Essays on Incentives and Leadership

Martin Bech Holte
Abstract

This thesis consists of three self-contained essays.

**Taxation, Career Concerns and CEO Pay.** This paper proposes a simple dynamic model of equilibrium CEO compensation. The focus of the theory is on the quality of talent identification in the economy and the efficiency in the assignment of managers to firms. Motivated by the strengthened career incentives stemming from the fall in the top income tax rates over the past decades, in particular in the US, I study the implications of a model where the quality of talent identification depends on how hard individuals are willing to work in order to be among the winners in the contest for managerial positions. It is shown how the compensation of CEOs can be interpreted in this light, both across time, across industries, and across countries, and I provide some evidence showing that the predictions of the model are in line with several important empirical developments over the past decades. In essence, the paper shows how stiffer competition for promotions in the labor market as a response to lower tax rates offers an explanation for the growth in CEO compensation.

**Incentives under Communism: The Value of Low-Quality Goods.** The theory of this paper provides a novel explanation for the economic rise and decline of Communism. The emphasis of the theory is on the efficiency with which centrally planned regimes can provide incentives across different stages of economic development. In particular, I study how the regime can exploit its control over access to products of different qualities, and the attractiveness of an incentive system based on exclusive provision of high-quality goods to high-ranked members of society. At low levels of economic development, a self-interested regime can exploit such an incentive system to reduce the cost of providing incentives and raise their level. However, such an incentive system generally loses its attractiveness to the regime as the economy grows. The economic performance of the centrally planned economies is
then analyzed in light of this result. I provide evidence that justifies the assumptions of the model and that is in line with its predictions.

**The Business of Troubled Autocrats.** Many autocrats control resource rents. Typically, these regimes rely on these rents in order to buy political support. In this paper, I study how such autocrats behave in product and capital markets, in particular at times of financial distress. The objective of this paper is to construct a theory that provides answers to the following two questions: First, how does the asset position of an autocrat affect his behavior as a producer in a market with monopoly rents? What are the implications for prices? Second, from whom does the autocrat obtain financing in order to get out of difficulties? In particular, is it possible for the autocrat to exploit those who have a stake in his regime in order to get a debt contract with better terms than in a perfectly competitive capital market? I show that when the asset position of the autocrat drops below a certain threshold, output drops below the level of a standard monopolist. Under some conditions, the autocrat can obtain less expensive financing domestically, implying that there is zero foreign debt in equilibrium.
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Completing a PhD tests you in many ways. It is a test of your persistence, your patience, your creativity, to name some of them. Fortunately, I have had many people around me that have made the process easier, more interesting and more enjoyable.

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Chapter 1

Introduction

This thesis consists of three self-contained essays. All essays have a similar structure; while the research questions are empirically motivated, the analysis is primarily theoretical. Further, although the topics of the essays are diverse, there are some recurring themes, in particular moral hazard and incentives.

Chapters 2 and 3 concern the design and the effects of incentives in the labor market. In Chapter 2, I focus on how the strength of individuals’ career incentives affects the income distribution, in particular the level of compensation for those who reach the top of the ladder. In Chapter 3, on the other hand, I focus on how effectively labor market incentives can be designed under central planning.

Both Chapters 3 and 4 concern the behavior of non-democratic regimes. In Chapter 3, I analyze how the Communist regimes could exploit their complete control over the allocation of resources to provide incentives in a cost-effective manner during the era of central planning. In Chapter 4, on the other hand, I study how autocrats in control of resource rents behave in product and capital markets.

In what follows, I give a brief introduction to each of the three main chapters of the thesis.

Chapter 2: Taxation, Career Concerns and CEO Pay. Between 1970 and 2003, the ratio between CEO compensation and average income increased from about 40 to about 200. This general tendency towards higher incomes at the top can also be found in other occupations, like in law and finance, and in other countries. The favorite explanations for these tendencies include agency problems, technological change and globalization. In this paper, however, I approach the question of increasing pay differences from a new perspective. My point of departure is based on two other trends that have been observed during
the same period, in particular in the US. First, there have been steep cuts in the top marginal income tax rates. For a given compensation structure, this development has raised the payoff for being among the winners in the contest for top positions in the labor market. Second, those who are competing for the top positions in the labor market have significantly increased their supply of labor. For instance, the share in the top hourly income quintile working more than 50 hours per week doubled from 1980 to 2001. The question I ask is the following: Is the rise in CEO compensation, at least partly, the result of greater intensity in the contest for top jobs?

In the theoretical framework I construct, there is perfect competition in the market for managerial talent. However, the talent of individuals is imperfectly known and based on noisy performance signals. The driving assumption of the theory is the following: As individuals fight harder for managerial positions, the noise of the signal falls. Intuitively, it should be harder to rank individuals if they are sitting around the coffee table all day long rather than performing tasks that reveal their talent. The implication is that the distribution of expected talent spreads out as competition for positions becomes more fierce. As this happens, there are two main reasons for why pay is affected. First, as those on the demand side for managerial talent are more confident in the actual quality of the individuals, competition for those at the top is intensified. This implies that the sensitivity of pay with respect to the market value of the firm increases. Second, improvements in the identification of talents imply that the matching of firms’ assets and managers becomes more efficient. This raises the valuation of firms, implying that the willingness to pay for a manager of a given quality increases.

In the empirical part of the essay, I show that the predictions of the model are consistent with several trends that have been observed over the last few decades. First, for data from the US, I show how changes in the elasticity of CEO pay with respect to the market value of the firm have been systematically and significantly related to changes in the top marginal income taxes. I also provide evidence for how pay systematically differs across industries with different levels of capital intensity, in ways that are in line with the theoretical predictions. Finally, using data from a set of eleven industrialized countries, I show that changes in CEO pay across countries have been significantly related to changes in top marginal income tax rates.
Chapter 3: Incentives under Communism: The Value of Low-Quality Goods. In a capitalist system, incentives primarily stem from profit opportunities created by decentralized markets. In centralized economies, on the other hand, all incentives must be designed from above. The objective of this chapter is to understand how well a self-interested regime manages to solve this fundamental problem and how this affects the economic performance of centrally planned economies across the different stages of economic development.

Incentives depend on the existence of a utility wedge induced by the rewards in different states of the world, where the probability of these states can be affected by the agent. The level of incentives depends on the cost of creating these utility wedges.

The analysis is motivated by the actual design of incentives during the era of central planning in the Soviet Union and Eastern Europe. It is well known that the selective provision of high-quality goods and services was a basic element of the incentive systems implemented in these countries. In particular, all members of the working class were generally excluded from consuming goods of high quality, and were instead provided with an irregular supply of low-quality goods. It is easy to see how this system makes it possible to provide incentives at a low cost. Low-quality products as the outside option for high-ranked members of society make it possible to create high-powered incentives by only providing a modest amount of high-quality products to those who deserve them. Still, there are limits to how badly the regime can and wants to treat the working class. First, an unhappy working class threatens the stability of the regime and will possibly stage a revolt. Second, workers who are too hungry and unhealthy are unproductive. In other words, the regime is constrained in its design of the outside option for those further up in the hierarchy.

In this essay, I formally show how and when an incentive system based on the exclusive provision of high-quality goods to certain groups is optimal from the perspective of the regime. The analysis yields the following set of main results: First, if no credible threat of a revolt exists, the regime always wants to offer workers products of low quality. Second, given that a credible threat of a revolt exists, it is only at levels of consumption where the supply of labor responds to changes in consumption that the regime wants to offer workers products of low quality. In this range, the attractiveness of giving the workers fewer goods of higher quality is moderated by its negative effect on labor supply. Whenever the supply of labor is no longer responsive to increases in consumption, it is
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no longer optimal to base incentives on the exclusive provision of high-quality goods. Only in a special case, when the technology the regime employs to deter the workers from revolting is sufficiently effective, will the strategy of exclusive provision of high-quality goods continue to be optimal as the economy develops. Third, at low levels of development, the ability of the regime to offer low-quality products to the workers raises the optimal level of incentives. On the other hand, whenever the regime is forced to increase the rewards to workers in order to secure its power, the optimal level of incentives for managers falls.

In the final part of the essay, I use these results to account for the evolution of centrally planned economies over time.

Chapter 4: The Business of Troubled Autocrats. Many autocrats control resource rents, for instance those of the oil-producing countries in the Middle East and elsewhere. These regimes typically rely on resource revenues to buy political peace. In this paper, I study how autocrats behave in the product and capital markets, given this political constraint.

The essay is inspired by developments in one such country, namely the Kingdom of Saudi Arabia. In the late 1990s, the Saudi government was experiencing a fiscal crisis. Government debt was at 120 percent of GDP, after a long period of large public deficits and periods of low oil prices. With soaring public debt and low oil prices, spending on transfers to the citizens had to be cut. The Saudi regime had relied on transfers to the citizens to preserve power and there were fears of a collapse of the Saudi regime.

These developments in the political arena were coupled with the interesting developments in the oil and financial markets. As the financial situation of the regime deteriorated, Saudi Arabia cut its excess oil production capacity and advocated cuts in OPEC production. Note that oil revenues have consistently constituted about three fourths of the revenues of the Saudi government. Public debt, on the other hand, was solely financed from domestic sources, and the share of foreign assets of Saudi institutional investors fell rapidly. Saudi retail investors, on the other hand, were reluctant to hold long-term government debt.

With these developments in mind, the objective of this paper is to construct a theory that provides answers to the two following questions: First, how does the asset position of an autocrat affect his behavior as a producer in a market with monopoly rents? What are the implications for prices? Second, from whom does the autocrat obtain financing in order to get out of difficulties? In particular, is it possible for the autocrat to exploit those with a stake in his regime in order
to get a debt contract with better terms than in a perfectly competitive capital market?

Regarding production behavior, I show that there exists a threshold level of assets such that the autocrat behaves as a regular monopolist above this threshold, while he is restricting output to below the monopoly level when he is below the threshold. The intuition is straightforward. First of all, in the model, demand is uncertain and output is determined before the demand shock is realized. When the level of assets is low, the autocrat foresees that in the future, he will not be able to provide the citizens with the level of transfers he wishes. This is especially true if the realization of demand in the product market is low. This implies that the marginal value of profits for the autocrat is higher when demand is low than when it is high, which further implies that states with a low demand receive a greater weight in the production decision, which makes it optimal to restrict output below the level of a regular monopolist.

Next, I study the question of who the autocrat chooses as a counterpart for public debt. In the model, the autocrat can approach either a regular competitive capital market or a group of supporters of the regime. The income of these supporters depends on the survival of the regime, and I study whether the autocrat can exploit this dependence in order to obtain funding at a lower expense from them than from the competitive capital market. I show that if the supporters can overcome collective action problems, and if their stake in the continued existence of the regime is sufficiently large, the autocrat will, in fact, strike better deals with the supporters and only domestic debt is observed in equilibrium.
Chapter 2

Taxation, Career Concerns and CEO Pay\footnote{I am grateful to Torsten Persson for advice and comments, and for comments and suggestions from Bård Harstad, Estelle Cantillon, Mathias Dewatripont, Luis Garicano, Per Krusell, Kjetil Storesletten, Per Strömberg and seminar participants at the Econometric Society European Winter Meeting, IFN, the IIES Brown Bag Seminar, LSE and University of Pennsylvania. I also thank Kevin J. Murphy for providing data and Christina Lönnblad for editorial support. Financial support from The Jan Wallander’s and Tom Hedelius’ Foundation is gratefully acknowledged. All errors are mine.}

2.1 Introduction

In the last few decades, the compensation of top executives has increased substantially relative to the income of a typical worker. This development has been especially dramatic in the United States, as indicated in Figure 1 where I provide a plot of the ratio of CEO compensation to mean income over the years 1970 to 2003. In the early 1970s, a typical CEO of a top 500 US company earned about forty times the average income in the US. In the early 2000s, however, this ratio had increased to about 200.\footnote{Note that the Forbes data that cover the period 1970-1991 do not include the value of stock options prior to 1978. Thus, the compensation ratio for these early years is somewhat too low.}

The compensation of CEOs has increased rapidly in other countries as well, as can be seen in Table 1. This table shows the development in the ratio between CEO compensation and manufacturing wages for various countries from the mid 1980s to 2000. There has been a general upward trend for all countries. However, there is a great deal of variation. Whereas this ratio almost tripled in the US between 1984 and 2000, it only increased by about 20 percent in Germany and about 40 percent in Switzerland.
During the same period, there have been two other noteworthy tendencies. First, there has been a trend towards lower top marginal income tax rates. In the US, for instance, the top marginal income tax rate was more than halved from the early 1960s to the 1990s. There has also been a general tendency towards lower top marginal income tax rates in other countries. For a given compensation structure, this development has raised the payoff for being among the winners in the contest for top positions in the labor market. Second, at least in the US, many of those who are in the race for such positions seem to have increased their labor supply significantly. For instance, the share in the top hourly income quintile working more than 50 hours per week doubled from 1980 to 2001.\(^3\)

In this paper, I propose a theory of the relationship between these trends. The purpose is to propose answers to the following questions: What happens to the levels of compensation at the top end of the distribution when the value of successful careers increases as a result of a fall in top marginal tax rates? And, in particular, is the rise in CEO compensation, at least partly, the result of greater intensity in the contest for top jobs?

The main emphasis of the theory is on the quality of talent identification in the labor market. Allocating individuals to jobs in an efficient way is not a trivial matter. Hiring decisions are based on incomplete information about the quality of workers and obviously errors are made. In this paper, I present a theory of how well firms manage to solve this problem and, in particular, how the efficiency in the assignment of individuals to jobs affects the compensation of top managers. Motivated by the strengthened career incentives stemming from the fall in the income tax rates over the past decades, I study a model where the quality of talent identification depends on how hard individuals work in order to be among the winners in the contest for managerial positions. Whereas most studies of CEO pay take the distribution of managerial talent to be constant over time (see e.g. Lucas [1978], Terviö [2003], Gabaix and Landier [2007]), the hypothesis advanced here is that the difficulty in discovering talent in the labor market is related to the workers’ career incentives. If these incentives change, for instance through changes in the income tax rates, the quality of talent discovery is also affected, and this alters the compensation level of managers. In my model, which is built around the career-concern model of Holmström (1999), the effort level provided by the workers affects the precision with which talent is identified and thus, the distribution of expected talents at the time when managers are hired.

\(^3\)See Section 2 for the details.
At low effort levels, the distribution of expected talent is compressed, and it is hard for firms to separate talented individuals from untalented ones. As effort increases, the distribution of expected talent spreads out. It becomes clearer who the talented individuals are, and the competition for top talent creates upward pressure on pay.

There are two main reasons for why pay is affected as the distribution of expected talent spreads out. First, as those on the demand side for managerial talent are more confident in the actual quality of the individuals, competition for those at the top is intensified. This implies that the sensitivity of pay with respect to the market value of the firm increases. Second, improvements in the identification of talents imply that the matching of firms’ assets and managers becomes more efficient. My model follows the assignment approach used by Rosen (1982) and recent papers by Terviö (2003) and Gabaix and Landier (2007). Complementarities between firms’ assets and managerial talent imply that the best managers (in expectation) are hired by the firms with most assets. As untalented managers are weeded out and the efficiency in the assignment improves, the market value of the firms improves. This increase in the market values of firms is the second reason for why pay increases, as the firms have more to gain from getting a manager with a certain level of talent.

In the empirical part of the paper, I show that the predictions of the model are consistent with several trends that have been observed over the last few decades. First, for data from the US, I show how changes in the elasticity of CEO pay with respect to the market value of the firm have been systematically and significantly related to changes in the top marginal income taxes. I also provide evidence for how pay systematically differs across industries with different levels of capital intensity, in ways that are in line with the theoretical predictions. Finally, using data from a set of eleven industrialized countries, I show that changes in CEO pay across countries have been significantly related to changes in top marginal income tax rates.

To sum up, the contribution of the paper is as follows: First, I introduce a novel mechanism that brings together the strength of career incentives and the distribution of compensation in the labor market. In essence, I demonstrate that stiffer competition for promotions in the labor market as a response to lower

4More broadly, the model follows the approach in the literature of assortative matching (see. e.g. Becker [1973]). See Sattinger (1993) for an overview of assignment models.
tax rates offers an explanation for the growth in CEO compensation. Second, I derive new testable predictions for the structure of CEO compensation over time, across industries and across countries. Finally, I provide suggestive evidence in support of these predictions.

A few remarks are in order. First, the driving assumption of the theory is that the share of the variance in an individual’s output caused by random factors falls when individuals work harder. This assumption implies that the variance in the distribution of expected talent is a function of the effort level of workers. To me, at least, this seems to be a reasonable assumption. The results of the paper correspond to a setting where the number of tasks performed by an agent depends positively on his effort, and the noise in output across tasks is not perfectly correlated. Intuitively, if a worker sits around the coffee table most of the day, you have to base your judgement about his ability on fewer observations, and you also lose some of the ability of seeing him perform in stressful situations that might give important clues about the worker’s ability as a leader. Of course, hard evidence on the validity of this assumption is ultimately needed, but it is interesting in itself to understand the effects of an increase in the intensity in the competition for promotions when there are selection effects.

Second, even though I focus on the market for CEOs in this paper, the argument can easily be extended to other types of labor markets where reputations are important. For instance, the analysis might as well have been built around the careers of lawyers seeking to become partners of a law firm, or around those young aspiring investment bankers seeking to build a reputation and a network that makes it easier to attract money to their hedge fund or venture capital firm. In light of this, the study by Kaplan and Rauh (2007) is interesting. They study how the composition of the top end of the income distribution (e.g. the top 0.01%) has changed from 1994 to 2004. Whereas the share of CEOs at the top end of the income distribution has been relatively stable during this period, the share of partners in law firms and "Wall Street" type individuals seems to have increased.

Finally, even though I focus on tax rates as the catalyst of change, the reader should have in mind that this is not the only catalyst that would trigger the effects I describe. Other factors affecting labor supply would also do this, as would factors affecting the demand for talent. This is further discussed in the final section of the paper.
With its focus on the strength of career concerns and the quality of talent identification in the economy, this paper adds a novel argument to the debate on the causes for the increases in CEO pay in the last few decades. There are four other broad classes of explanations in the literature. The first is the explanation advanced by Gabaix and Landier (2007), namely that the growth in CEO pay is the result of an increase in the market value of the average firm in the economy. This explanation is related to Rosen’s (1981) superstar theory. Gabaix and Landier focus on how exogenous changes in the demand for talent have affected pay. The focus of my paper is instead on changes in the supply of talent, which have consequences for the demand side via the impact on the level and distribution of firm values.

The second explanation for higher CEO pay relates to the adoption of compensation packages with more high-powered incentives over the last two decades (see e.g. Murphy [1999] for an overview). Inderst and Mueller (2005) argue that a more competitive business environment has made it optimal to provide managers with greater incentives. Given that managers are risk averse, the greater risk in the compensation package requires that the expected value of compensation must rise in order to satisfy CEOs’ participation constraints. There is also a literature on taxation and the design of CEO pay (see e.g. Hall and Liebman [2000] and Katuscak [2006]), which studies how the details in the tax system affect how high-powered the incentives in an optimal compensation package will be. In this paper, I abstract from agency problems at the CEO level and the details of the compensation package. Rather, the focus is on the determinants of the reservation wage of CEOs and thus, on the level of expected compensation a company must pay in order to attract a manager with a certain level of expected talent.

Third, there is the literature on managerial rent-seeking and insufficient control by the CEOs’ principals (see e.g. Bertrand and Mullainathan [2001], Bebchuck and Fried [2004], and Hall and Murphy [2003]). In that literature, the rise in CEO compensation is explained by changing social norms, an increase in managerial entrenchment, or boards’ limited understanding of the cost of using financial instruments as stock options in the compensation package. I ignore such frictions in my paper and focus on another friction, namely the market’s limited ability to identify talent, and study how this friction has evolved over time.\footnote{Kaplan and Rauh (2007) argue that the rent-seeking view is at best incomplete, as income growth seems to have been even larger in law and finance, where it is hard to divert funds.}
Finally, there is a literature explaining the rise in CEO pay with changes in the nature of the CEO job, either due to changes in technology (Garicano and Rossi-Hansberg [2006]) or the generality of skills that is required (Frydman [2005]). In this literature, the multiplier that maps talent to pay is either the state of technological development or the number of firms that compete for an individual’s talent. In my paper, in contrast, the multiplier of talent is the strength of an individual’s career incentives.

At a more general level, the paper is also related to the literature on the sources of increased income inequality over the last few decades. Whereas others have focused on the effects of technical change (see e.g. Katz and Autor [1999] and Acemoglu [2002]), changes in labor market institutions (e.g. DiNardo, Fortin and Lemieux [1996]), or globalization, my paper offers a theory of how the (progressiveness of) the tax system affects the degree of (pre-tax) income inequality in society. In a recent paper, Levy and Temin (2007) argue that the income distribution in the US during the twentieth century has been shaped by the strength of redistributitional policies and institutions. This paper adds a formal theoretical framework to the view advocated by Levy and Temin.

The rest of the paper is organized as follows. In Section 2, I provide some further empirical and theoretical motivations for the theory presented in this paper. Section 3 presents the model and defines its equilibrium. The model is analyzed in Section 4. In Section 5, I provide some empirical evidence from the US, whereas some preliminary cross-country evidence can be found in Section 6. I discuss and conclude in Section 7.

### 2.2 Further Motivations

#### 2.2.1 Empirical Motivation

The argument in this paper rests on the assumption that individuals’ effort levels in the high-skilled end of the labor market have responded to changes in career incentives and, in particular, that effort has responded to the drop in marginal income tax rates.

In Figure 2, I provide a plot of the top marginal federal income tax rates in the US since 1960. From a high level of about 0.9, the top marginal tax rate started...
to fall in the late 1960s. It was cut substantially early in the 1980s, reaching a bottom around 1990. After an early tax increase by President Clinton, the rate has remained relatively stable in the 0.3 to 0.4 range.

There is a great deal of anecdotal evidence that the workweek has become significantly longer for those competing for high-paying jobs. Fortune magazine, for instance, claims that "[t]he 60-hour weeks once thought to be the path to glory are now practically considered part-time." (Fortune, Nov. 29, 2005). "In downtown Manhattan, black cars line up outside Goldman Sachs’ headquarters every weekday around 9. Employees who work that late get a free ride home, and there are plenty of them. Until 1976, a limousine waited at 4:30 p.m. to ferry partners to Grand Central Terminal." These days, "4:30 is the middle of the workday, not the end." (The New York Times, May 15, 2005). Harder statistical data also tell us that the workweek is longer today than in the 1960s, at least for the well-paid and highly educated. In Figure 3, I reproduce evidence presented in Kuhn and Lozano (2006). Based on US Census data over six decades, the figure shows the share of male (non self-employed) workers that put in 49 hours or more per week over the period 1940-2000. This share dropped somewhat from a level of 19 in 1940 to 17 in 1970. Since then, however, the share of male workers that puts in long workweeks has increased substantially, to a level of 26.5 in 1990, and an even higher level in the year 2000.\(^6\)

A second interesting thing to note is that there have been large differences in the labor-supply responses across skill levels and income groups. In Table 2, I reproduce some further evidence from Kuhn and Lozano (2006). This table shows that the increase in the fraction of workers that reports workweeks of more than 50 hours is positively correlated with the level of education. Between 1980 and 2001, there was essentially no change for individuals who have not completed high school, whereas the increase was the largest for college graduates. Across average hourly earnings quintiles, the development is even more pronounced. In the bottom quintile, the fraction of men working long hours fell by one third from 0.21 to 0.143, whereas it doubled for those in the top quintile, from 0.146 to 0.29.

Thus, even though the number of average hours worked in the US has been quite stable over time, this statistic hides the sizable increase in the variance of average hours, as well as the higher variance of hours for those in the top income brackets.

\(^6\)In the year 2000, the Census did not collect information on actual hours worked in the previous week. Thus, only data on the usual hours worked in the previous year are available. However, the relationship between actual and usual hours worked seems to be quite stable, and the fraction reporting usual hours above 48 hours increased significantly from 1990 to 2000.
hours worked across occupations and skill levels, and the fact that many of those who compete for or hold high-paying jobs have significantly increased their labor supply over the past decades. This paper deals with how this shift in career concerns has affected pay in equilibrium.

2.2.2 Theoretical Motivation

To see the logic of the model I formulate in the next section, it might be useful to take a brief look at a simple static competitive assignment model of CEO pay. The ranks of firms and managers are denoted by $r \in [0,1]$ and $m \in [0,1]$, with lower ranks implying a more valuable firm/more talented manager. Consider a firm ranked $r$, with organizational capital $O(r)$ that hires a manager with talent $T(m)$ at cost $W(m)$ and thus, earning a profit $\pi(r, m) = T(m)O(r) - W(m)$. The firm maximizes over $m$, the rank of its manager. The first-order condition for an optimum is given by $T'(m)O(r) - W'(m) = 0$. Due to the complementarity in the profit function, efficiency in the assignment process requires that $m = r$. Therefore, we get the following differential equation for the wage function:

$$W'(r) = T'(r)O(r).$$

Integrating this over some interval $[r, r^*]$ gives

$$W(r) = W(r^*) - \int_r^{r^*} T'(r)O(r)dr.$$ 

The expected talent of potential managers is a result of an initial qualifying round, where the competing workers have put in effort to provide signals of their talent. The ranking of expected talents is determined by the outcomes in this initial contest. The question in which I am interested is what is needed for higher effort in the qualifying round to increase the manager’s wage at any rank $r$. Assume the effect on the organizational capital of firms to be zero. Then, differentiation with respect to effort gives (ignore for the moment the effect on $W'(r)$, which is assumed to be relatively small)

$$\frac{\partial W(r)}{\partial e} = -\int_r^{r^*} \left( \frac{\partial T'(r)}{\partial e} O(r) \right) dr.$$ 

\(^7\)Hewlett and Luce (2006) provide further evidence of the extent of extreme workweeks among individuals that take part in the type of race for positions that I consider in this paper.
2.3. THE MODEL

It is evident that a sufficient condition for wages at rank \( r \) to increase in the effort level is that

\[
\frac{\partial T'(r)}{\partial e} < 0.
\]

As \( T'(r) \) is a downward-sloping function, this condition says that as effort increases, the slope of the talent spacing distribution must become steeper. In words, this means that an increase in effort must make the expected talent difference between two different managers greater, holding their ranks fixed. This condition will not be satisfied by distributions with a fixed upper bound of talent.\(^8\) With a uniform distribution, for instance, this will not be the case, as higher effort will tighten the distribution of managerial talent levels. In other words, to generate the positive effect of effort on wages, we need to work with a distribution where nobody’s talent is perfectly revealed. This is exactly what I do in my model of talent identification.

2.3 The Model

In this section, I present the model. Section 3.1 presents the basic components of the model economy, while the equilibrium is defined in Section 3.2.

2.3.1 Preliminaries

Consider an industry where the workforce is composed of two-period overlapping generations of individuals. In each period, a mass 1 of young individuals enters the labor market of this industry. These individuals work when they are young in order to provide signals about their talent level to the mass \( \phi \in (0, 1/2) \) of firms in the industry that will be competing to hire managers in the next period.\(^9\) No agent has any information ex ante about a certain individual’s actual level of talent, but after one period of work, the talent level can be imperfectly inferred by all agents. The fraction \( \phi \) of workers with the highest expected ability will be hired as managers by the firms in a setting with competitive assignment, where a firm with rank \( r \in [0, \phi] \) is assigned a manager with rank \( m \in [0, \phi] \). The rank of firms is based on the level of assets, which is determined in equilibrium. The rank of prospective managers is based on the rank of their output level when

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\(^8\)Gabaix and Landier (2007) advocate a talent distribution with a fixed upper. However, their argument is based on an assumption of an exogenous distribution of firm values, independent of managerial talent.

\(^9\)The assumption that \( \phi < 1/2 \) implies that only old individuals are hired as managers.
young. Individuals who do not qualify as managers remain workers.

**Preferences:** Individuals are risk neutral. When young, they put in effort in order to maximize the sum of expected wages minus the cost of providing effort.¹⁰ Formally,

\[ U_y = \max_e \{ E[w^y(\tau)|e] + E[w^o(\tau)|e] - C(e) \}, \]

where \( w^y \) is the wage when young and \( w^o \) is the wage when old. \( \tau \) is the tax schedule imposed by society, and \( C(\cdot) \) is an increasing convex function. For simplicity, the tax system is characterized by a proportional income tax rate, \( \tau \). Moreover, assume that \( C(0) = 0, C'(0) = 0 \) and \( \lim_{e \to \infty} C'(e) = \infty \). Further, I assume that old individuals supply a unit level of effort inelastically, so as to focus on the career incentives facing the young individuals. Thus, the only decision facing individuals is the choice of effort level to provide when they are young.

**Output and compensation of workers:** The output of young workers, \( q \), depends on their level of talent \( \theta \), the effort they provide and a noise term, \( \varepsilon \):

\[ q = e\theta + \varepsilon. \]

I assume talent to be normally distributed with mean \( \bar{\theta} \) and variance \( \sigma_\theta^2 \),

\[ \theta \sim N(\bar{\theta}, \sigma_\theta^2). \]

The noise term is also normally distributed with mean zero and variance \( \sigma_\varepsilon^2 \),

\[ \varepsilon \sim N(0, \sigma_\varepsilon^2). \]

Given the realization of output for young workers, the market estimates their expected talent. Let \( T = E[\theta|q] \) denote the expected talent of an individual given his output level when young. Based on the location of an individual in the distribution of expected talent, each individual is given a rank \( m \in [0,1] \). The expected talent of an individual with rank \( m \in [0,1] \) is denoted by \( T(m) \).

¹⁰When income increases, many individuals will take the opportunity to increase their amount of leisure. However, if you choose to enter the competition for top positions in the labor market, you can only afford such a luxury if others let you. It is the behavior of the individuals who have entered this competition that I study in this paper. For this group, the income effect will be weaker and I choose to disregard it altogether in the model.
Old individuals provide a unit of effort inelastically and earn a wage equal to their expected talent. Young individuals earn a wage equal to their output. Thus, the expected wage of a young individual is $e\theta$. I let the old workers supply effort inelastically because I do not want to focus on the design of incentive contracts, but rather on the force of career incentives and selection.\footnote{The exact type of contract offered to the workers does not affect the results of the paper qualitatively, even though the details of the discussion of for instance efficiency in Section 4.5. changes somewhat. However, the driving force behind the results is the process of talent identification, which is operational whenever (expected) compensation increases in the level of expected talent.}

**Industries:** The main purpose of introducing industries of different types is to investigate whether, in addition to a time series prediction for CEO pay, we can also obtain concrete predictions about how CEOs are paid across industries. This will potentially give us a much richer set of predictions to evaluate empirically.

One natural way of distinguishing industries in this setting is by the importance of human capital, and the CEO’s talent in particular, in the profit function.\footnote{Bennedsen, Perez-Gonzalez and Wolfenzon (2007) argue that the impact of CEOs is greater in human-capital intensive and fast-growing industries.} Let $\alpha$ define an industry’s talent intensity. My choice of production function below is guided by two requirements.

First, to guarantee that an increase in $\alpha$ increases talent intensity, the profit function (before CEO pay) must satisfy the following condition:

$$\frac{\partial}{\partial \alpha} \left( \frac{\partial \pi_w}{\partial T} \right) > 0,$$

(2.1)

where $\pi_w$ is defined as profits before CEO compensation. In words, the marginal value of talent increases when $\alpha$ increases.

Second, because my empirical proxy for talent-intensity is the price-book value of the industry, the price-book value of a representative firm in the industry must be an increasing function of $\alpha$. Let $p_b(\alpha)$ denote the price-book value of this firm.\footnote{The price-book value is defined as the market value of equity divided by the book value of equity.} Thus, I require that

$$\frac{\partial p_b(\alpha)}{\partial \alpha} > 0.$$ 

(2.2)

**Firms:** Firms produce output using two main factors of production, human capital and physical capital, $K$. An industry is defined by the relative importance
of these two factors, captured by the parameter $\alpha$. The (expected) level of human capital in a firm depends on two factors of production: The expected talent of the CEO, $T$, and the organizational capital of the firm, $O$.

The gross income of the firm is

$$I = (TO)^\alpha K^\alpha. \quad (2.3)$$

Physical capital is rented in the capital market at an interest rate $i$. A CEO with expected talent $T$ is paid $W(T)$, where $W(\cdot)$ is the wage function determined in equilibrium. The organizational capital of the firm in period $t + 1$ if the realized talent of the CEO was $\theta_t$ in period $t$ is given by

$$O_{t+1} = \bar{O} + \theta_t^m, \quad (2.4)$$

where $\bar{O}$ is a constant and $\theta_t^m$ is the actual talent level of the firm’s manager in the current period. In other words, firms are concerned about getting a good manager not only because it affects current profits, but also because it will affect future profits. Organizational capital has a broad interpretation in the model. It could, for instance, represent how efficiently the previous CEO managed to streamline the organization to make production efficient, the quality of the products he introduced, the size of the market reached by the products, and so on.

To simplify the analysis, I assume that the firm always rents the ex post optimal amount of capital. In other words, the talent of the CEO is revealed before capital is rented. The advantage of this assumption is that expected profits become a function of $T = E[\theta|q]$, rather than some other function $E[\theta^x|q]$ for $x \neq 1$. This makes the analysis simpler and more intuitive. Thus, given the talent level $\theta^m$ of its manager, firms rent capital optimally by maximizing $(\theta^m O)^\alpha K^\alpha - iK$ over $K$. This yields

$$K = \left(\frac{1 - \alpha}{i}\right)^{\frac{1}{\alpha}} \theta^m O$$

Inserting the optimal capital level, the firms’ expected profits are given by the expression

$$\pi = \int_\Theta f_T(\theta^m)\pi(\theta^m) = \alpha \left(\frac{1 - \alpha}{i}\right)^{\frac{1}{\alpha}} TO - W(T) \equiv \kappa(\alpha, i)TO - W(T), \quad (2.5)$$
where I have defined
\[ \kappa(\alpha, i) = \alpha \left( \frac{1 - \alpha}{i} \right)^{\frac{1-\alpha}{\alpha}}. \]

For the profit function to satisfy condition (2.1) above, it is necessary that \( \partial \kappa(\alpha, i) / \partial \alpha > 0 \). For this to be the case, we must have \( \alpha \geq \bar{\alpha} < 1 \), where \( \bar{\alpha} \) is defined as the level of \( \alpha \) that minimizes \( \kappa(\alpha, i) \). Thus, I only consider industries with \( \alpha \in [\bar{\alpha}, 1] \). Even though my assumptions imply that profits are an increasing function of human-capital intensity, \( \alpha \), this is not equivalent to assuming that CEO pay will be an increasing function of \( \alpha \), controlling for the value of the firm. In fact, several models of CEO pay would not yield such a prediction. An example is the model in Lucas (1978), where CEO pay is predicted to be proportional to firm values, and the proportionality factor is independent of \( \alpha \).

Since there is no separation between debt and equity in the model, the appropriate definition of the price-book value is the market value of assets (the present value of profits) divided by the book value of assets (the capital stock of the firm). Consider a typical firm of an industry, where we for a moment ignore the fact that the levels of talent and organizational capital fluctuate over time. The price-book value of such a firm is given by the present value of profits divided by the capital stock:
\[ p_b(\alpha) = \frac{\pi(\alpha)}{K(\alpha)} = \alpha \left( \frac{1-\alpha}{i} \right)^{\frac{1-\alpha}{\alpha}} TO = \frac{\alpha}{1-\alpha}. \] (2.6)

With my assumptions, the human-capital intensity of the industry is the single cause of variation in industry-average price-book values. Clearly, eq. (2.6) satisfies condition (2.2) above.

Finally, I can state the problem facing the firm. A firm of rank \( r \in [0, \phi] \) chooses a manager of rank \( m \in [0, 1] \) with expected talent \( T(m) \), in order to maximize the sum of current profits and the continuation value of the firm, given the wage function and the law of motion for organizational capital. Formally, the firm’s Bellman equation is given by
\[ V(r) = \max_m \{ \kappa(\alpha, i)T(m)O(r) - W(m) + \frac{E[V(r')|m]}{1+i} \}, \] (2.7)

where \( r' \) is the rank of the firm in the next period, which depends on the level of organizational capital built up by the CEO in this period.

To facilitate the analysis, some major simplifications have implicitly been
made regarding the firms’ production function. In particular, the only dimension that distinguishes firms within an industry in the model is the talent of their previous CEOs. No other firm-specific factors affect profits. This simplification makes it easy to rank firms according to their willingness to pay for managerial talent, and the assignment problem is simplified. However, since firms are equally willing to pay for the future value of the talent of today’s manager, competition between firms over the best CEOs probably becomes too fierce as compared to reality. By giving managers an unrealistic degree of bargaining power, some of the parameters in the managerial wage function become unrealistic.

To sum up, the timing of events within period $t$ is as follows: Stage (i): A young generation enters the labor market. Stage (ii): Members of the old generation are ranked based on their output in the previous period. Firms are ranked based on their level of organizational capital. Stage (iii): Firms hire managers. Stage (iv): Managerial talent is revealed. Stage (v): Firms rent capital. Stage (vi): Individuals put in (unobservable) effort. Individual (observable) and firm output realized and consumed. Stage (vii): Members of the old generation exit the labor market.

I can now move on to define the equilibrium of the model.

2.3.2 Definition of Equilibrium

**Definition 1:** An equilibrium in this model (for a given tax system, $\tau$) is defined by a wage function $W(m)$, an assignment function $M(r)$, a value function $V(r)$, a cutoff output level $q^*(\phi)$, and a common effort level $e$, such that

- The effort level $e$ solves the workers’ problem given $W(m)$, $M(r)$, $V(r)$, and $q^*(\phi)$
- The assignment function $M(r)$ is optimal: Given $W(m)$ and $V(r)$, no firm wants to hire another manager than the one it has
- The cutoff output level $q^*(\phi)$ is such that a mass $\phi$ of young workers produces an output level at or above $q^*(\phi)$
- The wage function $W(m)$ is consistent with optimal firm behavior and satisfies the participation constraint of potential managers, i.e. $W(r) \geq T(r)$
2.4 Analysis

In order to solve the model, I proceed as follows: The individual’s problem and the firm’s problem depend on the distribution of expected talents and the distribution of talent in the population of managers, as the ranks of individuals and firms for given output and asset levels depend on these distributions. Thus, before I proceed to analyzing these problems, I need to characterize these distributions for a given symmetric effort level, $e^*$. Then, I study the young individual’s choice of effort given that the other individuals put in effort $e^*$. The wage and value functions are determined by the firm’s problem and the distribution of expected talent in the population of old individuals, also for a given equilibrium effort level. Finally, together with the worker’s first-order condition, these two functions are sufficient for characterizing the equilibrium effort level.

2.4.1 Preliminaries

Expected talent of prospective managers: Firms estimate the talent of prospective managers based on their output when young. It is critical that effort and talent are complements; this is what makes it possible for the effort level to affect the precision in the identification of talent.

For a given equilibrium effort level $e^*$ and given the assumptions on the distribution of $\theta$ and $\varepsilon$ stated above, output $q$ of young workers is distributed as $q \sim N(e\tilde{\theta}, (e^*)^2\sigma_{\theta}^2 + \sigma_{\varepsilon}^2)$. Let $f_q$ and $F_q$ denote the pdf and cdf of $q$.

Given $e^*$, the expected talent level of an individual who produced output $q$ is given by

$$E[\theta|q] = E[\theta] + \frac{\text{Cov}(\theta, q)}{\text{Var}(q)} (q - E[q]) = \tilde{\theta} + \frac{e^* \sigma_{\theta}^2}{(e^*)^2 \sigma_{\theta}^2 + \sigma_{\varepsilon}^2} (q - e^* \tilde{\theta}),$$

and the variance around this expected value is given by

$$\text{Var}(\theta|q) = \frac{\sigma_{\theta}^2 \sigma_{\varepsilon}^2}{(e^*)^2 \sigma_{\theta}^2 + \sigma_{\varepsilon}^2}.$$

For each firm to obtain a manager, a measure $\phi \in (0, 1)$ of each generation will be recruited as managers when old. Since expected talent is strictly increasing in the output level, this implies that all young individuals with an output level above some threshold value $q^*(\phi)$ become managers when old. This cutoff level
is implicitly defined by
\[ \phi = \int_{q^*(\phi)}^{\infty} f_q(q) dq. \]  

(2.8)

**How effort affects the distribution of expected talent:** Define expected talent given output \( q \) as \( \hat{T}(q) = E[\theta|q] \). Note that the distribution of \( \hat{T} \) is given by
\[ \hat{T} \sim N \left( \theta, \left( e^* \sigma^2_{\theta} + \sigma^2_{\varepsilon} \right)^2 \right), \]
such that the variance of \( \hat{T} \) is increasing in the effort level, or
\[ \frac{\partial \text{Var}(\hat{T})}{\partial e^*} = 2e^* \frac{(\sigma^2_{\theta})^2 \sigma^2_{\varepsilon}}{\text{Var}(q)^2} > 0. \]
Moreover, as expected, we have
\[ \lim_{e \to \infty} \text{Var}(\hat{T}) = \sigma^2_{\theta}, \]
which implies that as effort increases, the distribution of expected talents converges to the distribution of actual talents. At low effort levels, the distribution of expected talents has most of its mass around the mean, but as effort increases, the difference in talent across individuals becomes clearer, and we move towards the actual distribution given to us by nature. It is important to note that this effect is due to the complementarity between effort and talent in the production function. In the standard additive Holmström (1999) model, talents can be as easily identified with zero effort as with a high effort level. It is as if everyone could get to know Roger Federer’s brilliance even if he never stepped onto a tennis court.

**Rank of managers:** As expected talent is a strictly increasing function of output, prospective managers are ranked based on their output. Let \( q(m) \) be the output of a potential manager with rank \( m \). This level is defined by
\[ m = \int_{q(m)}^{\infty} f_q(q) dq. \]
Obviously, only those with rank \( m \leq \phi \) become managers.

Given that a manager has rank \( m \), his expected talent is given by \( T(m) = E[\theta|q(m)] \), which is clearly increasing in effort. As \( \hat{T} \) is normally distributed, it is straightforward to find the talent level of a manager with rank \( m \). Define the
standard normal variable

\[ z(m) = \frac{T(m) - \bar{\theta}}{\sqrt{\text{Var}(T)}}. \]

Manipulating this slightly, the talent level of a manager with rank \( m \) is given by

\[ T(m) = \bar{\theta} + z(m)\sqrt{\text{Var}(T)}, \]

where \( \text{Var}(T) = \text{Var}(\hat{T}) \). The expected talent of a manager with rank \( m \) depends positively on the variance of expected talents. From this equation, we can derive the following useful result:

**Lemma 1:** The difference in expected talents across ranks increases in effort:

\[ \frac{\partial T'(m)}{\partial e^{\ast}} < 0 \]

**Proof:** See Part A1 of the Appendix.

Intuitively, the complementarity between effort and talent implies that the output signal becomes less noisy when the effort level increases, as more of the variation in output stems from talent rather than noise. This raises the variation in the distribution of expected talents.

**The Distribution of Organizational Capital:** A firm’s organizational capital depends on the actual talent level of its previous manager. The distribution of actual talents in the group of managers satisfies

\[
f_{\theta^{\text{m}}}(\theta) = f(\theta | \theta \geq q^*(\phi)) = \frac{f_{\theta}(\theta)(1 - F_{\phi}(q^*(\phi) - e^\ast \theta))}{\int f_{\theta}(u)(1 - F_{\phi}(q^*(\phi) - e^\ast \theta))d\theta} = \frac{f_{\theta}(\theta)(1 - F_{\phi}(q^*(\phi) - e^\ast \theta))}{\phi}.
\]

The density at a given talent level is the product of the density of that particular talent level in the population multiplied by the probability that an individual with this talent level produces a level of output higher than the threshold \( q^*(\phi) \).

Given that a firm has a stock of organizational capital, \( O \), what is its rank, \( r \)? This is given by the mass of managers with higher talent than that of the firm’s previous manager

\[ r = \int_{O(r) - O}^{\infty} f_{\theta^{\text{m}}}(\hat{O} + \theta)d\theta. \]

This equation implicitly defines the function \( O(r) \).
To continue with the characterization of the solution of the model, the following result is useful:

**Lemma 2:** As \( r \to 0 \), the distribution of organizational capital can be approximated by the normal distribution.

**Proof:** See Part A2 of the Appendix.

Lemma 2 is useful because I will mainly be concerned with wages at the extreme right tail of the distribution. Intuitively, the most talented individuals almost always make the cut, so that the density of these types in the population of managers closely approximates their density in the general population, and this density is normal given the distributional assumptions in Section 3.

### 2.4.2 The Individual’s Problem

The individual puts in effort given the market’s rational expectations of what he is doing, and given that he expects all other individuals to put in effort \( e^* \). In order to understand how the equilibrium effort level affects the distribution of expected talents and wages, and thus also the individual’s incentives, the following corollary of Lemma 1 is useful:

**Corollary 1:** \( \frac{\partial T(m)}{\partial e^*} < 0 \) for \( m > \frac{1}{2} \) and \( \frac{\partial T(m)}{\partial e^*} > 0 \) for \( m < \frac{1}{2} \).

**Proof:** This follows directly from Lemma 1 and the fact that \( T(\frac{1}{2}) = \bar{\theta} \), independently of effort. From Lemma 1 we have \( \frac{\partial T(m)}{\partial e^*} < 0 \), which implies that expected talent must fall for all those with rank higher than \( \frac{1}{2} \), and rise for all those with a rank lower than \( \frac{1}{2} \).

**Corollary 2:** An increase in effort causes wages to fall for workers with a rank above \( \frac{1}{2} \), while they rise for those with a rank lower than \( \frac{1}{2} \).

\[
\frac{\partial w(r; e^*)}{\partial e^*} = \begin{cases} 
< 0 & \text{if } r > \frac{1}{2} \\
> 0 & \text{if } \phi < r < \frac{1}{2} 
\end{cases},
\]  

(2.9)

**Proof:** This is a direct consequence of Corollary 1 and the fact that the wage of old workers equals their expected talent level.

The wage function facing individuals of different ranks can be written as:

\[
w(r) = \begin{cases} 
T(r) & \text{if } r > \phi \\
W(T(r)) & \text{if } r \leq \phi 
\end{cases},
\]  

(2.10)
2.4. ANALYSIS

Given the effort of everyone else, a young individual provides the effort level that solves:

$$\max_e \{(1 - \tau)e\bar{\theta} + \int_0^1 (1 - \tau)w(r)f(r, e^*; e)dr - C(e)\}, \quad (2.11)$$

where

$$f(r, e^*; e) = -q'(r) \int \theta f(\theta)f_e(q(r; e^*) - e\theta)d\theta.$$  

In equilibrium, when $e = e^*$, this is a constant function $f(r, e^*; e^*) = 1$ (see the Appendix).

The first-order condition of the worker’s problem is

$$(1 - \tau)\bar{\theta} + (1 - \tau)\int_0^1 w(r)\frac{\partial f(r, e^*; e^*)}{\partial e}dr - C'(e) = 0. \quad (2.12)$$

The second-order condition is given by

$$D = (1 - \tau)\int_0^1 w(r)\frac{\partial^2 f(r, e^*; e^*)}{\partial e^2}dr - C''(e) < 0.$$  

For now, simply assume that this is satisfied in equilibrium (see Section 4.4 for details).

The main comparative static in which I am interested is how effort responds to taxes. Differentiating the FOC with respect to $e$ and $\tau$, I get

$$\left[\bar{\theta} + \int_0^1 w(r)\frac{\partial f(r, e^*; e^*)}{\partial e}dr\right]d\tau = \left[(1 - \tau)\int_0^1 w(r)\frac{\partial^2 f(r, e^*; e^*)}{\partial e^2}dr - C''(e)\right]de,$$

which implies

$$\frac{de}{d\tau} = \frac{\bar{\theta} + \int_0^1 w(r)\frac{\partial f(r, e^*; e^*)}{\partial e}dr}{D} < 0.$$  

Thus, effort is decreasing in the tax rate, provided that the second-order condition is satisfied in equilibrium.

2.4.3 The Firm’s Problem

A firm with rank $r$ hires a manager with rank $m$ and pays him a wage $W(m)$. Having plugged in optimally chosen capital, the Bellman equation of the firm is

$$V(r) = \max_m \{\kappa(\alpha, i)T(m)O(r) - W(m) + \frac{E[V(r')|m]}{1 + i}\},$$
subject to the law of motion for the firm’s rank,

\[ r' = r(\theta^m). \]

where \( \theta^m \) is the actual realization of talent of the manager hired by the firm.

The first-order condition of the firm is

\[ \kappa(\alpha, i)O(r)T'(m) - W'(m) + (1 + i)^{-1} \frac{\partial E[V(r')|m]}{\partial m} = 0 \]

To proceed, we need the following result:

**Lemma 3:** The equilibrium assignment function is given by \( M(r) = r \).

In equilibrium, the assignment of managers to firms is efficient. This requires that the ranks of managers and firms are matched. In other words, the equilibrium assignment function satisfies \( M(r) = r \). The reason for this is simple: The assignment is immaterial for the sum of continuation values. Thus, efficiency in the assignment process is simply determined by maximizing the sum of current profits. Due to the complementarity between talent and organizational capital, this sum is maximized by setting \( m = r \).

Lemma 3 implies that, in equilibrium, the firm’s FOC is

\[ \kappa(\alpha, i)T'(r)O(r) - W'(r) + (1 + i)^{-1} E[V(r')|r] = 0, \tag{2.13} \]

and its value function is

\[ V(r) = \kappa(\alpha, i)T(r)O(r) - W(r) + (1 + i)^{-1} E[V(r')|r]. \]

Now, taking the derivative of the value function with respect to \( r \), and combining this with the first-order condition (2.13), the differential equation for the firm’s value function can be written as

\[ V'(r) = \kappa(\alpha, i)T(r)O'(r) \]

\[ \text{Note that the property of positive assortative matching will also hold for more general functions of } O(r). \text{ In particular, it will hold whenever the expected marginal impact on continuation values from current talent depends (weakly) positively on the rank in the current period, } \frac{\partial E[V(r')|m,r]}{\partial m}. \text{ This condition ensures that talent and organizational capital are supermodular, which implies positive assortative matching (see, for instance, Becker [1973]).} \]
Integrating this over the interval \([r, r^*]\) gives us an expression of the firm’s value function:

\[ V(r) = V(r^*) - \kappa(\alpha, i) \int_r^{r^*} O'(u) T(u) du. \]  

(2.14)

Finally, by integrating the equilibrium FOC, the managerial wage function can be written as

\[
W(r) = W(r^*) - \kappa(\alpha, i) \int_r^{r^*} T'(u) O(u) du \\
+ (1 + i)^{-1} E[V(r')|r] - E[V(r')|r^*].
\]

(2.15)

Together with the individual’s first-order condition (2.12), eqs. (2.14) and (2.15) are the two key equations characterizing the equilibrium of the model. To make them empirically useful, I will approximate the functions \(T(\cdot)\) and \(O(\cdot)\) by applying some basic results from extreme value theory (see Section 4.6).

### 2.4.4 Properties of Equilibrium

For a given effort level, we can find \(V(r; e^*)\), and \(q^*(\phi; e^*)\) as described above. Having \(V(r; e^*)\), we can find \(W(r; e^*)\). To complete the characterization of the equilibrium, the equilibrium effort level, \(e^*\), needs to be pinned down. If we plug the wage function (2.15), which is a function of \(e^*\), into the individual’s first-order condition (2.12), we can solve for the equilibrium effort level. It is not difficult to show that an equilibrium exists, but sufficient conditions for its uniqueness require some relatively weak assumptions on the cost-of-effort function \(C(\cdot)\).

Let \(w^m(r)\) be the CEO premium at rank \(r\), defined by

\[ w^m(r; e^*) = w(r; e^*) - T(r; e^*) \geq 0. \]

Thus, we can write \(w(r; e^*) = T(r; e^*) + w^m(r; e^*)\). Then, we can rewrite the individual’s FOC (2.12) as

\[
\frac{C'(e^*)}{(1 - \tau)} = \bar{\theta} + \int_0^1 T(r; e^*) \frac{\partial f(r; e^*; e^*)}{\partial e} dr + \int_0^\phi w^m(r; e^*) \frac{\partial f(r, e^*; e^*)}{\partial e} dr. \quad (2.16)
\]

I will call the right-hand side of this equation the marginal benefit of effort, \(MB(e)\). Using \(T(r, e^*) = \bar{\theta} + z(r) \sqrt{Var(T)}\), and the fact that \(\int_0^1 \frac{\partial f(r; e^*; e^*)}{\partial e} dr = 0\),
(2.16) can be rewritten as
\[
\frac{C'(e^*)}{(1-r)} = \tilde{\theta} + \sqrt{Var(T)} \int_0^1 z(r) \frac{\partial f(r, e^*; e^*)}{\partial e} dr + \int_0^{\phi} w^m(r; e^*) \frac{\partial f(r, e^*; e^*)}{\partial e} dr.
\]

It is clear that the right-hand side of this equation equals \( \tilde{\theta} \) for \( e^* = 0 \), as \( T(r; 0) = \tilde{\theta} \), and \( w^m(r; 0) = 0 \). Moreover, the right-hand side converges to \( \tilde{\theta} \) (from above) as \( e^* \to \infty \). A proof of this is found in Part A5 of the Appendix. These two facts, in addition to the fact that the right-hand side is continuous in \( e^* \), imply that an equilibrium exists given that the marginal cost of effort, \( C'(e^*) \), is a continuous and increasing function of \( e^* \).

Next, I want to characterize how the marginal benefit of effort, \( MB(e) \), changes in \( e^* \). Taking the derivative of the right-hand side of (2.17) with respect to \( e^* \), we get
\[
\frac{\partial MB(e^*)}{\partial e^*} = \sqrt{Var(T)} \int_0^1 z(r) \frac{\partial f(r, e^*; e^*)}{\partial e} dr \left[ \frac{\partial Var(T)}{\partial e^*} + \int_0^1 z(r) \frac{\partial^2 f(r, e^*; e^*)}{\partial e \partial e^*} dr \right] + \int_0^{\phi} w^m(r; e^*) \frac{\partial f(r, e^*; e^*)}{\partial e} \left[ \frac{\partial w^m(r; e^*)}{\partial e^*} - \frac{\partial^2 f(r, e^*; e^*)}{\partial e \partial e^*} \right] dr
\]

In order to characterize how the marginal benefit of effort develops as \( e^* \) increases, the following results are useful.

**Lemma 4:** \( \frac{\partial f(r, e^*; e^*)}{\partial e} = \left( \tilde{\theta} + \frac{z(r)\sqrt{Var(\theta)}}{\sqrt{Var(q)}} \right) \frac{z(r)\sqrt{Var(e)}}{\sqrt{Var(q)}} + \frac{e\sqrt{Var(\theta)}\sqrt{Var(e)}}{\sqrt{Var(q)}} \)

**Proof:** See Part A6 of the Appendix.

**Corollary 3:**
(i) \( \int_0^1 z(r) \frac{\partial f(r, e^*; e^*)}{\partial e} dr = \frac{\tilde{\theta}Var(e)}{\sqrt{Var(q)}} \).
(ii) For any \( z(r) > 0 \), there exists a finite \( \tilde{e}(z(r)) \) such that \( \frac{\partial^2 f(r, e^*; e^*)}{\partial e \partial e^*} \) for all \( e > \tilde{e}(z(r)) \geq 0 \)

**Proof:** Follows directly from Lemma 4.

With Corollary 3, (2.18) can be rewritten as
\[
\frac{\partial MB(e^*)}{\partial e^*} = \frac{\tilde{\theta}Var(e)\sqrt{Var(\theta)}}{Var(q)^2} \left[ Var(e) - (e^*)^2 Var(\theta) \right] + \int_0^{\phi} w^m(r; e^*) \frac{\partial f(r, e^*; e^*)}{\partial e} \left[ \frac{\partial w^m(r; e^*)}{\partial e^*} - \frac{\partial^2 f(r, e^*; e^*)}{\partial e \partial e^*} \right] dr
\]
2.4. ANALYSIS

The first term in this sum is single-peaked and eventually converges to zero as $e^*$ increases. Next, consider each term of the integral separately. Given that the wage premium $w^m$ converges monotonically as $e^* \to \infty$, it is clear that for all $r$ there exists an $\tilde{e}(r; w^m)$ such that the term in the bracket is negative for all $e > \tilde{e}(r; w^m)$. In this case, there exists an $\hat{e}$ such that $MB(e^*)$ is decreasing for all $e > \hat{e}$, and a sufficient condition for a unique equilibrium is that $C'(e) < \tilde{\theta}$ for all $e < \hat{e}$, such that the cost function intersects the marginal benefit on its downward sloping section. Such an equilibrium is illustrated in Figure 4. An increase in the tax level from $\tau_1$ to $\tau_2$ makes the function $C'(e)/(1 - \tau)$ shift upward and the equilibrium effort level falls.\footnote{The second-order condition (SOC) can only be violated in equilibrium if there exists more than one equilibrium, and in particular another equilibrium where the SOC is satisfied. Consider an increase in the tax rate as in Figure 4. If the SOC is violated, it implies that a shift from $C'(e)/(1 - \tau_1)$ to $C'(e)/(1 - \tau_2)$ yields an increase in the effort level. But if this happens, the marginal benefit curve is above $C'(e)/(1 - \tau_1)$ for effort levels higher than the initial equilibrium. As the marginal benefit converges to $\tilde{\theta}$ as $e$ increases, it must intersect $C'(e)/(1 - \tau_1)$ at least once more, and at least once from above such that the SOC is satisfied.}

Intuitively, at low levels of effort, the marginal benefit of effort is increasing, as the dispersion in wages across talent levels increases and because the individuals’ incentive to increase their effort to affect the market’s judgement of them is large. As the effort level increases, however, the temptation to affect the market’s judgement falls, as the distribution of output becomes so spread out that a marginal change in effort is unlikely to change the position in the ranking to any considerable extent. In addition, much of the scope for wage dispersion due to talent identification eventually becomes exhausted. These effects cause the marginal benefit of effort to have an inverse U-shape.

2.4.5 Efficiency

In this section, I will outline how the market outcome differs from the outcome the individuals would have chosen had it been possible to enforce a coordinated common effort level. The individuals have no control over the level of taxation and I will not discuss optimal policy. The coordinated solution is nevertheless referred to as the planner’s solution.

Consider the planner’s problem of maximizing the utility of individuals:

$$\max_e \{ (1 - \tau) \left( e \tilde{\theta} + \int_0^\theta \pi(r, e) dr + \int_0^1 w(r, e) dr \right) - C(e) \}.$$
Let $w^m(r)$ be the managerial wage premium at rank $r$, i.e., the amount a manager earns in excess of what he would have earned as a worker, $T(r)$, that is,

$$w^m(r) = w(r) - T(r).$$

Note that for the planner, the density $f(r)$ is a constant and equal to 1. For the planner, $f(r)$ is simply the density of the population across ranks, which is a constant by definition. For an individual, however, $f(r)$ denotes the probability density for that particular individual across ranks, which depends on his effort level relative to the effort level of the other individuals. This implies that the wages of the old in the planner’s problem can be rewritten as follows:

$$Z_0^1 w(r, e^p) dr = Z_0^1 (w^m(r) + T(r)) dr = \int_0^\phi w^m(r) + \int_0^1 T(r) dr = \int_0^\phi w^m(r) + \tilde{\theta}.$$

The problem can then be rewritten as

$$\max_e \{ (1 - \tau) \left( e \tilde{\theta} + \int_0^\phi (\pi(r, e) + w^m(r)) dr + \tilde{\theta} \right) - C(e) \}. $$

Let $e^p$ denote the planner’s solution. The first-order optimality condition is

$$\tilde{\theta} + \int_0^\phi \left( \frac{\partial \pi(r, e^p)}{\partial e} + \frac{\partial w^m(r, e^p)}{\partial e} \right) dr = \frac{C'(e^p)}{1 - \tau}. \quad (2.19)$$

For simplicity, ignore industries and consider the simple profit function discussed in Section 2,

$$\pi(r, e) = T(r)O(r) - W(r).$$

This implies that

$$\pi(r, e) + w^m(r, e) = T(r, e)(O(r, e) - 1). \quad (2.20)$$

Thus, the term $\pi(r, e) + w^m(r, e)$ equals the output of a firm of rank $r$, $T(r, e)O(r, e)$, minus the opportunity cost for having the manager in this firm, $T(r)$.

For convenience, I restate the first-order condition for individuals in the market:

$$\tilde{\theta} + \int_0^\phi w^m(r, e) \frac{\partial f(r, e^*; e^*)}{\partial e} dr + \int_0^1 T(r, e) \frac{\partial f(r, e^*; e^*)}{\partial e} dr = \frac{C'(e^*)}{1 - \tau}. \quad (2.21)$$

Comparing equations (2.19) and (2.21), we see that consumption for the
young enters the problem in the same way for both the individual and the planner. Thus, there is no distortion involved in how effort affects the consumption of the young. Whereas the planner chooses the effort level to maximize the surplus of the firms (output minus the opportunity cost), individuals do not internalize this effect. Workers put in effort in order to affect the market’s judgement of their talent level. In one sense, this is a self-defeating zero-sum game. But it has positive side-effects: it improves the identification of talent and raises the aggregate output of firms. Without this efficiency gain, too much effort would be exerted relative to the optimum. In fact, from the planner’s point of view, the term $\int_0^1 T(r) f(r) \, dr$ is independent of effort, so the individuals’ struggle to affect their rank is a complete waste. That effort is overprovided in positional zero-sum games has been emphasized by many authors, for instance in the classic career-concerns model by Holmström (1999). In the setting considered in this paper, however, it is not clear that effort is overprovided, due to the productive side-effects of the race for top positions. If the quality of talent identification improves sufficiently when effort increases, effort is actually underprovided by the market.

It is difficult to establish precise conditions for when the planner’s effort level is higher or lower than that provided in the market. However, we can identify the important factors affecting this balance.

Specifically, we have $e^p \leq e^*$ if the individual’s marginal benefit of effort in the market is higher than the planner’s benefit of effort at $e^p$ (the planner’s optimum), i.e. if

$$
\int_0^\phi w^m(r, e^p) \frac{\partial f(r, e^p; e^p)}{\partial e} \, dr + \int_0^1 T(r, e) \frac{\partial f(r, e^p; e^p)}{\partial e} \, dr \\
\geq \int_0^\phi \left( \frac{\partial \pi(r, e^p)}{\partial e} + \frac{\partial w^m(r, e^p)}{\partial e} \right) \, dr.
$$

Using eq. (2.20), we have

$$
\left( \frac{\partial \pi(r, e^p)}{\partial e} + \frac{\partial w^m(r, e^p)}{\partial e} \right) = \frac{\partial T(r, e^p)}{\partial e} (O(r, e^p) - 1) + T(r, e^p) \frac{\partial O(r, e^p)}{\partial e}.
$$

By inspecting these expressions, I can identify three factors in particular that make it more likely that the market’s effort level exceeds the optimal effort level:

(i) $\phi$ is small. In this case, the wasteful competition for positions is more likely to dominate. The potential efficiency gain is too small since there are few
managerial positions that must be filled.

(ii) $O(r), \ r \in (0, \phi)$ is small. In this case, organizational capital is relatively unimportant, i.e., the gains from equipping talented managers with organizational capital are small relative to what these individuals could produce by themselves. In other words, Rosen's (1981) superstar effect is weak, as the most talented individuals do not have control over any significantly larger amount of resources than the less talented ones.

(iii) $\text{Var}(\varepsilon)$ is small. If this is the case, $\frac{\partial T(r,e^p)}{\partial e}$ and $\frac{\partial O(r,e^p)}{\partial e}$ are small. This implies that the potential for improvements in talent identification is small relative to the individuals' incentives to affect their positions. The planner understands well that if talent is already well identified, there is no point in pushing up effort. However, in isolation, the individuals have incentives to increase effort even when $\text{Var}(\varepsilon)$ is small. Formally, $\int_0^1 T(r, e) \frac{\partial f(r,e^p,e^p)}{\partial e}$ falls relatively slowly to zero when effort increases, even when $\text{Var}(\varepsilon)$ is small. This is because the effort level is unobserved, so that an individual always has an incentive to try to make the market believe that he is more talented than he really is, even though the market sees through this and discounts it completely in equilibrium.

### 2.4.6 Approximations Based on Extreme Value Theory

To get analytical expressions for the firm's value function and the market wage function, it is necessary to make a few approximations. As in Gabaix and Landier (2007), I will base these approximations on results from extreme value theory. For all "regular" continuous distributions, there exist constants $B > 0$ and $\xi$ such that the spacings in the upper tail of the distribution can be approximated by

$$T'(r) = -Br^{-\xi-1}.$$  

This equation may hold exactly or up to a slowly varying function. $\xi$ is the tail index and distributions with fatter tails will have weakly larger $\xi$ (see Gabaix, Li and Laibson [2006] for details).

All functions $F$ with regular distributions belong to one of three domains of attraction, which is characterized by the sign of $\xi$, the tail index. When a random variable $t$ is normally distributed, it belongs to the domain of attraction of the Gumbel, where the distribution function of $t$ in the upper tail is approximated by

$$H(t) = \exp(-\exp(-\frac{t - \mu}{\sigma})), \quad (2.22)$$
where $\mu$ is the location parameter and $\sigma$ is the scaling parameter. The tail index of the distributions belonging to the domain of attraction of the Gumbel is zero. When we draw from a normal distribution with mean $\bar{X}$ and variance $Var(\bar{X})$, the location parameter is given by $\mu = \bar{X}$ and the scaling parameter by $\sigma^2 = \frac{6}{\pi^2} Var(\bar{X})$.

Further, consider two normally distributed random variables $X_1$ and $X_2$, with $E[X_1] = E[X_2]$ and $Var(X_1) < Var(X_2)$. Then, in the approximate distribution of the upper tail, the parameters $\mu$ and $\sigma$ will be such that $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$. In particular, in the setting considered in this paper, when effort increases, the mean of the expected talent distribution is unaffected, but the variance, the scaling parameter, increases.

To see how we arrive at the function approximating expected talent, $T(r)$, one can go through the following four steps: First, let $r$ be the rank corresponding to talent level $t$. Then eq. (2.22) can be rewritten as

$$r = 1 - \exp(-\exp(-\frac{t-\mu}{\sigma})).$$

Second, a few manipulations of this equation give

$$T(r) = t = \mu - \sigma \ln(-\ln(1-r)).$$

Third, as we consider the case where $r \to 0$, we have $\ln(1-r) = -r$, such that

$$T(r) = \mu - \sigma \ln(r).$$

Finally, this gives us

$$T'(r) = -\sigma r^{-1}.$$

This implies that we can approximate the function $T(r)$ by

$$T(r) = C_T - B_T(e) \ln(r), \quad (2.23)$$

where $C_T = \bar{\theta}$ and the scaling parameter $B_T(e) = \frac{6}{\pi^2} \sqrt{Var(T)}$ depends positively on the level of effort:

$$B_T'(e) > 0. \quad (2.24)$$

Finally, we can approximate the distribution of organizational capital with the normal distribution where both the scaling and location parameters are unaffected by the effort level (Lemma 2). Thus, the function $O(r)$ can be approxi-
mated by the function
\[
O(r) = C_O - B_O \ln(r),
\]
(2.25)
where \(C_O = E[O]\) and \(B_O = \frac{\sigma_2}{\tau} \sqrt{\text{Var}(O)}\).

As the tail index \(\xi = 0\) is constant, the convenience of using the normal distribution, or other distributions belonging to the domain of attraction of the Gumbel, is obvious. The changes in the distribution are simply captured by the changes to the scaling parameter, \(B\).

### 2.4.7 Approximate Solution

Using the approximations of \(T\) and \(O\), the value function (2.14) becomes

\[
V(r) = V(r^*) - \kappa(\alpha, i) \int_r^{r^*} T(u)O'(u)\,du
\]
\[
= V(r^*) + \kappa(\alpha, i) \left[ \int_r^{r^*} \frac{B_O}{u} (C_T - B_T(e) \ln(u))\,du \right]
\]
\[
= V(r^*) + \kappa(\alpha, i) B_O C_T \left[ \ln(r^*) - \ln(r) \right]
\]
\[
- \kappa(\alpha, i) B_O B_T(e) \left[ \ln(r^*)^2 - \ln(r)^2 \right] / 2
\]
(2.26)

As I only consider the largest firms, let \(r/r^* \to 0\). This leads us to the final version of the value function (a formal proof is available in the Appendix):

\[
V(r) = \kappa(\alpha, i) B_O \left[ \frac{B_T(e)}{2} \ln(r)^2 - C_T \ln(r) \right].
\]
(2.27)

This function shows that firms gain when talent is better identified, i.e., when \(B_T(e)\) increases. Further, firm values increase in the mean level of talent and in the variance of organizational capital. The latter finding is intuitive: A lower variance of firm assets implies that the competition for managerial talent increases. This implies that more rents are acquired by the managers and less by the firms.

Next, plugging in the approximations in the wage equation, we can rewrite eq. (2.15) as

\[
W(r) = W(r^*) + \kappa(\alpha, i) B_T(e) \left[ C_O \left( \ln(r^*) - \ln(r) \right) - \frac{B_O}{2} \left( \ln(r^*)^2 - \ln(r)^2 \right) \right]
\]
\[
+ (1 + i)^{-1} \left( E[V(r^*) | r] - E[V(r^* | r^*)] \right).
\]
By adding and subtracting $\kappa(\alpha, i)B_O C_T(\ln(r^*) - \ln(r))$ to this equation, (2.26) can be used to rewrite it as follows:

$$W(r) = W(r^*) + \kappa(\alpha, i)(B_T(e)C_O - B_O C_T)[\ln(r^*) - \ln(r)] + V(r) - V(2.28) + (1 + i)^{-1}(E[V(r')|r] - E[V(r')|r^*]).$$

For $r$ small, we can use (2.26) to find the following relationship between $\ln(r^*) - \ln(r)$ and $V(r) - V(r^*)$:

$$\ln(r^*) - \ln(r) = \sqrt{\frac{2(V(r) - V(r^*))}{\kappa(\alpha, i)B_O B_T(e)}}.$$

We can use this to rewrite eq. (2.28) as follows:

$$W(r) = W(r^*) + \kappa(\alpha, i)(B_T(e)C_O - B_O C_T)\sqrt{\frac{2(V(r) - V(r^*))}{\kappa(\alpha, i)B_O B_T(e)}} + V(r) - V(r^*) + (1 + i)^{-1}(E[V(r')|r] - E[V(r')|r^*]).$$

Since we are only considering the largest firms, I let $E[V(r')|r^*] = W(r^*) = V(r^*) = 0$. Moreover, I approximate $E[V(r')|r']$ by $V(r)$. Finally, then, the CEO compensation function can be written as

$$W(r) \approx \left[\sqrt{2\kappa(\alpha, i)B_O B_T(e)}\left(\frac{C_O}{B_O} - \frac{C_T}{B_T(e)}\right)\right] \sqrt{V(r)} + (1 + (1 + i)^{-1})V(r).$$

For equation (2.29) to have one fundamental and empirically attractive feature, namely that managerial pay is concave in the value of the firm, the following condition is sufficient:

**Assumption 1**: $\frac{\partial \pi}{\partial T} \frac{dT}{dr} < \frac{\partial \pi}{\partial O} \frac{dO}{dr}.$

Assumption 1 implies that $\left(\frac{C_O}{B_O} - \frac{C_T}{B_T(e)}\right) > 0$ in (2.29). Assumption 1 requires that the marginal profits of new talent are greater than the marginal impact of inherited talent (i.e., organizational capital). The assumption implies that firms bid up the prices for good managers because the willingness to pay for talent drops quite slowly as one moves down the rank ladder for organizational talent.

16The proof can be found in part A8 of the Appendix.
capital. How much firm \( X \) has to pay depends on the willingness to pay of all firms with less assets. The more slowly the asset level drops further down the ranks, the more firm \( X \) has to pay, and the more concave is the pay function. Further, if the expected level of talent drops rapidly down the managerial ranks, firms with few assets are willing to participate in the competition for talents, and this competition creates a more concave pay function.

2.4.8 Predictions

With these preliminaries in hand, I am ready to state the predictions of the model.

Proposition 1 (General Properties): Given Assumption 1, the managerial wage function has the following properties:

1. CEO pay is concave in the market value of the CEO’s firm: \( \frac{\partial^2 W}{\partial V^2} < 0 \).
2. CEO pay is increasing in the price-book ratio of the industry: \( \frac{\partial W}{\partial \alpha} > 0 \).
3. The sensitivity of CEO pay to the market value of the firm is increasing in the price-book ratio of the industry: \( \frac{\partial^2 W}{\partial V \partial \alpha} \).

Proof: Properties 1-3 follow directly from inspection of (2.29).

Proposition 1.1 is an attractive feature of the model, since the concavity of CEO pay to market value might be the most well-known stylized fact in the literature (see e.g. Roberts [1959] or Kostiuk [1990]). Typically, in a regression \( \ln(w_{it}) = \alpha + \beta \ln(V_{it}) + \varepsilon \), the estimate of \( \beta \) is around 0.3. The reason why this concavity emerges even in a model with Gaussian signals is that the talents of the present and previous manager interact in the profit function.

The prediction in Proposition 1.2 has, to the best of my knowledge, not previously been made in the literature. The result that CEO pay should increase in the talent-intensity of the industry emerges as a result of the equilibrium wage function. The required assumption for Proposition 1.1 to hold also implies that Proposition 1.2 holds. Assumption 1 requires that the marginal profits of new talent are greater than the marginal impact of inherited talent. This causes firms with low levels of capital to bid intensely for good managers, and this effect will be stronger in talent-intensive industries. Similarly, to the best of my knowledge, Proposition 1.3 is new.

Proposition 2 (Properties relating to effort/taxes): Given Assumption 1, the managerial wage function has the following properties:
1. CEO pay is an increasing function of effort/decreasing function of taxes: 
\[ \frac{\partial W}{\partial e} > 0, \frac{\partial W}{\partial \tau} < 0. \]
2. Higher effort/lower taxes increase the sensitivity of CEO pay to the firm’s market value: 
\[ \frac{\partial^2 W}{\partial V \partial e} > 0, \frac{\partial^2 W}{\partial V \partial \tau} < 0. \]
3. Higher effort/lower taxes strengthen the effect of the industry’s price-book value on CEO pay: 
\[ \frac{\partial^2 W}{\partial a^2} > 0, \frac{\partial^2 W}{\partial a \partial \tau} < 0. \]

**Proof:** Properties 1-3 follow directly from inspection of (2.29) and the fact that \( \frac{\partial e^*}{\partial \tau} < 0. \)

Proposition 2.1 states that CEO pay increases when effort increases (taxes go down). This is due to two main factors: On the one hand, CEO pay grows because firm values increase as the talents of managers improve. On the other hand, CEO pay grows because the sensitivity of CEO pay to the firm’s market value is increasing in the effort level (Proposition 2.2). In an assignment model, a manager’s bargaining power is determined by the talents of those with poorer rank than himself, as these are the ones who can replace him as manager. When talent identification improves, the bargaining power of a given managerial rank improves as the gap to those behind grows, so that firms become less willing to substitute him for someone else. Thus, Proposition 2.2 predicts that higher effort/lower taxes will force the firms to give more of the surplus to the managers.

Proposition 2.3 predicts that the result in Proposition 1.2 should be strengthened if effort goes up/taxes fall.

Note that the increase in the factor share going to CEOs is limited by the increase in the variance of the expected talent distribution \( B_T(e) \). Since we have \( \lim_{e \to \infty} B_T(e) = 0 \), factor shares will eventually stabilize, even as the hours the contestants put in continue to increase from high levels.

Note that the model yields no prediction of the kind investigated by Gabaix and Landier (2007), namely how firm size affects pay. In my model, firm size is endogenous. I suspect that models with an exogenously given distribution of market values of firms will yield the same kind of prediction as that found by Gabaix and Landier.

Finally, the model also yields some predictions about what should happen to the distribution of firms’ market values and profits.

The major impact of improved talent identification is that the mean talent of the population of managers increases. This increase in the mean talent is not primarily caused by more participation of the most talented types (remember
that the distribution of actual talent of managers in the extreme right tail is approximately normal), but by types that are just below the top. In particular, consider a sample of the largest firms, i.e. firms with organizational rank \( r < \hat{r} \) with \( \hat{r} \) small. We know that \( \partial O(\hat{r})/\partial e > 0 \) (the level of organizational capital at a given rank increases when effort increases), and \( \lim_{r \to 0} \partial O(r)/\partial e = 0 \). This implies that the variance of actual talents of managers in the sample of large firms goes down when talent identification improves. As realized profits are a function of actual talents only, this implies that the variance of realized profits across firms goes down. Further, over time, it implies that smaller firms should grow relatively faster than larger firms.

For market values, the same mechanism is of importance.Inspecting the expression for the value function in (2.14), it can be seen that as the actual talent of the CEO is realized, and since the variance of actual talents falls as effort increases (taxes fall), there will be a fall in the variance of market values when effort increases (taxes fall). It is an interesting property of the model that the forces that make the distribution of expected talent more spread out when effort increases also make the distribution of actual talents of managers tighter.

I can now state the following

**Proposition 3 (Compression in the variance of profits and market values):**

1. The variance of profits falls when effort increases/taxes fall.
2. The variance of market values falls when effort increases/taxes fall.

Note that I abstracted away from the change in the distribution of \( O \) in the approximation of the managerial wage function. Since I only looked at the extreme right tail of the talent distribution, I could ignore the increases in organizational capital that took place somewhat further down in the distribution. Including this effect in the theoretical analysis would only serve to strengthen the results. A higher mean and a lower variance in actual talent levels make the competition between firms for talent move one more step towards Bertrand-style competition, as \( O'(r) \) falls. In particular, these changes make Assumption 1 more likely to hold, as \( dO/dr \) falls, which once more implies that the term \( \left( \frac{CO}{BO} - \frac{CT}{BT} \right) \) in the managerial pay function (2.29) increases.
2.5 Some Evidence From the US

In this section, I will provide some evidence concerning a number of the above predictions. Primarily, I will look at the determinants of CEO pay in the US and how these have evolved over time, but I will also give some evidence of how the distribution of market values has changed over time.

2.5.1 Taking the Model to the Data

Obviously, it is difficult to directly test the underlying hypothesis of this paper, namely that talent is easier to identify in a more competitive labor market. To evaluate this hypothesis, one would want direct evidence of the changes in the relative quality of managers over time. One solution could be to have time-series evidence on the quality of managerial practice, based on a set of objective standards. A recent paper by Bloom and van Reenen (2007) provides such evidence across countries. They find that management practices are, on average, better in the US than in Europe, and these practices tend to be weak when product-market competition is weak and when family-owned firms follow the tradition of primo geniture (i.e., passing control over to the eldest son). To the best of my knowledge, however, no good time-series data exist.

Instead, I will provide some indirect evidence by studying whether the theoretical implications for the CEO pay function are consistent with the data. One important question is then whether my theoretical variables have clear empirical counterparts.

In the model, the valuation of firms takes a very simple form, and is completely driven by the quality of managers. Thus, the model’s notion of firm values might best be understood as the valuations that are driven by the quality of its managers. Naturally, many other factors affect the value of firms, so it is not clear a priori that there is a close mapping between the firm valuations in the model and the firm valuations in the data. However, if one assumes that there is a positive relationship between the valuation of the firm and the amount by which a CEO can affect a firm’s valuation, this is not a big problem.

In the data, I will let human-capital intensity (α in the model) be proxied by the ratio of the market value of equity to the book-value of equity (PB value). In the model, where there is no difference between debt and equity, the proper measure is the market value divided by the book-value of assets (MA value). These ratios might differ systematically due to different levels of leverage across
industries. For a high debt-asset industry, it might be the case that the PB value is high while the MA value is low, just because there is little equity relative to total assets. For instance, firms in the financial sector tend to have very high debt-to-asset measures, so that the MA value is lower than the PB value. In practice, however, these two measures are strongly positively correlated (see Figure 5). Moreover, it is reasonable to believe that the PB value might be more relevant for compensation than the MA value. After all, compensation packages are determined by equity owners. In any case, the point estimates of my regressions are unaffected by using the MA value (not reported below) instead of the PB value, although the standard errors are slightly larger. Furthermore, the results essentially remain unchanged when I control for the debt-to-assets ratio in the regressions below.

Finally, I will test the predictions using a log-specification. This is not completely in line with the model, as some predictions hold in levels but not necessarily in logs. Empirically, the results are similar for the level and log specifications, and the interpretation of results is facilitated using the latter.

2.5.2 Data

The data come from three sources. Firm data are collected from the Compustat database. Data on executive compensation come from the Execucomp database, which covers the years 1992-2005, and the Forbes’ Survey of Executive Compensation, which covers the years 1970-1991. There is one major problem with combining these two datasets, namely how stock options are treated, if at all. In fact, prior to 1978, the value of stock options was not included at all in the Forbes’ dataset at all. From 1978 to 1991, stock options are included, but only as they are exercised. Finally, Execucomp treats stock options in the way that is most suitable for my purposes, namely by computing the Black-Scholes value of the options as they are granted.

From the Compustat database, I choose, for every year, the 500 largest companies by total market value, i.e., debt plus the market value of equity. For every year, I then merge this dataset with either the Forbes’ database or the Execucomp database. Not all top 500 companies are found in these two data sets, so in each year I have data on fewer than 500 CEOs.
2.5. SOME EVIDENCE FROM THE US

2.5.3 Definition of Variables

Before going to the results, I will briefly explain how I compute some of the variables used in the regressions.

The measure of CEO compensation is "totalpay" when stemming from the Forbes database. From Execucomp, I use the TDC1 measure of compensation. The difference between these two measures is the way in which stock options are treated, as explained above.

The measure of the market value of firms is given by the market capitalization of equity plus total assets minus common equity minus deferred taxes, as in Gabaix and Landier (2007).17 Like Gabaix and Landier, I also use the size of the firm ranked as number 250 as the representative of aggregate firm size. All nominal quantities are converted into 2000 USD using the GDP deflator of the Bureau of Economic Analysis.

To compute the book value of equity, I follow the definitions in Davis, Fama and French (2000). I then compute the price-book value for each industry and each year. This price-book value is computed as the sum of the market value of equity across the firms in the industry, divided by the sum of the book value of equity across the firms in the industry. To compute the average price-book value of an industry, I take the average of the industry price-book value across all years.

Finally, as pay should be determined by fundamentals, I take the relevant market fundamentals affecting pay received in period $t$ to be the market fundamentals in period $t - 1$.

2.5.4 Results

General Properties of the CEO Pay Function

In Table 3, I report the results from a number of basic regressions on CEO pay. I run regressions of the form

$$\ln(w_{i,j,t}) = \alpha_t + \beta_{1,t} \ln(V_{i,t-1}) + \beta_3 \ln(PBV_j) + \varepsilon_{i,j,t},$$

or some version of this. In words, regress the log of total compensation ($\ln(w_{i,j,t})$) in firm $i$, industry $j$ in year $t$ on a year fixed effect ($\alpha_t$), the log of the market value of the firm in year $t - 1$ ($\ln(V_{i,t-1})$), and the log of the industry price-book

17 The exact formula is given by: mvalue=data25*data199+data6-data10-data60
value ($\ln(PBV_j)$). In some of the specifications, instead of the year fixed effect, I control for the market value of the reference firm, the 250th largest firm in the US, as advocated by Gabaix and Landier (2007). Robust standard errors, clustered by industry, are in parentheses. Relative to the predictions of Proposition 1, these regressions tell us the following.

**Evidence related to Proposition 1.1:** On average over the past decades, the elasticity of CEO pay to the firm’s market value has been about 0.3, a number which has been found by many others (Roberts [1956], Kostiuk [1990] and Gabaix and Landier [2007]). Thus, the concave relationship between pay and firm value is confirmed.

**Evidence related to Proposition 1.2:** The industry’s price-book value is positively and significantly related to CEO compensation across a number of specifications. Thus, the prediction of Proposition 1.2 is consistent with the data. Note that it is the average price-book value of an industry that seems to be of importance, not the price-book value of the individual company or the price-book value of the company in a given year. The latter two are individually positively related to pay, but when I include all three, only the average industry price-book value survives (see columns (4),(5), and (7) in Table 3). Thus, there does indeed seem to be a systematic tendency for the CEO pay function to vary across industries according to the underlying industry price-book value, as predicted by the model.

**Evidence related to Proposition 1.3:** Next, I want to check whether the prediction of Proposition 1.3 is consistent with the data. In particular, is the elasticity of CEO pay to market value higher in high price-book value industries? To check this, I do the following:

First, I run the following regression

$$\ln(w_{ijt}) = \alpha_t + \beta_t \ln(V_{ijt}) + \beta_j(I_j * \ln(V_{ijt})) + \epsilon_{ijt},$$

where the $I_j$’s are a set of industry dummies, such that the $\beta_j$’s give us the point estimates of the industry-specific differential in pay elasticity.

Second, I correlate these pay elasticity differentials with the price-book value of the industry. The correlation is always positive, but the significance of the

---

18. The significance of the results tend to be slightly stronger if I cluster the standard errors by year instead of by industry.
correlation increases appreciably when I drop the industries with very few observations (see Table 4). This is not surprising, since we get a very imprecise measure of the average price-book value of an industry when few firms are present. These measurement problems are reduced when the industries with a small number of observations are dropped, and the significance of the correlation then becomes stronger. On average, the results indicate that going from an industry with $p_b = 1$ to one with $p_b = 6$ raises the CEO pay elasticity to firm value by 15-25%.

In Figure 6, I plot the relationship between pay sensitivity and price-book value. Besides indicating the positive correlation, we also see that the industries with the lowest price-book values are utilities, autos, and transportation - typical capital-intensive industries, while the industries with the highest price-book values are the pharmaceutical industry, computers, and business services, all areas where innovation and human capital are dominant. This evidence suggests that the mechanism driving my theoretical results does have empirical significance.

**How is the CEO Pay Function Related to Taxation?**

**Evidence related to Proposition 2.2:** The theory emphasized how the CEO pay elasticity is affected by the tax system via the career incentives for individuals and the effort they choose to put in. Is there any evidence of this mechanism? Suppose there is a reduction in the top marginal tax rates in year $t$. Given that this change was unexpected, it will affect the effort individuals put in from year $t$ and onwards. The improvements in talent identification and the effects on prices will be largest some years later, when those who were early in their careers (say around 30 years old) when taxes were changed become potential CEOs (say around 40-45 years old). This is because these individuals have been in the competitive contest the longest, so for this group talent is most precisely identified. In other words, the impact of a tax change on the CEO pay function kicks in with a lag.

To judge how long it takes for the effect of taxes to affect prices in the data, I look at the correlation between the top marginal tax rate and the CEO pay elasticity across years, where the CEO pay elasticity is lagged by a certain number of years to take into account the lagged effect of taxes on the pay function.

In order to estimate the pay elasticity across years, I run the regression

$$\ln(w_{ijt}) = \alpha_t + \beta_t \ln(V_{ijt}) + \gamma(PBV_j) + \varepsilon_{ijt},$$
or some versions of this. The results are reported in Table 5 and Figure 7. Figure 7 shows how the point estimate $\beta_1$ has been changing over time, relative to its level in 1970, together with the 95% confidence intervals around this estimate. The results do not vary much across specifications. In the figure, I have also plotted the results when all options are excluded from the Forbes’ data. The elasticity of pay to market value was quite stable during the 1970s. From the early 1980s until the year 2000, the pay elasticity trended upwards, before decresing somewhat in the early 2000s.

Given the potential problem with how options are treated, it is comforting that Frydman and Saks (2007) report a similar finding from their dataset, where options are consistently treated across all time periods. All elasticities in Table 5 indicate that there has been an upward shift in the elasticity of CEO pay from the period 1946-1975 to the period 1976-2005.19

In Figure 8, I plot the top marginal tax rate and the CEO pay elasticity lagged by ten years for the years 1960-1995. The correlation coefficient between these two series is -0.85. If we compute this correlation for various lags of the CEO pay elasticity, we can see how the correlation differs across lags. In Figure 9, I plot the $R^2$ when I regress the CEO pay elasticity lagged by $t$ years on the top marginal tax rate, for $t \in \{0, 1, 2, ..., 15\}$. It is clear from the figure that the correlation is the largest for lags in the interval six to eleven years, while it is substantially lower for shorter and longer lags. If this is taken literally, it takes somewhere between 6 and 11 years for a tax change to have its full impact on the CEO pay function.

The theory predicts that the effect of taxes on CEO pay runs through the pay elasticity with respect to the firm’s market value. To test whether taxes enter the CEO pay function in this way, I run various regressions of the form

$$\ln(w_{i,j,t}) = \alpha + \beta_1 \ln(V_{i,t}) + \beta_2 (tax_{t+f} \times \ln(V_{i,t})) + \beta_3 X_t,$$

where I am particularly interested in the coefficient on the interaction variable between the log of market value and the top marginal tax rate forwarded by $f$ years. $X$ is a set of other variables, either fixed year or industry effects, or other covariates. I let $f$ equal 10. If the tax rate has a negative effect on the (lagged) pay elasticity negatively, $\beta_2$ should be negative. The results are

19Murphy (1999, p. 6) notes that the elasticity of pay to firm revenues has declined over time. However, this model predicts that it is the value of a firm that should matter, not the revenues.
2.5. SOME EVIDENCE FROM THE US

presented in Table 6. In columns 1 and 2, I use the full sample. Robust standard errors, clustered by industry, are in parentheses. The coefficient on the interaction variable between the log of market value and the top marginal tax rate is negative, and significant at the 1% level. When I include the tax rate itself as an explanatory variable, this turns up negative, but insignificant. The coefficient on the interaction variable is quite stable and remains highly significant. The results therefore indicate that the effect of taxes on pay runs through the elasticity of pay with respect to the firm’s market value.

To tackle problems with serial correlation, I have also repeated the above regressions with data for five-year intervals. More specifically, I ran the same regressions when I only included the years 1970, 1975, 1980, and so on. The results, reported in columns 3 and 4, are very similar to the results with the full sample.

Evidence related to Proposition 2.3: Next, I investigate how the effect of the price-book value on CEO pay has changed over time, as taxes have changed.

In Figure 10, I plot the estimate of the coefficient $\gamma_t$ of the regression $\ln(w_{ijt}) = \alpha_t + \beta \ln(V_{ijt}) + \gamma_t(PBV_j) + \varepsilon_{ijt}$. Except a shift upward during the 1970s, there has been no notable trend in the effect of the price-book value over time.

In columns 5 and 6 of Table 6, I test whether there is a positive interaction effect between the tax rate (forwarded by ten years) and the industry price-book value. For the full period, 1970 to 2005, the estimate is negative but insignificant. Further, when excluding the years prior to 1978, the effect is actually slightly positive, although insignificant. Thus, the prediction in Proposition 2.3 does not find much support in the data. Given that there is no time trend in the effect of the price-book value after the late 1970s, this is not very surprising.

What might account for this lack of a stronger effect of the price-book value on pay as taxes have fallen? The above model is a partial equilibrium model of pay, where there is no mobility of workers across industries. In a general equilibrium setting, it is not clear that the predictions discussed in this setting would hold. In particular, in the longer run, it is reasonable to assume that the workers enter contests such that the expected value of contests is equalized across sectors. Thus, high-paying industries will draw more contestants, which will make the average talent higher in the high-paying sectors. Talent-spacings will therefore fall in these sectors (as the density increases at the top of the distribution), which causes the growth on compensation to be less rapid than what would otherwise have been the case. Alternatively, given that potential
managers have some information about their level of talent, they will seek high-paying sectors, and this will also cause wage growth to be less rapid than it would otherwise have been. This might be one reason why the results here are weak.

Changes in the Distribution of Market Values

Evidence related to Proposition 3.2: Finally, I will take a brief look at how the distribution of market values across firms has been changing over time. The prediction from the theory is that as taxes fall, smaller firms should grow faster relative to larger firms, such that the distribution becomes more compressed.

Indeed, this is what we see in the data. Let firm #250 be the reference firm, whose value is indexed to 1 for all years. Second, to remove some of the noise in the data, I split the sample into five-year periods. In the first five-year period (1966-1970), I index the market value of all firms to 1. Then, I compute how quickly firms across different ranks among the top 500 US firms have grown compared to firm #250. The results are shown in Figure 11, which shows the cumulative changes from the first five-year period for the period 1971-1975 and 2001-2005.20 There is a clear pattern over these 35 years. Except for some large fluctuations in the value of the top 25 firms, the value of smaller firms has systematically tended to grow faster than the value of larger firms. For instance, the value of firm #475 relative to firm #250 has increased by a factor of about 2.7, while the value of firm #125 relative to firm #250 has fallen by more than 25%.

This trend is quite interesting and it is probably driven by many forces. In the 1960s and the 1970s, it is probably related to the dismantling of the conglomerates (see e.g. Chandler). Technological developments might also make the advantage of scale less important than it used to be. However, it is also possible that the trend owes something to the explanation proposed in this paper.

2.6 Some Cross-Country Evidence

Further suggestive evidence for my theory can be obtained by looking at cross-country data. Provided that the elasticity of the variance of the distribution of expected talent with respect to effort does not differ too much across countries,
Proposition 2 implies that changes in marginal tax rates and changes in the compensation of executives should be negatively correlated across countries. As we shall see, this has indeed been the case over the past couple of decades.

2.6.1 Data

CEO compensation data come from various versions of Towers Perrin’s Worldwide Total Remuneration Survey, and cover 11 countries for the years 1984, 1992, 1996 and 2000. Towers Perrin estimates pay in firms with similar values of annual sales across a range of countries.\footnote{In 1984 and 1992 this value was $250 million. In the latter periods, it was $500 million.} The measure of executive pay includes base salary, bonus, compulsory benefits, perquisites, and long-term compensation such as stock options and stock grants. To facilitate comparisons across countries, I normalize executive pay with the pay of manufacturing workers, as is done in Towers Perrin’s survey, and test whether changes in this ratio are systematically related to changes in marginal tax rates.

Since the compensation data are collected from equally sized companies across countries, for the US for instance, we are dealing with moderately large companies, relatively far down in the distribution, whereas for smaller countries we are dealing with relatively large companies. In terms of the theory, relatively better managers will lead companies in the small countries. Thus, comparing levels of pay across countries gives us limited insights, as we are comparing the most talented managers in a small country with less talented managers in a large country. However, this problem largely disappears when looking at differences across time, since the size of the firm, and thus also the differences across countries in terms of the location in the talent distribution, are differenced out.

I use two measures of marginal taxes. Let $\tau_w$ be the marginal tax on income, $\tau_{ss}$ the marginal payroll tax, and $\tau_c$ the VAT or sales tax. The first measure, including the sales tax, is given by

$$\tau_1 = 1 - \frac{(1 - \tau_c)(1 - \tau_w)}{(1 + \tau_{ss})},$$

while the second, excluding the sales tax, is given by

$$\tau_2 = 1 - \frac{(1 - \tau_w)}{(1 + \tau_{ss})}.$$
employee (the first for consumption, the second for saving). For simplicity, I ignore the taxation of profits and capital income, as this would require a machinery involving saving and portfolio decisions. This task is left to future research.

Data for marginal taxes in 1980, 1984, 1990 and 1995 are presented in Table 7. The tax-rate data come from several sources. Top marginal income tax rates are from Gwartney and Lawson (2001); consumption taxes are from OECD (2006); marginal payroll taxes in 1980 are from McKee, Visser and Saunders (1986); the remaining tax data are found in Abowd and Bognanno (1995).

2.6.2 Results

In Figure 12, I provide a plot of the relationship between the relative change in tax rates and the relative change in compensation. As in Section 5.4.2, I compare the changes in compensation with lagged changes in the tax rates. Thus, the tax measure is the marginal tax in 1995 relative to the marginal tax in 1980, whereas the compensation measure is given by the compensation in year 2000 relative to the level of compensation in 1984. The figure shows that there is a clear negative relationship between the two variables. The correlation coefficient is $-0.73$, and the relationship is significant at the 1% level. This negative relationship is very robust across different time periods for this sample of countries.

To go somewhat beyond this simple correlation, let $w_t$ and $\tau_t$ be the compensation ratio and the marginal tax rate in period $t$, respectively. Further, let $\delta_i$ be a country fixed effect, and $\delta_t$ a year fixed effect. Assume that pay in country $i$ in year $t$ is given by

$$w_{i,t} = \delta_i + \delta_t + \beta \tau_{i,t-x} + \varepsilon_{i,t}.$$  

In Table 8, I report the results from estimating this equation for different values of $x$. The results in Table 8 confirm that the systematic negative relationship between the levels of marginal taxes and executive compensation remains after controlling for country fixed effects, time fixed effects, and time trends. The relationship is somewhat stronger when using a five-year lag for tax rates rather than a lag of zero or ten years. Finally, the results are very similar for my two measures of the marginal tax rates.
2.7 Discussion and Conclusions

The general objective of this paper has been to improve our understanding of how the tax system affects the cost of providing incentives and how this, in turn, affects overall income inequality in society. The particular focus of this paper has been on how taxes affect the career incentives of individuals, how the effort they put in to reach the top responds to these incentives and how this, in turn, affects talent identification and pay.

It would be interesting to explore other channels through which taxes may affect the quality of talent identification in society. For instance, less progressive tax systems make high-powered incentive schemes a more attractive form of compensation\(^\text{22}\). Thus, it becomes easier to motivate subordinates to work through monetary incentives. In a model à la Aghion and Tirole (1997) of formal and real authority in organizations, more responsibility will be delegated to subordinates, and superiors will learn more about their quality. At a general level, it would be interesting to study how the cost of providing incentives (e.g., due to the tax system) affects the organization of firms, in particular whether the decentralization of firms and the increased use of profit centers where the performance of individuals is easy to measure are natural consequences of a lower cost of incentive provision.\(^\text{23}\)

In this paper, I have focused on taxes as the driving force of change in the market for talent. Naturally, this is not the only plausible source of change. In particular, two other major trends over the past decades may also be important in this regard: deregulation and globalization. Until around 1980, competition in many industries was restricted by heavy regulation. In the US, for instance, the banking, air traffic and telecommunications industries had heavy restrictions on interstate operations. This implied that local rents were protected and quite secure, but also that the potential for the most talented CEOs to expand the scope of activity was very limited. In other words, even a relatively poor manager would manage to secure decent profits in the absence of competition, and a good manager would not be allowed to fully exploit his talents. In terms of my model, such regulation will reduce the marginal benefit of talent for all firms in the distribution, and push the compensation of managers down. However, as regulation is removed, the forces I have focused on will take effect, starting

\(^{22}\)For evidence of the increased use of performance-based pay, see Lemieux (2006).

\(^{23}\)See Harstad (2007) for an analysis of how the organization of firms changes when competition in the product market increases. See Rajan and Wulf (2006) for evidence on the trend towards flatter and less hierarchical firms.
from the demand side rather than the supply side; the firms' marginal benefit of talent goes up, pay is bid up by the firms, the competition to become a CEO becomes stiffer as the reward improves, talents are better identified, pay and firm values increase further, and so on. The effects of globalization are similar. Globalization increases competition locally and makes rents less secure, while it also increases the market size for the most talented individuals and thus, the value of such individuals for firms. Eeckhout and Jovanovic (2007) develop a theory related to this argument. In a globalized economy, the span of control of the most talented individuals increases as they get access to more labor, which causes their productivity and income to increase. The countries with the best pool of talented individuals have the most to gain from this, and wages at the top increase significantly here as restrictions on international trade are removed.

Regarding the specific model presented in this paper, it would be interesting to extend it to include more generations of workers. This would allow us to study the effects on the gradient of inequality across cohorts, on the age of managers, and so on. In particular, it would be interesting to see whether these extensions yield results consistent with the data, namely that inequality has been increasing relatively more for younger cohorts in the labor market than for the old, and that the average age of CEOs of S&P500 firms has been declining over the past decade (also when excluding the "dot.com CEOs"). Further, introducing more generations of individuals will make it possible to produce sharp predictions about how long it takes for changes in the tax system to have their full impact on the economy.
2.8 Appendix

A1: Proof of Lemma 1

Lemma 1: $T'(m)$ falls when effort increases.

Proof: We have $T'(m) = z'(m) \sqrt{Var(T)} < 0$, as $z'(m) < 0$ given the way ranks are defined. Thus, $\frac{\partial T'(m)}{\partial e^*} = z'(m) \frac{1}{2} Var(T) - \frac{1}{2} \frac{\partial Var(T)}{\partial e^*} < 0$, as $\frac{\partial Var(T)}{\partial e^*} > 0$. QED.

A2: Proof of Lemma 2

Lemma 2: As $r \rightarrow 0$, the distribution of organizational capital can be approximated by the normal distribution.

Proof: I want to show that the ratio of the countercumulative distribution of organizational capital and the countercumulative distribution of talent converges to 1 as $r \rightarrow 0$:

$$\lim_{r \rightarrow 0} \frac{\int_{T(r)}^{\infty} f_\theta(\theta)(1 - F_\varepsilon(q^* - e\theta))d\theta}{\int_{T(r)}^{\infty} f_\theta(\theta)d\theta} = 1.$$ 

In fact,

$$\lim_{r \rightarrow 0} \frac{\int_{T(r)}^{\infty} f_\theta(\theta)(1 - F_\varepsilon(q^* - e\theta))d\theta}{\int_{T(r)}^{\infty} f_\theta(\theta)d\theta} = \lim_{r \rightarrow 0} \frac{\int_{T(r)}^{\infty} f_\theta(\theta)d\theta - \int_{T(r)}^{\infty} f_\theta(\theta)F_\varepsilon(q^* - e\theta)d\theta}{\int_{T(r)}^{\infty} f_\theta(\theta)d\theta}$$

$$= 1 - \lim_{r \rightarrow 0} \frac{\int_{T(r)}^{\infty} f_\theta(\theta)F_\varepsilon(q^* - e\theta)d\theta}{\int_{T(r)}^{\infty} f_\theta(\theta)d\theta}$$

$$= 1 - \lim_{r \rightarrow 0} \frac{f_\theta(T(r))F_\varepsilon(q^* - eT(r))T'(r)}{f_\theta(T(r))T'(r)}$$

$$= 1 - \lim_{r \rightarrow 0} F_\varepsilon(q^* - eT(r))$$

$$= 1,$$

where I have applied l’Hôpital’s rule in the third to last line. This implies that organizational capital is normally distributed in the limit. QED.

A3: Derivation of $f(r; e, e^*)$

I want to show that the expression for $f(r; e, e^*)$ is given by

$$f(r; e, e^*) = -q'(r; e, e^*) \int f(\theta)f_\varepsilon(q(r; e^*) - e\theta)d\theta.$$
We have

\[ F(r; e, e^*) = \Pr(r' \leq r | e, e^*) = \int_\Theta f_\theta(\theta) \Pr(r' \leq r | \theta, e, e^*) d\theta = \int_\Theta f_\theta(\theta) \Pr(e \geq q(r; e^*) - e\theta) d\theta = \int_\Theta f_\theta(\theta)(1 - F_\varepsilon(q(r; e^*) - e\theta)) d\theta = 1 - \int_\Theta f_\theta(\theta) F_\varepsilon(q(r; e^*) - e\theta) d\theta = 1 - \int_\Theta f_\theta(\theta) \left( \int_{-\infty}^{q(r; e^*) - e\theta} f_\varepsilon(\varepsilon) d\varepsilon \right) d\theta. \]

Taking the derivative wrt. \( r \) yields

\[ f(r; e, e^*) = F'(r; e, e^*) = -q'(r; e^*) \int_\Theta f_\theta(\theta) f_\varepsilon(q(r; e^*) - e\theta)) d\theta. \]

**A4: Proof that** \( f(r; e^*, e^*) = 1 \)

**Proof:** I need to show that for any \( r \in (0, 1) \), we have

\[ f(r; e^*, e^*) = -q'(r) \int_\Theta f_\theta(\theta) f_\varepsilon(q(r; e^*) - e^*\theta) d\theta = 1, \]

where

\[ q(r; e^*) = e^* \bar{\theta} + z(r) \sqrt{\text{Var}(q)}. \]

This implies that

\[ q'(r; e^*) = z'(r) \sqrt{\text{Var}(q)}. \]

Thus, we need

\[ \int_\Theta f_\theta(\theta) f_\varepsilon(q(r; e^*) - e^*\theta) d\theta = z'(r) \sqrt{\text{Var}(q)}. \]

We have

\[ z'(r) = -\frac{1}{f_n(z(r))}, \]
where \( f_n(\cdot) \) is the pdf of the standard normal variable. Thus:

\[
\begin{align*}
\int \! f_n(\theta) f_\varepsilon(q(r; e^*) - e^* \theta) d\theta \\
= -z'(r) \sqrt{\text{Var}(q)} \int \! f_\theta(\theta) f_\varepsilon(q(r; e^*) - e^* \theta) d\theta \\
= \sqrt{\text{Var}(q)} \int \! f_\theta(\theta) f_\varepsilon(q(r) - e^* \theta) d\theta.
\end{align*}
\]

Expanding this expression, we find

\[
\begin{align*}
f(r; e^*, e^*) &= \sqrt{\text{Var}(q)} \int_{\Theta} f_\theta(\theta) \frac{1}{\sqrt{2\pi\text{Var}(\varepsilon)}} \exp\left( -\frac{(q(r) - e^* \theta)^2}{2\text{Var}(\varepsilon)} \right) d\theta \\
&= \sqrt{\text{Var}(\varepsilon)} \int_{\Theta} f_\theta(\theta) \exp\left( -\frac{(q(r) - e^* \theta)^2}{2\text{Var}(\varepsilon)} + \frac{\varepsilon^2}{2} \right) d\theta \\
&= \frac{\text{Var}(\varepsilon) \varepsilon^2}{2\text{Var}(\varepsilon)} \int_{\Theta} f_\theta(\theta) \exp\left( -\left( \sqrt{\text{Var}(q)} \varepsilon - e^* (\theta - \bar{\theta}) \right)^2 \right) d\theta \\
&= \text{Var}(\varepsilon) \varepsilon^2 - \text{Var}(q) \varepsilon^2 - e^2(\theta - \bar{\theta})^2 + 2e(\theta - \bar{\theta})\sqrt{\text{Var}(q)} \varepsilon.
\end{align*}
\]

Now, remember that

\[
\text{Var}(q) = e^2 \text{Var}(\theta) + \text{Var}(\varepsilon).
\]

Rewrite:

\[
f(r; e^*, e^*) =
\]
\[
\frac{\sqrt{\text{Var}(q)}}{\sqrt{\text{Var}(\varepsilon)}} \times \\
\int_{\theta} \frac{1}{\sqrt{2\pi \text{Var}(\theta)}} \exp \left( -\frac{(\theta-\bar{\theta})^2}{2\text{Var}(\theta)} \right) d\theta
\]
= \frac{1}{\sqrt{2\pi \text{Var}(\varepsilon)}} \frac{\sqrt{\text{Var}(q)}}{\sqrt{\text{Var}(\theta)}} \times \\
\int_{\theta} \exp \left( -\frac{\text{Var}(q)(\theta-\bar{\theta})^2 - 2e\text{Var}(\theta)(\theta-\bar{\theta})\sqrt{\text{Var}(q)z(r)} + e^2\text{Var}(\theta)z(r)^2}{2\text{Var}(\varepsilon)\text{Var}(\theta)} \right) d\theta
= \frac{1}{\sqrt{2\pi \text{Var}(\varepsilon)}} \frac{\sqrt{\text{Var}(q)}}{\sqrt{\text{Var}(\theta)}} \times \\
\int_{\theta} \exp \left( -\frac{e^2\text{Var}(\theta)z(r)^2}{2\text{Var}(\varepsilon)} \right) d\theta
= \frac{1}{\sqrt{2\pi \text{Var}(\varepsilon)}} \frac{\sqrt{\text{Var}(q)}}{\sqrt{\text{Var}(\theta)}} \times \\
\int_{\theta} \exp \left( -\frac{e^2\text{Var}(\theta)z(r)^2}{2\text{Var}(\varepsilon)} \right) d\theta
\]

We can exploit the following rule: For given \(a\) and \(b \in \mathbb{R}\), we have
\[
\int_{-\infty}^{\infty} e^{-ax^2} e^{2bx} dx = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{a}}.
\]
(2.30)

Let
\[
a = \frac{\text{Var}(q)}{2\text{Var}(\varepsilon)\text{Var}(\theta)}
\]
\[
b = \frac{e\sqrt{\text{Var}(q)z(r)}}{2\text{Var}(\varepsilon)}
\]

Using the rule in (2.30), we find that
\[
\sqrt{\frac{\pi}{a}} e^{\frac{b^2}{a}} = \sqrt{\frac{2\pi \text{Var}(\varepsilon)\text{Var}(\theta)}{\text{Var}(q)}} \exp \left( -\frac{e^2\text{Var}(\theta)z(r)^2}{4\text{Var}(\varepsilon)^2\text{Var}(\theta)} \right)
\]
\[
= \sqrt{\frac{2\pi \text{Var}(\varepsilon)\text{Var}(\theta)}{\text{Var}(q)}} \exp \left( \frac{e^2\text{Var}(\theta)z(r)^2}{2\text{Var}(\varepsilon)} \right).
\]
Plugging this back into the expression for \( f(r; e^*, e^*) \), we find that
\[
\begin{align*}
f(r; e^*, e^*) &= \frac{1}{\sqrt{2\pi \text{Var}(e)}} \sqrt{\frac{\text{Var}(q)}{\text{Var}(\theta)}} \times \exp\left(-\frac{e^2 \text{Var}(\theta) z(r)^2}{2 \text{Var}(e)}\right) \sqrt{\frac{2\pi \text{Var}(e) \text{Var}(\theta)}{\text{Var}(q)}} \exp\left(\frac{e^2 \text{Var}(\theta) z(r)^2}{2 \text{Var}(e)}\right) \\
&= \exp(0) \\
&= 1.
\end{align*}
\]

QED.

**A5: Proof that** \( \lim_{e^* \to \infty} MB(e^*) = \bar{\theta} \).

**Proof:** \( MB(e^*) \) is defined by
\[
MB(e^*) = \bar{\theta} + \sqrt{\text{Var}(T)} \int_0^1 z(r) \frac{\partial f(r, e^*, e^*)}{\partial e} dr + \int_0^\phi w^m(r; e^*) \frac{\partial f(r, e^*, e^*)}{\partial e} dr.
\]

In Lemma 4, I show that \( \lim_{e^* \to \infty} \frac{\partial f(r, e^*, e^*)}{\partial e} = 0 \). This implies that \( \lim_{e^* \to \infty} \int_0^1 z(r) \frac{\partial f(r, e^*, e^*)}{\partial e} dr = 0 \). To make sure that \( \lim_{e^* \to \infty} MB(e^*) = \bar{\theta} \), I need to make sure that the other terms are bounded in the limit. As \( \sqrt{\text{Var}(T)} \) converges to a finite upper bound, I just need to make sure that \( w^m(r; e^*) \) is bounded for \( r \in (0, \phi) \). We have
\[
w^m(r; e^*) = W(r) - T(r) = T(\phi) - T(r) - \kappa(\alpha, i) \int_r^\phi T'(u) O(u) du + E[V(r')|r] - E[V(r')|r^*].
\]

This can be rewritten as
\[
w^m(r, e^*) = \sqrt{\text{Var}(T)} [z(\phi) - z(r)] - \kappa(\alpha, i) \sqrt{\text{Var}(T)} \int_r^\phi z'(u) O(u) du + E[V(r')|r] - E[V(r')|\phi] \\
= \sqrt{\text{Var}(T)} \left( [z(\phi) - z(r)] - \kappa(\alpha, i) \int_r^\phi z'(u) O(u) du \right) + E[V(r')|r] - E[V(r')|\phi].
\]

It is clear that the first term of this expression has an upper bound equal to
\[
\sqrt{\text{Var}(\theta)} \left( [z(\phi) - z(r)] - \kappa(\alpha, i) \int_r^\phi z'(u) O(u) du \right),
\]
as \( e^* \to \infty \), where \( O(u) \) in the limit is derived from the distribution of actual talent, cut off at \( T(\phi) \) (The expression is positive provided that the distribution
of $O$ has a sufficiently high mean. Note that $\int_{r}^{\phi} z'(u)O(u)du$ is increasing in $e^*$, as $O(u)$ is (weakly) increasing in $e^*$. Finally, then, I need to make sure that $E[V(r')|r] - E[V(r')|\phi]$ has a finite upper bound for all $r \in (0, \phi)$. Let $f_{O}(r'|r)$ be the density function at rank $r'$ in the next period for a firm that hires a manager with rank $r$ today. Define $E[\Delta V] = E[V(r')|r] - E[V(r')|r^*]$. We then have

$$E[\Delta V] = \int_{0}^{\phi} [f_{O}(r'|r) - f_{O}(r'|r^*)] V(r')dr'$$

$$= \int_{0}^{\phi} [f_{O}(r'|r) - f_{O}(r'|r^*)] \left( V(r^*) - \kappa(\alpha, i) \int_{r'}^{\phi} O'(u)T(u)du \right) dr'$$

$$= -\kappa(\alpha, i) \int_{0}^{\phi} [f_{O}(r'|r) - f_{O}(r'|r^*)] \left( \int_{r'}^{\phi} O'(u)T(u)du \right) dr'$$

$$= -\tilde{\theta}\kappa(\alpha, i) \int_{0}^{\phi} [f_{O}(r'|r) - f_{O}(r'|r^*)] \left( \int_{r'}^{\phi} O'(u)du \right) dr' - \sqrt{Var(T)}\kappa(\alpha, i) \int_{0}^{\phi} [f_{O}(r'|r) - f_{O}(r'|r^*)] \left( \int_{r'}^{\phi} z(u)O'(u)du \right) dr'$$

For $e^* \to \infty$, we have $f_{O}(r'|r) = 0$ for all $r' \neq r$. Thus, in the limit, the expression

$$\int_{0}^{\phi} [f_{O}(r'|r) - f_{O}(r'|r^*)] (O(r') - O(r^*)) dr'$$

equals $O(r) - O(r^*)$, where $O(\cdot)$ in the limit is derived from the distribution of actual talent. Finally,

$$\int_{0}^{\phi} [f_{O}(r'|r) - f_{O}(r'|r^*)] \left( \int_{r'}^{\phi} z(u)O'(u)du \right) dr' = \left( \int_{r'}^{\phi} z(u)O'(u)du \right),$$

where $O(\cdot)$ in the limit is derived from the distribution of actual talent. Both of these terms are bounded. QED.

A6: Proof of Lemma 4

Lemma 4: $\frac{\partial f(r, e^{*}; e^{*})}{\partial e} = \left( \tilde{\theta} + \frac{z(r)e^{*}Var(\theta)}{\sqrt{Var(q)}} \right) \frac{z(r)Var(e)}{\sqrt{Var(q)}} + \frac{e^{*}Var(\theta)Var(e)}{Var(q)}$

Proof: First some notation. We have

$$f(r, e; e^{*}) = -q'(r) \int_{\theta} f_{\theta}(\theta)f_{e}(q(r; e^{*}) - e\theta)d\theta,$$
2.8. APPENDIX

so, evaluated at \( e = e^* \), we have

\[
\frac{\partial f(r, e^*; e^*)}{\partial e} = -q'(r) \int_{\Theta} f_\theta(\theta) \frac{\partial f_z(q(r; e^*) - e^*\theta)}{\partial e} d\theta
\]

\[
= -\frac{q'(r)}{Var(\epsilon)} \int_{\Theta} \theta(q(r; e^*) - e^*\theta) f_\theta(\theta) f_z(q(r; e^*) - e^*\theta) d\theta
\]

\[
= -\frac{q'(r)}{Var(\epsilon)} \int_{\Theta} \theta(z(r) \sqrt{Var(q)} - e^*(\theta - \bar{\theta})) f_\theta(\theta) f_z(q(r; e^*) - e^*\theta) d\theta
\]

Rewriting this, we have

\[
\frac{\partial f(r, e^*; e^*)}{\partial e} = \left( \int \frac{\sqrt{Var(q)} f_\theta(\theta) f_z(q(r; e^*) - e\theta)}{f_n(z(r))} \frac{\theta(q(r; e^*) - e\theta) \theta(q(r; e^*) - e\theta)}{Var(\epsilon)} d\theta \right)
\]

where \( f_n(\cdot) \) is the pdf of the standard normal distribution.

Define \( f(\theta; z(r)) = \frac{\sqrt{Var(q)} f_\theta(\theta) f_z(q(r; e^*) - e\theta)}{f_n(z(r))} \). The next two results provide properties of this function.

**Lemma A5.1** (Symmetry): \( f(\theta; z(r)) \) is symmetric around \( \theta = \bar{\theta} + \frac{z(r) Var(\theta)e^*}{\sqrt{Var(q)}} \).

**Proof**: Consider \( \theta_1 \) and \( \theta_2 \) with \( \theta_1 > \theta_2 \) such that \( (\theta_1 - \bar{\theta}) - \frac{z(r) Var(\theta)e}{\sqrt{Var(q)}} = (\theta_2 - \bar{\theta}) \). I need to show that \( g(\theta_1 - \bar{\theta}) = g(\theta_2 - \bar{\theta}) \). The definitions of \( \theta_1 \) and \( \theta_2 \) imply that \( (\theta_1 - \bar{\theta}) = 2 \frac{z(r) Var(\theta)e}{\sqrt{Var(q)}} - (\theta_2 - \bar{\theta}) \). It is easily shown that

\[
g(\theta_1 - \bar{\theta}) = g(2 \frac{z(r) Var(\theta)e}{\sqrt{Var(q)}} - (\theta_2 - \bar{\theta})) = g(\theta_2 - \bar{\theta}),
\]

which completes the proof. QED.

**Lemma A5.2** (Constant variance for all \( r \)): Consider

\[
f(\theta; z(r)) = \frac{\sqrt{Var(q)} f_\theta(\theta) f_z(q(r; e^*) - e\theta)}{f_n(z(r))}
\]

for \( z(r_1) \) and \( z(r_2) \). These functions take on identical values for all values of \( \theta \) around their respective means, which are given by \( \bar{\theta}(z(r_1)) = \bar{\theta} + \frac{z(r_1) Var(\theta)e^*}{\sqrt{Var(q)}} \). In other words, if \( \theta_1 \) and \( \theta_2 \) are such that the distance from the mean is the same, i.e.

\[
\theta_1 - \bar{\theta} - \frac{z(r_1) Var(\theta)e^*}{\sqrt{Var(q)}} = \theta_2 - \bar{\theta} - \frac{z(r_2) Var(\theta)e^*}{\sqrt{Var(q)}},
\]

then \( f(\theta_1; z(r_1)) = f(\theta_2; z(r_2)) \).
CHAPTER 2. CEO PAY

**Proof:** I need to show that

\[
\exp \left( -\frac{(e^*)^2 \text{Var} \theta z(r_1)^2}{2 \text{Var} \varepsilon} \right) \exp \left( -\frac{\text{Var} \theta (\theta_1 - \bar{\theta})^2}{2 \text{Var} \theta \text{Var} \varepsilon} + \frac{z(r_1) \sqrt{\text{Var} \theta} e^* (\theta_1 - \bar{\theta})}{\text{Var} \varepsilon} \right) = \exp \left( -\frac{(e^*)^2 \text{Var} \theta z(r_2)^2}{2 \text{Var} \varepsilon} \right) \exp \left( -\frac{\text{Var} \theta (\theta_2 - \bar{\theta})^2}{2 \text{Var} \theta \text{Var} \varepsilon} + \frac{z(r_2) \sqrt{\text{Var} \theta} e^* (\theta_2 - \bar{\theta})}{\text{Var} \varepsilon} \right)
\]

whenever \( \theta_1 - \frac{z(r_1) \text{Var} \theta e^*}{\sqrt{\text{Var} \theta}} = \theta_2 - \frac{z(r_2) \text{Var} \theta e^*}{\sqrt{\text{Var} \theta}} \). This implies that

\[
\theta_1 = \theta_2 + \frac{(z(r_1) - z(r_2)) \text{Var} \theta e^*}{\sqrt{\text{Var} \theta}}
\]

Plugging in:

\[
\exp \left( -\frac{(e^*)^2 \text{Var} \theta z(r_1)^2}{2 \text{Var} \varepsilon} \right) \exp \left( -\frac{\text{Var} \theta (\theta_2 - \bar{\theta})^2}{2 \text{Var} \theta \text{Var} \varepsilon} + \frac{z(r_1) \sqrt{\text{Var} \theta} e^* (\theta_2 - \bar{\theta})}{\text{Var} \varepsilon} \right) = \exp \left( -\frac{(e^*)^2 \text{Var} \theta z(r_2)^2}{2 \text{Var} \varepsilon} \right) \exp \left( -\frac{\text{Var} \theta (\theta_2 - \bar{\theta})^2}{2 \text{Var} \theta \text{Var} \varepsilon} + \frac{z(r_2) \sqrt{\text{Var} \theta} e^* (\theta_2 - \bar{\theta})}{\text{Var} \varepsilon} \right)
\]

which is what I wanted to show. QED.

With these two results in hand, I can go back and evaluate \( \frac{\partial f(r, e^*; e^*)}{\partial e} \).

We have

\[
\frac{\partial f(r, e^*; e^*)}{\partial e} = \int \theta (z(r) \sqrt{\text{Var} \theta} + e^* \bar{\theta} - e^* \theta) f(\theta; z(r)) d\theta = (z(r) \sqrt{\text{Var} \theta} + e^* \bar{\theta}) \int \theta f(\theta; z(r)) d\theta - e^* \int \theta^2 f(\theta; z(r)) d\theta = (z(r) \sqrt{\text{Var} \theta} + e^* \bar{\theta}) \theta(z(r), e) - e^* \int \theta^2 f(\theta; z(r)) d\theta. \quad (2.31)
\]
Now, we have
\[ \text{Var}(\theta; e^*) = E[\theta^2 | f(\theta; z(r))] - \bar{\theta}(z(r), e^*)^2, \]
which, given the results above, is constant across \( r \in (0, 1) \) and converges to zero as \( e^* \to \infty \). Remember that \( \bar{\theta}(z(r), e) = \bar{\theta} + \frac{z(r)e\text{Var}(\theta)}{\sqrt{\text{Var}(q)}} \). Thus, eq. (2.31) can be written as
\[
\frac{\partial f(r, e^*; e^*)}{\partial e} = (z(r)\sqrt{\text{Var}(q)} + e^*\bar{\theta})(z(r), e^*) - e^* (\bar{\theta}(z(r), e^*)^2 + \text{Var}(\theta; e^*))
\]
\[
= \bar{\theta}(z(r), e^*)(z(r)\sqrt{\text{Var}(q)} + e^*\bar{\theta} - e^*\bar{\theta}(z(r), e)) - e^*\text{Var}(\theta; e^*)
\]
\[
= \bar{\theta}(z(r), e^*) \left( z(r)\sqrt{\text{Var}(q)} - \frac{z(r)(e^*)^2\text{Var}(\theta)}{\sqrt{\text{Var}(q)}} \right) - e^*\text{Var}(\theta; e^*)
\]
\[
= \bar{\theta}(z(r), e^*) \frac{z(r)\text{Var}(\varepsilon)}{\sqrt{\text{Var}(q)}} - e^*\text{Var}(\theta; e^*).
\]

We get
\[
\frac{\partial f(r, e^*; e^*)}{\partial e} = \left( \bar{\theta} + \frac{z(r)e\text{Var}(\theta)}{\sqrt{\text{Var}(q)}} \right) \frac{z(r)\text{Var}(\varepsilon)}{\sqrt{\text{Var}(q)}} + e^*\text{Var}(\theta; e^*).
\]
The variance \( \text{Var}(\theta; e^*) \) induced by \( f(\theta; z(r)) \) is independent of \( r \), so in particular it is the same for \( r = 1/2 \), or \( z(r) = 0 \) as for any other \( r \). The density at \( z = 0 \) is
\[
f(\theta; 0) = \frac{1}{\sqrt{2\pi\text{Var}(\theta)\text{Var}(\varepsilon)}} \exp \left( -\frac{(\theta - \bar{\theta})^2}{2\text{Var}(\theta)\text{Var}(\varepsilon)} \right).
\]
This implies that the variance \( \text{Var}(\theta; e^*) \) is given by
\[
\text{Var}(\theta; e^*) = \frac{\text{Var}(\theta)\text{Var}(\varepsilon)}{\text{Var}(q)},
\]
so we have
\[
e^*\text{Var}(\theta; e) = \frac{e^*\text{Var}(\theta)\text{Var}(\varepsilon)}{(e^*)^2\text{Var}(\theta) + \text{Var}(\varepsilon)} = \frac{e^*\text{Var}(\theta)\text{Var}(\varepsilon)}{\text{Var}(q)}.
\]
Therefore,
\[
\frac{\partial f(r, e^*; e^*)}{\partial e} = \left( \bar{\theta} + \frac{z(r)e\text{Var}(\theta)}{\sqrt{\text{Var}(q)}} \right) \frac{z(r)\text{Var}(\varepsilon)}{\sqrt{\text{Var}(q)}} + e^*\text{Var}(\theta)\text{Var}(\varepsilon) \frac{1}{\text{Var}(q)}.
\]
This completes the proof. QED.
CHAPTER 2. CEO PAY

A7: Approximation when $r/r^* \to 0$

For $V(r) = V(r^*) + \kappa(\alpha, i) \left[ B_OC_T[(\ln(r^*) - \ln(r)) - \frac{B_0B_T(e)}{2}[\ln(r^*)^2 - \ln(r)^2]] \right]$, I want to show that $\lim_{r/r^* \to 0} V(r) = \kappa(\alpha, i)B_O \left[ \frac{B_T(e)}{2} \ln(r)^2 - C_T \ln(r) \right]$.

First, I show that $\lim_{r/r^* \to 0} (\ln(r^*) - \ln(r)) = -\ln(r)$. We have

$$\ln(r^*) - \ln(r) = \ln(r) \left( \frac{\ln(r^*)}{\ln(r)} - 1 \right) = \ln(r) \left( \frac{r}{r^*} - 1 \right).$$

Letting $r/r^* \to 0$, we get

$$\lim_{r/r^* \to 0} \ln(r) \left( \frac{r}{r^*} - 1 \right) = -\ln(r).$$

Now, can we also ignore $V(r^*)$ in the expression for $V(r)$? In the limit, the ratio between $V(r)$ and $V(r) - V(r^*)$ is given by

$$\lim_{r/r^* \to 0} \frac{V(r)}{V(r) - V(r^*)} = \frac{\lim_{r/r^* \to 0} V(r)}{\lim_{r/r^* \to 0} (V(r) - V(r^*))}$$

$$= \lim_{r/r^* \to 0} \frac{V(r^*) + \kappa(\alpha, i) \left[ B_OC_T[(\ln(r^*) - \ln(r)) - \frac{B_0B_T(e)}{2}[\ln(r^*)^2 - \ln(r)^2]] \right]}{\kappa(\alpha, i) \left[ B_OC_T[(\ln(r^*) - \ln(r)) - \frac{B_0B_T(e)}{2}[\ln(r^*)^2 - \ln(r)^2]] \right]}$$

$$= 1 + \lim_{r/r^* \to 0} \frac{V(r^*)}{\kappa(\alpha, i) \left[ B_OC_T \ln(r)^2 - B_OC_T \ln(r) \right]} = 1.$$

Thus, when we are considering the absolutely largest firms, the error from ignoring the term $V(r^*)$ is arbitrarily small.
A8: Proof that \( \frac{\partial \pi}{\partial T} \frac{dT}{dr} < \frac{\partial \pi}{\partial O} \frac{dO}{dr} \iff \left( \frac{C_O}{B_O} - \frac{C_T}{B_T(e)} \right) > 0 \)

\[
\begin{align*}
\frac{\partial \pi}{\partial T} \frac{dT}{dr} & < \frac{\partial \pi}{\partial O} \frac{dO}{dr} \\
\downarrow & \\
-\kappa(\alpha, i)(C_O - B_O \ln(r))B_T(e) & < -\kappa(\alpha, i)(C_T - B_T(e) \ln(r))B_O \\
\downarrow & \\
-C_OB_T(e) & < -C_TB_O \\
\downarrow & \\
\left( \frac{C_O}{B_O} - \frac{C_T}{B_T(e)} \right) & > 0
\end{align*}
\]

QED.

A9: Tables and Figures

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<tbody>
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<td>23.6</td>
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<td>11.0</td>
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### Table 2: Fraction of Men Usually Working Long (>=50) Hours

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<th>1990</th>
<th>2001</th>
</tr>
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<tbody>
<tr>
<td>All Men</td>
<td>0.147</td>
<td>0.192</td>
<td>0.185</td>
</tr>
<tr>
<td>Full Time Men (&gt;=30 hours)</td>
<td>0.15</td>
<td>0.198</td>
<td>0.203</td>
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**Across Level of Education:**

<table>
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<th>Education</th>
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<th>1990</th>
<th>2001</th>
</tr>
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<tbody>
<tr>
<td>College Graduates</td>
<td>0.222</td>
<td>0.298</td>
<td>0.305</td>
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<tr>
<td>Some College</td>
<td>0.152</td>
<td>0.188</td>
<td>0.185</td>
</tr>
<tr>
<td>High School Graduates</td>
<td>0.123</td>
<td>0.154</td>
<td>0.148</td>
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<tr>
<td>Less than High School</td>
<td>0.109</td>
<td>0.124</td>
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**Across Average Hourly Earnings Quintiles:**

<table>
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<th>1990</th>
<th>2001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quintile 1 (highest wage)</td>
<td>0.146</td>
<td>0.235</td>
<td>0.29</td>
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<tr>
<td>Quintile 2</td>
<td>0.121</td>
<td>0.201</td>
<td>0.216</td>
</tr>
<tr>
<td>Quintile 3</td>
<td>0.121</td>
<td>0.18</td>
<td>0.195</td>
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<tr>
<td>Quintile 4</td>
<td>0.155</td>
<td>0.192</td>
<td>0.179</td>
</tr>
<tr>
<td>Quintile 5 (lowest wage)</td>
<td>0.21</td>
<td>0.189</td>
<td>0.143</td>
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### Table 4: Correlation between pay sensitivity and price-book value across industries

<table>
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<th>Industry Price-Book Value</th>
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<th>(3)</th>
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<tr>
<td></td>
<td>0.0045</td>
<td>.0067 **</td>
<td>.0082 ***</td>
</tr>
<tr>
<td></td>
<td>(.0028)</td>
<td>(.0029)</td>
<td>(.0028)</td>
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<table>
<thead>
<tr>
<th>Obs.</th>
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<th>31</th>
<th>21</th>
</tr>
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<tr>
<td>R2</td>
<td>0.033</td>
<td>0.098</td>
<td>0.191</td>
</tr>
</tbody>
</table>

Note: Dependent variable is Industry Pay Elasticity. ***: sign at the 1% level. **: sign at the 5% level. *: sign at the 10% level. Standard errors in parenthesis.

(1): All industries.
(2) Drop industries with less than an average of two observations per year.
(3) Drop industries with less than an average of four observations per year.
Table 3: General Properties of the CEO Pay Function

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(a)</th>
<th>(b)</th>
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<tbody>
<tr>
<td>ln(Market value)</td>
<td>.362**</td>
<td>.296**</td>
<td>.296**</td>
<td>.292**</td>
<td>.290**</td>
<td>.301**</td>
<td>.293**</td>
<td>.310**</td>
<td>.302**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.046)</td>
<td>(.007)</td>
<td>(.030)</td>
<td>(.030)</td>
<td>(.031)</td>
<td>(.032)</td>
<td>(.030)</td>
<td>(.032)</td>
<td>(.032)</td>
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<tr>
<td>ln(Market value of firm # 250)</td>
<td>.775***</td>
<td>1.094***</td>
<td>1.127***</td>
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<td>(.068)</td>
<td>(.040)</td>
<td>(.043)</td>
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</tr>
<tr>
<td>ln(Industry price-book value)</td>
<td>.404**</td>
<td>.351***</td>
<td>.350***</td>
<td>.279**</td>
<td>.389***</td>
<td>.317***</td>
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</table>

Note: Dependent variable ln(Total compensation). Standard errors, clustered by industry, are in parenthesis. ** sign. at the 1% level, * sign. at the 5% level, * sign. at the 10% level. For the years 1970-1991, the compensation data are from Forbes Survey of Executive Compensation, while for the years 1992-2004 the compensation data are from the Compustat's Execucomp database. All firm data are from Compustat.
(a) Only include Forbes years with data on options.
(b) Excluded options pay from the Forbes data.
Table 5: Time Trend in the Elasticity of CEO Pay

<table>
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<td>yes</td>
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<td>Year FE</td>
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Note: Dependent variable ln(Total compensation). Standard errors in parenthesis. *** sign. at the 1% level, ** sign. at the 5% level, * sign. at the 10% level. For the years 1970-1971, the compensation data are from Forbes Survey of Executive Compensation, while for the years 1992-2004 the compensation data are from the Compustat’s Execucomp database. All firm data are from Compustat. (a) Excluded options pay from the Forbes data.

Table 7: Marginal Tax Rates (Incl. Cons. Taxes)

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<td>86.9</td>
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Sources: See text.
Table 6: Effect of Taxes on CEO Pay

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<td>.501 ***</td>
<td>.504 ***</td>
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<td>(.062)</td>
<td>(.060)</td>
<td>(.081)</td>
<td>(.066)</td>
<td>(.085)</td>
<td>(.091)</td>
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<td>(.081)</td>
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Note: Dependent variable is ln(Total compensation). Standard errors, clustered by industry, are in parenthesis. *** sign. at the 1% level, ** sign. at the 5% level, * sign. at the 10% level. For the years 1970-1991, the compensation data are from the Forbes Survey of Executive Compensation, while for the years 1992-2004, the compensation data are from Compustat's Execucomp database. All firm data are from Compustat.


(b) Excluding data prior to 1978.
## Table 8: Cross-Country Correlations of Taxes and Executive Compensation

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<th>10 years</th>
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<td>-.270 *** (.062)</td>
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<td>.8178</td>
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Note: Left-hand side variable is the ratio of executive pay to manufacturing wages. Standard errors are in parenthesis. *** denotes significance on the 1% level, ** on the 5% level. Explanation of lags: No lag implies that I regress compensation in year \( t \) on taxes in year \( t \). A lag of \( x \) years implies that I regress compensation in year \( t \) on taxes in year \( t - x \).
Figure 1: CEO pay to income per capita in the United States over the years 1970-2003. Mean CEO pay is defined as the mean CEO pay of the 500 most valuable US companies. Data on CEO pay are collected from Forbes’ Executive Compensation Survey and from Execucomp. For details, see Section 5.

Figure 2. The figure shows the top marginal US income tax rate over the years 1955-2005.
CHAPTER 2. CEO PAY

Figure 3. Source: Kuhn and Lozano (2006)

Figure 4: The equilibrium.
Figure 5. The figure shows the correlation between the price-book value of an industry and the ratio between market value and total assets. Excl. industries with less than an average of two observations per year.

Figure 6. The figure plots the industry-specific elasticity of CEO pay against the price-book value of the industry. I have excluded industries with less than three observations per year.
Figure 7. The figure shows the time trend of the elasticity of CEO pay to market value across years, relative to the base year 1970. The dashed lines are the bounds of the 95% confidence interval of the year-specific effect.

Figure 8. The figure shows the top marginal income tax rate (left axis) and the elasticity of CEO pay, lagged by ten years (right axis), across years.
Figure 9. The figure shows the $R^2$ of a regression of the elasticity of CEO pay to market value on the tax rate, forwarded by 0-15 years.

Figure 10. The figure plots the effect of the price-book value on CEO pay over time, relative to the effect in 1970.
Figure 11. The figure plots the cumulative growth of firms across different ranks relative to the market value of firm #250. The value of all firms is indexed to 1 for the period 1966-1970.

Figure 12. The figure plots the relationship between the relative change in marginal taxes and the relative change in executive compensation. The data for taxes are based on the taxes in year 1995 relative to 1980. The data for executives are based on compensation in year 2000 relative to 1984.
Chapter 3

Incentives under Communism: The Value of Low-Quality Goods

3.1 Introduction

With hindsight, it is easy to say that communist central planning was doomed from the beginning. Still, one has to bear in mind that until the mid 1970s, the centrally planned economies of Eastern Europe and the Soviet Union did reasonably well economically. They had been growing faster than the US for several decades, and they had grown faster than the typical country with the same level of income. But if the system was doomed, how come that they actually performed decently economically for quite a long period? This paper seeks to provide a new answer to this question by studying the cost of incentive provision under central planning. In market economies, incentives to direct efforts toward productive activities primarily stem from profit opportunities created by decentralized markets. In centralized economies, on the other hand, all incentives must be designed from above. The objective of this paper is to understand how well a self-interested regime manages to solve this fundamental problem, given the instruments at its disposal and the constraints it faces.

1I am grateful to Torsten Persson for advice and comments, and for comments from Daron Acemoglu, Giovanni Favara, Erik Lindqvist, Kjetil Storesletten, Fabrizio Zilibotti and seminar participants at the University of Oslo, the IIES Macro Lunch, and the University of Zürich. I am grateful to Christina Lönnblad for editorial assistance. Financial support from The Jan Wallander and Tom Hedelius Foundation is gratefully acknowledged. All errors are mine.
As the economies of the Communist bloc started to fall apart in the 1980s, it became increasingly apparent that one of their major deficiencies was the inability to provide citizens with products of decent quality. The regimes had been concerned about this for some time, but found it difficult to find a solution to the problem. To keep the masses under control, the substitute for high-quality goods and services was costly deterrence.2

At the same time, an important instrument for regimes of centrally planned economies was the control they had over the types of goods and services that were supplied to individuals of different ranks. It is well-known that the selective provision of high-quality goods and services was a basic element of the incentive systems designed in communist countries (see e.g. Matthews [1978] and Voslensky [1984]). In particular, all members of the working class were generally excluded from consuming goods of high quality, and were instead provided with an irregular supply of low-quality goods. In terms of incentives, this poor state of affairs for the working class makes the threat of being demoted from the favored class of citizens quite severe, and can therefore reduce the cost to the regime of rewarding the members who preserve their membership in the favored class.

Thus, on the one hand, low-quality products make the masses unhappy, and the regime must keep them quiet by other costly means. On the other hand, low-quality products as the outside option for high-ranked members of society make it inexpensive to provide them with incentives, which leaves more resources available for investment and growth. This paper is about the balance between these two effects across different stages of economic development.

In the first part of this paper, I study how a self-interested regime can exploit its control over the product mix and its ability to selectively offer benefits to groups of citizens to maximize the share of output it can appropriate. In the model, there are two types of citizens, managers and workers. On the production side, the regime must solve a traditional moral hazard problem, such that managers must be given sufficient incentives to provide costly effort. Managers who

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2In Section 3, I describe the characteristics of the centrally planned economies in more detail.
perform poorly are demoted to the working class. In order to provide incentives at a low cost, then, the regime would like to make this outside option as bad as possible, for instance by rewarding the workers with products of low quality. At the same time, however, the harshness of the treatment of the working class is limited by two factors. First, the workers can stage a revolt against the regime if they are treated too badly and the regime does not compensate bad treatment with sufficient investments in deterrence. Second, the effective supply of labor by the working class depends, at least up to some limit, on its health and thus, on its level of consumption. Giving the workers low-quality products involves a loss as well as a potential gain for the regime. The loss is that it becomes more costly to keep the workers sufficiently satisfied, while the gain is that it becomes cheaper to satisfy the managers’ incentive constraint.

The main results of the analysis are as follows. First, if no credible threat of a revolt exists, the regime always wants to offer workers products of low quality. Second, given that a credible threat of a revolt exists, it is only at levels of consumption where the supply of labor responds to changes in consumption that the regime wants to offer workers products of low quality. In this range, the attractiveness of giving the workers fewer goods of higher quality is moderated by its negative effect on labor supply. Whenever the supply of labor is no longer responsive to increases in consumption, it will not be optimal to base incentives on the exclusive provision of high-quality goods to the managers if it is possible to easily expand the production of high-quality products. Only in a special case, when the technology the regime employs to deter the workers from revolting is sufficiently effective, will the strategy of exclusive provision of high-quality goods continue to be optimal as the economy develops. Third, at low levels of development, the ability of the regime to offer low-quality products to the workers raises the optimal level of incentives. On the other hand, whenever the regime is forced to increase the rewards to workers in order to secure its power, the optimal level of incentives for managers falls.

In the second part of the paper, I use these results to account for the evo-
lution of centrally planned economies over time. At a relatively low level of
development, the opportunity to centrally fix the product mix reduces the incen-
tive costs, and enables the regime to command a greater share of output than
otherwise. Over time, as the productive potential of the economy grows, workers
become healthier and more demanding, and the cost of oppressing the people by
only giving them low-quality goods starts to grow. If feasible, the regime would
want to phase in high-quality products in the bundles given to the workers. His-
tory and economic theory tell us that this transition towards mass-production
of high-quality goods will be difficult within the framework of central planning.
I base the analysis of long-run development on this premise. Whereas a benev-
olent planner would always have to struggle with these inefficiencies of central
planning, the self-interested regime actually prefers these inefficiencies in the ini-
tial stages of development. The inefficiencies of central planning eventually get
their revenge, however, by making it difficult for the regime to make the trans-
ition to high quality. Instead, the workers have to be silenced in an inefficient
and costly manner, while the more demanding workers also improve the outside
option for managers and thus, make the incentives more costly to create. The
regime gradually commands less output. Two factors reduce the efficiency level
in production: more resources are diverted from regular production in order to
silence the workers, and the regime finds it optimal to reduce the level of in-
centives. Both these factors are exacerbated by the difficulty in providing the
workers with high-quality goods.

One remark about the scope of the paper should be noted. In this paper,
unlike e.g. Hayek (1945), I do not provide a theory of why it was difficult
to produce high-quality consumer goods under central planning. Rather, the
questions I seek to provide answers to are the following: How can the regime
exploit the existence of low-quality products to create high-powered incentives?
Is it in the interest of the regime to provide the masses with products of high
quality? If yes, what are the costs of not being able to provide them? And how
do these costs evolve as the economy develops?
3.2. RELATED LITERATURE

The rest of the paper is organized as follows. In the next section, I review some relevant literature, and put the present theory into context. In section 3.3, I present some background information on characteristics of the societies of the Eastern bloc. The model is presented in section 3.4 and analyzed in section 3.5. In section 3.6, I discuss the implications of the theory for long-run economic development. In Section 3.7, I conclude.

3.2 Related Literature

Central planning is the most extreme and complete form of government intervention in the economy. Economists usually employ one of two general methods to evaluate the effects of such intervention. One method is the traditional social-planner method. The appeal of this approach is discussed in Banerjee (1997), where the effects of interventions by a benevolent social planner facing agency problems are studied. The second general method, employed by for instance Olson (1993) and Acemoglu (2005), but also more generally in the modern political economy literature, is to view all agents, also the leaders and policy makers, as purely self-interested.

With the growth of modern political economy, the second method has become the dominating approach in economics. Academic work on central planning, however, has largely been occupied with understanding why benevolent central planners would encounter efficiency problems. Following the socialist calculation debate (see Barone [1908], Lange [1936], Lerner [1938], and von Mises [1920]), most of the analysis of the economic inefficiencies under communism has been interpreted in the light of institutional difficulties of effective central planning.\(^3\) Hayek (1945), for instance, argued that no mechanism in a centrally planned system could compensate for the central informational role played by prices in a market system. Further, as argued by Kornai (1986) and later by Maskin and Dewatripont (1995), a centrally planned system is prone to suffer from so-called soft

\(^3\)The meaning of 'inefficiencies' will throughout the paper be inefficiencies at the societal level, in contrast to inefficiencies at the regime level.
budget constraints. Shleifer and Vishny (1992) focus on the perversely distorted incentives of government agents who could earn rents by creating shortages, thus obstructing the objectives of the central government.

In the tradition of the social-planner method, economists have identified several inefficiencies that will be more severe in the enormous hierarchy created under central planning than under capitalism (in the examples above: information problems, commitment problems, agency problems). My position in this paper will be that these inefficiencies are important and unavoidable characteristics of central planning. However, since I assume that the leadership is not benevolent but rather purely self-interested, it is not clear a priori what the impact of the inefficiencies will be. Indeed, to serve its self-interest, the leadership might optimally want to create some of the same inefficiencies that would frustrate a benevolent central planner. It will do so since these inefficiencies minimize the cost of incentive provision and thereby maximize the rents to the leadership. Even though the inefficiencies are costly at the societal level, they are valuable to the regime. The optimality of such inefficiencies might be a temporary phenomenon, however. If and when they are no longer optimal, the cost of the inefficiencies will reappear.

From the perspective of modern political economy, the idea of studying outcomes in terms of the incentives of the political leadership is very natural. The study of how the political elite sets policy in order to serve its own interests and how these policies interact with institutions has received much attention recently. Reasons for studying the communist countries from this perspective are provided in Anderson and Boettke (1997). More generally, this paper is part of an emerging literature that studies how political elites consciously create inefficiencies to serve their own interests. The literature is synthesized in Acemoglu (2005).

There are, of course, other theories that try to explain the path of economic development in Communist countries. The most potent is perhaps that of Berliner (1978) who studies the difficulties of rapid innovation under central planning. As the speed of innovation went up during the 1970s and 1980s, these
difficulties meant that the centrally planned economies could no longer keep the same pace as the capitalist economies. On the other side of the coin, absent positive shocks to the pace of innovation, centrally planned resource allocation could be reasonably effective. The emphasis in Berliner (1978) is similar to that in Acemoglu, Aghion and Zilibotti (2006), which models the process of development as a gradual movement toward the world technological frontier. As the economy develops, creating efficient selection mechanisms of firms and talented people ensuring local innovation becomes more important, as the gains from simply adopting existing technologies diminish. My theory is a complement to this approach. Whereas Berliner and Acemoglu et al. argue that the importance of proper incentives grows over time, I argue that the cost of providing them increases. Thus, these two forces reinforce each other, making the initial stage of development relatively smooth, but the subsequent stages all the more difficult.

3.3 Background

The purpose of this section is to highlight six stylized characteristics of the communist societies that will play a key role in the theory presented below, namely:

(i) The quality level of goods offered to the workers was generally low, while members in important positions had exclusive access to high-quality goods. Efforts to improve the general quality level of consumer goods were absent early on, but increased somewhat over time.

(ii) An incentive system based on exclusion and selective benefits was prevalent.

(iii) There was a significant rate of turnover among managers. Many were demoted, and thus, the threat of facing workers’ conditions was credible.

(iv) Massive amounts of resources were devoted to intelligence and deterrence.

(v) Up until the 1960s, there was a steady increase in the physical health of citizens in communist countries, as measured by life expectancy at birth.
Thereafter it stagnated.

(vi) Initially, the Communist bloc countries performed relatively well economically. However, the performance deteriorated over time.

(i) Product Quality

It is well known that the general level of product quality in the Eastern bloc was moderate, to say the least. It is also true that this was not a temporary phenomenon. In 1937, Trotsky wrote about it in his work *The Revolution Betrayed* (Trotsky [1990]). "[N]either Europe nor America ever heard of such low-grade tobacco as makhorka"\(^4\), he lamented, and more generally he was complaining about widening inequalities, and the poor quality of products targeted to the working class (bread, butter vs. margarine, apartments, etc.). As Communism in Eastern Europe collapsed, low product quality was once again on display. In East Germany, for instance, only 8% of the products could be sold on world markets (Akerlof et al. [1991]). From 1989 to 1990, output in DDR collapsed by 54%, and there was massive substitution of Western goods for domestic goods (even though aggregate consumption did not grow). Local goods disappeared from the stores almost immediately. Kornai (1992) gives a general characterization of the level of product quality:

> When the purchase and consumption has taken place at last, the satisfaction is greatly reduced by the fact that there is frequently something wrong with the quality of the product or service: it does not do its job, it easily goes wrong, it soon wears out, it is out of date, its appearance is ugly, and so on. (p. 307)

There are several traditional explanations for this low level of quality, summarized in Kornai (1992, pp. 307-310).\(^5\) Kornai discusses four classes of explanations. First, there is the difficulty of providing incentives along quality

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\(^4\)Makhorka was the low grade tobacco offered to the masses.

\(^5\)See also Roland (1988) for a discussion on product quality under socialism.
dimensions in the absence of markets. The contracts given to the agents were incomplete, and the agents knew how to exploit this, for instance by fulfilling contracts specifying production in tons by producing few but heavy units. Second, it was difficult to centrally set prices that reflected quality differences. This implied that producers had few incentives to focus on quality. Third, the growth obsession of the communist leaders led them to focus on quantity rather than quality, and on the production of investment goods rather than consumption goods. Finally, Kornai argues that the general shortage of products created a seller’s market. Consumers were happy as long as they could satisfy their basic needs, and they did not have much bargaining power relative to producers.

These explanations mainly focus on the difficulties of a principal in implementing a system where agents produce high-quality goods in the absence of market discipline. According to Kornai, the major difficulty seems to be that buyers did not have the exit strategy often available in market economies, implying that contractual problems had greater negative consequences in a centrally planned system.

Especially from the 1970s and onward, it seems clear that at least in some countries, there was a political desire to improve the quality of consumer goods. However, through several reform periods the communist regimes experienced how hard it was to implement high quality standards in the absence of markets, at least in a cost-effective manner. In the 1970s, bowing to pressure from the public to increase product quality, the Polish government allowed a moderate-sized private sector to grow in order to alleviate some of the problems of lacking public support (Matthews [1978]). In the Soviet Union, price differentiation in consumer goods to take quality differences into account was not introduced until 1969 (Gorlin [1981]). Starting in 1971, a system of quality certification was introduced, where consumer goods of high quality were awarded with a seal of quality. Progress in quality was slow, however, and quality standards remained higher for producer goods, even though they were lacking also in this area.
Both theoretical reasoning and this cursory reading of the empirical evidence indicate that it was hard to substantially improve the level of product quality on a general basis, even when the political willingness existed.

(ii) Incentives in the Eastern Bloc

By the standards of the early twentieth century, even a member of the Inner Party lives an austere, laborious kind of life. Nevertheless, the few luxuries that he does enjoy - his large well-appointed flat, the better texture of his clothes, the better quality of his food and drink and tobacco, his two or three servants, his private motorcar or helicopter - set him in a different world from a member of the Outer Party, and the members of the Outer Party have a similar advantage in comparison with the submerged workers. George Orwell (Nineteen Eighty-Four, 1949, p. 192)

It is often claimed that the (eventual) lack of economic success in the Soviet Union and other former communist countries was due to the lack of proper incentives. However, incentive systems were widely used. I will now take a quick look at the structure of the incentive systems that prevailed in the Eastern bloc.

Almost immediately after the 1917 revolution, the Soviet regime realized the necessity of providing adequate incentives to agents in critical positions. These agents primarily consisted of top bureaucrats, managers of large enterprises, members of the technical and artistic intelligentsia and high-level military personnel. By 1923, for instance, a system of personal bonuses for specialists based on enterprise profits was implemented. Later, when Communism reached Eastern Europe, the broad patterns of the Soviet incentive system were introduced also in the satellite states.

Incentives in the Communist economies operated in a way that was different from those we regularly experience in market economies. Rewards in market economies primarily take the form of financial assets. In the Soviet Union, however, rewards were often received in kind (a (new) car, luxurious apartments,
access to special health clinics, Beluga caviar, vacations to the Black Sea, etc.).
The value of such in kind rewards was all the higher since such goods were very
difficult to obtain via other channels. Matthews (1978) suggests that in the
Soviet Union, the ration system became more important over time, as ‘Party
packets’ (envelopes filled with cash) were substituted for so-called ‘Kremlin ra-
tions’ (coupons for consumer goods). This ration system made it easy to exclude
the workers from the consumption of high-quality products, while providing easy
and reliable access for the elite. Note also that the use of the ration system
made it harder for members of the elite to smooth consumption if they were to
be demoted. This makes the threat of demotion more severe.

Matthews (1978) describes in detail the instruments used by the Soviet state
to provide the elite with selective benefits. From special restaurants within pub-
lic offices or via special distributors and currency shops in the cities, the privi-
leged groups were able to obtain more or less any luxury they desired. Indeed, as
long as the willingness and outside pressure existed, the Soviet leadership seemed
quite capable of acquiring goods of high quality, at least on a limited scale:

High-quality goods, incidentally, need to be produced with more
than average care, and it seems that special production units ex-
ist to service the restricted distribution system. Special dairy herds
are known to be kept in agricultural enterprises near Moscow. The
Mikoyan Meat Processing Combine is said to have a separate produc-
tion unit for high-quality meat: and finer bread is evidently baked in
Moscow (to the same weight and size as the ordinary loaf) for favored
customers. Matthews (1978, p.41)

Matthews also presents data on wages for managers across industries in the
period 1960-1970. Managers in the textile and food industries had significantly
lower wages than those in the heavy industries. Further, managers of collective
and state farms, which employed 30 percent of the labor force in 1970, all had

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Matthews’ book is based partly on interviews with 61 emigrants of various positions from
the Soviet Union. Thus, he had access to detailed information from ex-insiders.
very moderate wages. Managers of big heavy-industry enterprises, on the other hand, had incomes several times higher than the average wage across the Eastern bloc. So, even though incentives clearly existed, they were not directed towards the mass production of fine consumer goods.

(iii) Demotions of Managers

In the model that will be presented below, poorly-performing managers will be punished by being demoted to the working class. The little evidence on managerial turnover that I have come across suggests that such demotions were quite common. Matthews (1978, p. 156) refers to a study from Volgograd in 1965 and 1972 which found that within a three-year period, about 35 percent of the directors were replaced, while the figures for main engineers and deputy directors were 56 and 34 percent, respectively. Among those who were replaced, about 45 percent were promoted, while the rest were presumably demoted. This would imply that within a three-year span, about one sixth of all directors would be dismissed due to poor performance. Lazarev (2005) provides some evidence that the turnover rates among members of the elite fell over time.

(iv) Intelligence and Deterrence

All communist regimes have relied heavily on deterrence and intelligence activities in order to prevent workers from voicing their discontent and mobilizing a common opposition toward the regime. Eastern Germany is, of course, the most notable country in this respect. Some noteworthy figures have been recovered from the Stasi archives (see Koehler [1999]). As the Berlin Wall fell, the Eastern Germany Ministry for State Security, the Stasi, had 102 000 full-time officials employed, and a further estimated 0.5-2 million part-time informants. For a population of just 17 million, this implied that there was one full-time official per 166 citizens, and a far greater density if we count the part-time informants. In

\footnote{I have not managed to find data on the size of the intelligence operations over time. However, my reading of for instance Koehler (1999) indicates that the size grew substantially over time.}
the Soviet Union, on the other hand, the KGB employed 480,000 officials in order to contain a population of 280 million, giving a density of just one agent per 5,830 citizens. We might use Nazi Germany as a measurement stick. Göring’s Gestapo employed at most 40,000 officials to control a population of 80 million, thus giving a density of 2,000 citizens per agent.\textsuperscript{8} This shows that deterrence required an enormous amount of resources, especially in Eastern Germany. I will return to this point in the discussion below.

\textbf{(v) Physical Health}

In the model below, I will argue that the optimality of an incentive scheme involving low-quality goods in general\textsuperscript{9} depends on there being gains from improving the physical health of the workers such that they can supply more labor. In the Soviet Union, Lenin immediately understood that when it came to medicine, ideology had to be put aside. He rewarded doctors handsomely (Matthews [1978]), and campaigns were carried out against typhus, cholera and malaria. Table 1 shows that in Russia/the Soviet Union, average life expectancy at birth grew from about 33 years in the pre-World War I period to 47 years in 1938, before it stabilized at about 68 years around 1960. In Poland under Communism, average life expectancy grew from 55 years in 1950 to about 66 years in 1965, where it was stabilized. Note that the data are the average life expectancy across sexes, and that life expectancy for women in general was around 5-7 years higher than for men.

Figure 1 shows a plot of food supply in Russia/USSR over the period 1895-1989, as measured by the amount of calories per person per day. The figure shows that the amount of calories per person grew from about 2000 at the beginning of the twentieth century to a level around 3300 at the end of the 1950s, before it flattened out at about 3400 calories per person per day in the 1970s. In comparison, in India in 1988-1990, there were about 2230 calories available per day.

\textsuperscript{8}Of course, Gestapo was not Hitler’s only deterrent. If we had included SA this number would have been smaller.

\textsuperscript{9}If intelligence and deterrence are extremely effective, this will not be needed.
person per day, while the calorie consumption in Western Europe by the late twentieth century was around 3400 (see Allen [2006], pp. 134-136).

These data indicate that during the initial stages of communism, there was much to gain in labor productivity from providing the people with a sufficient amount of basic goods.

(vi) The Economic Performance of the Eastern Bloc

The economies of the Eastern bloc performed tolerably well for several decades. The data from Maddison (2003) show that relative to the United States, the GDP per capita of the Communist bloc was increasing until the mid 1970s (see Figure 2).

Relative to Western Europe the comparison is less advantageous, especially for the Soviet Union, as the economies of Western Europe recovered from World War II. Note, however, that the communist countries of Eastern Europe countries grew as quickly as Western Europe between the end of World War II and the mid 1970s. Even though proper quality adjustments of the data would no doubt worsen the relative performance of the communist countries, we are still left with a picture of a performance that deteriorates over time.\(^\text{10}\) Further evidence of this can be seen by comparing Figures 3 and 4. Figure 3 shows the annual growth in GDP per capita between 1950 and 1975, given the initial level of output. The solid line shows the average performance for a given initial output. During this period, we can see that the Eastern European countries and the Soviet Union performed better than average.\(^\text{11}\) However, when we look at the period from 1975 through 1989 in Figure 4, the economic performance is below average. A discussion of the deteriorating growth performance of the Soviet Union can be found in Easterly and Fischer (1994), for instance. Akerlof et al. (1991) contrast productivity levels in Eastern and Western Germany in 1990. For a discussion

\(^{10}\)Such an adjustment would most likely make the initial period look even better relative to the latter, as the quality gap grew larger over time.

\(^{11}\)For a detailed account of the relatively successful industrialization process in the Soviet Union, in particular, see Allen (2003).
3.4 The Model

In the model there are three groups of agents; a self-interested regime, a group of managers and a group of workers. I will regard the regime as a single decision-making unit. It should be thought of as the top political elite. The regime wants to maximize the value of output it can appropriate, but faces a set of constraints. First, there is a moral hazard problem in production, so the managers must be given incentives to exert effort. Second, the workers will oust the regime if they are treated too badly and the regime does not back up its harsh treatment with investments in deterrence. Third, and finally, the workers’ supply of labor depends on the level of consumption, at least up to some limit.

I will be interested in how the regime exploits its opportunity to centrally control the product mix in order to provide incentives in the most cost-effective manner. In one sense, I will give the regime quite a bit of flexibility in designing the reward systems. In particular, products of different quality levels can be produced at the same cost. In another sense, by appealing to plausible real-world complications, I will restrict the set of feasible contracts the regime can design.

The model in this section should be considered as a model where the regime can take advantage of the control provided by central planning while, at the same time, avoiding the production inefficiencies it creates, particularly when it comes to producing high-quality goods on a large scale. In the next section, I will compare the optimal incentive systems in this utopian world with the incentive systems created by a regime that does suffer from the inefficiencies of central planning. It is only when a mismatch arises between these two incentive systems that the inefficiencies of central planning will materialize.
3.4.1 Production

There are two types of consumer goods, one of high quality, another of low quality, with subscripts \( g \) and \( b \), respectively. Production of the high- and low-quality goods takes place in factories using labor and capital as inputs. The regime has a capital stock \( K \) at its disposal. There is a measure 1 of managers, each supervising production in one factory, and a mass \( N \) of workers.

**Managers’ input:** Each manager chooses how much effort to exert at work. Initially, the set of possible effort levels has two elements only, \( e \in \{e_h, e_l\} \), with \( e_h > e_l \). Effort is costly to provide and is unobserved by the regime. I denote the cost of providing effort at level \( e_i \) by \( \psi(e_i) \), normalized such that \( \psi(e_h) > \psi(e_l) = 0 \). The effort level affects the factory’s productivity level, which can take on two values, \( \gamma \in \{\gamma_H, \gamma_L\} \), where \( \gamma_H > \gamma_L \). For simplicity, let \( \gamma_H = 1 \) and \( \gamma_L = 0 \). Also, let \( p_i = \Pr(\gamma = \gamma_H| e = e_i) > 0 \) denote the probability of reaching the high productivity level given an effort level \( e_i \).

**Workers’ input:** Workers are able to exert effort according to their physiological health. Health depends on the amount of goods consumed, that is, on \( c = c_b + c_g \). The supply of labor that can be extracted from each worker is given by the function

\[
\ell = L(c),
\]

which is increasing and concave.\(^{12}\) What I want to capture is that at least at low levels of consumption, the more the workers consume, the less they will be sick, the longer they will live, and the more productive effort the regime can extract from them. Moreover, a long working life means that accumulated experience will not be wasted. The regime will take these effects into consideration since it is the residual claimant of output. Aggregate labor supply is given by \( L(c)N \).

Note that I am abstracting from workers’ incentive problems. In hierarchies,

\(^{12}\)One could allow the labor supply effects to be greater for high-quality goods than for low-quality goods. The critical assumption is that, compared to low-quality products, high-quality products are relatively better at satisfying preferences than at increasing the supply of labor. That is, if we assume that \( L = \ell(c_b^\alpha + c_g^\zeta) \), with \( \zeta > 1 \), the critical assumption is that \( \zeta < \alpha \) (see eq. ?? for the definition of \( \alpha \)).
3.4. THE MODEL

the effort of the agents at higher levels of the system will be of most importance (see e.g. Qian, 1994), and the focus here will be on solving this incentive problem. I am also abstracting from the multiple layers of the hierarchy, but this makes no qualitative difference for the results that follow.

**Factory output:** In a factory where the manager exerts effort \( e_i \), the capital stock is \( K \) and there are \( N \) workers, the expected output is

\[
y_i = p_i (L(c)N)^\eta K^{1-\eta} \equiv p_i L(c)^\eta y.
\]  

(3.2)

**Aggregate output:** The realizations of productivity levels across factories are independent. The principal divides the capital stock and the supply of labor between the two sectors of production, with a fraction \( \lambda_b \) given to the low-quality good sector. Since there are constant returns to scale in production and output shocks are independent across factories, the planner will optimally allocate an equal amount of capital and labor to all factories. In order to bias the case against production of low-quality goods, I will assume that for a given effort level, the unit cost of production is the same for both quality levels. In a market economy with perfect competition, low-quality products could never exist, as the producers of these goods would be unable to remain in business and, at the same time, sell any products. Low-quality goods are only produced in a setting where the product mix is centrally determined and as long as they serve the interests of the regime.\(^{13}\)

Given a high effort level by the managers and using the expression in eq. 3.2, the output levels in the two sectors can be written as

\[
Y_b = \lambda_b y_h, \quad (3.3)
\]

\[
Y_g = (1 - \lambda_b) y_h. \quad (3.4)
\]

Note that I have implicitly assumed that it takes the same effort level from the

\(^{13}\)In reality, low-quality goods are most often cheaper to produce, and this cost-advantage is another cause of their existence. Still, the point here is to show that to the regime, there is a value of low-quality products over and above this underlying value.
managers to ensure a high level of expected output in both the low-quality and the high-quality sector. This will also make managerial wages equal in the two sectors. This is a simplification. Data from the Soviet Union show that managers in low-priority sectors (consumer goods, for instance) were given lower incentives than managers in high-priority sectors. Still, managers in low-priority sectors were given substantial incentives. By allowing lower costs of effort in the low-quality sector, the allocation where workers only receive low-quality products would become more appealing, but the qualitative insights would remain the same.

3.4.2 Preferences

Workers and managers: Let the consumption levels of workers and managers be denoted by superscripts $w$ and $m$, respectively. The utility function of an agent of type $j \in \{w, m\}$ who consumes $c^j_b$ units of the low-quality product and $c^j_g$ units of the high-quality product is given by

$$u(c^j_b, c^j_g) = v(c^j_b + \alpha c^j_g),$$

(3.5)

where $\alpha > 1$ and the function $v(\cdot)$ is increasing and strictly concave. There is perfect substitutability between the two types of goods. In other words, the marginal rate of substitution between a high-quality product and a low-quality product is $\alpha$.

There might be other more realistic ways of specifying preferences over quality and quantity. In particular, one could imagine that the marginal utility of the high-quality good relative to the low-quality good is increasing in the level of consumption. However, this would take the focus away from the main issue and also complicate the analysis.

The regime: How to specify a clear objective function for a political regime is far from self evident, but I will here follow the 'Leviathan' tradition of assuming that the regime wants to maximize the value of its own consumption. Such an
objective function seems to be especially relevant for the communist countries involved in the Cold War. Rapid industrialization was both a stated objective and perhaps also a necessary condition for sustaining power in the long run (by deterring foreign aggression). In a more elaborate model, one could model the surplus as a function of the regime’s consumption and the likelihood of avoiding a defeat against foreign aggressors. In the present model, I instead introduce the possibility of a domestic uprising. The critical assumption is that the regime has an interest in a high level of economic output.

The regime has a constant marginal utility of consumption of each good but, like its subjects, the regime prefers the high-quality good to the low-quality good. The preferences of the regime are represented by the function

\[ U^r(c^b_r, c^g_r) = c^b_r + \alpha c^r_g. \]  

Note that the linearity of the regime’s objective function is irrelevant for the results that follow. What is required is simply that the regime’s payoff is increasing in its consumption level, and that the high-quality good is preferred to the low-quality good.

### 3.4.3 Constraints

**Incentive compatibility (IC):** I have already noted that the effort level of the managers is unobservable. I will be assuming that the regime wants the managers to exert the high effort level and, therefore, the regime will have to compensate the managers for the cost of the effort they are providing.

Let the consumption level of the good of quality \( q \in \{ g, b \} \) for a manager who produced at productivity level \( \gamma \in \{ \gamma_H, \gamma_L \} \) be denoted by \( c^m_{q, \gamma} \). Next, I have to specify the compensation received the poorly-performing managers. I assume that such managers are rewarded in the same manner as workers. By this assumption, I want to capture the higher-ranked individuals’ fear of being thrown back down to the workers. Thus, we can think of the regime excluding
a poorly-performing (or disloyal) official from the special stores and the fancy restaurants. Instead, he will have to wait in line together with the unfortunate workers for the products distributed to them. The critical element is that the reward to the managers in the poor state is related to the reward given to the working class, and it is the gap between the high reward in the good state, and the low reward in the bad state that creates incentives.

This gives us the following incentive-compatibility constraint (IC) for managers

\[ p_h v(c_b^m + \alpha c_g^m) + (1 - p_h)v(c_b^w + \alpha c_g^w) - \psi(e_h) \geq \]

\[ p_l v(c_b^m + \alpha c_g^m) + (1 - p_l)v(c_b^w + \alpha c_g^w). \]

Defining \( \Delta p = p_h - p_l \), this can be rewritten as

\[ v(c_b^m + \alpha c_g^m) - v(c_b^w + \alpha c_g^w) \geq \frac{\psi(e_h)}{\Delta p}. \]  

This condition simply tells us that the compensation structure offered by the regime must be such that the agent prefers to exert the high effort level.

In assuming this to be the relevant incentive structure, I have restricted the set of contracts that the regime can implement. Ideally, the regime would like to give high-quality products to the workers (this will make it inexpensive to prevent an uprising), while punishing the managers by giving them low-quality products. There are two difficulties with such a system. First, it would be difficult to administer in practice. The regime would have to force the managers into some special stores where only low-quality products were offered, while also pre-
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venting them from entering stores with high-quality products and from receiving
tastes of the good life with the help of well-fed friends and family members. Sec-
ond, there also exists a participation constraint for managers who can choose to
stay among the workers. Hence, if the punishment of the unlucky hardworking
managers is too bad, the reward for the lucky ones will have to compensate for
this. Essentially, two conditions must thus be satisfied (to simplify the notation,
let \( v^m_\gamma \) denote the utility of a manager with productivity level \( \gamma \in \{ \gamma_H, \gamma_L \} \) and
\( v^w \) the utility level of a worker):

\[
\begin{align*}
  v^m_{\gamma_H} - v^m_{\gamma_L} & \geq \frac{\psi(e_h)}{\Delta p} \\
  v^m_{\gamma_H} + (1 - p_h)v^m_{\gamma_L} & \geq v^w + \psi(e_h).
\end{align*}
\]

The first of these represents the incentive constraint for effort, while the second
represents the participation constraint, which requires that managers cannot be
worse off than members of the working class. Suppose that both these equations
are binding. The two equations imply that \( p_l v^m_{\gamma_H} + (1 - p_l)v^m_{\gamma_L} = v^w \), or that a lazy
manager gets the same payoff as a worker. This solution requires that a manager
who is demoted must be worse off than a worker, as \( v^m_{\gamma_H} - v^m_{\gamma_L} > 0 \). Such schemes
can be ruled out by allowing unlucky managers to always reinvent themselves as
workers, if it is optimal for them to do so. If the managers are given such an
opportunity, the regime can never threaten the managers with something worse
than the reward workers are getting. In the following, schemes where managers
are worse off than workers will therefore be excluded from the feasible set.

**No-revolution constraint (NRC):** For a given deterrent capacity, there
are limits to how badly the workers can be treated. If the regime treats them
too badly, they will manage to overcome the collective action problem and stage
a revolution. I assume that such a revolt succeeds with probability \( P \) (this will
be endogenized later), and gives the people a payoff of \( V^R \) if successful.\(^{17}\) If

\(^{17}\)Below, this probability will depend on the resources spent on deterrence. \( 1 - P \) is thus the
deterrent capacity of the regime.
unsuccessful, the regime simply recycles its plan and implement it. The cost of the revolution is that the payoffs will be received at a later stage, so they are discounted. For the regime to avoid a revolution, it then follows that the following condition must hold:

\[ v(c_b^w + \alpha c_g^w) \geq \beta [PV^R + (1 - P)v(c_b^w + \alpha c_g^w)], \]

which can be rewritten as

\[
v(c_b^w + \alpha c_g^w) \geq \frac{\beta P}{1 - \beta(1 - P)} V^R \equiv \bar{v}. \quad (3.9)
\]

This constraint will be called the 'no-revolution constraint’, and is a quite standard element of models involving the conflict between a dictatorial regime and an oppressed class of citizens. I will assume that the regime always wants to avoid a revolution, for instance because members of the political elite perceive that the probability of getting killed or ousted even in an ultimately unsuccessful revolution is high. For an overview of models using such constraints, see Acemoglu and Robinson (2006).

It is important to recognize that in the analysis that follows, a non-binding no-revolution constraint always implies that the workers will be rewarded with low-quality products only. If the workers are sufficiently servile, the cost of providing them with low-quality products disappears, and only the gains from reduced incentive costs remain.

**Resource constraints:** Even dictators cannot avoid resource constraints. In our case, these tell us that the aggregate consumption of each good of the workers, managers and the regime can be no greater than the total output of these goods. Or, in other words, the restrictions are

\[
(N + 1 - p_h)c_b^w + p_h c_b^m + \bar{c}_b \leq Y_b \quad (3.10)
\]

\[
(N + 1 - p_h)c_g^w + p_h c_g^m + \bar{c}_g \leq Y_g. \quad (3.11)
\]
3.5 Analysis

We are now ready to analyze the problem faced by the regime, namely the maximization of the regime’s payoff subject to the incentive constraint (eq. 3.8), the no-revolution constraint (eq. 3.9), the labor-supply relation (eq. 3.1) and the resource constraints (eqs. 3.10 and 3.11).

3.5.1 Constant Labor Supply

I will start out with a brief discussion of the problem when the labor supplied by each worker is fixed, i.e. we fix $L(c)$ at some constant level. This will serve as a benchmark case, and can be interpreted as the problem facing the regime in a developed economy where the regime is forced to reward the workers relatively generously.

The problem of the regime is how to structure the reward system so as to minimize the cost of satisfying the incentive constraint of managers and the no-revolution constraint of the workers. Giving the workers low-quality products involves a loss and a potential gain for the regime. The loss is that it becomes more costly to satisfy the no-revolution constraint, while the potential gain is that it may become cheaper to satisfy the managers’ incentive constraint by depressing the workers’ utility. In the case under discussion here, the no-revolution constraint and the incentive constraint will always be binding. As the incentive constraint is binding, we need $\Delta v = v^m - v^w = \psi(e_k)/\Delta p$. The utility of the workers is fixed by the no-revolution constraint, which implies that $v_w$ is determined by $P$, $\beta$, and $V^R$. This makes the outside option for managers independent of the choices of the regime, which means that there cannot be any gains from introducing low-quality products to the masses. Only the additional resource cost of fulfilling the no-revolution constraint remains. Therefore, it is evident that no capital will be allocated to the production of the low-quality good in this situation. We can state this as a proposition:

**Proposition 1**: If the labor supply of the workers is fixed, it will never be...
optimal to produce a positive amount of the low-quality good.

**Proof:** See Part A1 of the Appendix.

The implication of this proposition is that if low-quality goods serve no other purpose than to satisfy the no-revolution constraint (the social contract), there is no reason to allocate factors to the low-quality sector. The regime will always want to move from providing a quantity $c_b$ of low-quality goods to the less expensive $c_g = c_b/\alpha$ units of the high-quality good, since this creates the same utility for the workers. This is illustrated in Figure 5. The solid line starting in point A shows the indifference curve of a worker given the reservation utility $\bar{v}$ (i.e., this is the NRC), while the dashed line starting in point B shows the indifference curve of a manager who is compensated with the utility differential $\Delta v = v(\alpha c_g^m) - v(c_b^m + \alpha c_g^m)$. There will never be any reasons to provide managers with low-quality products, so they will end up at B. Further, at point A, where the workers are only given high-quality goods, the regime will minimize the cost of satisfying the no-revolution constraint.

### 3.5.2 Labor Supply Effects

The results that follow next rest on a claim that low-quality goods indeed do serve another purpose, namely the following: Even though it is not very pleasant to sleep in uncomfortable beds, go to work using low-quality public transportation, or eat food that is not exactly of exquisite quality, such goods do keep you alive and make you able to exert a decent amount of effort at work. Why would this matter? As long as low-quality goods are relatively better at buying health than pleasure, and if the NRC is relatively easy to satisfy, it is not given that the regime will always want to provide a low quantity of high-quality goods instead of a higher quantity of low-quality goods. In fact, providing low-quality goods to the workers and high-quality goods to the managers can be optimal. Even though the cost of satisfying the NRC is higher than what is necessary, the regime will be happy about this since they are getting labor supply from the low-quality
goods, while incentive costs are reduced. I will now show this possibility more formally.

Consider the problem of the planner when the labor supplied by the workers responds to the quantity of consumption offered to them. The regime will now maximize output minus wage costs subject to the managerial incentive constraint, the resource constraints, and the no-revolution constraint, while also respecting that labor supply is given by the strictly increasing and concave function $L(c)$.

I assume that the leadership gets positive consumption in equilibrium. To simplify the notation, let $N_2 = N + 1 - p_h$, and $\beta_p = 1 - \beta(1 - P)$. The regime wants to maximize $U^r = c_b^r + \alpha c_g^r = [\lambda_b y_h - N_2 c_b^{w} + \alpha (1 - \lambda_b) y_h - N_2 c_g^{w} - p_h c_g^{m}]$. The first term disappears as the regime will never decide on a production level of the low-quality good in excess of the amount given to the workers (i.e., the regime will never produce low-quality goods for its own consumption). Thus, as workers and unlucky managers consume all low-quality goods, we have $N_2 c_b^{w} = \lambda_b y_h$, or $\lambda_b = \frac{N_2 c_b^{w}}{y_h}$. Using the expression for $\lambda_b$ in the second term, the regime’s problem can be written as:

$$\max_{\{c_b^{w},c_g^{w},c_g^{m}\}} \{\alpha [y_h - N_2 c_b^{w} - N_2 c_g^{w} - p_h c_g^{m}]\}$$

s.t.

NRC: $v(c_b^{w} + \alpha c_g^{w}) \geq \frac{\beta}{\beta_p} PV^R$

IC: $v(\alpha c_g^{m}) - v(c_b^{w} + \alpha c_g^{w}) \geq \frac{\psi(e_h)}{\Delta p}$

$c_b^{w} \geq 0, c_g^{w} \geq 0, y_h = p_h (L(c) N)^{\eta} K^{1-\eta}$

I will assume that the relative risk aversion coefficient of the utility function $v(\cdot)$ is below some level $\bar{R}_r$. If the level of risk aversion is too high, it will be extremely costly to provide the managers with incentives as the consumption level of the workers increases. In this case, the dictator might find it worthwhile to keep the workers with the low-quality bundle, even though the gains to labor supply from increasing the quantity of consumption are extremely low.\(^{18}\)

\(^{18}\) A reasonable alternative way to rule out this is to let the positive effects on labor supply
By studying this problem, I find the following result:

**Proposition 2:** The no-revolution constraint will be binding for some set of values of $V^R$ such that $V^R \in [V_R^L, \infty)$. If the no-revolution constraint does not bind in equilibrium ($V^R < V_R^L$), the workers will be provided with low-quality products only. With a binding no-revolution constraint, it will be optimal to provide the workers with a bundle of only low-quality products for some interval of values of $V^R$ such that $V^R \in [V_R^L, V_R^R]$. When $V^R > V_R^R$, the regime will want to gradually phase out the supply of low-quality products, and, eventually, as $V^R$ reaches some level $V^{R*}$, the workers will only be given products of high quality.

**Proof:** See Part A2 of the Appendix.

The intuition behind the result is the following. Suppose first that the revolutionary outcome $V^R$ is low, such that the no-revolution constraint will not be binding in equilibrium. To increase labor supply, the dictator then supplies the labor force with goods in excess of what is needed to satisfy the NRC. In this scenario, it will be optimal to only provide low-quality goods to the workers. The reason is that this makes the outside option as bad as possible for the managers and, by concavity of the utility function, it becomes cheaper to satisfy the incentive compatibility constraint. In this case, the optimal consumption levels for workers and managers are determined by the incentive constraint and the equation for optimal labor supply$^{19}$:

$$
\eta L^{n-1} L' y - N_2 - \frac{p_h}{\alpha} v'_w = 0.
$$

(3.13)

A marginal increase in consumption for workers increases output by $\alpha \eta L^{n-1} L' y$. The costs of such a change consist of two components: First, the wage of $N_2$ agents must be marginally increased, and second, it will become more costly to satisfy the incentive constraint for managers. Thus, to keep wage costs down, the workers are mistreated in two ways: First, they only get low-quality products disappear completely after some threshold level of consumption is reached.

$^{19}$To save on notation, I let $v_j \equiv v(c^j_b + \alpha c^j_g)$ and $v'_j \equiv v'(c^j_b + \alpha c^j_g)$. 
and, second, the quantity is reduced so that the wage costs for managers can also be kept low.

Now, as $V^R$ increases and the dictator sticks to the above reward system, at some level $V^R = V_R$ of the revolutionary outcome, the no-revolution constraint will become binding. Since the incentive constraint for managers always binds in equilibrium, the shadow price on this constraint ($\phi_2$) is always strictly positive, with a minimum level defined by its level when the NRC does not bind. This means that as $V^R$ hits the level such that the NRC starts to bind, the shadow price on the NRC ($\phi_1$) is lower than the shadow price on the incentive constraint. As long as we have $\phi_1 < \phi_2$, the dictator keeps giving the workers low-quality products only. In this range, the dictator introduces a distortion in the compensation to workers in order to keep the more important managerial costs down. The reason why he is willing to do so is that if he were to shift consumption from low quality goods to high quality goods, keeping the quantity constant, he would give the workers a payoff in excess of the outside option, thereby increasing the cost of providing incentives to the managers, while giving no offsetting gains. Alternatively, he could just satisfy the NRC by giving the workers high-quality goods only, keeping managerial costs fixed, but he would then suffer a fall in the supply of labor, with a cost in excess of the gain of satisfying the NRC in a more cost-effective manner. The compensation awarded to the workers is determined by the NRC, while the IC defines the compensation to managers. The labor supply decision is now given by

$$\eta L^{n-1} L' y - N_2 - (\phi_2 - \phi_1) \frac{v_w^L}{\alpha} = 0,$$

where we have $(\phi_2 - \phi_1) \frac{v_w^L}{\alpha} < \frac{p_N v^L}{\alpha v_m}$. Thus, as $\phi_1 \to \phi_2$, the second source of inefficiency in the treatment of workers disappears: The dictator moves closer to a situation where the marginal product of consumption on labor supply equals the resource cost for the regime.

Eventually, as $V^R$ continues to grow, the loss from giving the workers low-quality goods in a situation where the NRC is binding starts to increase, which
makes $\phi_1$ grow relative to $\phi_2$. At the point where the two shadow prices are equal, at a level $V^R = V^R$, the regime starts to phase in high-quality products to the workers. The consumption levels continue to be pinned down by the NRC and the IC, while the constraints now force labor supply to be determined by

$$\eta L^{y-1}L'y - N_2 = 0,$$

i.e. the marginal product of consumption equals the resource cost. For this interval of $V^R$, the volume of consumption must thus be fixed, the only change being that there is a decrease in the amount of low-quality products, which will be substituted one-for-one with high-quality products.

Finally, as $V^R$ continues to increase, at some point $V^{R*}$ only high-quality goods are awarded to the workers, and the regime is forced to provide consumption to the workers such that $\eta L^{y-1}L'y - N_2 < 0$.

The result in Proposition 2 shows that as long as the benefit of a successful revolution is limited, an optimal incentive scheme will consist of the provision of low-quality products to the workers. The result also hinges on the revolution payoff to be low relative to the payoff to the regime from increasing labor supply by handing out more consumption to the workers. In reality, of course, the regime has another weapon at hand, as it can spend resources on reducing the expected payoff from a revolution. This possibility might make the incentive scheme involving low-quality goods more attractive. I now turn to this case.

### 3.5.3 Deterrence

In reality, the value of an uprising is far from independent of the regime’s actions. It is clear that the regimes under consideration were concerned about such variables as $P$ and $V^R$. The intelligence bureaus invested heavily in preventing counter-revolutionaries from gaining a foothold and spreading their message (reducing $P$), and they also indoctrinated the people about the attractiveness of life under Communism in the hope of reducing their perceptions of $V^R$. 
In this subsection, I will let the regime invest in military assets in order to reduce the likelihood of a successful uprising. Such investments make it optimal to more often exploit the low-quality goods in the incentive system. I will provide conditions for when such investments make an incentive system based on low-quality products to workers optimal, no matter how tempting it is for the workers to revolt, that is, no matter the level of $V^R$. In the next subsection, I will let the value of a successful revolution depend on the level of the capital stock in the economy. Using the results from this subsection, I will then discuss how the optimal (static) incentive system develops as the economy grows.

The military technology works as follows. By allocating a part $\lambda_d$ of the capital and workers to the production of military goods, the deterrence capability will be defined by $D = \lambda_d y_h$. The likelihood of a successful revolution will be a function $P(D)$, where $P(D)$ is a decreasing and convex function of $D$. The assumption that the technology producing deterrence is the same as the one producing output is of no qualitative importance for the result that follows, and is just made for simplicity.

In addition to the investment in military technology, I will make one other modification. Instead of letting the supply of labor be a continuously increasing function in the level of consumption, it will be defined by

$$L(c) = \begin{cases} 
1 & \text{if } c_b + c_y \geq \bar{c} \\
0 & \text{otherwise} 
\end{cases}$$

(3.15)

This equation defines the labor supply constraint for the regime, which captures the importance for the regime of having a relatively healthy work force. At the cost of analytical complexity, one could use the specification with continuous labor supply above and avoid this knife-edge interpretation. The results would be qualitatively the same.

First, note that a result similar to that in Proposition 1 continues to hold:

**Proposition 3:** If the workers’ labor supply is independent of the level of consumption, it will never be optimal to produce a positive amount of the low-
quality good, even if the regime can invest in military technology and affect the no-revolution constraint.

**Proof**: See Part A3 of the Appendix.

Even though the outside option for the workers is now affected by the level of military spending, it is still the case that low-quality goods serve no other purpose than satisfying the no-revolution constraint. Military spending only affects the level of the outside option for the workers, but it will always be optimal to provide the workers with this payoff level in the most cost-effective manner. Therefore, the intuition behind this result is identical to that behind Proposition 1.

To rationalize the use of low-quality products, the labor-supply constraint must be introduced. To make the exposition simpler, I will rule out solutions where the no-revolution constraint is satisfied even when the regime spends nothing on military production while, at the same time, it gives the workers low-quality products only. The assumptions needed to rule out this case are (i) $V_R > v(c)$, and (ii), $P(0) > \frac{1-\beta}{\beta} \frac{v(c)}{V_R - v(c)}$. Given these two assumptions, I can state the following proposition:

**Proposition 4**: When the labor-supply constraint does not bind, the outcome will be as in Proposition 3. If it binds, there are three types of solutions: the workers are given either (i) high-quality products only, (ii) a mix of high-quality and low-quality products, or (iii) low-quality products only. The solution where the workers are given only low-quality products will be the equilibrium when effort costs are high, the quality gap is not too great, and the deterrence technology is effective.

**Proof**: See Part A4 of the Appendix.

The different types of solutions are illustrated in Figure 6. In all solutions, both the incentive constraint for managers and the no-revolution constraint for workers will be binding.

In the type-(S) (slack) allocation, the consumption level is sufficiently high to make the labor-supply constraint non-binding, and only high-quality products
will be offered to the workers. For this allocation to constitute an equilibrium, some of the following ingredients are required: A high payoff from a successful revolution, low effort costs, large quality differences, or costly deterrence. In the other allocations, the labor-supply constraint is binding. What varies is the proportion of high-quality products in the bundle given to workers. In the type-HQ (high quality) allocation, workers only get high-quality products. In an allocation of type MQ (mixed quality), workers get a mix between high-quality and low-quality products, while in the type-LQ (low quality) allocation, they only get low-quality products.

To better understand the determinants of the regime’s optimal solution, consider the following experiment. Suppose that the labor-supply constraint is binding and that it becomes marginally stricter. The direct cost for the regime of this change is $\alpha N_2$ in foregone utility. However, there are also two indirect effects, and the relative size of these two effects will determine whether the type-HQ, the type-MQ, or the type-LQ allocation will solve the regime’s problem. First, more consumption to the workers implies that less resources must be invested in deterrence in order to satisfy the no-revolution constraint. Second, more consumption to the workers implies that the reward to managers must increase to respect the incentive constraint. The type-HQ allocation will be the optimal solution when the shadow price on the labor-supply constraint takes on values in the range $(0, \alpha N_2)$. In other words, this allocation will be realized when the savings from less deterrence are greater than the additional cost of satisfying the incentive constraint for managers. A type-MQ allocation, on the other hand, where a mix of high and low-quality goods is offered, will prevail when the shadow price on the SLC is exactly equal to $\alpha N_2$, in other words when the two indirect effects cancel out. Finally, the Type-LQ equilibrium, with only low-quality goods given to the workers, will be the solution to the regime’s problem when the incentive costs are large relative to the deterrence costs, i.e. when the shadow price on the SLC is greater than $\alpha N_2$.

Given that the low-quality-good allocation is the most attractive one, the
regime will invest in deterrence to push the NRC inward such that it justs binds at the point where the workers are given a quantity of low-quality products such that the labor supply constraint binds. In this way, incentives will be cheap to buy.

In order to understand the long-term dynamics of the optimal incentive scheme, I am especially interested in how the optimal allocation changes as $V^R$ increases. If an increase in $V^R$ makes the type-HQ allocation more attractive relative to the type-LQ allocation, the regime will want to phase in high-quality products to the workers over time. In this case, the optimal bundle will converge to a completely high-quality one and, eventually the workers will be granted a level of consumption such that the labor supply constraint no longer binds. On the other hand, if the deterrence technology is sufficiently effective, it might be optimal for the regime to keep the workers stuck at the type-LQ allocation.

The regime’s consumption levels in the type-HQ and type-LQ allocations are given by

$$c_{HQ}^r = \lambda_g^{HQ} y_h - N_2 \bar{c} - p_h c_m^{HQ},$$

$$c_{LQ}^r = \lambda_g^{LQ} y_h - p_h c_m^{LQ},$$

where $c_m^t$ denotes the managers’ consumption in allocation $t$, defined by the incentive constraint. The difference in the consumption levels for the regime between the two allocations is then

$$c_{LQ}^r - c_{HQ}^r = (\lambda_g^{LQ} - \lambda_g^{HQ}) y_h + N_2 \bar{c} + p_h [c_m^{HQ} - c_m^{LQ}],$$

where

$$\lambda_g^{LQ} - \lambda_g^{HQ} = \frac{1}{y_h} \left[ P^{-1} \left( \frac{(1 - \beta)v(\alpha \bar{c})}{\beta(V^R - v(\bar{c}))} \right) - P^{-1} \left( \frac{(1 - \beta)v(\bar{c})}{\beta(V^R - v(\bar{c}))} \right) \right] - \lambda_b^{LQ}. \quad (3.16)$$

If we were allowing labor supply to be continuous in this Section, this situation would have been the one where the productivity gains to the Party from an increase in consumption does not justify the resource costs.
The difference in consumption levels between the allocations consists of two parts: First, in the low-quality-good allocation, it will be more expensive to satisfy the NRC as more deterrence is needed. Second, there is a potentially offsetting benefit, since lower utility to the workers implies that it will be cheaper to satisfy the IC. For sufficiently low values of $V^R$, the difference in the deterrence cost will be marginal as the NRC can be satisfied at minimal costs. However, the gain in terms of lower incentive costs in the low-quality-good allocation is a constant. This implies that for such low levels of $V^R$, the dictator will always prefer the low-quality allocation to the high-quality allocation. To evaluate what will happen in the ranking of these two alternatives as $V^R$ increases, by eq. (3.16) it is sufficient to evaluate the derivative $\frac{\partial (LQ - HQ)}{\partial V^R}$. If this is negative, the high-quality allocation will gradually improve relative to the low-quality allocation. But when will this be the case? In order to make the discussion as clean as possible, I will make the following technical assumption on the function $P(\cdot)$:

Assumption 1: Consider two allocations $X$ and $Z$ where the workers are given a bundle $x$ in the former and a bundle $z$ in the latter. Assume that the workers prefer bundle $z$ to $x$. Let spending on deterrence in allocation $\chi \in \{X, Z\}$ be defined from the no-revolution constraint by $D_\chi = P^{-1}\left(\frac{1-\beta v(x)}{\beta (V^R - v(x))}\right)$. Keeping $x$ fixed, I will then assume that the sign of the inequality

$$\frac{P'(D_z)}{P'(D_x)} \leq \frac{v(z)}{v(x)} \left(\frac{V^R - v(x)}{V^R - v(z)}\right)^2$$

(3.17)

is constant for all $V^R$ (s.t. interior solutions) and all $z \succ x$.

Assumption 1 is not very restrictive, the reason being that the arguments in $D_x$ and $D_z$ are exactly the same as those that can be found on the right-hand side of eq. (3.17). For instance, it is satisfied when $P(\cdot)$ is defined by either the exponential function, $P(D) = \exp(-kD)$, or the Pareto distribution, $P(D) = (1 + D)^{-b}$. The assumption prevents the effectiveness of the military technology from suddenly changing character as $V^R$ increases.

Given Assumption 1, the evolution of the optimal incentive scheme as $V^R$
CHAPTER 3. INCENTIVES UNDER COMMUNISM

increases is given by the following proposition:

**Proposition 5**: As $V^R$ increases, one of two things will happen:

(a) If the deterrence technology is sufficiently effective, which is when we have
$$\frac{P'(D_{HQ})}{P'(D_{LQ})} < \frac{v(z)}{v(x)} \left( \frac{(V^R - v(\alpha \tilde{c}))}{(V^R - v(\tilde{c}))} \right)^2,$$ the type-LQ-allocation with only low-quality goods is an absorbing solution to the regime’s incentive problem.

(b) If the deterrence technology is not sufficiently effective, which is when we have
$$\frac{P'(D_{HQ})}{P'(D_{LQ})} > \frac{v(z)}{v(x)} \left( \frac{(V^R - v(\alpha \tilde{c}))}{(V^R - v(\tilde{c}))} \right)^2,$$ there exist threshold levels of $V^R$, $V^R < V^R$, such that only low-quality products are offered for $V^R < V^R$, a mix of high-quality and low-quality products is offered for $V^R \in (V^R, V^R)$, while only high-quality products are offered when $V^R \geq V^R$. For $V^R > V^R$, the labor-supply constraint will be slack.

**Proof**: See Part A5 of the Appendix.

The proposition tells us that if the military technology is sufficiently effective, it will be optimal for the regime to preserve the quality-differentiated incentive system even when $V^R$ increases substantially and it becomes more costly to silence the workers. The condition for this to take place is that the marginal value of deterrence spending does not lose steam too rapidly as spending increases. If deterrence spending loses its effectiveness relatively fast, on the other hand, it will be optimal to move step-by-step from the type-LQ allocation, via the type MQ and type-HQ allocations, to the type-(S) allocation as $V^R$ grows. In other words, the regime will want to completely remove low-quality products from the incentive system over time. An illustration of this can be found in Figure 7.

Two simple examples will serve to illustrate the applicability of the two different regimes. First, consider the Pareto distribution, where we let $P(D) = (1 + D)^{-b}$ for $b > 0$. Suppose that $b < 1$. Then, the inequality (3.17) becomes

$$\frac{P'(D_{HQ})}{P'(D_{LQ})} = \left( \frac{v(\alpha \tilde{c})}{v(\tilde{c})} \left( \frac{(V^R - v(\alpha \tilde{c}))}{(V^R - v(\tilde{c}))} \right) \right)^{1/b} > \frac{v(\alpha \tilde{c})}{v(\tilde{c})} \left( \frac{V^R - v(\tilde{c})}{V^R - v(\alpha \tilde{c})} \right)^2.$$

In this example, case (b) of Proposition 5 is the relevant one. As the deterrence technology loses effectiveness, the optimal incentive system will move away from
the low-quality solution.

Next, consider the exponential distribution, where $P(D) = \exp(-kD)$ for $k > 0$. For any $k$, the inequality becomes

$$
\frac{P'(D_{HQ})}{P'(D_{LQ})} = \frac{v(\alpha \tilde{c})}{v(\tilde{c})} \left( \frac{V^R - v(\tilde{c})}{V^R - v(\alpha \tilde{c})} \right) < \frac{v(\alpha \tilde{c})}{v(\tilde{c})} \left( \frac{V^R - v(\tilde{c})}{V^R - v(\alpha \tilde{c})} \right)^2.
$$

Thus, in this example, case (a) of Proposition 5 is the one that survives. The deterrence technology is so efficient that the low-quality equilibrium is an absorbing state.

### 3.5.4 The Level of Incentives

Up until now, the analysis has proceeded under the assumption that there are only two levels of effort, and that the regime always finds it optimal to give the managers high-powered incentives. I will now discuss the implications of letting effort be a continuous variable, such that there will be a continuum of incentive levels that can be offered to the managers. I will show that events that force the regime to raise the utility of the workers will also make the regime reduce the incentive levels for managers, as the cost of incentives increases. Thus, production becomes less efficient when the demands of the workers increase and the growth of output slows down.

Let there be a continuum of effort levels, s.t. $e \in [\underline{e}, \bar{e}] = E$. The cost of effort is a continuous, increasing and convex function $\psi(e)$. There are still only two possible output levels. The probability of the high output level is some function $p(e)$, with $p'(.) > 0$, with $p(\underline{e}) > 0$ and $p(\bar{e}) < 1$. For simplicity, I let $p(e) = e$. For a given contract offered by the regime, the managers choose the $e \in E$ that solves their maximization problem:

$$
e^* \in \arg\max_{e \in E} \{e v^m + (1 - e)v^w - \psi(e)\}.
$$
The necessary and sufficient optimality condition is given by

\[ \psi'(\epsilon^*) = v^m - v^w, \]

where the marginal cost of effort equals the marginal gain from effort, which is the utility differential between being a manager and a worker. For simplicity, I will assume that the cost-of-effort function is given by \( \psi(\epsilon) = \kappa \epsilon^2 \). The effort level induced by a certain incentive system can then be written as

\[ \epsilon^* = \frac{v^m - v^w}{\kappa}. \]

If we include these aspects into the above model, two basic but important points appear. These are stated in the two following Propositions.

**Proposition 6:** The ability of the regime to offer low quality products to the workers raises the optimal level of incentives for low levels of \( V^R \).

**Proof:** See Part A6 of the Appendix.

If it is true, as claimed by Trotsky, that the quality level of products offered to the workers declined after the introduction of Communism, this Proposition offers an explanation for such a development. The introduction of low-quality products lowers the price of incentives, and causes the regime to purchase more of them. As long as the workers pose little danger to the regime, the cost of treating the masses badly is low. Therefore, the regime can primarily care about production efficiency, and will in these circumstances raise the levels of incentives.

**Proposition 7:** Suppose there is a positive shock to \( V^R \). Then, the optimal level of incentives offered to managers will fall whenever the increase in \( V^R \) causes the regime to increase the utility of the workers.

**Proof:** See Part A7 of the Appendix.

This proposition is of general concern to all dictators, Communist or not.
3.6 DISCUSSION: LONG-RUN IMPLICATIONS

An increase in the outside option for the workers without any compensating increase in output will give the regime a double whammy. First, they will be forced to reward the workers more handsomely. Second, the effect of better conditions for workers is to improve the outside option for managers. This will increase the cost of incentives and make it optimal to reduce the incentives for the managers, thereby making production less efficient. Thus, the amount of output appropriable by the regime will fall both due to a drop in output as incentive levels fall and due to the higher costs of silencing the workers.

3.6 Discussion: Long-Run Implications

In the previous sections, I laid the foundations for a discussion of the long-run properties of centrally planned economies. Two issues must now be settled. First, the focus so far has been on characterizing optimal incentive systems in a utopian world, where the regime was essentially allowed to adopt the power and control provided by socialism with the ability to produce high-quality goods provided by capitalism. Now, it is time to bring reality back. One of the main drawbacks of socialist organization was its inability to produce high-quality goods on a large scale (see Section 3.3). In terms of the above model, this can be characterized by decreasing returns to scale in the high-quality goods sector. That is, the production function of high-quality products would be written as

\[ Y_g = S(\lambda_g)y_h, \]

with \( S(0) = 0, S(1) < 1, S' \geq 0 \text{ and } S'' \leq 0 \). Relative to the utopian model, this adds a new motive for the provision of low-quality products: They are cheaper to produce. The effect of this will be that the regime will phase in the provision of high-quality products to workers more slowly than in the utopian model. Note, however, that up until the point where the utopian regime begins to phase in
high-quality products, the incentive schemes of the real and utopian regimes will be identical. Importantly, in the initial phase of development, the regime does not suffer much from being restricted to the socialist production technology, as it is less exposed to its weaknesses.

Second, I have characterized how the optimal incentive scheme evolves as the revolutionary outcome for the workers, $V_R$, increases, but little has been said about the factors that affect its level. Naturally, $V_R$ is intimately related to basic economic factors. In particular, two factors seem essential. First, it is reasonable to assume that it is an increasing function of the economy’s productive potential, in particular the capital stock. In the event that the regime is overthrown, a higher level of the capital stock implies that the expected consumption for workers will be higher. Second, $V_R$ is an increasing function of the workers’ knowledge of the relative superiority in wealth and social organization of the rest of the world. I will capture this by letting the variable $A^*$ denote the workers’ knowledge of the relative attractiveness of organizing the economy according to capitalist rather than communist principles.\textsuperscript{21} With these two factors in mind, I write $V_R = V_R(K, A^*)$, where $V_R$ is an increasing function of both elements.\textsuperscript{22}

The dependence of $V_R$ on $K$ makes it more costly to keep the payoff for the workers down in a capital-abundant economy. In particular, it becomes harder to enforce the equilibrium with low-quality products only, and the share of output going to the regime will decline. The regime is squeezed from two sides. First, the increasing demands of the population make it more costly to make them quiet. Second, higher consumption for the workers implies that the cost of incentive

\textsuperscript{21}The prediction will be that all else equal, countries close to the West will have to invest more heavily in deterrence or reward workers with higher levels of consumption in order to satisfy the NRC (given that proximity to the West makes it more costly to prevent citizens from obtaining information about $A^*$). The relative size of the Stasi and the KGB is consistent with such a prediction.

\textsuperscript{22}One can argue whether $V_R$ is increasing in the entire capital stock or only in the part that is invested in the high-quality sector. It might be the case that the capital invested in the low-quality sector is completely useless ex post, as no products stemming from this sector can ever be sold in free markets and because it might be very costly to upgrade the facilities. If so, this will make it even more beneficial for the regime to invest in low-quality goods. Even though this scenario is quite plausible, I will abstract from this here, and simply assume that $V_R$ increases in the aggregate level of $K$. 

provision increases.

In Proposition 5, we saw above that if the deterrence technology is sufficiently effective, the regime will not want to provide high-quality goods to the workers for any $V^R$. Naturally, the history of communism in Eastern Europe is not completely uniform but the general picture seems to be that a (forced) willingness to improve product quality started to arise around 1970 (see Section 3.3). This is an indication that the low-quality equilibrium had become too costly to sustain, and that a movement from the type-LQ allocation towards the type-HQ allocation was the optimal response by the regimes. Thus, for these countries, I will take case (b) in Proposition 5, where it is optimal to phase out low-quality products as $V^R$ grows, to be the relevant one.

As the regimes tried to make a transition to the high-quality allocation, they were exposed to the true weaknesses of socialism. Suddenly, the regime wants to behave as if it were benevolent, and introduce high-quality goods to the mass market. But the regime will be disappointed in how hard it is to make this transition. Evidence from the Eastern European countries as Communism collapsed indicates that the efforts to improve quality were more or less a complete failure.

In light of this, imagine the following situation: Consider an economy where the regime allocates its share of output to consumption and investment in each period. It also optimally solves the incentive problem on the production side. Since I assume case (b) in Proposition 5 to be the relevant one, at some level of $K$ (for a given $A^*$) the regime wants to introduce high-quality goods to the workers. At this point, a share $\lambda_b$ of the production capital is allocated to the low-quality sector. Assume that $S(\cdot)$ has the following extreme shape:

$$S(1 - \lambda_b) = \begin{cases} 1 - \lambda_b & \text{if } \lambda_b \geq \hat{\lambda}_b \\ 1 - \hat{\lambda}_b & \text{if } \lambda_b < \hat{\lambda}_b \end{cases}.$$  

In other words, due to e.g. imports of high-quality products (financed by oil revenues) and the limited scale of the high-quality sector, there are no efficiency losses up until the point where the utopian regime would want to phase in the
high-quality products. The cost of further expanding the high-quality sector is prohibitive, however.

This implies that system failures prevent the regime from making the desired transition, so if the consumption level to workers is to increase, it must take place by further increases in the supply of low-quality products. This is a very costly way of increasing the workers’ utility level. Given this restriction on the regime’s possibilities, one can then study how the maximum level of output that the regime can appropriate evolves as the capital stock of the economy grows. Assume that capital accumulation only takes place through investments by the regime.\(^{23}\) Let capital depreciate by a factor \(\delta \in (0, 1)\). Finally, I let the regime discount the future by a factor \(\beta\).

In this setting, there will be a regime production function that I will denote by \(Y_r(K)\), which incorporates the nature of the optimal contract at this level of capital. This production function will be given by total output from the high-quality sector, minus wage costs for managers and the high-quality products given to workers and unlucky managers. This can be formally written as

\[
Y_r(K) = \lambda_y(K)y_h(K) - p_h c^m_y(K) - N_2 c^m_y(K) .
\] (3.18)

The capital accumulation equation is then

\[
K_{t+1} = Y_r(K_t) - c_r(K_t) + (1 - \delta)K_t ,
\] (3.19)

where \(c_r\) is the consumption level of the regime. Given these assumptions, the regime will optimally want to accumulate capital until the point where the following version of the Ramsey condition is satisfied:

\[
\beta (1 + Y_r'(K) - \delta) = 1. \tag{3.20}
\]

\(^{23}\)It is evident that consumers will save nothing as the consumption levels handed out by the regime will be increasing over time (as the capital stock will grow over time). Also assume that managers, who fears being demoted, cannot save, as the managerial rewards are given in kind and are non-storable (alternatively, they are afraid of expropriation).
3.6. DISCUSSION: LONG-RUN IMPLICATIONS

Since the regime ignores all positive benefits of an increase in capital accumulation for workers and managers, the economy always stabilizes at a level with too little capital (in the high-quality sector) compared to the market outcome, ceteris paribus.

At the level of $K$ when the regime would ideally want to introduce high-quality products, it is instead forced to optimally balance an increase in the supply of low-quality products with an increase in military spending. At this stage, this has turned into being a costly way of respecting the workers' no-revolution constraint. As long as $V^R$ increases sufficiently rapidly in $K$ (alternatively, if $A^*$ also tends to increase over time), it is not difficult to come up with examples where the production function of the regime will eventually be a decreasing function of the capital stock of the economy. It becomes so costly to deter the workers and buy their loyalty with the low-quality products that less and less will be left for regime consumption, even as the wealth of the economy increases. Such a development is illustrated in Figure 8. The solid line in the picture represents the regime output when it is restricted to offer the workers low-quality products. The regime will stop accumulating capital and shut down the growth of the economy at some point before point A in the figure is reached, where the Ramsey condition holds. This level will be highly inefficient, as much of the resources will be spent on deterrence and the production of low-quality products.

By recapitulating some of the evidence from Section 3.3, I will substantiate the claims of this theory. In Section 3.3, it was shown that an incentive system built around quality differentiation was actively used by the regimes. Its optimality for Communist countries in the initial stages of central planning is plausible given that the population as a whole was quite poor at this point, as indicated by the initial low levels of life expectancy and the complete lack of effort by the regimes to improve product quality. At this stage, the supply of labor was responsive to the provision of basic consumption goods, making it relatively less useful to purchase utility for the workers by substituting fewer high-quality goods for a given amount of low-quality goods. The introduction of an inefficient product
mix in the economy made it possible to create incentives inexpensively, and the regime could command a greater proportion of output than what would otherwise have been the case. Given that this was an objective, it could also industrialize more rapidly than it could otherwise have done. This explains the initial rapid growth. Popular demand increased over time, making this tradeoff less favorable for the existing arrangements. Whereas physical health as captured by life expectancy leveled out in the 1960s, periods of quality reforms started around 1970. This sudden attention to quality indicates that a shift away from the low-quality incentive system was optimal. However, the efforts were frustrated by the inefficiencies of the system. Instead, lots of resources had to be spent on silencing the workers with even more low-quality products and deterrence activities. There are two main reasons why this would cause the efficiency level of the economies to decline over time. First, since the workers had to be silenced in an inefficient manner, more resources had to be allocated to deterring and silencing them. Second, the gradual increase in the costs of providing incentives made it optimal to decrease the incentive level for managers. Data show that the turnover among government officials fell over time, indicating that the incentive levels indeed became weaker.\footnote{Another option that was used to reduce the discontent of the masses, and which probably only served to raise the awareness of the inferiority of central planning and thereby exacerbate the problems, was to allow markets to operate in a limited fashion, either formally or underground (see Grossman, 1977, Kornai, 1992, and Matthews, 1978).}

Further, Bergson’s (1984) estimates of income inequality in the Soviet Union indicate that inequality started to decline from the 1960s and onwards, which is also an indication that economy-wide incentive levels were declining. Especially from the middle of the 1970s and onwards, the performance of these economies deteriorated rapidly.

### 3.7 Conclusion

A fundamental challenge in centrally planned systems is how to design incentives in the absence of profit opportunities stemming from decentralized market transactions. In this paper, I have analyzed how a self-interested regime would...
face this challenge and, in particular, what the relationship between the optimal and feasible incentive system is.

I have shown how the instruments available to the regime interact with the constraints it faces to make the cost of incentive provision low at early stages of economic development and gradually higher as the economy develops. This increasing cost of incentive provision can, in itself, give an explanation for the path of economic development of Communist countries. It is also a complement to the theories of Berliner (1978) and Acemolgu et al. (2006) which emphasize that the importance of proper incentives grows over time. These two forces, the increasing cost of incentives and the increasing importance of incentives, reinforce each other, thus exacerbating the difficulties of central planning in the long run.
3.8 Appendix

A1: Proof of Proposition 1

There are positive costs and no benefits associated with setting \( c_m^w > 0 \). This will make it unnecessarily costly to satisfy the incentive constraint of managers, while there are no effects on other constraints. Thus, I will at once set \( c_m^w = 0 \). Further, the dictator will not produce more low-quality goods than what is necessary, so we will have \((N + 1 - p_h)c_b^w = \lambda_b y_h\), or \( c_b^w = \frac{\lambda_b y_h}{(N + 1 - p_h)}\).

The regime’s problem (it is maximizing the sum of consumption of the two goods, where the high-quality product is valued at a multiple higher than the low-quality good) is then

\[
\begin{align*}
\text{max} \quad & \{ \lambda_b [y_h - y_k] + \alpha [(1 - \lambda_b) y_h - (N + 1 - p_h)c_g^w - p_h c_g^m] \} \\
\text{s.t.} \quad & \text{NRC (} \phi_1 \text{): } (1 - \beta (1 - P)) v \left( \frac{\lambda_b y_h}{N + 1 - p_h} + \alpha c_g^w \right) \geq \beta PV^R \\
& \text{IC (} \phi_2 \text{): } \Delta p \left[ v(\alpha c_g^m) - v \left( \frac{\lambda_b y_h}{N + 1 - p_h} + \alpha c_g^w \right) \right] \geq \psi(e_h) \\
& \quad (\mu): \lambda_b \geq 0.
\end{align*}
\]

The first-order conditions are then

\[
\begin{align*}
\lambda_b & : - \alpha y_h + \phi_1 \frac{(1 - \beta (1 - P)) y_h}{N + 1 - p_h} v'_w = 0 \\
\phi_2 & \Delta p \frac{y_h}{N + 1 - p_h} v'_w + \mu = 0 \\
c_g^w & : - \alpha (N + 1 - p_h) + \phi_1 (1 - \beta (1 - P)) \alpha v'_w - \phi_2 \Delta p \alpha v'_w = 0 \\
c_g^m & : - \alpha p_h + \phi_2 \alpha v'_m = 0.
\end{align*}
\]

This clearly shows that \( \phi_2 > 0 \), as we have \( \phi_2 = \frac{p_h}{v'_m} \) from eq. (3.23). Reformulate equations (3.21) and (3.22) as follows:

\[
\begin{align*}
\alpha & = \frac{v'_w}{N + 1 - p_h} [\phi_1 (1 - \beta (1 - P)) - \phi_2 \Delta p] + \mu' \\
(N + 1 - p_h) & = v'_w [\phi_1 (1 - \beta (1 - P)) - \phi_2 \Delta p].
\end{align*}
\]

These two equations imply that

\[
\frac{1}{\alpha} [\phi_1 (1 - \beta (1 - P)) - \phi_2 \Delta p] + \frac{\mu'}{\alpha} = v'_w [\phi_1 (1 - \beta (1 - P)) - \phi_2 \Delta p],
\]

so for \( \alpha > 1 \), the solution requires that \( \mu > 0 \), i.e. \( \lambda_b = 0 \). QED.
A2: Proof of Proposition 2

From the problem defined by the set of equations in (3.12), we can derive the following first-order conditions:

\[ c_w^c : \alpha nL^{n-1}L'y - \alpha N_2 + \phi_1 v'_w - \phi_2 v'_w + \mu_1 = 0 \]  
(3.24)

\[ c_g^w : \alpha nL^{n-1}L'y - \alpha N_2 + \alpha \phi_1 v'_w - \alpha \phi_2 v'_w + \mu_2 = 0 \]  
(3.25)

\[ c_m^m : -ph + \phi_2 v'_m = 0 \]  
(3.26)

First note that eq. (3.26) gives \( \phi_2 = \frac{ph}{v'_m} \). The set of variables we need to solve for is \( \{ c_w^w, c_b^w, c_g^m, \phi_1, \mu_1, \mu_2 \} \).

The NRC can be binding or slack in equilibrium (the regime might value labor so much that the workers will be rewarded with consumption levels making them strictly better off than by revolting).

(i) Slack NRC

Candidate 1: First, consider the case where the NRC is slack, such that \( \phi_2 = 0 \). From (3.24) and (3.25) we then get that \(-\phi_2 v'_w + \mu_1 = -\alpha \phi_2 v'_w + \mu_2 \), or \((\alpha - 1)\phi_2 v'_w = \mu_2 - \mu_1 \). We clearly cannot have both \( \mu_1 \) and \( \mu_2 \) positive since the implication will be that the workers get zero consumption, \( c_g^w = c_m^w = 0 \), and there will be no output. The only feasible solution then requires that \( \mu_2 > 0 \) as the left-hand side is positive. Thus, in this case, we can say for sure that \( c_b^w > 0 \) and \( c_g^w = 0 \). Whenever the NRC is slack, the workers will only consume low-quality products. Rewriting (3.24), we have

\[ \alpha nL^{n-1}L'y - \alpha N_2 - \frac{ph}{v'_m} v'_w = 0. \]  
(3.27)

This tells us that the marginal product of an additional unit of consumption to all workers should equal the resource cost for the regime, \( N_2 \), plus the additional cost of satisfying the incentive constraint, \( \frac{ph}{v'_m} v'_w \).

We have two unknowns (\( c_b^w \) and \( c_m^m \)) and two equations to determine these (the IC and eq. (3.27)). From the binding IC, we can find \( c_m^m \) as a function of \( c_b^w \). Eq. (3.27) then gives us \( c_b^w \). Using the resulting value for \( c_b^w \) we can check if, in fact, the NRC (eq. (3.12)) is slack. If it is not, we have to check for the solutions where the NRC is binding.

Let \( PV^R \) be defined such that the value of \( c_b^w \) that just satisfies the NRC, \( c_b^w(PV^R) \), coincides with the solution of eq. (3.27). For values of \( V^R \) below \( V^R \), \( c_b^w(V^R) \) will be lower than that satisfying eq. (3.27). And, vice versa, for values of \( V^R \) above \( V^R \), \( c_b^w(V^R) \) will be higher than that satisfying eq. (3.27). This implies that, all else equal, the no-revolution constraint will be binding for some set of values of \( V^R \) such that \( V^R \in [V^R, \infty) \).

(ii) Binding NRC

Now suppose that the NRC is binding, so that \( \phi_1 > 0 \). For this case, there are three different solutions: First, a solution where only low-quality products is offered. Second, interior solutions where the workers are offered a mix of the
two types of products, and third a solution where the workers are only offered high-quality products. From (3.24) and (3.25), we then get that

\[(\alpha - 1)v_w[p_h - \frac{1}{\nu_m} - \phi_1] = \mu_2 - \mu_1 \tag{3.28}\]

having used the expression for \(\phi_2\). The three possible cases are:

Candidate 2: \(\phi_1 < p_h \frac{1}{\nu_m} = \phi_2\). In this case, the shadow price associated with the IC is higher than that associated with the NRC. We will then have \(\mu_2 > 0\), so \(c_g^w = 0\) and \(c_b^w > 0\). Since the value for the regime of slackening the IC is higher than for the NRC, the regime will try to provide incentives in an inexpensive manner. This is done by providing low-quality goods to the workers. From the NRC we can pin down \(c_g^w\). The IC gives us \(c_g^m\). We then have to determine \(\mu_2\) and \(\phi_1\), using eqs. (3.24) and (3.25). We have \(\mu_2 = (\alpha - 1)v_w[\phi_2 - \phi_1]\), and 

\[\alpha \eta L^{\nu-1}L'y - \alpha N_2 + \alpha \phi_1 v_m - \alpha \phi_2 v_m + \mu_2 = 0.\]

Thus, 

\[|\phi_2 - \phi_1| = \frac{\alpha \eta L^{\nu-1}L'y - \alpha N_2}{v_m}\]

Plug this back into the first equation, which gives us 

\[|\phi_2 - \phi_1| = \alpha (\alpha - 1)(\eta L^{\nu-1}L'y - N_2).\]

For this solution, the NRC will be binding, and the regime will only provide the workers with low-quality products. We can see that the marginal product of consumption is still above its resource cost in equilibrium, as 

\[\alpha \eta L^{\nu-1}L'y - \alpha N_2 = v_w[\phi_2 - \phi_1] > 0.\]

Candidate 3: \(\phi_1 = p_h \frac{1}{\nu_m} = \phi_2\). In this case, both the NRC and the IC are binding, and the shadow price on the two constraints are the same. Since we cannot have both \(\mu_1\) and \(\mu_2\) positive, we get \(\mu_1 = \mu_2 = 0\). The marginal product of consumption is pinned down by the relation 

\[L^{\nu-1}L'y = N_2,\]

which says that the marginal product of an additional unit of consumption to all workers should equal the resource cost for the regime, \(N_2\). Note that the incentive constraint no longer affects the labor-supply decision of the regime. This is because the NRC is binding, so the cost of satisfying the incentive constraint cannot be affected; it is given by the NRC and the IC. This relationship for optimal labor supply pins down \(c_g^w + c_b^w\). Further, the NRC pins down \(c_b^w + \alpha c_g^w\), and using this information we can pin down \(c_b^w\) and \(c_g^w\) separately. \(c_g^m\) is then determined by the IC.

Candidate 4: \(\phi_1 > p_h \frac{1}{\nu_m} = \phi_2\). In this case, the value of slackening the NRC is higher than that associated with the IC. We will then have \(\mu_1 > 0\) and \(\mu_2 = 0\), so that \(c_g^w > 0\) and \(c_b^w = 0\). For this equilibrium, the regime is forced to set the volume of consumption at a level where the marginal product of consumption provided to the workers is lower than the resource cost. In equilibrium, we have 

\[|\phi_1 - \phi_2| = \frac{\eta L^{\nu-1}L'y - N_2}{v_m} > 0.\]

Next, to show that the evolution of compensation as \(V^R\) increases is as stated in the proposition, we only need to show that \((\phi_1 - \phi_2)\) is weakly increasing in \(V^R\).

Now, in the equilibria where the NRC is binding but only low-quality goods are provided, we have 

\[|\phi_2 - \phi_1| = \frac{\alpha (\eta L^{\nu-1}L'y - N_2)}{v_m}.\]

This gives us the following
derivative of the difference of the shadow prices:

\[
\frac{\partial(\phi_2 - \phi_1)}{\partial V^R} = \alpha \left( \frac{[(\eta - 1)L^{-1}(L')^2 + L''\eta L^{n-1}yv_w' - (\eta L^{n-1}L'y - N_2)v_w'']}{(v_w')^2} \right) \frac{d\bar{c}_w}{dV^R}.
\]

(3.29)

We have \( \frac{d\bar{c}_w}{dV^R} > 0 \) as the workers must be given a higher consumption level when the outside option improves. Thus, the sign of the derivative depends on the sign of the expression \( [(\eta - 1)L^{-1}L' + L''\eta L^{n-1}yv_w' - (\eta L^{n-1}L'y - N_2)v_w'] \). For \( \phi_1 \) to grow relative to \( \phi_2 \) as \( V^R \) grows, we need \( [(\eta - 1)L^{-1}L' + L''\eta L^{n-1}yv_w' - (\eta L^{n-1}L'y - N_2)v_w'] < 0 \), or

\[
-\frac{v_w'c_b'}{v_w'} < \frac{[(1 - \eta)L^{-1}(L')^2 - L''\eta L^{n-1}yv_w']}{(\eta L^{n-1}L'y - N_2)}.
\]

The left-hand side of this equation gives us the relative risk aversion coefficient of \( v(\cdot) \). The numerator of the right-hand side is positive, as is the denominator (as, by assumption, we are in the equilibrium where \( \phi_2 > \phi_1 \)). Thus, there exists some level of relative risk aversion, \( R'_c \), of \( v(\cdot) \) such that \( \phi_1 \) is increasing relative to \( \phi_2 \) as \( V^R \) increases whenever \( R_{c} < R'_c \).

Eventually, as \( V^R \) increases further, at some point we will have \( \phi_2 = \phi_1 \). Further increases of \( V^R \) will then only lead to a substitution of low-quality for high-quality goods. Eventually, however, \( V^R \) will reach a point \( V^{R_0} \) such that \( \eta L^{n-1}L'y - N_2 < 0 \), and we are then in the equilibrium where only high-quality products are offered. At this stage, we have \( (\phi_1 - \phi_2) = -\frac{\eta L^{n-1}L'y - N_2}{v_w'} \), so the derivative of the difference between the two shadow prices is given by

\[
\frac{\partial(\phi_2 - \phi_1)}{\partial V^R} = \left( \frac{[(\eta - 1)L^{-1}(L')^2 + L''\eta L^{n-1}yv_w' - \alpha(\eta L^{n-1}L'y - N_2)v_w'']}{(v_w')^2} \right) \frac{d\bar{c}_w}{dV^R}.
\]

As we have \( \eta L^{n-1}L'y - N_2 < 0 \), this expression will always be negative, so in this set of equilibria we will always have that \( \phi_1 \) is increasing relative to \( \phi_2 \) as \( V^R \) increases. Finally, let \( R_{c} = R'_c \). QED.
A3: Proof of proposition 3

Initially, we will solve the problem by assuming a fixed labor supply. The problem to be solved by the regime is now defined by

\[
\max_{\{\lambda_g, \lambda_b, c_g^w, c_g^m\}} \{\alpha[\lambda_g y_h - N_2 c_g^w - p_h c_g^m]\}
\]

s.t.

\[
\text{NRC (}\phi_1\text{): } (1 - \beta(1 - P(D))) v \left( \frac{\lambda_b y_h}{N_2} + \alpha c_g^w \right) \geq \beta P(D) V^R \\
\text{IC (}\phi_2\text{): } \Delta p \left[ v(\alpha c_g^m) - v \left( \frac{\lambda_b y_h}{N_2} + \alpha c_g^w \right) \right] \geq \psi(e_h)
\]

(\mu): \lambda_b \geq 0. \\
(\psi): c_g^m \geq 0. \\
D = (1 - \lambda_b - \lambda_g) y_h

Let \( v_w \) denote the utility of workers. We then have the following first-order conditions:

\[
\begin{align*}
\lambda_g : \alpha - \phi_1 \beta P'(D)[v_w - V^R] &= 0 \quad (3.30) \\
\lambda_h : \phi_1 [(1 - \beta(1 - P(D))) v'_{w} c_g^w - \beta P'(D)[v_w - V^R]] - \\
\phi_2 \frac{\Delta p}{N_2} v'_{w} + \frac{\mu}{y_h} &= 0 \quad (3.31) \\
c_g^w : -N_2 + \phi_1 (1 - \beta(1 - P(D))) v'_{w} - \phi_2 \Delta p v_{w} + \frac{\psi}{\alpha} &= 0 \quad (3.32) \\
\end{align*}
\]

From (3.33) we find that \( \phi_2 = \frac{p_h}{\Delta p v_{w}} > 0 \). Further, from (3.30) we see that \( \phi_1 = \frac{\alpha}{\beta P'(D)[v_w - V^R]} > 0 \). Now, suppose that \( \mu = 0 \). Then, we can write (3.31) as

\[
\frac{\alpha(1 - \beta(1 - P(D))) v'_{w}}{N_2 \beta P'(D)[v_w - V^R]} - \frac{v'_{w} p_h}{v_{m} N_2} = \alpha
\]

In (3.32) we have

\[
\frac{\alpha(1 - \beta(1 - P(D))) v'_{w}}{N_2 \beta P'(D)[v_w - V^R]} - \frac{v'_{w} p_h}{v_{m} N_2} = 1 - \frac{\psi}{\alpha}.
\]

We see that these two conditions are incompatible, as we have \( \psi \geq 0 \) by the Karush-Kuhn-Tucker theorem. Thus, there does not exist an equilibrium with positive production of the low-quality good when there are no labor-supply effects. QED.
A4: Proof of Proposition 4

Since the planner will never produce more low-quality goods than what will be given to the workers, we can write $c^w_b = \frac{\lambda_b y_h}{N_2}$. Moreover, assume that the regime ends up with positive consumption in equilibrium, so that the resource constraint for high-quality products will not be binding. We have the following problem:

$$\max_{\{\lambda_y, \lambda_b, c^w_g, c^m_g\}} \{\alpha [\lambda_g y_h - N_2 c^w_g - p_h c^m_g]\}$$

s.t.

NRC ($\phi_1$): $(1 - \beta (1 - P(D)))v \left(\frac{\lambda_b y_h}{N_2} + \alpha c^w_g\right) \geq \beta P(D)V^R$

IC ($\phi_2$): $v(\alpha c^m_g) - v \left(\frac{\lambda_b y_h}{N_2} + \alpha c^w_g\right) \geq \frac{\psi(e_h)}{\Delta p}$

LSC ($\phi_3$): $\frac{\lambda_b y_h}{N_2} + c^w_g \geq \bar{c}$

($\mu$): $\lambda_b \geq 0$.

($\xi$): $c^w_g \geq 0$.

$D = (1 - \lambda_b - \lambda_g)y_h$

Note that we cannot have both $\mu > 0$ and $\xi > 0$ simultaneously, since the workers would then get zero consumption. So either only one is positive, or both are zero. The first-order conditions are as follows:

$$\lambda_y : \alpha - \phi_1 \beta P'(D)[v - V^R] = 0 \tag{3.34}$$

$$\lambda_b : \phi_1 [(1 - \beta (1 - P(D))) \frac{1}{N_2} v'_w - \beta P'(D)[v - V^R]] -$$

$$\phi_2 v'_w N_2 + \phi_3 \frac{1}{N_2} + \mu y_h = 0 \tag{3.35}$$

$$c^w_g : -N_2 + \phi_1 (1 - \beta (1 - P(D))) v'_w - \phi_2 v'_w + \frac{\phi_3}{\alpha} + \frac{\xi}{\alpha} = 0 \tag{3.36}$$

$$c^m_g : -p_h + \phi_2 v'_m = 0 \tag{3.37}$$

From eq. (3.34) one finds $\phi_1 = \frac{\alpha \beta P'(D)[v - V^R]}{v - v(\bar{c})}$. With the assumption that $V^R > v(\bar{c})$ and $P(0) > \frac{1 - \beta}{\beta} v(\bar{c})$ the NRC will always be binding because this rules out an equilibrium where the workers are given the subsistence level in low-quality products and the dictator invests nothing in deterrence (the second condition ensures a violation of the NRC). From eq. (3.37) one also finds $\phi_2 = \frac{p_h}{v'_m} > 0$, so the IC will also be binding. Using this, eqs. (3.35) and (3.36) can be rewritten as follows:

$$\frac{\alpha (1 - \beta (1 - P(D))) v'_w}{N_2 \beta P'(D)[v - V^R]} - \alpha - \frac{p_h}{N_2} v'_w + \frac{\phi_3}{\alpha} N_2 + \frac{\mu}{y_h} = 0$$

$$\frac{\alpha (1 - \beta (1 - P(D))) v'_w}{N_2 \beta P'(D)[v - V^R]} - 1 - \frac{p_h}{N_2} v'_w + \frac{\phi_3}{\alpha} N_2 + \frac{\xi}{\alpha N_2} = 0,$$
Setting the two left-hand sides equal to each other, one finds

$$\frac{\mu}{y_h} - \frac{\xi}{\alpha N_2} = (\alpha - 1)[1 - \phi_3 \frac{1}{\alpha N_2}]$$

This equation gives four possible cases:

(i) $\phi_3 = 0$. Since we cannot have both $\psi$ and $\mu$ positive, this will imply that $\mu > 0$, and we are back to the solution of Proposition 3. The LSC is slack.

(ii) $0 < \phi_3 < \alpha N_2$. Then, we will still have $\mu > 0$, so that the workers consume high-quality goods at the subsistence level.

(iii) $\phi_3 = \alpha N_2$. This requires that $\mu = \xi = 0$. Here, the consumption bundle given to the workers will consist of both kinds of products.

(iv) $\phi_3 > \alpha N_2 > 0$. The solution will then require that $\xi > 0$ and $\mu = 0$, so there will be zero consumption of the high-quality good for the workers.

These are four different types of solutions, which I will now solve for. The set of variables to solve for is given by the tuple $\{\lambda_b, \lambda_g, \phi_3, c_g^m, c_g^w, \xi, \mu\}$.

Type S: Here $\phi_3 = 0$, so the LSC is not binding. Then, the workers only consume high-quality goods, so we have $\xi = 0, \mu = (\alpha - 1)y_h$, and $\lambda_b = 0$. From the IC, we have $\Delta \rho (v_m - v_w) = \psi(e_h)$, or, (i), $v_m = \frac{\psi(e_h)}{\Delta \rho}$ and from the NRC we have, (ii), $v_w = \frac{\beta \rho \rho (D - V)}{(1 - \beta (1 - P(D)))}$. Next, from the remaining FOC we get, (iii), $\frac{\alpha (1 - \beta (1 - P(D)))c_g^m - \rho h_v c_w^m}{N_2 \beta \rho \rho (D - V)} = 1$. We are left with three unknowns ($\lambda_g, c_g^m, \text{and } c_g^w$) and three equations ((i), (ii), and (iii)). These characterize the equilibrium. This equilibrium will occur if effort costs are low, quality differences are high, and deterrence is costly. Its feasibility depends on equations (ii) and (iii) returning a solution for $c_g^w$ greater than $\bar{c}$.

Type HQ: Here $\phi_3 > 0$ but small enough such that also $\mu > 0$. Then, $\xi = 0$, and $\lambda_b = 0$. Further $c_g^w = \bar{c}$. From the IC, we get $v(\alpha c_g^m) = \frac{\psi(e_h)}{\Delta p} + v(\alpha \bar{c})$, which gives us $c_g^m$. From the NRC, we can find $\lambda_g$. We have the following relationship between $\mu$ and $\phi_3$: $\mu = y_h(\alpha - 1)[1 - \phi_3 \frac{1}{\alpha N_2}]$. Inserting this expression for $\mu$ in the remaining FOC, we get $\phi_3 = \alpha N_2 + \alpha p_h, \frac{v_c}{v_p} - \frac{\alpha^2 (1 - \beta (1 - P(D)))}{\beta \rho \rho (D - V)} = 0$. These two equations allow us to find $\mu$ and $\phi_3$. Note that for the solution to be feasible s.t. $\mu > 0$, we need $\phi_3 < \alpha N_2$, or $\rho h_v \frac{v_c}{v_m} < \frac{\alpha (1 - \beta (1 - P(D)))}{\beta \rho \rho (D - V)}$, while for $\phi_3 > 0$ we need $\rho h_v \frac{v_c}{v_m} > \frac{\alpha (1 - \beta (1 - P(D)))}{\beta \rho \rho (D - V)} - N_2$.

Type MQ: Here we have $\phi_3 = \alpha N_2$, and $\mu = \xi = 0$. Further from the LSC we get, (i), $c_g^w = y_h N_2 = \bar{c} - c_g^w$, while the NRC and the IC then become, (ii), $v(\bar{c} + (\alpha - 1)c_g^w) = \frac{\beta \rho \rho (D - V)}{(1 - \beta (1 - P(D)))}$ and, (iii), $v(\alpha c_g^m) = \frac{\psi(e_h)}{\Delta p} + v(\bar{c} + (\alpha - 1)c_g^w)$. Finally, the remaining FOC gives us (iv), $\frac{\alpha (1 - \beta (1 - P(D)))}{\beta \rho \rho (D - V)} = p_h \frac{1}{v_m}$. These four equations characterize the equilibrium.

Type LQ: Here $\phi_3 > 0$ and is sufficiently large so that $\mu = 0$ and $\xi > 0$. Then $c_g^w = \bar{c}$ and we find $\lambda_b = \frac{N_2}{y_h}$. From the IC we get $v(\alpha c_g^m) = \frac{\psi(e_h)}{\Delta p} + v(\bar{c})$. Finally,
from the NRC we can find $\lambda_g$. We have the following relationship between $\xi$ and $\phi_3$: 

$$\xi = (\alpha - 1) [\phi_3 - \alpha N_2].$$

Inserting this in the remaining FOC, we get

$$\phi_3 = \alpha N_2 + p_h \frac{\nu}{\nu_m} - \frac{\alpha^2(1 - \beta(1 - P(D)))\nu}{\beta P^2(D)[V^R - \nu_w]}.$$ 

These two equations allow us to find $\xi$ and $\phi_3$. For this solution to be feasible (i.e. $\xi > 0$ and $\phi_3 > 0$) we need

$$p_h \frac{\nu}{\nu_m} > \frac{\alpha^2(1 - \beta(1 - P(D)))\nu}{\beta P^2(D)[V^R - \nu_w]}$$

(3.38)

Given that (3.38) is satisfied, there will exist an equilibrium where the workers are only offered low-quality products. From the NRC for Candidate 4, $D$ is given by $P^{-1} \left( \frac{(1 - \beta)v(c)}{\beta(V^R - \nu_w)} \right)$. Plugging this into (3.38), we can see that this equilibrium becomes more likely when $\alpha$ is not too large and when $\frac{\nu}{\nu_m}$ is large, that is, when the effort costs are high. QED.

### A5: Proof of Proposition 5

The consumption levels of the regime in the type-HQ and type-LQ allocations are given by (super- and subscripts HQ and LQ represent variables associated with allocations of type HQ and LQ, respectively)

$$c_{HQ}^L = \lambda_g^{HQ} y_h - N_2 \bar{c} - p_h c_{HQ}^m$$

$$c_{LQ}^L = \lambda_g^{LQ} y_h - p_h c_{LQ}^m,$$

where the $\lambda_g$’s are defined by the NRC, $P(D) = \frac{(1 - \beta)v(c)}{\beta(V^R - \nu_w)}$, and $c_{HQ}^m$ is defined by the equation $v(\alpha c_{HQ}^m) = v(\alpha \bar{c}) + \frac{v(\alpha \bar{c})}{\beta P^2(D)}$, where $\alpha_t = 1$ if we are in Type HQ and $\alpha_t = \alpha$ if we are in Type LQ. From the NRC, we find that

$$\lambda_g^{HQ} = 1 - \frac{1}{y_h} P^{-1} \left( \frac{(1 - \beta)v(\alpha \bar{c})}{\beta(V^R - v(\alpha \bar{c}))} \right)$$

and

$$\lambda_g^{LQ} = 1 - \frac{1}{y_h} P^{-1} \left( \frac{(1 - \beta)v(\bar{c})}{\beta(V^R - v(\bar{c}))} \right) - \lambda_b^{LQ}.$$ 

where we have $\lambda_b^{LQ} = \bar{c} N_{HQ}/y_h$. The difference in the consumption levels is then

$$c_{LQ}^L - c_{HQ}^L = (\lambda_g^{LQ} - \lambda_g^{HQ}) y_h + N_2 \bar{c} + p_h [c_{HQ}^m - c_{LQ}^m],$$

where

$$\lambda_g^{LQ} - \lambda_g^{HQ} = \frac{1}{y_h} \left[ P^{-1} \left( \frac{(1 - \beta)v(\alpha \bar{c})}{\beta(V^R - v(\alpha \bar{c}))} \right) - P^{-1} \left( \frac{(1 - \beta)v(\bar{c})}{\beta(V^R - v(\bar{c}))} \right) \right] - \lambda_b^{LQ}. $$

For $V^R \rightarrow v(\bar{c})$, we will have $(\lambda_g^{LQ} - \lambda_g^{HQ}) y_h + N_2 \bar{c} \rightarrow -\lambda_b y_h + N_2 \bar{c} = 0$ as $D_{HQ} \rightarrow 0$ and $D_{LQ} \rightarrow 0$. In this case, we will have $c_{LQ}^L - c_{HQ}^L > 0$ as there
will be essentially no extra cost of satisfying the NRC in the low-quality-good allocation, while the reduced cost of the IC is fixed at a positive level \( p_h[c_{HQ}^m - c_{LQ}^m] \). Thus, for sufficiently low levels of \( V^R \), the low-quality-good allocation will always dominate. To determine which allocation becomes relatively more attractive as \( V^R \) increases, it is sufficient to sign the following derivative (where I have used the inverse function theorem):

\[
\frac{\partial (\lambda_y^{LQ} - \lambda_y^{HQ})}{\partial V^R} = \frac{1 - \beta}{\beta Y_h} \left[ -\frac{1}{P'(D_{HQ}) (V^R - v(\alpha \tilde{c}))^2} + \frac{1}{P'(D_{LQ}) (V^R - v(\tilde{c}))^2} \right].
\]

This derivative is negative if (this needs to be the case if the allocation with good products is to become more attractive as the outside option of the workers improves):

\[
-\frac{1}{P'(D_{HQ}) (V^R - v(\alpha \tilde{c}))^2} < -\frac{1}{P'(D_{LQ}) (V^R - v(\tilde{c}))^2} \left( \frac{P'(D_{HQ})}{P'(D_{LQ})} > \frac{v(\alpha \tilde{c})}{v(\tilde{c})} \left( \frac{V^R - v(\tilde{c})}{V^R - v(\alpha \tilde{c})} \right)^2 \right) \tag{3.39}
\]

where \( D_j \) is defined by \( P^{-1} \left( \frac{(1 - \beta) v(\alpha \tilde{c})}{\beta (V^R - v(\alpha \tilde{c}))} \right) \), s.t. \( \alpha_j = \alpha \) for \( D_{HQ} \) and \( \alpha_j = 1 \) for \( D_{LQ} \). We know that \( D_{LQ} > D_{HQ} \), so, as \( P(\cdot) \) is a decreasing and convex function we have \( P'(D_{LQ}) < P'(D_{HQ}) \). Thus, the left-hand side of eq. (3.39) is greater than 1. Further, \( V^R - v(\tilde{c}) > V^R - v(\alpha \tilde{c}) \), so that also the right-hand side is greater than 1. Therefore, how the optimal allocation changes as \( V^R \) increases will depend on the deterrence technology.

Suppose now that \( \frac{\partial (\lambda_y^{LQ} - \lambda_y^{HQ})}{\partial V^R} < 0 \). Then there exists \( \tilde{V}^R \) s.t. for \( V^R < \tilde{V}^R \) we have \( c_{LQ}^m - c_{HQ}^m > 0 \), while for \( V^R > \tilde{V}^R \) the opposite holds. In some intermediate range, however, the allocation will be of type MQ. To check that a similar condition to eq. (3.39) also holds in this case, consider an allocation of type MQ associated with some level of high-quality consumption \( c_g^u \in (0, \tilde{c}) \). The difference in consumption for the regime between the type-MQ allocation and the type-LQ allocation is given by

\[
c_{MQ} - c_{LQ}^m = D_{LQ} - D_{MQ} + p_h[c_{LQ}^m - c_{MQ}^m],
\]

where \( D_{LQ} - D_{HQ} \) gives the difference in deterrence costs, and \( p_h[c_{LQ}^m - c_{MQ}^m] \) is the difference in the cost of satisfying the incentive constraint. We have \( D_{MQ} = P^{-1} \left( \frac{(1 - \beta) v(c_g^u)}{\beta (V^R - v(c_g^u))} \right) \) and \( D_{LQ} = P^{-1} \left( \frac{(1 - \beta) v(\tilde{c})}{\beta (V^R - v(\tilde{c}))} \right) \). Now, if any allocation of type MQ is to dominate an allocation of type LQ, we need \( D_{LQ} - D_{MQ} \) to grow
as $V^R$ grows. The derivative is

$$\frac{\partial (D_{LQ} - D_{MQ})}{\partial V^R} = \frac{1 - \beta}{\beta} \left[ -\frac{1}{P'(D_{LQ})} \frac{v(\tilde{c})}{(V^R - v(\tilde{c}))^2} + \frac{1}{P'(D_{MQ})} \frac{v(\alpha c^w_g + (\tilde{c} - c^w_g))}{(V^R - v(\alpha c^w_g + (\tilde{c} - c^w_g)))^2} \right],$$

which is positive if

$$\frac{P'(D_{MQ})}{P'(D_{LQ})} > \frac{v(\alpha c^w_g + (\tilde{c} - c^w_g))}{v(\tilde{c})} \left( \frac{V^R - v(\tilde{c})}{V^R - v(\alpha c^w_g + (\tilde{c} - c^w_g))} \right)^2,$$

which is the equivalent condition to eq. (3.39). A similar argument can be given to find the relationship between the type-S allocation relative to the other allocation. Given the assumption in eq. (3.17), these results imply that the relative attractiveness of the various allocations changes monotonically as $V^R$ increases. For some allocations $z \succ x$, if $z$ improves relative to $x$ when $V^R$ increases, some $z' \succ z$ will improve relative to both $x$ and $z$. On the other hand, if $x$ improves relative to $z$, then $z$ will improve relative to $z'$, and also $x$ will also improve to $z'$. The result follows from this. QED.

**A6: Proof of Proposition 6**

I will only prove this for the case analyzed in proposition 4, and when the workers are only given low-quality products. Thus, $c^w_b = \tilde{c}$ and $c^w_g = 0$. In this case, we have $\mu = 0$, and $\xi > 0$. After some manipulations, the problem reduces to

$$\max_{\{c^m_g\}} \left\{ e \left[ 1 - \frac{(N + 1 - e)c}{ey_h} \right] y_h - c^m_g \right\} \tag*{\dagger}$$

$$\max_{\{c^m_g\}} \left\{ ey_h - (N + 1 - e)\tilde{c} - c^m_g \right\} \tag*{\dagger}$$

$$\max_{\{c^m_g\}} \left\{ \frac{v^m - v^w}{\kappa} y_h - (N + 1 - \frac{v^m - v^w}{\kappa})\tilde{c} - c^m_g \right\} \tag*{\dagger}$$

$$\max_{\{c^m_g\}} \left\{ \frac{v^m}{\kappa} (y_h + \tilde{c}) - c^m_g \right\}.$$

We get the FOC

$$\frac{\alpha}{\kappa} (y_h + \tilde{c}) v'_m = 1.$$
This implies that we have
\[
dc_g^m \over d\alpha = c \frac{1 - R_r(\alpha c_g^m)}{\alpha R_r(\alpha c_g^m)}.
\]
Thus, we have
\[
de^* \over d\alpha = \frac{v'_m c_g^m}{\kappa R_r(\alpha c_g^m)} > 0.
\]
In other words, equilibrium effort is an increasing function of \(\alpha\). This implies that the quality differentiation in rewards enables the regime to increase the level of incentives. One can easily extend the result to show that this will be the case as long as it is optimal to reward workers with at least some low-quality products. QED.

A7: Proof of Proposition 7

The effort level exerted by the managers is given by \(e = \frac{v^m - v^w}{\kappa}\). Thus, in order to show that the effort level drops, it is sufficient to show that \(d(v^m - v^w) < 0\).

Consider the model without military spending. The regime gets a consumption level \(e((1 - \lambda_b)y_h - c_g^m + c_g^w) - (N + 1)c_g^w\), where I have used that \(p(e) = e\), which gives us
\[
\frac{de}{dc_g^w} = -\frac{1}{\kappa} \frac{v'}{v'} \frac{de}{dc_g^w} = -\frac{\alpha}{\kappa} \frac{v'}{v'} \frac{de}{dc_g^m} = \frac{1}{\kappa} v'
\]
As before, only workers and failed managers will consume low-quality goods, if any, so from the resource constraint for the low-quality goods, we have \(p_b(e)\lambda_b y_h = (N + 1 - p_b(e))c_b^w\). Thus, \(\lambda_b = \frac{(N+1-e)c_b^w}{eY_h} = \lambda_b(c_b^w, e)\). This gives us
\[
\frac{d\lambda}{de} = -\frac{c_b^w e y_h - y_h(N + 1 - e)c_b^w}{e^2 y_h^2} = -\frac{(N + 1)c_b^w}{e^2 y_h^2}
\]
The planner has to choose \(c_b^w, c_g^w, c_g^m\).

The problem faced by the regime is the following:
\[
\max_{\{c_b^w, c_g^w, c_g^m\}} \left\{ \alpha \left[ e \{(1 - \lambda_b(c_b^w, e)) y_h - c_g^m + c_g^w\} - (N + 1)c_g^w \right] \right\}
\]
s.t.
\[
\text{NRC (}\phi_1\text{): } v(c_b^w + \alpha c_g^w) \geq \frac{\beta P}{1 - \beta(1 - P)} V^R
\]
\[
\text{LSC (}\phi_3\text{): } c_b^w + c_g^w \geq \bar{c}
\]
\[
(\mu): c_b^w \geq 0.
\]
\[
(\xi): c_g^w \geq 0.
\]
The optimality conditions are as follows:

\[
  c^w_b : \alpha \frac{de}{dc^w_b} \{ \cdot \} - \alpha e \frac{d\lambda_b}{dc^w_b} y_h - \alpha e \frac{d\lambda_b}{dc^w_b} y_h + \phi_1 v'_w + \phi_3 + \mu = 0
\]

\[
  c^w_g : \alpha \frac{de}{dc^w_g} \{ \cdot \} - \alpha e \frac{d\lambda_b}{dc^w_g} y_h - \alpha (N + 1 - e) + \phi_1 \alpha v'_w + \phi_3 + \xi = 0
\]

\[
  c^m_g : \frac{de}{dc^m_g} \{ \cdot \} - e \frac{d\lambda_b}{de} y_h = e. \tag{3.40}
\]

Using the above derivatives, we can rewrite eq. 3.40 as

\[
  v^m - v^w = \frac{y_h + \bar{c}}{\alpha c^m_g} + \frac{c^w_b + c^w_g}{\alpha c^m_g} - \frac{1}{\alpha}. \tag{3.41}
\]

Consider now the consequences of a positive shock to \( V^R \). First, note that if the no-revolution constraint does not bind, the regime will not have to change the rewards given to workers. Thus, there will be no effects on optimal incentives for managers either. Next, consider the Type-(iii) equilibrium, where workers are offered a mix between high and low quality products, with their level of consumption given by \( \bar{c} \). Let the share of low-quality products in the workers’ bundle be denoted by \( \rho \), such that the workers have a utility level \( v(\alpha \bar{c} + \rho (\alpha - 1) \bar{c}) \).

We can rewrite eq. 3.41 as

\[
  v(c^m_g) - v(\alpha \bar{c} + \rho (\alpha - 1) \bar{c}) = \frac{y_h + \bar{c}}{\alpha c^m_g} - \frac{1}{\alpha}. \tag{3.42}
\]

From the binding no-revolution constraint, we can see that a change in \( V^R \) gives the following change in utility to the workers

\[
  \frac{dv_w}{dV^R} = \frac{\beta P}{1 - \beta (1 - P)}
\]

Differentiating eq. 3.42 wrt. \( c^m_g \) and \( V^R \), I get

\[
  \alpha v'_m dc^m_g - \frac{dv_w}{dV^R} dV^R = -\frac{y_h + \bar{c}}{\alpha (c^m_g)^2} dc^m_g,
\]

which implies that

\[
  \frac{dc^m_g}{dV^R} = \frac{\frac{\beta P}{1 - \beta (1 - P)}}{\frac{y_h + \bar{c}}{\alpha (c^m_g)^2} + \alpha v'_m}.
\]

The change in utility for the manager is therefore

\[
  \frac{dv_m}{dV^R} = \alpha v'_m \frac{dc^m_g}{dV^R}.
\]
Therefore, we get

\[
\frac{dv_m}{d\bar{V}^R} - \frac{dv_w}{d\bar{V}^R} = \alpha v_m' \frac{de_m}{d\bar{V}^R} \frac{\beta P}{1 - \beta(1 - P)} = \alpha v_m' \frac{\beta P}{1 - \beta(1 - P)} \left[ \frac{1}{\varphi \alpha c_y^{\delta} + 1} - 1 \right].
\]

Clearly, we have \( \frac{1}{\varphi \alpha c_y^{\delta} + 1} - 1 < 0 \), which means that \( \frac{dv_m}{d\bar{V}^R} - \frac{dv_w}{d\bar{V}^R} < 0 \). In other words, the optimal level of incentives declines after such a shock. The proof for the other allocations is analogous. QED.

A8: Tables and Figures

<table>
<thead>
<tr>
<th>Table 1: Average Life Expectancy at Birth</th>
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<tr>
<td><strong>Russia/Soviet Union</strong></td>
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<tr>
<td><strong>Year</strong></td>
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<tr>
<td>1896</td>
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<td>1926</td>
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<td>1969</td>
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* The data from 1896 is for Russia only.  
Sources: Pressat (1985) for Russia/Soviet Union, Nolte et al. (2000) for Poland.
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Figure 1. Source: Allen (2006)

Figure 2. GDP per capita for Eastern Europe and the USSR relative to the USA.
CHAPTER 3. INCENTIVES UNDER COMMUNISM

Annual Growth Rates Between 1950 and 1975 Given GDP per Capita in 1950
(Removed Persian Gulf Oil Countries)

Data from Maddison (2003)

Eastern Europe
USSR

Figure 3. Data from Maddison (2003), 1990 Geary-Khamis Dollars.

Annual Growth Rates Between 1975 and 1989 Given GDP per Capita in 1975
(removed Persian Gulf Oil Countries)

Data from Maddison (2003)

Eastern Europe
USSR

Figure 4. Data from Maddison (2003), 1990 Geary-Khamis Dollars.
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**Figure 5.** The incentive scheme under constant labor supply

**Figure 6.** An illustration of the four types of solutions for optimal consumption bundles to the masses.
Figure 7. The optimal proportion of high-quality goods awarded to the masses across levels of $V^R$. The figure illustrates the two alternative regimes described in Proposition 5.
Figure 8. The upper envelope of the three lines represents the amount of output that the regime can appropriate when it is unconstrained in its choice of incentive systems. The solid line (LQ) shows the output appropriated by the regime when only low-quality products is provided. The dotted line (MQ) represents the regime output for the interval of capital when a mix between high-and low-quality products is optimal, while the dashed line (HQ) shows the regime’s output when only high-quality products are offered to both workers and managers.
Chapter 4

The Business of Troubled Autocrats

4.1 Introduction

Many autocrats control resource rents, for instance those of the oil-producing countries in the Middle East and elsewhere. These regimes typically rely on resource revenues in order to buy political peace. In this paper, I study how autocrats behave in the product and capital markets, given this political constraint.

The paper is inspired by developments in one such country, namely the Kingdom of Saudi Arabia. There are four main developments in this country that I bring together in the analysis. First, from the early 1980s until the late 1990s, the Saudi government ran large budget deficits, in part due to lower oil prices, but also due to expenses related to the Iran-Iraq war and Gulf wars. As the oil price collapsed in 1998, the public debt of Saudi Arabia reached a level of 120% of GDP. Second, data from the Saudi Arabia Monetary Agency show that oil revenues have consistently constituted about three-fourths of the revenues of the Saudi government. Thus, without other government assets to compensate for a fall in oil revenues, government spending must be cut. The Saudi regime has relied on transfers to the citizens to preserve power (see e.g. Amuzegar [1998] Jaffe

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1I am grateful to Torsten Persson for advice and comments and Christina Lönnblad for editorial assistance. Financial support from the Jan Wallander and Tom Hiedelius Foundation and the Mannerfelt Foundation is gratefully acknowledged. All remaining errors are mine.

2This is an estimate made by SAMBA, The Saudi American Bank.
and Manning [2000] and Gause [2000]). As public finances deteriorated during
the late 1990s, transfers fell, and there were fears of a collapse of the Saudi regime
(see e.g. Gause [2000] and Rouleau [2002]). Third, Saudi Arabian behavior has
typically been a guarantee to prevent skyrocketing oil prices, drawing on the
country’s significant spare oil production capacity. However, as the financial
situation of the regime worsened, there was a substantial drop in this excess
production capacity and Saudi Arabia advocated cuts in OPEC production (see
e.g. Gause [2000]). Fourth, public debt has solely been financed from domestic
sources, and the share of foreign assets in the portfolios of Saudi institutional
investors fell from 50% in 1995 to 10% in 2000 (Mihaljek et al. [2002]). Saudi
retail investors, on the other hand, are very reluctant to hold long-term govern-
ment debt (Al-Jasser and Banaf [2002]). Interestingly, debt is purely domestic
also in other oil-producing countries (see e.g. Fouad et al. [2007]).

With these developments in mind, the objective of this paper is to construct
a theory that provides answers to the following questions: How does the as-
set position of an autocrat affect his behavior as a producer in a market with
monopoly rents? What are the implications for prices? From whom does the
autocrat obtain financing in order to get out of difficulties? In particular, is it
possible for the autocrat to exploit those who have a stake in his regime in order
to get a debt contract with better terms than in a perfectly competitive capital
market?

I construct a model where the autocrat is a monopolist producer. Demand
is stochastic, so the level of profits for a given supply is uncertain. The survival
of the autocrat depends on the level of transfers to the citizens, and the likeli-
hood that the autocrat keeps power is an increasing function of these transfers.
However, the uncertainty of profits implies that the transfers the autocrat can
give the citizens in each period are uncertain. I analyze how this uncertainty
affects the production behavior of the autocrat, depending on his initial asset
position. The main result is that the autocrat behaves as a regular monopolist

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3 One reason for this is to prevent substitution to alternative sources of energy (see e.g. Gause [2000]).
above a certain threshold level of assets, while he is restricting output to below the monopoly level when he is below that threshold. The intuition is straightforward. When the level of assets is low, the autocrat foresees that in the future, he will not be able to provide the citizens with the level of transfers he wishes. This is especially true if the realization of demand in the product market is low. This implies that the marginal value of profits for the autocrat is higher when demand is low than when it is high, which further implies that states with low demand receive a greater weight in the production decision, which makes it optimal to restrict output below the level of a regular monopolist.

Next, I study the question of who the autocrat chooses as a counterpart for public debt. In the model, the autocrat can approach either a regular competitive capital market or a group of supporters of the regime. The income of these supporters depends on the survival of the regime, and I study whether the autocrat can exploit this dependence in order to obtain funding at a lower expense from them than from the competitive capital market. I show that if the supporters can overcome collective action problems, and if their stake in the continued existence of the regime is sufficiently large, the autocrat will, in fact, strike better deals with the supporters and only domestic debt is observed in equilibrium.

This paper is related to the literature on the workings of resource cartels (see e.g. Alhajji and Huettner [2000] and Mason and Polasky [2005]). The results of this paper suggest that it is important to understand the interactions between political and product markets, at least when the success of a politician in the political market depends on his behavior in the product market. For instance, some authors argue that the capacity and production choices of OPEC members are characterized by risk aversion (see e.g. Reynolds (1999)). In this paper, I show how such risk-averse behavior might arise endogenously due to influences from the political arena. Sinn (2008) argues that one needs to take the response of the supply side into account when designing policies aimed at reducing the demand for polluting exhaustible resources. In particular, he is interested in policies that provide the suppliers with incentives to reduce the
speed of extraction. The results of this paper suggests that autocrats behave as Sinn wishes only when they are in financial difficulties. When this is the case, the autocrats in charge of supply reduce output to make sure that they stay in power even when demand is low.

The paper is also related to the literature on the behavior and survival of autocratic regimes. Acemoglu, Robinson and Verdier (2004) and Padro-i-Miquel (2007) try to explain why autocrats who produce inefficient economic outcomes often survive politically for long periods of time. They argue that ethnic or other divisions in the population make it possible for the ruler to play different groups against each other. The mutual fear that the opposing group will grab power when the ruler is ousted enables the ruler to exploit the supporting group. In this paper, I study societies with access to resource rents. When the regime faces difficulties, it can exploit the vested interests of those whose rents depend on the political survival of the regime.

The paper is also related to soft budget constraints in the literatures on central planning and corporate finance (Kornai [1986], Dewatripont and Maskin [1995]). Dewatripont and Maskin show that lack of commitment at the refinancing stage of projects implies that centralized systems are less efficient in weeding out bad projects at the initial stage (as the entrepreneurs, who have private information about the project quality, know that they will get refinancing at later stages). In my paper, those who have a vested interest in the autocrat’s regime end up "refinancing" the autocrat on terms that are advantageous for the autocrat. In the final section of the paper, I discuss how my model might be useful for studying issues in corporate finance.

Finally, the paper is related to the literature on sovereign debt. The model I construct is closely related to that of Gertler and Rogoff (1990), but applied and extended to a new setting. Typically, large parts of the debt of emerging economies are denominated in foreign currencies, potentially contributing to the instability of the economies (Eichengren and Hausmann [1999]). The theory of this paper, where I argue that the composition of debt (domestic vs. foreign) is
affected by characteristics of the political landscape, complements the theories on lack of monetary credibility (see e.g. Jeanne [2005]). The paper is also related to Drazen (1998), which sets up a model where a politician can be punished politically by debt holders if he takes actions that are detrimental to the value of the debt. According to this theory, the composition of domestic versus foreign debt depends on the differences in how this political punishment mechanism can be used by domestic and foreign debt holders.

The rest of the paper is organized as follows. In Section 2, I introduce the model. In Sections 3 and 4, I study the production decision of the autocrat, while I study the debt financing problem in Section 5. Section 6 concludes.

4.2 The Model

4.2.1 Preliminaries

An autocrat is in control of the production of a good from which he extracts rents. However, to increase the likelihood that he survives as the political leader, he must transfer some of the rents to the public. These transfers reduce the probability that the autocrat is overthrown. Demand for the good is stochastic, implying that the rents the dictator can extract for a given output level are uncertain. Further, there are constraints on how much the ruler can borrow. In the first part of the paper, I will consider a very simple setting with an exogenously imposed no-borrowing constraint. Later, I endogenize this constraint. The problem of the autocrat is to determine the optimal levels of production and transfers.

Endowments and Preferences of the Autocrat: The autocrat enters the current period with a level of assets, \( A \in \mathbb{R} \), which he can dispose of as he wishes. The autocrat is risk neutral, and his objective is to maximize the (discounted) sum of current and future consumption. In the most basic model,
the autocrat gets a fixed price $V$ if he survives to the next period, and 0 if he is
overthrown, but I also consider more general setups.

In each period, the autocrat must determine the level of production, $y$, the
level of transfers, $T$, and the level of savings, $A'$.

**Production and Profits:** The autocrat is a monopolist who has to choose
his output level before demand is realized. World demand for the good is stochas-
tic, and given by the continuous and differentiable function $y = y(p; s)$, where $s$
$\in S = [s^l, s^h]$ is the stochastic variable affecting demand. The demand shocks
are distributed according to the density function $f(s)$. For a given price, a higher
$s \in S$ increases the demand for the good. From this demand function, we find an
inverse demand function $p = p(y; s)$, where, once more, for a given $y$, a higher
$s \in S$ increases the price the buyers are willing to pay. The marginal cost of
production is constant and given by $c$. When supply is $y$ and the realization of
the shock is $s$, profits are given by

$$\pi(y; s) = (p(y; s) - c)y.$$ 

Even for the worst state of demand, I assume that the autocrat can ensure that
he makes a profit. More precisely, there exists $\bar{y} > 0$ such that $\pi(y, s^l) > 0$
whenever $y < \bar{y}$. Clearly, given the assumption, profits increase in $s$. Marginal
profits are given by $\frac{\partial \pi(y; s)}{\partial y} = y\frac{\partial p(y; s)}{\partial y} + p(y; s) - c$. Then, the impact of $s$ on
marginal profits is given by $\frac{\partial^2 \pi(y; s)}{\partial s} = y\frac{\partial^2 p(y; s)}{\partial p \partial y} + \frac{\partial p(y; s)}{\partial s}$. I assume that $\frac{\partial^2 \pi(y; s)}{\partial s} > 0$
holds. The second-order condition for profit-maximization amounts to $\frac{\partial^2 \pi(y; s)}{\partial y^2} = 2\frac{\partial p(y; s)}{\partial y} + y\frac{\partial^2 p(y; s)}{\partial y^2} < 0$, and I also assume this to be satisfied.

The assumption that the autocrat has to produce the good before the demand
shock is realized is important; If production takes place after the state of demand
is known, a producer always chooses the level of output maximizing monopoly
profits, no matter what his other interests are.

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4For instance, this expression is positive for any isoelastic demand function given that the
elasticity of demand with respect to price, $\eta$, is greater than 1.
4.2. THE MODEL

Note that even though oil production was the motivating example of this paper, I abstract from issues related to the optimal extraction of exhaustible resources (see Hotelling [1931]).

**Transfers and Revolution Technology**: The revolution technology is very stylized. If nature decides that there is a revolution, it succeeds with probability 1. If the autocrat is overthrown, he gets a payoff of 0 forever after. If he remains in power, the continuation value is \( V \).\(^5\) The probability that nature decides on a revolution is governed by a function \( 1 - P(T) \), where \( T \) is the level of transfers. Thus, \( P(T) \) is the probability that the autocrat remains in power. This function is assumed to be continuous and differentiable. Moreover, I assume \( \lim_{T \to 0} P(T) = 0, \ P'(T) > 0, \ P''(T), \) and \( \lim_{T \to 0} P'(T) = +\infty \).

I am the first to admit that this political environment is very sterile. For instance, relative to the work of, for instance, Acemoglu and Robinson (2001, 2004), I abstract from the issue of how the autocrat might promote a transition to democracy in order to keep some of the rents and avoid an outright revolution. Still, I think that this simple reduced-form way of modeling the political incentives of the autocrat is useful, and that it captures essential elements of reality.

**Capital Market Imperfections**: Following the debt crisis of the 1980s, a large literature emerged on repudiation risk in international lending (see e.g. Cohen and Sach [1986], Bulow and Rogoff [1989] and Gertler and Rogoff [1990]). Due to limited commitment and enforcement mechanisms to ensure repayment, this literature argues that borrowing countries face an upper bound on their foreign debt, or that the borrowing costs are an increasing function of the level of debt.

In the first part of the paper, I impose an exogenous borrowing constraint for the autocrat. This simple model yields the basic insight about how the autocrat behaves as a producer in troubled times. In the second part of the paper, I set up

\(^5\)Below, I consider cases where \( V \) is both exogenous and endogenous.
a model with endogenous borrowing costs. This model yields insights into how the autocrat finances his way out of trouble. Qualitatively, the insights regarding his production behavior carry over to this setting. In the model with endogenous borrowing costs, I assume that debt is repudiated if the regime is overthrown. One way of interpreting this assumption is that the new political regime suffers little from debt repudiation, for instance because the sanctions imposed by the creditors affect the previous autocrat and his supporters the most, as they are the ones who have accumulated assets that can be seized (see Bulow and Rogoff [1989]). Further, given that the export good of the economy (e.g. crude oil) is in high demand internationally, trade sanctions are not credible.

**Timing:** The timing of events within a period is as follows.

Stage 0: The autocrat enters with a given level of assets, $A$.

Stage 1: The autocrat sets the production level, $y$.

Stage 2: The state, $s$, of demand is revealed, and profits are realized.

Stage 3: Given the level of profits, the autocrat decides on levels of saving, $A_s'$, and transfers, $T_s$.

Stage 4: Nature determines whether there is a revolution or not, given the transfers from the autocrat to the people.

Stage 5: either the ruler is thrown out of office obtaining a payoff of zero, or he remains in power earning some continuation value.

The problem of the autocrat is then the following: At stage 3, given ‘cash on hand’ $K(y; A, s) = A + \pi(y; s)$, he sets $A_s'$ and $T_s$ to solve

$$W(K(y; A, s)) = \max_{C, T_s, A_s'} \{C + \beta P(T)(V_2(A_s'))\}$$

subject to

$$C + T_s + A_s' \leq K(y; A, s).$$

---

6Jayachandra and Kremer (2006) argue that odious debt *should* be forgiven when dictators are overthrown, since this raises the borrowing costs of dictators and thus their ability to retain power. In this paper, I assume that the market participants, for whatever reason, expect not to be repaid if the dictator is overthrown.
At stage 1, given initial assets $A$, the autocrat sets to production level $y$ to solve

$$V(A) = \max_y \{ \int_S f(s) W(A + \pi(y; s)) ds \}.$$  

In the next section, I analyze the behavior of the autocrat under various assumptions about the payoff for the autocrat from staying in power. In particular, I study three cases. The first two take place in a two-period environment. In the first of these, there will be a fixed prize for surviving to period 2. In the second, the size of the prize depends on the level of assets that are carried over from period 1 to period 2. The third model is a recursive model with an infinite horizon. In these models, I impose an exogenous borrowing constraint, an assumption that will be relaxed on the subsequent section.

### 4.3 Exogenous Borrowing Constraints

#### 4.3.1 $V_2$ Is Independent of the State $s \in S$

The main objective of this section is to show how an autocrat behaves relative to a regular monopolist, given that he faces borrowing constraints in some states of the world. In order to get some intuition, I start out with a simple two-period model. The setup is a simple two-period model where the autocrat gets a fixed prize $V_2$ if he survives to period 2. If he is overthrown, he gets 0. Thus, the value of this prize is independent of the level of assets he carries over from period 1 to period 2. The autocrat enters period 1 with a level of assets $A$. An exogenous no-borrowing constraint is imposed, implying that the autocrat cannot spend more on transfers than the sum of initial assets and (uncertain) profits he makes in period 1. The autocrat maximizes the sum of present consumption and the expected discounted value of the future prize. The problem of the autocrat is thus to determine an output level $y$ and optimally trade off his private consumption and transfers to the public:
\[ V_1 = \max_{y,T_s} \left\{ \int_s [A + \pi(y; s) - T_s] f(s) ds + \beta V_2 \int_s P(T_s) f(s) ds \right\} \]

\[ s.t. \]

\[ T_s \leq A + \pi(y; s), \forall s \in S = [s^l, s^h]. \]

Letting \( \lambda_s \) be the multiplier on the no-borrowing constraint in state \( s \), the Lagrangian of this problem is

\[ \mathcal{L} = \int_s [A + \pi(y; s) - T_s] f(s) ds + \beta V_2 \int_s P(T_s) f(s) ds + \int_s \lambda_s [A + \pi(y; s) - T_s] ds. \]

The first-order conditions are as follows

\[ y : \int_s \frac{\partial \pi(y; s)}{\partial y} f(s) ds + \int_s \lambda_s \frac{\partial \pi(y; s)}{\partial y} ds = 0 \]

\[ T_s : -f(s) + \beta V_2 P'(T_s) f(s) - \lambda_s = 0. \]

Define \( \mu_s = \lambda_s / f(s) \), such that \( \lambda_s = \mu_s f(s) \) and \( \mu_s \) is the probability adjusted shadow price on the state-\( s \) constraint. Rewriting the FOCs, we get

\[ \int_s \frac{\partial \pi(y; s)}{\partial y} (1 + \mu_s) f(s) ds = 0 \quad (4.1) \]

\[ \beta V_2 P'(T_s) = 1 + \mu_s \quad (4.2) \]

The following preliminary result, showing that the shadow price is monotonically decreasing in \( s \), is useful:

**Lemma 1:** The shadow prices on the borrowing constraint are characterized as follows:

(i) Suppose \( \exists \hat{s} \in S \) s.t. \( \mu_{\hat{s}} = 0 \). Then, \( \mu_s = 0 \) for all \( s > \hat{s} \).
(ii) Suppose \( \exists \hat{s} \in S \) s.t. \( \mu_{\hat{s}} > 0 \). Then, \( \mu_s > 0 \) for all \( s < \hat{s} \).

(iii) Suppose \( \exists \hat{s} \in S \) s.t. \( \mu_{\hat{s}} > 0 \). Then, \( \mu_s \) is strictly decreasing in \( s \) for \( s \geq \hat{s} \).

**Proof:** See Part A1 of the Appendix.

Suppose now that \( \exists \hat{s} \in S \) such that \( \mu_{\hat{s}} > 0 \), but \( \mu_s = 0 \) \( \forall s > \hat{s} \). Thus, the budget constraint is binding over the interval \([s', \hat{s}] \subseteq S\). Eq. (4.1) can be rewritten as follows (where the last equality follows from eq. (4.2)):

\[
\int_s^{s'} \frac{\partial \pi(y; s)}{\partial y} f(s) ds = - \int_{s'}^{\hat{s}} \mu_s \frac{\partial \pi(y; s)}{\partial y} f(s) ds = - \int_{s'}^{\hat{s}} \frac{\partial \pi(y; s)}{\partial y} [\beta V_{\hat{s}} P'(T_s) - 1] f(s) ds
\]

Now, let \( y^A \) and \( y^M \) be the production levels of the autocrat and a regular monopolist, respectively. The following results hold:

**Lemma 2:**

(i) \( y^A < y^M \) \( \iff \int_{s'}^{\hat{s}} \mu_s \frac{\partial \pi(y^A; s)}{\partial y} f(s) ds < 0 \)

(ii) If \( \mu_s = 0 \) \( \forall s \in S \), then \( y^A = y^M \)

**Proof:** See Part A2 of the Appendix.

We are now ready to state the main result of this subsection:

**Proposition 1:**

Part (i) Suppose \( \exists \hat{s} \in S \) s.t. \( \mu_{\hat{s}} > 0 \), with \( \mu_s = 0 \), \( \forall s > \hat{s} \). Then \( y^A < y^M \).

Part (ii) There exists a level of initial assets, \( \bar{A} \), such that \( y^A < y^M \) for all \( A < \bar{A} \)

Part (iii) There exists a level of initial assets, \( \bar{A} \), such that the price in state \( s \in S \) is such that \( \frac{\partial p_s}{\partial A} < 0 \) for all \( A < \bar{A} \).

**Proof:** See Part A3 of the Appendix.

This result tells us that if the autocrat expects to be constrained in his transfers to the citizens in some states of the world, he optimally chooses to produce a
lower output than a regular profit-maximizing monopolist. Thus, for sufficiently low levels of initial assets, such that the autocrat knows that he will be constrained in his transfers in some states of demand in the future, the autocrat reduces the output so that it is below the level that maximizes the monopoly profits. The intuition for this result is that the marginal value of profits is higher when the borrowing-contraint prevents him from increasing the transfers. The autocrat optimally decides to increase profits in these poor states of the world, at the cost of lowering profits in good states, where the marginal value of profits is lower.

Based on these results, the empirical predictions are clear, as illustrated in Figure 1. Output and prices are independent of the autocrat’s financial situation as long as the level of assets is sufficiently high. However, when a certain threshold level of assets has been passed, and the situation deteriorates further, output falls and prices increase.

4.3.2 \( V_2 \) Varies Over the States \( s \in S \)

It is reasonable to assume that the value of \( V_2 \) depends on the realization of profits in period 1, since this affects the level of assets in period 2. These assets can be used to buy political support and thus, a prolonged life in office. We will now assume that \( V_2 \) is an increasing and strictly concave function of the assets with which the dictator enters period 2, i.e. \( V_{2,s} = V_2(A_{s}') \). For convenience, let the gross return on the savings equal \( R = \beta^{-1} \), which implies that \( A_{s}' = R(\pi(y; s) - T_s - C_s) \), where \( C_s \) is the level of consumption in the first period. The strict concavity of the value function is assumed due to the concavity of \( P(\cdot) \), which implies that there are diminishing returns when you use assets to buy political support. This extension is interesting, since the incentives for making profits in bad states diminish when the potential prize you can win in bad states is low relative to the prize in good states. The dictator now has three variables to control: The level of output, the level of consumption, and the level of transfers. \( A_{s}' \) follows as a residual. We must now impose nonnegativity constraints on both
consumption and assets in all states. With these changes (and when we maximize over $A_s'$ instead of $C_s$), the Lagrangian of the dictator’s problem becomes

$$
\mathcal{L} = \int_s [A + \pi(y; s) - T_s - R^{-1}A_s']f(s)ds + \beta \int_s V_2(A_s')P(T_s)f(s)ds + \int_s \lambda_s[A + \pi(y; s) - T_s - R^{-1}A_s']ds + \int_s \gamma_s A_s'ds.
$$

The first-order conditions are now given by

$$
y : \int_s \frac{\partial \pi(y; s)}{\partial y}f(s)ds + \int_s \lambda_s \frac{\partial \pi(y; s)}{\partial y}ds = 0
$$

$$
T_s : -f(s) + \beta V_2(A_s')P'(T_s)f(s) - \lambda_s = 0.
$$

$$
A_s' : -R^{-1}f(s) + \beta V_2'(A_s')P(T_s)f(s) - R^{-1}\lambda_s + \gamma_s = 0.
$$

For sufficiency, we need the problem to be concave in $T_s$ and $A_s'$. This requires that the determinant of the Hessian of $T_s$ and $A_s'$ is positive. The condition $V_2(A_s')P(T_s)V_2''(A_s)P''(T_s) - V_2'(A_s)^2P'(T_s)^2 > 0$ is thus assumed to hold.

We now get some new terms in the first-order conditions, since current decisions affect the value of the future prize, and not only the probability of getting that prize. Using the definition $\mu_s = \lambda_s/f(s)$, such that $\lambda_s = \mu_s f(s)$, and now also $\psi_s = \gamma_s/f(s)$, we can rewrite the FOCs (using $R\beta = 1$) to get

$$
y : \int_s \frac{\partial \pi(y; s)}{\partial y}[1 + \mu_s]f(s)ds = 0 \quad (4.4)
$$

$$
T_s : \beta V_2(A_s')P'(T_s) = 1 + \mu_s \quad (4.5)
$$

$$
A_s' : V_2'(A_s')P(T_s) = 1 + \mu_s - R\psi_s. \quad (4.6)
$$

I will go through the same steps as in Section 3.1 to characterize the optimal solution of the dictator. For now, assume that the nonnegativity constraint on $A_s'$ is always satisfied (for instance due to an Inada condition on the slope of $V$
as \( A' \) approaches zero). The first-order conditions then tell us that either the marginal benefits from consumption, transfers, and saving should all be the same, or that consumption should be zero and the marginal benefits from transfers and saving should be equalized.

The analogue of Lemma 1 holds also in this setting:

**Lemma 3**

Part (i): Suppose \( \exists \hat{s} \in S \) s.t. \( \mu_{\hat{s}} = 0 \) with \( C_{\hat{s}} > 0 \). Then \( \mu_s = 0 \) for all \( s > \hat{s} \).

Part (ii): Suppose \( \exists \hat{s} \in S \) s.t. \( \mu_{\hat{s}} > 0 \). Then \( \mu_s > 0 \) for all \( s < \hat{s} \).

Part (iii): Suppose \( \exists \hat{s} \in S \) s.t. \( \mu_{\hat{s}} > 0 \). Then, \( \mu_s \) is strictly decreasing in \( s \) for \( s \geq \hat{s} \).

**Proof:** See Part A4 of the Appendix.

As above, I will now suppose that \( \exists \hat{s} \in S \) such that \( \mu_{\hat{s}} > 0 \), but \( \mu_s = 0 \) \( \forall s > \hat{s} \). Thus, as I have proved, the budget constraint will be binding over the interval \( [s', \hat{s}] \subset S \). The results above imply that Proposition 1 goes through also in this framework. Since expression (4.4) is equivalent to expression (4.1), the proof from Section 3.1 can be recycled in the following proposition:

**Proposition 2:** Suppose that \( \exists \hat{s} \in S \) s.t. \( \mu_{\hat{s}} > 0 \), with \( \mu_s = 0, \forall s > \hat{s} \). Then \( y^A < y^M \).

Thus, given that the ruler, in some states of the world, will be constrained in his transfer and savings decisions, he still reduces the level of output below the regular monopoly level. Note, however, that in the above proofs I have made heavy use of the assumption of strict concavity. This might not always be a reasonable assumption. It is possible that the value function is strictly concave for low asset levels, but linear for higher asset levels when the risk of being constrained has disappeared. In the next section, I analyze the problem in a complete recursive model.
4.3. EXOGENOUS BORROWING CONSTRAINTS

4.3.3 A Recursive Model

The setup in the previous sections was quite stylized. In this section, I illustrate how the autocrat behaves in a full recursive model. This enables us to see how the precautionary motives work out in a setting where there are no ex ante assumptions on the shape of the value function.

There is a discrete set $S$ of demand shocks. For simplicity, I assume that the demand shocks are iid, and the probability of state $s \in S$ is denoted by $q_s$. Naturally, with more persistent shocks, a distressed autocrat who has faced a history of low demand would have an even stronger precautionary motive.

The autocrat must determine the level of production and, for a given realization of profits, the level of transfers and savings. The gross return on savings is $R$ and I maintain the assumption that $R\beta = 1$. The problem of the autocrat can be stated as follows:

$$V(A) = \max_{C_s,y,T_s} \left\{ \sum_s q_s [C_s + \beta P(T_s) V(A'_s)] \right\}$$

subject to

$$A'_s = R[A + \pi(y; s) - T_s - C_s], \forall s \in S$$

$$A'_s \geq 0, \quad C_s \geq 0, \quad T_s \geq 0, \forall s \in S.$$ 

Using the budget constraint, this can be reformulated in the following way:

$$V(A) = \max_{y,T_s,A'_s} \left\{ \sum_s q_s [A + \pi(y; s) - T_s - R^{-1}A'_s + \beta P(T_s) V(A'_s)] \right\}$$

subject to

$$A'_s \geq 0, T_s \geq 0, A + \pi(y; s) - T_s - R^{-1}A'_s \geq 0, \forall s \in S.$$
The first-order conditions for a maximum are given by

\[
y : \sum_s q_s \frac{\partial \pi(y; s)}{\partial y} + \sum_s \lambda_s \frac{\partial \pi(y; s)}{\partial y} = 0 \quad (4.7)
\]

\[
T_s : -q_s [1 - \beta P'(T_s) V(A'_s)] - \lambda_s = 0 \quad (4.8)
\]

\[
A'_s : -q_s [1 - P(T_s) V'(A'_s)] - \lambda_s + R \gamma_s = 0 \quad (4.9)
\]

As our complementary slackness conditions, we have \( \gamma_s A'_s = 0 \) and \( \lambda_s [A + \pi(y; s) - T_s - R^{-1} A'_s] = 0, \forall s \in S \). (Due to the assumptions on \( P(\cdot) \), the autocrat always sets \( T > 0 \).)

To ensure that the autocrat might face a binding constraint in the level of transfers to the citizens, I make the following assumption:

**Assumption 1:** \( \beta P'(\pi(y^M; s^I))(\sum_s q_s \pi(y^M; s)) - 1 > 0 \).

**Lemma 4:** Given Assumption 1, there exist initial asset levels \( A \) such that \( \lambda_s > 0 \) for some \( s \in S \) when \( y^A = y^M \)

**Proof:** The assumption implies that the borrowing constraint binds whenever the autocrat enters a period with zero assets, produces the monopoly output, and experiences the worst state of demand, \( s^I \). Why is this the case? First, note that \( V(0) \) must be at least as great as the one-period expected profits for the monopoly output. The autocrat always has the option to produce \( y^M \), consume the profits, and then run away. But this implies that in eq. (4.8), it must be the case that \( \lambda_s / q_s = \beta P'(\pi(y^M; s^I))V(0) - 1 \geq \beta P'(\pi(y^M; s^I))(\sum_s q_s \pi(y^M; s^I)) - 1 > 0 \). QED.

I now characterize the solution to this problem. Let \( B \) be the Bellman operator, \( B : f \rightarrow f \).

**Lemma 5:** \( B \) is a contraction mapping.

**Proof:** To show this, we just have to check that Blackwell’s conditions are satisfied.
4.3. EXOGENOUS BORROWING CONSTRAINTS

(i) Monotonicity: This requires that if $f$ and $g$ are continuous, and $f(A) \leq g(A)$ for all $A \geq 0$, then $(Bf)(A) \leq (Bg)(A)$ for all $A \geq 0$. The following argument implies that $B$ satisfies this requirement. Let $x^*$ denote an optimal choice under $f$, while $x^{**}$ denotes an optimal choice under $g$. We have

$$
Bf(A) = \sum_s q_s [A + \pi(y^*; s) - T_s^* - R^{-1}A_s^* + \beta P(T_s) f(A_s^*)]
$$

$$
\leq \sum_s q_s [A + \pi(y^*; s) - T_s^* - R^{-1}A_s^* + \beta P(T_s) g(A_s^*)]
$$

$$
\leq \sum_s q_s [A + \pi(y^{**}; s) - T_s^{**} - R^{-1}A_s^{**} + \beta P(T_s) g(A_s^{**})]
$$

$$
= Bg(A).
$$

(ii) Discounting: This requires that there exists $\bar{\beta} \in (0, 1)$ s.t $B(f + a)(A) \leq Bf(A) + \bar{\beta}a$ for all $f \in C(X)$, where $B(f + a) = f(A) + a$. The following argument implies that $B$ satisfies this requirement. Let $\bar{\beta} = \beta$. As $P(T) \leq 1$, we have

$$
B(f + a)(A) = \sum_s q_s [A + \pi(y^*; s) - T_s^* - R^{-1}A_s^* + \beta P(T_s) (f(A_s^*) + a)]
$$

$$
= Bf(A) + \beta P(T_s)a
$$

$$
\leq Bf(A) + \beta a.
$$

This completes the proof. QED.

This Lemma implies that there is a unique solution for the value function, $V$, and that this can be found through value function iteration. Next, it is useful to define the following set of functions:

Definition: Let $F$ be the set of continuous, positive, and increasing functions defined over $A \in [0, \infty]$ such that all $f \in F$

(i) are strictly concave with bounded slope over the interval $[0, \hat{A})$,

(ii) have a slope converging to unity as $A \to \hat{A}$, and

(iii) are linear with a slope of unity over the interval $[\hat{A}, \infty)$.

The following proposition tells us that the operator $B$ maps functions in $F$
CHAPTER 4. THE BUSINESS OF TROUBLED AUTOCRATS

Proposition 3: Let $f \in F$. Then, $Bf \in F$.

Proof: Consider the first-order conditions (eqs 4.7-4.9) of the problem. Let $K_s = A + \pi(y; s)$ denote cash on hand. Given the assumptions on $f$, the autocrat behaves as follows: For sufficiently low $K_s$, it is obvious that the autocrat sets $K_s = T_s$, as $P'(T_s)$ is sufficiently high for low values of $T_s$. As $K_s$ increases, at some point, the autocrat sets $A'_s > 0$, but still $\lambda_s > 0$. Note that the autocrat would never set $A'_s > \hat{A}$, as this would violate eq. (4.9). Further, there exists an upper bound of $K_s$, $\bar{K}$, such that $C_s > 0$ for all $K_s$ greater than this upper bound. Why? Suppose that this is not the case. Then, at some sufficiently high level of $K_s$, since $K_s = T_s + R^{-1}A'_s$, either eq. (4.8) or eq. (4.9) would be violated. Indeed, there exists $\bar{K}$, such that $C_s = K_s - \bar{K}$ whenever $K_s > \bar{K}$. Thus, all assets are used for consumption above some threshold level of cash on hand, while all assets are used for transfers and saving below this level of assets. Define the value of cash on hand as $W(K_s) = \max \{C_s + P(T_s)f(A'_s)\}$. I now show that $W \in F$. For low levels of $K_s$ when all resources are allocated to $T_s$, this is clearly strictly concave. When $K_s$ increases further to the point where $A'_s > 0$, we still have that $W(K_s)$ is strictly concave. The condition for this is that $P''(T_s)f(A'_s)(1 - \beta \frac{\partial A'_s}{\partial K_s}) + P'(T_s)f(A'_s)\frac{\partial A'_s}{\partial K_s} < 0$. But, from the argument in Lemma 3, we know that $\frac{\partial \lambda_s}{\partial K} < 0$ whenever the constraint binds, which implies that we must have $P''(T_s)f(A'_s)(1 - \beta \frac{\partial A'_s}{\partial K_s}) + P'(T_s)f(A'_s)\frac{\partial A'_s}{\partial K_s} < 0$ for eq. (4.8) to hold. Finally, $W(K_s)$ is linear for $K_s \geq \bar{K}$ as all marginal resources are allocated to consumption, with a marginal value that is constant and equal to one. This implies that $W \in F$. We can write $f(A) = \max_y \{\sum_s q_s W(A + \pi(y; s))\}$. For $K_s < \bar{K}$, we have $\lambda_s > 0$, and $\partial \lambda_s / \partial s \leq 0$. This must be the case since for all relevant $A'_s$, $f(\cdot)$ is strictly concave, so we can apply the results from Lemma 3. Let $\bar{A} = \bar{K} - \pi(y^M; s)$. The previous results imply that $y^A < y^M$ for all $A < \bar{A}$. Clearly, $f(A)$ has a unity derivative for all $A \geq \bar{A}$, since then the monopoly output is then always produced, $K_s \geq \bar{K}$ for all $s$, so that all marginal wealth is used for consumption. For $A < \bar{A}$, there exist states where $K_s < \bar{K}$, implying
that the slope is greater than unity in at least some states of the world, and since
the slope of $f(A)$ equals the expected slope across states, it must be greater than
one. In other words, $Bf$ inherits all the properties of $f$. QED.

Thus, since $B$ is a contraction mapping, implying that the value function is
unique, and since $B$ preserves all functions $f \in \mathbf{F}$, we know that $V \in \mathbf{F}$. In other
words, the value function of the autocrat looks like the function depicted in the
upper panel of Figure 2. Further, the proof of Proposition 3 also tells us that
whenever $A < \bar{A}$, we have $y^A < y^M$, which is illustrated in the lower panel of
Figure 2. In other words, the empirical predictions regarding output and prices
are the same as in Sections 3.1 and 3.2.

The behavior of the autocrat is characterized by another interesting property.
Given that the autocrat starts out with $A < \bar{A}$, the autocrat never saves up to the
point where $A \geq \bar{A}$. In other words, the autocrat always leaves public finances
in a state such that there is a positive probability that he will be constrained in
his transfers to the citizens. This is because the autocrat is better off diverting
resources to himself than saving up to the point where the slope of the value
function converges to unity, which happens at $A = \bar{A}$.

### 4.4 Endogenizing the Borrowing Constraint

In Section 3, I looked at models with an exogenously given borrowing constraint.
Naturally, one would like this constraint to be generated by the characteristics
of the economic environment. For this purpose, I will now return to the simple
setup with a two-period model and a constant period-two prize. The timing of
events is as follows: First, the autocrat decides upon a production level. Second,
the demand shock and profits are realized. Third, the autocrat determines the
levels of consumption, saving, and transfers. If he finds that a borrowing contract
is needed, he approaches a lender, and the contract is signed. Fourth, nature
determines whether there is a revolution or not, given the level of transfers.
Sixth, and finally, payoffs are realized. Crucially, as discussed in Section 4.2,
debt is only repaid when the autocrat survives.

My model is similar to that of Gertler and Rogoff (1990), where an international lender has to decide on how much to lend to an entrepreneur. The investment opportunity of the entrepreneur gives a stochastic return, and the probability of a good outcome depends on the level of investment in the project. The entrepreneur is financially constrained, and borrows funds to put more money into the project. However, he can also divert resources, so if he is offered a regular full-commitment contract, this will be exploited by the entrepreneur, and the lender suffers a loss. Thus, there is a need to design an incentive-compatible borrowing contract.

The model in this paper fits nicely into the Gertler-Rogoff framework. The entrepreneur is the autocrat, the project is the preservation of his rule, and the investments are the level of transfers to the citizens. The stochastic return is given by the survival probability times the discounted value of the prize. The moral hazard problem arises because instead of investing the borrowed funds in transfers, the autocrat can divert them for private consumption.\footnote{Alternatively, and perhaps more realistically, the autocrat can divert the resources to a secret foreign bank account.}

\subsection*{4.4.1 Lack of Commitment}

We are now ready to go into the details. To save on notation, let $A = 0$. Suppose that state $s \in S$ was realized, and profits $\pi(y; s)$ earned. The ruler now has the amount $\pi(y; s)$ available for transfers. The efficient amount to invest for the unconstrained planner, $\bar{T}$, is implicitly given by the first-order condition $-1 + \beta P'(\bar{T})V = 0$. I assume that $\bar{T} > \pi(y^M; s^I)$, which implies that the autocrat is constrained in some states of the world if he produces the monopoly output. In such a state, the autocrat confronts the lender and asks on which terms he can borrow some resources. The task is to create a contract that is incentive compatible and earns the expected return $R$; the opportunity cost of the lender.

The lender knows $s \in S$ and $\pi(y; s)$. The autocrat proposes a contract to the
lender, which consists of a four-tuple \((\psi_{s,R}, \psi_{s,N}, T_s, B_s)\). Here, \(\psi_R\) and \(\psi_N\) are the amounts that the autocrat has to repay the supporters if there is a revolt or no revolt, respectively, and \(B_s\) is the amount borrowed by the autocrat in state \(s\). Since the payoff to the autocrat is 0 in the event of a revolution, I let \(\psi_R = 0\). Under commitment, the ruler would want to borrow \(\bar{T} - \pi(y; s)\) and invest everything. Suppose that the autocrat is offered this contract by the lender. The problem is that the ruler has some discretion in how to spend these borrowed funds. The objective function of the autocrat is

\[
\max_{T_s} \{\pi(y; s) + B_s - T_s + P(T_s)[V - \psi_{s,R}]\},
\]

yielding the first-order condition

\[
-1 + P'(T_s)[\alpha V - \psi_{s,R}] = 0. \tag{4.10}
\]

The unconstrained choice for the ruler is implicitly given by \(-1 + P'(\bar{T})V = 0\); thus, due to the strict concavity of \(P(\cdot)\), we see from eq. (4.10) that the ruler invests less than what is his unconstrained best choice, \(\bar{T}\). His incentives to invest in his own power have diminished, since he gets a smaller piece of the cake if he survives. Thus, if the lender offers the autocrat the full commitment contract, the autocrat will violate the terms of the contract and invest too little, and the return to the lender will be lower than \(R\). Thus, we have to search for a different contract which satisfies the arbitrage condition for the lender and, at the same time, is incentive compatible for the ruler. For this purpose, I search for contracts \((\psi_{s,R}, \psi_{s,N}, B_s, T_s)\) proposed by the ruler that maximize his expected return with respect to the transfer level, subject to incentive compatibility and a lender participation constraint.

### 4.4.2 The Contract With an International Lender

As a benchmark case, I look at a borrowing contract that is in the spirit of that proposed in Gertler and Rogoff (1990). Suppose that the autocrat approaches
the international capital market when he finds that he is short of funds. There is
perfect competition in the international market, so the lender’s expected return
on the contract is given by the gross world interest rate, $R$. For simplicity,
let $\beta R = 1$. Ex ante, there is full information, so the lender has access to all
information that is available to the autocrat (in particular the level of assets).

I let the autocrat suggest a contract to the lender. The contract consists of a
four-tuple $(\psi_{s,R}, \psi_{s,N}, B_s, T_s)$, where $\psi_{s,R}$ and $\psi_{s,N}$ are the repayments in the
case of a revolution or no revolution, respectively. $B_s$ is the amount borrowed
by the autocrat, and $T_s$ is the amount he has to invest in transfers. We set
$\psi_{s,R} = 0$, since the ruler will have no funds in this event. The contract must
respect the incentive compatibility constraint $(-1 + P'(T_s)[V - \psi_{s,N}] = 0)$, a
resource constraint $(T_s \leq \pi(y; s) + B_s)$, and a zero-profit constraint for lenders
$(P(T_s)\psi_{s,N} = RB_s)$. This means that the implied interest rate on a repaid loan
is given by

$$R^A_s = \frac{\psi_{s,N}}{B_s} = \frac{R}{P(T_s)}.$$  

The problem is the following:

$$\max_{D_s, T_s, \psi_{s,N}} \{\pi(y; s) + B_s - T_s + \beta P(T_s)[V - \psi_{s,R}]\}$$

s.t

$$(IC): \quad -1 + \beta P'(T_s)[V - \psi_{s,N}] = 0, \quad (\mu)$$

$$(BC): \quad T_s \leq \pi(y; s) + B_s, \quad (\gamma)$$

$$(ZP): \quad P(T_s)\psi_{s,N} = RB_s, \quad (\phi),$$

which gives the Lagrangian

$$\mathcal{L} = \pi(y; s) + B_s - T_s + \beta P(T_s)[V - \psi_{s,N}] + \mu\{-1 + \beta P'(T_s)[V - \psi_{s,N}]\} +$$
$$\gamma\{\pi(y; s) + B_s - T_s\} + \phi\{\beta P(T_s)\psi_{s,N} - \beta RB_s\}.  $$
4.4. ENDOGENIZING THE BORROWING CONSTRAINT

Setting $B_s, T_s$, and $\psi_{s,N}$ optimally yields the following first-order conditions and complementary slackness condition on the $(BC)$ constraint

\begin{align*}
B_s : & \quad 1 + \gamma - \phi = 0 \tag{4.11} \\
T_s : & \quad - (1 + \gamma) + \{\beta P'(T_s) + \mu \beta P''(T_s)\}[V - \psi_{s,N}] + \\
\phi \beta P'(T_s) \psi_{s,N} = & \quad 0 \\
\psi_{s,N} : & \quad (\phi - 1)\beta P(T_s) - \mu \beta P'(T_s) = 0 \tag{4.13} \\
CS_{(BC)} : & \quad \gamma(\pi(y; s) + B_s - T_s) = 0. \tag{4.14}
\end{align*}

The solution to this program can be summarized as follows:

**Proposition 4:** The contract between the autocrat and the international lender is such that

(i) there is no diversion in equilibrium ($T_s = \pi(y; s) + B_s$).

(ii) the level of borrowing is implicitly given by $1 = \beta P'(T_s) \left(\alpha V - \frac{[T_s - \pi(y; s)]}{\beta P(T_s)}\right)$.

(iii) for a constrained autocrat, transfers fall when $\pi(y; s)$ falls.

(iv) for a constrained autocrat, the borrowing costs, $R^A_s$, increase when $\pi(y; s)$ falls.

**Proof:** See Part A5 of the Appendix.

The key insight of this proposition is that when the asset level of the autocrat drops below a threshold level $\bar{T}$, it becomes gradually more expensive for the autocrat to obtain funds.

4.4.3 Production Behavior With the Endogenous Borrowing Constraint

Define the payoff for the autocrat under a contract with the international lender:
\[
W^I_s,\eta_s(\pi(y; s)) = \beta P(T_s) \left( V - \frac{(T_s - \pi(y; s))R}{P(T_s)} \right) = \frac{P(T_s)}{P'(T_s)},
\]
where \(T_s\) is implicitly given by

\[
1 = \beta' P'(T_s) \left( V - \frac{(T_s - \pi(y; s))R}{\beta P(T_s)} \right).
\]

Given this payoff function, it is straightforward to see that Proposition 4 implies that the payoff for the autocrat is strictly concave whenever \(\pi(y; s) < \tilde{T}\), as transfers fall when profits fall. Further, for \(\pi(y; s) \geq \tilde{T}\), the payoff is linear, with a slope equal to unity, as all marginal profits are used for consumption. Thus, it should be clear that the predictions for the autocrat’s production behavior are qualitatively the same in this model as in the above model with an exogenous borrowing constraint. The strict concavity of the payoff function whenever the autocrat faces a binding constraint implies that the results of Section 3 hold up.

### 4.5 Who Ends Up Financing the Rescue Operation?

Until now, I have studied the question of how financial distress affects the production behavior of the autocrat. In this section, I study how the autocrat optimally chooses his creditors and, in particular, whether the debt is raised in domestic or international capital markets. The main questions are: Who comes to rescue when the autocrat needs funding? And on what terms does this happen?

I assume that when liquidity constrained, the autocrat can approach either the domestic or the international capital market for funds. To this end, I have to expand the setup of the Gertler and Rogoff (1990) model slightly, to allow for a domestic group of individuals supporting the ruler. I assume that the income of this group depends on the continued existence of the autocrat’s regime. Given a profit of \(\pi\), I assume that the supporting group manages to grab \((1 - \alpha)\pi\).
For instance, the supporters could be managing the production of the good, giving them the opportunity to lure away this fraction of profits in the process. I assume that this share \((1 - \alpha)\) is exogenously given and not subject to uncertainty or bargaining (given that the autocrat remains in power). For simplicity, it is assumed that the supporters’ welfare is unaffected by the transfers the ruler gives to the masses. The objective of the supporters is to maximize their second period expected consumption level.

For simplicity, I assume that \(\bar{T} < \pi(y; s)\) for all \(s \in S\). In other words, in all states of the world, it is possible to reach the optimal amount (from the ruler’s point of view) of transfers through a borrowing contract with the supporters.

Due to the supporters’ stake in the continuation of the autocrat’s rule, they have an added incentive to lend the ruler funds in order to increase the likelihood of the ruler remaining in power. The main question I am interested in is whether the autocrat can exploit this in order to obtain funding inexpensively. Initially, I assume that the supporters act together and avoid collective action problems. Later, I discuss the consequences of such problems.

I will first go through a few introductory steps to make clear why the supporters might be willing to lend funds to the autocrat. Let \(B^D\) be the amount that the ruler borrows from the supporters in period 1. The return on government debt is given by \(R^G\), so that \(R^GD = \psi_N\) is the amount the autocrat pays if he remains in power. Further, let \(K\) be the amount that the supporters save in the international market, where the rate of return is \(R = 1 + r > 1\). If the autocrat remains in power, the supporters get a share \((1 - \alpha)\) of the prize \(V\). Let \(a^r_2\) and \(a^n_2\) represent the assets of the supporters in period 2 in the event that the autocrat is ousted or not, respectively. If there is a revolution, I assume that the government defaults on its debt, but that the supporters keep and enjoy the funds invested abroad. Given these assumptions, the supporters have the following budget constraints.
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\[ B^D + K \leq (1 - \alpha)\pi(y) \]
\[ c^n_2 \leq a^n_2 = RK \]
\[ c^n_2 \leq a^n_2 = (1 - \alpha)V + R^G D + RK \]

Assume that there is a probability \( P \) that the government survives, where as previously, \( P \) depends on the level of transfers, \( T \). Given that the government defaults in the event of a revolution, the expected amount of resources in period 2 is given by

\[ E[a_2] = P(T)[(1 - \alpha)V + R^G B^D] + RK. \]

The marginal return on investments in foreign assets is \( R \) with certainty, while the expected marginal return on investment in domestic government bonds is given by

\[
\frac{\partial E[a_2]}{\partial B^D} = \frac{\partial P(T)}{\partial B^D}[(1 - \alpha)V + R^G D] + P(T)R^G = \frac{\partial P(T)}{\partial T} \frac{dT}{dB^D}[(1 - \alpha)V + R^G B^D] + P(T)R^G. \quad (4.15)
\]

Here, the term \( \frac{dT}{dB^D} \) is the critical one. By how much does the autocrat increase his investments in transfers, given that there is a marginal increase in his borrowing opportunities? As the autocrat spends the borrowed funds in the way that pleases him the most, it is critical that the contract is incentive compatible, such that the hidden action problems do not make the lenders abstain from lending.

It can also be noted from eq. (4.15) that the supporters’ return on investments in government debt increases in \((1 - \alpha)\), i.e. the share of rents going to the supporters. This is intuitive; the more of the gains from the autocrat’s survival that accrues to the supporters’, the greater are the incentives to help him survive.
When the autocrat borrows $D^I$ from the international market, the zero-profit condition for the international lender is

$$P(T^I)\psi^I_N = RB^I,$$

or, since there is no diversion in equilibrium,

$$P(T^I)\psi^I_N = R(T^I - \alpha \pi(y)).$$

As we saw above, the zero-profit constraint for the supporter consists of another crucial element, as the lenders also gain from increasing the probability that they will earn rents, $(1-\alpha)V$, in the second period. Thus, for the supporters to provide funds $B^D$ to the autocrat, the participation constraint is given by

$$[P(T^D) - P(\alpha \pi(y))](1-\alpha)V + P(T)\psi^D_N \geq RB^D.$$

The expected payoff for the ruler when he borrows from $i \in \{I, D\}$ is given by

$$P(T^i)[\alpha V - \psi^i_N].$$

The autocrat always borrows from the counterparty that provides him with the best combination of $T$ and $\psi_N$. If the contract with the supporters is such that the participation constraint binds with equality, we can see that the supporter can accept a contract with both a lower $\psi_N$ and a higher $T$ and still participate. This is because marginal changes in $\psi_N$ and $T$ only have second-order effects on the supporters’ payoff, while the term $[P(T) - P(\alpha \pi)](1-\alpha)V$ has a first-order effect. Thus, the loss from a worse borrowing-part of the contract is compensated by the gain from the shareholder stake of the supporter.

This argument seems to indicate that the supporters can offer a contract with lower repayment $\psi^D_N$ and more lending $B$, and still earn a return $R$ due to the shareholder effect. However, since the autocrat can strike a deal in the
international market, the disagreement point between the domestic supporters and the ruler is not autarky, but rather a situation with a contract between the autocrat and an international lender. Let $C_I^s = (\psi_{s,R}^I, \psi_{s,N}^I, B_s^I, T_s^I)$ and $C_D^s = (\psi_{s,R}^D, \psi_{s,N}^D, B_s^D, T_s^D)$ be the contracts with the international lender and the domestic supporters, respectively. Taking the contract with the international lender as the disagreement point, the participation constraint for the supporters is

$$[P(T_D) - P(T_I)](1 - \alpha)V + P(T)\psi_N^D \geq RB.$$  

The supporters’ contract must satisfy incentive compatibility, which requires that $-1 + \beta P'(T_I)[\alpha V - \psi_N^D] = 0$. For the autocrat to prefer $C_D^s$ over $C_I^s$, the payoff for the ruler must be higher for $C_D^s$. Thus, the contract $C_D^s$ must either involve a lower $\psi_N$ or a higher $T$ relative to the levels in the contract of the international lender. If we can show that there exists such an equilibrium where also the participation constraint of the supporters is satisfied, we will have shown that a group with vested interests can make the ruler abandon the international market when issuing debt.

### 4.5.1 Optimal Contracts With the Supporters

As above, I let the autocrat find the optimal contract that satisfies all the relevant incentive and participation constraints. This contract is the one he proposes to the supporters, and since it satisfies the lender’s participation constraint, the supporters agree to it. The problem solved by the autocrat is:
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\[
\max_{B_s^D, T_s^D, \psi_{s,N}} \{ \alpha \pi(y; s) + B_s^D - T_s^D + \beta P(T_s^D)[\alpha V - \psi_{s,N}^D]\}
\]

s.t

(\text{IC}) : \quad -1 + \beta P'(T_s^D)[\alpha V - \psi_{s,N}^D] = 0, \quad (\mu)

(\text{BC}) : \quad T_s^D \leq \alpha \pi(y; s) + B_s^D, \quad (\gamma)

(\text{PC}^D) : \quad [P(T_s^D) - P(T_s^I)](1 - \alpha)V + \beta P(T_s^D)\psi_{s,N}^D \geq RB_s^D, \quad (\phi). (4.16)

This gives us the Lagrangian (where I have used \(\beta R = 1\))

\[
\mathcal{L} = \alpha \pi(y; s) + B_s^D - T_s^D + \beta P(T_s^D)[\alpha V - \psi_{s,N}^D] + \
\mu \{-1 + \beta P'(T_s^D)[\alpha V - \psi_{s,N}^D]\} + \
\gamma \{\alpha \pi(y; s) + B_s^D - T_s^D\} + \phi \{\beta P(T_s^D) - \
P(T_s^I)](1 - \alpha)V + \beta P(T_s^D)\psi_{s,N}^D - B_s^D\}.
\]

Choosing \(B_s^D, T_s^D,\) and \(\psi_{s,N}^D\) optimally yields the following first-order conditions and complementary slackness condition on the (BC) constraint:

\[
B_s^D : 1 + \gamma - \phi = 0 \quad (4.17)
\]

\[
T_s^D : -(1 + \gamma) + \{\beta P'(T_s^D) + \mu \beta P''(T_s^D)\}[\alpha V - \psi_{s,N}^D] + \phi \beta P'(T_s^D)[(1 - \alpha)V + \psi_{s,N}^D] = 0 \quad (4.18)
\]

\[
\psi_{s,N}^D : (\phi - 1)\beta P(T_s^D) - \mu \beta P'(T_s^D) = 0 \quad (4.19)
\]

\[
CS_{(BC)} : \gamma (\alpha \pi(y; s) + B_s^D - T_s^D) = 0. \quad (4.20)
\]

I now characterize the solution to this problem. First, eq. (4.17) implies that \(\phi > 0,\) so (PC\(^D\)) holds with equality. The constraints (IC), (BC) and (PC\(^D\)) together yield solutions for the unknowns \(T_s\) and \(\psi_{s,N},\) given that constraint (BC) holds with equality. This is what I show next. First, note that this holds
if we have $\gamma > 0$, which is implied by the following result:

Next, note the following:

**Lemma 6:** An optimal contract is such that $\gamma > 0$, and hence, $T_s^D = \alpha\pi(y; s) + B_s^D$.

**Proof:** See Part A6 of the Appendix.

Also:

**Lemma 7:** The contract will be such that $B_s^D > 0$

**Proof:** Suppose, on the contrary, that $B_s^D \leq 0$. Then, we have $T_s^D \leq \alpha\pi(y; s) < T_I$. From (PC$^D$), we can see that this implies that $\psi^D < 0$. We also know that the ruler is constrained in autarky, thus $-1 + \beta P'(\alpha\pi(y; s))\alpha V > 0$. But with $T_s^D \leq \alpha\pi(y; s)$ and $\psi^D < 0$, we will also have $-1 + \beta P'(T_s^D)[\alpha V - \psi^D_{s,N}] > 0$ so that (IC) is not satisfied and this cannot be a solution. **QED.**

The contract $C_s^D = (\psi^D_{s,N}, 0, T_s^D, B_s^D)$ is therefore determined by the constraints (IC),(BC) and (PC$^D$). Use (BC) to eliminate $B_s^D$ from (PC$^D$), and then use (IC) and (PC$^D$) to determine the equilibrium. Rewriting these two conditions, such that $\psi^D_{s,N}$ becomes a function of $T_s^D$, yields

\[
\text{(IC)-line:} \quad (\psi^D_{s,N})_{IC} = \alpha V - \frac{1}{\beta P'(T_s^D)}
\]

\[
\text{(PC)-line:} \quad (\psi^D_{s,N})_{PC} = \frac{(T_s^D - \alpha\pi_s(y) - \beta[P(T_s^D) - P(T_s^I)](1 - \alpha)V)}{P(T_s^D)}.
\]

A comparison between the contracts with international and domestic lenders gives us the following result.

**Proposition 5:** Given that an equilibrium exists for an international-lender contract (ILC), there always exists an equilibrium for the domestic-supporter contract (DSC). This equilibrium corresponds precisely to the equilibrium under the international-lender contract.

**Proof:** See Part A7 of the Appendix.
This implies that if the equilibrium is unique, the presence of supporters with vested interests has no effect on the borrowing contracts the ruler is able to sign. In effect, the ruler will be indifferent between borrowing internationally or domestically. However, in general, the equilibrium is not unique. Multiple equilibria arise for the following reason. To satisfy the participation constraint of the supporters, for low levels of borrowing, repayments $\psi_s^D$ fall as borrowing increases. This is because the increase in transfers induced by more borrowing raises the likelihood of the supporters getting rents in period 2, and this effect is so large that it more than compensates for the increase in lending. However, this effect is only present if the supporters have a sufficiently large stake in future rents, i.e. $\alpha$ must be below a certain threshold $\bar{\alpha}$. If this is the case, the (PC) line intersects the (IC) line (at least) twice, with one equilibrium involving more borrowing and lower repayments than the equilibrium with the international lender. The following proposition holds.

**Proposition 6:** There exists $\bar{\alpha} \in (0, 1)$ such that for all $\alpha < \bar{\alpha}$, the autocrat can offer the domestic supporters an incentive-compatible contract that makes him better off relative to the contract with the international lender. Moreover, there exists a continuum of contracts that make both the autocrat and the supporters better off relative to the contract with the international lender.

**Proof:** See Part A8 of the Appendix.

The two equilibria are illustrated in Figure 3. The equilibrium with the international lender is given by the intersection of the $(PC^I)$ line and the (IC) line in point $A$. There are two equilibria in the contract with the supporters, given by the intersection between the $(PC^D)$ line and the (IC) line in points $A$ and $B$. The shaded region in between the two equilibria illustrates all potential contracts that dominate the international lender contract for both the supporters and the autocrat. Thus, given bargaining between the supporters and the autocrat, a contract somewhere in this region would be chosen. For implementability, however, the contract must be along the (IC) line. Thus, given multiple equilibria,
there exists a continuum of implementable contracts on the interval of the (IC) line in between points $A$ and $B$. When the autocrat is given all the bargaining power, the equilibrium in point $B$ is chosen.

Here, I have made it clear when a ruler is able to exploit the presence of vested interests to his own advantage and that this could lead to the in-equilibrium absence of foreign borrowing and debt. The reason why this is possible is the following: When the ruler operates in isolation, he only considers the effects on the transfers upon his own future payoff. Thus, by not considering the positive effects for the supporters of higher transfers, the maximum aggregate payoff of the two parties is not achieved. However, when the ruler and supporters enter into a borrowing contract, the ruler internalizes the additional effect, since this positive effect allows him to reduce the repayment and increase his borrowing. Thus, the higher investments in transfers increase aggregate expected future payoff, and additional rents are created. This makes us wonder why the ruler does not always sign a contract with the supporters, given that they can realize efficiency gains. Note that when the ruler can satiate his preferred level of transfers using his own assets only, incentive compatible contracts can only be implemented if the supporters subsidize the ruler in both periods. Since the supporters might not have the commitment technology to make this credible, negotiations over such contracts might not succeed.

### 4.5.2 Collective Action Problems

Above, I assumed that all the supporters would act as one cohesive body. This assumption is critical. Without it, the autocrat might find himself stuck with a problem of the following type. Suppose that the autocrat proposes a borrowing contract where he shares the gains relative to the contract with the international lender with the supporters. Next, suppose that the supporters come together to decide whether to accept the borrowing contract or not. Assume that the supporters are homogenous, and that there is a number $N$ of them. For collusion to be sustainable, it must be individually rational to provide the funds promised
in the contract. That is, even if the supporters formally have agreed upon a contract among themselves, they also have to ensure that it will be implemented.

Suppose that the supporters have collectively agreed to fund the ruler, lending him $D$ and being repaid $\psi$. Thus, if one of the members deviates and refuses to lend money to the autocrat, the others have to pay up. Each supporter gets a fraction $\frac{1}{N}(1 - \alpha)V$ of the future prize, and he lends a fraction $\frac{1}{N}$ of the total amount borrowed by the autocrat. For it to be individually rational to stick to the agreement, the following condition must hold

$$P(T) \frac{1}{N}[(1 - \alpha)V + \psi] + R \frac{(1 - \alpha)\pi - D}{N} \geq P(T) \frac{1}{N}[(1 - \alpha)V] + R \frac{(1 - \alpha)\pi}{N}.$$ 

This condition simplifies to

$$P(T)\psi \geq RB,$$

which is independent of $N$. If this condition looks familiar, it is because we saw the same condition (only with an equality) in the section of the international lender model. The condition simply says that on the margin, the return on lending to the autocrat must be at least as good as the alternative return, $R$. Thus, the individual’s participation constraint says that the contract must provide at least as good terms as the international lender contract. If not, the individual will free-ride, as he expects the other individuals to stick to the agreement. However, this implies that it will be impossible to create a contract that the autocrat prefers to the contract with the international lender. Remember that the shaded region in Figure 3 represents contracts that dominate the ILC for both the autocrat and the supporters. Thus, all parties lose from free-riding relative to a situation where the collective action problem is overcome. To sum up, I can state the following

**Proposition 7:** The set of incentive-compatible contracts under collective action problems is empty.

It follows that the domestic capital market is preferred to the international
market only in situations where collective action problems among the supporters can be solved. At least two factors can be expected to facilitate cooperation. First, if there are few members in the group of supporters, this would be expected to make it easier to overcome the free-riding problem. Second, if the autocrat can control access to rents, he can punish individuals who do not cooperate in a state of emergency. This would enable the autocrat to reduce the free-riding problem, and facilitate efficient contracts. Empirically, one would thus expect to see little foreign debt in countries where rents are very concentrated among few individuals, and where the regime has control over the access to rents.

4.6 Conclusions

I have presented two main results concerning the behavior of autocrats as producers and a borrowers: First, an autocrat who needs to secure revenue to buy political goodwill will behave in a more cautious manner than a regular producer in situations where the autocrat fears that he will be financially constrained. This effect increases in the severity of the credit frictions. Second, when financing his way out of a crisis, I have shown that there exist equilibria where an autocrat can exploit the vested interests of his supporters when designing debt contracts, such that the market for international borrowing and lending will not be used in equilibrium. The implementability of such contracts depends on coordination among the supporters of the autocrat.

The main empirical predictions of the paper can be summed up as follows. First, output and prices are independent of the autocrat’s financial situation as long as the level of assets is sufficiently high. However, when a certain threshold level of assets has been passed, and the situation deteriorates further, output falls and prices increase. Second, the issuance of debt should be biased towards domestic debt when the autocrat is in financial difficulties and wealth and rents are concentrated among a relatively few individuals.

Several interesting questions are left unanswered.
First, it would be interesting to analyze how political considerations affect the stability of a cartel like OPEC. The question would be whether the political aspect would strengthen the incentives of collusion. Compared to a cartel with regular producers, there is more interdependency in a cartel where politics is involved. Since all parties prefer a cartel to competition, and since the cooperation of a new regime if an existing one is overthrown is more uncertain, the parties seem to have incentives to allocate cartel profits depending on the political fragility of a country. Thus, other members could formally or tacitly allow a country in difficulties to increase its level of production, and they will view this more as an investment in the reduction of future uncertainty than as a loss of current profits.

Second, given that the autocrat relies on the supporters to facilitate his way out of difficulties, what are the implications for how the autocrat sets up a system with rents in the economy in the first place? To reduce collective action problems among the supporters, the autocrat would want to set up a system that concentrates rents to as few individuals as possible. Further, control over access to the sources of rents increases the likelihood that the regime’s supporters will bail him out of trouble. Even though I do not develop this argument further in this paper, it seems as if this could be a fruitful path for better understanding the sources of the resource curse (see e.g. Sachs and Warner [2001] and Mehlum, Moene and Torvik [2006]).

Third, the theory of how the ruler may be able to exploit the vested interest of the supporters seems to be applicable in other settings, in particular corporate finance. Suppose, for instance, that there is a firm with two shareholders. One of them is the entrepreneur, while the other is an outside investor. The objective of the firm could, for instance, be to develop a new drug or a computer game. Like in Dewatripont and Maskin (1995), at some stage, the manager realizes that the firm needs more funds for a project. Given a suitable distribution of the ownership of the company, the entrepreneur could manipulate the outside investor just like
the ruler manipulates his supporters. Thus, instead of going to the bank for money, a new contract is signed between the firm and the outside investor which makes the investments into the project larger, such that the aggregate surplus of the two is higher than under a regular contract with the bank. Naturally, under this contract, the entrepreneur has to give up less of the ownership of the firm than if he were approaching a new outside investor. A rational investor would realize this at the initial stage when he first becomes involved with the firm, and the inability of the entrepreneur to commit not to exploit him at a later stage has implications for what the initial contract looks like. One solution for the investors is to ensure that ownership is sufficiently dispersed so that collective-action problems at the refinancing stage prevent the entrepreneur from exploiting them. However, note that refinancing is efficient, so that an optimal solution can clearly not be reached by creating collective action problems. It would be interesting to delve deeper into these issues. Empirically one might be able to distinguish projects according to the risk of ‘capture’ at the refinancing stage (for instance, some products take longer to develop than others, and the development of some products involves more risk than others), in order to see whether and how these issues are resolved. One might also wonder whether the model is able to explain the presence of cross-ownership, dual-class equity and so on. The costs of such arrangements are illustrated in, for instance, Bebchuk, Kraakman and Triantis (2000). They are most widespread in countries with concentrated ownership, and it is argued that they create enormous agency costs. My model might be able to shed light on how and why such arrangements come into place.
4.6. CONCLUSIONS

Appendix

A1: Proof of Lemma 1

**Proof of part (i):** \( \mu_s = 0 \) implies that \( \beta V_2 P'(T_s) = 1 \), and \( T_s \leq \pi(y; \hat{s}) \). For \( s > \hat{s} \) we have \( \pi(y; s) > \pi_s(y; \hat{s}) \). Suppose on the contrary that \( \mu_s > 0 \) for \( s > \hat{s} \). We would then need \( T_s > T_{\hat{s}} \). Due to the strict concavity of \( P(T) \), we would then have \( 1 + \mu_s = \beta V_2 P'(T_s) < \beta V_2 P'(T_{\hat{s}}) = 1 \), a contradiction. QED.

**Proof of part (ii):** \( \mu_{\hat{s}} > 0 \) implies that \( T_{\hat{s}} = \pi(y; \hat{s}) \). For \( s < \hat{s} \) we have \( \pi(y; s) < \pi(y; \hat{s}) \). Suppose on the contrary that \( \mu_s = 0 \) for \( s < \hat{s} \). Then \( T_s \leq \pi(y; s) < \pi(y; \hat{s}) = T_{\hat{s}} \). Then we have \( 1 = \beta V_2 P'(T_s) > \beta V_2 P'(T_{\hat{s}}) = 1 + \mu_{\hat{s}} > 1 \), a contradiction. QED.

**Proof of part (iii):** Given that \( \mu_s > 0 \) \( \forall s \leq \hat{s} \), the budget constraint is binding, so \( T_s = \pi(y; s) \), and \( \mu_s = \beta V_2 P'(\pi(y; s)) - 1 \). We then have \( \frac{\partial \mu_s}{\partial s} = \beta V_2 P'(\pi(y; s)) \frac{\partial \pi(y; s)}{\partial s} < 0 \), since \( P(\cdot) \) is strictly concave and \( \frac{\partial \pi(y; s)}{\partial s} > 0 \). Thus, \( \mu_s \) is strictly decreasing in \( s \) over \([s', \hat{s}]\). QED.

A2: Proof of Lemma 2

**Proof of part (i):** First, note that the necessary condition for profit maximization for a regular producer is \( \int_s \frac{\partial \pi(y; s)}{\partial y} f(s)ds = 0 \). We know that \( \frac{\partial \pi(y; s)}{\partial y} \) is strictly decreasing in \( y \). Thus, \( y^D < y^P \Rightarrow \frac{\partial \pi(y^A; s)}{\partial y} > \frac{\partial \pi(y^B; s)}{\partial y} \). \( \forall s \in S \). Thus, \( y^A < y^M \Leftrightarrow \int_s \frac{\partial \pi(y^A; s)}{\partial y} f(s)ds > 0 \), and the latter inequality holds if and only if \( \int_s \mu_s \frac{\partial \pi(y^A; s)}{\partial y} f(s)ds < 0 \). QED.

**Proof of part (ii):** If \( \mu_s > 0 \) \( \forall s \in S \), then \( \int_s \mu_s \frac{\partial \pi(y^A; s)}{\partial y} f(s)ds = 0 \), so the first-order condition of the dictator is identical to the first-order condition of the regular producer. QED.

A3: Proof of Proposition 1

**Proof of part (i):** Suppose on the contrary that \( y^A \geq y^M \). Then, since \( \int_s \frac{\partial \pi(y^A; s)}{\partial y} f(s)ds = 0 \), we would have \( \int_s \frac{\partial \pi(y; s)}{\partial y} f(s)ds \leq 0 \), and by (4.3) we would also have that \( \int_s \mu_s \frac{\partial \pi(y^A; s)}{\partial y} f(s)ds \geq 0 \). \( \pi_s(y) \) is strictly increasing in \( s \). Thus, since \( \int_s \frac{\partial \pi(y^A; s)}{\partial y} f(s)ds \leq 0 \), we also need \( \int_s \mu_s \frac{\partial \pi(y^A; s)}{\partial y} f(s)ds \leq 0 \). For \( s' < s'' \), we have \( \frac{\partial \pi(y^A; s')}{\partial y} < \frac{\partial \pi(y^A; s'')}{\partial y} \) and \( \mu_s > \mu_{s''} \). Suppose \( \frac{\partial \pi(y^A; s')}{\partial y} < 0 \). Then, since \( \mu_s > 0 \) \( \forall s < \hat{s} \), it is trivial to see that we cannot have \( \int_s \mu_s \frac{\partial \pi(y^A; s)}{\partial y} f(s)ds \geq 0 \), so we have a contradiction. So suppose instead that \( \frac{\partial \pi(y^A; s')}{\partial y} > 0 \). Since \( \int_s \frac{\partial \pi(y^A; s)}{\partial y} f(s)ds \leq 0 \), there must exist \( s < \hat{s} \) s.t. \( \frac{\partial \pi(y^A; s)}{\partial y} < 0 \), and since \( \frac{\partial \pi(y^A; s)}{\partial y} \) is continuous in \( s \), there must also exist \( s^* \) s.t. \( \frac{\partial \pi(y^A; s^*)}{\partial y} = 0 \) by the intermediate value theorem.

Now, we have \( \int_s \mu_s \frac{\partial \pi(y^A; s)}{\partial y} f(s)ds = \int_s \mu_s \frac{\partial \pi(y^A; s)}{\partial y} f(s)ds + \int_s \mu_s \frac{\partial \pi(y^A; s)}{\partial y} f(s)ds < \)


and\
\[ \int_{s}^{s^*} \mu_s \frac{\partial \pi(y^A(s))}{\partial y} f(s) ds + \int_{s}^{s^*} \mu_s \frac{\partial \pi(y^A(s))}{\partial y} f(s) ds = \mu_s \int_{s}^{s^*} \frac{\partial \pi(y^A(s))}{\partial y} f(s) ds \leq 0. \]
Thus,
\[ \int_{s}^{s^*} \mu_s \frac{\partial \pi(y^A(s))}{\partial y} f(s) ds < 0, \]
and we have a contradiction. QED.

**Proof of part (ii):** The autocrat is unconstrained if he can transfer \( \tilde{T} \) to the masses, where \( \tilde{T} \) is implicitly given by \( \beta V_2 P'(\tilde{T}) = 1 \). Let \( \bar{A} \) be defined by \( \bar{T} = \pi(y^M; s_i) + \bar{A} \), where \( \pi(y^M; s_i) \) represents profits for output \( y^M \) in the state with the lowest possible demand. For \( s = s_i \), whenever \( A < \bar{A} \), we have \( T < \tilde{T} \) for \( y = y^M \), and thus, the constraint is binding in this state. As the shadow prices are greater for bad states than for good states, the autocrat reduces output to raise profits in those states, implying that \( y^A < y^M \) whenever \( A < \bar{A} \). QED.

**Proof of part (iii):** We have \( \partial p / \partial A < 0 \), and as \( \partial y / \partial A \neq 0 \) for \( A < \bar{A} \), it directly follows that \( \partial p / \partial A < 0 \) whenever \( A \) is in this range. QED.

**A4: Proof of Lemma 3**

**Proof of part (i):** Suppose to the contrary that \( \mu_s > 0 \) for some \( s \in (\hat{s}, \hat{s}^h] \). This implies that \( C_s = 0 \), \( A'_s = A + \pi_s(y) - T_s \), and we also have \( \pi(y; s) \) \( \pi(y; \hat{s}) \). Note that the marginal benefit from consumption is fixed at 1, so in \( \hat{s} \) all returns are equal to one. Now, for \( s > \hat{s} \), the budget set has increased in size, and the claim is that we will set consumption to zero and spend the entire budget on transfers and saving. This implies that the returns on the latter must be larger than one, since \( \mu_s > 0 \). First note that if this is to be true, we can see from the fist-order conditions that we need \( T_s > \hat{T}_s \) and \( A'_s > \hat{A}'_s \). Further, since it is optimal to set \( \hat{C}_s > 0 \), we have \( \hat{C}_s + \beta V_2(\hat{A}'_s)P(\hat{T}_s) > \beta V_2(A'_s + \alpha \hat{C}_s)P(T_s + (1 - \alpha)\hat{C}_s) \) for all \( \alpha \in [0,1] \). Since \( \mu_s > 0 \), we also have the inequality \( \beta V_2(A'_s)P(T_s) > \epsilon + \beta V_2(A'_s - \epsilon)P(T_s - (1 - \epsilon)\epsilon) \) for all \( \epsilon > 0 \) and for all \( \alpha \in [0,1] \), and, in particular, for \( \epsilon = \hat{C}_s \). Rewriting these inequalities, we have \( \hat{C}_s > \beta [V_2(A'_s + \alpha \hat{C}_s)P(T_s + (1 - \alpha)\hat{C}_s) - V_2(A'_s)P(\hat{T}_s)] \) and \( \beta [V_2(A'_s)P(T_s) - V_2(A'_s - \alpha \hat{C}_s)P(T_s - (1 - \alpha)\hat{C}_s)] \). Now, to apply the concavity of the function \( f(A', T) = V_2(A')P(T) \), these inequalities cannot hold simultaneously, since this would require that the returns of an increased budget are higher in \( s \) than in \( \hat{s} \). This means that we have a contradiction. QED.

**Proof of part (ii):** Suppose on the contrary that \( \mu_s = 0 \) for some \( s \in [\hat{s}, \hat{s}^h] \). Going through a similar process as in part (i), we end up with a similar set of inequalities and once more we find a contradiction by applying the concavity of the problem. QED.

**Proof of part (iii):** The budget constraint is binding \( \forall s \in [\hat{s}, \hat{s}^h] \), so \( A'_s = \pi(y; s) - T_s \) in this region. From (4.5) we have \( \mu_s = \beta V_2(A + \pi_s(y) - T_s)P'(T_s) - 1 \). Differentiating this, we get \( \frac{\partial \mu_s}{\partial s} = \beta \frac{d \pi(y,s)}{ds} \{P'(T_s)V'_2(A + \pi(y; s) - T_s) + [V_2(A + \pi(y; s) - T_s)P''(T_s) - P'(T_s)V'_2(A + \pi(y; s) - T_s)] \frac{d \hat{T}_s}{d \pi(y,s)} \}. \) We have \( \frac{d \hat{T}_s}{d \pi(y,s)} \in (0,1) \), since in the range where \( \mu_s > 0 \), both \( T \) and \( A' \) will have to change as \( \pi(y; s) \) varies for both first-order conditions to hold. Write \( \frac{d \hat{T}_s}{d \pi(y,s)} \equiv z_s \). Then,
for $\frac{\partial \mu_s}{\partial s} < 0$ to hold, we need $P'(T_s)V_2'(A + \pi(y; s) - T_s)(1 - z_s) + V_2(A + \pi(y; s) - T_s)P''(T_s)z_s < 0$. From (4.6) we have $\mu_s = V_2'(A + \pi(y; s) - T_s)P(T_s) - 1$. Differentiating this one, we have $\frac{d\mu_s}{ds} = \frac{d\pi(y; s)}{ds} \{V_2'(A + \pi(y; s) - T_s)P(T_s) - [V_2''(A + \pi(y; s) - T_s)P(T_s) - P'(T_s)V_2'(A + \pi(y; s) - T_s)] \frac{dP(T_s)}{d\pi(y; s)}\}$, and the condition here becomes $P(T_s)V_2''(A + \pi(y; s) - T_s)(1 - z_s) + P'(T_s)V_2'(A + \pi(y; s) - T_s)z_s < 0$. To save on notation, we will write these conditions as $P'V'(1 - z) + VP''z < 0$, and $PV''(1 - z) + PV'z < 0$. These are two expressions for $\partial \mu / \partial s$, so they have to be equal. From these two expressions, we can solve for $z$. Doing this, we find that $z = \frac{z'}{2PV'-VP''z}$. Use this expression to substitute for $z$ in $PV'(1 - z) + VP''z$. This now becomes $P'V'(1 - z) + VP''z = \left(\frac{P'(z)}{P'(z)}\right)^2 - VP''z - \left(\frac{PV'}{VP''z}\right)$. The numerator we know is negative, since the condition for concavity is that $VP'V''P'' - (V')^2 > 0$. Further, we can see that the denominator is positive. Hence, we can conclude that $\frac{\partial \mu_s}{\partial s} < 0$. QED.

A5: Proof of Proposition 4

Proof: First, I show that $\gamma > 0$, s.t. $T_s = \pi(y; s) + B_s$, i.e. there is no diversion of resources in equilibrium. First, if $B_s = 0$, then this trivially holds, as there are no borrowed funds to divert. So, let $B_s > 0$, and suppose on the contrary that $\gamma = 0$. Then, by eq. (4.11), $\phi = 1$, and eq. (4.12) yields $-1 + \beta P'(T_s)V + \mu \beta P'(T_s)[V - \psi_{s,N}] = 0$. Using $[V - \psi_{s,N}] = \frac{1}{\beta P'(T_s)}$ from constraint (IC) implies that $\mu = \frac{P'(T_s)[1 - \beta P'(T_s)V]}{P'(T_s)}$. Now, from (4.13) we see that we need $\mu = 0$. This once more requires that we need $1 - \beta P'(T_s)V = 0$. Then, from constraint (IC) we see that this is possible only if we have $\psi_{s,N} = 0$. But then we cannot have a contract with $B_s > 0$. Thus, we have a contradiction, and we conclude that $T_s = \pi(y; s) = B_s$.

Next, I show that $T_s$ increases in $\pi(y; s)$. From constraint (ZP) we have $(\psi_{s,N})_{ZP} = \frac{(T_s - \pi(y; s))R}{P(T_s)}$. I denote this as the (ZP)-line. From constraint (IC) we have $(\psi_{s,N})_{IC} = V - \frac{1}{\beta P'(T_s)}$, which I denote the (IC)-line. The (IC) and (ZP) constraints yield solutions for $T_s$ and $\psi_{s,N}$. Note that the (IC) line is independent of the profit level. However, the (ZP)-line shifts when $\pi(y; s)$ increases. In particular, since $\psi_{s,N} = \frac{[T_s - \pi(y; s)]}{\beta P'(T_s)}$, the (ZP)-line shifts to the south-east in the $(T_s, \psi_{s,N})$ plane when profits increase. Since the (ZP)-line is strictly increasing in $T_s$ and the (IC)-line is strictly decreasing in $T_s$, the equilibrium moves to the south-east and thus, $T_s$ increases and $\psi_{s,N}$ falls.

Finally, the interest rate on borrowing is given by $R_s^A = \frac{\psi_{s,N}}{B_s} = \frac{R}{P(T_s)}$. As $T_s$ increases in $\pi(y; s)$, $R_s^A$ falls when $\pi(y; s)$ increases. Thus, borrowing costs fall in the level of profits. QED.

A6: Proof of Lemma 6

Proof: Suppose on the contrary that $\gamma = 0$. From (4.17) we then have $\phi = 1$. For a proposal to be a solution, we need (IC) to hold. Thus, we can use (IC)
to replace \( \alpha V - \psi_{s,N}^D \) in (4.18). Doing this, we find an expression for \( \mu_s \) given by 
\[
\mu_s = \frac{P_s(T^P_s)}{P_s(T^D_s)} \left[ 1 - \beta P_s(T^P_s) V \right].
\]
From (4.19) we see that we need \( \mu = 0 \). This can only be true if \( 1 - \beta P_s(T^D_s) V = 1 \). Since (IC) must hold, this implies that we need \( \psi_{s,N}^D = -(1 - \alpha)V \). But then (PC) can only be satisfied if \( B^D < 0 \), which I have shown not be true. Thus, conclude that \( \mu_s \neq 0 \), and we have a contradiction. QED.

A7: Proof of Proposition 5

**Proof:** First, a little bit of notation. Let \( T_s^I \) be feasible transfers under the ILC, while \( T_s^I^* \) is the equilibrium transfer under the ILC. Then, the (PC) line under the ILC is given by \( (\psi^I_{s,N})_{PC} = \frac{(T_s^I - \alpha \pi_s(y))^R}{P_s(T^I_s)} \), while the (PC) line under the DSC is given by \( (\psi^D_{s,N})_{PC} = \frac{T^D_s - \alpha \pi_s(y) - \beta(P_s(T^D_s) - P(T^I_s))^R}{\beta P_s(T^D_s)} \). The (IC) line is the same under both contracts. Note that (PC) and (IC) intersect at least once under the ILC, namely for \( T^I_s = T^I_s^* \). However, at this point, the (PC) line takes on the same value under both the ILC and DLC, and since the (IC) line is the same, this must also be an equilibrium for the DSC. QED.

A8: Proof of Proposition 6

**Proof:** First, I rewrite the constraints slightly in order to facilitate comparative statics on \( \alpha \) while keeping the equilibrium in the ILC fixed. Define \( \hat{V} = \alpha V \) and \( \hat{\pi} = \alpha \pi_s(y) \). The incentive constraint, which is identical for both the ILC and the DLC, becomes \( (\psi^I_{s,N})_{IC} = \hat{V} - \frac{1}{\beta P_s(T^I_s)} \). The participation constraint under the ILC is \( \psi^I_{s,N} = \frac{(T_s^I - \hat{\pi})^R}{P_s(T^I_s)} \), while the participation constraint under the DLC is 
\[
(\psi^D_{s,N})_{PC} = \frac{\hat{V}^R}{P_s(T^D_s)} - \frac{(P(T^D_s) - P(T^I_s))^R}{P(T^D_s)} \frac{(1 - \alpha)V^R}{P(T^D_s)}.
\]
Thus, changes in \( \alpha \) now only affect the (PC) line for the supporters. \( T_s^I^* \) denotes equilibrium transfers under the ILC.

Note that the incentive constraint \( (\psi^D_{s,N})_{IC} = \alpha V \) for \( T = 0 \), it slopes downward in \( T_s \), and goes to \( -\infty \) in the limit as \( T_s \) increases without bound.

Further, the participation constraint under the ILC is upward sloping. This can be shown as follows. We have \( \frac{\partial \psi^I_{s,N}}{\partial T_s} > 0 \) iff \( P(T_s) - P'(T_s)(T_s - \hat{\pi}) > 0 \). As \( P'(T_s) > 0 \), a sufficient condition for this is that \( P(T_s) - P'(T_s)T_s > 0 \), or \( P(T_s)/T_s > P'(T_s) \). This must be true due to the assumption of \( P''(T_s) < 0 \). Thus, given that there is an equilibrium under the ILC, it must be unique.

Next, consider the participation constraint for the DLC. Define \( G(T_s) = -\frac{(P(T^D_s) - P(T^I_s))^R}{P(T^D_s)} \frac{(1 - \alpha)V^R}{P(T^D_s)} \). \( G(T_s) \) is characterized as follows. Clearly, \( G(T^I_s^*) = 0 \). Moreover, note that \( G(T_s) < 0 \) for \( T^D_s > T^I_s^* \), and \( G(T_s) > 0 \) for \( T^D_s < T^I_s^* \). In other words, the participation constraint for the DLC is above the same constraint for the ILC when \( T^D_s < T^I_s^* \), and vice versa when \( T^D_s > T^I_s^* \). Further, \( \lim_{T_s \to -\infty} G(T_s) = -[1 - P(T^I_s)] \frac{(1 - \alpha)V^R}{P(T^I_s)} \), implying that \( \lim_{T_s \to -\infty} (\psi^D_{s,N})_{PC} = +\infty \).
This implies that if the slope of the participation constraint of the DLC is sufficiently steep and negative at $T^*_I$, more specifically more negative than the slope of the incentive constraint, then there exists (at least) one other equilibrium contract involving greater transfers and lower repayments for the autocrat. It then remains to find a condition for establishing when this is the case. I need

$$\frac{\partial (\psi^D_{s,N})_{PC}}{\partial T^D_s} \bigg|_{T^D_s = T^*_I} < \frac{\partial (\psi^D_{s,N})_{IC}}{\partial T^D_s} \bigg|_{T^D_s = T^*_I}.$$ 

By inspecting the expression for $(\psi^D_{s,N})_{PC}$, we see that the slope at $T^*_I$ decreases monotonically and without a lower bound as $\alpha$ falls. As the slope $\frac{\partial (\psi^D_{s,N})_{IC}}{\partial T^D_s} \bigg|_{T^D_s = T^*_I}$ is bounded, this implies that there exists $\bar{\alpha} \in (0, 1)$ such that there exist multiple equilibria whenever $\alpha < \bar{\alpha}$. (Note that the participation constraint under the DLC can intersect the incentive constraint also to the 'northwest' of the equilibrium with the international lender. This happens for $\alpha \in (\bar{\alpha}, \alpha'')$ for $1 > \alpha'' > \bar{\alpha}$. However, such a contract would violate the participation constraint of the autocrat, and does not constitute an equilibrium here.)

This contract is constructed such that all gains under the DLC are appropriated by the autocrat. However, for implementability, the contract only has to respect the incentive compatibility constraint of the autocrat with equality. The participation constraint of the supporters may, of course, be slack. Thus, any point on the incentive compatibility constraint between the equilibrium with the international lender and the second equilibrium with the domestic supporters, is implementable, and involves gains for both the supporters and the autocrat. **QED.**
Figure 1: The lower panel illustrates the production decision of the autocrat for different levels of initial assets. The upper panel illustrates the effect on prices.
4.6. CONCLUSIONS

Figure 2: The upper panel illustrates the shape of the autocrat’s value function across levels of initial assets. The lower panel illustrates the autocrat’s policy function for output.
Figure 3: The figure illustrates the equilibrium the autocrat achieves with the international lender in point $A$, and the equilibrium he can obtain with the domestic supporters in point $B$. 
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