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CURRENCY PRICES, TERMS OF TRADE,
AND INTEREST RATES: A GENERAL EQUILIBRIUM
ASSET-PRICING CASH-IN-ADVANCE APPROACH

by

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1. Introduction

The literature on international finance has not yet provided a satisfactory general equilibrium synthesis where real, financial and monetary aspects are integrated (see the survey by Adler and Dumas (1983)). The literature that applies models and tools of the finance literature to international finance, for instance Kouri (1977), Fama and Farber (1979), Hodrick (1981) and Stulz (1983), has indeed integrated monetary aspects only to the extent that real balances are used as an aspect in the utility functions. This literature, using the continuous-time stochastic differential equations approach, is also not fully general equilibrium, in the sense that the stochastic processes of some prices and interest rates are exogenously given.

A more general equilibrium approach is the discrete-time pure-exchange two-country two-currency two-good model in the seminal paper of Lucas (1982), where asset prices, exchange rates and interest rates are endogenously derived, with only the stochastic processes of output/endowments and money stocks exogenously given. In Lucas' model, the demand for money is derived via the cash-in-advance constraint associated with Clower (1967) and thoroughly discussed by Kohn (1981). Unfortunately, like most of the cash-in-advance literature (for instance Wilson (1979), Helpman (1981) and Persson (1982)), the elaborate
set-up leads in equilibrium to the trivial constant-velocity quantity equation, which we know is an unsatisfactory demand function for money.²

In Svensson (1983) a closed-economy variant of Lucas' (1982) model is developed, with a crucial modification that leads to a more reasonable demand for money. In Lucas' model consumers decide upon their cash holdings after they know the current state and hence after they know their current consumption. The result is that they acquire exactly the amount of cash they need to buy their consumption, which in equilibrium results in the quantity equation. The important modification in Svensson (1983) is that consumers must decide on their cash balances before they know the current state.³ This gives rise to a combined transactions, precautionary and store-of-value demand for money, with a non-unitary income velocity of money. Hence, with this single modification, money becomes an asset that is held for the liquidity services it provides, where the latter are endogenously determined as the shadow price of the liquidity constraints. Then money can be priced in symmetry with other assets, and a general-equilibrium synthesis of monetary, financial and real aspects results.⁴ In Svensson (1983) the model is used to examine the demand for money as a function of temporary and permanent disturbances in output and money supply, to discuss optimum monetary policy, and to discuss the Fisher relation and the premium on nominal bonds.

In the present paper, the model is extended to the original two-country two-good two-currency world of Lucas (1982), still with the modification which gives a more reasonable demand for money. The model is used to examine the effects on terms of trade, exchange rates and interest rates of temporary and permanent disturbances in the two countries' money supply. It is shown, for instance, that a temporary increase in the rate of monetary expansion in one country deteriorates its terms of trade (when liquidity constraints are binding) and depreciates its currency. A permanent increase in the rate of
monetary expansion has larger effects than a temporary one. For a temporary increase in the rate of monetary expansion in one country the nominal interest rates of both countries are unaffected, whereas the nominal interest rate increases for the country with a permanent increase in the rate of monetary expansion. A country's (own-good) real interest rate falls with a temporary increase in its rate of monetary expansion (if the consumers are cash-constrained). The real interest rate response to permanent increases in the rate of monetary expansion is ambiguous.

These results differ from those of Lucas and the quantity equation, since then terms of trade and the real interest rates are independent of the rate of monetary expansion, and the exchange rate is equally affected by temporary and permanent monetary disturbances.

The paper is organized as follows: Section 2 presents the model. Section 3 examines the effects of temporary monetary disturbances and Section 4 the effects of permanent monetary disturbances. Section 5 discusses the concept of terms of trade somewhat, and suggests a different interpretation of the model where currencies are traded continuously. Section 6 includes some conclusions, a comparison with corresponding results of Lucas, and a discussion of some limitations and possible extensions. An Appendix compares in some detail with Lucas (1982).

2. The Model

We consider a two-country, two-good, two-currency world. The home (foreign) country has a stochastic non-storable endowment/output \( y_t \) \((y^*_t)\) of home (foreign) goods in period \( t \), \( t = \ldots, -1, 0, 1, \ldots \). The total supplies of home and foreign currency, \( \bar{M}_t \) and \( \bar{N}^*_t \), respectively, follow

\[
\bar{M}_{t+1} = \omega_t \bar{M}_t \quad \text{and} \quad \bar{N}^*_{t+1} = \omega^*_t \bar{N}^*_t ,
\]
where (the gross rates of) monetary expansion of home and foreign currency $\omega_t$ and $\omega^*_t$ are stochastic. Let $s_t = (y_t, y^*_t, \omega_t, \omega^*_t)$ be the state in period $t$. It follows a Markov process with the distribution of $s_{t+1}$ given by the distribution function $F(s_{t+1} | s_t)$. That is, the probability distribution of $s_{t+1}$ depends on the realization of $s_t$.

The two countries each have an identical representative consumer with preferences

$$
E_t \sum_{t=0}^{\infty} \beta^{t-t} u(c_{ht}, c_{ft}), \quad 0 < \beta < 1,
$$

in period $t$, where $u(c_{ht}, c_{ft})$ is a concave utility function of consumption of home goods ($c_{ht}$) and of foreign goods ($c_{ft}$), and $E_t$ is the expectations operator conditional upon information available at $t$.

We shall first look at the decision problem of the home consumer. We assume that there exist traded claims to endowments of home and foreign goods, and to transfers of home and foreign money. The timing of events is crucial: The home consumer enters period $t$ with predetermined holdings $M_t$ and $N_t$ of home and foreign money, predetermined shares $z_{ht}$ and $z_{ft}$ of claims to endowments of home and foreign goods, and predetermined shares $x_{Mt}$ and $x_{Nt}$ of claims to transfers of home and foreign money. He/she learns the current state $s_t$ and then has the opportunity to purchase home and foreign goods. Home currency is used to pay for home goods at the price $P_{ht} = P_{ht}(s_t, M_t, N_t)$, whereas foreign currency is used to pay for foreign goods at the price $P^{*}_{ft} = P^{*}_{ft}(s_t, M_t, N_t)$. His/her purchases must obey the liquidity constraints

$$
(2.3) \quad P_{ht} c_{ht} \leq M \quad \text{and} \quad P^{*}_{ft} c_{ft} \leq N.
$$

After the goods markets are closed, he/she receives his/her share of the home and foreign cash from sales of goods endowments, $P_{ht} y_t z_{ht}$ and $P^{*}_{ft} y^*_t z_{ft}$.
as well as his/her share of the money transfers, \( \omega t \tilde{M}_t x_t \) and \( \omega t \tilde{N} x_t \). At the end of period \( t \), currencies and shares are traded. The home consumer then faces the budget constraint

\[
M_{t+1} + e_t N_{t+1} + Q_{ht} z_{ht, t+1} + Q_{ft} z_{ft, t+1} + R_{Mt} x_{M, t+1} + R_{Nt} x_{N, t+1} \leq \\
(M_t - P_{ht} c_{ht}) + e_t (N_t - P_{ft} c_{ft}) + (Q_{ht} + P_{ht} y) z_{ht} + (Q_{ft} + e_t P_{ft} y) z_{ft} + \\
+ [R_{Mt} + (\omega t - 1) M_t] x_M + [R_{Nt} + e_t (\omega t - 1) N_t] x_N .
\]

Here \( M_{t+1} \) and \( N_{t+1} \) are new home and foreign currency holdings, and \( z_{ht, t+1} \), \( z_{ft, t+1} \), \( x_{M, t+1} \) and \( x_{N, t+1} \) are new shares of claims to goods and endowments and money transfers, to be carried into period \( t+1 \). The exchange rate is

\[ e_t = e(s_t, \tilde{M}_t, \tilde{N}_t^*) \]

and share prices (in terms of home currency) are

\[ Q_{ht} = Q_h(s_t, \tilde{M}_t, \tilde{N}_t^*), \quad Q_{ft} = Q_f(s_t, \tilde{M}_t, \tilde{N}_t^*), \quad R_{Mt} = R_M(s_t, \tilde{M}_t, \tilde{N}_t^*) \]

and

\[ R_{Nt} = R_N(s_t, \tilde{M}_t, \tilde{N}_t^*). \]

We introduce the (real) prices of home and foreign currency (in terms of home goods), \( \pi_{Mt} = 1/P_{ht} \) and \( \pi_{Nt} = e_t/P_{ht} \), the relative price of foreign goods, \( p_t = e_t P_{ft}/P_{ht} \), and the real prices of assets, \( q_{ht} = Q_{ht}/P_{ht} \), \( q_{ft} = Q_{ft}/P_{ht} \), \( r_{Mt} = R_{Mt}/P_{ht} \) and \( r_{Nt} = R_{Nt}/P_{ht} \). For simplicity we let primed variables denote variables in period \( t+1 \) at state \( s_{t+1} \) and money stocks \( \tilde{M}_{t+1} \) and \( \tilde{N}^*_{t+1} \), whereas non-primed variables denote variables in period \( t \) at state \( s_t \) and money stocks \( \tilde{M}_t \) and \( \tilde{N}^*_t \). Then the budget constraint (2.4) and the liquidity constraints (2.3) can be written as

\[
c_h + p_c f + \pi_{M} M' + \pi_{N} N' + q_{ht} z_h' + q_{ft} z_f' + r_{M} x_M' + r_{N} x_N' \leq 
\]

\[
(2.5a) \quad \pi_{M} M' + \pi_{N} N' + (q_{ht} + y) z_h' + (q_{ft} + py) z_f' + [r_{M} + \pi_{M} (\omega - 1) \tilde{M}] x_M + \\
+ [r_{N} + \pi_{N} (\omega - 1) \tilde{N}] x_N \equiv w ,
\]

\[ c_h \leq \pi_{M} M' \quad \text{and} \]

\[ p_c f \leq \pi_{N} N' . \]
Here the right-hand side of (2.5a) defines real wealth in period \( t, w \). Real wealth in period \( t+1 \) will then be given by

\[
(2.5c) \quad w' = \pi_M^t M' + \pi_N^t N' + (q_h^t + y')z_h^t + (q_f^t + p'y^t')z_f^t
\]

\[
\quad + [r_M^t + \pi_M^t(\omega-1)\bar{M}']x_M^t + [r_N^t + \pi_N^t(\omega^* - 1)\bar{N}^*']x_N^t.
\]

(Note that the variables \( M', N', z_h^t, z_f^t, x_h^t \) and \( x_f^t \) are all choice variables in period \( t \) and hence, in equilibrium, functions of \( s, \bar{M} \) and \( \bar{N}^* \) rather than of \( s', \bar{M}' \) and \( \bar{N}^{*'} \).)

The home consumer will maximize (2.2) subject to (2.3) and (2.4). The foreign consumer will face exactly the same optimization problem. We assume the existence of a unique stochastic stationary equilibrium, that is, an equilibrium where the probability distributions of the endogenous variables are independent of \( t \). Due to the symmetry of the set-up, the two consumers will then end up consuming half of the endowment of each good, and owing half of each currency and half of the share of each asset. That is, we will have

\[
c_{ht} = c_{ht}^* = y_t/2,
\]

\[
c_{ft} = c_{ft}^* = y_t^*/2,
\]

\[
(2.6) \quad M_t = M_t^* = \bar{M}_t/2,
\]

\[
N_t = N_t^* = \bar{N}_t^*/2, \quad \text{and}
\]

\[
z_{ht} = z_{ft} = z_{ht}^* = z_{ft}^* = x_{ht} = x_{ft} = x_{ht}^* = x_{ft}^* = 1/2,
\]

where the foreign variables are denoted by an asterisk.

In a stochastic stationary equilibrium, the solution to maximizing (2.2) subject to (2.3) and (2.4) gives the value function \( v(w, M, N, s, \bar{M}, \bar{N}^*) \) defined implicitly as the maximum of

\[
(2.7) \quad u(c_h, c_f) + \beta\int v(w', M', N', s', \bar{M}', \bar{N}')dF(s'|s)
\]
over \( c_h, c_f, M', N', z_h', z_f', x_M' \) and \( x_N' \), subject to (2.5). We let \( \lambda, \mu \) and \( \nu \) denote the Lagrange multipliers of (2.5a-b). Then by standard properties we know that the value function fulfills

\[
(2.8) \quad v_w = \lambda, \quad v_M = \mu \pi_M \quad \text{and} \quad v_N = \nu \pi_N.
\]

Combining (2.6) and (2.8) with the first-order conditions for maximizing (2.7) subject to (2.5), we can finally write the equations that define an equilibrium in period \( t \), for each \( s, \bar{M} \) and \( \bar{N}^* \), as

\[
y \leq \pi_M \bar{M} \quad [\mu \geq 0],
\]

\[
p y^* \leq \pi_N \bar{N} \quad [\nu \geq 0],
\]

\[
u_h(y/2, y*/2) = \lambda + \mu,
\]

\[
u_f(y/2, y*/2) = (\lambda + \nu)p,
\]

\[
\lambda \pi_M = \beta E[(\lambda' + \mu')\pi_M'],
\]

\[
(2.9) \quad \lambda \pi_N = \beta E[(\lambda' + \nu')\pi_N'],
\]

\[
\lambda q_h = \beta E[\lambda'(q_h' + y')],
\]

\[
\lambda q_f = \beta E[\lambda'(q_f' + p y^*)],
\]

\[
\lambda r_M = \beta E[\lambda'(r_M' + \pi_M'(\omega'-1)\bar{M}')]
\]
\text{and}
\[
\lambda r_N = \beta E[\lambda'(r_N' + \pi_N'(\omega^*'-1)\bar{N}^*)].
\]

Here non-primed variables are functions of \( s, \bar{M} \) and \( \bar{N} \), and \( E f' \) denotes the conditional expectation \( \int f(s', \bar{M}', \bar{N}')dF(s'|s) \) (all expectations throughout this section are conditional).

The equations in (2.9) determine the endogenous variables \( \lambda, \mu, \nu, p, \pi_M', \pi_N', q_h', q_f', r_M \) and \( r_N \) as functions of the exogenous variables \( s, \bar{M} \) and \( \bar{N} \). Then the nominal goods prices and the exchange rate are given by
(2.10) \[ P_h = \frac{1}{\pi_M}, \quad P_f^* = \frac{p}{\pi_N} \quad \text{and} \quad e = \frac{\pi_N}{\pi_M}. \]

Let us interpret the equations somewhat. By (2.8) \( \lambda \) is the marginal utility of wealth, and \( \mu \) and \( \nu \) can be interpreted as the marginal utility of real balances of home and foreign currency, when real balances are measured in home goods. Note that \( \mu \) and \( \nu \) are the partial, liquidity services component of the marginal utility of real balances. Real balances are also wealth, hence the total marginal utility of real balances of home and foreign currency is, by the third and fourth equations in (2.9), \( \lambda + \mu = u_h \) and \( \lambda + \nu = u_f/p \), respectively. The total marginal utility of real balances of home currency - measured in home goods - equals the marginal utility of consumption of home goods. Analogously, the total marginal utility of real balances of foreign currency - measured in foreign goods - equals the marginal utility of consumption of foreign goods \( ((\lambda + \nu)p = u_f) \). The first and second equations of (2.9) show that the value of output of home and foreign goods cannot exceed real balances of home and foreign currency, respectively. The notation \([\mu \geq 0]\) and \([\nu \geq 0]\) refers to the usual complementary slackness conditions according to which a liquidity constraint that is not binding implies a zero marginal utility of the relevant real balances, and a positive marginal utility of real balances implies a binding liquidity constraint. The existence of a binding liquidity constraint hence drives a wedge between the marginal utility of wealth and the marginal utility of consumption, the former falling short of the latter by the marginal utility of real balances. Hence, in a cash-in-advance economy we have to distinguish between the marginal utility of wealth and of consumption.

The seventh and eighth equations in (2.9) are the standard asset pricing equations for claims to output of home and foreign goods, respectively. They are completely standard, except that the marginal utility of wealth that enters the equation is not equal to the marginal utility of consumption. Clearly, the existence of liquidity constraints affects the pricing of real assets.
The ninth and tenth equations in (2.9) give the prices of the claims to money transfers. Of more interest to us are the fifth and sixth equations. They give the prices of home and foreign currency. As we see, the marginal utility of real balances, the liquidity services, enter there as dividends.7

Henceforth, we will not be concerned about the prices of claims to output and to money transfers, but instead concentrate on the determination of currency prices. We note that we only need the first six equations for that.

It will be practical to rewrite them somewhat. First, we note that the equations

$$y \leq \pi_M^\infty \quad [\mu \geq 0],$$

(2.11a) \quad $$u_h = \lambda + \mu \quad \text{and}$$

$$\lambda \pi^\infty_M = \beta E[u^t_{nM}]$$

are sufficient for determining \(\lambda, \mu\) and \(\pi^\infty_M\). By symmetry, the equations

$$y^* \leq \pi^\infty_{N^*} \quad [\nu^* \geq 0],$$

(2.11b) \quad $$u_f = \lambda^* + \nu^* \quad \text{and}$$

$$\lambda^* \pi^*_{N^*} = \beta E[u^t_{fN}]$$

are sufficient for determining the variables \(\lambda^*, \nu^*\) and \(\pi^*_{N^*}\). These variables are the marginal utility of wealth and foreign real balances and the price of foreign money, all measured in foreign goods. The variables in (2.11 a, b) then determine \(p, \nu\) and \(\pi_N^*\) by

(2.11c) \quad $$p = \lambda^*/\lambda = \nu^*/\nu = \pi_N^*/\pi^*_{N^*}.$$  

Second, we introduce real home and foreign balances \(m, n\) and \(n^*\) by

(2.12) \quad $$m = \pi_M^\infty, \quad n = \pi_N^\infty, \quad \text{and} \quad n^* = \pi^*_{N^*}.$$
That is, \( m \) and \( n \) are real balances of home and foreign currency measured in home goods, whereas \( n^* \) is real balances of foreign currency measured in foreign goods. Then the equations in (2.11) can be rewritten as

\[
(2.13a) \quad y \leq m \quad [\mu \geq 0],
\]

\[
u_h = \lambda + \mu \quad \text{and} \quad \lambda m = \beta E[u_h'm']/\omega; \quad \text{and} \quad \lambda^*m^* = \beta E[u_f^*n^*]/\omega^*.
\]

\[
(2.13b) \quad y^* \leq n^* \quad [\nu^* \geq 0],
\]

\[
u_f = \lambda^* + \nu^* \quad \text{and} \quad \lambda^*n^* = \beta E[u_f^*n^*]/\omega^*.
\]

Here equation (2.13a) determines \( \lambda, \mu \) and \( m \), and equation (2.13b) determines \( \lambda^*, \nu^* \) and \( n^* \). The advantage with (2.13) is that it is easy to show that the variables are functions of the state \( s \) only, and not of the money stocks \( \tilde{m} \) and \( \tilde{n}^* \). Then \( \pi_M, \pi_N^* \) and \( \pi_N \) can be determined by (2.12). They will evidently depend on the money stocks in addition to the state.

We note that real balances here are beginning-of-period real balances, which do not include the current monetary transfer since the latter by assumption is paid at the end of the period. Hence end-of-period real balances are \( \tilde{m} = \omega m, \tilde{n} = \omega n^* \) and \( \tilde{n}^* = \omega n^* \). Since the concept of real balances is only an intermediate step to understand the determination of the price levels and inflation rates, the choice of definition is purely a matter of convenience and ease of interpretation.

We also note that the assumption that monetary transfers are paid at the end of the period, rather than in the beginning, implies that claims to monetary transfers are symmetric to other assets in that that their dividends cannot be used to buy consumption in the same period. Put differently,
monetary transfers are not more liquid than the dividends of other assets. When monetary transfers are negative, that is they are net taxes, consumers are not required to hold cash as a precaution for random taxes. A consequence of the assumption of timing of money transfers is that $\bar{M}_t$ refers to beginning-of-period money supply in period $t$, whereas $\bar{M}_{t+1} = \omega_t \bar{M}_t$ refers to end-of-period money supply in period $t$ (and beginning-of-period money supply in period $t+1$).

Let us finally emphasize that the set-up here implies that currencies are traded only at the end of the period. The alternative when currencies are traded also in the beginning of the period is considered in Section 5.

As in Lucas (1982), the price of any arbitrary asset can be determined if only its return as a function of the state is specified. This can be done even if equilibrium holdings of the asset are zero. We will be concerned with nominal and real interest rates. We consider first a nominal bond that pays one certain unit of home currency at the end of period $t+1$. In each state $s'$ it then pays $\pi'_M$ units of real wealth, the marginal utility of which is $\lambda' \pi'_M$. The expected utility in period $t$ of the bond is $\beta E[\lambda' \pi'_M]$, the real present value of it is $\beta E[\lambda' \pi'_M]/\lambda$, and the nominal present value of it is $\beta E[\lambda' \pi']/\lambda \pi_M$, which equals one over one plus the home nominal interest rate, $i$.

Hence

$$1/(1+i) = \beta E[\lambda' \pi'_M]/\lambda \pi_M = E[\lambda' \pi'_M]/E[u_h' \pi'_M],$$

where we have used (2.11a). In complete symmetry, the foreign nominal interest rate, $i^*$, will be given by

$$1/(1+i^*) = E[\lambda^* \pi^*_N]/E[u_f' \pi^*_N],$$

where the marginal utility of wealth and the price of foreign currency enter, both measured in foreign goods.
The home real rate of interest, $\rho$, is defined from the present value in period $t$ of an indexed bond that pays one unit of real wealth at the end of period $t+1$, that is

\[(2.15a) \quad 1/(1 + \rho) = \beta E[\lambda]/\lambda.\]

Similarly, the foreign real rate of interest, $\rho^*$, is defined by

\[(2.15b) \quad 1/(1 + \rho^*) = \beta E[\lambda^*/\lambda^*].\]

We note that a home indexed bond does not pay one home good at the end of period $t+1$. Rather, since this is a monetary economy, it is assumed to pay cash corresponding to one unit of real wealth, meaning $1/\pi^t_M$ units of home currency (or $1/\pi^t_N$ units of foreign currency). Similarly, a foreign indexed bond pays $1/\pi^t_N$ units of foreign currency.

We also note that these interest rates are "own" interest rates. We can also consider a real "consumer" interest rate, derived from an exact consumer price index, if the utility function $u(c^n, c^x)$ is homothetic (see for instance Svensson and Razin (1983)).

3. **Temporary Monetary Disturbances**

We shall see how the terms of trade, the exchange rate and nominal and real interest rates are affected by temporary and permanent disturbances in monetary expansion in the two countries. We will not be concerned with effects of temporary and permanent disturbances in the two countries' output, although the analysis is easy to extend (see Svensson (1983) for analysis of temporary and permanent output disturbances in a closed economy). For simplicity we then assume throughout the paper that output is serially uncorrelated. With respect to monetary expansion, we shall let serially uncorrelated disturbances represent temporary disturbances, whereas disturbances with positive serial correlation will represent permanent disturbances.
We first deal with temporary disturbances of monetary expansion. With no serial correlation, the probability distribution fulfills

\[(3.1) \quad F(s'|s) = F(s') ,\]

that is, the probability distribution of $s'$ is independent of the realization of $s$. Put differently, there is no information content in the current realization, and the probability distribution of future variables is unaffected by the current realization.

Under these circumstances, the solution to (2.13) is rather simple, since the expected total "utility-value" of home and foreign real balances, the terms $E[u_h' n']$ and $E[u_f' n^*']$, are constant and independent of $s$. Then the solution to (2.13a) can be written as follows:

For $\omega < \omega(y, y^*)$ we have

\[(3.2a) \quad \lambda(s) = u_h(y/2, y^*/2) \quad \text{and} \quad \mu(s) = 0 ;\]

whereas, for $\omega \geq \omega(y, y^*)$, we have

\[(3.2b) \quad \lambda(s) = \beta A/\omega \quad \text{and} \quad \mu(s) = u_h(y/2, y^*/2) - \beta A/\omega \geq 0 ;\]

where the constant $A$ and the function $\omega(y, y^*)$ are given by

\[(3.2c) \quad A = E[u_h'(y'/2, y^{*'}/2)m(s')] \quad \omega(y, y^*) = \beta A/[u_h'(y/2, y^*/2)y] .\]

This solution is intuitive: For a small (gross rate of) monetary expansion of home currency, the liquidity constraint is not binding, and home real
balances exceed home output. Then the marginal utility of real balances is zero and the marginal utility of wealth equals the marginal utility of consumption of home goods, as seen in (3.2a). Home real balances are then decreasing in home monetary expansion. For a sufficiently large home monetary expansion, home real balances fall to hit the liquidity constraint. For larger home monetary expansion, the liquidity constraint is binding and home real balances equal home output. The marginal utility of real balances is positive and increasing in monetary expansion, and the marginal utility of wealth is less than marginal utility of consumption of home goods and decreasing in monetary expansion.

We note in particular that \( m, \lambda \) and \( \mu \) in (3.2) are determined by \( y, y^* \) and \( \omega \), but independent of \( \omega^* \). That is, foreign monetary expansion has no effect on home real balances and the marginal utility of home real balances. The solution to (2.13b) is similarly independent of home monetary expansion and completely symmetric:

For \( \omega^* < \tilde{\omega}^*(y, y^*) \), we have

\[
\begin{align*}
n^*(s) &= \frac{\beta A^*}{u_x(y/2, y^*/2)\omega^*} > y^*, \\
\lambda^*(s) &= u_x(y/2, y^*/2) \quad \text{and} \\
\mu^*(s) &= 0; 
\end{align*}
\]

(3.3a) whereas, for \( \omega^* \geq \tilde{\omega}^*(y, y^*) \), we have

\[
\begin{align*}
n(s^*) &= y^*, \\
\lambda(s^*) &= \frac{\beta A^*}{y^*\omega^*} \quad \text{and} \\
\mu(s^*) &= u_x(y/2, y^*/2) - \frac{\beta A^*}{y^*\omega^*} \geq 0; 
\end{align*}
\]

(3.3b) where the constant \( A^* \) and the function \( \tilde{\omega}(y, y^*) \) are given by
\[ A^* = E[u_f(y'/2, y^*/2)n^*(s')] \] and
\[ \tilde{\omega}(y, y^*) = \beta a/[u_f(y/2, y^*/2)y] . \]

This solution has the same interpretation as (3.2), of course.

Let us now see how the relative price of foreign goods, the terms of trade, depends on temporary disturbances in home and foreign monetary expansion.

From (2.9) and (2.11a) we can write
\[ (3.4) \quad p/(u_f/u_h) = (1 + \mu/\lambda)/(1 + v^*/\lambda^*) . \]

In a barter economy, the relative price of foreign goods would equal the marginal rate of substitution in consumption, \( u_f/u_h \). In our monetary economy this is not so. The relative price of foreign goods also depends on monetary factors, since it depends on the ratios of marginal utility of real balances to marginal utility of wealth for home and foreign currency. Since the marginal utility of home real balances is increasing in home monetary expansion, it follows that the relative price of foreign goods increases with home monetary expansion. Put differently, a temporary increase in home monetary expansion deteriorates the home country's terms of trade.

More precisely, we note that for \( \omega < \tilde{\omega}(y, y^*) \) and \( \omega^* < \tilde{\omega}^*(y, y^*) \), the relative price of foreign goods fulfills \( p = u_f/u_h \) and is independent of home and foreign monetary expansion. For \( \omega \geq \omega(y, y^*) \) and \( \omega^* \geq \tilde{\omega}^*(y, y^*) \) the price of foreign goods is increasing in home monetary expansion and decreasing in foreign monetary expansion.

What can we say about the exchange rate. From (2.10), (2.11c), (2.12) and (2.13) we have
\[ \begin{align*}
e &= \pi_w/\pi_M = (n/m)(\bar{M}/\bar{N}^*) = (p_n^*/m)(\bar{M}/\bar{N}^*) = (\lambda^*n^*/\lambda m)(\bar{M}/\bar{N}^*) = \\
&= \{E[u_f'n^*/']/E[u_h'm']\}(\omega/\omega^*)(\bar{M}/\bar{N}^*),
\end{align*} \]
which by (3.2) and (3.3) can be written

\[(3.6) \quad e = \left(\frac{A^*}{A}\right)\left(\frac{\omega}{\omega^*}\right)\left(\frac{\bar{M}^*}{\bar{N}^*}\right).\]

Hence, the exchange rate is proportional to the ratio of home monetary expansion to foreign monetary expansion. A temporary home monetary expansion depreciates the home currency with a unitary elasticity.

The rate of depreciation of the home currency between period \(t\) and \(t+1\) is, by (3.6)

\[(3.7) \quad e'/e = \frac{\left[(\omega'/\omega^{*'})\left(\bar{M}'/\bar{N}^{*'}\right)\right]}{\left[(\omega/\omega^*)\left(\bar{M}/\bar{N}^*\right)\right]} = \omega'/\omega^{*'},\]

where we have used (2.1) Hence, the expected (rate of) depreciation is

\[(3.8) \quad E[e'/e] = E[\omega'/\omega^{*'}],\]

which is constant and independent of \(s\). The rate of depreciation depends on the next period's monetary expansion only, and is hence independent of the current state.

How can we understand these results? Consider the home consumer in an initial equilibrium, when he/she has learned the current state. Suppose that he/she instead learns that current monetary expansion is slightly higher. That means that he/she will get slightly more home cash at the end of the period. Suppose the consumer is not liquidity constrained with respect to home currency. At constant prices the consumer's current real wealth is higher and he/she will then like to spend the extra cash on consumption, on assets and on foreign currency. This bids up the home currency prices of goods, assets and foreign currency. A new equilibrium is reached when all these prices have risen in proportion to the increased monetary expansion. Then the consumer's real wealth is unchanged and his/her real demands for goods, assets and currency are unchanged. The price of foreign goods in terms of foreign currency remains
constant during this experiment, since it depends on foreign monetary expansion only. Hence, since in the end the exchange rate has depreciated and the price of home goods has risen in the same proportion to the monetary expansion, the relative price of foreign goods remains unchanged.

Suppose instead that the consumer is cash-constrained in home currency. When he/she learns of the increase in home currency transfers to be received at the end of the period, the consumer cannot at constant prices spend more on home goods, since the consumer is cash-constrained. The increase in cash is spent on assets, foreign currency and — if the consumer is not cash-constrained in foreign currency — on foreign goods. A new equilibrium is reached where home currency has depreciated but the home currency price of home goods is unchanged, as well as the foreign currency price of foreign goods. Hence, the relative price of foreign goods has increased.

The nominal interest rates, given by (2.14) are clearly constant and independent of temporary disturbances in monetary expansion. The real rates of interest, given by (2.15) vary with the corresponding marginal utility of real wealth. Hence, for low home monetary expansion, the home real interest rate is independent of home monetary expansion; whereas for high monetary expansion it is decreasing in monetary expansion. The home real interest rate is independent of foreign monetary expansion. By symmetry, the converse holds for the foreign real interest rate.

4. Permanent Monetary Disturbances

Next, we shall examine the effect on the same endogenous variables of permanent monetary disturbances. We choose to model this as follows: Monetary expansion is given by the product of two stochastic processes, according to

\[ w_t = \tilde{w}_t \theta_t \quad \text{and} \quad w_t^* = \tilde{w}_t^* \theta_t^*. \]
Here $\theta_t$ and $\theta^*_t$ are serially uncorrelated and correspond to temporary disturbances, whereas $\bar{\omega}_t$ and $\bar{\omega}^*_t$ are positively serially correlated and correspond to permanent monetary disturbances. Let now $s_t = (y_t, y^*_t, \theta_t, \theta^*_t, \bar{\omega}_t, \bar{\omega}^*_t)$ denote the state in period $t$. The probability distribution of $s' = s_{t+1}$ hence depends on the realization of $\bar{\omega}_t$ and $\bar{\omega}^*_t$ according to

\[(4.2) \quad F(s_{t+1} | s_t) = F(y_{t+1}, y^*_{t+1}, \theta_{t+1}, \theta^*_{t+1}, \bar{\omega}_{t+1}, \bar{\omega}^*_{t+1}, \bar{\omega}_t, \bar{\omega}^*_t).\]

We choose the specially simple situation when

\[(4.3) \quad \bar{\omega}_{t+1} = (\bar{\omega}_t)^a \xi_{t+1}, \quad 0 < a < 1, \quad \text{and} \quad \bar{\omega}^*_{t+1} = (\bar{\omega}^*_t)^b \tilde{\xi}^*_{t+1}, \quad 0 < b < 1,

with constant "autocorrelation" coefficients $a$ and $b$, and with $\xi_{t+1}$ and $\xi^*_{t+1}$ serially uncorrelated and jointly distributed with $(y_t, y^*_t, \theta_t, \xi_t)$ according to the distribution function $G(y_{t+1}, y^*_{t+1}, \theta_{t+1}, \theta^*_{t+1}, \xi_{t+1}, \xi^*_{t+1})$. The constants $a$ and $b$ must be less than unity in absolute value to ensure that the stochastic process is stationary. They are positive since we only consider positive serial correlation. Then the distribution function (4.2) fulfills

\[(4.4) \quad F(s_{t+1} | s_t) = G(y_{t+1}, y^*_{t+1}, \theta_{t+1}, \theta^*_{t+1}, \bar{\omega}_{t+1}/(\bar{\omega}_t)^a, \bar{\omega}^*_{t+1}/(\bar{\omega}^*_t)^b).\]

The solution to (2.13) will now be like (3.2) and (3.3), except that the previous constants $A$ and $A^*$ in (3.2) and (3.3) are now functions of $\bar{\omega}$ and $\bar{\omega}^*$ according to

\[A(\bar{\omega}) = E[u_h(y'/2, y^*/2)m(s') | \bar{\omega}, \bar{\omega}^*] \quad \text{and} \quad (4.5a) \quad A^*(\bar{\omega}^*) = E[u_f(y'/2, y^*/2)n*(s') \bar{\omega}, \bar{\omega}^*].\]

Also, the conditions $\omega < \bar{\omega}(y, y^*)$, etc., should now be written $\omega < \bar{\omega}(\bar{\omega}, y, y^*)$, $\omega \geq \bar{\omega}(\bar{\omega}, y, y^*)$, $\omega^* < \bar{\omega}^*(\bar{\omega}^*, y, y^*)$ and $\omega^* \geq \bar{\omega}^*(\bar{\omega}^*, y, y^*)$, where the functions $\bar{\omega}(\bar{\omega}, y, y^*)$ and $\bar{\omega}^*(\bar{\omega}, y, y^*)$ are given by
\[ \tilde{\omega}(\tilde{\omega}, y, y^*) = \beta A(\tilde{\omega})/[u_h(y/2, y^*/2) y] \quad \text{and} \]

\[ \tilde{\omega}^*(\tilde{\omega}^*, y, y^*) = \beta A^*(\tilde{\omega}^*)/[u_f(y/2, y^*/2) y^*]. \]

We note that from (4.3) more precisely that permanent home monetary disturbances are serially correlated with previous home monetary disturbances, but uncorrelated with previous foreign monetary disturbances it follows that \( A, m, \lambda \) and \( \mu \) will be independent of foreign monetary disturbances. Similarly, \( A^*, m^*, \lambda^* \) and \( \mu^* \) will be independent of home monetary disturbances.

It is not difficult to show that \( A(\tilde{\omega}) \) and \( A^*(\tilde{\omega}^*) \), the expected total utility-value of real balances, are decreasing in permanent monetary disturbances. Intuitively, a large permanent monetary expansion in period \( t \) implies that next period's monetary expansion will on the average be higher, which implies that next period's real balances will on average be lower, which decreases the expected utility-value of next period's real balances. (Note that it is crucial that the total marginal utility of real balances \( u_h \) and \( u_f \) depend on output only.)

With \( A(\tilde{\omega}) \) and \( A^*(\tilde{\omega}^*) \) decreasing functions, it follows directly from (3.2) and (3.3) that the variability of \( A \) and \( A^* \) reinforces the previous effects on the endogenous variables of monetary disturbances. Hence we can conclude that permanent monetary disturbances have a larger effect on real balances, marginal utility of wealth and marginal utility of real balances, than temporary monetary disturbances. It is then straightforward to show that the relative price of foreign goods is more sensitive in permanent than temporary monetary disturbances. Hence a permanent increase in monetary expansion deteriorates a country's terms of trade more than a temporary increase. Similarly, a permanent increase in monetary expansion depreciates the exchange rate more than a temporary increase.
What about expected depreciation of the exchange rate? We can write

\[(4.6) \, e' / e = \left[ (A(\bar{\omega}^*) / \omega^*) / (A(\bar{\omega}') / \omega') \right] \left[ A(\bar{\omega}^*) / A(\bar{\omega}) \right].\]

Since an increase in \(\bar{\omega}\) will decrease \(A(\bar{\omega})\), and on the average decrease \(A(\bar{\omega}') / \omega'\), it follows that expected depreciation, \(E[e' / e | \bar{\omega}, \omega^*]\), will indeed be increasing in permanent home monetary expansion.

The home nominal interest rate can be shown to be increasing in permanent home monetary expansion, and the foreign nominal interest rate is increasing in permanent foreign monetary expansion. The intuition is that a large current permanent monetary expansion implies that monetary expansion is likely to be large next period, which implies relatively large marginal utility of real balances and relatively low marginal utility of wealth. By (2.14) this serves to increase the nominal rate of interest.

The real interest rates have an ambiguous response to permanent monetary expansion in each respective currency. A permanent monetary expansion decreases marginal utility of wealth (if \(\omega \geq \omega(\bar{\omega}, y)\) but it also decreases expected future marginal utility of real balances. In general the net effect is ambiguous.

5. The Terms of Trade and Continuous Currency Trade

We shall discuss the concept of terms of trade somewhat, and in this context suggest an alternative interpretation of our model:

The relative price of foreign goods \(p\), what we call the terms of trade, have been defined as \(p = eP_f / P_h\). One might object that this is not the appropriate concept of terms of trade, the reason being that it does not convey how many physical units of home goods are traded for one physical unit of foreign goods. This is so since goods are not directly exchanged: When selling one unit of the foreign good one receives foreign cash, and one
cannot in this model immediately exchange that foreign currency into home currency and then buy the appropriate number of physical units of home goods. This is so since by assumption currencies cannot be exchanged until the end of the period, when the goods markets are closed.

It might then be argued that the relative price of home goods is a pure ex post accounting entity, computed from observed nominal prices and exchange rates, and that it does not convey the proportions in which goods actually could be exchanged ex ante. My feeling is that most actual empirical terms of trade computations might have the same problem. Nevertheless, one might want to use some ex ante, forward-looking, concept of terms of trade. Of course one can use the expectation Ep', but this is arbitrary. The problem is that due to the inherent uncertainty in the model, there is no given ex ante proportion according to which goods are traded. Hence, the alternative to our ex post definition is some very arbitrary definition. Due to this we restrict the discussion to p, what we continue to call the relative price of foreign goods and the terms of trade. Any reader can of course choose his own definition and examine the effects on monetary disturbances on it.

Underlying the previous discussion is, however, the assumption that currency markets are closed during the beginning of the period, when consumption goods are purchased. This in turn can be related to the implicit assumption that transaction costs in currency markets and in other asset markets are of the same order of magnitude, and it is the existence of such transaction costs that make asset and currency markets open only at the end of the period. However, it might be argued that transactions costs in currency markets are significantly less than in other asset markets, and that this manifests itself in more frequent currency trading than other asset trading. This assumption is made in Helpman and Razin (1982). If we modify our model along the same lines, this amounts to allowing currency trade also in the beginning of the period, after the state is known. This we call "continuous currency trade".
This can be modelled as follows. Consider a consumer that enters the period with home and foreign currency, $M$ and $N$ respectively, and learns the state $s_t$ and the nominal prices $P_{ht}$ and $P^{*}_{ft}$ of home goods in home currency, and foreign goods in foreign currency. He can then trade currencies on the currency market, and use the currencies so obtained to purchase home goods with home currency and foreign goods with foreign currency. This is represented as

$\tilde{M}_t + \tilde{e}_t \tilde{N}_t = M_t + \tilde{e}_t N_t$,  $P_{ht} \tilde{c}_{ht} \leq \tilde{M}_t$ and $P^{*}_{ft} \tilde{c}_{ft} \leq \tilde{N}_t$,

where $\tilde{e}_t$ is the exchange rate (in the beginning of period $t$, after the state $s_t$ is known) and $\tilde{M}_t$ and $\tilde{N}_t$ are the new holdings of currencies after the trade on the currency market and before the purchases of consumption goods. At the end of the period the budget constraint for the currency and asset trade is as (2.4), except that $\tilde{M}_t$ and $\tilde{N}_t$ are substituted for $M_t$ and $N_t$. Note that the exchange rate at the end of the period with the trade in asset markets is $e_t$, which a priori is not identical to the exchange rate $\tilde{e}_t$ in the beginning of the period.

In an equilibrium, in addition to (2.6) also

$\tilde{M}_t = \frac{M_t}{2}$ and $\tilde{N}_t = \frac{N_t}{2}$

must hold. It is then not difficult to show that, with these modifications, the stochastic stationary equilibrium of the model is identical to the stochastic stationary equilibrium of the model without continuous asset trade. Hence the nominal prices $P_{ht} = P_h(s_t, \tilde{M}_t, \tilde{N}_t)$ and $P^{*}_{ft} = P^{*}(s_t, \tilde{M}_t, \tilde{N}_t)$ are the same, as well as the exchange rate $e_t = e(s_t, \tilde{M}_t, \tilde{N}_t)$ on the asset and currency markets at the end of the period. What about the exchange rate $\tilde{e}_t$ in the beginning of the period? It is easy to see that it must fulfill

$\tilde{e}_t P^{*}_{ft} / P_{ht} = u_f(y_{t}/2, y^{*}_{t}/2) / u_h(y_{t}/2, y^{*}_{t}/2)$.
The relative price of foreign goods \( \tilde{P}_t = \tilde{e}_t \tilde{P}^*_t / \tilde{P}^*_h \), computed with the beginning-of-period exchange rate \( \tilde{e}_t \) will of course equal the marginal rate of substitution of home goods for foreign goods. It follows from (3.4) and the definition of \( P_t = e_t P^*_t / P^*_h \) that the two exchange rates are related according to

\[
(5.4) \quad \tilde{e}_t = \frac{(1 + \nu^* / \lambda^*)}{(1 + \nu / \lambda)} e_t.
\]

In the absence of currency trade in the beginning of the period, \( e_t \) in (5.4) can be interpreted as a shadow exchange rate at which the consumer would voluntarily hold his predetermined balances of home and foreign currency. If currency is traded, this exchange rate of course becomes the explicit exchange rate. Note that in general the beginning-of-period and end-of-period exchange rates differ!

With continuous currency trade, there is no ambiguity about which is the appropriate terms of trade. It is of course the relative price of foreign goods computed with the beginning-of-period exchange rate, since that shows the proportions in which the goods can be exchanged. Hence, these terms of trade are equal to the marginal rate of substitution in consumption, and independent of monetary expansion. Instead of causing a difference between the terms of trade and the marginal rate of substitution, monetary expansion causes a difference between the beginning-of-period and end-of-period exchange rates. Home monetary expansion depreciates home currency at the end of the period, but somewhat less at the beginning of the period (to keep the relative price of the beginning of the period constant).

Note also the jumps in the exchange rate from the beginning of the period to the end of the period cannot be excluded by the usual arbitrage argument, even though the jump is known with certainty in the beginning of the period after the state is known (since both \( \tilde{e}_t \) and \( e_t \) are known at the beginning of the period). Suppose \( \tilde{e}_t < e_t \). It seems that a consumer with a positive
balance of home currency after his purchases of home goods, \( \tilde{M}_t - P_t C_{ht} > 0 \), would exchange all his remaining home currency into foreign currency to receive a certain capital gain. However, by (5.4) and the fact that marginal utility of real balances is non-negative, \( \tilde{e}_t < e_t \) implies \( \mu > 0 \) and \( \tilde{M}_t = P_t C_{ht} \), and the consumer has no remaining home cash after his consumption purchases.

6. Conclusions, Limitations and Extensions

The paper presents a general-equilibrium asset-pricing analysis of a two-country monetary world, where the demand for money is derived via cash-in-advance constraints. Since consumers must choose cash balances before they know the current state and their current planned consumption, the demand function for money ends up having variable monetary velocity, in contrast to most of the cash-in-advance literature. In this model the effects on terms of trade, exchange rates and interest of temporary and permanent increases in monetary expansion in the two countries are derived. A temporary increase in the rate of monetary expansion in the home country deteriorates its terms of trade and depreciates its currency, more so for a permanent increase in the rate of monetary expansion. A temporary increase in the rate of monetary expansion in the home country has no effect on the two continuous nominal interest rates, the foreign country's real interest rate, but it decreases the home real interest rate (if consumers are cash-constrained in home currency). A permanent increase in the home rate of monetary expansion has no effect on the foreign country's nominal and real interest rate, increases the home interest rate, and has an ambiguous effect on the home country's real interest rate.

As a contrast, in Lucas' (1982) case with unitary monetary velocity, as shown in the Appendix terms of trade and real interest rates are independent of monetary expansion. The effect on the exchange rate is the same for temporary and permanent monetary expansions.
We recall that the result that terms of trade determinate for a monetary expansion depends crucially on the assumption that transactions costs in currency trading are the same as in other asset trading. If they are lower and currency is continuously traded, the terms of trade are independent of monetary policy.

Several extensions of the analysis are possible. Effects of temporary and permanent output disturbances can easily be incorporated, as in Svensson (1983). The parity conditions often used in international finance (cf. for instance Roll and Solnik (1979), Fama and Farber (1979) and Kouri (1983)) can be more thoroughly analyzed. The advantage with the present general equilibrium formulation is that these conditions can be expressed in terms of the exogenous stochastic processes of output and monetary expansion, rather than only in the endogenous stochastic processes of prices and interest rates (see Svensson (1983)). It should also be possible to incorporate fiscal and monetary policy, by introducing public sector and outstanding public debt, say along the lines of Lucas and Stokey (1983). Another possibility is a modification of the cash-in-advance constraint to incorporate transactions demand also from asset trade, see on this Helpman and Razin (1983a). As is well known, the general most serious limitation of this kind of analysis is the restriction to identical consumers, absence of investment, and a perfectly pooled equilibrium, where each consumer holds the same share of all outside assets. This construction, although convenient when constructing the equilibria, excludes interesting and relevant situations when consumers have different wealth effects and different marginal propensities to consume.
Appendix: Comparison with Lucas (1982)

Our timing of events is different from Lucas'. In his model, the consumer enters period $t$ with shares and possibly with money (although we shall see that in equilibrium no money is carried over). The consumer learns the state, receives his/her share of the cash from the sales of last period's endowment, receives money transfers, and trades cash and shares. After these transactions are completed, the consumer buys goods and pays with cash, and the period ends.

The period $t+1$ money supply is defined including the period $t+1$ transfer (since the transfer occurs in the beginning of the period),

$$M_{t+1} = \omega_{t+1} M_t \quad \text{and} \quad \bar{N}^*_{t+1} = \omega^*_{t+1} \bar{N}^*_t$$

(we use our notation throughout).

The period $t$ budget constraint can be written (in our notation)

$$\pi^M M + \pi^N N + q_h z'_h + q_f z'_f + r_M x'_M + r_N x'_N = w,$$

where $w$ is wealth in the beginning of period $t$. The liquidity constraints are

$$c_h \leq \pi^M M \quad \text{and} \quad \bar{p}c_f \leq \pi^N N.$$  

Wealth at the beginning of next period is

$$w' = \pi^M (M - c_h / \pi^M) + \pi^N (N - \bar{p}c_f / \pi^N) +

(A.4) \quad [q_h' + \pi^M(y / \pi^M)] z'_h + [q_f' + \pi^N(y^* / \pi^N)]

+ [r_M' + \pi^M(\omega' - 1)\bar{M}] x'_M + [r_N' + \pi^N(\omega^* - 1)\bar{N}^*] x'_N$$

The first two terms on the right-hand side of (A.4) are the real values of home and foreign currency not spent in period $t$ and carried over into period $t+1$. In equilibrium these terms will be zero, since no money will be carried over by the consumer, for the following reason: Period $t$ cash holdings are
determined after the state is known. Then the consumer chooses to hold cash exactly equal to his planned consumption, and the liquidity constraints (A.3) are always binding. (This is if nominal interest rates are positive). In equilibrium consumption equals output, and hence real balances will in contrast to our case, always equal output,

\[(A.5) \quad y = \frac{\pi}{M} \text{ and } py^* = \frac{\pi}{N} \tilde{N}.\]

The second and third terms on the right-hand side of (A.4) are the total returns on the claims to output. Here \(y/\pi\) is cash from the sale of present home output, and \(\pi(y/\pi)\) is the real value of it in next period. Now since by (A.7) \(y/\pi = \tilde{M}, \pi(y/\pi) = \tilde{M}/y\) and \(\tilde{N}' = \omega \tilde{M}\), this term can be written \(y'/\omega\).

The claim to future output is really a claim to cash, and the real value of that cash is diluted by monetary expansion next period. This does not occur in our case, since with our timing cash from the sale of output is distributed within the same period. Similarly, \(y^*/\pi^*_N\) is foreign cash from the sale of foreign output. The home goods value of this is \(\pi^*_N(y^*/\pi^*_N)\). Since \(\pi^*_N = p\pi^*_N\) if follows from (A.5) that this term can be written \(p'y^*/\omega^*\).

With this the budget constraint and next period's wealth in Lucas' model can be written

\[(A.6a) \quad c_h + pc_f + q_hz_h + q_hz_f + r_Mx_M + r_Nx_N = w \quad \text{and} \]
\[(A.6b) \quad w' = (q_h + y'/\omega')z_h + (q_f + p'y^*/\omega^*)z_f + [r_M + \pi_M(\omega - 1)\tilde{M}]x_M + [r_N + \pi_N(\omega^* - 1)\tilde{N^*}]x_N.\]

It follows that the equations describing the equilibrium, in addition to (A.5), are, for each s, \(\tilde{M}\) and \(\tilde{N^*},\)
\[ u_h = \lambda, \]
\[ u_f = \lambda p, \]
\[ \lambda q_h = \beta E[\lambda'(q_h + y'/\omega')], \]
\[ \lambda q_f = \beta E[\lambda'(q_f + p'y^s'/\omega^s')], \]
\[ \lambda r_M = \beta E[\lambda^*(r_M^* + \pi_M^*(\omega^s - 1)\bar{M})] \quad \text{and} \]
\[ \lambda r_N = \beta E[\lambda'(r_N^* + \pi_N^*(\omega^s - 1)\bar{N^s})], \]

where all expectations are conditional upon s, and where \( \lambda \) is the multiplier of (A.6a) and hence the marginal utility of wealth. Since shares and cash are traded in the beginning of the period when the state is known, and the cash then is used to buy the planned consumption, there is no wedge between the marginal utility of wealth and the marginal utility of consumption of home goods, in contrast to our case.

The pricing equations for the claims to output in (A.7) is different from ours in (2.9), since in our equation (2.9) the marginal utility of wealth is not identical to the marginal utility of consumption of home goods, and since in Lucas' case the real value of the return on the claim is diluted by monetary expansion.

The pricing of currencies in our case fundamentally differs from that of Lucas'. In our case, currencies are held to provide future liquidity services, they are a store of value and part of wealth, and they are priced by a capital-asset-pricing equation in complete symmetry with other assets, once its direct return - its liquidity services - have been specified.

In Lucas' case, money is not held by the consumer between periods, it is not a store of value, and it is not part of wealth. It is acquired and disposed of by the consumer within the period, only. Its real price is given
directly by the quantity equation (A.5). There is no marginal utility of money and a capital-asset-pricing equation can of course not be derived. Hence, expectations of future states have no effect on the current value of money.

The relative price of foreign goods is given by

(A.8) \[ p = \frac{u_f(y/2, y^*/2)}{u_h(y/2, y^*/2)}, \]

the marginal rate of transformation of home goods for foreign goods, and is independent of monetary expansion.

The exchange rate is

(A.9) \[ e = \frac{\pi_N}{\pi_M} = \frac{(p^*y^*/y)(\omega/\omega^*)}{(1 - \lambda)(\bar{M} - \bar{N}^*)}. \]

Hence temporary and permanent monetary disturbances have the same effect on the exchange rate.

The home nominal interest rate is, using (A.1), (A.5), (A.7) and the definition \( (1 + i) = \lambda \pi_M/\beta \mathbb{E}[\lambda \pi_M'] \),

(A.10a) \[ 1 + i = \frac{u_h(y/2, y^*/2) \mathbb{E}[u_h(y'/2, y^*/2) y'/\omega']}{u_h(y'/2, y^*/2) y'/\omega'}. \]

We realize that the nominal interest rate will be independent of temporary monetary disturbances but increasing in permanent home monetary expansion. The foreign nominal interest rate will by symmetry be

(A.10b) \[ 1 + i^* = \frac{u_f y^*/\beta \mathbb{E}[u_f y^*/\omega^*]}{u_f y^*/\omega^*}. \]

The home goods and foreign goods real interest rate are

(A.11a) \[ 1 + \rho = \frac{u_h/\beta \mathbb{E}[u_h']}{u_h'}, \]

(A.11b) \[ 1 + \rho^* = \frac{u_f/\beta \mathbb{E}[u_f']}{u_f'}. \]
hence independent of monetary policy.

Note that these interest rates are "own-good" real interest rates. Lucas defines the real interest rate differently, namely as

\[(A.12) \quad 1 + \bar{\rho} = (u_h'y + u_f' y^*)/\beta E[u_h' y' + u_f' y^{*'}].\]

See Svensson (1983) for further discussion of this definition. Another real interest is the one corresponding to an exact consumer price index: Assume that \(u(c_h, c_f)\) is homothetic and can be written \(U(\phi(c_h, c_f))\), where \(\phi(c_h, c_f)\) is linearly homogenous. Then the exact CPI interest rate would be \(1 + \bar{\rho} = U_\phi'/\beta E[U_\phi']\).
Footnotes

* This paper is a considerable revision and extension of some international aspects of a paper with a different title that was presented at the Workshop in International Economics at Tel-Aviv University, July 11-13, 1983. I am grateful for comments by participants of the Workshop, and in particular for several discussions with Elhanan Helpman. On the present version I have received helpful comments by participants in an IIES seminar, especially Lars Calmfors. Remaining errors and obscurities are my own.

1. To be precise, Fama and Farber (1979) and Stulz (1983) assume that consumption goods input and real balances jointly produce consumption services, which in turn enter the utility function.

2. The ambitious analysis by Kouri (1983) also relies on a cash-in-advance constraint which results in the quantity equation (see also the discussion by Fischer (1983a)).

3. This modification is suggested in the conclusion of Lucas (1982).

4. Previous papers with cash-in-advance models giving rise to non-unitary monetary velocity include Goldman (1974), Lucas (1980), Stockman (1980), Helpman and Razin (1982), Krugman, Persson and Svensson (1983), and Obstfeld and Stockman (1983). Although these papers derive a combined transactions, precautionary and store-of-value money demand, they do not specify a full general equilibrium stationary stochastic equilibrium with several assets, and they do not exploit the capital-asset-pricing equations.

The present model is rather close to that of Helpman and Razin (1982), in particular in their informal discussion. Their formal analysis is within a two period framework with very asymmetric periods, which makes it impossible to consider the distinction between temporary and permanent disturbances — a weakness shared by Krugman, Persson and Svensson (1983).

The cash-in-advance tradition represents one way of several to provide a somewhat better microfoundation of money (than just plugging money into the
utility function), by assuming that transactions costs are zero with money and infinite without money. Most of this tradition seems too simplified in that the trivial quantity equation emerges as determining the price level. The present paper, as Krugman, Persson and Svensson (1983), can be seen as an attempt to improve on the cash-in-advance models. Another tradition is the overlapping-generations one, like Wallace (1980), which by many seems inadequate in that money is given a role only as a store of value, and can in equilibrium not be rate-of-return dominated by other assets. The Baumol-Tobin tradition has previously been partial equilibrium only. Some recent work by Jovanovic (1982), Grossman and Weiss (1982) and Rotemberg (1982) provide very interesting general equilibrium extensions of it.

It has been argued that using cash-in-advance constraints is no different from using real balances in the utility function (cf. Fischer (1983 b)). In Svensson (1983) it is argued that the two approaches are indeed more fundamentally different.

5. As discussed in Lucas (1982), there is a certain arbitrariness in what assets are assumed traded. The important aspect is that there are enough assets so as to allow a stationary equilibrium. This requires trade in claims to transfers of home and foreign currency.

6. Since these prices are assumed to be functions of the state and the money stocks, but not explicitly of time, we are already presuming the existence of a stationary equilibrium, to be specified below.

We assume that the origin of goods determines which currency is used in transactions. An alternative assumption is that the means of payment is determined by the destination. See Helpman and Razin (1983 b) for the different implications of these two assumptions.

7. The stochastic difference equation for the price of claims to home output can be solved forward in the usual way - disregarding bubble solutions - to
express the price as the present discounted expected marginal utility of the stream of future output divided by present marginal utility of wealth. Similarly the stochastic difference equation for the price of home currency can be solved to express this price as the present discounted expected marginal utility of the stream of future liquidity services divided by present marginal utility of wealth.

8. Note that by (2.14a) and (2.9) we can write the nominal interest rate as $i = E[\lambda^r \pi^r_h] / E[\lambda^r \pi^r_m]$. This is the closest analogue to the familiar expression $i = U_m(c, m) / U_c(c, m)$ in a model where real balances enter the direct utility function, that is the expression according to which the nominal interest rate is simply the marginal rate of substitution of real balances for consumption. See Svensson (1983) for further discussion.

9. The elasticity $\varepsilon A / \varepsilon w \equiv \bar{w} \Delta_w / A$ fulfills $\varepsilon A / \varepsilon w = a E[u' \lambda m' / \varepsilon \bar{w} | \bar{w}, \bar{\omega}]$, where $\varepsilon M / \varepsilon \bar{w} = \varepsilon A / \varepsilon \bar{w} = 1$ for $\omega < \bar{w}(\bar{w}, y, y^*)$ and $\varepsilon w / \varepsilon \bar{w} = 0$ for $\omega \geq \bar{w}(\bar{w}, y, y^*)$. By the same argument as in Svensson (1983) it can be shown that $\varepsilon A / \varepsilon \bar{w}$ is the fix-print of a contraction mapping which maps negative fractions into negative functions. Hence $\varepsilon A / \varepsilon \bar{w} \leq 0$.

Note that $A(\bar{w})$ decreasing implies that the conditions $\omega < \bar{w}(\bar{w}, y, y^*)$ (low $\omega$) and $\omega \geq \bar{w}(\bar{w}, y, y^*)$ (high $\omega$) are well-defined.
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