Seminar Paper No. 296

ON UNION PREFERENCES AND LABOUR
MARKET MODELS: NEGLECTED CORNERS

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Revised version
August 1984

* This paper was written while I was on leave at Princeton University, and was revised during my stay at the Institute for International Economic Studies at the University of Stockholm. Both institutions provided the warmest of hospitality. I am also grateful to John Pencavel for helping me to correct an error in an earlier version, and to David Card for many useful discussions.

Seminar Papers are preliminary material circulated to stimulate discussion and critical comment.

October, 1984

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Abstract

Most of the recent work on the economics of trade unions has used an expected utility (or utilitarian) function to represent the preferences of a labour group. The paper shows that it has been wrongly formulated, presents the corrected version, proves that that is a semi-strictly quasi-concave function, and incorporates it into two labour market models. Corner solutions become important; conventional results change; current testing procedures begin to look more suspect. The models are also used to illustrate and to criticize the "insider-outsider" explanation of persistent unemployment, to discuss possible asymmetries over the trade cycle, and to show why in an efficient bargain the wage may be equal to the value of marginal product.
On Union Preferences and Labour Market Models

1. Introduction

The microeconomics of trade union behaviour is now a lively area in labour economics. Recent papers by Farber (1978), McDonald and Solow (1981), Dertouzos and Pencavel (1981), Oswald (1982), Ashenfelter and Brown (1983), Card (1983), Grossman (1983), MacCury and Pencavel (1984) and others have all attempted to write down a formal model of the union, and some have tackled the hard but important task of testing the model against the data.¹ Many economists are becoming interested in the area.

This paper shows that the most widely used model in the literature has been specified incorrectly. The proper formulation is given, and its properties are derived. What the correction amounts to is the recognition that the indifference curves of a labour group or union are kinked at an employment level equal to current membership. Once all the members of the group have a job, in other words, that group no longer attaches much (if any) weight to its employment goal, and becomes concerned almost entirely with the task of raising the wage rates earned by its members. The consequence for theoretical models is that corner solutions then occur (at kink points around the existing membership level), and these have different characteristics from conventional equilibria.² Although this point appears never to have attracted attention in the formal literature, an idea related to it - or, to be more precise, one suggested by this way of thinking about union preferences - has recently been suggested. Lindbeck and Snower (1984) and
Solow (1984) both discuss the distinction between "insiders" and "outsiders" in models of union wage setting. A later section of the paper illustrates and assesses their conclusions by applying the kinked utility function to an otherwise standard framework.

It is natural to expect the comparative static predictions of the corrected union model to differ, when there are corner outcomes, from those in the orthodox framework. That certainly happens, but is not sufficient to ensure that the correction is an important one. One might argue that all economic models can generate unusual corner equilibria and that in practice such solutions are of little relevance to how economic agents act (we are rarely at corners, in short). This view would be difficult to assess without empirical testing. Nevertheless, one feature of the world might count as evidence against a criticism of this kind. That is the fact that in the real world a typical union's level of membership appears to be very close to the employment level: see, for one example, John Pencavel's discussions of the International Typographical Union in his articles estimating union models. This suggests that corners at employment equal to membership may be of more than passing interest.

Another reason to focus upon the corrected union utility function is that it leads to models which contain intriguing asymmetries. Whilst notions of asymmetric wage and employment behaviour have been discussed informally for decades, it seems fair to say that the idea has never been captured successfully in an explicit theoretical model. This paper makes only limited progress in that direction; but it can generate a certain kind of
asymmetry, and this is a direct result of the introduction of the kinked union utility function. Consider, for example, a stylized trade cycle of unpredictable booms and slumps. Imagine that in an early boom period there is a trade union of size $m$ which finds itself in a state of full employment. Say that a severe slump then occurs, and that employment falls well below $m$. Assume that only current members of the union have the right to vote — or be represented in some other way — at the meetings which are held to decide the size of wage claim to be made. If the slump is extremely short and predictable, and membership remains at $m$, the next time a boom comes the original equilibrium might well be replicated exactly. One might think of this as symmetry over booms and slumps, or as stability of the underlying equilibrium rate of unemployment. If, however, one slump were to persist for an unusually long time, and workers were badly informed about the likelihood of a boom, it is only reasonable to think of some of the unemployed union members as leaving their trade union to seek work elsewhere. This would lower membership — Bain and Elsheikh (1976) discuss this process as it has occurred in Britain — within the trade union. Once the boom were eventually to come, therefore, the voting group of current members would be smaller. Corner equilibria describing full employment would then exist at a level of jobs rather lower than in the previous boom, because the existing trade union members would be reluctant to vote for wage rates requiring employment to exceed the now smaller current membership. In other words, demand increases following from this boom are likely to be translated into greater wage rises, and smaller increases in employment, than in the previous upswing in the cycle.
The union utility function described in the paper has another implication. Because there are situations (namely where employment is above current membership) where the labour group is locally indifferent to the employment level, union indifference curves have horizontal segments. An efficient wage bargain, in the conventional Pareto sense, can then exist in which a concave iso-profit contour is tangential to a flat union indifference contour. Because the profit-maximising labour demand curve is the locus of these points, moreover, it is clear that in this framework an efficient labour contract can coincide with an equilibrium on the labour demand function. Conventional theory is rather different. Leontief (1946) and McDonald and Solow (1981), among others, show that an efficient outcome lies above and to the right of the atomistic demand curve.\(^3\) This feature has been used to test empirically for the existence of efficient or monopoly solutions. The key test in, for example, MaCurdy and Pencavel (1984) involves an attempt to identify whether observed outcomes in the printing industry can be characterized as points at which the wage equals the marginal product of labour. That is taken as the criterion for inefficiency. Yet, if the argument sketched above is valid, labour demand equilibria of this type can be efficient. Although the paper shows that this result holds only for a growing union, that is not, in many countries, an excessively rare phenomenon.\(^4\)

Most of the analysis to come uses an expected utility (or utilitarian) objective function to describe trade union preferences. This is normally justified by an appeal to the idea that unemployment is like a random draw that imposes the same risk of job loss
on each worker. The majority of the recent theoretical work on trade unions and labour contracts makes this assumption. A different form of utility function, however, has been proposed in Dertouzos and Pencavel (1981), Macurdy and Pencavel (1984) and Pencavel (1984), namely the Stone-Geary function familiar from consumer theory; and this appears to do well empirically. Other authors, moreover, have suggested further union maximands. Hence it may be useful to stress here that the basic point to be made in this paper does not depend on any particular functional form for labour union preferences. As long as we are willing to make the usual assumption that individual workers are rational and selfish, and to hold the assumptions that union leaders represent their members and that future union members have no voting rights, there will be an employment level (equal to current voting membership) at which union indifference curves become horizontal.5

Might not altruism make a difference? Is it not the case that, if union workers care about the welfare of those outside their union, we might expect union indifference contours to have a negative gradient for all points on the employment axis? There is no doubt that this is a logical possibility. However, few economists would want to take an axiom of unselfishness as the foundation stone upon which to construct a theory of trade union actions. It is more common to take selfishness — at least in this realm — as the general case, which is the reason for the paper’s style of presentation, and to take altruism to be the special case. There is another reason for the paper’s strategy. As long as altruistic individuals place less weight on others'
utilities than on their own, the union utility function will still have kinks at employment equal to membership. Ian MacDonald (1984) has made a similar point about the possibility that a union might use different welfare weights for its employed and unemployed members. A feature of these models is that the gradient of the preference contour changes abruptly at certain points, but never becomes zero as is assumed in the next few sections of the paper. Less extreme kinks of this kind would preserve most of the results to be proved (although not the possibility of efficient equilibria on the labour demand function).

The paper is organised in the following way. Section 2 sets out the union utility function and shows why corner solutions are important. An efficient bargain model is outlined in Section 3. Section 4 analyses a monopoly union model, discusses the wage preference path and demonstrates that conventional results can break down. Section 5 deals with implications. The principal conclusions are given in Section 6, and the Appendix proves some of the properties of the union utility function.

2. The Union Utility Function

The most commonly used union utility function is a kind of expected utility formulation, which is equivalent, with membership fixed, to the assumption that the union has utilitarian preferences. This function can be written either as

\[ U = nu(w) + (m-n)u(b) \]  

or as
\[ U' = \frac{n}{m} u(w) + (1 - \frac{n}{m}) u(b), \] (2)

where \( w \) is the wage rate, \( b \) is unemployment benefit or an alternative wage, \( n \) is employment, \( m \) is membership and \( u(.) \) is the individual worker's utility function. Functions like these are used in Ashenfelter and Brown (1983), Card (1983), McDonald and Solow (1981), Oswald (1982), Sampson (1983) and, in a median-voter framework, by Farber (1978) and Grossman (1983).\(^7\)

There is a difficulty with formulations (1) and (2). When employment, \( n \), is greater than membership, \( m \), these utility functions are mis-specified. The correct way to write (1) and (2), which has not been done in the literature, is as

\[ U = mu(w) + (u(b) - u(w)) \max [0, m-n] \] (3)

and

\[ U' = u(w) + (u(b) - u(w)) \max [0, \frac{m-n}{m}]. \] (4)

Once everyone in the union has a job, the members are indifferent to changes in employment. This is where (1) and (2) go wrong.

For the purposes of this paper it will make no difference which of these alternative forms, (3) or (4), is chosen, so we shall concentrate on the first and slightly neater specification. Equation (3) is a semi-strictly quasi-concave function, but not a concave one, on \((w,n)\), as long as it is assumed that \( u(.) \) is concave. The proof is in the Appendix. It is also shown there that equation (3) describes a utility function which is continuous, non-decreasing and differentiable almost everywhere. The wage is a normal good and employment is a non-inferior good. Figure 1
Figure 1

The union utility function
sketches the function. Indifference curves are denoted $I_0$, $I_1$ and $I_2$ and all have a non-differentiable 'kink' at $n=m$. For that employment level it is shown in the Appendix that the left-hand gradient of the indifference curves is an increasing function of the wage rate.

Because $U$ is semi-strictly quasi-concave, the typical maximisation problem which occurs in the microeconomics of unions may still be well-behaved. Interior solutions like those in the papers cited above look like equilibrium $e$ in Figure 2 (the shaded area is some general constraint), but there is now an extra kind of solution, at a corner, which is depicted in Figure 3.

There are at least three reasons why corner equilibria of the sort in the diagram may be important. First, the comparative static properties of the model are different from those studied in the published literature. Second, most real world trade unions run a union shop (in US terminology) or an open shop (the British equivalent) in which every new worker must join the union. In these circumstances membership tends to be close or equal to employment, which is not an idea that is easily fitted into the main models of the literature. Third, so far all econometric work on the microeconomics of union behaviour has ignored the possibility of corner solutions. Hence the structural models in Farber (1978), Dertouzos and Pencavel (1981), Ashenfelter and Brown (1983), Carruth and Oswald (1983) and MacCurdy and Pencavel (1984) run the risk of being mis-specified, because all study only tangency optima. To make these points clearly, however, it is necessary to specify a complete microeconomic model of the labour market.
Figure 2

An interior solution
A corner optimum
3. A Simple Efficient Bargain Model

A new and important line of research has now begun on the question of whether real unionized labour markets are better characterized as efficient bargains or monopoly equilibria. The main papers are Ashenfelter and Brown (1983) and Macurdy and Pencavel (1984). Using data on the International Typographical Union the authors use the fact that, by orthodox theory, it is only at monopoly outcomes that the wage rate is equal to the marginal product of labour. A significant difficulty with this kind of exercise is that, as this section tries to show, a growing union will set an efficient wage bargain which is at a point where the wage rate equals the worker's marginal product. This result does not seem to be widely known.

Imagine an industry in which one large union negotiates with one large employer. Let the firm maximise profits \( \bar{\pi} = pf(n) - wn \), where \( f(n) \) is a strictly concave production function and \( p \) is the parametric price of output. Assume that the union can force the firm to accept profit \( \bar{\pi} \) and that the two agents together fix a Pareto optimal bargain. The efficient wage and employment combination is the solution to the problem:

\[
\max_{w,n} \quad U = mu(w) + (u(b) - u(w)) \max [0, m-n] \quad (5)
\]

subject to \( pf(n) - wn \geq \bar{\pi} \) \quad (6)

The second derivative of the constraint, in \((w,n)\) space, is

\[
\frac{\partial^2 w}{\partial n^2} = \left( npf''(n) + 2[w-pf'(n)] \right) / n^2 , \quad (7)
\]
which cannot be signed, so that the constraint set is not necessarily convex. We shall ignore this, as the literature had done, and note simply that there is a neighbourhood around \( w = pf'(n) \) in which there is a local solution to (5) and (6). For the case \( n < m \) the contract curve then slopes upwards (McDonald and Solow (1981)) with gradient

\[
\frac{dw}{dn} = -\frac{f''(n)u'(w)}{u''(w)[pf'(n)-w]} > 0. \tag{8}
\]

At \( m = n \), however, the contract curve becomes vertical. Figure 4 therefore describes the complete contract curve when membership is fixed at \( m \) and there is a given output price. But as soon as we bear in mind that a trade union and a firm can, if they wish, expand both membership and employment, it becomes clear that the locus in Figure 4 is not the only form which is possible. Consider, for example, the contract curve in Figure 5. It also represents a set of Pareto-efficient wage bargains between the two sides of industry. The distinguishing feature of this locus is that it includes tangency points (against the horizontal segments of the union's indifference curves) which lie to the right of existing membership. Hence a big market shock - one shifting the iso-profit curves sufficiently - can induce the union and employer to expand employment above the current membership of the labour group.

The principal conclusion to be drawn from Figure 5 is that efficient bargains can lie on the employer's demand curve (point \( e' \), for example), because the labour demand function is the locus of turning points of the iso-profit contours which are marked.
The contract curve $CC^1$ (given $m$ and $p$)
Figure 5

An alternative contract curve $CC^1$
(given $m$ and $p$)
This is also, trivially, the contract curve, because union indifference curves in this region are horizontal. More intuitively, each current worker and union member is assured of a job when employment exceeds membership, so that each individual (and his or her trade union) then wishes only to maximise the wage rate. This means that the union is locally indifferent to the total number of jobs. An efficient labour agreement will thus set employment at the level desired by the firm (which is not indifferent about employment), and this point must therefore lie on the employer's labour demand function.

The model produces a paradox, however, and it is one which also occurs in another branch of economics. A fall in the price of output is necessary to produce growth in membership beyond  

This result, noted by Vanek (1970), for example, in the formally equivalent literature on labour managed firms, follows from the fact that the turning point of the \( \pi \) iso-profit contour moves down and to the right when the price of the product declines. To prove this, let the wage be

\[
 w = \frac{pf(n) - \pi}{n},
\]  

(9)

which the union wishes to maximise. At an optimum, then, the appropriate choice of employment, \( n^* \), requires that the wage equal the value of the marginal product of labour. Hence

\[
 \frac{\partial n^*}{\partial p} = \frac{f(n) - nf'(n)}{n pf''(n)},
\]  

(10)

which is unambiguously negative by the concavity of the production function. Moreover, if
\[ w^*(p, \pi) = \max_n \frac{p \pi(n) - \pi}{n}, \]  

(11)

we have that

\[ \frac{\partial w^*}{\partial p} = \frac{f(n)}{n}, \]  

(12)

which is unambiguously positive.

It is difficult to know how one should react to this feature of the model. We are more used, of course, to the idea that it is a boom in product prices which will bring forth expansions of employment. Yet the model's contrary conclusion is unambiguous and apparently firmly based on sensible axioms. For the purpose of empirical work the result has the merit that it gives a sharp prediction: with profits held constant a fall in the product price is required to induce an increase in union membership and employment. Our intuitions about the world might suggest that this test is likely to fail, but that view may be unreasonably influenced by the fact that in reality the level of profit is not constant. The matter can only be settled empirically.

4. A Monopoly Union Model

Assume now that the firm and the union do not set an efficient wage/employment bargain, but rather that the union maximises against the firm's labour demand curve. Do the same kinds of corner solutions occur?

The union knows in this case that its wage rate is the value of the marginal product of labour. Hence the maximisation problem which it faces is
Maximise \( J = \mu(u'f'(n)) + (u(b) - u(p'f'(n))) \max [0, m-n] \) \( (13) \)

Let the solution be \( n^* \). If \( m > n^* \), and the problem is well-behaved, there will be a unique optimum characterised by the first-order condition

\[
J_n = u'(w)p'f''(n) + u(w) - u(b) = 0. \tag{14}
\]

Then it follows immediately that small changes in union membership, \( m \), leave \( n^* \) unchanged. The union's marginal benefit from raising employment is \( u(w) - u(b) \), the difference between an employed member's utility and an unemployed member's utility. Its marginal cost is \( u'(w)p'f''(n)n \), the fall in utility of each employed person multiplied by the number of those with jobs. Both marginal benefit and marginal cost are independent of total union membership; hence \( dn^*/dm \) is zero.

This result, proved in Oswald (1982) and Farber (1983), and implicit in many other studies, is no longer true when corner solutions are considered. Say \( J_n = 0 \) occurs at \( n > m \). Then equation (14) cannot describe the solution. Because the wage rate of an individual is always a declining function of employment, no union worker in this monopoly framework would ever vote for an expansion of membership. Thus the problem can be written more precisely as

Maximise \( R = \nu(u'f'(n)) + (m-n)u(b) \) \( (15) \)

\[ \max J_n \]

subject to \( m - n \geq 0 \) \( (16) \)

with solution (assuming \( n \) strictly positive)
\[ u'(w)pf''(n)n + u(w) - u(b) - \phi = 0 \] (17)

\[ \phi \geq 0, \ m-n \geq 0 \ (\text{comp. slack}), \] (18)

where \( \phi \) is a multiplier. At employment levels below membership, \( m \), it is well known that it is not possible to prove that a rise in the output price will always raise employment. However, if workers have relative risk aversion greater than unity, it does follow that \( \frac{dn}{dp} > 0 \). To see why, assume \( m > n \). Then, by a conventional method, we know that

\[ \text{sign} \frac{dn}{dp} = \text{sign} \left\{ f''(n)[u'(w) + u''(w)w] + u'(w)f'(n) \right\} \] (19)

where the second derivative \( R_{nn} \) is negative for a maximum. Hence, if \( [u'(w) + u''(w)w] \) is negative, which is the requirement that relative risk aversion be above unity, the comparative static result is established. It will be convenient for the next diagram to make this assumption about workers' aversion to risk.

Figure 6 presents a diagram of the union's optimal wage and employment path as output prices increase. It is not necessarily the case that wages are raised by higher product demand, but the diagram assumes that. One sensible name for the locus in wage-employment space thus generated would be 'the output price expansion path'. Following Cartter (1959), however, it can be thought of as a wage preference path. It is interesting to note that it has a similar shape to the ones drawn - on the basis of entirely different and intuitive arguments - in Cartter's book.\(^{13}\)

If \( m - n \geq 0 \) is a binding constraint, as at point \( c \), then it follows trivially that a rise in membership will also raise employment. But that membership change would have to be forced upon the
Figure 6

The wage preference path for a monopoly union
union. Unlike the efficient bargain example, a labour monopoly model will not produce any endogenous growth in trade union size. Finally, it can be checked that other normal comparative static results, like those in McDonald and Solow's (1981) first section, or in Oswald (1982), need no longer hold. Two unambiguous results in the conventional framework are that a small rise in unemployment benefit reduces employment and that a small fall in the income tax threshold raises employment. Neither is true at corner solutions of the sort considered in this section.

5. **Membership and the Insider- Outsider Distinction**

The kinked union utility function makes clear the sharp distinction between those inside and those outside the union. Union members' interests are represented in the trade union's preferences; those of other workers are not. This notion, which is simply another way of saying that it is the actual rather than potential members of a group who have the right to vote, has been discussed recently in Lindbeck and Snower (1984) and Solow (1984). Robert Solow's paper can be interpreted particularly simply within the analytical framework developed in earlier sections. He makes three main points.

(i) In a monopoly union model the insiders will wish to exclude outsiders. Demand increases will then be translated purely into wage rises.

(ii) In an efficient bargain model things are less clear, but it is possible to make a case for the "proposition that the wage-bargaining market form is more favorable
than the wage-setting market form to the employment of outsiders".

(iii) There is "a credible case for the following proposition: one reason for the persistence of unemployment over a wide range of fluctuations of aggregate demand is the willingness and ability of insiders to convert higher demand into higher wages for themselves rather than increased access to jobs for outsiders".

It is possible to make an argument against Solow's third and most important point. Deferring that for the moment, however, points (i) and (ii) are almost immediate corollaries of the analysis described earlier. Conclusion (i) follows from Figure 6. Once employment has hit the membership constraint any further increase pushes wages up and keeps employment fixed. Conclusion (ii) is illustrated by Figure 5 and the result, proved earlier in the text, that a sufficiently large market shock (but, paradoxically, a price fall) could generate a new equilibrium to the right of existing membership. In this case outsiders are permitted to enter the group. Both of Solow's propositions therefore emerge in a straightforward way – perhaps even in a simpler way than in his own model with training and two labour types – from the pictures suggested by the kinked union utility function.

Is the persistence of unemployment something to do with the power of insiders to keep out outsiders? It may be, but two objections must first be faced. One can be seen by considering Figure 6. While a demand rise at $n = m$ certainly pushes up wages and not employment, a demand fall from the higher equilibrium
will act in an exactly symmetric manner. In other words, a slump should produce a reduction in wages sufficient to maintain full employment of the union membership. The difficulty with this model of unemployment, therefore, is that it has quite the opposite characteristics of those we have normally seen ourselves (see McDonald and Solow (1981), for example) as seeking. The model has perfect wage flexibility rather than wage rigidity, and it predicts that large demand booms will generate no employment increases (at least, to be precise, beyond current membership).

The second question mark over an insider-outsider explanation of unemployment is rather different. Consider a group of insiders who are told by their firm that, because of a recession, one tenth of members are to be laid off. This is now a common event in the world, and one would naturally expect a theory of unemployment to take it as the main fact to be explained. Yet the insider-outsider distinction is apparently not relevant here: the outsiders play no role. The difficult task is to explain why current insiders agree that some proportion of their ranks will have to be dismissed. This is again, of course, the question of why wages are sticky downwards, and it is not one with which an "insider-outsider" model of involuntary unemployment can especially help us.

So far nothing has been said about the effects of changes in union membership. Yet, as Figure 1 makes clear, alterations in the size of the labour group must shift the position of the kinks in union indifference curves. One place to start is by considering a world in which membership has been allowed to grow, an efficient bargain has been established, and the new point is a long run equilibrium. Figure 7(i) shows this position before any growth in
membership. Assume that new members have the same rights and
degree of influence as old ones, and that the new outcome is a
steady state. Then this stationary equilibrium has the form
sketched in Figure 7(ii). Equilibrium is at e; the membership
change causes the locus of kink points to move out; the outcome
is on the labour demand curve (not drawn).

To look at the same phenomenon in a different way, the
preferences of a labour group are endogenous rather than exogenous.
This is less revolutionary than it sounds, because each individual
here has a normal and unchanging utility function defined on his
or her wage rate. What alters is not how a single worker values
income, but the number of those individuals whose valuations count.
This produces the possibility of asymmetric behaviour: if 'preferences'
change irrevocably at some point in the trade cycle, the upswing may
not merely be a mirror image of the downswing. There are two
channels through which this might work (the relevant diagrams are
straightforward enough to be omitted). First, if new members are
taken on, so that the locus of kinks at m shifts rightwards,
this forever biases employment towards levels higher than the old
membership size. The new contract curve now has a longer upward
sloping segment. Second, say that employment is reduced well
below membership. Say too that union members become tired of
waiting for a chance to work again in this sector and, perhaps
because they believe employment will never pick up, leave the
union to go elsewhere. This forever biases employment downwards,
because the number of insiders shrinks. Whether these kinds of
asymmetries lead anywhere interesting is not entirely clear. They
do seem, nevertheless, to suggest microeconomic possibilities that
might matter for macroeconomics, and they follow merely from what this paper has argued is a correct specification of trade union preferences.

6. Conclusion

Recent research efforts on the economics of trade unions have attempted to construct and to test microeconomic models of how unions behave. The purpose of this paper has been to show that the most commonly used model is incorrectly specified. The paper has suggested a reformulation of the union utility function, has shown that that function is reasonably well-behaved (it is semi-strictly quasi-concave), and has incorporated it into efficient bargain and labour monopoly models. Corner solutions turn out to be important and some well-known results change.

The main economic idea in the paper is that it is the current members of a trade union whose preferences affect the utility of, and the choices made by, that union. Hence the commonly drawn kind of convex union indifference contours become horizontal (unless members are altruistic towards outsiders) at employment levels equal to and above union membership. As there is some evidence that, in practice, employment is equal or rather close to membership, the possibility of equilibria at corner points may be worth taking seriously. Even if union members put some weight on the welfare of non-union workers, the indifference curves will still be kinked at current membership, and this will preserve most of the characteristics of the more extreme and (in its assumption of selfishness) more conventional form of model proposed in this paper.
One interesting conclusion from the analysis is that an efficient bargain can be a point at which the wage and the marginal product of labour are equal. This is a direct result of the newly formulated union objective function. It has indifference curves with horizontal segments, so that tangency points with iso-profit curves can occur where the wage is the same as the value of marginal product. This means that it is potentially misleading to test for efficiency of labour contracts by studying the discrepancy between these two variables. However, and more constructively, such estimation procedures should be appropriate when the industry is declining in size, or in other situations in which membership is greater than employment.\textsuperscript{14} It is for an expanding sector that the technique looks most suspect.\textsuperscript{15}

The paper has shown that union preferences may be endogenous rather than exogenous. This is because the size of the labour group matters, and that can be raised or reduced by trade union actions. As membership, \( m \), varies, the positions of the kinks (see Figure 1, say) must also change. It is this endogeneity of tastes which leads to the possibility, discussed in the paper, that wage and employment behaviour may not be as symmetric as conventional neoclassical models have always suggested.

The paper may also have some bearing on the "insider-outsider" explanation of involuntary unemployment which Lindbeck and Snower (1984) and Solow (1984) have recently proposed. A straightforward application of the union utility function of Section 2 produces the conclusion that those who are currently members of a union will wish to allow in extra "outsiders" only under special circumstances. The paper's kinked utility function suggests diagrams
that capture this in a particularly simple way. Nevertheless, it has been argued here that there are flaws in this as a new theoretical explanation of the persistence of aggregate unemployment. First, at corner equilibria it is the wage that is flexible and employment that is sticky. This is the reverse of our macroeconomic presumptions. Second, the model's explanation of lay-offs among insiders is exactly that which can be found in earlier models of unionized labour markets. Neither criticism, however, invalidates the idea that there is an important distinction between "insiders" and "outsiders". Indeed this paper's correction of standard theory rests upon just that.
Appendix

Part I

Assume that the individual worker's utility function is a scalar-valued concave function \( u(.) \) which is continuous, increasing and bounded on \([0, \infty)\). Let its argument be denoted the wage, \( w \). Define employment as \( n \in (0, \infty) \) and membership as \( m \in (0, \infty) \). The union's utility function, which maps \((0, \infty) \times (0, \infty) \to \mathbb{R}^+\), is

\[
U = m u(w) + (u(b) - u(w)) \max [0, m-n]
\]

(20)

It will also be assumed that \( u(w) \geq u(b) \). We wish to prove that this is (i) a semi-strictly quasi-concave function which is (ii) continuous, (iii) differentiable almost everywhere on \([0, \infty) \times [0, \infty)\) and (iv) non-decreasing.

The first property is the only one which requires a detailed proof.

(i). It is only necessary to prove that \( nu(w) \) is quasi-concave. Semi-strict quasi-concavity will follow from this and the linearity in the rest of the function.

Let \( \nu \) be an arbitrary level of \( nu(w) \). Define two wages, \( w' \) and \( w'' \), such that

\[
u(w') - u(w'') > 0
\]

(21)

Hence define \( n' \) and \( n'' \) such that

\[
u(n')u(w') = \nu
\]

(22)

\[
u(n'')u(w'') = \nu
\]

(23)
so that \( n' \leq n'' \). We need to prove that

\[
u(n + (1-\gamma)n') \geq \gamma u(w') + (1-\gamma)u(w'')\]

(24)

for any \( \gamma \in [0,1] \).

Now, by the concavity of \( u(.) \),

\[
u(n + (1-\gamma)n') \geq \gamma u(w') + (1-\gamma)u(w'') \]

(25)

Multiply through by \( \gamma n' + (1-\gamma)n'' \) to give

\[
u(n + (1-\gamma)n') [\gamma n' + (1-\gamma)n''] \geq \gamma u(w') + (1-\gamma)u(w'')\]

(26)

To prove (24) we have to establish that the right hand side of (26) is greater than or equal to \( \nu \). By definition,

\[
u = \gamma \nu + (1-\gamma) \nu \]

\[
= \gamma u(w')n' + (1-\gamma)u(w'')n''
\]

(27)

Hence it is only necessary to prove

\[
[\gamma u(w') + (1-\gamma)u(w'')][\gamma n' + (1-\gamma)n''] \]

\[
\geq \gamma u(w')n' + (1-\gamma)u(w'')n'',
\]

(28)

that is,

\[
\gamma u(w')[\gamma n' + (1-\gamma)n'' - n']\]

\[
+ (1-\gamma)u(w'')[\gamma n' + (1-\gamma)n'' - n''] \geq 0.
\]

(29)
Simplifying terms, this is just

\[ u(w') - u(w'') \geq 0, \]  

which we know from (21).

(ii). Continuity can be proved by using theorem 1-8 of Spivak (1965) and noting that every utility level can be mapped back into an open set \( V \subseteq [0,\infty) \times [0,\infty) \).

(iii). There is an interval of non-differentiable points defined by the set \( Z = \{(w,n) \mid n=m\} \), which lie on a straight line and can be covered by closed rectangles so as to make the sum of their volumes arbitrarily small. Hence \( Z \) has measure zero.

(iv). \( U \) is non-decreasing by the assumptions \( u(w) \geq u(b) \) and \( u(.) \) increasing.

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**Part II**

The paper asserts that the wage rate is a normal good, the employment level is a non-inferior good, and the indifference curves' gradient at \( n=m \) is an increasing function of the wage. It is helpful to take the latter result first.

Define the slope of the indifference curves at \( n < m \) as a function

\[ \mu(w,n) = \frac{u(b) - u(w)}{nu'(w)} < 0. \]

Its partial derivative with respect to the wage is, at \( n=m \),
\[ \mu_w(w,m) = -\frac{1}{m} - \frac{u(b) - u(w)}{[mu'(w)]^2} u''(w) < 0, \]

which establishes that higher indifference curves are steeper.

This immediately also proves the second of the two remaining claims: employment cannot be an inferior good. To check the first, that the wage rate is a normal good, it is sufficient to show that the cross partial of the union utility function is positive. This is just

\[ U_{wn} = u'(w) > 0, \]

which proves the result for \( n < m \). For membership equal to employment the normality of the wage is clear.
Footnotes


2. There may be an analogy here with modern developments in the economics of labour supply.

3. This assumes that the union has no private unemployment insurance scheme. Oswald (1984) documents American evidence on this.

4. Trade unions in the U.S. have typically been declining in size, of course.

5. John Pencavel has tried to convince me that these are not compelling assumptions.

6. Labour contract theory often uses the same form of a labour group’s preferences (see Baily (1974) and Azariadis (1975), for example), and the criticisms to come apply equally well to that literature.

7. Dertouzos and Pencavel (1981) and Macurdy and Pencavel (1984) use a Stone-Geary union utility function. This is not derived from microeconomic axioms, but seems open to the same kinds of criticism as equations (1) and (2). Card (1983) suggests an ingenious utility specification which includes (1) and (2) as special cases, and his results favour those cases.

8. Theoretical papers include McDonald and Solow (1981) and Oswald and Ulph (1983).
9. Price uncertainty and state-contingent contracts would produce the same results.

10. Note that monopoly rents can exist under this specification.

11. In fact, because there is a region of corner solutions to be overcome, the fall needs to be a sufficiently large one.

12. One way to criticize the assumptions, however, is to point to the importance of seniority in real contracts.

13. But Cartter did not draw the right hand segment as exactly vertical.

14. This also applies to sectors which have expanded in the past (and then held the same size), not just to currently growing industries.

15. By chance, in fact, empirical work has so far used data sets drawn from contracting sectors: the coal industry in the USA (Farber (1978)) and Britain (Carruth and Oswald (1983)), and the printing industry in the USA (Dertouzos and Pencavel (1981), Pencavel (1984) and MacCurdy and Pencavel (1984)).
References


