching between the two fixpoints at \( y = 0 \) and \( y = \pi R \). Fields, say \( \Phi \), are then given Neumann (or Dirichlet) boundary conditions \( \partial_5 \Phi = 0 \) (or \( \Phi = 0 \)), so that they become even (or odd) functions along the \( y \)-direction.

### 5.2 Kaluza-Klein Parity

The circular compactification \( S^1 \) breaks global Lorentz invariance, but local invariance is preserved. By Noether’s theorem, local translational invariance corresponds to momentum conservation along the extra dimension. A set of suitable base functions are thus \( e^{i \frac{ny}{R}} \), which are the eigenstates of the momentum operator \( i \partial_y \) (the integer \( n \) is a conserved quantity called the KK number). For example, a scalar field \( \Phi \), is expanded as

\[
\Phi(x^\mu, y) = \sum_{n=-\infty}^{\infty} \Phi^{(n)}(x^\mu) e^{i \frac{ny}{R}}, \tag{5.1}
\]

We might expect that the fifth component of the momentum should be a conserved quantity in UED. However, the orbifold compactification \( S^1 / \mathbb{Z}_2 \), with its fixpoints, breaks translational symmetry along the extra dimension, and KK number is no longer a conserved quantity. However, as long as the fixpoints are identical there is a remnant of translation invariance, namely translation of \( \pi R \), which takes one fixpoint to the other. Rearranging the sum in Eq. (5.1) into terms that are eigenstates to the orbifold operator \( \mathcal{P}_{\mathbb{Z}_2} \) (i.e., \( \Phi(y) \) is even or odd) and simultaneously KK mass eigenstates (i.e., momentum squared, or \( \partial_y^2 \), eigenstates) gives:

\[
\Phi(x^\mu, y) = \Phi^{(0)}_{\text{even}}(x^\mu) + \sum_{n=1}^{\infty} \Phi^{(n)}_{\text{even}} \cos \frac{ny}{R} + \Phi^{(n)}_{\text{odd}} \sin \frac{ny}{R}. \tag{5.2}
\]
Depending on KK level, the terms behave as follows under the translation $y \rightarrow y + \pi R$:

\[
\cos \frac{ny}{R} \rightarrow \cos \frac{n(y + \pi R)}{R} = (-1)^n \cos \frac{ny}{R}, \quad (5.3a)
\]

\[
\sin \frac{ny}{R} \rightarrow \sin \frac{n(y + \pi R)}{R} = (-1)^n \sin \frac{ny}{R}. \quad (5.3b)
\]

Hence, a Lagrangian invariant under the $\pi R$ translation can only contain terms that separately sum their total KK level to an even number. In other words, every term in the UED Lagrangian must have $(-1)^{n_{\text{tot}}}$, where $n_{\text{tot}}$ is the sum of all the KK levels in a particular term. In an interaction term, split $n_{\text{tot}}$ into ingoing and outgoing particles, such that $n_{\text{tot}} = n_{\text{in}} + n_{\text{out}}$, and we have $(-1)^{n_{\text{in}}} = (-1)^{n_{\text{out}}}$. This is known as the conservation of KK parity $(-1)^n$, and will be essential in our discussion of KK particles as a dark matter candidate.

### 5.3 The Lagrangian

In the UED model, all the SM particles, with its three families of fermions, force carrying gauge bosons and one Higgs boson, are allowed to propagate in the extra $S^1/Z_2$ dimension. In such a higher dimensional Lagrangian, the gauge, Yukawa and quartic-Higgs couplings have negative mass dimensions, and the model is non-renormalizable [3]. Therefore, should the UED model be viewed as an effective theory, applicable only below some high-energy cutoff scale $\Lambda_{\text{cut}}$. With the compactification scale $1/R$ distinctly below the cutoff $\Lambda_{\text{cut}}$, a finite number of KK states appears in the effective four-dimensional theory. If only KK states up to $\Lambda_{\text{cut}}$ are considered, the UED model is from this perspective a perfectly valid four-dimensional field theory. In the minimal setup, all coupling strengths in the UED model are fixed by the measured four-dimensional SM couplings, and the only new parameters are the cutoff and the compactification scale, $\Lambda_{\text{cut}}$ and $R$, respectively.

For later chapters, mainly the electroweak part of the Lagrangian is relevant. Therefore the fermions, the $U(1) \times SU(2)$ gauge bosons, the Higgs doublet, and their interactions will be discussed in some detail, whereas the Quantum Chromo Dynamics (QCD) sector, described by the $SU(3)$ gauge group, is left out.

Furthermore, with the expectation that much of the interesting phenomenology can be captured independently of the number of extra space dimensions, we will in the following only consider the addition of one extra space dimension.

* The reason for the QCD sector being of little importance in this thesis is because it contains neither any dark matter candidate, nor do the gluons (i.e., gauge bosons of QCD) interact directly with photons (which means they are not relevant for the gamma-ray yield calculations made in PAPER II,III).
The UED Lagrangian under consideration will be split into the following parts:

$$\hat{\mathcal{L}}_{\text{UED}} = \hat{\mathcal{L}}_{\text{Bosons}} + \hat{\mathcal{L}}_{\text{Higgs}} + \hat{\mathcal{L}}_{\text{Fermions}}. \quad (5.4)$$

In general, the inclusion of gauge fixing terms will additionally result in a $\hat{\mathcal{L}}_{\text{ghost}}$-term including only unphysical ghost fields. In what follows, all these parts are reviewed separately.

**Gauge Bosons**

The first four components of the $SU(2)$ and $U(1)$ gauge fields, denoted $A^r_M$ and $B^r_M$ respectively, must be even under the orbifold projection to retain the ordinary four-dimensional gauge fields as zero modes in their Fourier series:

$$P_{\mathbb{Z}_2} A^r_M(x^\mu, y) = A^r_M(x^\mu, -y). \quad (5.5)$$

A four-dimensional gauge transformation of such a gauge field is given by

$$A^r_\mu(x^\mu, -y) = A^r_\mu(x^\mu, y) \rightarrow A^r_\mu(x^\mu, y) + \frac{1}{\hat{g}} \partial_\mu \alpha^r(x^\mu, y) + f^{rst} A^s_\mu \alpha^t, \quad (5.6)$$

where $f^{rst}$ is the structure constant, defining the lie algebra for the generators, $[\sigma_r, \sigma_s] = if^{rst} \sigma_t$, and $\hat{g}$ is a five-dimensional coupling constant. From Eq. (5.6) we read off that the function $\alpha(x^\mu, y)$ has to be even under $\mathbb{Z}_2$. Therefore $\partial_y \alpha^r(x^\mu, y)$ is odd, and hence

$$A^r_5(x^\mu, y) = -A^r_5(x^\mu, -y) \quad (5.7)$$

to keep five-dimensional gauge invariance. The Fourier expansions of the gauge fields in the extra dimension are then:

$$A^i_\mu(\hat{x}) = \frac{1}{\sqrt{2\pi R}} A^i_\mu(0) + \frac{1}{\sqrt{\pi R}} \sum_n A^i_\mu(n)(x^\mu) \cos \frac{ny}{R}, \quad (5.8a)$$

$$A^i_5(\hat{x}) = \frac{1}{\sqrt{\pi R}} \sum_n A^i_5(n)(x) \sin \frac{ny}{R}. \quad (5.8b)$$

Note that the five-dimensional gauge invariance automatically implies the absence of the unwanted zero mode scalars $A^r_5$ and $B_5$ in the four-dimensional theory.

The kinetic term for the gauge fields in the Lagrangian reads

$$\hat{\mathcal{L}}_{\text{gauge}} = -\frac{1}{4} F^M_N F^{MN} - \frac{1}{4} \hat{g} F^r_M F^{rMN}, \quad (5.9)$$

with the $U(1)$ and $SU(2)$ field strength tensors given by

$$F_{MN} = \partial_M B_N - \partial_N B_M, \quad (5.10a)$$

$$F^i_{MN} = \partial_M A^i_N - \partial_N A^i_M + \hat{g} \epsilon^{ijk} A^j_M A^k_N. \quad (5.10b)$$
Inserting the field expansions (5.8) and integrating out the extra dimension results in the SM Lagrangian accompanied by a KK tower of more massive states. For example, the $U(1)$ part of $\hat{L}_{\text{gauge}}$ becomes
\begin{align}
L_{4D}^{\text{gauge}} & \supset -\frac{1}{4} \int_0^{2\pi R} dy \, F_{MN} F^{MN} \\
& = -\frac{1}{4} \left( \partial_\mu B^{(0)}_\nu - \partial_\nu B^{(0)}_\mu \right) \left( \partial^\mu B^{(0)}_\nu - \partial^\nu B^{(0)}_\mu \right) \\
& - \frac{1}{4} \sum_{n=1}^{\infty} \left( \partial_\mu B^{(n)}_\nu - \partial_\nu B^{(n)}_\mu \right) \left( \partial^\mu B^{(n)}_\nu - \partial^\nu B^{(n)}_\mu \right) \\
& + \frac{1}{2} \sum_{n=1}^{\infty} \left( \partial_\mu B^{(n)}_5 + \frac{n}{R} B^{(n)}_\mu \right) \left( \partial^\mu B^{(n)}_5 + \frac{n}{R} B^{(n)}_\mu \right),
\end{align}
(5.11)

and with a similar expression for the kinetic part of the $SU(2)$ gauge fields. At each KK level, a massive vector field $B^{(n)}_\mu$ with mass $nR$ appears. The scalars $B^{(n)}_5$ are, however, not physical, because by a gauge transformation $B_\mu \to B_\mu^{(n)} - (R/n)\partial_\mu B^{(n)}_5$ the scalar field terms $\partial_\mu B^{(n)}_5$ can be removed. This actually becomes apparent already from a naïve counting of degrees of freedom: A massive four-dimensional vector field has three degrees of freedom, which is the same as the number of polarization directions for a massless five-dimensional gauge field. Once the Higgs mechanism with electroweak symmetry breaking is added (discussed in the next section), the additional Goldstone scalar fields will form linear combinations with the vector field’s fifth component to form both physical as well as unphysical scalar fields.

For the non-abelian gauge fields there will also be cubic and quartic interaction terms, and we can identify the ordinary four-dimensional $SU(2)$ coupling constant with
\begin{equation}
g \equiv \frac{1}{\sqrt{2\pi R}} \hat{g}.
\end{equation}
(5.12)

This mapping between the four-dimensional couplings in the SM and the five-dimensional couplings is general and will hold for all coupling constants.

**The Higgs Sector**

The electroweak masses of the SM gauge fields are generated by the usual Higgs mechanism. The Higgs field is a complex $SU(2)$ doublet that is a scalar under Lorentz transformations:
\begin{equation}
\phi \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \chi^2 + i\chi^1 \\ H - i\chi^3 \end{pmatrix}, \quad \chi^\pm \equiv \frac{1}{\sqrt{2}} (\chi^1 \mp i\chi^2).
\end{equation}
(5.13)

\footnote{Sometimes also called the Brout-Englert-Higgs mechanism, Higgs-Kibble mechanism or Anderson-Higgs mechanism.}
To have the SM Higgs boson as the zero mode, the expansion must be even under the $\mathbb{Z}_2$ orbifold
\begin{equation}
\phi(\hat{x}) = \frac{1}{\sqrt{2\pi R}} \phi^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n} \phi^{(n)}(x) \cos \frac{ny}{R}.
\end{equation}

To make the $H$ field electromagnetically neutral, the hypercharge is set to $Y = 1/2$ for the Higgs doublet and its covariant derivative is
\begin{equation}
D_M = \partial_M - i\hat{g}A^r_M \sigma_r - iY \hat{g}_Y B_M,
\end{equation}
where $\hat{g}_Y$ and $\hat{g}$ are the higher dimensional $U(1)$ and the $SU(2)$ coupling constants, respectively, and $\sigma_r$ are the usual Pauli matrices:
\begin{align*}
\sigma_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, & \sigma_2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, & \sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\end{align*}

The full Higgs Lagrangian is written as
\begin{equation}
\hat{L}_{\text{Higgs}} = (D_M \phi)^\dagger (D^M \phi) - V(\phi),
\end{equation}
where the potential $V(\phi)$ is such that spontaneous symmetry breaking occurs. That is
\begin{equation}
V(\phi) = \hat{\mu}^2 \phi^\dagger \phi + \hat{\lambda}(\phi^\dagger \phi)^2,
\end{equation}
where the values of the parameters are such that $-\hat{\mu}^2, \hat{\lambda} > 0$. This ‘Mexican hat’ potential has a (degenerate) minimum at
\begin{equation}
|\phi|^2 = \frac{-\hat{\mu}^2}{2\hat{\lambda}^2} = \frac{\hat{v}^2}{2}.
\end{equation}

By choosing any specific point in the minimum as the vacuum state, around which the physical fields then are expanded, the symmetry is said to be spontaneously broken. With the vacuum expectation value chosen to lie along the real axis, $\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \hat{v} \end{pmatrix}$, the Higgs field $H$ should be replaced by $h + \hat{v}$ so that $h$ is a perturbation around the vacuum and hence represents the Higgs particle field.

Electroweak gauge boson mass terms will now emerge from the kinematic part of Eq. (5.17). As in the standard Glashow-Weinberg-Salam electroweak theory, three of the gauge bosons become massive,
\begin{align*}
W_M^\pm &= \frac{1}{\sqrt{2}} \left( A^1_M \mp iA^2_M \right) \quad \text{with mass} \quad m_W = \frac{\hat{g}\hat{v}}{2},
\end{align*}
\begin{align*}
Z_M &= c_W A^3_M - s_W B_M \quad \text{with mass} \quad m_Z = \frac{m_W}{c_W},
\end{align*}
\begin{align*}
W^\pm_M &= \frac{1}{\sqrt{2}} \left( A^1_M \mp iA^2_M \right) \quad \text{with mass} \quad m_W = \frac{\hat{g}\hat{v}}{2},
\end{align*}
\begin{align*}
Z_M &= c_W A^3_M - s_W B_M \quad \text{with mass} \quad m_Z = \frac{m_W}{c_W},
\end{align*}
and one, orthogonal to $Z$, is massless

$$A_M = s_W A_M^3 + c_W B_M.$$  \hspace{1cm} (5.21)

The Weinberg angle $\theta_W$ that appears above is (at tree level) given by

$$s_W \equiv \sin \theta_W = \frac{g_Y}{\sqrt{g_Y^2 + g_Y^2}}, \quad c_W \equiv \cos \theta_W = \frac{g}{\sqrt{g^2 + g^2}}.$$  \hspace{1cm} (5.22)

This relates the electromagnetic and weak coupling constants in such a way that the gauge field $A_\mu$ becomes the photon with its usual coupling to the electric charge $e = s_W g = c_W g_Y$.

In the quadratic part of the kinetic Higgs term

$$\hat{L}^{(\text{kin})}_{\text{Higgs}} \supset \left\{ \frac{1}{2} \left( \partial_M h^2 \right) + \frac{1}{2} \left( \partial_M \chi^3 - m_Z Z_M \right)^2 + \left| \partial_M \chi + m_W W_M^+ \right|^2 \right\},$$  \hspace{1cm} (5.23)

there are unwanted cross-terms that mix vector fields and scalar fields (e.g., $Z^\mu \partial_\mu \chi^3$). This is the same type of unwanted terms that appeared in the Lagrangian of Eq. (5.11). To remove these unwanted terms properly, five dimensional gauge-fixing terms are added to the Lagrangian:

$$\hat{L}_{\text{gaugefix}} = - \frac{1}{2} (G_Y')^2 - \sum_i \frac{1}{2} (G_i')^2,$$  \hspace{1cm} (5.24)

$$G_i = \frac{1}{\sqrt{\xi}} \left[ \partial_\mu A_i^\mu - \xi (-m_W \chi^i + \partial_5 A_5^i) \right],$$  \hspace{1cm} (5.25a)

$$G_Y = \frac{1}{\sqrt{\xi}} \left[ \partial_\mu B_\mu - \xi (s_W m_W \chi^3 + \partial_5 B_5^i) \right].$$  \hspace{1cm} (5.25b)

which is a generalization of the $R_\xi$ gauge [3]. These are manifestly five-dimensional Lorentz breaking terms. However, this should not be worrying since in the effective theory we are restricted to four-dimensional Poincaré transformations anyway, and the above expressions are still manifestly four-dimensionally Lorentz invariant. If we add up the scalar contributions from the gauge bosons in Eq. (5.9), the kinetic part of the Higgs sector in Eq. (5.17) and the gauge fixing terms in Eq. (5.24), and then integrate over the internal dimensions, the result is:

$$\hat{L}^{(\text{kin})}_{\text{scalar}} \equiv \sum_{n=0}^{\infty} \left\{ \frac{1}{2} \left( \partial_\mu h^{(n)} \partial^\mu h^{(n)} - M_h^{(n)} h^{(n)} \right)^2 + \frac{1}{2} \left( \partial_\mu G_0^{(n)} \partial^\mu G_0^{(n)} - \xi M_Z^{(n)} G_0^{(n)} \right)^2 + \left( \partial_\mu G_+^{(n)} \partial^\mu G_-^{(n)} - \xi M_W^{(n)} G_+^{(n)} G_-^{(n)} \right) \right\} + \ldots
\[
\sum_{n=1}^{\infty} \left\{ \frac{1}{2} \left( \partial_\mu a_0^{(n)} \partial^\mu a_0^{(n)} - M_x^{(n)} a_0^{(n)} \right) \right. \\
+ \left( \partial_\mu a_+^{(n)} \partial^\mu a_-^{(n)} - M_w^{(n)} a_+^{(n)} a_-^{(n)} \right) \\
+ \left. \frac{1}{2} \left( \partial_\mu A_5^{(n)} \partial^\mu A_5^{(n)} - \frac{\xi n^2}{R^2} A_5^{(n)} \right) \right\}. \tag{5.26}
\]

The above appearing mass eigenstates in the four-dimensional theory are given by:

\[a_0^{(n)} = \frac{M^{(n)}}{M_z^{(n)}} \chi_3^{(n)} + \frac{M_z}{M_z^{(n)}} Z_5^{(n)}, \tag{5.27a}\]

\[a^{\pm(n)} = \frac{M^{(n)}}{M_w^{(n)}} \chi^{\pm(n)} + \frac{M_w}{M_w^{(n)}} W_5^{\pm(n)}, \tag{5.27b}\]

\[G_0^{(n)} = \frac{M_z}{M_z^{(n)}} \chi^{3(n)} - \frac{M^{(n)}}{M_z^{(n)}} Z_5^{(n)}, \tag{5.27c}\]

\[G^{\pm(n)} = \frac{M_w}{M_w^{(n)}} \chi^{\pm(n)} - \frac{M^{(n)}}{M_w^{(n)}} W_5^{\pm(n)}, \tag{5.27d}\]

where

\[M_{X}^{(n)} = \sqrt{m_0^2 + \left( \frac{n}{R} \right)^2}, \quad M^{(n)} = \frac{n}{R}. \tag{5.28}\]

Now, let us count the number of physical degrees of freedom in the bosonic sector. At KK zero-level, we recover the SM with one physical Higgs field \(h^{(1)}\) together with the three massive gauge fields \(Z^{(0)}, W^{\pm(0)}\) that have eaten the three degrees of freedom from the three unphysical Goldstone bosons \(G^{(0)}_{0,\pm}\). At each higher KK mode, there are four additional scalar degrees of freedom coming from the fifth components of the gauge bosons. These scalars form, together with four KK mode scalars from the Higgs doublet, four physical scalars \(h^{(n)}, a_0^{(n)}\) and \(a^{\pm(n)}\) and four unphysical Goldstone bosons \(A_5^{(n)}, G_0^{(n)}\) and \(G^{(n)}_{\pm}\) that have lost their physical degrees of freedoms to the massive KK vector modes \(A_\mu^{(n)}, Z_\mu^{(n)}\) and \(W_\mu^{\pm(n)}\), respectively.

**Ghosts**

In the case of non-Abelian vector fields, or when Abelian vector fields acquire masses by spontaneous symmetry breaking, the gauge-fixing terms in Eq. (5.24) are accompanied by an extra Faddeev-Popov ghost Lagrangian term [3]. These ghost fields can be interpreted as negative degrees of freedom that serve to cancel the effects of unphysical timelike and longitudinal polarization states of gauge bosons. The ghost Lagrangian is determined from the
gauge fixing terms in Eq. (5.24) as

\[ \hat{L}_{\text{ghost}} = -\bar{c}^a \frac{\delta G^a}{\delta \alpha^b} c^b, \quad (5.29) \]

where \( a, b \in \{ i = 1, 2, 3, Y \} \). The ghost fields \( c^a \) are complex, anticommuting Lorentz scalars. The bar in the expression denotes Hermitian conjugation. The ghosts are set to be even under the \( \mathbb{Z}_2 \) orbifold. The functional derivatives in Eq. (5.29) are found by studying infinitesimal gauge transformations:

\[ \delta A^i_M = \frac{1}{\hat{g}} \partial^\alpha_i \alpha^i + \epsilon^{ijk} A^j_M \alpha^k, \quad (5.30) \]
\[ \delta B_M = \frac{1}{\hat{g}_Y} \partial_M \alpha^Y, \quad (5.31) \]

and

\[ \delta \phi = \left[ \frac{i \alpha^Y}{2} + \frac{i \alpha^i}{2} \right] \phi = \frac{1}{\sqrt{2}} \left( \delta \chi^2 + i \delta \chi^1 \right), \quad (5.32) \]

with

\[ \delta \chi^1 = \frac{1}{2} \left[ \alpha^1 H - \alpha^2 \chi^3 + \alpha^3 \chi^2 + \alpha^Y \chi^2 \right], \quad (5.33a) \]
\[ \delta \chi^2 = \frac{1}{2} \left[ \alpha^1 \chi^3 + \alpha^2 H - \alpha^3 \chi^1 - \alpha^Y \chi^1 \right], \quad (5.33b) \]
\[ \delta \chi^3 = \frac{1}{2} \left[ -\alpha^1 \chi^2 + \alpha^2 \chi^1 + \alpha^3 H - \alpha^Y H \right]. \quad (5.33c) \]

The ghost Lagrangian then becomes, after the ghost fields have been rescaled according to \( c^a \rightarrow (\hat{g}_a \sqrt{\xi})^{1/2} c^a, \)

\[ -\bar{c}^a \frac{\delta G^a}{\delta \alpha^b} c^b = \bar{c}^a \left[ -\partial^2 \delta^a b - \xi (M^{ab} - \partial^2_5 \delta^{ab}) \right] c^b \]
\[ + \bar{c}^a \left[ \hat{g} \epsilon^{ijk} \delta^{kb} \delta^{ia} (\xi \partial_5 A^j_M - \partial^\mu A^j_M) - \xi \frac{\hat{g}^2 \hat{g}^b v}{4} I^{ab} \right] c^b, \quad (5.34) \]

where

\[ I = \begin{pmatrix} h & -\chi^3 & \chi^2 & \chi^2 \\ \chi^3 & h & -\chi^1 & -\chi^1 \\ -\chi^2 & \chi^1 & h & -h \\ \chi^2 & -\chi^1 & -h & h \end{pmatrix}, \quad M = \frac{\hat{g}^2}{4} \begin{pmatrix} \hat{g}^2 & 0 & 0 & 0 \\ 0 & \hat{g}^2 & 0 & 0 \\ 0 & 0 & \hat{g}^2 & -\hat{g} \hat{g}_Y \\ 0 & 0 & \hat{g}_Y & \hat{g}^2 \end{pmatrix}. \]

Integrating out the extra dimensions, the kinetic part of the ghost Lagrangian becomes

\[ L_{\text{ghost}}^{(\text{kin})} = \sum_{n=0}^{\infty} \bar{c}^{a(n)} \left\{ -\partial^2 \delta^{ab} - \xi M^{ab(n)} \right\} c^{b(n)}, \quad (5.35) \]
where the mass matrix $M^{ab(n)}$ now is given by

$$
M^{(n)} = \begin{pmatrix}
M_{W}^{(n)} & 0 & 0 & 0 \\
0 & M_{W}^{(n)} & 0 & 0 \\
0 & 0 & M_{W}^{(n)} & -\frac{1}{4}v^2ggY \\
0 & 0 & -\frac{1}{4}v^2ggY & \frac{1}{4}v^2g_Y^2 + \left(\frac{\eta}{\sqrt{2}}\right)^2
\end{pmatrix}.
$$

(5.36)

The same rotation as in Eq. (5.20)-(5.21) diagonalizes this matrix and the ghosts mass eigenstates become $c^\pm \equiv \frac{1}{\sqrt{2}}(c^1 \mp c^2)$, $c^\gamma$, and $c^\gamma$, with the corresponding masses $\sqrt{\xi}M_{W}^{(n)}$, $\sqrt{\xi}M_{Z}^{(n)}$ and $\sqrt{\xi}M_{\gamma}^{(n)}$.

As for the Goldstone bosons, the gauge dependent masses indicate the unphysical nature of these fields.

**Fermions**

Fermions can appear both as singlets and as components of doublets under $SU(2)$ transformations. Furthermore, in the massless limit fermions have a definite chirality in even spacetime dimensions. In the SM, all fermions in doublets $\psi^{(0)}_d$ have, from observation, negative chirality, whereas all singlets $\psi^{(0)}_s$ have positive chirality:

$$
\gamma^5\psi^{(0)}_d = \psi^{(0)}_d \quad \text{and} \quad \gamma^5\psi^{(0)}_s = -\psi^{(0)}_s,
$$

(5.37)

where $\gamma^5 \equiv i\Gamma^0\Gamma^1\Gamma^2\Gamma^3$ is the four-dimensional chirality operator constructed from the generators of the Clifford algebra. The $d$-dimensional Clifford algebra reads:

$$
\{\Gamma^M, \Gamma^N\} = 2\eta^{MN}.
$$

(5.38)

For an even number of spacetime dimensions ($d = 2k + 2$) the fundamental representation of the Clifford algebra’s generators are $2^{k+1} \times 2^{k+1}$-matrices, whereas for odd spacetime dimensions ($d = 2k + 3$) they are the same matrices as for the case of one less spacetime dimension and with the addition of

$$
\Gamma^{2k+2} \equiv -i^{1+k}\Gamma^0\Gamma^1 \ldots \Gamma^{2k+1}
$$

(5.39)

Note that our four-dimensional chirality operator $\gamma^5$ is the same as $i\Gamma^4$ in five dimensions, i.e., the chirality operator $\gamma^5$ is a part of the Clifford algebra’s generators in five dimensions.

For an even number of spacetime dimensions we can always reduce the Dirac representation into two inequivalent Weyl representations, characterized by their spinors’ chirality. For odd spacetime dimensions, this is not possible, since the $\Gamma^M$ matrices then form an irreducible Dirac representation and they

\[\text{Note that } c^+ \text{ and } c^- \text{ are not each other’s Hermitian conjugates.}\]
mix under Lorentz transformations. However, we can still always artificially split up spinors with respect to their four-dimensional chirality operator. That is

$$\psi = P_R \psi + P_L \psi \equiv \psi_R + \psi_L .$$  \hspace{1cm} (5.40)

where $P_{R,L}$ are the chirality projection operators:

$$P_{R,L} \equiv \frac{1}{2} \left( 1 \pm \gamma^5 \right) , \quad P_{R,L}^2 = P_{R,L} , \quad P_L P_R = P_R P_L = 0 .$$  \hspace{1cm} (5.41)

Five-dimensional Lorentz transformations mix $\psi_R$ and $\psi_L$, but if restricted to only four-dimensional Lorentz transformations, as in the effective KK theory, they do not mix. Therefore it makes sense to assign different orbifold projections depending on a fermion’s four-dimensional chirality when constructing an effective four-dimensional model. To recover the chiral structure of the SM, the fermion fields should thus be assigned the following orbifold properties:

$$P_{Z^2} \psi_d(y) = -\gamma^5 \psi_d(-y) \quad \text{and} \quad P_{Z^2} \psi_s(y) = \gamma^5 \psi_s(-y) .$$  \hspace{1cm} (5.42a)

With these orbifold projections, the doublets and singlets have the following expansions in KK modes

$$\psi_d = \frac{1}{\sqrt{2\pi R}} \psi_{d_L}^{(0)} + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left( \psi_{d_L}^{(n)} \cos \frac{n y}{R} + \psi_{d_R}^{(n)} \sin \frac{n y}{R} \right) ,$$  \hspace{1cm} (5.43a)

$$\psi_s = \frac{1}{\sqrt{2\pi R}} \psi_{s_R}^{(0)} + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left( \psi_{s_R}^{(n)} \cos \frac{n y}{R} + \psi_{s_L}^{(n)} \sin \frac{n y}{R} \right) ,$$  \hspace{1cm} (5.43b)

where (as wanted) the zero mode doublets are left-handed and the zero mode singlets are right-handed.

The fermion Lagrangian has the following structure:

$$\hat{\mathcal{L}}_{\text{fermion}} = i \begin{pmatrix} \bar{\psi}_{d,U} & \bar{\psi}_{d,D} \end{pmatrix} \not\!D \begin{pmatrix} \psi_{d,U} \\ \psi_{d,D} \end{pmatrix} + \hat{\mathcal{L}}_{\text{Yukawa}} ,$$  \hspace{1cm} (5.44)

where $U$ and $D$ denote up-type ($T_3 = +1/2$) and down-type ($T_3 = -1/2$) fermions in the $SU(2)$ doublet (where $T_3$ is the eigenvalue to the $SU(2)$ generator $\sigma_3$ operator), respectively, and the covariant derivative is

$$\not\!D \equiv \not\!\partial_M \left( \partial_M - i \hat{g} A_M^{\alpha} \frac{\sigma_\alpha}{2} - i Y \hat{g}_Y B_M \right) ,$$  \hspace{1cm} (5.45)

where $Y \in \{ Y_{d,U}, Y_{d,D}, Y_s \}$ is the hypercharge of the fermion in question. Note that the $A_M^{\alpha}$ term is absent for the singlet part, since $\psi_s$ does not transform under $SU(2)$ rotations.

The fermions have Yukawa-couplings to the Higgs field,

$$\hat{\mathcal{L}}_{\text{Yukawa}} = -\hat{\lambda}_D \left[ (\bar{\psi}_{d,U}, \bar{\psi}_{d,D}) \cdot \phi \right] \psi_{s,D} - \hat{\lambda}_U \left[ (\bar{\psi}_{d,U}, \bar{\psi}_{d,D}) \cdot \phi \right] \psi_{s,U} + \text{h.c.} ,$$  \hspace{1cm} (5.46)
from which, after the spontaneous symmetry breaking in the Higgs sector, the fermions get their electroweak fermions masses. The conjugate of the Higgs doublet is defined by \( \tilde{\phi}_a \equiv \epsilon_{ab}\phi_b^\dagger \). The electroweak masses will become 

\[
m^{U,D}_{\text{EW}} = \left( \lambda^{U,D} \hat{v} \right)/\sqrt{2}.
\]

When several generations of quarks are present, there can be couplings that mix quark generations. It is still always possible to diagonalize the Higgs couplings, which is also the base that diagonalizes the mass matrix. However, this diagonalization causes complications in the gauge couplings regarding quarks and will lead to the need for introducing a quark-mixing matrix \( V^{ij} \).

Let \( \psi_{d,U} = (\psi_{d,U}^u, \psi_{d,U}^c, \psi_{d,U}^t) \) and \( \psi_{d,D} = (\psi_{d,D}^u, \psi_{d,D}^c, \psi_{d,D}^t) \).

\[
\psi_{d,U}^i = (\psi_{d,U}^u, \psi_{d,U}^c, \psi_{d,U}^t)
\quad \text{and} \quad
\psi_{d,D}^i = (\psi_{d,D}^u, \psi_{d,D}^c, \psi_{d,D}^t),
\]

where \( V \equiv U^\dagger U_U U^\dagger U_D \) is known as the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, \( \sigma_\pm = (\sigma_1 \pm i\sigma_2) \) are the usual step operators in \( SU(2) \), and the charge \( Q \) is related to the weak isocharge and the hypercharge by \( Q = T_3 + Y \). For the terms containing the photon and the \( Z \) boson, the unitary matrices \( U_U \) and \( U_D \) effectively disappear because they appear only in the combination \( U^\dagger U = 1_{3\times3} \). In the absence of right-handed neutrinos \( (\psi_{s,U}) \) the CKM matrix can also be made to vanish for the couplings to the \( W_\pm \) and the replacement \( V_{ij} \rightarrow \delta_{ij} \) should be done in the leptonic sector [3].

Let us now finally integrate out the extra dimension and see how the KK masses arise from the kinetic term:

\[
\int_0^{2\pi R} dy \left( i\bar{\psi}_d \gamma^\mu \partial_\mu \psi_d + i\bar{\psi}_s \gamma^\mu \partial_\mu \psi_s \right) = i\bar{\psi}^{(0)} \gamma^\mu \partial_\mu \psi^{(0)}
\]

\[
\quad + \sum_{n=1}^{\infty} \left[ \bar{\psi}_d^{(n)} \left( i\gamma^\mu \partial_\mu - \frac{n}{R} \right) \psi_d^{(n)} + \bar{\psi}_s^{(n)} \left( i\gamma^\mu \partial_\mu + \frac{n}{R} \right) \psi_s^{(n)} \right],
\]

where

\[
\psi^{(0)} \equiv \psi^{(0)}_{dL} + \psi^{(0)}_{sR}, \quad \psi^{(n)}_d \equiv \psi^{(n)}_{dL} + \psi^{(n)}_{dR}, \quad \psi^{(n)}_s \equiv \psi^{(n)}_{sL} + \psi^{(n)}_{sR}.
\]
Hence, for each SM fermion $\psi^{(0)}$ there are two fermions at each KK level $\psi_{s}^{(n)}$ and $\psi_{d}^{(n)}$ (this can also be seen directly from Eq. (5.43)). Note that the singlet fields get the ‘wrong’ sign for their KK masses. Adding the Yukawa masses $m_{\text{EW}}$ to the KK masses, the mass matrix becomes

$$
\begin{pmatrix}
\bar{\psi}_{d}^{(n)} & \bar{\psi}_{s}^{(n)} \\
\frac{n}{R} + \delta m_{d}^{(n)} & m_{\text{EW}} \\
m_{\text{EW}} & -\frac{n}{R} + \delta m_{s}^{(n)}
\end{pmatrix}
\begin{pmatrix}
\psi_{d}^{(n)} \\
\psi_{s}^{(n)}
\end{pmatrix},
$$

where $\delta m_{d}^{(n)}$ and $\delta m_{s}^{(n)}$ are additional radiative loop corrections that appear (these corrections will be further discussed in Section 5.5). By a rotation, and a correction for the ‘wrong’ sign of the singlets’ mass terms, the following mass eigenstates are found:

$$
\begin{pmatrix}
\xi_{d}^{(n)} \\
\xi_{s}^{(n)}
\end{pmatrix} =
\begin{pmatrix}
1 & 0 \\
0 & -\gamma^{5}
\end{pmatrix}
\begin{pmatrix}
\cos \alpha^{(n)} & \sin \alpha^{(n)} \\
-\sin \alpha^{(n)} & \cos \alpha^{(n)}
\end{pmatrix}
\begin{pmatrix}
\psi_{d}^{(n)} \\
\psi_{s}^{(n)}
\end{pmatrix},
$$

where the mixing angle is

$$
\tan 2\alpha^{(n)} = \frac{2m_{\text{EW}}}{2\frac{n}{R} + \delta m_{d}^{(n)} + \delta m_{s}^{(n)}}. \quad (5.54)
$$

The physical masses for these states are

$$
m_{d,s}^{(n)} = \pm \frac{1}{2}(\delta m_{d}^{(n)} - \delta m_{s}^{(n)}) + \sqrt\left[\frac{n}{R} + \frac{1}{2}(\delta m_{d}^{(n)} + \delta m_{s}^{(n)})\right]^2 + m_{\text{EW}}^2, \quad (5.55)
$$

For all fermions, except for the top quark, the mixing angles $\alpha^{(n)}$ are very close to zero.

### 5.4 Particle Propagators

The five-dimensional propagators get modified by the orbifold compactification. There are two differences compared to an infinite Lorentz invariant spacetime. The first is trivial and comes from the compactification itself, which implies that the momentum in the extra dimension is quantized and is given by $p_{y} = n/R$, where $n$ takes only integer values. The other is related to the reflection symmetry of the orbifold boundaries, which breaks momentum conservation in the direction of the extra dimension and therefore allows for a sign flip of $p_{y}$ in the propagator.

By the introduction of unconstrained auxiliary fields, the momentum space propagators can be found [195, 196]. For example, a scalar field $\Phi$ can be expressed in terms of an unconstrained auxiliary field $\chi$ as:

$$
\Phi(x^{\mu},y) = \frac{1}{2} (\chi(x^{\mu},y) \pm \chi(x^{\mu},-y)), \quad (5.56)
$$
where the ±-sign depends on whether the scalar is constrained to be even or odd under the orbifold transformation. Note that since the unconstrained fields contain both $y$ and $-y$, the propagator $\langle \Phi(x^n, y)\Phi(x'^n, y') \rangle$ will (after a Fourier transformation into momentum space) depend on $p_y - p'_y$, as well as $p_y + p'_y$. The five-dimensional propagator for a massless scalar field becomes [195, 196]

\[
\langle \Phi(p\mu, p_y)\Phi^*(p'\mu, y) \rangle = i \left( \frac{\delta_{pp_y} \pm \delta_{p_y p'_y}}{p^2 - p'^2} \right) .
\]

In the effective four-dimensional theory, the propagators for the KK particles take the usual form (see also Appendix A):

\[
\langle \Phi(p\mu)\Phi^*(p'\mu) \rangle = \frac{i}{p^2 - m^2} ,
\]

where $m$ is the mass of the KK particle.

### 5.5 Radiative Corrections

A main characteristic of KK theories is that at each KK level the particle states are almost degenerate in mass. This is true even after generation of electroweak masses $m_{EW}$, at least as long as the compactification scale $1/R$ is much larger than $m_{EW}$. The mass degeneracy implies that all momentum-conserving decays are close to threshold and radiative corrections will determine if certain decay channels are open or not. For cosmology, these loop corrections are essential in that they determine which of the fields is the lightest KK particle (LKP) and hence if the model has a natural dark matter candidate.

For example, in the massless limit of five-dimensional QED, the reaction

\[
e^{(1)} \rightarrow e^{(0)} + \gamma^{(1)}
\]

is exactly marginal at three level. After inclusion of the electroweak electron mass $m_e$, the reaction becomes barely forbidden as $M_e^{(1)} \equiv \sqrt{1/R^2 + m_e^2} < 1/R + m_e$. Radiative mass corrections are, however, naïvely expected to be larger than $m_{EW}$, because they are generically of the order of $\alpha \sim 10^{-2}$, which is much larger than $m_{EW}/m^{(n)}$ (which ranges from $10^{-12}$ for electrons to $10^{-2}$ for the top quarks at the first KK level). The study in [196] of one-loop radiative corrections show that radiative corrections generically open up decay channels, like the one in Eq. (5.59), so that all first-level KK modes can decay into the LKP and SM particles.

Radiative corrections to masses arise from Feynman loop diagrams contributing to the two-point correlation functions. These contributions can in the UED model be artificially split into three types: (1) five-dimensional Lorentz invariant loops (2) winding modes and (3) orbifold contributions.
I will only briefly outline what goes into the calculation of the radiative mass corrections (the interested reader is referred to [196], and references therein). The five-dimensional dispersion relation reads

\[ p^\mu p_\mu = m^2 + Z_5 p_y^2 = m_{\text{phys}}^2 + Z_5 / Z m_{(n)}^2 , \]  

(5.60)

where \( Z \) and \( Z_5 \) are potential radiative quantum corrections. In the SM, the (divergent) quantum correction \( Z \) is absorbed into the physically observed masses \( m_{\text{phys}} \). Hence, any extra mass corrections to the KK masses must come from extra contributions to \( Z_5 \). In general, both \( Z \) and \( Z_5 \) receive (divergent) loop radiative quantum corrections, but in a five-dimensional Lorentz-invariant theory they are protected to stay equal, \( Z = Z_5 \), to preserve the usual dispersion relation. At short distances (away from the orbifold fixpoints), the compactification is actually not perceptible, and localized loops should therefore preserve Lorentz invariance. In fact, all five-dimensional Lorentz invariance preserving self-energy contributions can therefore be absorbed into the renormalization of the zero mode mass \( m \to m_{\text{phys}} / Z \) so these do not give any additional radiative mass correction to the KK modes.

Although local Lorentz invariance still holds under a \( S^1 \) compactification, global Lorentz invariance is broken. Such non-local effects appear in those Feynman diagrams that have an internal loop that winds around the compact dimension, as shown in Fig. 5.1. The radiative corrections from such winding propagators can be isolated by the following procedure. Because of the \( S^1 \) compactification, the momenta \( p_y \) in the compact dimension are quantized in units of \( 1/R \), and therefore the phase space integral over internal loop momenta reduces to a sum over KK levels. This sum can be translated into a sum over net winding \( n \) around the compactified dimension, where the first term with \( n = 0 \) exactly corresponds to an uncompactified five-dimensional theory. The non-winding loops \( (n = 0) \) can be dealt with in the same way as described in the previous paragraph. The sum of the remaining winding modes \( (n \neq 0) \) turns out to only give finite and well-defined radiative corrections to each KK mass.
Finally, the third kind of radiative correction appears due to the orbifold compactification $\mathbb{S}^1/\mathbb{Z}_2$. This is a local effect, caused by the orbifold’s fixpoints that break translational invariance in the $y$-direction. As shown in Section 5.4, this leads to modified five-dimensional propagators. Thus we must redo the two previously described calculations, but now instead with the correct orbifold propagators. The finite contribution stated in the previous paragraph remains the same (although divided by two because the orbifold has projected out half of the states), but also new, logarithmically divergent terms localized at the orbifold fixpoints appear. This means that counter terms should be included at the boundaries to cancel these divergences and that these calculations strictly speaking determine only the running behavior. In order to keep unknown contributions at the cutoff scale $\Lambda_{\text{cut}}$ under control, they are assumed to be small at that high energy cutoff scale—a self-consistent assumption since the boundary terms are generated only by loop corrections. If large boundary terms were present, they could induce mixing between different KK modes [196].

5.6 Mass Spectrum

Including these radiative corrections, we can calculate the mass spectra of the KK particles. In most cases, the electroweak mass can be ignored, with the prominent exceptions of the heavy Higgs and gauge bosons, and the top quark. In the case of fermions, these radiative corrections were already included in the fermionic mass matrix Eq. (5.32). Similarly this can be done also for the bosons. Let us take a closer look at the cosmologically important mass matrix for the neutral gauge bosons. In the $B, A^3$ basis, including radiative corrections $\delta m_B^{(n)}$ and $\delta m_{A^3}^{(n)}$, the mass matrix reads

$$
\begin{pmatrix}
\left(\frac{R}{R}\right)^2 + \delta m_B^{2(n)} + \frac{1}{4}v^2 g_Y^2 \\
-\frac{1}{4}v^2 g g_Y & \left(\frac{R}{R}^2 + \delta m_{A^3}^{2(n)} + \frac{1}{4}v^2 g^2\right)
\end{pmatrix}
$$

By diagonalizing this matrix, the physical KK photon mass and $Z$-boson mass can be found. The $n^{\text{th}}$ KK level ‘Weinberg’ angle for the diagonalizing rotation is given by

$$
\tan 2\theta^{(n)} = \frac{v^2 g g_Y}{2 \left[\delta m_{A^3}^{2(n)} - \delta m_B^{2(n)} + \frac{v^2}{4} (g^2 - g_Y^2)\right]}
$$

For small compactifications scales, this Weinberg angle is driven to zero since generally $\delta m_{A^3}^{2(n)} - \delta m_B^{2(n)} \gg v^2 g g_Y$. For example, for $R^{-1} \gtrsim 500$ GeV and $\Lambda_{\text{cut}} R \gtrsim 20$, $\theta^{(n)}$ is less than $10^{-2}$. Therefore, the KK photon $\gamma^{(n)}$ is very well approximated by the weak hypercharge gauge boson $B^{(n)}$ and these two states are often used interchangeably.

Figure 5.2 shows the spectrum for the first KK excitations of all SM particles, both at (a) tree level and (b) including one-loop radiative corrections.
We find that the lightest SM KK particle is the first KK level photon $\gamma^{(1)}$. Since unbroken KK parity $(-1)^n$ guarantees the LKP to be stable, it provides a possible dark matter candidate. Due to the loop corrections, the mass degeneracy will be lifted enough so that all other first KK level states will promptly cascade down to the $\gamma^{(1)}$.

Instead of the minimal UED model – defined by using the described one-loop expressions, with boundary terms negligible at the cutoff scale $\Lambda_{\text{cut}}$ (unless specified otherwise, $\Lambda_{\text{cut}} = 20R^{-1}$) – you could take a more phenomenological approach and treat all the KK mass corrections as independent input to the theory (see, e.g., [197]). With such an approach, other KK particles than the $\gamma^{(1)}$ can become the LKP. With the $\gamma^{(1)}$ not being the LKP, the LKP still has to be electrically neutral to be a good dark matter candidate [198, 199]. Therefore, the only other potentially attractive LKP options are $h^{(1)}$, $Z^{(1)}$, $\nu^{(1)}$, $g^{(1)}$ and if going beyond SM particles also the first KK level of the graviton. At least naïvely, some of these can be directly excluded: the high electroweak Higgs mass naturally excludes the Higgs particles; the gluons are color charged and thus excluded by their strong interactions [199]; the $Z^{(1)}$ is expected to be heavier than $\gamma^{(1)}$ since usually both electroweak and one-loop order correction contribute to the mass matrix in a way that
the lighter state is almost $B^{(1)}$ \textit{(i.e., $\gamma^{(1)}$)}. When it comes to the graviton, it is interesting to note that for $R^{-1} \lesssim 800$ GeV the first KK graviton $G^{(1)}$ in the minimal UED model becomes lighter than the $\gamma^{(1)}$. This result is under the reasonable assumption that the radiative mass corrections to the graviton is very small, since it only couples gravitationally, and therefore the mass of $G^{(1)}$ is well approximated by $R^{-1}$. The $G^{(1)}$ will be a superweakly interacting particle, as it interacts only gravitationally. Therefore, it is hard, if not impossible, to detect it in conventional dark matter searches. However, the presence of $G^{(1)}$ could cause effects both on the BBN and the CMB radiation predictions [200] (see also [201–203]). In the case of a KK neutrino, thermal relic calculations show that the favored mass range is between 0.8 TeV and 1.3 TeV [197]. This is in strong conflict with direct detection searches that exclude $\nu^{(1)}$ masses below 1000 TeV [204, 205] [206, 207].

For the remaining chapters, $\gamma^{(1)} \approx B^{(1)}$ will be taken to be the LKP.
Numerous phenomenological constraints must be fulfilled in order to make a particle a viable dark matter candidate. In the UED model, it turns out that the first Kaluza-Klein excitation of the photon $\gamma^{(1)}$ is an excellent dark matter candidate. If $\gamma^{(1)}$ ($\approx B^{(1)}$) is the lightest Kaluza-Klein state and its mass is about 1 TeV, then this candidate can make up all the dark matter. It is therefore of importance to find direct and indirect ways to detect such a candidate. The properties of this Kaluza-Klein dark matter particle are discussed, and observational constraints are reviewed. The chapter concludes by discussing the results of Paper II and Paper III, where characteristic indirect detection signals were predicted from pair-annihilation of these dark matter particles. So-called final state radiation, producing very high energy gamma rays, and a monochromatic gamma-ray line, with an energy equal to the mass of the dark matter particle, produce distinctive signatures in the gamma spectrum.

### 6.1 Relic Density

In the early Universe, the temperature $T$ was higher than the compactification scale ($T > R^{-1} \sim 1$ TeV), and KK particles were freely created and annihilated and kept in thermal and chemical equilibrium by processes like:

$$X_i^{(n)} X_j^{(n)} \leftrightarrow x_k x_l$$  \hspace{1cm} (6.1a)

$$X_i^{(n)} x_j \leftrightarrow X_k^{(n)} x_l$$  \hspace{1cm} (6.1b)

$$X_i^{(m)} \rightarrow X_j^{(n)} x_k \ldots$$  \hspace{1cm} (6.1c)

where $X_i^{(n)}$ is the $n$th-level excitation of a SM particle $x_i$ ($i, j, k, l = 1, 2, \ldots$). Processes of the type (6.1a) are usually referred to as annihilation when $i = j$ and coannihilation when $i \neq j$ (in fact, coannihilation processes usually refer
to all pair annihilations that not exclusively include the dark matter candidate under consideration).

As the Universe expands, the temperature drops and the production of KK states soon becomes energetically suppressed, leading to their number density dropping exponentially. This happens first for the heavier KK particles, which by inelastic scattering with numerous SM particles (Eq. (6.1b)) and decays (Eq. (6.1c)) transform into lighter KK particles and SM particles. Due to conservation of KK parity \((-1)^n\) (\(n\) being the KK level), the lightest of the KK particles is not allowed to decay and can therefore only be destroyed by pairwise annihilation (or potentially by coannihilation if there is a sufficient amount of other KK particles around). Hence, at some point the LKPs annihilation rate cannot keep up with the expansion rate, the LKP density leaves its chemical equilibrium value, and their comoving number density freezes. What is left is a thermal remnant of non-relativistic LKPs that act as cold dark matter particles. That only the LKPs contribute to the dark matter content today is of course only valid under the assumption that all level-1 KK states are not exactly degenerate in mass and can decay into the LKP. Generically this is the case for typical mass splittings, as in e.g., the minimal UED model at the one-loop level.

As stated in the previous chapter, the lightest of the KK particles is in the minimal UED model the first KK excitation of the photon \(\gamma^{(1)}\), which to a good approximation is equal to \(B^{(1)}\). In practice, there is a negligible difference between using \(\gamma^{(1)}\) and \(B^{(1)}\) as the LKP (See Section 5.6). Henceforth, I switch notation from \(\gamma^{(1)}\) to \(B^{(1)}\) for the KK dark matter particle, as this is the state actually used in the calculations.

Quantitatively, the number density of a dark matter particle is described by the Boltzmann equation [208, 209]:

\[
\frac{dn}{dt} + 3Hn = \langle \sigma_{\text{eff}} v \rangle \left[ n^2 - (n^{\text{eq}})^2 \right], \tag{6.2}
\]

where \(n = \sum n_i\) is the total number density including both the LKP and heavier states that will decay into the LKP, \(H\) is the Hubble expansion rate and \(n^{\text{eq}}\) the sum of the chemical equilibrium number densities for the KK particles, which in the non-relativistic limit are given by

\[
n^{\text{eq}}_{X_i^{(1)}} \simeq g_i \left( \frac{m_{X_i^{(1)}} T}{2\pi} \right)^{3/2} \exp \left( \frac{-m_{X_i^{(1)}}}{T} \right), \tag{6.3}
\]

where \(g_i\) is the internal number of degrees of freedom for the particle in question. Finally, the effective annihilation (including coannihilation) cross section times the relative velocity (or more precisely, the Møller velocity [210]) of the annihilating particles \(\langle \sigma_{\text{eff}} v \rangle\) is given by

\[
\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j} \left( \frac{n_i^{\text{eq}} n_j^{\text{eq}}}{n^{\text{eq}} n^{\text{eq}}} \right) \langle \sigma_{ij} v_{ij} \rangle \tag{6.4}
\]
where the thermal average is taken for the quantity within brackets $\langle \rangle$.

In general, a lower effective cross section means that the comoving number density freezes out earlier, when the density is higher, and therefore leaves a larger relic abundance today. A high effective cross section, on the other hand, means that the particles under consideration stay in chemical equilibrium longer and the number density gets suppressed. The exact freeze-out temperature and relic densities are determined by solving Eq. (6.2), but a rough estimate is that the freeze-out occurs when the annihilation rate $\Gamma = n \langle \sigma v \rangle$ fall below the Hubble expansion $H$. As mentioned in Chapter 1, a rule of thumb is that the relic abundance is given by

$$\Omega_{\text{WIMP}} h^2 \approx 3 \cdot 10^{-27} \text{ cm}^{-3} \text{ s}^{-1} \langle \sigma_{\text{eff}} v \rangle.$$  \hfill (6.5)

At freeze-out, typically around $T \sim m_{\text{WIMP}} / 25$, there are roughly $10^{10}$ SM particles per KK excitation that by reaction (6.1b) can keep the relative abundance among KK states in chemical equilibrium ($n_i / n_j = n_i^{\text{eq}} / n_j^{\text{eq}} \propto \exp(-\Delta M / T)$). Coannihilation is therefore only of importance when the mass difference between the LKP and the other KK states is smaller or comparable to the freeze-out temperature. Since the masses of the KK states are quasi-degenerated by nature, we could expect that coannihilation are especially important for the UED model. In general, the presence of coannihilations can both increase and decrease relic densities. In the case of UED, we could also expect that second-level KK states could significantly affect the relic density if the appear in s-channel resonances, see [211, 212].

To see how differences in the KK mass spectrum affect the relic density, let us take a look at three illustrative examples. These cases set the mass range for which $B^{(1)}$ is expected to make up the observed dark matter.

First, consider a case where we artificially allow only for self-annihilation. This would mimic the case when all other KK states are significantly heavier than the LKP. The effective cross section in Eq. (6.4) then reduces to $\sigma_{\text{eff}} = \sigma_{B^{(1)}, B^{(1)}}$. The relic density dependence on the compactification scale, or equivalently the LKP mass, is shown by the dotted line in Fig. 6.1. In this case the $B^{(1)}$ mass should be in the range 700 GeV to 850 GeV to coincide with the measured dark matter density.

Imagine, as a second case, that coannihilation with the strongly interacting KK quarks and gluons are important. This can potentially cause a strong enhancement of the effective cross section if these other states are not much heavier than the LKP. Physically, this represents the case when other KK particles deplete the LKP density by keeping it longer in chemical equilibrium through coannihilations. Thus this effect will allow for increased $B^{(1)}$ masses. As an illustrative example, take the mass spectrum to be that of the minimal UED model, except that the KK gluon mass $m_{g^{(1)}}$ is treated a free parameter. For a mass difference $\Delta g^{(1)} \equiv (m_{g^{(1)}} - m_{B^{(1)}}) / m_{B^{(1)}}$ as small as 2% an allowed relic density can be obtained for $B^{(1)}$ masses up to about 2.5 TeV (see the
Figure 6.1: Relic density of the lightest Kaluza-Klein state $B^{(1)}$ as a function of the inverse compactification radius $R^{-1}$. The dotted line is the result from considering $B^{(1)}B^{(1)}$ annihilation only. The dashed line is the when adding coannihilation with the Kaluza-Klein gluon $g^{(1)}$, using a small mass split to the $B^{(1)}$: $\Delta_{g^{(1)}} \equiv (m_{g^{(1)}} - m_{B^{(1)}}) / m_{B^{(1)}} = 0.02$. The solid line is from a full calculation in the minimal UED model, where all coannihilations have been taken into account. The gray horizontal band denotes the preferred region for the dark matter relic density $0.094 < \Omega_{\text{CDM}}h^2 < 0.129$. Figure constructed from results in [213].

As a final case, let the mass spectrum be that of the minimal UED model, and include all coannihilation effects with all the first level KK states. The minimal UED model is here set to have a cutoff scale equal to $\Lambda_{\text{cut}} = 20R^{-1}$. The relic density result is shown as the solid line in Fig. 6.1. At first sight the displayed result might look surprising, since in the previous example including coannihilations with quarks and gluons reduced the relic density. However, in the minimal UED scenario the KK quarks and gluons get large radiative corrections, and therefore they are more than 15% heavier than $B^{(1)}$ and are not very important in the coannihilation processes. Here, a different effect becomes important. Imagine that the coannihilation cross section $\sigma_{B^{(1)},X^{(1)}}$ with a state $X^{(1)}$ of similar mass is negligibly small. Then the effective cross
section in Eq. (6.4) for the two species, in the limit of mass degeneration, tends towards

$$\sigma_{\text{eff}} \approx \sigma_{B^{(1)}, B^{(1)}} g_{B^{(1)}}^2 / (g_{X^{(1)}} + g_{B^{(1)}}) + \sigma_{X^{(1)}, X^{(1)}} g_{X^{(1)}}^2 / (g_{X^{(1)}} + g_{B^{(1)}}). \quad (6.6)$$

If $\sigma_{X^{(1)}, X^{(1)}}$ is small then $\sigma_{\text{eff}}$ may be smaller than the self-annihilation cross section $\sigma_{B^{(1)}, B^{(1)}}$. This is exactly what happens in the minimal UED model: the mass splittings to the leptons are small (about 1% to the $SU(2)$-singlets and 3% to the $SU(2)$-doublets). Contrary to the case of coannihilating KK quarks and gluons (and the usual case in supersymmetric models with the neutralino as dark matter), the coannihilation processes are here weak (or actually of similar strength as the self-annihilation) and the result is therefore an increase of the relic density. The physical understanding of this situation is that the two species quasi-independently freeze-out, followed by the heavier state decaying thus enhancing the LKP relic density.

In summary, relic density calculations [197, 213, 214] show that if the KK photon $B^{(1)}$ is to make up the observed amount of dark matter, it must have a mass roughly in the range from 500 GeV up to a few TeV.

While freeze-out is the process of leaving chemical equilibrium, it is not the end of the $B^{(1)}$ interacting with the much more abundant SM particles. Elastic and inelastic scattering keeps the LKPs in thermal equilibrium until the temperature is roughly somewhere between 10 MeV and a few hundred MeV (see, e.g., [205, 215, 216]). The kinetic decoupling that occurs at this temperature sets a distance scale below which dark matter density fluctuations get washed out. In other words, there is a cutoff in the matter power spectrum at small scales. This means that there is a lower limit on the size of the smallest protohalos created, whose density perturbations later go non-linear at a redshift between 40 and 80. The consequence is that the smallest protohalos (or dark matter clumps) should not be less massive than about $10^{-3}$ to $10^3$ Earth masses. Whether these smallest clumps of dark matter have survived until today depends critically on to which extent these structures are tidally disrupted through encounters with, e.g., stars, gas disks, and other dark matter halos [217–219].

### 6.2 Direct and Indirect Detection

With the $B^{(1)}$ as a dark matter candidate, the experimental exclusion limits on its properties and its prospect for detection should be investigated.

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* Species never really freeze-out completely independently of each other because of the presence of the processes $6.15$ that usually efficiently transform different species into each other.
Accelerator Searches

High-energy colliders are in principle able to produce heavy particles below their operating center of mass energy. The absence of a direct discovery of any non-SM particle therefore sets upper limits on production cross sections, which in the case of UED translates into a lower limit on KK masses. Because of KK parity conservation, KK states can only be produced in pairs. For the Large Electron-Positron Collider (LEP) at CERN, this means that only masses less than $E_{CM} \sim 100$ GeV could be reached. The circular proton-antiproton Tevatron accelerator at the Fermi laboratory can reach much higher energies as it is running at a center of mass energy of $E_{CM} \sim 2$ TeV. Non-direct detection (i.e., no excess of unexpected missing energy events) in the Tevatron sets an upper limit on one extra-dimensional radius of $R \lesssim (0.3 \text{ TeV})^{-1}$ [126, 220–222]. The future LHC experiment will be able to probe KK masses up to about 1.5 TeV [205]. Suggested future linear electron-positron colliders could significantly improve measuring particle properties such as masses, couplings and spins of new particles discovered at the LHC [156, 223], but will probably not be a discovery machine for UED unless they are able to probe energies above $\sim 1.5$ TeV.

Physics beyond the SM can manifest itself not only via direct production, but also indirectly by its influence on other observables such as magnetic moments, rare decays, or precision electroweak data [205]. For a $\sim 100$ GeV Higgs mass, electroweak precision data limits $1/R$ to be $\gtrsim 800$ GeV, which weakens to about 300 GeV for a $\sim 1$ TeV Higgs. Strong indirect constraints also come from data related to the strongly suppressed decay $b \rightarrow s \gamma$, which is less dependent on the Higgs mass, and gives $1/R \gtrsim 600$ GeV [224].

Direct Detection

Direct detection experiments are based on the idea of observing elastic scattering of dark matter particles that pass through the Earth’s orbit. The searched signal is that a heavy WIMP depositing recoil energy to a target nucleus. There are three common techniques to measure this recoil energy (and many experiments actually combine them). (i) Ionization: in semiconductor targets, like germanium (Ge) [206, 225, 226], silicon (Si) [206, 226] or xenon (Xe) [207], the recoil energy can cause the surrounding atoms to ionize and drift in an applied electric field out to surrounding detectors. (ii) Scintillation: for example, sodium iodide (NaI) crystals [227–229] or liquid/gas Xenon (Xe) scintillators [207, 230] can produce fluorescence light when a WIMP interaction occurs. This fluorescence light is then detected by surrounding photon detectors. (iii) Phonon production: cryogenic (i.e., low temperature) crystals like germanium and silicon detectors, as in [206, 225, 226], look for the phonon (vibration) modes produced by the impulse transfer due to WIMP scattering.

To achieve the required sensitivity for such rare scattering events, a low background is necessary. Operating instruments are therefore well shielded.
and often placed in underground environments. Features that are searched for are the recoil energy spectral shape, the directionality of the nuclear recoil, and possible time modulations. The time modulation in the absolute detection rate is expected due to the Earth’s spin and velocity through the dark matter halo (see Section 9.1 for a short comment on the DAMA experiment’s claim of such a detected annual modulation).

The elastic scattering of a WIMP can be separated into spin-independent and spin-dependent contributions. The spin-independent scattering can take place coherently with all the nucleons in a nucleus, leading to a cross section proportional to the square of the nuclei mass \[ \sigma_{SI} \propto m_N^2 \] [42]. Due to the available phase space, there is an additional factor proportional to the square of the reduced nuclei mass \( m_r = m_{DM} m_N / (m_N + m_{DM}) \) [where \( m_N \) and \( m_{DM} \) are the nuclei and dark matter particle mass, respectively]. The present best limits on the spin-independent cross section comes from XENON [207] and is for WIMP masses around 1 TeV of the order of \( \sigma_{SI} \lesssim 10^{-6} \) pb per nucleon (that is per proton or neutron, respectively), whereas for WIMP masses of 100 GeV it is somewhat better, \( \sigma_{SI} \lesssim 10^{-7} \) pb. This translates roughly to \( m_{B^{(1)}} \gtrsim 0.5 \) TeV [204,231], when assuming a mass shift of about 2% to the KK quarks (the limits gets weakened for increased mass shifts).

The spin-dependent cross-section limits are far weaker, and in the same WIMP mass range they are at present roughly \( \sigma_{SD} \lesssim 0.1 - 1 \) pb. These limits are set by CDMS (WIMP-neutron cross section) [226] and NaIAD (WIMP-proton cross section) [229].

**Indirect Detection**

Indirect searches aim at detecting the products of dark matter particle annihilation. The most promising astrophysical indirect signals seem to come from excesses of gamma-rays, neutrinos or anti-matter. As the annihilation rate is proportional to the number density squared, nearby regions of expected enhanced dark matter densities are the obvious targets for studies.

WIMPs could lose kinetic energy through scattering in celestial bodies, like the Sun or the Earth, and become gravitationally trapped. The concentration of WIMPs then builds up until an equilibrium between annihilation and capture rate is obtained. In the case of \( B^{(1)} \) dark matter, equilibrium is expected inside the Sun, but not in the center of the Earth. The only particles with a low enough cross section to directly escape the inner regions of the Sun and Earth, where the annihilations rate into SM particles is the highest, are neutrinos. Current experiments are not sensitive enough to put any relevant limits, and at least kilometer size detectors, such as the IceCube experiment under construction, will be needed [204,231,232]. Although only the neutrinos can escape the inner parts of a star, it has been proposed that in optimistic scenarios the extra energy source from dark matter annihilation in the interior of stars and white dwarfs could change their temperatures in a
detectable way [233–235].

Another way to reveal the existence of particle dark matter would be to study the composition of cosmic rays, and discover products from dark matter annihilations. Unfortunately, charged products are deflected in the galactic magnetic fields, and information about their origin is lost. However, dark matter annihilation yields equal amount of matter and antimatter, whereas anti-matter in conventionally produced cosmic rays is expected to be relatively less abundant (this is because anti-matter is only produced in secondary processes, where primary cosmic-ray nuclei – presumably produced in supernova shock fronts – collide with the interstellar gas). A detected excess in the anti-matter abundance in cosmic rays could therefore be a sign of dark matter annihilation.

In this manner, the positron spectrum can be searched for dark matter signals. In the UED model, 20% of the $B^{(1)}$ annihilations are into monochromatic electron-positron pairs, and 40% into muons or tau pairs. Muons and taus can subsequently also decay into energetic electrons and positrons. A sizable positron flux from KK dark matter, with a much harder energy spectrum than the expected background, could therefore be looked for [231, 236]. No such convincing signal has been seen, but the High-Energy Antimatter Telescope (HEAT) [237], covering energies up to a few 10 GeV, has reported a potential small excess in the cosmic positron fraction around 7-10 GeV (see, *e.g.*, [238] for a dark matter interpretation, including KK states, of the HEAT data).

In the UED scenario, the large branching ratio into leptonic states makes the expected antiprotons yield – mainly produced from the quark-antiquark final states – relatively low compared to the positron signal. Today antiproton observations do not provide any significant constraints on the UED model [239, 240]. However, the PAMELA (Payload for Antimatter Matter Exploration and Light-nuclei Astrophysics) [241] satellite, already in orbit, might find a convincing energy signature that deviates from the conventionally expected antiproton or positron spectrum, as it will collect more statistics as well as probe higher energies (up to some 100 GeV).

### 6.3 Gamma-Ray Signatures

In addition to the above-mentioned ways to detect WIMP dark matter, there is the signal from annihilation into gamma rays. In general, this signal has many advantages: (i) gamma rays point directly back to their sources, (ii) at these energies the photons basically propagate through the galactic interstellar medium without distortion [242] [243,244], (iii) they often produce characteristic spectral features that differ from conventional backgrounds, (iv) existing techniques for space and large ground-based telescopes allow to study gamma rays in a large energy range – up to tens of TeV.

Dark matter annihilations into photons can produce both a continuum of
gamma-ray energies, as well as monochromatic line signals when $\gamma\gamma$, $\gamma Z$ or $\gamma h$ are the final states.

**Gamma-Ray Continuum**

At tree level, with all first KK levels degenerated in mass, $B^{(1)}$ dark matter annihilates into charged lepton pairs (59%), quark pairs (35%), neutrinos (4%), charged (1%) and neutral (0.5%) gauge bosons, and the Higgs boson (0.5%).

The fraction of $B^{(1)}B^{(1)}$ that annihilates into quark pairs will in a subsequent process of quark fragmentation produce gamma-ray photons, mainly through the decay of neutral $\pi^0$ mesons:

$$B^{(1)}B^{(1)} \rightarrow q\bar{q} \rightarrow \pi^0 + \ldots \rightarrow \gamma\gamma + \ldots$$  \hspace{1cm} (6.7)

These chain-processes result in differential photon multiplicities, i.e., the number density of photons produced per annihilation, and is commonly obtained from phenomenological models of hadronization. In PAPER III, we use a parametrization of the differential photon multiplicity $dN^\gamma_q/dE_\gamma$ published in [245] for a center of mass energy of 1 TeV. This parametrization was based on the Monte Carlo code PYTHIA [246], which is based on the so-called Lund model for quark hadronization. Since PYTHIA are able to reproduce experimental data well, and $dN^\gamma_q/dE_\gamma$ is fairly scale invariant at testable high energies, we judge it to be reliable to use PYTHIA results up to TeV energies.

The massive Higgs and gauge bosons can also decay into quarks that produce photons in their hadronization process. For $B^{(1)}$ dark matter, this gives a negligible contribution due to the small branching ratios into these particles.

The remaining and majority part of the $B^{(1)}$ dark matter annihilations result in charged lepton pairs: electrons $(e)$, muons $(\mu)$ and tau $(\tau)$ pairs, each with a branching ratio of about $20\%$. The only lepton heavy enough to decay into hadrons is the $\tau$. Hence, $\tau$ pairs generate quarks that, as described above, subsequently produce gamma rays in the process of quark fragmentation. The photon multiplicity $dN^\gamma_\tau/dx_\gamma$ for this process is taken from reference [245]. Due to the high branching ratio into $\tau$-pairs for $B^{(1)}$ annihilation, this contribution is very significant at the highest photon energies (see Fig. 6.3 at the end of this section).

An even more important contribution to the gamma-ray spectrum at the highest energies comes from final state radiation. This process is a three-body final state, where one of the charged final state particles radiates a

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† This is radically different from the neutralino dark matter candidate in supersymmetric theories, where annihilation into light fermions are helicity suppressed (see the discussion in Section 7.2).

‡ PYTHIA partly takes final state radiation into account, but certainly not as specifically as in PAPER II and PAPER IV (see also [247]). This is particularly true for internal bremsstrahlung from charged gauge bosons, treated in Chapter 7 and in PAPER IV.
photons. The relevant Feynman diagrams for $B^{(1)}$ annihilation with a final state photon is depicted in Fig. 6.2. In principle, diagrams with an s-channel Higgs bosons also exists. These diagram can safely be neglected since the Higgs boson couple very weakly to light leptons ($\propto m_{\ell}$) and are typically far enough from resonance. Typically, the galactic velocity scale for WIMPs is $10^{-3}c$, and the cross section can be calculated in the zero velocity limit of $B^{(1)}$. As found in Paper II, the differential photon multiplicity can be well approximated by:

$$\frac{dN_{\gamma}}{dx} \approx \frac{\alpha}{\pi} \frac{\alpha (x^2 - 2x + 2)}{x} \ln \left[ \frac{m_{B^{(1)}}^2}{m_{\ell}^2} (1 - x) \right], \quad (6.8)$$

where $x \equiv E_\gamma/m_{B^{(1)}}$ and $m_{\ell}$ is the mass of the lepton species in consideration.

The factor $\alpha/\pi$ arises due to the extra vertex and the phase space difference between two- and three-body final states. The large logarithm $\ln(m_{B^{(1)}}^2/m_{\ell}^2)$ appears due to a collinear divergence behavior of quantum electrodynamics. This effect is easy to see from the kinematics. Consider the propagator of the lepton that emits a photon in the first (or third) diagram of Fig. 6.2. Denoting the outgoing photon [lepton] momentum by $k^\mu = (E_k, \vec{k})$ $[p^\mu = (E_p, \vec{p})]$, the denominator of the propagator is

$$(p + k)^2 - m_{\ell}^2 = 2p \cdot k = 2E_k(E_p - |\vec{p}| \cos \theta) \quad (6.9)$$

where $\theta$ is the angle between the outgoing photon and lepton. This expression shows that for a highly relativistic lepton ($|\vec{p}| \to E_p$) and a collinear ($\theta \to 0$) outgoing photon the lepton propagator diverges, meaning that leptons tend to rapidly lose their energy by emission of forward-directed photons.

Let us be slightly more quantitative and investigate the cross section. In next to lowest order, $\cos \theta \to 1 + \theta^2/2$ and $|\vec{p}| \to E_p(1 - m_{\ell}^2/2E_p^2)$, and the denominator of the propagator becomes $E_kE_p(\theta^2 + m_{\ell}^2/E_p^2)$. The photon vertex itself gives in this limit a contribution $\bar{u}(p)\gamma^\mu u(p + k)$, where the approximation consists of treating the virtual, almost on shell, electron with momentum $p + k$, as a real incoming electron. Squaring this part of the amplitude, and
taking the spin sum, leads to:

\[
\sum_{\text{spin}} |\epsilon_\mu \bar{u}(p) \gamma^\mu u(p + k)|^2 = -\text{Tr}[(\not{p} + \not{k} + m_\ell) \gamma^\mu (\not{p} + m_\ell) \gamma_\mu]
\]

\[
= 8 (p + k)^\mu \rho_\mu \approx 8 (E_k E_p - \vec{k} \vec{p}) \approx 8 E_k E_p (1 - \cos \theta) \approx 8 E_k E_p \theta^2,
\]

where the standard notation that \( u(p) \) and \( \bar{u} \equiv u^\dagger \gamma^0 \) are dirac spinors, and \( \epsilon_\mu \) is the photon polarization. In the second line, the lepton mass is set to zero, \( m_\ell = 0 \). The final ingredient including \( \theta \) is the phase space factor \( \int d^3k \). In the small \( \theta \) limit we have \( d^3k = 2\pi p_\perp dk_\parallel dk_\perp \rightarrow 2\pi E_k \theta dE_k d\theta \) and therefore the photon multiplicity should scale as:

\[
\frac{dN_\gamma^f}{dx} \propto \int_0^{\theta_{\text{max}}} d\theta \frac{\theta^3}{[\theta^2 + m_\ell^2/E_p^2]^2} \approx \frac{1}{2} \ln(E_p^2/m_\ell^2), \tag{6.10}
\]

where only the leading logarithm is kept, and \( \theta_{\text{max}} \) is an arbitrary upper limit for which the used collinear approximations hold. In the collinear limit, energy conservation in the vertex of the radiating final state photon gives \( E_p = \sqrt{s}/2 - E_\gamma \) (\( \sqrt{s} \) being the center of mass energy), which for incoming non-relativistic (dark matter) particles reduces to \( E_p \approx m_{\text{WIMP}}(1 - x) \). This is qualitatively the result we obtained for the UED model above. I would like to stress that these arguments are very general. Thus, for any heavy dark matter candidate, with unsuppressed couplings to fermions, high-energy gamma rays, as in Eq. (6.8), are expected. This means that a wide class of dark matter particles should by annihilation produce very hard gamma spectra with a sharp edge feature, with the flux dropping abruptly at an energy equal to the dark matter mass.

The general behavior of final state radiation from dark matter annihilations was later studied also in [247], where they further stress that annihilation into any charged particles, \( X \) and \( \bar{X} \), together with a final state radiated photon takes a universal form:

\[
\frac{d\sigma(\chi\chi \rightarrow X\bar{X}\gamma)}{dx} \approx \frac{\alpha Q_X^2}{\pi} F_X(x) \ln \left( \frac{s(1 - x)}{m_X^2} \right) \sigma(\chi\chi \rightarrow X\bar{X}), \tag{6.11}
\]

where \( Q_X \) and \( m_X \) are the electric charge and the mass of the \( X \) particle, respectively. The splitting function \( F \) depends only on the spin of the \( X \) particles. When \( X \) is a fermion:

\[
F_f(x) = \frac{1 + (1 - x)^2}{x}, \tag{6.12}
\]

\[\text{§ I am definitely a bit sloppy here. A full kinematically and gauge invariant calculation (including all contributing Feynman diagrams) would, however, give the same result. If we so wish, we could be a bit more correct and imagine the electron-positron pair (momentum \( p_1 = p + k \) and \( p_2 \)) to be produced directly from a scalar interaction, where the \( p_1 \) particle radiates a photon with momentum \( k \). The exact spinor part of the amplitude then becomes \( \epsilon_\mu \bar{u}(p) \gamma^\mu (p_1) u(p_2) \), which again leads to \( -\text{Tr}[(p_2 p_1 \gamma_\mu \not{p}_\mu \not{p}_1) \propto \theta^2 \) in lowest order in \( \theta \).}
Figure 6.3: The total number of photons per $B^{(1)}B^{(1)}$ annihilation (solid line), multiplied by $x^2 = (E_\gamma / m_{\mu^{(1)}})^2$. Also shown is what quark fragmentation alone would give (dashed line), and adding to that $\tau$ lepton production and decay (dotted line). Here a $B^{(1)}$ mass of 0.8 TeV and a 5% mass split to the other particles first Kaluza-Klein level are assumed – the result is, however, quite insensitive to these parameters. Figure from Paper II.

whereas if $X$ is a scalar particle,

$$\mathcal{F}_s(x) = \frac{1 - x}{x}.$$  \hfill (6.13)

If $X$ is a $W$ boson, the Goldstone boson equivalence theorem implies that $\mathcal{F}_W(x) \approx \mathcal{F}_s(x)$ [248]. Unfortunately, a sharp endpoint is not obvious in the scalar or $W$ boson case. According to Eq. (6.13), $\lim_{x \to 1} \mathcal{F}_s(x) = 0$ and the flux near the endpoint might instead be dominated by model-dependent non-collinear contributions [247] (see Section 7.2 and PAPER IV for internal bremsstrahlung in the case of neutralino annihilations).

Including also this final state radiation, the total observable gamma spectrum per $B^{(1)}B^{(1)}$ annihilation is finally given by:

$$dN^{\text{eff}}_\gamma / dx \equiv \sum_i \kappa_i dN_i^{\gamma} / dx,$$  \hfill (6.14)

where the sum is over all processes that contribute to primary or secondary gamma rays, and $\kappa_i$ are the corresponding branching ratios. The result is shown as the solid line in Fig. 6.3.
There are also other sources of photon production accompanying the LKP annihilation. For example, the induced high-energy leptons will Compton scatter on the CMB photons and starlight. Although these processes produce gamma rays, they are expected to give small fluxes [249]. Another source of photon fluxes emerge when light leptons lose energy by synchrotron radiation in magnetic fields [249]. With moderate, although uncertain, assumptions for the magnetic fields in the galactic center, the synchrotron radiation could give a significant flux of photons, both at radio [250] and X-ray [251] wavelengths.

**Gamma Line Signal**

Since the annihilating dark matter particles are highly non-relativistic, the processes $B^{(1)}B^{(1)} \rightarrow \gamma\gamma$, $B^{(1)}B^{(1)} \rightarrow Z\gamma$, and $B^{(1)}B^{(1)} \rightarrow H\gamma$ result in almost mono-energetic gamma-ray lines with energies $E_\gamma = m_{B^{(1)}}$, $E_\gamma = m_{B^{(1)}}(1 - m_Z^2/4m_{B^{(1)}}^2)$ and $E_\gamma = m_{B^{(1)}}(1 - m_h^2/4m_{B^{(1)}}^2)$, respectively. With the expectation that dark matter being electrically neutral, these annihilation processes are bound to be loop suppressed (since photons couple only to electric charge). On the other hand, such gamma-ray lines would constitute a ‘smoking gun’ signature for dark matter annihilations if they were to be observed, since it is hard to imagine any astrophysical background that could mimic such a spectral feature.

In the case of $B^{(1)}$ dark matter, with unsuppressed couplings to fermions, it could be expect that loops with fermions should dominate the cross section. This is a naïve expectation from the tree-level result of a 95% branching ratio into charged fermions (but this expectation should be true if no particular destructive interference or symmetry suppressions are expected to occur at loop-level).

In Paper III, we calculated the line signal $B^{(1)}B^{(1)} \rightarrow \gamma\gamma$. The pure fermionic contributions give in total $2 \times 12$ different diagrams for each charged SM fermion that contributes to $B^{(1)}B^{(1)} \rightarrow \gamma\gamma$; see Fig. 6.4. The calculation of the Feynman amplitude of $B^{(1)}B^{(1)} \rightarrow \gamma\gamma$, within the QED sector, is described in detail in Paper III. The basic steps in obtaining the analytical result are as follows:

1. The total amplitude
   \[ M = \epsilon_1^{\mu_1}(p_1)\epsilon_2^{\mu_2}(p_2)\epsilon_3^{\mu_3}(p_3)\epsilon_4^{\mu_4}(p_4) \mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4), \]  
   (6.15)
   is formally written down from the Feynman rules given in Appendix A.

2. Charge invariance of the in and out states and a relative sign between vector and axial couplings,
   \[ C\bar{\psi}\gamma^\mu\psi C^{-1} = -\bar{\psi}\gamma^\mu\psi, \quad C\bar{\psi}\gamma^\mu\gamma^5\psi C^{-1} = \bar{\psi}\gamma^\mu\gamma^5\psi, \]  
   (6.16)

\footnote{Remember, at a given KK level the fermionic field content is doubled as compared to the SM.}
means that an odd number of axial couplings in the Feynman amplitudes must automatically vanish (i.e. no $\gamma^5$ in the trace). Splitting the amplitude into a contribution that contains only vector-like couplings $\mathcal{M}_v$ and terms that contains only axial vector couplings $\mathcal{M}_a$ is therefore possible. In this specific case, where the axial part has equal strength as the vector part in couplings between $B^{(1)}$ and fermions (see Appendix A or Paper III) we also have $\mathcal{M}_a = \mathcal{M}_v$ and the full amplitude can be written as

$$\mathcal{M}^{\mu_1\mu_2\mu_3\mu_4} = -2i\alpha_{em}\alpha_Y Q^2(Y_s^2 + Y_d^2)\mathcal{M}_v^{\mu_1\mu_2\mu_3\mu_4},$$

where $\alpha_{em} \equiv e^2/4\pi$, $\alpha_Y \equiv g_Y^2/4\pi$, and $Q$ and $Y$ are the electric and hypercharge, respectively, of the KK fermions in the loop.

3. From momentum conservation in the zero velocity limit of the $B^{(1)}$s ($p_1 = p_2 = (m_{B^{(1)}}, 0)$), and transversality of the polarization vectors, we can decompose the amplitude into the following Lorentz structure:

$$\mathcal{M}_v^{\mu_1\mu_2\mu_3\mu_4} = \frac{A}{m_{B^{(1)}}^2} p_3^{\mu_1} p_4^{\mu_2} p_\mu^3 p_\mu^4 + \frac{B_1}{m_{B^{(1)}}^2} g^{\mu_1\mu_2} p_3^{\mu_3} p_\mu^4$$
$$+ \frac{B_2}{m_{B^{(1)}}^2} g^{\mu_1\mu_3} p_3^{\mu_2} p_\mu^4 + \frac{B_3}{m_{B^{(1)}}^2} g^{\mu_1\mu_4} p_3^{\mu_2} p_\mu^3 + \frac{B_4}{m_{B^{(1)}}^2} g^{\mu_2\mu_3} p_3^{\mu_1} p_\mu^4$$
$$+ \frac{B_5}{m_{B^{(1)}}^2} g^{\mu_2\mu_3} p_3^{\mu_1} p_\mu^3 + \frac{B_6}{m_{B^{(1)}}^2} g^{\mu_3\mu_4} p_3^{\mu_1} p_\mu^2$$
$$+ C_1 g^{\mu_1\mu_2} g^{\mu_3\mu_4} + C_2 g^{\mu_1\mu_3} g^{\mu_2\mu_4} + C_3 g^{\mu_1\mu_4} g^{\mu_2\mu_3}. \quad (6.18)$$

4. The expressions for the coefficients $A$, $B$, and $C$ are determined by identification of the amplitude found in step 1 above. However, by Bose symmetry and gauge invariance of the external photons, the whole amplitude can be expressed by means of only three coefficients, which are chosen to be $B_1$, $B_2$ and $B_6$. 

Figure 6.4: Fermion box contributions to $B^{(1)}B^{(1)} \rightarrow \gamma\gamma$ including the first KK levels. Not shown are the additional nine diagrams that are obtained by crossing external momenta. Figure from Paper III.
5. These coefficients, $B_1$, $B_2$, and $B_6$, are linear combinations of tensor integrals

$$D_0; D_{\mu}; D_{\mu\nu}; D_{\mu\nu\rho}; D_{\mu\nu\rho\sigma} (k_1, k_2, k_3; m_1, m_2, m_3, m_4)$$

$$= \int \frac{d^n q}{i \pi^2} \frac{1}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)(q_4^2 - m_4^2)}, \quad (6.19)$$

where

$$q_1 = q, \quad q_2 = q + k_1, \quad q_3 = q + k_1 + k_2, \quad q_4 = q + k_1 + k_2 + k_3. \quad (6.20)$$

All of these can in turn be reduced to scalar loop integrals [252], for which closed expressions exist [253]. However, for incoming particles with identical momenta the original reduction procedure [252] of Passarino and Veltman breaks down, and we therefore used the LERG program [254], which has implemented an extended Passarino-Veltman scheme to cope with such a case.

6. With the first-level fermions degenerate in mass, we finally found

$$(\sigma v)_{\gamma\gamma} = \frac{\alpha^2 \alpha_{\text{EM}}^2 g_{\text{eff}}^4}{144 \pi m_{\text{e}(1)}^2} \left\{ 3 |B_1|^2 + 12 |B_2|^2 + 4 |B_6|^2 - 4 \text{Re} [B_1 (B_2^* + B_6^*)] \right\}, \quad (6.21)$$

where

$$g_{\text{eff}}^2 = \sum_{\text{SM}} Q^2 (Y_s^2 + Y_d^2) = \frac{52}{9}. \quad (6.22)$$

The sum runs over all charged SM fermions, and the analytical expressions of $B_1$, $B_2$, and $B_6$ can be found in the appendix of Paper III.

Figure 6.5 shows the annihilation rate $$(\sigma v)_{\gamma\gamma}$$ as a function of the mass shift between the $B^{(1)}$ and the KK fermions.

In addition to the fermion box diagrams, there will be a large number of Feynman diagrams once $SU(2)$ vectors and scalar fields are included. There are 22 new diagram types that are not related by any obvious symmetry, and they are shown in Fig. 6.6. Obviously, it would be a tedious task to analytically calculate all these contributions by hand. Instead, we took another approach, and implemented the necessary Feynman rules into the FeynArts software [255]. FeynArts produces a formal amplitude of all contributing diagrams, that can then be numerically evaluated with the FormCalc [256] package (which in turn uses the FORM code [257] and LoopTools [256] to evaluate tensor structures and momentum integrals). This numerical method could also be used to check our analytical result of the fermion loop contribution. Adding the Feynman diagrams including bosons as internal propagators, it was numerically found that they make only up a slight percentage of the...
Figure 6.5: The annihilation rate into two photons as a function of the mass shift between the $B^{(1)}$ and first Kaluza-Klein level fermions $\xi^{(1)}$. This is for $m_{B^{(1)}} = 0.8 \text{ TeV}$, but the dependence on the $B^{(1)}$ mass is given by the scaling $(\sigma v)_{\gamma \gamma} \propto m_{B^{(1)}}^{-2}$. A convenient conversion is $\sigma v = 10^{-4} \text{ pb} = 10^{-4} \text{ c pb} \approx 3 \cdot 10^{-30} \text{ cm}^3 \text{ s}^{-1}$. Figure from Paper III.

total cross section. This is in agreement with the naïve expectation, previously mentioned, that the fermionic contribution should dominate. Higher KK levels also contribute; however, the larger KK masses in their propagators suppress these contributions. By adding second-level fermions, we could confirm that our previous result only changed by a few percent. In conclusion, the analytical expression (6.21) is a good approximation for two photon production and $(\sigma v)_{\gamma \gamma} \sim \text{few} \times 10^{-30} \text{ cm}^3 / \text{s} (1 \text{ TeV}/m_{B^{(1)}})^2$.

The two other processes that can give mono-energetic photons, $B^{(1)}B^{(1)} \rightarrow Z\gamma$ and $B^{(1)}B^{(1)} \rightarrow H\gamma$, have so far not been fully investigated. The fermionic contribution to $B^{(1)}B^{(1)} \rightarrow Z\gamma$ can at this stage easily be calculated. The only difference to the two photon case is here that the $Z$ boson has both a vector and axial part in its coupling to fermions. From the values of these couplings, we have analytically, and numerically, found that this $Z\gamma$ line should have a cross section of about 10% compared to the $\gamma\gamma$ line.

The $H\gamma$ line\[l could also enhance the gamma line signal. If the Higgs mass is very heavy, it could also potentially be resolved as an additional line at energy $E_\gamma = m_{B^{(1)}} (1 - m_h^2/4m_{B^{(1)}}^2)$). The contributing diagrams can have significantly different structure than in the $B^{(1)}B^{(1)} \rightarrow \gamma\gamma$ process. Although

\[l In supersymmetry, the $H\gamma$ line is inevitably very week as its forbidden in the limit of zero velocity annihilating neutralinos.
**Figure 6.6:** The 22 different types of bosonic diagrams, in addition to the fermion loop type in Fig. 6.4, that contribute to $B^{(1)} B^{(1)} \rightarrow \gamma \gamma$.

it is not expected to give any particularly strong signal**, an accurate analysis of this processes has not yet been carried out.

### 6.4 Observing the Gamma-Ray Signal

The characteristic dark matter signature in the gamma-ray spectrum – a hard spectrum with a sharp drop in the flux, and potentially even a visible gamma line, at an energy equal to the mass of the $B^{(1)}$ – is something that can be searched for in many experiments. Due to the large uncertainties in the dark matter density distribution, the expected absolute flux from different sources, such as the galactic center, small dark matter clumps and satellite galaxies,

** Estimates including only fermion propagators indicate that these contributions are not very significant.**
as well as the diffuse extragalactic, is still very difficult to predict with any certainty.

The velocity scale for cold dark matter particles in our Galaxy halo is of the order of $v \sim 10^{-3}c$. For an ideal observation of the monochromatic gamma line, this would lead to a relative smearing in energy of $\sim 10^{-3}$ due to the Doppler shift. This narrow line is much too narrow to be fully resolved with the energy resolution of current gamma-ray telescopes. To compare a theoretically predicted spectrum to experimental data, the predicted signal should first be convolved with the detector’s response. The actual detector response is often unique for each detector and is often rather complicated. To roughly take the convolution into account, it is reasonable to use a simple convolution/smearing of the theoretical energy spectrum before comparing with published data. For a Gaussian convolution function with an energy resolution $\sigma(E')$, the predicted experimental flux is given by

$$
\frac{d\Phi_{\text{exp}}}{dE} \bigg|_{E} = \int_{-\infty}^{\infty} dE' \frac{d\Phi_{\text{theory}}}{dE} \bigg|_{E'} e^{-(E' - E)^2/(2\sigma^2)} \sqrt{2\pi\sigma E'}.
$$

(6.23)

As an illustrative example, let us compare our theoretically predicted spectrum with the TeV gamma-ray signal observed from the direction of the galactic center, as observed by the air Čerenkov telescopes H.E.S.S. [258], Magic [259], VERITAS [260], and CANGAROO [261]. The nature of this source is partly still unknown. Because the signal does not show any apparent time variation, is located in a direction where a high dark matter density concentration could be expected, and is observed to be a hard spectrum up to high gamma-ray energies (i.e., the flux does not drop much faster than $E^{-2}$), it has been discussed if the gamma flux could be due to dark matter annihilations (see, e.g., [262] and references therein).

In Fig. 6.7, the predicted gamma spectrum from annihilating $B^{(1)}$s with masses of 0.8 TeV, smeared with an energy resolution of $\sigma = 0.15E'$, is shown together with the latest H.E.S.S. data. Annihilation of such low mass dark matter particles can certainly not explain the whole range of data. However, it is interesting to note that the flux comes out to be of the right order of magnitude for reasonable assumptions about the dark matter density distribution. For the flux prediction, an angular acceptance of $\Delta\Omega = 10^{-5}$ sr and a boost factor $b \approx 200$ to the NFW density profile were used in the flux equation (2.16). With a better understanding of backgrounds and more statistics, it might be possible to extract such a dark matter contribution; especially since there is a sharp cutoff signature in annihilations spectrum to look for. When the first data were presented from the H.E.S.S. collaboration, it was noticed that the spectral energy distribution of gamma rays showed a very similar hard spectrum as predicted from final state radiation from light leptons. To point this out, we suggested in PAPER II a hypothetical case with $M_{B^{(1)}} \sim 10$ TeV. A good match to the 2003 year data (solid boxes in Fig. 6.7) [263] was then found. Later observations during 2004 [258] did not match the prediction
Section 6.4. Observing the Gamma-Ray Signal

Figure 6.7: The H.E.S.S. data (open boxes: 2003 data [263]; solid triangles: 2004 data [258]) compared to the gamma-ray flux expected from a region of $10^{-5}$ sr encompassing the galactic center, for a $B^{(1)}$ mass of 0.8 TeV, a 5% mass splitting at the first Kaluza-Klein level, and a boost factor $b \sim 200$ (dotted line). The solid line corresponds to a hypothetical 9 TeV WIMP with similar artificial couplings, a total annihilation rate given by the WMAP relic density bound, and a boost factor of around 1000. Both signals have been smeared to simulate an energy resolution of 15%, appropriate for the H.E.S.S. telescope.

for such a heavy $B^{(1)}$ particle; neither was any cutoff at the highest energies found. Although it is in principle possible to reconstruct the observed spectral shape with dark matter particles of tens of TeV, as illustrated in [262] with non-minimal supersymmetry models, this does not work for the most simple and common models of dark matter [258, 262]. With more statistics in the 2004 data, a search for any significant hidden dark matter signal on top of a simple power law astrophysical background was performed, but no significant typical dark matter component could be found [258]. When it comes to suggested astrophysical explanations of the observed TeV signal from the galactic center, the main proposed sources are particle accelerations in the Sagittarius A supernova remnant, processes in the vicinity of the supermassive black hole Sagittarius A*, and the detected nearby pulsar wind nebula (see [264] and references therein).

When it comes to experimental searches for the monochromatic gamma line signal from $B^{(1)}$ annihilation, a much better energy resolution would be needed. Nevertheless, with an energy resolution close to the mentioned natural Doppler width of $10^{-3}$ the line could definitely be detectable. In
Figure 6.8: The gamma line signal as expected from $B^{(1)}$ dark matter annihilations. The line-signal is superimposed on the continuous gamma-ray flux (solid line) and is roughly as it would be resolved by a detector with a Gaussian energy resolution of 1% (dashed), 0.5% (dotted) and 0.25% $E'$ (dash-dotted), respectively. The actual line width of the signal is about $10^{-3}$, with a peak value of $1.5 \times 10^{-7} \text{ m}^{-2} \text{ s}^{-1} \text{ TeV}^{-1}$. The example here is for a $B^{(1)}$ mass $m_{B^{(1)}} = 0.8 \text{ TeV}$, and a mass shift $m_{\xi^{(1)}}/m_{B^{(1)}} = 1.05$. An angular acceptance of $\Delta \Omega = 10^{-5} \text{ sr}$ and a boost factor of $b = 100$ to a NFW profiles were assumed. Figure from PAPER III.

The expected gamma-ray spectrum around a 0.8 TeV mass $B^{(1)}$ is shown for three different detector resolutions; an energy resolution better than 1% would be needed to resolve the line signal. Typically the energy resolution of today’s detectors is a factor of ten larger.
The most well known dark matter candidate is the neutralino, a WIMP that appears in supersymmetry theories. Although the detection signals for this dark matter candidate have been well studied, the internal bremsstrahlung contribution to the gamma-ray spectrum for typical heavy neutralinos has previously not been investigated. In our study in Paper IV we found that internal bremsstrahlung produce a pronounced signature in the gamma-ray spectrum, in the form of a very sharp cutoff, or even a peak, at the highest energies. This signal can definitely have a positive impact on the neutralino detection prospects, as it not only possesses a striking signature, but also significantly enhances the expected total gamma-ray flux at the highest energies. With the energy resolution of current detectors, this signal can even dominate the monochromatic gamma-ray lines (\(\gamma\gamma\) and \(Z\gamma\)) that previously been shown to provide exceptional strong signals for heavy neutralinos.

7.1 Supersymmetry

Supersymmetry relates fermionic and bosonic fields, and is thus a symmetry mixing half-integer and integer spins. The generators of such a symmetry must carry a half integer spin and commute with the Hamiltonian. These generators transform non-trivially under Lorentz transformations and the internal symmetries interact non-trivially with the spacetime Poincaré symmetry. This might seem to violate a no-go theorem by Coleman and Mandula, stating that any symmetry group of a consistent four-dimensional quantum field theory can only be a direct product of internal symmetry groups and the Poincaré group (otherwise the scattering-matrix is identically equal to 1, and no scattering is allowed). However, the new feature of having anticommuting, spin
half, generators for a symmetry turn out to allow for a nontrivial extension of the Poincaré algebra. The details of the construction of a supersymmetric theory are beyond the scope of this thesis, and only some basic facts and the most important motivations for supersymmetry will be mentioned here.

Since it is not possible to relate the fermions and bosons within the SM, each SM fermion (boson) is instead given new bosonic (fermionic) supersymmetric partner particles. The nomenclature for superpartners is to add a prefix ‘s’ to the corresponding fermion name (e.g., the superpartner to the electron is called the selectron), whereas superpartners to bosons get their suffix changed to ‘ino’ (e.g., the superpartner to the photon is called the photino).

Superpartners inherit mass and quantum numbers from the SM particles. Only the spin differs. Since none of these new partners have been observed, supersymmetry must be broken, and all supersymmetric particles must have obtained masses above current experimental limits. The actual breaking of supersymmetry might introduce many new unknown parameters. Explicit symmetry breaking terms can introduce more than 100 new parameters [265]. In specific constrained models, where the supersymmetry is broken spontaneously, the number of parameters is usually much smaller. For example, in the minimal supergravity (mSUGRA) model [266] the number of supersymmetry parameters is reduced to only five, specified at a high energy grand unification scale.

In the following, only minimal supersymmetric standard models (MSSM) are considered. They are minimal in the sense that they contain a minimal number of particles: the SM fields (now with two Higgs doublets \(^\star\)) and one supersymmetric partner to each of these.

Some Motivations

A main motivation for having a supersymmetric theory, other than for the mathematical elegance of a symmetry relating fermions and bosons, is that it presents a solution to the fine-tuning problem within the SM. Within the SM, the Higgs mass \(m_h\) gets radiative corrections that diverge linearly with any regulating ultraviolet cutoff energy. With the cutoff of the order of the Planck scale (10\(^{19}\) GeV), the required fine-tuning is of some 17 orders of magnitude to reconcile the Higgs mass \(m_h\) with the indirectly measured value \(m_h \sim 100\) GeV. This naturalness problem (or fine-tuning problem) of the SM is elegantly solved within supersymmetry by the fact that supersymmetric partners exactly cancel these divergent quantum contribution to the Higgs mass. Even if supersymmetry is broken, there is no need for extreme fine-tuning, at least not as long as the breaking scale is low and the supersymmetric partners have masses not much higher than the Higgs mass.

\(^\star\) This is a type II two Higgs doublet model (to be discussed more in Section 8.1), and is needed in supersymmetry to generate masses to both the up- and down-type quarks [267, 268]
A second motivation for supersymmetry is that the required spontaneous
symmetry breaking of the electroweak unification can be obtained through
radiative quantum corrections, where the quadratic Higgs mass parameter is
driven to a negative value.

A third intriguing attraction has been that the running of the three gauge
coupling constants of the electromagnetic, weak, and strong forces are modi-
ﬁed in such a way that they all become equal, within a unification scheme, at
an energy of about \(10^{16}\) GeV. That this force strength uniﬁcation occurs at
an energy scale signiﬁcantly below the Plank scale and where the theory still
is perturbatively reliable is far from trivial.

Finally, supersymmetry can provide dark matter candidates. Usually this
is the case in models with an additional symmetry called R-parity, which is an
imposed conserved multiplicative quantity. Every SM particle is given posi-
tive R-parity, whereas all supersymmetric particles are given negative. This is
very important for cosmology, because R-parity guarantees that the lightest
supersymmetric particle is stable since it cannot decay into any lighter state
having negative R-parity. The introduction of such a symmetry can be fur-
ther motivated as it automatically forbids interactions within supersymmetry
that otherwise would lead to proton lifetimes much shorter than experimental
limits.

The Neutralino

In many models, the lightest stable supersymmetric particle is the lightest
neutralino, henceforth just ‘the neutralino’. It is a spin-1/2 Majorana particle
and a linear combination of the gauginos and the Higgsinos

\[
\chi \equiv \tilde{\chi}_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W}^3 + N_{13} \tilde{H}_1^0 + N_{14} \tilde{H}_2^0.
\] (7.1)

With R-parity conserved, the neutralino is stable and a very good dark matter
candidate. This is the most studied dark matter candidate, and there are
many previous studies on its direct and indirect detection possibilities (see,
e.g., [25, 42], and references therein). We will here focus on a new type of
gamma-ray signature first discussed in PAPER IV.

7.2 A Neglected Source of Gamma Rays

Previous studies of the gamma-ray spectrum from neutralino annihilations
have mainly focused on the continuum spectrum, arising from the fragment-
tion of produced quarks and \(\tau\)-leptons, and the second order, loop-induced
\(\gamma\gamma\) and \(Z\gamma\) line signals [96, 269, 270]. For high neutralino masses, the almost
monochromatic \(\gamma\gamma\) and \(Z\gamma\) photon lines can be exceptionally strong, with
branching ratios that reach percent level despite the naïve expectation of be-
ing two to three orders of magnitude smaller. The origin of this enhancement
is likely due to nonperturbative, binding energy effects in the special situation of very small velocities, large dark matter masses, as well as small mass differences between the neutralino and the lightest chargino [271, 272].

The contribution from radiative processes, i.e., processes with one additional photon in the final state, should naïvely have a cross section two orders of magnitude larger than the loop-suppressed monochromatic gamma lines, since they are one order lower in the fine structure constant $\alpha_{\text{em}}$. As investigated in Paper IV, internal bremsstrahlung, in the production of charged gauge bosons from annihilating heavy neutralinos, results in high-energy gamma rays with a clearly distinguishable signature. This is partly reminiscent of the case of KK dark matter, where final state radiation in annihilation processes with charged lepton final states dominates the gamma-ray spectrum at the highest energies.

**Helicity Suppression for Fermion Final States**

In Paper II, we found that final state radiation from light leptons produced in $B^{(1)}$ annihilation gave an interesting signature in the gamma-ray spectrum. Neutralino annihilations into only two light fermion pairs have an exceptionally strong suppression, and typical branching ratios into electrons are often quoted to be only of the order of $10^{-5}$. The reason for this so-called *helicity suppression* can be understood as follows. The neutralinos are self-conjugate (Majorana) fermions obeying the Pauli principle and must therefore form an antisymmetric wave function. In the case of zero relative velocity, the spatial part of the two particles’ wave function is symmetric – i.e., an orbital angular momentum $L = 0$ state (s-wave) – and the spin part must form an antisymmetric ($S = 0$) singlet state. Thus the incoming state has zero total angular momentum. The contributing neutralino annihilation processes conserve chirality, so that massless (or highly relativistic) fermions and antifermions come with opposite helicities (i.e., handedness). Therefore the spin projection in the outgoing direction is one, which precludes s-wave annihilation. The conclusion must be that the annihilation cross section into monochromatic massless fermions is zero; for massive fermions it is instead proportional to $m_f^2/m_\chi^2$. An interesting way to circumvent this behavior of helicity suppress-

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1. Typical dark matter halo velocities are $v \sim 10^{-3}$, and p-wave annihilations would be suppressed by $v^2 \sim 10^{-6}$.
2. Both the Z-fermion-antifermion and fermion-sfermion-gaugino vertices conserve helicity. Contributions from Higgs-boson exchange, from Higgsino-sfermion-fermion Yukawa interactions, and from sfermion mixing violate chirality conservation, but they all include an explicit factor of the fermion mass $m_f$ [273].
3. Orbital angular momentum of the outgoing fermions can never cancel the spin component in the direction of the outgoing particles (this is clear because orbital angular momentum of two particles can never have an angular momentum component in the same plane as their momentum vectors lie in). Therefore the total angular momentum must be nonzero, in contradiction to the initial state of zero angular momentum.
sion is to have a photon accompanying the final state fermions [274]. This open up the possibility of a significant photon and fermion spectrum, where the first-order corrected cross section can be many orders of magnitude larger than the tree-level result. I will not pursue this interesting possibility here (see, however, [274] and the recent work of [275]), but instead discuss internal bremsstrahlung when \(W^\pm\) gauge bosons are produced by annihilating heavy Higgsinos [PAPER IV].

**Charged Gauge Bosons and a Final State Photon**

In order not to overclose the Universe, a TeV-mass neutralino must in general have a very large Higgsino fraction\(^\dagger\) \(Z_h\) (\(Z_h \equiv |N_{13}|^2 + |N_{14}|^2\)), ensuring a significant cross section into massive gauge bosons. A pure bino state with a TeV mass, on the other hand, does not couple to \(W\) at all in lowest order. This usually excludes TeV binos as they would freeze-out too early and over-produce the amount of dark matter. Let us therefore focus on a Higgsino-like neutralino with \(N_{11} \approx N_{12} \approx 0\) and \(N_{13} \approx \pm N_{14}\). The annihilation rate into charged gauge bosons often dominates, and radiation of a final state photon should be of great interest to investigate.

For a pure Higgsino, the potential \(s\)-channel exchanges of \(Z\) and Higgs bosons vanish, and the only Feynman diagrams contributing to the \(W^+W^−\gamma\) final states are shown in Fig. 7.1.

For the analytical calculation of these Feynman diagrams, there is one technicality worth noticing. Due to the Majorana nature of the neutralinos, the Feynman diagrams can have crossing fermion lines, and special care must be taken to deal with the spinor indices correctly. Proper Feynman rules have been developed (see, e.g., [276]), which also have been implemented in different numerical code packages (e.g., **FeynArts** and **FORMCalc** [277]). For manual calculations a practical simplifying technique can be adopted: in the limit of zero relative velocity, the two ingoing annihilating neutralinos must

\(^\dagger\) This is the case if the usual GUT condition \(M_1 \sim M_2/2\) is imposed; otherwise a heavy wino would also be acceptable. For a pure wino the results are identical to what is found for the anti-symmetric \(N_{13} = −N_{14}\) Higgsinos considered here; apart from a multiplicative factor of 16 in all cross sections.

**Figure 7.1:** Contributions to \(\chi\chi \rightarrow W^+W^−\gamma\) for a pure Higgsino-like neutralino (crossing fermion lines are not shown). Figure from PAPER IV.
form a $^1S_0$ state (as explained above) and the sum over all allowed spin state configurations of the two incoming Majorana particles can be replaced by the projector [278]

$$\mathcal{P}_{^1S_0} \equiv -\frac{1}{\sqrt{2}} \gamma^5 (m_\chi - p),$$  

(7.2)

where $p$ is the momentum of one of the incoming neutralinos. $\mathcal{P}_{^1S_0}$ is simply inserted in front of the gamma-matrices originating from the Majorana fermion line, and then the trace is taken over the spinor indices. All analytical calculations in Paper IV were performed both by this technique of using the $\mathcal{P}_{^1S_0}$ projector operator, and direct calculations by explicitly including all the diagrams with their crossing fermion lines. The calculations were also further checked by numerical calculations with the FeynArts/FormCalc numerical package.

The analytical result is rather lengthy, but up to zeroth order in $\epsilon \equiv m_W/m_\chi$ and retaining a leading logarithmic term, the resulting photon multiplicity is given by

$$\frac{dN_{\gamma}^{W}}{dx} \approx \frac{d(\sigma v)_{W\gamma}}{(\sigma v)_{WW}} \sim \frac{\alpha_{em}}{\pi} \left[ \frac{4(1-x+x^2)^2 \ln(2/\epsilon)}{(1-x)x} \right. $$

$$- \frac{2(4-12x+19x^2-22x^3+20x^4-10x^5+2x^6)}{(2-x)^2(1-x)x} $$

$$\left. + \frac{2(8-24x+42x^2-37x^3+16x^4-3x^5) \ln(1-x)}{(2-x)^3(1-x)x} \right] $$

$$+ \delta^2 \left( \frac{2x(2-(2-x)x)}{(2-x)^2(1-x)} + \frac{8(1-x) \ln(1-x)}{(2-x)^3} \right) $$

$$+ \delta^4 \left( \frac{x(x-1)}{(2-x)^2} + \frac{(x-1)(2-2x+x^2) \ln(1-x)}{(2-x)^3} \right),$$  

(7.3)

where $x \equiv E_\gamma/m_\chi$ and $\delta \equiv (m_{\chi^\pm} - m_\chi)/m_W$, with $m_{\chi^\pm}$ denoting the chargino mass.

Figure 7.2 shows the photon multiplicity together with a concrete realized minimal supersymmetric model example as specified in Table 7.1.

Two different effects can be singled out to cause increased photon fluxes at the highest energies. The first occurs for large mass shifts $\delta$ between the neutralino and the chargino, whereby the last two terms in Eq. (7.3) dominate. These terms originate from the longitudinal polarization modes of the charged gauge bosons. Such polarization modes are not possible for a $^1S_0$ state with

---

\[ \text{The MSSM parameters specify the input to DarkSUSY [279]. } M_2, \mu, m_A, \text{ and } m_\tilde{f} \text{ are the mass scales for the gauginos, Higgsinos, supersymmetry scalars, and fermions, respectively. } A_f (= A_t = A_b) \text{ is the trilinear soft symmetry breaking parameter, and } \tan \beta = v_u/v_rmd \text{ is the ratio of vacuum expectation values of the two neutral Higgs doublet. All values are directly given at the weak energy scale. For more details of this 7-parameter MSSM, see [280].} \]
**Section 7.2. A Neglected Source of Gamma Rays**

**Figure 7.2:** The photon multiplicity for the radiative process $\chi\chi \rightarrow W^+W^-\gamma$. The dots represent the minimal supersymmetric model given in Table 7.1 as computed with the FormCalc package [256] for a relative neutralino velocity of $10^{-3}$. The thick solid line shows the full analytical result for the pure Higgsino limit of the same model but with zero relative neutralino velocity. Also shown, as dashed and dotted lines, are two pure Higgsino models with a lightest neutralino (chargino) mass of $10\,\text{TeV}$ ($10\,\text{TeV}$) and $1.5\,\text{TeV}$ ($2.5\,\text{TeV}$), respectively. Figure from PAPER IV.

**Table 7.1:** MSSM parameters for the example model shown in Fig. 7.2 and the resulting neutralino mass ($m_{\chi}$), chargino mass ($m_{\chi^\pm}$), Higgsino fraction ($Z_h$), branching ratio into $W$ pairs ($W^\pm$), and neutralino relic density ($\Omega_{\chi}h^2$), as calculated with DarkSUSY [281] and micrOMEGAs [282]. Masses are given in units of TeV.

<table>
<thead>
<tr>
<th>$M_2$</th>
<th>$\mu$</th>
<th>$m_A$</th>
<th>$m_f$</th>
<th>$A_f$</th>
<th>$\tan\beta$</th>
<th>$m_{\chi}$</th>
<th>$m_{\chi^\pm}$</th>
<th>$Z_h$</th>
<th>$W^\pm$</th>
<th>$\Omega_{\chi}h^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>1.5</td>
<td>3.2</td>
<td>3.2</td>
<td>0.0</td>
<td>10.0</td>
<td>1.50</td>
<td>1.51</td>
<td>0.92</td>
<td>0.39</td>
<td>0.12</td>
</tr>
</tbody>
</table>

only two vector particles (remember that the initial state must be in this state due to the Majorana nature of the low velocity neutralino); but when an additional photon is added to the final state, this channel opens up and enhances the photon flux at high energies. Typically MSSM models are, however, not expected to have very large mass shifts $\delta$. The other effect is, on the other hand, dominated by transversely polarized photons. For heavy neutralino masses the $W$ bosons can be treated as light, and the cross section is thus expected to be enhanced in a similar way to the infrared divergence that appears in QED when low-energy photons are radiated away. For kinematical
Supersymmetry and a New Gamma-Ray Signal

Chapter 7

reasons, each low energy $W$ boson is automatically accompanied by a high energy photon. The resulting peak in the spectrum at the highest energies is hence an amusing reflection of QED infrared behavior also for $W$ bosons. The two different effects are illustrated in Fig. 7.2 by the dotted and dashed curves, respectively.

In addition to the internal bremsstrahlung discussed above, secondary gamma rays are produced in the fragmentation of the $W$ pairs, mainly through the production and subsequent decay of neutral pions. Similarly, production of $Z$-bosons (or quarks) results in secondary gamma rays; altogether producing a continuum of photons dominating the gamma flux at lower energies. Previous studies have also shown that there are strong line signals from the direct annihilation of a neutralino pair into $\gamma\gamma$ [269] and $Z\gamma$ [270]. Due to the high mass of the neutralino (as studied here), the two lines cannot be resolved but effectively add to each other at an energy almost equal to the neutralino mass. Adding all contributions, and using the model of Table 7.1, the total spectrum is shown in the left panel of Fig. 7.3.

The practical importance of internal bremsstrahlung is even clearer when taking into account an energy resolution of about 15%, which is a typical value for current atmospheric Čerenkov telescopes in that energy range. The result is a smeared spectrum as shown in the right panel of Fig. 7.3. We can

Figure 7.3: Left panel: The total differential photon distribution from $\chi\chi$ annihilations (solid line) for the minimal supersymmetric model of Table 7.1. Also shown separately is the contribution from internal bremsstrahlung $\chi\chi \rightarrow W^+W^-\gamma$ (dashed), and the fragmentation of mainly the $W$ and $Z$ bosons, together with the $\chi\chi \rightarrow \gamma\gamma$, $Z\gamma$ lines (dotted). Right panel: A zoom in of the same spectra as it would approximately appear in a detector with a relative energy resolution of 15 percent. Figures from PAPER IV.
see that the contribution from the internal bremsstrahlung enhances the flux in the peak at the highest energies by a factor of about two. The signal is also dramatically increased, by almost a factor of 10, at slightly lower energies, thereby filling out the previous ‘dip’ just below the peak. This extra flux at high energies improves the potential to detect a gamma-ray signal. It is worth pointing out that this example model is neither tuned to give the most extreme enhancements, nor is it only pure Higgsinos with $W$ final states that should have significant contributions from this type of internal bremsstrahlung. On the contrary, this type of internal bremsstrahlung can contribute 10 times more to high-energy photon flux than the gamma-ray line, see [275] for an extensive scan of MSSM parameters.

As in the case of the KK dark matter candidate $B^{(1)}$, the internal radiation of a photon in the neutralino annihilation case also gives a very characteristic signature in the form of a very sharp cutoff in the gamma-ray spectrum at an energy equal to the neutralino mass. This is a promising signal to search for and with current energy resolution and with enough statistics the shape of the gamma-ray spectra could even provide a way to distinguish between different dark matter candidates. Figure 7.4 illustrates this by comparing the gamma-ray spectrum of a 1.5 TeV neutralino (specified in Table 7.1) and a 1.5 TeV $B^{(1)}$ KK dark matter candidate.
A possible, and economical, way to incorporate new phenomenology into the standard model would be to enlarge its Higgs sector. One of the most minimal way to do this, which simultaneously gives rise to a dark matter candidate, is the so-called inert doublet model (or inert Higgs model), obtained by adding a second scalar Higgs doublet with no direct coupling to fermions. The lightest of the new appearing inert Higgs particles could, if its mass is between 40 and 80 GeV, give the correct cosmic abundance of cold dark matter. One way to unambiguously confirm the existence of particle dark matter and determine its mass would be to detect its annihilation into monochromatic gamma rays by current or upcoming telescopes. In PAPER VII, we showed that for the inert Higgs dark matter candidate the annihilation signal into such monochromatic $\gamma\gamma$ and $Z\gamma$ final states is exceptionally strong. The energy range and rates for these gamma-ray line signals therefore make them ideal to search for with upcoming telescopes, such as the GLAST satellite. This chapter reviews the inert Higgs dark matter candidate and discusses the origin of these characteristic gamma line signals.

### 8.1 The Inert Higgs Model

Let us start by shortly reviewing why there is a need for a Higgs sector in the first place. In the SM of particle physics, it is not allowed to have any explicit gauge boson or fermion mass terms since that would spoil the underlying $SU(2) \times U(1)$ gauge invariance and lead to a non-renormalizable theory. To circumvent this, we start with a fully gauge invariant theory – with no gauge field or fermion mass terms – and adds couplings to a complex, Lorentz scalar, \footnote{A fundamentally non-renormalizable theory would lack predicability as the canonically appearing divergences from quantum corrections can not be cured by the renormalization procedure of absorbing them into a finite number of measurable quantities.}
SU(2) doublet $\phi$, which spontaneously develops a non-vanishing vacuum expectation value, and thereby breaks the full $SU(2) \times U(1)$ gauge structure and generates particle masses. This is exactly what was technically described in Section 5.3 of the UED model (but, of course, now without the complications of having an extra spatial dimension). The Higgs Lagrangian written down in Eq. (5.17), with the potential (5.18), is in four dimensions the most general setup we can have with one Higgs field $\phi$. As explained in Section 5.3 this Higgs field has four scalar degrees of freedom. Three of the degrees of freedom are absorbed by the new polarization modes of the now massive gauge bosons, and thus only one degree of freedom is left as a physical particle $h$ (the Higgs particle). The mass of the Higgs particle $h$ is a completely free parameter in the SM (which can be measured and constrained).

Another way to express the need for the Higgs particle is that, without it, certain cross sections would grow with the center of mass energy (denoted by $E_{\text{cm}}$) beyond the unitarity limit for large enough $E_{\text{cm}}$. An example is the process $f \bar{f} \to W^+W^-$ into longitudinally polarized $W^\pm$ (those $W$’s that arose by the Higgs mechanism). If the Higgs particle is not included, the only contributing Feynman diagrams are from $s$-channel gauge bosons and $t$- or $u$-channel fermions, which give rise to a term that grows as $m_f^2 E_{\text{cm}}^2$. This is the piece that is exactly canceled by the $s$-channel Higgs boson that couples proportionally to $m_f$. This should make it clear that a physical Higgs, or a similar scalar interaction, must be included to have a sensible theory.

So why is there only one Higgs doublet in the standard model? The inclusion of just one Higgs doublet is the most economical way of introducing masses into the SM, but in principle nothing forbids models with more complicated Higgs sectors. This might at first sound as a deviation from the principle of Occam’s razor, but we will soon see that such extensions can be motivated by its potential to address several shortcomings of the SM and still satisfy theoretical and existing experimental constraints.

A minimal extension of the SM Higgs sector would be to instead have two Higgs doublets, $H_1$ and $H_2$. The Lagrangian for such so-called two-Higgs-doublet models can formally be written as

$$\left| D_\mu H_1 \right|^2 + \left| D_\mu H_2 \right|^2 - V(H_1, H_2), \quad (8.1)$$

The most general gauge invariant, renormalizable potential $V(H_1, H_2)$ that is also invariant under the discrete $Z_2$ symmetry

$$H_2 \to -H_2 \quad \text{and} \quad H_1 \to H_1, \quad (8.2)$$

can be written as

$$V(H_1, H_2) = \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 (H_1^\dagger H_2)^2 + \lambda_5 \text{Re}[\{H_1^\dagger H_2\}^2], \quad (8.3)$$
where $\mu_i^2, \lambda_i$ are real parameters. The latter constraint of a discrete $Z_2$ symmetry is related to the experimental necessity to diminish flavor-changing neutral currents (FCNCs) and CP-violations in the Higgs sector [139,267,268, 283].

The experimental limits on FCNCs are very strong and come from, e.g., studies of the neutral $K^0$ meson. $K^0$ is a bound state (containing a down quark and a strange anti-quark) that would, if FCNCs were mediated by $s$-channel $Z$ or Higgs bosons at tree level, rapidly oscillate into its antiparticle $\bar{K^0}$ or decay directly into lepton pairs. These processes are so rare that they are only expected to be compatible with loop-level suppressed reactions (or in some other way protected, as for example pushing the FCNC mediator to very high masses). In the SM, FCNCs are naturally suppressed as they are forbidden at tree level. Technically, this comes about since the diagonalization of the mass matrix automatically also flavor diagonalizes the Higgs-fermion couplings, as well as the fermion couplings to the photon and the neutral $Z$ gauge boson. This lack of FCNCs would in general no longer be true in the Higgs sector once additional scalar doublets are included that have Yukawa couplings to fermions. In a theorem by Glashow and Weinberg [284], it was shown that FCNCs mediated by Higgs bosons will be absent if all fermions with the same electric charges do not couple to more than one Higgs doublet. Adopting this approach to suppress FCNCs, the scalar couplings to fermions are constrained, but not unique. To specify a model, it is practical to imposes a discrete symmetry. Any such discrete symmetry must necessarily be of the form $H_2 \rightarrow -H_2$ and $H_1 \rightarrow H_1$ (or vice versa) [267]. The $Z_2$ symmetry can then be used to design which Yukawa couplings are allowed or not. This is exactly the $Z_2$ symmetry that was already incorporated in the potential given in Eq. (8.3).

In, what is called, a Type I two-Higgs-doublet model, the fermions couple only to the first Higgs doublet $H_1$, and there are no couplings between fermions and $H_2$. This is the same as saying that the Lagrangian is kept invariant under the $Z_2$ symmetry that takes $H_2 \rightarrow -H_2$ and leaves all other fields unchanged. This is the type of model we will be interested in here. A Type II model is when the down-type fermions only couple directly to $H_1$ and up-type fermions only couple directly to $H_2$ (corresponding to the appropriate choice for the $Z_2$ transformation of the right-handed fermion fields, $u_R \rightarrow -u_R$). The minimal supersymmetric models belongs to this Type II class of models. Other choices where quarks and leptons are treated in some asymmetrical way

---

$^\dagger$ An additional term is actually possible, but can always be eliminated by redefining the phases of the scalars [267,268].

$^\ddagger$ Actually, even the loop-level FCNCs in the SM need to be suppressed. In 1970 Glashow, Iliopoulos, and Maiani realized that this could be achieved if quarks come in doublets for each generation (the GIM mechanism) [139]. Their work was before the detection of the charm quark, and therefore predicted this new quark to be the doublet companion for the already known strange quark.
could in principle also be possible.

Coming back to the Type I model, where only Yukawa terms involving $H_1$ are allowed. This means that $H_1$ must develop a nonzero vacuum expectation value $v \neq 0$ to generate fermion and gauge masses. In other words, the potential must have a minimum for $H_1 \neq 0$. For $H_2$ there are, however, two choices for its potential. Either $H_2$ also develops a vacuum expectation value $v_{H_2} \neq 0$ by spontaneous symmetry breaking and the potential has a global minimum for $H_2 \neq 0$, or the $Z_2$ symmetry $H_2 \rightarrow -H_2$ is unbroken and the potential has a global minimum at $H_2 = 0$. (Note that in general this latter case is not the $v_{H_2} \rightarrow 0$ limit of the $v_{H_2} \neq 0$ case.)

The model that contains our dark matter candidate is the latter of the two Type I two-Higgs-doublet models that have $v_{H_2} = 0$. In other words, this is an ordinary two-Higgs-doublet model with the $H_2 \rightarrow -H_2$ symmetry unbroken. The $H_1$ field is identified as essentially the SM Higgs doublet – it gets a vacuum expectation value and gives masses to the $W$, $Z$ and fermions exactly as in the SM. On the other hand the $H_2$ does not get any vacuum expectation value and does not couple directly to fermions. This $H_2$ will be called the inert Higgs doublet and the model the inert doublet model (IDM).

The origin of this IDM goes back to at least the 1970s [285] when the different possibilities for the two-Higgs-doublet models were first investigated. The IDM has recently received much new interest. Besides providing a dark matter candidate [286, 287], this type of model has the potential to allow for a high Higgs mass [286], generate light neutrinos and leptogenesis (see, e.g., [288] and references therein), as well as break electroweak symmetry radiatively [289].

**The New Particles in the IDM**

Let us set up some notation and at the same time present how many free parameters and physical fields this inert doublet model contains. The two Higgs doublets will be parameterized according to

\begin{align*}
H_1 &= \frac{1}{\sqrt{2}} \left( G^+ \sqrt{v + h + iG^0} \right) \\
H_2 &= \frac{1}{\sqrt{2}} \left( H^+ \sqrt{H^0 + iA^0} \right) 
\end{align*}

(8.4)

(8.5)

where $G^\pm, H^\pm$ are complex scalar fields while $h, G^0, H^0$ and $A^0$ are real.\footnote{We can always use the freedom of $SU(2) \times U(1)$ rotations to get the vacuum}

The name inert Higgs doublet might be found misleading as it is neither completely inert (since it has ordinary gauge interactions) nor contributes to the Higgs mechanism to generate masses. The name dark scalar doublet has later been proposed, but we will here stick to the nomenclature used in PAPER VII and call it the inert Higgs or inert scalar.

\footnote{I have here slightly changed the notation for the scalar fields compared to Section 5.3}
expectation value $v$ for $H_1$ to be real valued and in the lower component of the doublet. $G^+$ and $G^0$ constitute Goldstone fields that in unitarity gauge can be fixed to zero. After giving mass to the gauge bosons, five out of the original eight degrees of freedom in $H_1$ and $H_2$ remain. Besides the SM Higgs particle ($h$), the physical states derived from the inert doublet $H_2$ are thus two charged states ($H^\pm$) and two neutral; one CP-even ($H^0$) and one CP-odd ($A^0$). The $h$ field in the $H_1$ doublet will be referred to as the SM Higgs particle and the particle fields in $H_2$ as the inert Higgs particles. The corresponding (tree level) masses are given by:

$$
\begin{align*}
    m_h^2 &= -2\mu_1^2 \\
    m_{H^0}^2 &= \mu_2^2 + (\lambda_3 + \lambda_4 + \lambda_5)v^2 \\
    m_{A^0}^2 &= \mu_2^2 + (\lambda_3 + \lambda_4 - \lambda_5)v^2 \\
    m_{H^\pm}^2 &= \mu_2^2 + \lambda_3 v^2.
\end{align*}
$$

Measurements of the gauge boson masses determine $v = 175$ GeV, and we are left with only 6 free parameters in the model. A convenient choice is to work with $m_h, m_{H^0}, m_{A^0}, m_{H^+}, \mu_2$ and $\lambda_2$.

**Heavy Higgs and Electroweak Precision Bounds**

One of the original motivations for the IDM was that it could incorporate a heavy SM Higgs particle. Let us briefly review why this might be of interest and how this is possible, in contrast to the SM and the MSSM.

In the SM, the Higgs boson acquires an ultraviolet divergent contribution from loop corrections which will be, at least, of the same size as the energy scale where potential new physics comes in to cancel divergences. With no such new physics coming in at TeV energies, a tremendous fine-tuning is required to keep the Higgs mass below the upper limit of 144 GeV (95% confidence level), determined by electroweak precision tests (EWPT). [290]**. Low-energy supersymmetry provides such new divergence-canceling physics; and this is one of the strongest reasons to expect that physics beyond the SM will be found by the LHC at CERN. However, in the MSSM, the lightest Higgs particle is naturally constrained to be lighter than $\sim 135$ GeV [152], and some amount of fine-tuning [291] is actually already claimed to be needed to fulfill the experimental lower bound of roughly 100 GeV from direct Higgs searches [152, 290]. This suggestive tension has motivated several studies on...
the theoretical possibilities to allow for large Higgs masses both within supersymmetry and other extensions of the SM (see, e.g., [286, 291] and references therein). In [286] it was shown that the IDM can allow for a heavy SM-like Higgs (i.e., \(h\)). This was a basic motivation for the model, as it meant that the need for divergence canceling physics could be pushed beyond the reach of the upcoming LHC accelerator without any need for fine-tuning. While this argument of less fine-tuning (or improved naturalness) [286] has been disputed [292], the mere fact that the IDM allows for the SM Higgs mass to be pushed up to about 500 GeV is interesting in itself, as it might provide a clear distinction from the SM and the MSSM Higgs (as well as having an impact on the expected gamma-ray spectrum from annihilation of \(H^0\)s as discussed in Section 8.2).

To allow for a heavy SM-like Higgs, the upper mass limit of about 144 GeV from electroweak precision tests must be avoided. The so-called Peskin-Takeuchi parameters, denoted \(S\), \(T\), and \(U\), are measurable quantities constructed to parameterize contributions (including beyond SM physics) to electroweak radiative corrections, such as the loop-diagram induced contribution to self-energies of the photon, \(Z\) boson, and \(W\) boson, and the Weinberg angle [293]. These \(S\), \(T\), and \(U\) parameters are defined such that they vanish for a reference point in the SM (i.e., a specific value for the top-quark and Higgs masses). Deviations from zero would then signal the existence of new physics, or set a limit on the Higgs mass when the SM is assumed. Instead of the \(T\) parameter the \(\rho\) parameter is sometimes used, which is defined as

\[
\rho = \frac{m^2_W}{m^2_Z \cos \theta_w (m_W)}.
\]

A deviation of \(\rho\) from 1 measures how quantum corrections alter the tree level SM link between the \(W\) and \(Z\) boson masses \([267, 293]\). In fact, in most cases \(T\) represents just the shift of the \(\rho\) parameter

\[
\Delta \rho = \rho - 1 = \alpha T.
\]

Electroweak precision measurements of the \(S\), \(T\) and \(U\) parameters limit the Higgs boson mass. A heavy \(h\) of a few hundred GeV would produce a too small value for the observable \(T\), whereas the \(S\) and \(U\) parameters are less sensitive to the Higgs mass [286], see Fig. 8.1. However, a heavy Higgs can be consistent with the electroweak precision tests if new physics produce a compensating positive \(\Delta T\). For a \(m_h = 400 - 600\) GeV the compensation \(\Delta T\) must be \(\Delta T \approx 0.25 \pm 0.1\) to bring the value back near the central measured point and within the experimental limits [286]. In [286] it was found that neither the \(S\) nor the \(U\) parameter is affected much by the extra contribution from the IDM, but that the \(T\) parameter is approximately\(^\dagger\) shifted according to:

\[
\Delta T \approx \frac{1}{24\pi^2\alpha v^2} \left( m_{H^+} - m_{A^0} \right) \left( m_{H^+} - m_{H^0} \right).
\]

We thus see that a heavy SM Higgs that usually produces too negative values of \(\Delta T\) can be compensated for by the proper choice of masses for the inert

\(^\dagger\) This approximation is within a few percent accuracy for

\[1 \leq m_{H^\pm}/m_{H^0}, m_{H^\pm}/m_{A^0}, m_{A^0}/m_{H^0} \leq 3\]
Figure 8.1: Dependence of the $S, T$ parameters on the Higgs mass ($m_h$) within the standard model. The thick black band marks $m_h = 400 - 600$ GeV. The top quark mass $m_t$ range within the experimental bounds. Figure adapted from [286].

Higgs particles. For example, for a $m_h = 500$ GeV the required compensation is $\Delta T \approx 0.25 \pm 0.1$, and the masses of the inert scalar masses in Eq. (8.7) should satisfy

\[(m_{H^+} - m_{A^0})(m_{H^+} - m_{H^0}) = M^2, \quad M = 120^{+20}_{-30} \text{GeV}. \tag{8.8}\]

This means that a Higgs mass $m_h$ of up to about 500 GeV can be allowed in the IDM if only the inert Higgs masses are such that they fulfill Eq. (8.8).

As the Higgs mass is increased, the quartic scalar interactions become stronger, and the maximal scale at which perturbation theory can be used decreases. To have a natural perturbative theory up to, say, 1.5 TeV (which is about the highest new energy scale we can have without fine-tuning the Higgs mass [286]) the Higgs mass cannot be heavier than about $m_h = 600$ GeV [286].

More Constraints

There are several other constraints that must be imposed, besides the electroweak precision measurements bounds discussed above. The following constraints on the six free parameters are also used (which are the same constraints as used in PAPER VII):
• Theoretically, the potential needs to be bounded from below in order to have a stable vacuum, which requires:

\[
\lambda_1, \lambda_2 > 0, \\
\lambda_3, \lambda_3 + \lambda_4 - |\lambda_5| > -2\sqrt{\lambda_1 \lambda_2}.
\] (8.9)

• To trust perturbation theory calculations, at least up to an energy scale of some TeV, the couplings strengths cannot be allowed to become too large. Here and in PAPER VII we followed the constraints found in [286], which can be summarized, as a rule of thumb, in that no couplings should become larger than \(\lambda_i \sim 1\) (see [286] for more details).

• In order not to be in conflict with the observed decay width of the \(Z\) boson we should impose that \(m_{H^0} + m_{A^0} \gtrsim m_Z\) (see PAPER VII and [294]).

• No full analysis of the IDM has been done with respect to existing collider data from the LEP and the Tevatron experiments. However, comparison with similar analyses of supersymmetry enable at least some coarse bounds to be found. The summed mass of \(H^0\) and \(A^0\) should be greater than about 130 GeV [295], or the mass split must be less than roughly 10 GeV [286, 295]. Similarly, the mass of the charged Higgs scalars \(H^\pm\) is constrained by LEP data to be above about 80 GeV [295, 296].

• To explain the dark matter by the lightest inert particle (LIP), its relic abundance should fall in the range \(0.094 < \Omega_{CDM} h^2 < 0.129\). See Section 8.2 for more details.

• Direct detection searches of dark matter set limits on scattering cross sections with nucleons. At tree level, there are two spin-independent interactions whereby \(H^0\) could deposit kinetic energy to a nuclei \(q\) in direct search detectors: \(H^0 q \xrightarrow{Z} A^0 q\) and \(H^0 q \xrightarrow{h} H^0 q\). The former process, with a \(Z\) exchange, is very strong and is forbidden by current experiment limits [286, 296, 297]. However, this process becomes kinematically forbidden if the mass splitting is more than a few 100 keV, as typically the kinetic energy of the dark matter candidate \(H^0\) would then not be enough to produce an \(A^0\) (this thus excludes the \(\lambda_5 \rightarrow 0\) limit; see Eq. 8.6). With this process kinematically excluded, the signals from Higgs-mediated scattering is roughly two orders of magnitude below any current limits. The next generation of detectors could potentially reach this sensitivity [296].

Also naturalness could be imposed, i.e., parameters should not be tuned to extreme precision. In PAPER VII, we applied the naturalness constraints found in [286], but to be less restrictive, we relaxed their parameter bounds.
by a factor two because of the somewhat arbitrariness in defining natural-
ness (this constraint is not crucial for any of the general results here or in
PAPER VII).

When it comes to the upcoming LHC experiment, the IDM should be seen
in the form of both missing transverse energy and an increased width of the
SM Higgs [286, 294, 295].

8.2 Inert Higgs – A Dark Matter Candidate

The existence of the unbroken $Z_2$ symmetry in the IDM, where the inert Higgs
doublet are attributed negative $Z_2$-parity and all SM particles have positive
parity, means that none of the inert Higgs particles can directly decay into
only SM particles. The lightest inert particle (LIP) is therefore a (cosmolog-
ically) stable particle, which is a first necessity for a dark matter candidate.
Furthermore, the LIP should be electric and color neutral to not violate any
of the strict bounds on charged dark matter [198, 199]. The only choices for an
inert Higgs dark matter particle are therefore $H^0$ or $A^0$. Although the roles
of $H^0$ and $A^0$ are interchangeable for all the results, let us for definiteness
choose $H^0$ as the LIP.

The next crucial step is to see if this $H^0$ candidate can give the right
relic density to constitute the dark matter. In [296] it was shown that $H^0$
can constitute all the dark matter if its mass is roughly $10 - 80$ GeV (or
above $500$ GeV if parameters are particularly fine-tuned). However, this study
was made for SM Higgs masses of 120 and 200 GeV which, although giving
higher gamma rates, deviates from one of the motivation for the model – a
raised Higgs mass [286]. The setup we had in PAPER VII is based on a 500
GeV SM Higgs. $H^0$ relic density calculations were therefore performed. This
was done by implementing the proper Feynman rules from the Lagrangian in
Eq. (8.1) into the Feynman diagram calculator FormCalc [256]. Cross-section
calculations with FormCalc were then interfaced with the DarkSUSY [298] relic
density calculator. This allowed us to accurately calculate the relic density
for any given choice of IDM parameters after imposing existing experimental
constraints. The correct relic density is still roughly obtained for masses in
the range of $40 - 80$ GeV, and we will next see why this result is almost
independent of the SM Higgs mass.

Typically the relic density is governed by the cross section for annihilating
two $H^0$. For masses $m_{H^0}$ above the SM Higgs $m_h$ ($> m_Z, m_W$) the annihi-
lation channels are given by the diagrams in Fig. 8.2. If $m_{H^0}$ is above the $W$
mass, then the cross sections from the middle row diagrams dominate. These
diagrams produce very large annihilation cross sections, and therefore the $H^0$
relic density becomes too small to constitute the dark matter. For masses
below the $W$ mass, only the diagrams in the bottom line will contribute (as
the heavier $W$ and $Z$ bosons are generically not energetically allowed to be
produced during freeze-out). For these ‘low’ $H^0$ masses, the tree-level anni-
hilation rates are small, especially for high SM Higgs masses, and you could tend to assume that the relic density would be far too high. However, coan- 
nihilations with the next-to-lightest inert scalar allow us to reach the correct relic abundance (the right-hand-side diagram in the bottom row of Fig. 8.2).

It is mainly this coannihilation process which regulates the relic density, and this process is completely independent of the SM Higgs mass.

This is an interesting aspect of the IDM – that the $H^0$ mass generically has to be just below the charged gauge boson mass because the relatively strong coupling to $W^+W^-$ would otherwise give a too low relic density to account for the dark matter – and, as we next will see, this will also affect the indirect detection signal from gamma rays.
8.3 Gamma Rays

Continuum

The dark matter particle in this model is thus the $H^0$, with a mass below $m_W$. This means that only annihilations into fermions lighter than $m_{H^0}$ are accessible at tree level, and the only contributing Feynman diagram is the bottom left one of Fig. 8.2. The annihilation rate is calculated to be

$$v_{rel\sigma_{ff}} = \frac{N_c \pi \alpha^2 m_f^2}{\sin^4 \theta_W m_W^4} \frac{(1 - \frac{4m_f^2}{s})^{3/2}(m_{H^0}^2 - \mu_2^2)^2}{(s - m_h^2)^2 + m_h^4 \Gamma_h^2},$$

(8.10)

where $N_c$ is a color factor (which equals 1 for leptons and 3 for quarks), $\sqrt{s}$ is the center of mass energy, $\alpha$ the fine-structure constant, $m_W$ the $W$ boson mass, $\theta_W$ the weak mixing angle, $\Gamma_h$ the decay width of $h$, and $m_f$ the final state fermion mass.

The heaviest kinematically allowed fermion state will dominate the tree-level annihilation channels, since the cross section is proportional to $m_f^2$. Hence, in our case of interest, the bottom quark final states are the most important process at tree level, and with some contributions from charm quarks and $\tau$ pairs. Although the running of lepton masses can be safely neglected, the QCD strong interaction corrections to quark masses might be substantial, and we therefore take the leading order correction into account by adjusting the running quark masses [267, 299] to their values at the energy scale of the physical process ($\sim 2m_{H^0}$). Quark pairs will, as already described in the case of the KK and supersymmetric dark matter, hadronize and produce gamma rays with a continuum of energies. Because of the much harder gamma spectrum from the decay of $\tau$-leptons, these could also contribute significantly at the highest energies, despite their much lower branching ratio. Pythia (version 6.4) [300] was used to calculate the photon spectrum in the process of hadronization.

**Gamma-Ray Lines**

As has been said, the $H^0$ couplings are relatively strong to $W^+W^-$ (i.e., ordinary gauge couplings), which forces the mass of $H^0$ to be below $m_W$ if it is to explain the dark matter. Virtual gauge bosons close to threshold could, on the other hand, significantly enhance loop processes producing monochromatic photons (see Fig. 8.3). In PAPER VII, we showed that this is indeed correct and found `smoking gun’ line signals for the $H^0$ dark matter from the final states $\gamma\gamma$ and, when kinematically allowed, $Z\gamma$. This, in combination with small tree-level annihilation rates into fermions, makes the gamma lines a most promising indirect detection signal.

Let us see what these important line signals from direct annihilation of $H^0$ pairs into $\gamma\gamma$ and $Z\gamma$ look like. First of all these spectral lines would show
Figure 8.3: Typical contributing Feynman diagrams for the annihilation process $H^0 H^0 \rightarrow \gamma \gamma$. Due to unsuppressed couplings to $W^\pm$, virtual $W^\pm$ in the intermediate states are expected to give the largest contribution to this process.

up as characteristic dark matter fingerprints at the energies $m_{H^0}$ and $m_{H^0} - m_Z^2/4m_{H^0}$, respectively. The $Z\gamma$ line might not be strictly monochromatic due to the Breit-Wigner width of the $Z$ mass, but can still be strongly peaked. The potential third gamma line from $h\gamma$ is forbidden for identical scalar particle annihilation, as in the IDM, due to gauge invariance.

We could also note that when the branching ratio into $Z\gamma$ becomes large, the subsequent decay of the $Z$ boson significantly contributes to the continuum gamma-ray spectrum. The full one-loop Feynman amplitudes were calculated by using the numerical FormCalc package [256] – after the Feynman rules for the IDM had been derived and implemented.

To show the strength of the gamma-ray lines and the continuum spectrum for different parameter choices, four IDM benchmark models are defined, shown in Table 8.1. The two models III and IV have a low Higgs mass and could therefore be directly comparable to the relic density calculations done in [296]. Annihilation rates, branching ratios and relic densities for these models are given in Table 8.2. As an illustrative example, Fig. 8.4 shows the predicted gamma spectrum for model I.

The spectral shape with its characteristic peaks in the hitherto unexplored energy range between 30 and 100 GeV is ideal to search for with the GLAST
Section 8.3. Gamma Rays

$\gamma = \frac{E_{\gamma}}{m_{H^0}}$

$10^{-4}$

$10^{-2}$

$10^{-4}$

$0.01$ $0.1$ $1$

$x = E_{\gamma}/m_{H^0}$

**Figure 8.4:** The total differential photon distribution from annihilations of an inert Higgs dark matter particle (solid line). Shown separately are the contributions from $H^0H^0 \rightarrow b\bar{b}$ (dashed line), $\tau^+\tau^-$ (dash-dotted line) and $Z\gamma$ (dotted line). This is for the benchmark model I in Table 8.1. Figure from Paper VII.

Experiment [301]. In Fig. 8.5 this is illustrated by showing the predicted fluxes from a $\Delta \Omega = 10^{-3}\text{sr}$ region around the direction of the galactic center together with existing observations in the same sky direction. In this figure, a standard NFW density profile, as specified in Table 2.1 and with a normalization density of 0.3 GeV/cm$^3$ at 8.5 kpc, is the underlying assumption for the dark matter halo for our Galaxy. With the notation of Eq. (2.15), this correspond to $J \times \Delta \Omega \sim 1$ for $\Delta \Omega = 10^{-3}\text{sr}$. Processes such as adiabatic compression, which we discussed in Chapter 2, could very well enhance the dark matter density significantly near the galactic center. Therefore, the predicted flux compared to a pure NFW profile could very well be scaled up by a large ‘boost factor’. The boost factors used for the shown signals are also displayed in Fig. 8.5. Since the continuum part of the expected spectrum is within the energy range covered by EGRET satellite, there is an upper limit on

<table>
<thead>
<tr>
<th>Table 8.1: IDM benchmark models. (In units of GeV.)</th>
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<tbody>
<tr>
<td>Model</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>I</td>
</tr>
<tr>
<td>II</td>
</tr>
<tr>
<td>III</td>
</tr>
<tr>
<td>IV</td>
</tr>
</tbody>
</table>
Figure 8.5: Predicted gamma-ray spectra from the inert Higgs benchmark models I and II as seen by GLAST (solid lines). The predicted gamma flux is from a $\Delta \Omega = 10^{-3}$ sr region around the direction of the galactic center assuming an NFW halo profile (with boost factors as indicated in the figure) and convolved with a 7% Gaussian energy resolution. The boxes show EGRET data (which set an upper limit for the continuum signal) and the thick line H.E.S.S. data in the same sky direction. The GLAST sensitivity (dotted line) is here defined as 10 detected events within an effective exposure of 1 m$^2$yr within a relative energy range of ±7%. Figure from Paper VII.

Table 8.2: IDM benchmark model results.

<table>
<thead>
<tr>
<th>Model</th>
<th>$v\sigma_{tot}^{\nu\nu}$ [cm$^3$s$^{-1}$]</th>
<th>Branching ratios [%]: $\gamma\gamma$, $Z\gamma$, $b\bar{b}$, $c\bar{c}$, $\tau^+\tau^-$</th>
<th>$\Omega_{CDM}h^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$1.6 \times 10^{-28}$</td>
<td>36 33 26 2 3</td>
<td>0.10</td>
</tr>
<tr>
<td>II</td>
<td>$8.2 \times 10^{-29}$</td>
<td>29 0.6 60 4 7</td>
<td>0.10</td>
</tr>
<tr>
<td>III</td>
<td>$8.7 \times 10^{-27}$</td>
<td>2 2 81 5 9</td>
<td>0.12</td>
</tr>
<tr>
<td>IV</td>
<td>$1.9 \times 10^{-26}$</td>
<td>0.04 0.1 85 5 10</td>
<td>0.11</td>
</tr>
</tbody>
</table>
fact, these signals would potentially be visible even without any boost at all (especially if the background is low, as might be the case if the EGRET signal is a galactic off-center source as indicated in [302]). Also shown in Fig. 8.5 are the data from the currently operating air Čerenkov telescope H.E.S.S. [258]. The H.E.S.S. data are within a solid angle of only $\Delta \Omega = 10^{-5}$ sr, but since the gamma-ray flux is dominated by a point source in the galactic center, a larger solid angle would not affect the total flux much. Future air Čerenkov telescopes with lower energy thresholds and much larger effective area than GLAST are planned and will, once operating, be able to cover the entire region of interest for this dark matter candidate.

To go beyond just a few example models, we performed in Paper VII a systematic scan over the parameters in the IDM, for a SM higgs mass $m_h = 500$ GeV, and calculated the cross section into gamma lines. The constraints mentioned in Section 8.1 allowed us to scan the full parameter space for dark matter masses below the $W$ threshold of 80 GeV. The dependence on $m_{H^\pm}$ and $\lambda_2$ is small, and we chose to set these equal to $m_{H^0} + 120$ GeV (to fulfill precision tests) and 0.1, respectively. Importantly, we note that the right relic density is obtained with a significant amount of early Universe coannihilations with the inert $A^0$ particle. The resulting annihilation rates into $\gamma\gamma$ and $Z\gamma$ are shown in Fig. 8.6. The lower and upper $m_{H^0}$ mass bounds come from the accelerator constraints and the effect on the relic density by the opening of the $W^+W^-$ annihilation channel, respectively. For comparison, the same figure also shows the corresponding annihilation rates for the neutralino ($\chi$) within MSSM. The large lower-right region is the union of the range of cross sections covered by the annihilation rates $2\sigma v\gamma\gamma$ and $\sigma vZ\gamma$ as obtained with a large number of scans within generous MSSM parameter bounds with the DarkSUSY package [298]. The stronger line signal and smaller spread in the predicted IDM flux are caused by the allowed unsuppressed coupling to $W$ pairs that appear as virtual particles in contributing Feynman loop diagrams. In the MSSM, on the other hand, high $\gamma\gamma$ and $Z\gamma$ rates are harder to achieve [301, 303–305], at least while still satisfying both relic density and LEP constraints for the masses of interest here.

The IDM’s true strength lies in its simplicity and its interesting phenomenology. The lightest new particle in the model typically gives a WIMP dark matter candidate, once coannihilations are included, and the model allows a SM Higgs mass of up to at least a few hundred GeV without contradicting LEP precision tests. These are two typical features of the model, but the IDM also shows the typical dark matter properties of having weak interactions and electroweak masses. The main reasons why this scalar dark matter model gives such particularly strong gamma lines are that: (1) The dark matter mass is just below the kinematic threshold for $W$ production in the zero velocity limit. (2) The dark matter candidate almost decouples from fermions (i.e., couples only via SM Higgs exchange), while still having ordinary gauge couplings to the gauge bosons. In fact, these two properties by
Figure 8.6: Annihilation strengths into gamma-ray lines $2\nu\sigma_{\gamma\gamma}$ (upper band) and $\nu\sigma_{Z\gamma}$ (middle band) from the scan over the IDM parameter space. For comparison the lower-right region indicates the corresponding results within the minimal supersymmetric standard model as obtained with the DarkSUSY package [298]. This lower region is the union of $N_{\gamma}\nu\sigma$ from $\chi\chi \rightarrow \gamma\gamma$ and $\chi\chi \rightarrow Z\gamma$. Figure from PAPER VII.

themselves could define a more general class of models for which the IDM is an attractive archetype because of its simplicity with only six free parameters (including the SM Higgs mass).
Have Dark Matter Annihilations Been Observed?

Over the past several years, observed anomalies in the spectra from cosmic photons and anti-particles have been suggested to originate from dark matter annihilations. One strongly promoted claim of a dark matter annihilation signal is based on the anomaly that the EGRET experiment found in the diffuse galactic gamma-ray emission. For gamma-ray energies above roughly 1 GeV the data seems to show, in all sky directions, an excess of flux compared to what is conventionally expected. It has been realized that this excess in the spectrum could be due to dark matter annihilations. De Boer and collaborators [1,306–308] have therefore proposed a dark matter distribution in our Galaxy to explain this observed gamma-ray anomaly. Internal consistency of such a dark matter explanation must, however, be investigated. Generically, the same physical process producing the diffuse gamma rays also produces antiprotons. In PAPER V, we therefore studied this proposed dark matter model to see if it is compatible with measured antiproton fluxes. Using current, and generally employed, propagation models for the antiprotons, we showed that this dark matter explanation is excluded by a wide margin when checked against measured antiproton fluxes.

9.1 Dark Matter Signals?

Observations that have been proposed to be the product of dark matter annihilations include the cosmic positron spectrum measured by HEAT, the 511 keV emission from the galactic Bulge measured by INTEGRAL, the microwave excess from the galactic Center observed by WMAP, and the diffuse galactic and extragalactic gamma-ray spectra measured by EGRET. All of these potential dark matter signals are still very speculative. For a recent review, and references, see, e.g., [309]. There has also been a claim of a direct
detection signal of dark matter by the DAMA collaboration [227, 228], but this result is very controversial as other similar experiments have not been able to reproduce their result [206, 207, 225, 226, 229, 230].

It is beyond the scope of this thesis to go through all these potential hints of a dark matter signal in detail. We will only focus on scrutinizing (as in PAPER V) the perhaps most strongly promoted claim in the last few years – that the GeV anomaly in the diffuse galactic gamma-ray spectrum, measured by the EGRET satellite, could be well explained by a signal from WIMP dark matter annihilations.

9.2 The Data

Between the years 1991 and 2000, the Energetic Gamma Ray Emission Telescope EGRET [310], onboard the Compton gamma ray observatory, took data. During this period, it made an all-sky survey of the gamma-ray flux distribution for energies mainly between 0.03 and 10 GeV.

Diffuse emission from the Milky Way completely dominates the gamma-ray sky. The main part of the emission originates from interactions of cosmic rays (mostly protons and electrons) with the gas and radiation fields in the interstellar medium. Any calculation of the galactic diffuse emission is therefore primarily dependent on the understanding of the cosmic-ray spectra and interstellar gas and radiation fields throughout our Galaxy. Cosmic rays are believed to originate mainly from acceleration processes in supernovae, and propagate through large parts of the Galaxy, whereas the radiation fields mainly come from the CMB and photons from stars inside the Galaxy. The physical processes involved in the cosmic-ray interactions, which produce the gamma rays, are mainly the production and subsequent decay of $\pi^0$, inverse Compton scattering, and bremsstrahlung.

The first detailed analysis of the diffuse gamma rays was done by Hunter et al. [311] (using EGRET data in the galactic plane: latitudes $|b| \leq 10^\circ$ in galactic coordinates). The main assumptions in their analysis were that the cosmic rays are of galactic origin, that there exists a correlation between the interstellar matter density and the cosmic-ray density, and that the cosmic-ray spectra throughout our Galaxy are the same as measured in the solar vicinity. Their result confirmed that the agreement between the EGRET observed diffuse gamma rays and the expectations are overall good. However, at energies above 1 GeV the measured emission showed an excess over the expected spectrum. This excess is known as the EGRET ‘GeV anomaly’.

Later Strong, Moskalenko, and Reimer [312–315] developed a numerical code, GALPROP [316], for calculating the cosmic-ray propagation and diffuse gamma-ray emission in our Galaxy. Their code includes observational data on the interstellar matter, and a physical model for cosmic-ray propagation. The model parameters are constrained by the different existing observations, such as cosmic-ray data on B/C (i.e., the Boron to Carbon ratio, which relates
secondary to primary cosmic rays). This makes it possible to derive a diffuse gamma-ray spectrum in all sky directions. In the ‘conventional scenario’ in [315] the existence of the EGRET GeV anomaly was confirmed. However, by allowing for a spatial variation of the electron and proton injection spectra, it was pointed out that an ‘optimized scenario’ gives a good description of the diffuse gamma-ray sky [315]. To explain the GeV anomaly, this optimized model allows for a deviation of the cosmic-ray spectrum (within observational uncertainties) from what is measured in the solar vicinity. The electron injection spectrum is made slightly harder, with a drastic drop at 30 GeV, and at the same time normalized upward with a factor of about 5 compared to the measured spectrum in the solar vicinity. The proton injection spectrum is also made harder, and the normalization is increased by a factor 1.8 at 100 GeV. The derived spectra, in the conventional and the optimized model, compared to observational data are shown in Fig. 9.1.

The origin of the potential GeV anomaly is still a matter of debate. There are mainly three proposed explanations of its origin: (i) it is of conventional astrophysical origin, like in the mentioned optimized cosmic-ray model or due to unresolved conventional sources, (ii) it is an instrumental artefact due to uncertainties in the instrument calibration, or (iii) it is caused by dark matter
annihilations.

That the GeV anomaly could be due to a systematic instrumental artefact has recently been discussed by Stecker et al. [317]. They argue that the lack of spatial structure in the excess related to the galactic plane, galactic center, anti-center, or halo, indicates that the GeV anomaly above $\sim$1 GeV is more likely due to a systematic error in the EGRET calibration. Although not at all in contradiction with a calibration problem, it is notably that in a recent reanalysis [318] of the EGRET instrument response the GeV anomaly was found to be even larger. This reanalysis was done by modifying the GLAST simulation software, to model the EGRET instrument, and indicated that previously unaccounted instrumental effects mistakenly lead to the rejection of some gamma-ray events.

The alternative explanation that the GeV anomaly, in all sky directions, is a result of dark matter annihilations has been promoted in a series of papers by de Boer et al., e.g., [1,306–308]. The idea to use the gamma-ray excess as a dark matter annihilations signal has a long history (at least [97,305,319,320]), but de Boer et al. have extended this idea to claim that all the diffuse galactic gamma rays detected above 1 GeV by EGRET, irrespective of the direction, has a sizeable dark matter contribution.

### 9.3 The Claim

Specific supersymmetric models have been proposed as examples of viable candidates that can explain the EGRET GeV anomaly [307]. The precise choice of dark matter candidate is in itself not crucial, as long as its dark matter particles are non-relativistic, have a mass between 50 and 100 GeV, and annihilate primarily into quarks that then produce photons in their process of hadronization. In these cases, the predicted gamma-ray spectrum has the right shape to be added to the ‘conventional’ cosmic-ray model in [315] in order to match the the GeV anomaly; see Fig. 9.2.

The price to pay is, however, a rather peculiar dark matter halo of the Milky Way, containing massive, disk concentrated rings of dark matter besides the customary smooth halo. The dark matter distribution de Boer et al. propose is a profile with 18 free parameters. With the given proposal, a best fit to the EGRET data is performed. This is possible to do because gamma rays have the advantage of pointing back directly to their sources in the Galaxy, and since the gamma-ray spectral shape from dark matter annihilations is presumed to be known (and distinct from the conventional background). The sky-projected dark matter distribution can therefore be extracted from the EGRET observations. The deduced dark matter profile in [1] has the following main ingredients:

- a triaxial smooth halo in the form of a modified isothermal sphere, but somewhat flattened in the direction of the Earth and in the $z$-direction ($i.e.$, the height above the galactic plane),
Figure 9.2: Fit of the shapes of background and dark matter annihilation signal to the EGRET data in the inner part of the galactic disk. The light shaded (yellow) areas indicate the background using the shape of the conventional GALPROP model [315], while the dark shaded (red) areas are the signal contribution from dark matter annihilation for a 60 GeV WIMP mass. The reduced $\chi^2$ [75] for the background only and the corresponding fit including dark matter is indicated in the figure. Note the smaller error bars in this figure compared to in Fig. 9.1 - this is due to the disagreement in how to take into account systematic and correlated errors (see PAPER V for details). Figure adopted from [1].

- an inner ring at about 4.15 kpc with a density falling off as $\rho \sim e^{-|z|/\sigma_z,1}$, where $\sigma_z,1 = 0.17$ kpc, and

- an outer ring at about 12.9 kpc with a density falling off as $\rho \sim e^{-|z|/\sigma_z,2}$, where $\sigma_z,2 = 1.7$ kpc.

Fig. 9.3 shows this dark matter profile: The strong concentration of dark matter to the disk (upper panel), as well as the ring structure of the model (lower panel), is clearly seen.

A 50-100 GeV dark matter candidate, with a distribution as described, constitutes the claimed explanation of the EGRET GeV anomaly. In addition, which will become important later, this model also has to boost the predicted gamma-ray flux, in all sky directions, by a considerable ‘boost factor’ of around 60. With these ingredients, a good all sky fit to the gamma-ray spectra, as in Fig. 9.2, is obtained.
Figure 9.3: The dark matter distribution in the halo model of de Boer et al. [1]. **Upper panel:** The concentration of dark matter along the galactic disk. The right figure displays the density dependence as a function of the vertical distance from the galactic plane – at the position of the outer ring (dotted/green), solar system (solid/black), inner ring (dashed/red) and galactic center (dash-dotted/blue). **Lower panel:** The dark matter surface mass density within 0.8 kpc from the galactic disk. The Earth’s location is marked with a x-sign. Figure from PAPER V.

9.4 The Inconsistency

Even though the dark matter halo profile by de Boer et al. explains the EGRET data very well, it is of great importance to check its validity with other observational data.

**Disc Surface Mass Density**

Note that the distribution of the dark matter in this model seems very closely correlated to the observed baryon distribution in the Milky Way – containing a thin and a thick disk and a central bulge (see, e.g., [321]). Since the dark halo is much more massive than the baryonic one, one of the first things we should investigate is whether there is room to place as much unseen matter
Table 9.1: Derived local surface densities $\Sigma_{|z|}$, within heights $|z|$, compared to the amount of dark matter in the model of de Boer et al. [1]. The amount of dark matter exceeds the allowed span for unidentified gravitational matter in the inner part of the galactic disk (i.e., around $z = 0$). [323, 324]

| Surface Density ($\Sigma_{|z|}$) | Dynamical ($M_{\odot}/pc^2$) | Identified ($M_{\odot}/pc^2$) | Unidentified ($M_{\odot}/pc^2$) | DM in [1] ($M_{\odot}/pc^2$) |
|-------------------------------|-----------------------------|------------------------|-------------------------------|------------------------|
| $\Sigma_{50\,pc}$             | 9 – 11                      | $\sim$ 9               | 0 – 2                         | 4.5                    |
| $\Sigma_{350\,pc}$            | 36 – 48                     | $\sim$ 34              | 2 – 14                        | 19                     |
| $\Sigma_{800\,pc}$            | 59 – 71                     | $\sim$ 46              | 13 – 25                       | 29                     |
| $\Sigma_{1100\,pc}$           | 58 – 80                     | $\sim$ 49              | 9 – 32                        | 35                     |

in the vicinity of the disk as in the model by de Boer et al.

Observations of the dynamics and density distribution of stars in the disk give a measure of the gravitational pull perpendicular to the galactic plane. This can be translated into an allowed disk surface mass density (a method pioneered in [322]). Observational data from the local surroundings in the galactic disk sets fairly good limits on the disk surface mass density at the solar system location [323]. Observations are well described by a smooth dark matter halo and a disk of identified matter (mainly containing stars, white and brown dwarfs and interstellar matter in form of cold and hot gases). Therefore, there is little room for a concentration of dark matter in the disk.

Table 9.1 shows the observed local surface mass density in both identified components and the total dynamical mass within several heights. Their differences give an estimate of the allowed amount of dark matter in the local disk – the result is an exclusion of such strong concentrations of unidentified/dark matter as used in the model of [1] to explain the EGRET data. For example these observations give room for only about $0.01 M_{\odot}/pc^3$ in unidentified matter, which should be compared to the dark matter density of $0.05 M_{\odot}/pc^3$ in the model of de Boer et al. [1]. We should keep in mind that the estimates of the possible amount of dark matter are somewhat uncertain and that the disk models also have uncertainties of the order of 10% in their star plus dwarf components and uncertainties as large as about 30% in their gas components. Also the de Boer et al. halo model could easily be modified to give a lower disk surface mass density at the solar vicinity. However, such a modification just to circumvent this problem seems fine-tuned. The model, as it now stands, already has made fine-tuning modifications, i.e., the rings are constructed so that they can be very massive, while keeping the local density low. Figure 9.3 clearly shows that our Sun is already located in a region with relatively low disk mass surface density.

The dark matter distribution does not at all resemble what would be
expected from dissipationless cold dark matter. The distribution should be much more isotropic than that of the baryonic disk material, which supposedly forms dissipatively with energy loss but very little angular momentum loss [325] (see also Chapter 2).

**Comparison with Antiproton Data**

Any model based on dark matter annihilations into quark-antiquark jets inevitably also predicts a primary flux of antiprotons (and an equal amount of protons) from the same jets. As discussed in Section 9.2, the propagation models of the antiprotons (i.e., cosmic rays) are observationally constrained to enable reasonably reliably predict antiproton fluxes at Earth. To find out what the antiproton flux would be in the model proposed by de Boer et al., we calculated the expected antiproton fluxes in detail in paper Paper V.

*Calculating the Antiproton Flux*

There is no need to repeat here the details of the procedure to calculate the antiproton flux, which can be found in Paper V. The main point is that we followed, as closely as possible, how de Boer et al. found the necessary annihilation rates to explain the EGRET data on gamma rays, and then calculated the antiproton flux based on these same annihilation rates.

For the background gamma flux, we used, as de Boer et al., both the conventional diffuse gamma-ray background and the optimized background shown in Fig. 9.1.

For predicting the signals, we used DarkSUSY [281] to calculate the dark matter annihilation cross sections, gamma-ray and antiproton yields. We normalized our boost factors to fit the diffuse gamma-ray data from the inner region of our Galaxy, from where most observational data exist. The dark matter profile was fixed, and defined by the 18 parameters found in the de Boer et al. paper [1]. A least $\chi^2$-fit [75] was made to the EGRET data in 8 energy bins in the energy range 0.07 to 10 GeV.

By this procedure we were able to reproduce the result in [1]. That is, we find a good fit to the EGRET data for WIMP masses between roughly 50 and 100 GeV, and that the required boost factors can be less than the order of 100. Our best fit $\chi^2$-values for different dark matter masses are shown in Fig. 9.4 for both the conventional (triangles) and the optimized (circles) diffuse gamma-ray background.

Note that the fits with the optimized background never get very bad for higher masses, as there is no real need for a signal with this model. One should also note that we used relative errors of only 7% for the gamma fluxes, although the overall uncertainty is often quoted to be 10-15% [315,326]. The

* The protons produced in dark matter annihilations would be totally swamped by the much larger proton flux from conventional sources.
true errors are still under debate, and we chose to followed de Boer et al. using their estimate of 7% for the relative errors in our $\chi^2$-fits. Our results are, however, not sensitive to this choice (apart from the actual $\chi^2$ values of course). The optimized background model produce in this case a reduced $\chi^2 \sim 22/6$, which corresponds to a probability of $P \sim 0.1\%$ (P-value) that the data would give this or a worse (i.e., greater) $\chi^2$ value if the hypothesis were correct [75]. If instead conventional/larger uncertainties of $\sim 15\%$ for EGRET’s observed gamma fluxes were adopted, the reduced $\chi^2$ was decreased to $\sim 5/6$, and a P-value of $P = 56\%$.

Boost factors were determined model-by-model. This means, for each supersymmetric model we demanded it to give an optimized fit to the gamma-ray spectrum, which thus gave us an optimized boost factor for each model. Based on the determined boost factors, the antiproton fluxes could be directly calculated for each model. Since the boost factor is assumed to be independent of location in the Galaxy, the same boost factor could be used for the antiproton flux as that found for the gamma rays. To be concrete, the analysis in PAPER V was done within the MSSM, but, as mentioned, the correlation between gamma rays and antiprotons is a generic feature and the results are more general. We used DarkSUSY [281] to calculate the antiproton fluxes for a generous set of supersymmetry models with the halo profile of de Boer et
Once we had the calculated antiproton fluxes at hand, we could compare it with antiproton measurements. We chose to primarily compare antiproton data in the energy bin at 0.40–0.56 GeV and using BESS (Balloon-borne Experiment with a Superconducting Spectrometer) data from 1998 [327]. The reason for using the BESS 98 data is because the solar modulation parameter is estimated to be relatively low ($\phi_F = 610$ MV) at this time, and the low-energy bin correspond to an energy range where the signal is expected to be relatively high compared to the background.

Figure 9.5 shows (using the conventional background) the antiproton flux when enlarged with the boost factor found from the fit to the EGRET data. Models with the correct mass, i.e., low $\chi^2$, clearly overproduce antiprotons. In the figure, we have imposed a cut on the boost factor, to only allow models with reasonably low boost factors. To be conservative, we have allowed the boost factor to be as high as 100, which is higher than expected from recent analyses (see e.g., [116, 328]). It is fairly evident that all the models with good fits to the EGRET data give far too high antiproton fluxes. We find that low-mass models (masses less than 100 GeV) overproduce antiprotons by

**Figure 9.5**: The antiproton fluxes boosted with the same boost factor as found for the gamma rays compared to the measured BESS data. The solid line indicate how far down we could shift the models by choosing an extreme minimal propagation model (see Section 9.4). Figure from Paper V.
a factor of around ten. Higher-mass models (above a few hundred GeV) have a lower antiproton rate, so the overproduction is slightly less. However, they hardly give any improvements to the fits to the gamma-ray spectrum.

Other dark matter candidates, like KK dark matter, would also give a similar behavior since the gamma rays and antiprotons are so correlated. However, for, e.g. KK dark matter in the UED model one would not improve the fits to EGRET data as only heavier models are favored by the relic density constraint. For the IDM the boost factor would have to be very large in all sky directions, at least as long as its gamma-ray continuum part is strongly suppressed by having only heavy Higgs coupling to quarks. In fact, since antiprotons and gamma rays are so strongly correlated in general, our results should be valid for any typical WIMP.

**Antiproton Propagation Uncertainties**

We could be worried about the well-known fact that the antiproton flux from dark matter annihilations is usually beset with large uncertainties relating to unknown diffusion parameters combined with uncertainties in the halo distribution. In [329] it is pointed out that the estimated flux may vary by almost a factor of 10 up or down, for models that predict the correct cosmic-ray features. However, the results of such a large uncertainty are only valid for a relatively smooth halo profile, where much of the annihilation occurs away from the disk and propagation properties are less constrained.

The main reason for the large uncertainties found in [329] is a degeneracy (for the secondary signal) between the height of the diffusion box and the diffusion parameter. If we increase the height of the diffusion box, we would get a larger secondary signal because cosmic rays can propagate longer in the diffusion box before escaping. This can be counterbalanced by increasing the diffusion coefficient to make the cosmic rays diffuse away faster from the galactic disk. Hence, for the secondary signal, which originates in the galactic disk, we can get acceptable fits by changing these parameters. For the dark matter in a smooth halo the effect of these changes is different. If we increases the height of the diffusion box, we also increase the volume in which annihilations occur, and the total flux increases more than can be counterbalanced by an increase in the diffusion coefficient. This is, however, not true to the same extent in the de Boer et al. profile where most of the dark matter is concentrated to the disk.

To investigate this effect on the antiproton flux from variations in the propagation models, we recalculated the expected antiproton fluxes with the propagation code in [329]. For illustration, let us look at a supersymmetric configuration for which the agreement with the EGRET data is good – a reduced $\chi^2$ of roughly 3/6, a neutralino mass of 50.1 GeV and a derived boost factor of 69. By varying the propagation parameters to be as extreme as allowed from other cosmic-ray data (details on what this correspond to can be
Figure 9.6: A supersymmetric model that provides a good fit to the EGRET data has been selected and its antiproton yield has been carefully derived. It is featured by the red solid line in the case of the median cosmic-ray configuration. Predictions spread over the yellow band as the cosmic-ray propagation parameters are varied from the minimal to maximal configurations (see Table 2 in Paper V). The long-dashed black curve is calculated with DarkSUSY for a standard set of propagation parameters [330]. The narrow green band stands for the conventional secondary component. As is evident from this figure, the antiproton fluxes for this example model clearly overshoots the data. Figure from Paper V.

found in Paper V), we get the range of allowed predicted antiproton fluxes. In Fig. 9.6 the yellow band delimits the whole range of extreme propagation model configurations. This gives an indication on how well the flux of neutralino induced antiprotons can be derived in the case of the de Boer et al. dark matter distribution. The red solid curve is a median cosmic-ray propagation model configuration which could be compared to the long–dashed black curve computed with the DarkSUSY package [330]. The conventional secondary
background, producing antiprotons, is indicated as the narrow green band as it was derived in [331] from the observed B/C ratio. For maximal cosmic-ray configuration the dark matter induced antiproton flux is observed to increase by a factor of 2.5, and for the minimal configuration a decrease of a factor of 2.6. The total uncertainty in the expected dark matter induced antiproton flux corresponds therefore to an overall factor of only $\sim 6.5$, to be compared to a factor of $\sim 50$ in the case of an NFW dark matter halo.

Even after these propagation uncertainties are included the yellow uncertainty band is at least an order of magnitude above the secondary green component. This was for one example model, but this argument can be made more general. In Fig. 9.5, a solid line represents how far down we would shift the antiproton fluxes by going to the extreme minimal propagation models. As can be seen, the antiprotons are still overproduced by a factor of 2 to 10 for the models with good fits to EGRET data. It is therefore difficult to see how this dark matter interpretation of the EGRET data could be compatible with the antiproton measurements.

Our conclusion is therefore that the proposal of de Boer et al. [1] is not viable, at least not without further fine-tuning of the model or by significant changes in generally employed propagation models.

### 9.5 The Status to Date

Currently, the uncertainties regarding the data of the EGRET GeV anomaly can still be debated. In any case, the dark matter model proposed by de Boer et al. to explain the potential GeV anomaly has a problem of severe overproduction of antiprotons. With the usual cosmic propagation models there does not seem to be a way out of this problem. However, attempts with anisotropic diffusion models have been proposed by de Boer et al. as a way to circumvent these antiproton constraints. These models feature anisotropic galactic winds which transport charged particles to outer space, and places our solar system in an underdense region with overdense clouds and magnetic walls causing slow diffusion. In such a scenario, it is claimed that the antiproton flux due to dark matter annihilations could very well be decreased by an order of magnitude [308, 332] (see also [333, 334]). This seems somewhat contrived at the moment, especially since the dark matter distribution itself is not standard. There are, of course, also other ways to tune the model to reduce the antiproton flux. For example, if we take away the inner ring, the antiproton fluxes goes down a factor of 2.0. This indicates that fine-tuning the distribution even more, by having the high dark matter density as far away as possible from our solar system, could potentially reduce the antiproton significantly without reducing the gamma-ray flux. Remember though that the disk density (although notably still quite uncertain) has already been tuned to have a dip

\[1\] Because diffusive reacceleration is not included in DarkSUSY, the flux falls more steeply close to the neutralino mass than in the median model of [329].
in order not to be in large conflict with stellar motion measurements in the solar neighborhood. All such attempts to avoid the antiproton contradiction should be judged against optimized cosmic-ray propagation models, that also have been shown to enable an explanation of the GeV anomaly, without the need of dark matter. The actual density profiles of the rings, with exponential fall-off away from the disk, is also not what is expect from WIMP dark matter, although mergers of dwarf galaxies could leave some traces of minor ring-like dark matter structures in the galactic plane [308]. In fact, the de Boer et al. model seems most likely to be a fit of the baryonic matter distribution of our Galaxy, and not the dark matter density. This is not at all to say that there cannot be any hidden dark matter annihilation signal in the gamma-ray sky, instead we have shown that standard propagation models and antiproton measurements constrain the possibilities for an all-sky WIMP dark matter signal in gamma rays.

What can be said for certain is that to date there is no dark matter annihilation signal established beyond reasonable doubt.

Many open questions, especially regarding the EGRET GeV anomaly, will presumably be resolved once the GLAST satellite has data of the gamma-ray sky. For example, we will then know more about the actual disk concentration of the gamma-ray distribution, if the GeV anomaly persists, and how the spectrum is continued up to higher energies. Before GLAST, the PAMELA satellite [241] will collect data on, e.g., antiprotons and positrons, which could even further enhance the understanding of the cosmic-ray sky and potential dark matter signals in it.

‡ It has been put forward that rotation curve [1] and gas flaring [335] data support the existence of a very massive dark matter ring at a galactocentric distance of about 10-20 kpc. However, this is controversial and is, e.g., not supported by the derived rotation curve in [336] and the recent analysis in [337].
Chapter 10

Summary and Outlook

The need to explain a wide range of cosmological and astrophysical phenomena has compelled physicists to introduce the concept of dark matter. Once dark matter, together with dark energy (in the simplest form of a cosmological constant), is adopted, conventional laws of physics give a remarkably good description of a plethora of otherwise unexplained cosmological observations. However, what this dark matter is made of remains one of the greatest puzzles in modern physics.

Any experimental signals that could help to reveal the nature of dark matter are therefore sought. This could either be in the form of an observational discovery of an unmistakable feature or the detection of several different kinds of signals that all can be explained by the same dark matter model.

In this thesis I have presented the background materials and research results regarding three different types of dark matter candidates. These candidates all belong to the class of weakly interacting massive particles, and are: the lightest Kaluza-Klein particle $\gamma^{(1)}$, the lightest neutralino within supersymmetry $\chi$, and the lightest inert Higgs $H^0$. They could be characterized as dark matter archetypes for a spin 1, 1/2, and 0 particle, respectively.

The first dark matter candidate that was studied originates from the fascinating possibility that our world possesses more dimensions than the observed four spacetime dimensions. After a discussion of multidimensional universes – where it was concluded that a stabilization mechanism for extra dimensions is essential – the particle content within the particular model of universal extra dimensions (UED) was investigated. The cosmologically most relevant aspect of this UED model is that it naturally gives rise to a dark matter candidate $\gamma^{(1)}$ ($\approx B^{(1)}$). This candidate is a massive Kaluza-Klein state of an ordinary photon – i.e. a photon with momentum in the direction of an extra dimension. In this thesis, prospects for indirect dark matter detection by gamma rays from annihilating $\gamma^{(1)}$ particles were explored. It was discovered that by internal bremsstrahlung, from charged final state fermions, very high-energy gamma rays collinear with the fermions are frequently produced. An expected signature from $\gamma^{(1)}$ annihilations is therefore a gamma-ray energy spectrum.
that is very prominent at high energies, and has a characteristic sharp cutoff at the energy equal to the dark matter particle’s mass. As a by-product it was realized that this is a quite generic feature for many dark matter candidates. The amplitude of a potentially even more characteristic signal, a monochromatic gamma line, was also calculated. Although in principle a very striking signal, its feebleness seems to indicate that a new generation of detectors are needed for a possible detection.

The second dark matter candidate studied was the neutralino, appearing from a supersymmetric extension of the standard model. Also here it was found that internal bremsstrahlung in connection to neutralino annihilation can give characteristic signatures in the gamma-ray spectrum. The reason for these gamma-ray signals is somewhat different from that of the UED model. High-mass neutralinos annihilating into $W^\pm$ can give rise to a phenomenon similar to the infrared divergence in quantum electrodynamics, that significantly enhances the cross section into low-energy $W^\pm$ bosons and high-energy photons. Another possible boosting effect is that the (helicity) suppression of neutralinos annihilating into light fermions will no longer be present if a photon accompanies the final state fermions. For neutralino annihilations, both these effects result in the same type of characteristic signature, a pronounced high energy gamma-ray spectrum with a sharp cutoff at the energy equal to the neutralino mass.

The third candidate arises from considering a minimal extension of the standard model by including an additional Higgs doublet. With an unbroken $Z_2$ symmetry, motivated by experimental constraints on neutral flavor changing currents, an inert Higgs emerges that constitutes a good dark matter candidate. This dark matter candidate turns out to have the potential to produce a ‘smoking gun’ signal in the form of a strong monochromatic gamma-ray line in combination with a low gamma-ray continuum. Such a tremendous signal could be waiting just around the corner and be detected by the GLAST satellite, to be launched later this year.

The anticipated absolute fluxes of gamma rays from dark matter annihilations, and thus the detection prospects of predicted signals, are still accompanied by large uncertainties. This is because of the large uncertainty in the dark matter density distribution. To learn more about the expected dark matter density distribution, we used numerical $N$-body/hydrodynamical simulations. The effect of baryons, i.e., the ordinary matter particles constituting the gas and stars in galaxies, upon the dark matter was found to be significant. In the center of the galaxy the dark matter gets pinched, and the overall halo changes shape from somewhat prolate to more oblate. The pinching of dark matter in the center of galaxies could cause a boost of dark matter annihilations which might be needed for detection of annihilation signals.

The claimed observation of a dark matter annihilation signal by de Boer et al. was also scrutinized. It was concluded that to date there is no convincing observational evidence for dark matter particle annihilations.
If dark matter consists of massive and (weakly) interacting particles, there is a chance that in the near future some or many of the existing and upcoming experiments will start to reveal the nature of the dark matter. The types of signal presented in this thesis have the potential to contribute significantly to this process as they might be detected both with the GLAST satellite and with existing and upcoming ground based air Čerenkov telescopes. Other experiments – like PAMELA, measuring anti-particle fluxes – will simultaneously continue to search the sky for dark matter signals. Once the Large Hadron Collider at CERN is running, if not earlier, many theories beyond standard model physics will be experimentally scrutinized to see whether they give a fair description of how nature behaves and whether they are able to explain dark matter.
This Appendix collects a complete list of Feynman rules for all physical fields and their electroweak interactions in the five-dimensional UED model. The extra dimension is compactified on a $S^1/Z_2$ orbifold (the fields’ orbifold boundary conditions are specified in Chapter 5). All interaction terms are located in the bulk and are KK number conserving. In principle, radiative corrections at the orbifold fixpoints could give rise to interactions that violate KK number conservation [231]. These types of interactions are loop-suppressed, and it is self-consistent to assume they are small; they will therefore not be considered here.

A.1 Field Content and Propagators

In the four-dimensional theory, the mass eigenstates of vector fields are $A^{(0)} \mu$, $Z^{(0)} \mu$, and $W^{(0)}_\pm \mu$ at the SM level, whereas their KK level excitations are $B^{(n)} \mu$, $A_3^{(n)} \mu$, and $W^{(n)}_\pm \mu$ ($n \geq 1$) (for the heavy KK masses the ‘Weinberg’ angle is taken to be zero as it is essentially driven to zero). Depending on the gauge, there will also be an unphysical ghost field $c$ associated to every vector field. From the Higgs sector, there is one physical scalar $h^{(0)}$ present at the SM-level, and three physical scalars $h^{(n)}$, $a_0^{(n)}$ and $a_+^{(n)}$ at each KK level. Depending on the gauge, the Higgs sector also contains the unphysical Goldstone bosons $\chi_3^{(0)}$ and $\chi_\pm^{(0)}$ at SM-level, and $G_0^{(n)}$, $G_+^{(n)}$, and $A_5^{(n)}$ at each KK level. These generate the longitudinal spin modes for the massive vector fields. Finally, for every SM fermion $\xi^{(0)}_s$ there are two towers of KK fermions $\xi^{(n)}_s$ and $\xi^{(n)}_{d}$ – except for the neutrinos, which only appear as a component in the $SU(2)$ doublet, both at zero and higher KK levels.

Following the conventions in [3], the propagator for an internal particle is
given by

for scalars: \[ \frac{i}{q^2 - m^2 + i\epsilon}, \]  
(A.1)

for fermions: \[ \frac{i(q + m)}{q^2 - m^2 + i\epsilon}, \]  
(A.2)

for vectors: \[ \frac{-i}{q^2 - m^2 + i\epsilon} \left( \eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2 - \xi m^2} (1 - \xi) \right) \]  
(A.3)

where \( \epsilon \) is a small positive auxiliary parameter, which is allowed to tend to zero after potential integrations over \( q \), and \( \xi \) is a gauge parameter (\( \xi = 0 \) being the Landau gauge and \( \xi = 1 \) being the Feynman-'t Hooft gauge). The same propagators apply also for the unphysical ghosts and Goldstone bosons, but with the masses replaced by \( \sqrt{\xi} m_V \) (where \( m_V \) is the mass of the associated vector boson).

In Chapter 5, mass eigenstates were expressed in the fields appearing directly in the Lagrangian, see Eq. (5.20), (5.21), (5.27), and (5.53). Sometimes the inverse of these relationships are also convenient to have at hand:

\[
\begin{align*}
A^3_M &= s_w A_M + c_w Z_M \\
B_M &= c_w A_M - s_w Z_M
\end{align*}
\]  
(A.4a, b)

\[
\begin{align*}
Z_5^{(n)} &= \frac{m_z}{M_z^{(n)}} a_0^{(n)} - \frac{M^{(n)}}{M_z^{(n)}} G_0^{(n)} \\
\chi^3(n) &= \frac{M^{(n)}}{M_z^{(n)}} a_0^{(n)} + \frac{m_z}{M_z^{(n)}} G_0^{(n)} \\
W_5^{(n)} &= \frac{m_w}{M_w^{(n)}} a_\pm^{(n)} - \frac{M^{(n)}}{M_w^{(n)}} G_\pm^{(n)} \\
\chi^\pm(n) &= \frac{M^{(n)}}{M_w^{(n)}} a_\pm^{(n)} + \frac{m_w}{M_w^{(n)}} G_\pm^{(n)}
\end{align*}
\]  
(A.5a, b, c, d)

\[
\begin{align*}
\psi_s^{(n)} &= \sin \alpha^{(n)} \xi_s^{(n)} - \cos \alpha^{(n)} \gamma^5 \xi_s^{(n)} \\
\psi_d^{(n)} &= \cos \alpha^{(n)} \xi_s^{(n)} + \sin \alpha^{(n)} \gamma^5 \xi_s^{(n)}
\end{align*}
\]  
(A.6a, b)

### A.2 Vertex Rules

All vertex rules can be expressed in terms of five independent quantities, \( e.g. \), the electron charge \( e (= -|e|) \), the Weinberg angle \( \theta_w \), the \( W \) gauge boson mass \( m_w \), the Higgs mass \( m_h \), and the compactification size of the extra
dimension $R$. To shorten some of the vertex-rule expressions, the following shorthand notations will be frequently used:

$$
\begin{align*}
c_w & \equiv \cos(\theta_w), \\
s_w & \equiv \sin(\theta_w), \\
M^{(1)} & \equiv 1/R, \\
M_X^{(1)} & \equiv \sqrt{M^{(1)}^2 + m_X^2},
\end{align*}
$$

(A.7a) (A.7b) (A.7c) (A.7d)

together with the quantities:

$$
\begin{align*}
m_Z &= m_W/c_w, \\
g_Y &= e/c_w, \\
g &= e/s_w, \\
\lambda &= \frac{g^2 m_W^2}{8 m_W^2}.
\end{align*}
$$

(A.8a) (A.8b) (A.8c) (A.8d)

In addition, all the fermion Yukawa couplings are free parameters, i.e., the fermion masses $m_\xi$. In the case of charged gauge boson interactions, there are also additional independent parameters in the Cabibbo-Kobayahi-Maskawa (CKM) $V_{ij}$ matrix*, whose elements contain information on the strengths of flavor-changing interactions. All momenta are ingoing in the vertex rules.

All vertex rules in the UED model for the physical fields, up to the first KK level, will now follow†. Additional vertex rules, including unphysical Goldstone and ghost fields, are displayed if they were explicitly used in our numerical calculation of the one-loop process $B^{(1)}B^{(1)} \rightarrow \gamma \gamma, Z \gamma$ which was done in the Feynman-'t Hooft gauge ($\xi \rightarrow 1$) in PAPER III. In unitarity gauge ($\xi \rightarrow \infty$) all such unphysical fields disappear.

**Vector-Vector-Vector Vertices**

These couplings between tree vector fields originating from the cubic terms in gauge fields that appear in the field strength part of the Lagrangian (5.9). The following four-dimensional Feynman vertex rules are derived:

---

* In the SM $V_{ij}$ is a $3 \times 3$ complex unitary matrix. Unitarity (9 conditions) and the fact that each quark field can absorb a relative phase (5 parameters reduction) leaves only $2 \times 3^2 - 9 - 5 = 4$ free parameters in the CKM matrix.

† Some of these vertex rules can also be found in [214, 338, 339]. Note: A few typos were identified in [214, 338, 339] [Personal communication with Graham Kribs and Torsten Bringmann].
Vector-Vector-Scalar Vertices

Couplings between two gauge fields and one scalar field appear both in the field strength part of the Lagrangian (5.9) and the kinetic part of the Higgs Lagrangian (5.17). The following four-dimensional Feynman vertex rules are derived:

\[ igV_1 ^\mu V_2 ^\nu S \cdot \eta ^{\mu \nu} \]

where

\[
\begin{align*}
g_{A^{(0)}} W_+ ^{(1,0)} W_+ ^{(1,0)} &= -e \\
g_{Z^{(0)}} W_+ ^{(0,1)} W_- ^{(0,1)} &= -c_w g \\
g_{A_3^{(1)}} W_+ ^{(0,1)} W_- ^{(1,0)} &= -g
\end{align*}
\]

(A.9) (A.10) (A.11)

\section*{Vector-Vector-Scalar Vertices}

Couplings between two gauge fields and one scalar field appear both in the field strength part of the Lagrangian (5.9) and the kinetic part of the Higgs Lagrangian (5.17). The following four-dimensional Feynman vertex rules are derived:

\[ igV_1 ^\mu V_2 ^\nu S \cdot \eta ^{\mu \nu} \]

where

\[
\begin{align*}
g_{Z^{(0)}} Z^{(0)} h^{(0)} &= \frac{g}{c_w} m_z \\
g_{W_+ ^{(0)} W_- ^{(0)} h^{(0)}} &= g m_w \\
g_{Z^{(0)}} W_+ ^{(1)} a_+ ^{(1)} &= \pm ig m_z \frac{M_1 ^{(1)}}{M_1 ^{(1)}} \\
g_{Z^{(0)}} A_+ ^{(1)} h^{(1)} &= g m_z \\
g_{Z^{(0)}} B_+ ^{(1)} h^{(1)} &= -g_Y m_z \\
g_{W_+ ^{(0)} A_+ ^{(1)} a_+ ^{(1)}} &= \mp ig m_w \frac{M_1 ^{(1)}}{M_1 ^{(1)}} \\
g_{W_+ ^{(0)} B_+ ^{(1)} a_+ ^{(1)}} &= \mp ig_Y m_w \frac{M_1 ^{(1)}}{M_1 ^{(1)}} \\
g_{W_+ ^{(0)} A_- ^{(1)} a_+ ^{(1)}} &= \mp ig m_w \frac{M_1 ^{(1)}}{M_1 ^{(1)}} \\
g_{W_+ ^{(0)} B_+ ^{(1)} a_+ ^{(1)}} &= \mp ig_Y m_w \frac{M_1 ^{(1)}}{M_1 ^{(1)}}
\end{align*}
\]

In unitarity gauge ($\xi \to \infty$) unphysical scalar fields are not present. However, in a general gauge there are also additional vertices including unphysical scalar fields. I here choose to include only those vertex rules that were explicitly used in PAPER III to calculate the one-loop process $B^{(1)} B^{(1)} \rightarrow \gamma \gamma$ in the Feynman-'t Hooft gauge ($\xi \to 1$):

$$g_{A(0)W^{(0)}{\chi}^{(0)}} = \mp ie m_w \quad (A.25)$$
$$g_{Z(0)W^{(0)}{\chi}^{(0)}} = \pm i g s_w^2 m_z \quad (A.26)$$
$$g_{A(0)W^{(1)}G^{(1)}} = \mp ie M_w^{(1)} \quad (A.27)$$
$$g_{W^{(0)}B^{(1)}G^{(1)}} = \mp ig_Y m_w m_w \quad (A.28)$$
$$g_{W^{(1)}B^{(1)}{\chi}^{(0)}} = \mp ig_Y m_w \quad (A.29)$$

**Vector-Scalar-Scalar Vertices**

Couplings between one gauge field and two scalar fields appear both in the field strength part of the Lagrangian (5.9) and the kinetic part of the Higgs Lagrangian (5.17). The following four-dimensional Feynman vertex rules are derived:

$$g_{\pm a_0^{(1)} a_0^{(1)}} = e \quad (A.30)$$
\[ g_{Z(0)} a_+ a_-^{(1)} = \frac{1}{2} \left( g_{c_w} - g_Y s_w \right) \frac{M^{(1)}}{M_w^{(1)}}^2 + g_{c_w} \frac{m_w^2}{M_w^{(1)}} \] (A.31)

\[ g_{Z(0)} h^{(1)} a_0^{(1)} = -i \frac{g}{2c_w} \frac{M^{(1)}}{M_z^{(1)}} \] (A.32)

\[ g_{W_\pm} a_+^{(1)} a_0^{(1)} = \mp \frac{g}{2} \frac{M^{(1)}}{M_w^{(1)}} \frac{M_z^{(1)}}{M_w^{(1)}} \left( 1 - 2 \frac{m_w^2}{M_w^{(1)}} \right) \] (A.33)

\[ g_{W_\pm} a_+^{(1)} h_0^{(1)} = i \frac{g}{2} \frac{M^{(1)}}{M_w^{(1)}} \] (A.34)

\[ g_{A_3} a_0^{(1)} h_0^{(1)} = i \frac{g}{2} \frac{M^{(1)}}{M_z^{(1)}} \] (A.35)

\[ g_{A_3} G_0^{(1)} h_0^{(1)} = i \frac{g}{2} \frac{m_z}{M_z^{(1)}} \] (A.36)

\[ g_{B_1} a_0^{(1)} h_0^{(1)} = -i \frac{g_Y}{2} \frac{M^{(1)}}{M_z^{(1)}} \] (A.37)

\[ g_{W_\pm} a_+^{(1)} h_0^{(1)} = i \frac{g}{2} \frac{M^{(1)}}{M_w^{(1)}} \] (A.38)

In unitarity gauge \((\xi \to \infty)\) unphysical scalar fields are not present. However, in a general gauge there are also additional vertices including unphysical scalar fields. I here choose to include only those vertex rules that were explicitly used in Paper III to calculate the one-loop process \(B^{(1)} B^{(1)} \to \gamma \gamma\) in the Feynman-’t Hooft gauge \((\xi \to 1)\):

\[ g_{A_0} a_+^{(0)} a_-^{(0)} = e \] (A.39)

\[ g_{Z(0)} a_+^{(0)} a_-^{(0)} = g \frac{c_w}{2} - g_Y \frac{s_w}{2} \] (A.40)

\[ g_{Z(0)} a_+^{(0)} h_0^{(0)} = i \frac{g}{2c_w} \] (A.41)

\[ g_{W_\pm} a_+^{(0)} h_0^{(0)} = i \frac{g}{2} \] (A.42)

\[ g_{W_\pm} a_+^{(0)} a_0^{(0)} = \mp \frac{g}{2} \] (A.43)

\[ g_{A_0} G_0^{(1)} G_0^{(1)} = e \] (A.44)

\[ g_{B_1} a_+^{(0)} a_0^{(1)} = \pm \frac{g_Y}{2} \frac{M^{(1)}}{M_w^{(1)}} \] (A.45)

\[ g_{B_1} a_+^{(0)} G_0^{(1)} = \pm \frac{g_Y}{2} \frac{m_w}{M_w^{(1)}} \] (A.46)

\[ g_{B_1} a_0^{(0)} h_0^{(1)} = -i \frac{g_Y}{2} \] (A.47)
\[ g_{B^{(1)}G_0^{(2)}h^{(0)}} = -ig_y \frac{m_Z}{2 M_Z^{(1)}} \]  
\[ g_{W_{\pm}^{(1)}G^{(1)}G^{(1)}} = \pm i \frac{g}{2 M_w} \]  

Scalar-Scalar-Scalar Vertices

Couplings between tree scalar fields appear in the kinetic and potential in the Higgs Lagrangian (5.17). The following four-dimensional Feynman vertex rules are derived:

\[ S_1 \quad \dashv \quad i g S_1 S_2 S_3 \]

where

\[ g_{h^{(0)}h^{(0,1)}h^{(0,1)}} = -3 \frac{g}{2} \frac{m_h^2}{m_w} \]  
\[ g_{h^{(0)}a_+^{(1)}a_-^{(1)}} = -g m_w \left( 1 + \frac{1}{2} \frac{m_h^2 M^{(1)}_w}{M_w^{(1)}} \right) \]  
\[ g_{h^{(0)}a_0^{(1)}a_0^{(1)}} = -\frac{g}{c_w} m_Z \left( 1 + \frac{1}{2} \frac{m_h^2 M^{(1)}_w}{M_w^{(1)}} \right) \]  

In unitarity gauge (\( \xi \to \infty \)) unphysical scalar fields are not present. However, in a general gauge there are also additional vertices including unphysical scalar fields. I here choose to include only those vertex rules that were explicitly used in PAPER III to calculate the one-loop \( B^{(1)}B^{(1)} \to \gamma \gamma \) process (which was done numerically in the Feynman-'t Hooft gauge (\( \xi \to 1 \)):

\[ g_{h^{(0)}\chi^+^{(0)}\chi^-^{(0)}} = -\frac{g}{2} \frac{m_h^2}{m_w} \]  
\[ g_{h^{(0)}\chi^3^{(0)}\chi^3^{(0)}} = -\frac{g}{2} \frac{m_h^2}{m_w} \]  
\[ g_{h^{(0)}a^{(1)}_\pm G^{(1)}G^{(1)}} = \frac{g}{2} M^{(1)} \left( 1 - \frac{m_h^2}{M_w^{(1)} M^{(1)}} \right) \]  
\[ g_{h^{(0)}G^{(1)}_\pm G^{(1)}_\mp} = -\frac{g}{2} m_w \frac{m_h^2}{M_w^{(1)}} \]
Fermion-Fermion-Vector Vertices

Couplings between two fermions and the gauge fields appear in the covariant derivative of the fermion fields, see the Lagrangian in (5.44). The notation below is that indices $i$ and $j$ indicate which SM generation a fermion belongs to, and $V_{ij}$ is the Cabibbo-Kobayashi-Maskawa matrix. In the case of leptons $V_{ij}$ simply reduces to $\delta_{ij}$ (and remember that there are no singlet neutrinos; $Q_U = Y_{s,U} = 0$ for neutrinos.) Furthermore, $\xi = U, D$ denotes the mass eigenstates for up ($T_3 = +1/2$) and down ($T_3 = -1/2$) type quarks, respectively. Electric charge is defined as usual as $Q \equiv T_3 + Y_d = Y_s$. With these additional notations, the following four-dimensional Feynman vertex rules are derived:

\[
\begin{align*}
{g}_A^{(0)} \xi^{(0)} \xi^{(0)} &= Q e \quad \text{(A.57)} \\
{g}_A^{(0)} \xi^{(1)} \xi^{(1)} &= Q e \quad \text{(A.58)} \\
{g}_Z^{(0)} \xi^{(0)} \xi^{(0)} &= (T_3 g c_w - Y_d g_Y s_w) P_L - Y_s g_Y s_w P_R \quad \text{(A.59)} \\
{g}_Z^{(0)} \xi^{(1)} \xi^{(1)} &= \frac{g}{c_w} \left( T_3 \cos^2 \alpha^{(1)} - Y_s s_w^2 \right) \quad \text{(A.60)} \\
{g}_Z^{(0)} \xi^{(1)} \xi^{(1)} &= \frac{g}{c_w} \left( T_3 \sin^2 \alpha^{(1)} - Y_s s_w^2 \right) \quad \text{(A.61)} \\
{g}_Z^{(0)} \xi^{(1)} \xi^{(1)} &= \frac{g}{c_w} T_3 \sin \alpha^{(1)} \cos \alpha^{(1)} \gamma^5 \quad \text{(A.62)} \\
{g}_A^{(3)} \xi^{(0)} \xi^{(0)} &= T_3 g \cos \alpha^{(1)} P_L \quad \text{(A.63)} \\
{g}_A^{(3)} \xi^{(0)} \xi^{(0)} &= -T_3 g \sin \alpha^{(1)} P_L \quad \text{(A.64)} \\
{g}_B^{(1)} \xi^{(0)} \xi^{(0)} &= Y_s g_Y \sin \alpha^{(1)} P_R + Y_d g_Y \cos \alpha^{(1)} P_L \quad \text{(A.65)} \\
{g}_B^{(1)} \xi^{(0)} \xi^{(0)} &= -Y_s g_Y \cos \alpha^{(1)} P_R - Y_d g_Y \sin \alpha^{(1)} P_L \quad \text{(A.66)} \\
{g}_W^{(0)} \xi^{(0)} D^{(0)} &= \frac{g}{\sqrt{2}} V_{ij} P_L \quad \text{(A.67)} \\
{g}_W^{(0)} \xi^{(0)} D^{(0)} &= \frac{g}{\sqrt{2}} V_{ij} \cos \alpha^{(1)} U_i \cos \alpha^{(1)} D_j \quad \text{(A.68)} \\
{g}_W^{(0)} \xi^{(0)} D^{(0)} &= \frac{g}{\sqrt{2}} V_{ij} \sin \alpha^{(1)} U_i \sin \alpha^{(1)} D_j \quad \text{(A.69)} \\
{g}_W^{(0)} \xi^{(0)} D^{(0)} &= \frac{g}{\sqrt{2}} V_{ij} \cos \alpha^{(1)} U_i \sin \alpha^{(1)} D_j \gamma^5 \quad \text{(A.70)}
\end{align*}
\]
The remaining vertex rules are given by \( g_V \xi_1 \xi_2 = g_{V^* \xi_2 \xi_1} \).

**Fermion-Fermion-Scalar Vertices**

Couplings between two fermions and a scalar originate from the covariant derivative in Eq. (5.44) and the Yukawa couplings (5.46). The same notation as for the ‘fermion-fermion-vector’ couplings are used. The following four-dimensional Feynman vertex rules are derived:

\[
\begin{align*}
g_{\bar{\xi}_1} \bar{\gamma}_5 \xi_2 &= g_{\bar{\xi}_2} \gamma_5 \xi_1 = g_{\bar{\xi}_2} \gamma_5 \xi_1, \\
\bar{\xi}_1 &\rightarrow S \rightarrow \bar{\xi}_2,
\end{align*}
\]

where

\[
\begin{align*}
g_{\bar{\xi}_2} \gamma_5 \xi_1 &= -g \frac{m_{\xi}}{2m_w}
\quad \text{(A.76)}
g_{\bar{\xi}_2} \gamma_5 \xi_1 &= -g \frac{m_{\xi}}{m_w} \sin \alpha^{(1)} \cos \alpha^{(1)}
\quad \text{(A.77)}
g_{\bar{\xi}_2} \gamma_5 \xi_1 &= -g \frac{m_{\xi}}{m_w} \sin \alpha^{(1)} \cos \alpha^{(1)}
\quad \text{(A.78)}
g_{\bar{\xi}_2} \gamma_5 \xi_1 &= -g \frac{m_{\xi}}{2m_w} \left(1 - 2\cos^2 \alpha^{(1)}\right) \gamma_5
\quad \text{(A.79)}
g_{\bar{\xi}_2} \gamma_5 \xi_1 &= -g \frac{m_{\xi}}{2m_w} \left(\sin \alpha^{(1)} P_R + \cos \alpha^{(1)} P_L\right)
\quad \text{(A.80)}
g_{\bar{\xi}_2} \gamma_5 \xi_1 &= g \frac{m_{\xi}}{2m_w} \left(\cos \alpha^{(1)} P_R - \sin \alpha^{(1)} P_L\right)
\quad \text{(A.81)}
g_{\bar{\xi}_2} \gamma_5 \xi_1 &= i m_z \left\{ (T_3 g_{c_w} - Y_d g_{s_w}) \cos \alpha^{(1)} P_R + Y_d g_{s_w} \sin \alpha^{(1)} P_L \right\}
- i g T_3 m_{\xi} \frac{M_{\xi}}{m_w} \left(\sin \alpha^{(1)} P_R - \cos \alpha^{(1)} P_L\right)
\quad \text{(A.82)}
\end{align*}
\]
\[ g_{a_0^{(1)}}^{(1)\xi(0)\xi_{L}^{(1)}} = i \frac{m_Z}{M_Z^{(1)}} \left\{ (T_3 g_c w - Y_d g_w s_w) \sin \alpha^{(1)} P_R + Y_d g_w s_w \cos \alpha^{(1)} P_L \right\} \]
\[+ i g T_3 \frac{m_\xi}{m_w} M_Z^{(1)} \left( \cos \alpha^{(1)} P_R - \sin \alpha^{(1)} P_L \right) \] (A.83)

\[ g_{a_0^{(1)}}^{(1)\xi(0)D_{a,j}^{(1)}} = - i \frac{g}{\sqrt{2}} M^{(1)}_w \delta_{ij} \left( \frac{m_{D_i}}{m_w} \sin \alpha^{(1)} P_R - \frac{m_{U_i}}{m_w} \cos \alpha^{(1)} P_L \right) \]
\[+ i \frac{g}{\sqrt{2}} m_w V_{ij} \cos \alpha^{(1)} P_R \] (A.84)

\[ g_{a_0^{(1)}}^{(1)\xi(0)D_{s,j}^{(1)}} = \frac{i g}{\sqrt{2}} M^{(1)}_w \delta_{ij} \left( \frac{m_{D_i}}{m_w} \cos \alpha^{(1)} P_R - \frac{m_{U_i}}{m_w} \sin \alpha^{(1)} P_L \right) \]
\[+ i \frac{g}{\sqrt{2}} m_w V_{ij} \sin \alpha^{(1)} P_R \] (A.85)

\[ g_{a_0^{(1)}}^{(1)\xi(0)D_{d,j}^{(1)}} = - i \frac{g}{\sqrt{2}} M^{(1)}_w \delta_{ij} \left( \frac{m_{D_i}}{m_w} \cos \alpha^{(1)} U_i P_R - \frac{m_{U_i}}{m_w} \sin \alpha^{(1)} U_i P_L \right) \]
\[+ i \frac{g}{\sqrt{2}} m_w V_{ij} \cos \alpha^{(1)} U_i P_L \] (A.86)

\[ g_{a_0^{(1)}}^{(1)\xi(0)D_{s,j}^{(1)}} = \frac{i g}{\sqrt{2}} M^{(1)}_w \delta_{ij} \left( \frac{m_{D_i}}{m_w} \sin \alpha^{(1)} U_i P_R - \frac{m_{U_i}}{m_w} \cos \alpha^{(1)} U_i P_L \right) \]
\[+ i \frac{g}{\sqrt{2}} m_w V_{ij} \sin \alpha^{(1)} U_i P_L \] (A.87)

The remaining symmetry related fermion-fermion-scalar vertex rules are found using \( g_S^{\xi_1 \xi_2} = g_{S}^{\xi_1 \xi_2} \xi_1 \xi_2 \) for scalar couplings, whereas for all pseudo-scalar coupling parts (i.e., the part of \( g_S^{\xi_1 \xi_2} \) that include a \( \gamma^5 \)) pick up an additional minus sign \( g_S^{\xi_1 \xi_2} = - g_{S}^{\xi_1 \xi_2} \xi_1 \xi_2 \) (this follows from the relation \((\psi_1 \gamma^5 \psi_2)^\dagger = -\bar{\psi}_2 \gamma^5 \psi_1 \) for these interaction terms in the Lagrangian).

In unitarity gauge (\( \xi \to \infty \)) unphysical scalar fields are not present. However, in a general gauge there are also additional vertices including unphysical scalar fields. Although we in Paper III calculated the one-loop \( B^{(1)} B^{(1)} \to \gamma \gamma \) process in the Feynmann-'t Hooft gauge (\( \xi \to 1 \)) no fermion-fermion-scalar vertexes come in at loop order for this process.

**Ghost-Ghost-Vector Vertices**

In unitarity gauge (\( \xi \to \infty \)), all ghosts disappear from the theory. For other gauges one can derive the vertex rules from the ghost Lagrangian in Eq. (5.34). I here choose to include only the (one) vertex rules that was explicitly used in Paper III to calculate the one-loop process \( B^{(1)} B^{(1)} \to \gamma \gamma \) in the Feynmann-'t Hooft gauge (\( \xi \to 1 \)):
Section A.2. Vertex Rules

\[ A^{(1) \mu} = \pm ie q^\mu \]

**Ghost-Ghost-Scalar Vertices**

As for the above ghost-ghost-vector couplings, only the (one) ghost-ghost-scalar Feynman rule explicitly needed in our calculation in Paper III of the process \( B^{(1)} B^{(1)} \rightarrow \gamma \gamma \) is listed:

\[ h^{(1)} = -ig \frac{m_w}{2} \xi \]

**Vector-Vector-Vector-Vector Vertices**

Couplings between four vector fields originate from the gauge field strength part in the Lagrangian \((5.9)\). The following four-dimensional Feynman vertex rules are derived:

\[ i g_{V_1 V_2 V_3 V_4} \cdot \left( 2 \eta^{\mu\nu} \eta^{\rho\sigma} - \eta^{\mu\rho} \eta^{\nu\sigma} - \eta^{\mu\sigma} \eta^{\nu\rho} \right) \]

where

\[
\begin{align*}
g_{W_+^{(0)} W_+^{(0)} W_+^{(0)} W_+^{(0)}} &= g^2 \\
g_{W_-^{(1)} W_-^{(1)} W_+^{(1)} W_+^{(1)}} &= \frac{3}{2} g^2 \\
g_{W_-^{(1,0)} W_-^{(1,0)} W_+^{(0,1)} W_+^{(0,1)}} &= g^2 \\
g_{W_-^{(1,0)} W_-^{(1,0)} W_+^{(1,0)} W_+^{(1,0)}} &= g^2 \\
g_{Z^{(0)} Z^{(0)} W_+^{(0)} W_+^{(0)}} &= -g^2 c_w^2
\end{align*}
\]
Vector-Vector-Scalar-Scalar Vertices

Couplings between two vector and two scalar fields originate both from the gauge field strength term (5.9) and the kinetic Higgs term (5.17) of the Lagrangian. The following four-dimensional Feynman vertex rules are derived:

\[
g_{Z(0)} Z(0) W^+ W^- = -g^2 c_w^2 \tag{A.93}
\]
\[
g_{A(0)} A(0) W^+ W^- = -e^2 \tag{A.94}
\]
\[
g_{A(0)} A(0) W^+ W^- = -e^2 \tag{A.95}
\]
\[
g_{Z(0)} A(0) W^+ W^- = -eg c_w \tag{A.96}
\]
\[
g_{Z(0)} A(0) W^+ W^- = -eg c_w \tag{A.97}
\]
\[
g_{A_3(1)} A_3(1) W^+ W^- = -\frac{3}{2}g^2 \tag{A.98}
\]
\[
g_{A_3(1)} A_3(1) W^+ W^- = -g^2 \tag{A.99}
\]
\[
g_{A_3(1)} Z(0) W^{(0,1)} W^{(1,0)} = -g^2 c_w \tag{A.100}
\]

\[
\eta^{\mu\nu} g V_1 V_2 S_1 S_2
\]

where

\[
g_{Z(0)} Z(0) h(0) h(0) = \frac{g^2}{2c_w^2} \tag{A.101}
\]
\[
g_{W^0(0) W^0(0) h(0) h(0)} = \frac{g^2}{2} \tag{A.102}
\]
\[
g_{B(0) B(0) h(1) h(1)} = \frac{3g_2^2}{4} \tag{A.103}
\]
\[
g_{B(0) B(0) a_6(1) a_6(1)} = \frac{3g_2^2 M_{(1)}^2}{4 M_{(1)}^2} \tag{A.104}
\]
\[
g_{A_3(1) A_3(1) h(1) h(1)} = \frac{3g^2}{4} \tag{A.105}
\]
\[
g_{A_3(1) A_3(1) a_6(1) a_6(1)} = \frac{3g^2 M_{(1)}^2}{4 M_{(1)}^2} \tag{A.106}
\]
\[ g_{W_{\pm}^{(1)}} W_{\pm}^{(1)} a_{\pm}^{(1)} = -g^2 \frac{m_w^2}{M_w^{(1)^2}} \]  
(A.107)

\[ g_{B^{(1)}B^{(1)} a_{\pm}^{(1)} a_{\mp}^{(1)}} = \frac{3g^2}{4} \frac{1}{M_w^{(1)^2}} \]  
(A.108)

\[ g_{A_3^{(1)} A_3^{(1)} a_{\pm}^{(1)} a_{\mp}^{(1)}} = \frac{3g^2}{4} \frac{M_w^{(1)^2} + 1/3m_w^2}{M_w^{(1)^2}} \]  
(A.109)

\[ g_{W_{\pm}^{(1)} h^{(1)} h^{(1)}} = \frac{3g^2}{4} \]  
(A.110)

\[ g_{W_{\pm}^{(1)} a_0^{(1)} a_0^{(1)}} = \frac{3g^2}{4} \frac{M_w^{(1)^2} + 1/3m_w^2}{M_z^{(1)^2}} \]  
(A.111)

\[ g_{W_{\pm}^{(1)} a_0^{(1)} a_0^{(1)}} = \frac{3g^2}{4} \frac{M_w^{(1)^2} - 1/3m_w^2}{M_w^{(1)^2}} \]  
(A.112)

\[ g_{B^{(1)} A_3^{(1)} h^{(1)} h^{(1)}} = \frac{3g_v g}{4} \]  
(A.113)

\[ g_{B^{(1)} A_3^{(1)} a_0^{(1)} a_0^{(1)}} = -\frac{3g_v g}{4} \frac{M^{(1)^2}}{M_w^{(1)^2}} \]  
(A.114)

\[ g_{B^{(1)} A_3^{(1)} a_{\pm}^{(1)} a_{\mp}^{(1)}} = \frac{3g_v g}{4} \frac{M^{(1)^2}}{M_w^{(1)^2}} \]  
(A.115)

\[ g_{B^{(1)} W_{\pm}^{(1)} h^{(1)} a_0^{(1)}} = \mp i \frac{3g_v g}{4} \frac{M^{(1)}}{M_w^{(1)}} \]  
(A.116)

\[ g_{B^{(1)} W_{\pm}^{(1)} a_0^{(1)} a_0^{(1)}} = -\frac{3g_v g}{4} \frac{M^{(1)^2}}{M_w^{(1)^2} M_z^{(1)^2}} \]  
(A.117)

\[ g_{A_3^{(1)} W_{\pm}^{(1)} a_0^{(1)} a_0^{(1)}} = -\frac{g^2}{2} \frac{M_w^2}{M_w^{(1)^2} M_z^{(1)^2}} \]  
(A.118)

\[ g_{B^{(1)} B^{(1)} h^{(0)} h^{(0)}} = \frac{g_v}{2} \]  
(A.119)

\[ g_{Z^{(0)} Z^{(0)} h^{(1)} h^{(1)}} = \frac{g^2}{2 c_w^2} \]  
(A.120)

\[ g_{A_3^{(1)} A_3^{(1)} h^{(0)} h^{(0)}} = \frac{g^2}{2} \]  
(A.121)

\[ g_{Z^{(0)} A_3^{(1)} h^{(0)} h^{(1)}} = \frac{g^2}{2 c_w} \]  
(A.122)

\[ g_{Z^{(0)} Z^{(0)} a_0^{(1)} a_0^{(1)}} = \frac{g^2}{2 c_w^2} \frac{M_z^{(1)^2}}{M_z^{(1)^2}} \]  
(A.123)
\[ g_{W_{\pm}^{(0)} W_{\pm}^{(0)} a_{\pm}^{(1)}} = -g^2 \frac{m_w^2}{M_w^{(1)2}} \]  
(A.124)

\[ g_{A^{(0)} A^{(0)} a_{\pm}^{(1)}} = 2e^2 \]  
(A.125)

\[ g_{Z^{(0)} Z^{(0)} a_{\pm}^{(1)}} = \frac{g^2 4c_w^4 m_w^2 + (c_w^2 - s_w^2)^2 M_w^{(1)2}}{2c_w^2} \]  
(A.126)

\[ g_{W_{+}^{(0,1)} W_{-}^{(0,1)} h_{(1,0)} h_{(1,0)}} = \frac{g^2}{2} \]  
(A.127)

\[ g_{W_{+}^{(0,1)} W_{-}^{(0,1)} h_{(0,1)} h_{(1,0)}} = \frac{g^2}{2} \]  
(A.128)

\[ g_{W_{\mp}^{(0,0)} W_{\mp}^{(0,0)} a_{\mp}^{(1)}} = \frac{g^2 M_w^{(1)2} + 3m_w^2}{2 M_z^{(1)2}} \]  
(A.129)

\[ g_{W_{\pm}^{(0,0)} W_{\mp}^{(0,0)} a_{\mp}^{(1)}} = \frac{g^2 M_w^{(1)2} + m_w^2}{2 M_w^{(1)2}} \]  
(A.130)

\[ g_{B^{(1)} A_3^{(1)} h^{(0)} h^{(0)}} = -\frac{g_Y g}{2} \]  
(A.131)

\[ g_{B^{(1)} Z^{(0)} h^{(1)} h^{(0)}} = \frac{e g_Y}{2c_w} \]  
(A.132)

\[ g_{A^{(0)} Z^{(0)} a_{\pm}^{(1)}} = \frac{eg 2c_w m_w^2 + (c_w^2 - s_w^2) M_w^{(1)2}}{c_w M_w^{(1)2}} \]  
(A.133)

\[ g_{A^{(0)} W_{\pm}^{(0,1)} h_{(1,0)} a_{\mp}^{(1)}} = \mp i \frac{e^2}{2s_w M_w^{(1)}} M_w^{(1)} \]  
(A.134)

\[ g_{B^{(1)} W_{\pm}^{(0)} h_{(0)} a_{\mp}^{(1)}} = \mp i \frac{e^2}{2c_w M_w^{(1)}} M_w^{(1)} \]  
(A.135)

\[ g_{A^{(0)} W_{\pm}^{(0)} a_{0}^{(1)} a_{\mp}^{(1)}} = -\frac{e^2}{2s_w M_w^{(1)}} M_w^{(1)} M_z^{(1)} \]  
(A.136)

\[ g_{Z^{(0)} W_{\pm}^{(0,1)} h_{(1,0)} a_{\pm}^{(1)}} = \pm i \frac{e^2}{2c_w M_w^{(1)}} M_w^{(1)} \]  
(A.137)

\[ g_{Z^{(0)} W_{\pm}^{(0)} a_{0}^{(1)} a_{\mp}^{(1)}} = \frac{e^2}{2s_w^2 c_w} \frac{s_w^2 M_w^{(1)2} - 2c_w^2 m_w^2}{M_w^{(1)} M_z^{(1)}} \]  
(A.138)

In unitarity gauge \((\xi \to \infty)\), unphysical scalar fields are not present. However, in a general gauge there are also additional vertices including unphysical scalar fields. I here choose to include only those vertex rules that were explicitly used in Paper III to calculate the one-loop process \(B^{(1)} B^{(1)} \to \gamma \gamma\) in the Feynman–t’Hooft gauge \((\xi \to 1)\):

\[ g_{A^{(0)} A^{(0)} G_{3}^{(1)} G_{-}^{(1)}} = 2e^2 \]  
(A.139)
Scalar-Scalar-Scalar-Scalar Vertices

Couplings between four scalar fields originate from the kinetic Higgs term (5.17) of the Lagrangian. The following four-dimensional Feynman vertex rules are derived:

\[
g_{B^{(1)}B^{(1)}\chi^{(0)}\chi^{(0)}} = \frac{g_\gamma^2}{2} \tag{A.140}
\]

\[
g_{B^{(1)}B^{(1)}G^{(1)}_+ G^{(1)}_-} = \frac{3g_\gamma^2}{4} \frac{m_w^2}{M_w^{(1)}^2} \tag{A.141}
\]

\[
g_{B^{(1)}B^{(1)}G^{(1)}_\mp a^{(1)}_\pm} = \frac{3g_\gamma^2}{4} \frac{m_w M^{(1)}}{M_w^{(1)}^2} \tag{A.142}
\]

\[
g_{B^{(1)}B^{(1)}\chi^{(0)}\chi^{(0)}} = \frac{g_\gamma^2}{2} \tag{A.143}
\]

\[
g_{B^{(1)}B^{(1)}G^{(1)}_0 G^{(1)}_0} = \frac{3g_\gamma^2}{4} \frac{m_z^2}{M_z^{(1)}^2} \tag{A.144}
\]

\[
g_{B^{(1)}B^{(1)}G^{(1)}_0 a^{(1)}_0} = \frac{3g_\gamma^2}{4} \frac{m_z M^{(1)}}{M_z^{(1)}^2} \tag{A.145}
\]

\[
g_{B^{(1)}A^{(0)}G^{(1)}_\pm \chi^{(0)}_{\mp}} = \pm ieg_\gamma \frac{m_w}{M_w^{(1)}} \tag{A.146}
\]

\[
g_{B^{(1)}A^{(0)}a^{(1)}_\pm \chi^{(0)}_{\mp}} = \pm ieg_\gamma \frac{M^{(1)}}{M_w^{(1)}} \tag{A.147}
\]

\[
g_{W_{\pm}^{(1)}A^{(0)}h^{(0)}G^{(1)}_\mp} = \mp i \frac{eg}{2} \frac{m_w}{M_w^{(1)}} \tag{A.148}
\]

where

\[
g_{h^{(0)}h^{(0)}h^{(0)}h^{(0)}} = -6\lambda \tag{A.149}
\]

\[
g_{h^{(1)}h^{(1)}h^{(1)}h^{(1)}} = -9\lambda \tag{A.150}
\]

\[
g_{a^{(1)}_0 a^{(1)}_0 a^{(1)}_0 a^{(1)}_0} = \frac{3g^2/c_w^2 m_z^2 M^{(1)}_w^2}{M_z^{(1)}^4} + 9\lambda M^{(1)}_w^4 \tag{A.151}
\]
\[ g_{h(1)h(1)a_0(1)a_0(1)} = -\frac{g^2 m_w^2/c_w + 6\lambda M(1)^2}{2M_Z(1)^2} \]  
(A.152)

\[ g_{h(1)h(1)a_+(1)a_-(1)} = -\frac{g^2 m_w^2 + 6\lambda M(1)^2}{2M_w(1)^2} \]  
(A.153)

\[ g_{a_0(1)a_0(1)a_+(1)a_-} = -\frac{g^2 m_w^2 M(1)^2 (1 + c_w^4)/c_w^4 + 6\lambda M(1)^4}{2M_w(1)^2 M_Z(1)^2} \]  
(A.154)

\[ g_{a_+(1)a_+(1)a_-(1)a_-} = -\frac{2g^2 m_w^2 M(1)^2 + 6\lambda M(1)^4}{M_w(1)^2} \]  
(A.155)

\[ g_{h(1)h(0)h(0)} = -\lambda \]  
(A.156)

\[ g_{h(0)h(0)a_0(1)a_0(1)} = -\frac{g^2 m_Z^2/c_w^2 + 4\lambda M(1)^2}{2M_Z(1)^2} \]  
(A.157)

\[ g_{h(0)h(0)a_+(1)a_-(1)} = -\frac{g^2 m_w^2 + 4\lambda M(1)^2}{2M_w(1)^2} \]  
(A.158)

In unitarity gauge \((\xi \to \infty)\), unphysical scalar fields are not present. However, in a general gauge there are also additional vertices including unphysical scalar fields. Although we in Paper III calculated the one-loop \(B(1)B(1) \to \gamma\gamma\) process in the Feynman-'t Hooft gauge \((\xi \to 1)\), no scalar-scalar-scalar-scalar vertexes come in at loop order for this process.

**A Short Note on Conventions in FeynArts**

Unfortunately, there is no consensus in sign conventions in the field theory literature. In our actual implementation of vertex rules into the **FeynArts** package [255], which we used in the numerical calculations in Paper III, we adopted the same convention as for the preimplemented vertex rules for the SM particles in **FeynArts**. This requires a minor change for the zero mode Goldstone bosons vertex rules compared to those given in this Appendix. Feynman rules in the **FeynArts** convention would be obtained from this Appendix if we changes the overall sign for each \(\chi_3(0)\) and multiply by \(\pm i\) for each \(\chi_{\pm}(0)\) that appears in the vertex rule.


[41] http://chandra.harvard.edu/photo/2006/1e0657/ Credit: X-ray: NASA/CXC/CfA/ [33]; Optical: NASA/STScI; Magellan/U.Arizona/ [34]; Lensing Map: NASA/STScI; ESO WFI; Magellan/U.Arizona/ [34].


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Part II

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