Seminar Paper No. 283

INTERNATIONAL BORROWING AND
TIME-CONSISTENT FISCAL POLICY

by
Torsten Persson
and
Lars E.O. Svensson

INSTITUTE FOR INTERNATIONAL ECONOMIC STUDIES

University of Stockholm
Seminar Paper No. 283

INTERNATIONAL BORROWING AND
TIME-CONSISTENT FISCAL POLICY*

by
Torsten Persson
and
Lars E.O. Svensson

*Presented at the conference, Growth and Distribution: Intergenerational Problems, in Uppsala, June 1984. The major part of this paper was written when we were visiting the National Bureau of Economic Research. Helpful comments were given by participants at the conference and in seminars at Western Ontario, Harvard, MIT, Columbia, and IIES. In particular, we want to thank Andy Abel, Robert Barro, Guillermo Calvo, Daniel Cohen, Gene Grossman, Jeremy Greenwood, Elhanan Helpman, Larry Kotlikoff, Pentti Kouri, Paul Krugman, Assar Lindbeck, Edmond Malinvaud, Maury Obstfeld, Assaf Razin, Alan Stockman, and Sweder van Wijnbergen. Financial support was given by the Swedish Central Bank Tercentenary Foundation and the Sweden-America Foundation.

Seminar Papers are preliminary material circulated to stimulate discussion and critical comment.

July, 1984

Institute for International Economic Studies
S-106 91 Stockholm
Sweden
International Borrowing and Time-Consistent Fiscal Policy

ABSTRACT

We discuss optimal fiscal policy in open economies, using an open-economy version of a model used in the recent work by Lucas and Stokey. An optimal allocation smooths out the tax distortions associated with financing a given sequence of government consumption, and it also smooths out private consumption of goods and leisure by borrowing (lending) on the international capital market in periods with high (low) government consumption. The main question we ask is how the optimal policy can be made time-consistent, when successive governments reoptimize with respect to current and future tax rates, but must honor the government debt obligations. We show that this requires government debt of sufficiently rich maturity to be issued.

First we treat a case with capital controls, where only the government can borrow and lend abroad. Then there is a unique restructuring scheme for the domestic debt that is necessary to give succeeding governments incentives to continue following the optimal policy (here we interpret and extend Lucas and Stokey's results). For a small economy, this scheme is also sufficient for time-consistency, but in an economy large enough to affect its terms of trade, it is also necessary to follow a unique restructuring scheme for the government's (and the country's) foreign debt.

When there are no capital controls, time-consistency is no longer a problem in a small economy. In a large economy, what matters is total government debt and total foreign debt (but not their composition), and again there are unique maturity structures necessary and sufficient for time-consistency. An interesting observation is that in the distorted world we consider, relaxing the capital controls actually deteriorates welfare.

Torsten Persson
Lars E.O. Svensson
Institute for International Economic Studies
S-106 91 Stockholm
SWEDEN

Telephones:
46-8-163066 (Persson)
46-8-163070 (Svensson)
I. Introduction

Recently the dynamic consistency of government policies has attracted a great deal of attention. Since the influential papers of Kydland and Prescott (1977) and Calvo (1978), it has become widely recognized that in a framework of rational expectations optimal policies are often not time-consistent. That is, a government that chose a particular policy at some date by maximizing its objective function will not find it optimal to continue following this policy when maximizing its objective function at some later date. It has also become recognized that some sort of commitments might improve matters relative to the "consistent" equilibrium, when the government reoptimizes as time elapses and the public bases its actions on the understanding of the government's objectives. Some writers have used that insight to advocate the use of totally precommitted policies in the form of fixed rules.

A recent study by Lucas and Stokey (1983) sheds new light on these issues. The (major part of their) paper deals with the following problem: Suppose there is a given path of government spending that can be financed either by distortionary taxes on labor income, or by borrowing. As is well-known from earlier studies by Barro (1979) and Brock and Turnovsky (1980) the optimal fiscal policy, in the sense of maximizing the representative citizen's welfare, involves smoothing the tax distortions over time. Suppose also that the government in each period is allowed to choose its own tax rates, but must honor the outstanding debt obligations held by the private sector. What Lucas and Stokey show is that the optimal policy can indeed be made time-consistent, provided that there exists debt of sufficiently rich maturity (and contingency in the case of uncertainty). Then the government can induce its successors to
continue the optimal policy by a unique restructuring of its debt.

This result is important for two reasons. First, it shows that although some sort of commitments are needed to make the optimal policy time-consistent, this need not involve precommitting all the actual policy instruments. Second, in the words of Fischer (1983), it provides an example of an anti-Modigliani-Miller theorem, where the maturity structure of the debt matters.

In this paper we extend Lucas and Stokey's analysis of optimal fiscal policy to open economics, thereby relaxing their assumption that all debt is a "debt to ourselves." We look both at a small open economy and at an economy large enough to affect the intertemporal terms of trade at which it can borrow and lend. The question we address is whether a time-consistent optimal fiscal policy still exists and if so whether that requires any systematic management of the maturity structure of the domestic and foreign debt.

Section II presents our model, a deterministic open-economy version of Lucas and Stokey's intertemporal general equilibrium model. In the first part of the paper we assume that only the government, but not the private sector, can engage in foreign borrowing and lending. We derive the optimal fiscal policy, as seen from an initial period, and discuss the properties of the resulting allocation. In the following two sections we go on to discuss under what conditions the optimal policy is in fact time consistent.

In Section III we show that time-consistency requires the government's domestic debt obligations to be restructured over time and we discuss the properties of the unique necessary restructuring scheme. That section also
provides an alternative interpretation of Lucas and Stokey's results, which relies heavily on the concept of government "cash-flow"; see also Svensson and Persson (1984).

Section IV discusses the requirements on the government's (and the country's) foreign debt. We show that in a small open economy there are no requirements (except solvency) at all on the foreign debt. In the large open economy, on the other hand, there is a unique maturity structure that sustains the optimal policy.

In Section V we relax the assumption that private agents can not trade on the international capital market. This turns out to modify the results considerably, both in the small and the large economy.

Section VI offers some concluding remarks.

II. Optimal policy

We look at an open-economy, one-good, deterministic version of the model in Lucas and Stokey (1983). Many goods and uncertainty can be added, at the cost of considerable complexity, but this adds nothing essential, at least to the time consistency discussion. As mentioned in the introduction, we start by considering an economy where the government, but not the private sector, can trade on the international capital market. We may therefore think of all transactions with the rest of the world, including imports, as carried out by the government.

The economy's production technology is linear: one unit of labor results in one unit of output. There is one representative consumer with a constant
endowment of labor, which we take to be unity, in each period. Private
collection of goods and leisure in period \( t \) is denoted by \( c_t \) and \( x_t \), while the
given amount of government consumption is \( g_t \). With net imports denoted by \( m_t \),
the economy's resource constraint is thus:

\[
(1) \quad c_t + g_t + x_t \leq 1 + m_t, \quad t = 0, 1, 2, \ldots
\]

Our representative consumer has additively separable preferences

\[
(2) \quad \sum_0^\infty \beta^t U(c_t, x_t), \quad 0 < \beta < 1,
\]

where \( U(\cdot) \) is strictly concave in \( c_t \) and \( x_t \).

There are proportional taxes on labor income, where \( \tau_t \) denotes the tax
rate in period \( t \). At the outset, government debt obligations to the consumer
are predetermined and described by the vector \( \beta = (0^b, 0^b_1, \ldots) \), where \( \beta_t \) is
the consumer's claim to goods in period \( t \) -- the sum of interest and repayment
of maturing domestic debt, or total debt service. Denoting the domestic present
value prices (interest rate factors), which do not necessarily equal the
international present value prices, by \( p = (p_0, p_1, \ldots) \), we can then express the
consumer's intertemporal budget constraint as

\[
(3) \quad \sum_0^\infty p_tC_t - \sum_0^\infty (1-\tau_t)(1-x_t) \leq \sum_0^\infty p_0^b_t.
\]

The consumer maximizes (2) subject to (3), which gives the following
first-order conditions:

\[
(4a) \quad (1-\tau_t)U_c(c_t, x_t) = U_x(c_t, x_t), \quad t = 0, 1, 2, \ldots, \text{ and}
\]

\[
(4b) \quad \beta^tU_c(c_t, x_t) = p_t, \quad t = 0, 1, 2, \ldots,
\]

where we have normalized the present value prices by using units of utility as
the numeraire.
The government's given sequence of consumption \( g = (g_0, g_1, \ldots) \), has to be financed either with the proportional taxes on labor or by borrowing. The borrowing can be done at home or abroad. On the domestic capital market the government trades at prices \( p \), while at the international market the present value prices are \( p^* = (p^*_0, p^*_1, \ldots) \).

Let us now define what we shall refer to as government cash-flow, namely the excess of tax income over total government outlays in a particular period; this concept will turn out to be extremely useful later on. Total cash-flow of the government (at \( t = 0 \)) in period \( t \), \( y_{0t} + z_{0t} \), is thus given by

\[
(5a) \quad y_{0t} + z_{0t} = \tau_t (1-x_t) - g_t - o^t - o^t* - \delta_t, \quad t = 0,1,2,\ldots,
\]

where \( o^t* \) is the claims on goods from the rest of the world in period \( t \) -- the government's foreign debt service. We note that government cash-flow, so defined, is not in general equal to the budget surplus;\(^1\) nor is it equal to net government savings. See Svensson and Persson (1984) for some further comments.

Total government cash-flow has two components: foreign cash-flow given by

\[
(5b) \quad z_{0t} = -m_t - o^t*, \quad t = 0,1,2,\ldots,
\]

that is net exports (the trade surplus) minus foreign debt service (the sum of interest and repayment of foreign debt), and domestic cash flow, which from (5a) and (5b) is given by

\[
(5c) \quad y_{0t} = \tau_t (1-x_t) + m_t - g_t - o_t - o^t, \quad t = 0,1,2,\ldots.
\]

With these in hand we may write the intertemporal budget constraints faced by the time 0-government in an economical way, namely
(5d) \[ \sum_{0}^{\infty} a_{t} y_{0t} > 0, \] and

(5e) \[ \sum_{0}^{\infty} p_{t} y_{0t} > 0. \]

Before going into the governments' decision problem, we have to say something about the behavior of the rest of the world, which we refer to as the foreign country. In principle, we model the foreign country in the same way as the home country, although we make a few simplifying assumptions. First, we assume that foreign consumption of leisure \( x \) is totally inelastic and constant over time. Then we can write the foreign representative consumer's utility as

(6) \[ \sum_{0}^{\infty} \beta^{t} U^{*}(c_{t}^{*}), \quad 0 < \beta < 1. \]

Second, we abstract from all government activity in the foreign country. Since foreign imports are the negative of home imports, we can then write the foreign resource constraint as

(7) \[ c_{t}^{*} < 1 - x_{t}^{*} - m_{t}, \quad t = 0,1,2, \ldots \]

The foreign country's intertemporal budget constraint may be expressed as

(8a) \[ \sum_{0}^{\infty} p_{t}^{*}(m_{t}^{*} + h_{t}^{*}) > 0. \]

Maximizing (6) subject to (7) and (8a), we get the relation

(8b) \[ p_{t}^{*} = \beta^{t} U^{*}(1-x_{t}^{*}-m_{t}) = P_{t}^{*}(m_{t}), \quad t = 0,1,2, \ldots, \]

where we have invoked the same type of normalization as above. These conditions define the foreign country's demand-price functions \( P_{t}^{*}(m_{t}) \). Equations (8) (where (8a) is fulfilled with equality) summarizes what we need to know about the foreign country's behavior.
In equilibrium any decisions of the government must be consistent with private maximizing behavior at home and abroad. Using (4) and (8) we may therefore rewrite (5) as

\begin{align}
\text{(9a)} & \quad U_c(c_t, x_t)(1-x_t - \varepsilon_t - \gamma_{0t} - b_t + m_t) + U_x(c_t, x_t)(x_t - 1) = 0 \\
\text{(9b)} & \quad \sum_p \rho_t^*(m_t)(-m_t - b_t^*) > 0, \\
\text{(9c)} & \quad \sum_t \beta^t U_c(c_t, x_t) y_{0t} > 0,
\end{align}

The government's objective is to choose $c_t$, $x_t$ and $m_t$, so as to maximize the representative consumer's welfare given by (2), subject to its budget constraints (9) and the economy's resource constraint (1). Carrying out the maximization, one obtains the first-order conditions

\begin{align}
\text{(10a)} & \quad \beta^t U_{ct} + \lambda_0 \beta^t [U_{cct}(1-\tau_t)(1-x_t) + U_{xct}(x_t - 1)] \\
& \quad + \lambda_0 \beta^t U_{cct} y_{0t} - \beta^t \mu_{0t} = 0, \quad t = 0, 1, 2, \ldots, \\
\text{(10b)} & \quad \beta^t U_{xt} + \beta^t \lambda_0 [U_{xct} - U_{ct} + U_{cxt}(1-\tau_t)(1-x_t) + U_{xxt}(x_t - 1)] \\
& \quad + \lambda_0 \beta^t U_{cxt} y_{0t} - \beta^t \mu_{0t} = 0, \quad t = 0, 1, 2, \ldots, \text{ and}
\end{align}

\begin{align}
\text{(10c)} & \quad \lambda_0 \beta^t U_{ct} - \gamma_0 (p^*_t - p^*_t m_{0t}) + \beta^t \mu_{0t} = 0, \quad t = 0, 1, 2, \ldots,
\end{align}

where we have employed the shorthand $U_{ct} = U_c(c_t, x_t)$, etc., where $\mu_{0t}$, $\lambda_0$, $\gamma_0$ are the Lagrange multipliers associated with (1), (9a) and (9b), and where we have used that $1-x_t - \varepsilon_t - \gamma_{0t} - b_t + m_t = (1-\tau_t)(1-x_t)$. In the following we shall assume that the first-order conditions are not only necessary but sufficient for an interior unique solution. However, as is well known from the
literature on optimal taxation, this is by no means an innocuous assumption (see Diamond and Mirrlees (1971), for instance).

To understand the different trade-offs in the optimization problem, let us multiply (10a), (10b) and (10c) by $dc_t$, $dx_t$ and $dm_t$, respectively, and add the resulting equations to get

$$
\beta^t (U_{ct} dc_t + U_{xt} dx_t) + \lambda_0 p_t d[\tau_t (1-x_t)] + \lambda_0 y_{0t} dp_t
$$

$$
+ \lambda_0^p p_t dm_t - \gamma_0 (p^* - p^*_{m0}) dm_t
$$

$$
- \beta^t \mu_{0t} (dc_t + dx_t - dm_t) = 0, \quad t = 0, 1, 2, \ldots
$$

The left-hand side of expression (11) thus shows the overall effect on welfare of an arbitrary change in the allocation in period $t$. Each term has a clearcut interpretation.

First, we have the "direct" effect on utility of the change in consumption and leisure, $\beta^t dU_t = \beta^t (U_{ct} dc_t + U_{xt} dx_t)$.

The changes in $c_t$ and $x_t$ also change tax revenue in period $t$ by $d[\tau_t (1-x_t)]$. The effect on utility of the tax change is equal to $\lambda_0 p_t d[\tau_t (1-x_t)]$; the second term in (9). Here, $\lambda_0$ measures the distortionary effect of proportional taxation, more precisely the marginal effect of switching from proportionally to lump-sum taxes, at constant government expenditure. In the sequel, we shall refer to $\lambda$ as the (level of) "tax distortion."

Next, the changes in consumption and leisure in period $t$ imply a change in that period's domestic present value price (interest rate factor) $dp_t$. As a consequence, government "domestic net wealth" $\sum_0^w p_t y_{ot}$ changes by $\gamma_{0t} dp_t$. If government wealth increases (decreases) taxes can be lowered (must be raised) in other periods. In effect then, a wealth revaluation or, alternatively, a debt
devaluation, is identical to a switch to lump-sum taxes. Consequently its effect on utility is \( \lambda \gamma_0 \sigma t \), the third term in (9).

An increase in imports \( \Delta t \), at given tax rates and government spending by (5b) and (5c) increases domestic cash-flow and decreases foreign cash-flow by the same amount. The increase in domestic cash-flow, by itself, increases utility by \( \lambda \sigma \Delta t \), the fourth term in (11). (Conversely, the decrease in foreign cash-flow decreases utility; as we shall see, this appears in the next term.)

Then we have a fifth term reflecting the cost of imports/foreign borrowing. The bracketed expression is the country's true international shadow price of goods in period \( t \), which is not equal to \( p^*_t \) since a change in imports alters intertemporal prices and hence the country's "foreign net wealth" by

\[ z_0 \sigma t \Delta t \] = \[ z_0 \sigma \Delta t \].

The expression \[ p^*_t - p^*_t \Delta t \] can thus be thought of as an "effective price" of imports. This effective price is multiplied by a Lagrange multiplier \( \gamma_0 \) -- the marginal cost of imports -- the value of which recognizes that an increase in imports involves foreign borrowing that ultimately must be repaid by raising distortionary taxes (cf. above).

Finally, the sixth term measures the resource cost of changing the allocation in period \( t \). For all reallocations that obey the economy's resource constraint (1), this term is zero, of course.

In an optimum, further reallocations cannot increase welfare and the sum of these six terms must be zero. Let us discuss briefly the character of the optimal allocation.

It can readily be verified from (10a) and (10c) that, in general \( p_t = b_u \sigma c_t \) is not proportional to \( p^*_t \); that is, it is not optimal to set domestic prices equal to the foreign prices. This is true even for an
economy that is small enough to take world market prices for given (so that $P^* = 0$ in (10c)). The wedge between the domestic and foreign price vector arises as a result of the distortionary taxes and of the possibility to change the real value of the domestic debt. If the economy is large enough to affect $P^*_t$ — its intertemporal terms of trade — there will be an "optimal intertemporal tariff" on top of this, implicitly defined in the optimal allocation.

The optimal allocation will in general smooth both labor supply and consumption. In a closed economy an optimal pattern of taxes financing a given sequence of government consumption should smooth tax distortions over time, which involves government borrowing (lending) in periods with high (low) government consumption; see the discussion and examples in Barro (1979) and in Lucas and Stokey (1983). Essentially, this result comes about because it is desirable to stabilize the wedge $U^*_{ct} - U^*_{xt}$ and keep the relatively stable marginal rate of substitution between consumption and leisure $U^*_{xt}/U^*_{ct}$ as close as possible to the marginal rate of transformation, which is $1$ by definition.

In an economy that can trade at the world capital market, it also becomes desirable to stabilize $U^*_{ct}$ over time. This will involve further smoothing of consumption and leisure than in a closed economy, where the (domestic) resource constraint forces government consumption to completely "crowd out" private consumption and leisure. From another angle, when government consumption is low there will be a high level of resources left for private consumption. Then the country will have a "comparative advantage" in such periods and engage in net exports. Conversely, there will be net imports in periods when government
consumption is high. The smaller the country is in relation to the world economy, the more it can smooth out the adjustment. This general argument, modified to allow for the possibility of devaluing government debt, is basically an argument for production efficiency since the foreign demand price functions are analogous to an intertemporal transformation surface in production. Thus the argument, due to Diamond and Mirrlees (1971), that optimal commodity taxes require production efficiency, holds also here.

The implication of all this is that the country will borrow abroad in periods with positive government borrowing and that the optimal allocation involves a positive correlation between the deficits in the government budget and the country's current account. This result is more general than the present model and would hold also if there were fluctuations in the productivity of the economy's endowment of labor.3

Apart from designing an optimal policy along the lines we have just discussed, the government at time 0 must also issue or retire some debt according to whether its cash-flow in period 0 \( y_{00} + z_{00} \) is positive or negative.

The government at time 0 inherited the domestic and foreign debt obligations \( (0_0, 0_0 b_1, \ldots) \) and \( (0_1 b^*_0, 0_1 b^*_1, \ldots) \) and chooses now in its turn new debt structures \( I_1 b = (I_1 b_1, I_1 b_2, \ldots), I_1 b^* = (I_1 b^*_1, I_1 b^*_2, \ldots) \) that the government at \( t = 1 \) will inherit. For the foreign country to accept the new debt structure, the value of the net issue of foreign debt must satisfy

\[
(12a) \quad \sum_{t=1}^{\infty} b_t (b^*_0 - b^*_t) / p_0^* = -z_{00}^*.
\]

Analogously, the net issue of domestic debt must fulfill
(12b) \[ \sum_{t=0}^{\infty} \frac{p_{t-1}b_{t}}{p_{0}} = -y_{00}. \]

If the government at \( t = 1 \) would be committed to set the future tax rates in accordance with the optimal policy as seen from \( t = 0 \), any debt structure that fulfills (12) would do. If, however, the next government is allowed to set its own tax rates, but must honor the debt obligations it inherits, the restructuring of the government debt becomes a more delicate matter. In fact, we shall see in the following sections that it is precisely by successive revisions of the maturity structure of the debt that the optimal policy can be made time-consistent.

III. Time consistency and domestic government debt

We now turn to describing the conditions under which succeeding governments find it optimal to choose the same allocation as did the government at \( t = 0 \). As we shall show, time consistency requires a successive restructuring of the government's debt obligations abroad as well as at home.

Since the problem turns out to be recursive, we may proceed in two steps. In this section, then, we describe how the government's domestic debt should be restructured over time. The management of foreign debt that maintains time consistency is treated in the next section.

The reader can verify from (10) that \( y_{0t} \) (and thereby \( b_{t} \)) only appears in the first-order conditions for \( c_{t} \) and \( x_{t} \). As a preliminary step, let us add (10a) and (10b), and divide the resulting expression by \((U_{cct} - U_{ct})\) to obtain
(13a) \[ \lambda_0 r_{0t} + \lambda_0 a_{0t} = b_t, \quad t = 0, 1, \ldots, \]

where \( A_t \) and \( B_t \) are given by

(13b) \[ A_t = A(c_t^t, x_t^t) = (U_{c_t^t} U_{c_t^t} - U_{x_t^t})(1 - \tau_t) + (U_{x_t^t} U_{x_t^t})(1 - \tau_t)(1 - x_t^t) - (U_{x_t^t} U_{x_t^t})(x_t^t - 1) / (U_{x_t^t} U_{x_t^t}) \]

and

(13c) \[ B_t = B(c_t^t, x_t^t) = -(U_{c_t^t} U_{c_t^t} - U_{x_t^t}) / (U_{x_t^t} U_{x_t^t}), \]

and where \( B_t \) is positive as long as taxes are positive and consumption and leisure are both normal.\(^4\) From equation (13), we can obtain an explicit solution for \( \lambda_0 \) for future reference. Multiplying (13a) by \( p_t \), adding for \( t = 0, 1, 2, \ldots \), and using (5a) with equality, we get

(14) \[ \lambda_0 = \sum_{t=0}^{\infty} p_t b_t / \sum_{t=0}^{\infty} p_t a_t. \]

Now, \( b_t \) measures the wedge between marginal rates of transformation and substitution in period \( t \) caused by the distortionary taxes, while it can be shown that \( a_t \) is the derivative of government tax income with respect to \( c_t \) and \( x_t \) (at constant intertemporal prices).\(^5\) Therefore \( \lambda_0 \) provides a natural measure of the level of the tax distortion in the economy (at \( t = 0 \)).

Let us now turn to the decision problem of the government at \( t = 1 \). It maximizes \( \sum_{t=0}^{\infty} b_t U(c_t^t, x_t^t) \) subject to (1) and the \( t = 1 \) analog to (9), which yields first-order conditions for \( c_t^t, x_t^t \) and \( m_t \) on the same form as in (10).

(15a) \[ \beta^t U_{c_t^t} + \lambda_1 \beta^t [U_{x_t^t}(1 - \tau_t)(1 - x_t^t) + U_{x_t^t}(x_t^t - 1)] \]

\[ + \lambda_1 \beta^t U_{c_t^t} y_{1t} - \beta^t u_{1t} = 0, \quad t = 1, 2, \ldots, \]
\[(15b) \quad \beta^t U_{xt} + \beta^t \lambda_1 [U_{xt} - U_{ct} + U_{ctx} (1-x_t) (1-x_t) + U_{xxt} (x_t-1)]\]
\[\quad + \lambda_1 \beta^t U_{ctx} y_{lt} - \beta^t \mu_{lt} = 0, \quad t = 1, 2, \ldots, \text{and}\]
\[(15c) \quad \lambda_1 \beta^t U_{xt} - \gamma_1 (p^*_t - p^*_tm^*l_t) + \beta^t \mu_{lt} = 0, \quad t = 1, 2, \ldots\]

Here \(\lambda_1, \mu_{lt}\) and \(y_{lt}\) are the Lagrange multipliers of the analogs to (9a), (1) and (9b) for the government at \(t = 1\), and the domestic and foreign cash-flows of the government at \(t = 1\), \(y_{lt}\) and \(z_{lt}\) fulfill
\[(16a) \quad y_{lt} = r_t (1-x_t) + m_t - s_t - b_t, \quad t = 1, 2, \ldots, \text{and}\]
\[(16b) \quad z_{lt} = -m_t - b^*_t, \quad t = 1, 2, \ldots,\]

where \(b_t\) and \(b^*_t\) are home and foreign debt inherited from the government at \(t = 0\). We can now see the source of the time-consistency problem. If the multipliers \(\lambda, \gamma,\) and \(\mu_{0t}\) are different than their time 0 counterparts, which, as we shall see, they are in general, then the time 1 government has a different incentive to change the value of its domestic and foreign debt and hence to choose another allocation than the government at \(t = 0\).

However, it turns out that time consistency can be preserved. Subtracting \(15b\) from \(15a\) and manipulating, we get
\[(17) \quad \lambda_1 y_{lt} + \lambda_1 A_t = B_t, \quad t = 1, 2, \ldots,\]

For time consistency, the government at \(t = 1\) obviously must have incentive to choose the same \(c_t\) and \(x_t\) for each \(t = 1, 2, \ldots\) as did the government at \(t = 0\). Since \(A_t\) and \(B_t\) in equations (13) and (17) depend only on \(c_t\) and \(x_t\), they must then be the same. Combining the two equations, we get
(18) \[ \lambda_1y_{1t} = \lambda_0y_{0t} - (\lambda_1 - \lambda_0)A_t, \quad t = 1,2,\ldots \]

Equations (5) and (16) together with (18) yield

(19) \[ B_t = 0 = - (y_{1t} - y_{0t}) = (1 - \lambda_0 / \lambda_1)(y_{0t} + A_t), \quad t = 1,2,\ldots \]

In other words, if the government at \( t = 0 \) restructures its domestic debt according to (19) it will give its predecessor an appropriate incentive to choose the same \( c_t \) and \( x_t \), which is necessary, but in general not sufficient, for a continuation of the optimal tax policy (the government at \( t = 1 \) must also have incentives to choose the same \( m_t \); see further below).

To understand what this rule involves, consider the change in the tax distortion from \( t = 0 \) to \( t = 1 \). We can solve for \( \lambda_1 \) as

(20) \[ \lambda_1 = \frac{\sum_{t=1}^{\infty} P_t B_t}{\sum_{t=1}^{\infty} P_t A_t}. \]

From this expression and (14) it is clear that \( \lambda_1 \) is smaller than \( \lambda_0 \), whenever \( B_t > \lambda_0 A_t \). We see from (13a) that such a decrease in the tax distortion must be associated with \( y_{00} > 0 \), that is a positive domestic cash-flow at \( t = 0 \).

Indeed, solving explicitly for \( \lambda_1 - \lambda_0 \), we get

(21) \[ \lambda_1 - \lambda_0 = - \lambda_0 y_{00} / \sum_{t=1}^{\infty} P_t A_t. \]

So we may deduce

(22) \[ \lambda_1 > \lambda_0 \quad \text{if and only if} \quad y_{00} < 0. \]

Also, from (13) and (15)

(23) \[ \lambda_0 (y_{0t} + A_t) = \lambda_1 (y_{1t} + A_t) = B_t > 0, \quad t = 1,2,\ldots, \]

so that
\( y_{1t} \leq y_{0t}, \text{ if and only if } \lambda_1 > \lambda_0, \quad t = 1, 2, \ldots \)

Hence, by (22) and (24), a positive domestic cash-flow in period 0 is distributed over all future periods so that the cash-flow \( y_{1t} \) is greater than \( y_{0t} \) in all \( t \). With tax revenue and government spending given, this corresponds to a decrease in interest payments and/or debt maturing in period \( t \) (\( b_t > 0 b_t \)). A positive (negative) cash-flow in period 0 should thus be used to buy up (sell) some debt of all maturities; and (19) states the precise way that this should be done.

As long as \( y_{00} \) is non-zero, the government at \( t = 1 \) has a different tax distortion than the government at \( t = 0 \) and therefore a different incentive to change taxes in all periods. The logic behind the debt restructuring is to change the domestic cash-flow, and therefore the "base" for domestic debt devaluations, in all periods so as to exactly match the different tax distortion. By revising the maturity structure of its domestic debt according to the principles we have just described, the government at time 0 can indeed "bind the hands" of its successor so that it has incentives to continue choosing the same \( c_t \) and \( x_t \) and hence the same tax rates.

Exactly the same reasoning can be applied to any pair of governments. A complete description of the sequence of debt restructurings that are necessary for time consistency is therefore obtained just by changing the subscripts 0 and 1 to s and s+1, respectively, in equations (13) through (24).

If the economy were closed, these necessary conditions would also be sufficient for time consistency of the optimal policy. In fact, the
restructuring scheme we have just derived is exactly the scheme derived in the closed-economy model of Lucas and Stokey (1983), although our derivation is different and our interpretation more intuitive (for a further comparison with Lucas and Stokey's results we refer the reader to Svensson and Persson (1984)).

Since we deal with an open economy, however, we must also find out whether succeeding governments have incentives to continue choosing the same $m_t$ for $t = 1, 2, \ldots$ If so, the optimal policy is indeed time-consistent.

IV. Foreign debt

It remains to examine whether the government at $t = 1$ has incentive to choose the same import levels $m_t$, $t = 1, 2, \ldots$, as the government at time 0. Let us first analyze the case where the country is small and cannot affect world market present value prices $p_t^*$, that is, where the import derivative of the foreign demand-price function is zero,

(25) \[ p_{tm}^* = 0, \quad t = 0, 1, \ldots \]

Under this assumption (10c) simplifies to

(26) \[ \lambda_0 \beta^t u_{ct} - \gamma_0 p_{t}^* + \beta^t u_{0t} = 0, \quad t = 0, 1, \ldots \]

from which it follows that $\gamma_0$ fulfills

(27) \[ \gamma_0 = (\lambda_0 \beta^t u_{ct} + \beta^t u_{0t}) / p_t^*, \quad t = 0, 1, \ldots \]

The decision problem for the government at $t = 1$ implies the corresponding first-order condition

(28) \[ \lambda_1 \beta^t u_{ct} - \gamma_1 p_{t}^* + \beta^t u_{1t} = 0, \quad t = 1, 2, \ldots \]

which can be solved for
(29) \[ \gamma_t = (\lambda_1 \beta^t U_{ct} + \beta^t \mu_{lt})/p^*_t, \quad t = 1, 2, \ldots \]

In Section 3 we have seen that a necessary condition for the government at \( t = 1 \) to choose the same \( c_t \) and \( x_t \) as the government at \( t = 0 \) is that \( \lambda_1 \) is given by (20) and \( \gamma_{1t} \) in turn by (18). Then \( \mu_{lt} \) is given by (15a) and (15b), and a unique \( \gamma_1 \) satisfies (29). Since the international cash-flows \( z_{lt} \) do not enter (29), the cash-flows necessary for time-consistency are not unique.

Indeed, any cash-flows that satisfy (12a) and (16b) will do, and the maturity structure of the foreign debt does not matter. Intuitively, since world market interest rates are given, there is no way the government in the small economy can manipulate those interest rates and devalue the foreign debt. Therefore, changes in the incentive to devalue the foreign debt as \( \gamma \) changes over time do not give rise to a time consistency problem.

Instead, consider the large country case, when world market interest rates can be affected, that is when the demand-price derivative \( p^*_{tm} \) is no longer equal to zero. The relevant first-order conditions are now (10c) and (15c) rather than (26) and (28). Multiplying (15c) by \( p^*_t \), adding for \( t = 1, 2, \ldots \), and using \( \Sigma_1^\infty p^*_{tm} z_{lt} = 0 \), we can solve for \( \gamma_1 \) and get

(30) \[ \gamma_1 = \Sigma_1^\infty [(\lambda_1 \beta^t U_{ct} + \mu_{lt}) p^*_t / p^*_m] / \Sigma_1^\infty (p^*_t / p^*_m). \]

Hence, a unique \( \gamma_1 \) fulfills (30) for given \( c_t, x_t, m_t, \lambda_1 \), and \( \mu_{lt} \). It follows that there is a unique sequence of foreign cash-flows \( z_{1t}, \quad t = 1, 2, \ldots \) consistent with (15c). Put differently, there is a unique foreign debt structure \( b^*_t, \quad t = 1, 2, \ldots \), that gives the government at \( t = 1 \) incentive to choose the same sequence of \( c_t, x_t, m_t, \quad t = 1, 2, \ldots \), as the government at \( t = 1 \).
Let us try to characterize the time-consistent sequence of foreign cash-flows. First, by subtracting (10c) from (15c) for \( t = 1,2,... \), dividing by \( P_{tm}^* \), multiplying by \( P_{tm}^* \) and adding for \( t = 1,2,... \), and using \( E_t^{\infty} z_{1t} = 0 \) and \( E_t^{\infty} P_{tm}^* z_{0t} = -P_{tm}^* z_{00} \), we get

\[
(31a) \quad \gamma_1 - \gamma_0 = \gamma_0 P_{tm}^* z_{00} / E_t^{\infty} (P_{tm}^* / P_{tm}^*) + (\lambda_1 - \lambda_0) E_t^{\infty} (P_{tm}^* D_{tm}^* / P_{tm}^*) / E_t^{\infty} (P_{tm}^* / P_{tm}^*) ,
\]

where we also have used (10a) and (15a) to substitute for \( \mu_{1t} - \mu_{0t} \), and where \( D_t \) is given by

\[
(31b) \quad D_t = D_t(c_t, x_t) = \beta_t^t [U_{ct} + U_{cct} (1 - \lambda_t) (1 - x_t) + U_{xct} (x_t - 1) - U_{cct} A_t].
\]

It can be shown that \( D_t \) is positive for \( t = 1,2,... \), if goods and leisure are normal goods.  

With some algebra one gets

\[
(32) \quad \gamma_1 (z_{1t} - z_{0t}) = (P_{tm}^* / P_{tm}^*) [\gamma_0 P_{tm}^* / E_t^{\infty} (P_{tm}^* / P_{tm}^*)] z_{00}
\]

\[
+ [(P_{tm}^* / P_{tm}^*) z_{0t} E_t^{\infty} (P_{tm}^* / P_{tm}^*) / E_t^{\infty} (P_{tm}^* / P_{tm}^*) - D_t] (\lambda_1 - \lambda_0)
\]

from (10c), (15c) and (31). Let us first consider the case when \( \lambda_1 = \lambda_0 \), that is when \( y_{00} = 0 \). Since \( P_{tm}^* / P_{tm}^* - z_{0t} \) is positive, so is the term multiplying \( z_{00} \) in (32). Therefore, when \( \lambda_1 = \lambda_0 \), we have

\[
(33) \quad z_{1t} > z_{0t}, \quad t = 1,2,..., \quad \text{if and only if} \quad z_{00} > 0.
\]

That is, a positive foreign cash-flow in period 0 means that the government at \( t = 1 \) should have all its foreign cash-flows for \( t = 1,2,... \) larger than the government at \( t = 0 \). In terms of the debt structure, the level of foreign debt the government at \( t = 1 \) inherits should be lower for all maturities, \( b_{1t}^* < b_{0t}^* \), \( t = 1,2,... \).

To see why, recall that when \( \lambda_1 = \lambda_0, \gamma_{00} = 0 \), and by (18) \( y_{1t} = y_{0t} \),
\[ \gamma_t (p^*_t - p^* z_{t-1}) = \gamma_t^* (p^* - p^*_t z_{t-1}), \quad t = 1, 2, \ldots, \]

that is, the product of the multiplier \( \gamma \) and the effective price of imports should be the same in each period for both governments. (Note that the effective price of imports is always positive.) Now, when \( z_{00} \) is positive, the remaining cash-flows \( z_{ot}, \quad t = 1, 2, \ldots, \) are on average negative, since then

\[ \sum_{t=0}^{\infty} p^* z_{ot} = -p^*_{00} < 0. \]

Hence, the effective prices of imports of the government at \( t = 1, \ldots, \) are on average lower than the effective prices of imports for the government at \( t = 0, \) \( p^*_t - p^* z_{ot}. \) It then follows that the multiplier \( \gamma_t^* \) exceeds \( \gamma_t. \)

Anyway, if \( \gamma_t^* \) is greater than \( \gamma_t, \) we see from (34) that for time-consistency, the effective price of imports for the government at \( t = 1 \) should not only be lower on average, but lower in each period. This in turn requires its foreign cash-flow \( z^*_{1t} \) to exceed \( z_{ot} \) in each period. The government at \( t = 0 \) should thus use a positive cash-flow \( z_{00} \) to buy up some foreign debt of all maturities.

The situation becomes more complex whenever \( \lambda_1 \neq \lambda_0. \) This is apparent from (32), since the term multiplying \( (\lambda_1 - \lambda_0) \) can be of any sign. If \( y_{00} > 0 \) and \( \lambda_1 < \lambda_0, \) say, we have \( y_{1t} < y_{0t}, \) and it can be shown that

\[ \lambda_1 \beta^{*ct} + \beta^{u1t} < \lambda_0 \beta^{*ct} + \beta^{u0t}. \]

From (10c) and (15c) we get

\[ \gamma_t (p^*_t - p^* z_{t-1}) < \gamma_t^* (p^* - p^*_t z_{t-1}). \]

With less tax distortion and less private debt, the benefit \( \lambda_1 \beta^{*ct} + \beta^{u1t} \) of additional imports for the government at \( t = 1 \) is lower, hence in equilibrium
the cost of imports $\gamma_1(p^*_t - p^*_m z_{1t})$ must be lower. Furthermore, from (31) we see that $\lambda_1 < \lambda_0$ contributes to make $\gamma_1$ less than $\gamma_0$. Intuitively, we can understand that by observing that the multiplier $\gamma$ incorporates that increased imports must eventually be paid by increasing taxes, the costs of which are higher, the larger is the tax distortion $\lambda$. Hence, with $\lambda_1 < \lambda_0$, we have $\gamma_1 < \gamma_0$. We then realize that (35) may hold for effective prices of imports for the government at $t = 1$, either greater or smaller than those for the government at $t = 0$ and we do not get an unambiguous relation between the two governments' foreign cash-flows. However, if the difference between $\lambda_1$ and $\lambda_0$ is sufficiently small relative to $z_{00}$ (that is $y_{00}$ is sufficiently small relative to $z_{00}$), the "pure" effect of $z_{00}$ will dominate and the characterization of the restructuring scheme in (33) still holds.

V. Private capital mobility

Let us now relax the capital controls and allow free private foreign borrowing and lending. With no taxes on international capital flows, home present value prices must be proportional to international prices, that is

$$p_t = \beta^r u^t c_t = \alpha p^*(m^t), \quad t = 0,1,\ldots,$$

for some $\alpha > 0$. This expression will be added as an extra constraint to the government's maximization problem. We may therefore suspect already at this stage that allowing private capital mobility will not necessarily improve welfare; see further below.

We must also distinguish government foreign cash-flows and import, $\tilde{z}_{0t}$ and $m^t$, from the economy's total foreign cash-flows and import, $z_{0t}$ and
m_t. Also, we should distinguish government foreign debt 0_t^* from total foreign debt 0_t, the difference 0_t^* = 0_t - 0_t being private foreign debt. We have the identities

(37a) \[ z_{0t} = -m_t - 0_t^*, \quad t = 0,1,2,..., \text{ and} \]

(37b) \[ \tilde{z}_{0t} = -m_t - \tilde{0}_t^*, \quad t = 0,1,2,... \]

The constraints (9) can now be replaced by

(38a) \[ \sum_0^\infty U_{0t} (x_{0t} + \tilde{z}_{0t}) = \sum_0^\infty U_{0t} y_{0t} + \sum_0^\infty \alpha_p^* (m_t) (-m_t - 0_t^*) > 0, \]

(38b) \[ \sum_0^\infty \alpha_p^* (m_t) (-m_t - 0_t^*) > 0, \text{ and} \]

(38c) \[ U_{0t} (1-x_t - g_t - 0_t - y_{0t} \tilde{m}_t) + U_{0t} (x_t - 1) = 0. \]

The first constraint is the overall government budget constraint (rewriting it using domestic and foreign prices separately is convenient if one wants to compare with the case analyzed in previous sections). The second is the economy's budget constraint relative to the foreign country. The third is the constraint expressing private maximizing behavior which follows since total government cash-flow fulfills

(39) \[ y_{0t} + \tilde{z}_{0t} = r_t (1-x_t) - g_t - 0_t^* - 0_t^* \]

and thus

(40) \[ (1-r_t)(1-x_t) = 1 - x_t - g_t - 0_t^* - y_{0t} + \tilde{m}_t, \]

which can be substituted into (4a) to give (38c).

Carrying out the maximization at t = 0, one gets first order conditions analogous to (10). As before, we assume that these are sufficient and that the optimal solution is unique with respect to c_t, x_t, m_t, and \( \mu_{0t}, \lambda_0 \text{ and } \gamma_0 \).
the multipliers in (1), (38a), and (38b). However, one may easily check
that the solution need not be unique with respect to \( y_{0t}, \tilde{m}_t \) and \( \pi_{0t} \) -- the
multiplier in (36). This follows since only the sum \( y_{0t} + \tilde{z}_{0t} \) and the differ-
ence \( y_{0t} - \tilde{m}_t \) enter in the constraints (38a) and (38c). Indeed, what is unique
is the difference between \( y_{0t} \) and \( \tilde{m}_t \), or alternatively, the sum of \( y_{0t} + \tilde{z}_{0t} \).
Thus total government cash-flow as given in (39) is unique, but not the
government's private and foreign cash-flows separately.

This result can also be seen by deriving the effect on the representative
consumer's welfare of an arbitrary reallocation at \( t \). Restricting the changes
in \( dc_t, dx_t \) and \( dm_t \) to those that fulfill the price constraint (36) and the
resource constraint (1), one gets

\[
\beta^t du_t + \lambda_0 ap^t d[\tau_t (1-x_t)] + \lambda_0 (y_{0t} + \tilde{z}_{0t}) adp^t - \gamma_0 (p^*_t - p^*_0) z_{0t} dm_t = 0,
\]

in analogy with (11). Here we see that total foreign cash-flow \( z_{0t} \) enters
and so does total government cash-flow, \( y_{0t} + \tilde{z}_{0t} \). Intuitively, with the same
prices at home and abroad, what is relevant for the wealth revaluation effects
are the total cash-flows, but not their composition.

Let us now investigate whether a time-consistent policy exists. We first
consider the small economy when

\[
p^*_t = 0, \ t = 0,1,\ldots,
\]

and \( p^*_t \) are given. Not surprisingly, fiscal policy is time-consistent in that
case: Condition (41) is simplified to

\[
\beta^t du_t + \lambda_0 ap^t d[\tau_t (1-x_t)] - \gamma_0 ap^t dm_t = 0
\]

for the government at \( t = 0 \), with an analogous expression for the government at
\( t = 1 \). Since no cash-flow terms enter in (43), there are no wealth revaluation
effects, and the debt structure no longer matters. This is of course quite obvious, since the whole time-consistency problem arises only when intertemporal prices can be affected and wealth revaluation effects are relevant.

What about the large economy problem? Consider the first-order conditions for the government at \( t = 1 \). Plug in the sequence of \( c_t, x_t \) and \( m_t \), \( t = 1,2,\ldots \), that solves the optimal tax problem for the government at time 0. This results in unique \( \lambda_1, \mu_1 \) and \( \gamma_1 \). Having done that, one can indeed find unique total government cash-flows \( y_{1t} + \tilde{z}_{1t} \) and total foreign cash-flows \( z_{1t} \), and non-unique cash-flows \( y_{1t} \) and \( \tilde{z}_{1t} \) (and multipliers \( \pi_{1t} \)) that fulfill the \( t = 1 \) first-order conditions. Hence, a time-consistent policy exists.

This time-consistent policy requires the government to choose the new total foreign debt structure according to

\[
(44) \quad 1_b^* = -z_{1t} - m_t, \quad t = 1,2,\ldots,
\]

and the total government debt structure

\[
(45) \quad 1_b^* + 1_b^* = r_t(1-x_t) - s_t - (y_{1t} + \tilde{z}_{1t}), \quad t = 1,2,\ldots
\]

We have unfortunately not been able to obtain simple characteristics of the relations between foreign cash-flows \( z_{0t} \) and \( z_{1t} \) (that is, the relation between total foreign debt \( 0_b^* \) and \( 1_b^* \)) or of the relations between government total cash-flows \( y_{0t} + \tilde{z}_{0t} \) and \( y_{1t} + \tilde{z}_{1t} \) (that is, the relation between the total government debt structure \( 0_b^* + 0_b^* \) and \( 1_b^* + 1_b^* \)).

Let us finally discuss the welfare properties of the optimal allocation. We have seen that government foreign cash-flow \( \tilde{z}_{0t} \) is not unique. Then it can be chosen equal to \( z_{0t} \), which means that private foreign cash-flow is zero, while maintaining the same level of utility. A restriction to zero private foreign
cash-flows is thus not binding. This in turn suggests (assuming private initial foreign debt $b_t^* - \tilde{b}_t^*$ equal to zero) that we can get the same solution if we start in the situation considered in previous sections, namely when private international borrowing is forbidden and home (relative) prices are not restricted to equal foreign (relative) prices, and then add the constraint that home and foreign prices are equal. Clearly, this implies that the level of utility cannot be higher with private international borrowing, and if the constraints (36) are binding, the level of utility is actually lower with private borrowing. In the distorted world we are considering, it is better to forbid private international borrowing, separate the home and foreign credit market, and allow home interest rates to differ from world interest rates.

VI. Concluding remarks

We have discussed the design of optimal fiscal policy in open economies. Apart from smoothing out the tax distortions associated with financing a given sequence of government consumption over time, an optimal allocation also smooths out private consumption of goods and leisure by borrowing (lending) on the international capital market in periods of high (low) government consumption.

The bulk of the paper dealt with if and how the optimal policy can be made time consistent, when successive governments are allowed to reoptimize with respect to current and future tax rates, but must honor the government debt obligations they inherit. We first treated the case when the government, but not the private sector, is allowed to trade (over time) at international markets. It is then necessary that governments are able to issue domestic
debt of sufficiently rich maturity. We found, and were able to characterize, a unique restructuring scheme of the debt that is necessary to give succeeding governments incentives to continue following the optimal policy. In a small open economy this scheme for domestic debt is sufficient for time consistency. For a large economy, however, it is also necessary to follow a unique restructuring scheme for the government's (and the country's) foreign debt.

When the private sector is allowed to borrow abroad the results changed considerably. For a small open economy, with no possibility to change world market prices, time inconsistency of the optimal policy is no longer a problem and, consequently, no particular pattern of foreign debt obligations is required. In a large economy what matters is total government debt and total foreign debt, but not their composition. We showed that there is a unique maturity structure, for government as well as foreign debt, necessary and sufficient for time consistency. Unfortunately, we were not able to characterize the implied restructuring schemes. In our particular model welfare is higher when private capital movements are prohibited. Whether this conclusion continues to hold under more general circumstances remains an open question.

It is, of course, central to our result that each government can induce its successors to continue the optimal policy that the debt obligations are always honored. As a consequence, we could not allow any taxation of interest income nor of international capital flows. These assumptions of no debt repudiation are, in fact, analogous to a binding commitment of (the sequence of) governments.
The problem arising from allowing taxation of interest income is identical to the classical problem of capital levies. Such levies constitute a non-distortionary and hence desirable form of taxation from a myopic point of view, but are bound to induce under-accumulation once it is understood that they will be used. It is clear then that the absence of capital in the model is crucial for time consistency of the optimal policy.

So is the absence of money, since in a monetary economy governments would have short-run incentives to engage in "surprise" inflations so as to deflate the real value of their outstanding nominal debt obligations. This issue is further discussed by Lucas and Stokey (1983), who show that time consistency in a monetary economy requires a binding commitment to a particular path of nominal prices. Such a commitment is essentially the same as the "honesty" constraint Aurenheimer (1974) imposed on a government maximizing the revenue from money creation, namely that the price level is not allowed to jump.

The requirements of no surprise inflations and no debt repudiation are thus analogous to a commitment not to engage in future capital levies. In fact, a promise not to repudiate the foreign debt is completely isomorphic to a promise not to levy taxes on existing capital in the present model, once the foreign demand price functions are viewed as an intertemporal transformation surface (cf. Section 2).

Although these different commitments might be indistinguishable from each other (and from a commitment to future tax rates, for that matter) in a theoretical model, it still seems that some commitments make more sense than others in the real world. Why this is so and whether any commitments are
credible enough to solve the time consistency problem are very interesting issues. They would clearly need to be addressed in a very different framework from the one employed here, however.\textsuperscript{11}

Opening up the possibility of default lends the problem of optimal foreign borrowing entirely new dimensions, as shown by the recent literature that allows for possible repudiation of foreign debt by sovereign borrowers.\textsuperscript{12} For instance, choosing the optimal maturity structure of the foreign debt then also involves influencing the expectations of foreign creditors in a proper way (as discussed by Cohen and Sachs (1984)). Such considerations would obviously limit the degrees of freedom to engage in debt restructuring schemes like those we have discussed here.

Finally, an important qualification to our results (in the large country case) is the neglect of policy in the rest of the world. Even though our assumption of no government activity could have been relaxed to allow for passive policies in the foreign country -- the standard assumption in trade theory -- that would still not be very reasonable. Strategic considerations abroad would lead directly to a full-fledged game-theoretic analysis of conflicting government policies.
References


Footnotes

1. Only in the special case when all government debt is in the form of consols, so that $b_t$ and $b^*$ consist only of interest payments, does cash-flow correspond to the budget surplus.

2. The second term in (11) comes from the second term in (10a) and (10b) since

\[ \lambda_0 \beta^t [(U_{ct} - U_{ct}) dx_t + (U_{ct} dc_t + U_{ct} dx_t)(1-x_t) + (U_{ct} dc_t + U_{ct} dx_t)(x_t - 1)] = \lambda_0 p_t [(U_{ct} - U_{ct}) dx_t / U_{ct} - dU_{ct} (1-x_t) / U_{ct} + dU_{ct} x_t (1-x_t) / (U_{ct})^2] = \lambda_0 p_t [d(\tau_t (1-x_t))]. \]

3. This is qualitatively the same result as that obtained by Razin and Svensson (1983), who look at optimal taxation in a small open economy subject to productivity shocks.

4. We have $U_{ct} - U_{xt} = \tau_t U_{ct} > 0$, if $0 < \tau_t < 1$. If goods are normal,

\[ (\beta/c)(U_{c} / U_{x}) = (U_{c} U_{cc} U_{xc}) / (U_{x}^2) < 0 \text{ for all } (c,x), \]

which implies $U_{cc} < 0$.


6. Evaluating $D_t$ we get $D_t = [(U_{xc} U_{xc} U_{xc}) + (U_{xc} U_{xx} U_{xc}) (x_t - 1)] / (U_{cc} U_{cc} U_{xc}).$

The first term within brackets and the denominator are negative when goods and leisure are normal, and $U_{cc} U_{xx} - U_{xc}^2$ is positive by the concavity of $U(\cdot)$.

7. We have $\lambda_1 \beta^t U_{ct} + \beta^t U_{ct} - (\lambda_0 \beta^t U_{ct} - \beta^t U_{ct} U_{ot}) = (\lambda_1 - \lambda_0) D_t$, by (10a), (10b) and the definition of $D_t$.

8. In an open economy context an alternative might be a (binding and credible) commitment to fixed exchange rates, provided the economy was small enough not to affect world prices.

9. We owe this point to Guillermo Calvo.
10. This analogy was suggested to us by Elhanan Helpman.
