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INTERGENERATIONAL ASPECTS OF PUBLIC TRANSFERS,
BORROWING AND DEBT

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Abstract

The paper analyzes intergeneration distributional effects of taxes, transfer payments, public borrowing and debt. A two-period life-cycle model with overlapping generations is constructed. The model has two specific features: private wealth in both periods enter as arguments in the preference function, and the individuals (inelastically) supply labor in both periods. It turns out that some fiscal policy actions favor both of the currently living generations, possibly at the expense of future generations, while other actions generate conflicts between the currently living generations. The analysis also shows that it makes considerable difference whether the "burden" of public spending, debt and borrowing is defined in terms of wealth, consumption or utility. The different implications of foreign vs. domestic debt are examined.

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I. Introduction

Even though the literature on the "burden" of public borrowing and debt has advanced considerably since the controversies in the fifties and sixties (as summarized in Ferguson (1964)), the most well-known models in this field have not been specifically designed to illuminate intergenerational aspects. In contrast, the present paper attempts to do just that. A generation is then defined as a group of people that are born, or start to work, in the same period.

The policy instruments analyzed are lump-sum transfers, income taxes and public borrowing. We consider the possibility that current taxes and public transfers influence (the expectations concerning)

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future taxes and transfers, and we also bring out the distinction between public borrowing in a period and of entering the period with public debt, domestic or foreign. Distributional consequences of various fiscal policy actions are evaluated in terms of their effects on wealth, consumption and utility. As will be seen, there is not always a one-to-one correspondence between these effects, which confirms James Buchanan's insistence (1958) that it may be misleading to discuss the burden of public borrowing and debt by looking only at a "real" variable such as consumption rather than utility.

To analyze these various issues, we have constructed a life-cycle model with overlapping generations along the lines pioneered by Peter Diamond (1965), a model which was in turn inspired by Paul Samuelson's consumption loan model (1958). More specifically, the present analysis is built on a neoclassical growth model with a constant population of identical individuals living for two periods, a constant returns to scale production sector, and a public sector that pays

lump-sum transfers, levies proportional taxes on labor and capital earnings, and borrows or lends in the capital market. Monetary policy is not considered since there is only one good in the model. The capital stock is owned by the older generation, and the younger generation acquires ownership of it by way of saving (including saving out of inheritance).

Generally speaking, the present model has two main features as compared to other overlapping-generations' models: (i) private wealth holdings are arguments in the preference function, which implies that each generation may end life with positive wealth that is inherited by the next generation; and (ii) the individuals supply labor in both periods of life, which, as we shall see, implies that policy actions in the first period of life of a generation influences the gross wage rate that enters the second-period budget equation of that same generation.

(i) The first-mentioned special feature is in our opinion a realistic psychological extension. Indeed, there is really no good theoretical or empirical reason for the traditional notion in conventional economic theory that just physical consumption (and leisure) is the ultimate source of satisfaction of individuals in the context of economic decision-making. For example, it may be argued that the appreciation of an individual in society, and by himself, is higher if he(she) holds more wealth, in the present as well as at the end of life. It is also quite conceivable that individuals have an "acquisitive instinct" - like Walt Disney's Uncle Scrooge. Moreover, wealth holdings provide a feeling of security and freedom for the individual (cf. Lindbeck (1963)). Final asset holdings in the utility function may also represent the satisfaction of giving bequest (without

explicit reference to the welfare of the heirs). Moreover, the individual may want to create expectations among relatives about inheritance because he hopes to be more kindly treated by presumptive heirs during his life-time.

Preferences as these would certainly reduce the interest of the individual to decumulate wealth late in life. As is well known, available empirical studies show that such a decumulation is quite limited (see King (1983) for a survey). Another explanation for the limited decumulation of wealth at old age is, of course, bequest motives explicitly based on "altruism" towards the heirs, as expressed for instance in Barro's (1974) version of the Ricardian "equivalence theorem".¹ However, since we do not want to exclude the possibility of conflicting interests among generations, we abstract away altruistic bequest motives of this type which by their very nature preclude such conflicts.

Also Peter Diamond's seminal paper de facto assumes away inter-generational conflicts. The possibility of private conflicts is excluded by not allowing intergenerational transfers of private resources (each generation is assumed to end life with zero net wealth), and political conflicts are excluded by focusing on steady states, in which all generations live under identical conditions.

Because of the prominent role played by Barro's approach in the discussion of public debt, it is important to recall his "public debt neutrality" proposition and to pinpoint two critical assumptions behind it.² The first assumption concerns preferences. Barro assumes generally operative "perfect" altruism, in the sense that (a) the utility of an heir enters each individual's utility function, and (b) all individuals in fact choose to leave a positive bequest (i.e. an interior solution is assumed).³ Together with the assumption of a

perfect credit market, this implies that a marginal exogenous shift in the distribution of income between two generations leave their consumption plans unaffected. In other words, intergeneration redistributive neutrality emerges, which strikes us as a very special behavior pattern. However, for deriving the Ricardian equivalence theorem, essentially stating that tax-financed and debt-financed public spending have identical real effects on the economy, it is necessary also to assume that "the representative member" of a generation has perfect information about how increased public borrowing today affects the present value of future tax payments not only for himself but also for his descendants in all future periods. Only with this additional assumption is it possible to infer that a debt-financed public transfer to a generation will be entirely offset by a corresponding increase in the bequest to the next generation, implying that the above mentioned "intergenerational redistribution neutrality" results in "public debt neutrality". Such an assumption about the information set strikes us as stretching the idea of "rational behavior" too far. In the present model, by contrast, private inheritance is introduced without making Barro's extreme assumptions which lead to unrealistic knife-edge properties of his analysis.

(ii) The second characteristic feature of the present model is that the political and economic actions of a generation partly influence the gross wage rate that enters its own budget set, since the individual works in both periods of life. For example, if one takes the time span between two successive generations to be 25 years, and the average life-time to be 75 years, then the first life-period in a two-period overlapping generations model would cover ages 25-50 and the second ages 50-75. Denoting by λ the fraction of life-time

work that is performed in the first period, a reasonable empirical conjecture may be that λ is in the range 0.6 to 0.8. As λ in other overlapping generations models is set to one, each generation in those models influences the real wage rate not for itself but only for the next generation, who so to speak just "inherits" its wage rate from the present generation via the stock of real capital transferred from the present period. On the other hand, with $\lambda < 1$, each generation is confronted with a trade-off between increased consumption "today", for instance via higher public transfers, and higher wage rates "tomorrow" via increased capital accumulation.

If an internationally integrated capital market exists, economic agents cannot choose between domestic and foreign borrowing, as the distribution of the loan among domestic and foreign lenders becomes an endogenous variable rather than a decision-parameter. One way to come around this analytical problem is to assume that the capital market is regulated, in the sense that only the government is allowed to borrow abroad, and that domestic private agents are not allowed to repurchase government debt instruments that have been issued abroad. (This approach is, in fact, taken by Peter Diamond (1965).) In the present paper, the distinction between domestic and foreign debt is instead introduced by letting a generation "inherit" alternative amounts of foreign or domestic public debt - the debt then being treated as an outcome of the economic performance in previous periods, hence as an exogenous initial condition.⁴

In other respects we follow the conventions in most general equilibrium long-term models, by assuming full employment equilibrium and perfect capital markets. To emphasize the role of public borrowing and debt, we neglect the specific issues that are related to

public consumption and public investment. We will focus on the two polar cases of economies with closed and ultra-open small capital markets, respectively - the latter being referred to as a "small open economy", characterized by a capital market that is exposed to a given, worldwide interest rate.

The organization of the paper is as follows. The model is presented in Section 2, and it is used in Section 3 to analyze the effects of various fiscal policy experiments when future taxes and transfers are unchanged. Section 4 shifts from an analysis of policies that change public borrowing in a period to the issue of "inheriting" public debt (domestic and foreign) from previous

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periods. Shifts in future taxes and transfers are discussed in Section 5, where we also take up the possibility of debt neutrality. The results are summarized in a final section.

II. The Model

II.1 Assumptions

The model is composed of a consumption sector, a production sector, and a public sector.

The individual, who lives in two periods, has an endowment of one unit of labor, of which he supplies inelastically the fraction λ in the first period and $1-\lambda$ in the second, where $\lambda \in (0,1]$ is fixed and equal for all generations. (All quantities in the model are measured in per capita units relative to a generation, all generations being of the same size.) There is a single good which can be used either for consumption or investment. With superscripts for generations and subscripts for time periods, the life-cycle consumption pattern and asset holdings of a member of generation t are denoted $c^t =$

(c_t^t, c_{t+1}^t) and $a^t = (a_t^t, a_{t+1}^t)$. Let w_t and r_t be the real wage rate and interest rate, respectively, in period t , and let τ_t be the income tax rate, and T_t^{t-1} and T_t^t the lump-sum transfers to the old and young individuals in period t (where $0 < \tau_t < 1$, $T_t^{t-1} > 0$ and $T_t^t > 0$).

In period t the representative member of the younger generation receives his/her first (net of tax) wage earnings and lump-sum transfer along with the inheritance from the currently older generation, the inheritance being identical with the older generation's final asset holdings. (There are no inheritance taxes.) In the same period, each member of the younger generation makes a decision about how much to consume now and how much to save for the next period. In the second period, the individual receives his second (net of tax) wage earnings and lump-sum transfer along with the principal and (net of tax) interest earnings on savings from the first period.

More precisely, in period t , every young individual chooses $(c^t, a^t) \geq 0$ in order to maximize $u^t(c^t, a^t)$ subject to the constraints

$$(1) \quad c_t^t + s_t^t = (1-\tau_t)\lambda w_t + T_t^t + a_t^{t-1}$$

$$(2) \quad c_{t+1}^t + a_{t+1}^t = (1-\tilde{\tau}_{t+1})(1-\lambda)w_{t+1} + (1+(1-\tilde{\tau}_{t+1})r_{t+1})s_t^t + \tilde{T}_{t+1}^t,$$

where $\tilde{\tau}_{t+1} \in (0,1)$ and $\tilde{T}_{t+1}^t > 0$ are the expected taxes and transfers in period $t+1$ (both being point expectations).⁵ Moreover, s_t^t is the individual's saving for his second period, and $(1-\tilde{\tau}_{t+1})r_{t+1}$ the after-tax (or "effective") interest rate that connects periods t and $t+1$ from the viewpoint of the individual. The chosen current asset (wealth) holding a_t^t , which enters the utility function of generation t , is defined as savings plus the present value of expected incomes in the second period of life, i.e.

$$(3) \quad a_t^t = s_t^t + [(1-\tilde{\tau}_{t+1})(1-\lambda)w_{t+1} + \tilde{T}_{t+1}^t] / (1 + (1-\tilde{\tau}_{t+1})r_{t+1}) .^6$$

We assume the earnings in the second period of life to be positive, i.e. $(1-\tilde{\tau}_{t+1})(1-\lambda)w_{t+1} + \tilde{T}_{t+1}^t > 0$. Consequently, $a_t^t > s_t^t$.

Assuming a perfect capital market, equations (1) and (2) may be consolidated to a single life-time budget constraint. With the current consumption good as numeraire, let ω_t^t and p_{t+1} denote the (expected) life-time wealth and the (expected, intertemporal) price of the next-period good:

$$(4) \quad \omega_t^t = (1-\tau_t)\lambda w_t + T_t^t + a_t^{t-1} + [(1-\tilde{\tau}_{t+1})(1-\lambda)w_{t+1} + \tilde{T}_{t+1}^t] / (1 + (1-\tilde{\tau}_{t+1})r_{t+1}),$$

$$(5) \quad p_{t+1} = (1 + (1-\tilde{\tau}_{t+1})r_{t+1})^{-1} .$$

The representative consumer's decision problem may then be written

$$(6) \quad \max u^t(c^t, a^t)$$

$$s.t. \quad c^t, a^t \geq 0$$

$$c_t^t + (c_{t+1}^t + a_{t+1}^t)p_{t+1} = \omega_t^t$$

$$c_t^t + a_t^t = \omega_t^t .$$

We assume the utility function to be strictly quasi-concave and differentiable, with a positive marginal utility associated with each argument. Since the budget set is compact, convex and non-empty for all $\omega_t^t, p_{t+1} > 0$, this program then has a unique solution, which we assume to be interior. Hence, necessary first-order conditions are

$$(7) \quad \partial u^t / \partial a_{t+1}^t = \partial u^t / \partial c_{t+1}^t ,$$

$$(8) \quad \partial u^t / \partial c_{t+1}^t + p_{t+1} \partial u^t / \partial a_t^t = p_{t+1} \partial u^t / \partial c_t^t .$$

In other words, at optimum the marginal utility of future assets is equal to the marginal utility of future consumption. If the marginal

utility of current assets (a_t^t) were zero, then eq. (8) would be the usual marginal condition with respect to current and future consumption. However, since by assumption current assets influence utility directly, a shift from current to future consumption results in a double gain - utility being enjoyed not only from more future consumption but also from larger current asset holdings. Eq. (8) shows that (with decreasing marginal utilities of current and future consumption) the inclusion of the argument a_t^t in the utility function promotes future consumption and asset holdings at the expense of current consumption. For our further analysis we note (from eq. (8) and the assumption that the marginal utility of future consumption is positive) that at optimum the marginal utility of current consumption must exceed the marginal utility of current assets: $\partial u^t / \partial c_t^t > \partial u^t / \partial a_t^t$.

Let us turn to the production side. When used as capital in production, together with labor, the good is assumed to be non-depreciating. Let k_t denote the capital-labor ratio used in production for period t , and $y_t = f(k_t)$ the corresponding output (per unit of labor). We assume f to be twice continuously differentiable with the usual properties $f' > 0$, $f'' < 0$, and f' ranging from plus infinity to zero. The producers hire capital and labor for production for the next period at real prices r_{t+1} and w_{t+1} per unit, respectively.⁷ (Factor payments are made in period $t+1$, when output becomes available.) Because of constant returns to scale, profits are zero in equilibrium, $y_{t+1} = w_{t+1} + r_{t+1}k_{t+1}$, and profit maximization requires the factor prices to lie on the factor-price frontier ϕ defined by

$$(9) \quad w = \phi(r) \quad \text{iff} \quad \begin{cases} (r = f'(k) \\ (w = f(k) - kf'(k) \end{cases} \quad \text{for some } k > 0.$$

As usual, the derivative of ϕ equals the negative of the corresponding capital intensity: $\phi'(r) = -k$. The demand for capital goods in period t is hence determined by the marginal condition $r_{t+1} = f'(k_{t+1})$, or, equivalently, $k_{t+1} = K(r_{t+1})$ where the capital demand function is down-sloping by the assumed properties of f : $K' = 1/f'' < 0$.

With D_t denoting the per-capita stock of public debt in period t , the budget identity for the consolidated public sector is

$$(10) \quad D_t = (1+r_t)D_{t-1} + T_t^t + T_t^{t-1} - (w_t + r_t s_{t-1}^{t-1})\tau_t.$$

In other words, the public sector pays lump-sum transfers, receives income taxes from current factor payments, and pays the principal and interest of the net debt in the previous period, all bonds having a maturity of one period.

Consumers hold their savings in the form of government bonds or claims to capital goods, the two forms being regarded as perfect substitutes. In a closed economy, equilibrium in the capital market therefore requires

$$(11) \quad s_t^t = k_{t+1} + D_t,$$

i.e. private savings, which in this type of overlapping-generations model is supplied in the capital market by the young generation only (although based in part on the inheritance received from the currently old), should equal the demand for capital plus the stock of issued government bonds. In fact, eq. (11) defines general equilibrium in this (barter) economy since the labor market is assumed always to clear - implying that if the capital market equilibrates so does the product market by Walras' law. More exactly, under the hypothesis that the capital market was in equilibrium in period $t-1$, eq. (11) can be shown to be equivalent to

$$(12) \ y_t = c_t^{t-1} + c_t^t + k_{t+1} - k_t ,$$

i.e. output equals consumption plus investment.

In steady state the equilibrium condition (12) boils down to $y_t = c_t^{t-1} + c_t^t$, an obvious balance identity, as no investment is necessary in steady-state equilibrium with non-depreciating capital.⁸ However, since all generations live under identical conditions in steady-state, an inquiry into intergenerational aspects requires, of course, examination of equilibria also outside steady-state paths. Before embarking on that analysis, however, we briefly consider some general equilibrium and welfare aspects of the model.

II.2 Existence and uniqueness of equilibrium

For the purpose of this analysis, and also for later convenience, let the wealth function Ω be defined by

$$(13) \ \Omega(r_{t+1}) = (1-\tau_t)\lambda w_t + T_t^t + a_t^{t-1} + \frac{(1-\tilde{\tau}_{t+1})(1-\lambda)\phi(r_{t+1}) + \tilde{T}_{t+1}^t}{1 + (1-\tilde{\tau}_{t+1})r_{t+1}} .$$

In other words, for any $r_{t+1} > 0$, $\Omega(r_{t+1})$ is the life-time wealth ω_t^t of a member of generation t , granted that the corresponding wage rate is on the factor-price frontier, cf. eq. (4). Clearly Ω is differentiable and decreasing:

$$(14) \ \Omega'(r_{t+1}) = - (1-\tilde{\tau}_{t+1})p_{t+1}[(1-\lambda)k_{t+1} + (a_t^t - s_t^t)] < 0 .$$

In the square bracket, the first term represents the (negative) effect via the falling future wage rate and the second term the (also negative) effect via the falling present value, due to the higher interest rate, of future incomes ($a_t^t - s_t^t > 0$ by assumption.). Thus, the life-time wealth of an individual falls if the factor prices in the second period of his life move along the factor-price frontier in the

direction of an increasing interest rate (and hence falling wage rate).

Secondly, let the consumption function C^t be defined by

$$(15) \quad C^t(\omega_t^t, r_{t+1}) = c_t^t(\omega_t^t, [1 + (1 - \tilde{\tau}_{t+1})r_{t+1}]^{-1}),$$

i.e. C^t specifies the current consumption by the younger generation corresponding to a given combination of wealth ω_t^t and (gross) interest rate r_{t+1} , at any given value of $\tilde{\tau}_{t+1}$. (Recall that c_t^t is a function of (ω_t^t, p_{t+1}) by program (6).) Assuming differentiability, let C_ω^t and C_r^t denote the partial derivatives of C^t , and let the corresponding (reduced form) savings function S be defined by $S(r_{t+1}) = (1 - \tau_t)\lambda w_t + T_t^t + a_t^{t-1} - C^t(\Omega(r_{t+1}), r_{t+1})$, cf. eq. (1).

For definiteness in subsequent discussions, we assume

$$\underline{A1.} \quad 0 < C_\omega^t < 1 \quad \text{and} \quad C_r^t < |K'|,$$

where the first condition is satisfied if current and future consumption are normal goods and final asset holdings non-inferior (cf. budget equation in (6)). The second part of A1 is fulfilled if the substitution effect of interest-rate changes on current consumption dominates the income effect. If instead the income effect dominates, then A1 requires capital demand to be more sensitive to the interest rate than is current consumption - a conventional requirement of partial ("Hicksian") stability in the capital market. For the comparative-statics exercises to follow it is sufficient that A1 holds locally, i.e. at the equilibrium under consideration.

Finally, let the function Ψ define the excess supply of capital, i.e. the excess of private saving over issued public bonds and demanded productive capital, as a function of r_{t+1} , i.e.

$$(16) \Psi(r_{t+1}) = S(r_{t+1}) - K(r_{t+1}) - D_t$$

(cf. eq. (11)). Thus, a price pair (w_{t+1}, r_{t+1}) defines general equilibrium (in a closed economy) iff $w_{t+1} = \phi(r_{t+1})$ and $\Psi(r_{t+1}) = 0$.

Since Ψ is a continuous function on $(0, +\infty)$, negative for r_{t+1} sufficiently small, and $K(r_{t+1})$ decreases to zero as r_{t+1} tends to infinity, it follows that a sufficient condition for the existence of equilibrium is that the limit superior of $S(r_{t+1})$, as r_{t+1} tends to infinity, exceeds D_t . In other words, at a sufficiently high interest rate, private savers should be willing to absorb all public bonds

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issued in the same period. In particular, this requires the resources available for generation t (cf. eq. (1)) to exceed the public debt in that period, and reversely, if this is not the case then no equilibrium exists in the closed economy since private savers cannot absorb all public bonds.

A sufficient condition for an equilibrium interest rate r_{t+1} to be locally unique is $\Psi'(r_{t+1}) > 0$. (If $\Psi' > 0$ everywhere, then "global" uniqueness is guaranteed.) Now

$$(17) \Psi' = -\Omega' \cdot C_{\omega}^t - C_r^t - K',$$

so in view of the negativity of Ω' and K' we have $\Psi' > 0$ under A1.

In other words: the normality assumption $C_{\omega}^t > 0$ and partial stability assumption $C_r^t < |K'|$ in A1 together imply stability in the capital market with respect to simultaneous movements of both factor prices along the factor-price frontier.

II.3 Welfare

Let $v_t^t(\omega_t^t, p_{t+1})$ denote the (ex-ante) indirect utility function associated with program (6). Differentiation of the constraints with respect to wealth and price gives:

$$(18) \partial v_t^t / \partial \omega_t^t = \partial u^t / \partial c_t^t > 0 ,$$

$$(19) \partial v_t^t / \partial p_{t+1} = - (c_{t+1}^t + a_{t+1}^t) (\partial u^t / \partial c_t^t - \partial u^t / \partial a_t^t) < 0 .$$

Hence, at optimum a marginal increase in life-time wealth, at a fixed future price p_{t+1} , increases utility in proportion to the marginal utility of current consumption - as in standard consumer models. However, a marginal increase in the price of future consumption (and hence also of future assets), at a fixed level of wealth, influences utility not only via the marginal utility of current consumption but also via the marginal utility of current asset holdings. The net effect is negative as usual, since $\partial u^t / \partial c_t^t >$

$\partial u^t / \partial a_t^t$ at optimum (cf. eq. (8)).

Of importance for the subsequent analysis of welfare effects associated with changes in the tax-transfer structure is the relationship between the factor prices (w_{t+1}, r_{t+1}) and the corresponding utility level $v_t^t(\omega_t^t, p_{t+1})$ of generation t . In order to highlight this the reduced-form indirect utility function V be defined by $V(r_{t+1}) = v_t^t(\Omega(r_{t+1}), (1 + (1 - \tilde{\tau}_{t+1})r_{t+1})^{-1})$. In other words, V maps each point on the factor-price frontier on the corresponding welfare of a member of generation t . Its derivative is composed of two terms, one reflecting the effect via changed wealth and the other reflecting the effect via the changed (future) price:

$$(20) V'(r_{t+1}) = \Omega'(r_{t+1}) \partial v_t^t / \partial \omega_t^t - (1 - \tilde{\tau}_{t+1}) p_{t+1}^2 \partial v_t^t / \partial p_{t+1} .$$

By eqs. (18) and (14), the first term in eq. (20) is negative and, by eq. (19), the second positive. The total effect is hence ambiguous in general, and can be written

$$(21) \quad V'(r_{t+1}) = (1-\tilde{\tau}_{t+1})p_{t+1}[(s_t^t - (1-\lambda)k_{t+1})\partial u^t/\partial c_t^t - a_t^t \partial u^t/\partial a_t^t] .$$

However, it is unambiguous in three important limiting cases.

First, in standard overlapping-generations models (in which $\lambda=1$, $s_t^t > 0$ and $\partial u^t/\partial a_t^t = 0$) we note $V' > 0$, i.e. the utility rises as the factor prices move along the factor price frontier in the direction of a higher interest rate. This seems intuitively clear since then the labor earnings of generation t (w_t , determined in the preceding period) are unaffected while the return on their savings increases with a rising interest rate.⁹

Secondly, and more generally, if the marginal utility of current asset holdings still is minimal ($\partial u^t/\partial a_t^t = 0$) but λ arbitrary, then V' has the same sign as the quantity $s_t^t - (1-\lambda)k_{t+1}$. In other words, the utility effect then is positive iff the savings of generation t is sufficiently high to offset the negative effect of the falling wage rate in period $t+1$ (recall that $\phi'(r_{t+1}) = -k_{t+1}$). However, the quantity $s_t^t - (1-\lambda)k_{t+1}$ equals $D_t + \lambda k_{t+1}$ in equilibrium (the existence of which requires the first-period income of generation t to exceed D_t , cf. subsection II.2), so in the empirically relevant case of a public sector in net debt, we conclude that $V' > 0$, just as in standard overlapping-generations models.

Thirdly, in the opposite extreme case of a maximal marginal utility of current asset holdings ($\partial u^t/\partial a_t^t = \partial u^t/\partial c_t^t$), we instead observe $V' < 0$, i.e. that the utility falls as the equilibrium interest rate rises, independently of λ and the size of the public net debt ($s_t^t < a_t^t$ by assumption). In this limiting case, the price effect vanishes (cf. eqs. (19) and (20)), so the total effect on utility boils down to the negative indirect wealth effect.

For convenience in later interpretations we will take $V' > 0$ to be the typical case.

III. Policy effects when future taxes and transfers are fixed

We are now in a position to study the consequences of marginal changes in current taxes and transfers for the two living generations. We postpone (to Section 5) an analysis of the role of future taxes and transfers.

The subsequent discussion is based on the assumption that there exists an equilibrium in period t and that condition A1 holds - implying $\Psi' > 0$. The analysis concentrates on (qualitative) comparative statics effects on wealth, current consumption and welfare. The effects are summarized in Table 1, with the shift parameters in the left-hand column and the endogenous variables in the top row. The "lower-level" signs within parenthesis refer to a small open economy whenever the effects differ from the case of a closed economy.

Table 1: Comparative-statics effects of policy shifts in period t

	dD_t	dr_{t+1}	$d\omega_t^t$	dc_t^t	dv_t^t	$d\omega_t^{t-1}$	dc_t^{t-1}	da_t^{t-1}	dv_t^{t-1}
1. $dT_t^t > 0$: +	+	+	+	+	+	0	0	0	0
	(0)								
2. $dT_t^{t-1} > 0$: +	+	amb	amb	amb*	+	+	+	+	+
	(0)	(+)	(+)	(+)					
3. $d\tau_t < 0$: +	+	amb	amb	amb*	+	+	+	+	+
	(0)	(+)	(+)	(+)					
4. $dT_t^{t-1} > 0$: 0	+	-	amb**	amb	+	+	+	+	+
	(0)		(-)	(-)					
$dT_t^t = -dT_t^{t-1}$									

* positive if $V' > 0$.

** negative if substitution effect dominates income effect of dr_{t+1} .

III.1 Transfers to the currently young

Let us first look at the effects of marginally increased transfers to the currently young generation ($dT_t^t > 0$). In the case of such a ceteris paribus increase in transfers there is a matching change in public borrowing, as required by the public sector budget identity. The induced increase in private saving being smaller than the reduction in public saving, the interest rate connecting periods t and $t+1$ is driven up as noted in Table 1:

$$(22) \quad dr_{t+1}/dT_t^t = C_\omega^t/\Psi' \quad (> 0 \text{ by A1}) .$$

As a consequence, part of the real capital stock is "crowded out" ~~and the real wage rate is lowered. Less evidently, the private wealth~~ of generation t increases, as, by the "stability" assumption A1, the positive direct effect of the transfer dominates the negative indirect effect:

$$(23) \quad d\omega_t^t/dT_t^t = 1 + \Omega' dr_{t+1}/dT_t^t = (|K'| - C_r^t)/\Psi' \quad (> 0 \text{ by A1}) .$$

Also current consumption of generation t will increase, i.e. the positive effect via the wealth variable cannot (by A1) be entirely offset by the rise in the interest rate:

$$(24) \quad dc_t^t/dT_t^t = C_\omega^t d\omega_t^t/dT_t^t + C_r^t dr_{t+1}/dT_t^t = C_\omega^t |K'|/\Psi' \quad (> 0 \text{ by A1}) .$$

Finally, the total effect on the utility of generation t is unambiguously positive by way of the rise in wealth and the fall in the future price, each implying a larger budget set:

$$(25) \quad dv_t^t/dT_t^t = [(d\omega_t^t/dT_t^t)(\partial v_t^t/\partial \omega_t^t) + (dp_{t+1}/dT_t^t)(\partial v_t^t/\partial p_{t+1})] = \\ = [(|K'| - C_r^t)\partial u^t/\partial c_t^t + (1-\tilde{\tau}_{t+1})a_t^t C_\omega^t \partial u^t/\partial c_{t+1}^t]/\Psi' \quad (> 0 \text{ by A1}) .$$

(Here equations (8), (18) and (19) have been used.)

These qualitative conclusions are independent of how labor supply is divided between the two periods of life (as expressed by λ) and of the marginal utilities with respect to current and future assets (as long as equilibrium exists and A1 holds).

In our simple model there are no effects on the currently old.

III.2 Transfers to the currently old

The increased transfer to the currently old ($dT_t^{t-1} > 0$), ceteris paribus, is assumed to have been unanticipated by the members of that generation when they made their consumption/saving decision in period $t-1$.¹⁰ In period t , they consequently face the decision problem

$$(26) \quad \max u_t^{t-1}(c_t^{t-1}, a_t^{t-1})$$

$$\text{s.t. } c_t^{t-1}, a_t^{t-1} \geq 0$$

$$c_t^{t-1} + a_t^{t-1} = \omega_t^{t-1},$$

where ω_t^{t-1} is their actual (ex post) wealth in period t ,

$$(27) \quad \omega_t^{t-1} = (1-\tau_t)(1-\lambda)w_t + (1+(1-\tau_t)r_t)s_{t-1}^{t-1} + T_t^{t-1},$$

which deviates from their ex ante wealth as perceived in period $t-1$

(cf. eq.(2)), since their previous expectation $\tilde{T}_t^{t-1}$ turned out to be

wrong.¹¹ Now c_{t-1}^{t-1} and s_{t-1}^{t-1} are exogenous and fixed and we take a_{t-1}^{t-1}

to be the ex ante quantity defined in eq. (3).¹² Program (26) then

has a unique solution under the assumptions concerning preferences

made earlier; we assume this solution to be interior. Since the

predetermined arguments c_{t-1}^{t-1} and a_{t-1}^{t-1} are functions of $(\omega_{t-1}^{t-1}, p_t)$,

the ex post solution (c_t^{t-1}, a_t^{t-1}) is a function of ω_{t-1}^{t-1} , p_t and

ω_t^{t-1} .¹³

The quantities ω_{t-1}^{t-1} and p_t being fixed, we write $C^{t-1}(\omega_t^{t-1}) = c_t^{t-1}(\omega_{t-1}^{t-1}, p_t, \omega_t^{t-1})$, and assume both c_t^{t-1} and a_t^{t-1} to be normal "goods":

$$\underline{A2.} \quad 0 < C_{\omega}^{t-1} < 1.$$

We are now in a position to study the effects of the shift $dT_t^{t-1} > 0$. First, one may note the simple direct effects for the currently older generation: they experience a rise in wealth, consumption, final asset holdings and utility; see eqs. (a1)-(a3) in the Appendix.¹⁴ Moreover, assuming A1 and A2, the interest rate will increase, and more so than in the case of Section III.1 as now both the old and the young generations (the latter because of the inheritance) will increase their consumption, implying a smaller increase in aggregate private saving; see eq. (a4).

It is readily verified that the utility effect for the young is positive in the "typical" case $V' > 0$, independently of the consumption propensity of the old (cf. eq. (a7)). The effects on the young generation's wealth and consumption, by contrast, are ambiguous.

However, in the extreme case when the old have a minimal (marginal) propensity to consume, i.e. $C_{\omega}^{t-1} = 0$, they in fact hand over to the young the full transfer received, so all effects on the young become identical to those discussed in the preceding subsection. As the marginal consumption propensity C_{ω}^{t-1} of the old rises from zero to one, all direct effects on the young fall from positive values to zero, while at the same time the magnitude of the indirect effects increases. In the extreme case $C_{\omega}^{t-1} = 1$, all effects on the young are purely indirect, and we note $d\omega_t^t < 0$, and $dc_t^t < 0$ if $C_r^t < 0$.

It may finally be noted that in a small open economy, where the factor prices are unaffected, there is always a positive effect on wealth, current consumption and utility of the young due to the increased inheritance from of the old generation.

III.3 Balanced budget redistribution

Let us finally look at the consequences of balanced-budget lump-sum transfer payments to the old, i.e. $dT_t^{t-1} > 0$, financed either by a lump-sum tax on the young, i.e. $dT_t^t = -dT_t^{t-1}$, or by an increased tax rate on all labor and capital earnings in the period, i.e. $d\tau_t = dT_t^{t-1} / (w_t + r_t s_{t-1}^{t-1})$. In both cases, the public budget is balanced (cf. eq. (10)). ~~As in the preceding subsection it is assumed that the~~ present public policy changes were unanticipated by the currently older generation in period $t-1$.

First note that, by the ceteris paribus hypothesis (comprising the income-tax rate in period $t+1$), an upward shift in τ_t is equivalent to lump-sum taxation of both living generations (cf. eqs. (a8)-(a14)). In particular, it is readily verified that in the second policy alternative mentioned above, the equivalent lump-sum tax on the young and lump-sum transfer to the old is βdT_t^{t-1} , where $\beta = \lambda w_t / (w_t + r_t s_{t-1}^{t-1})$.¹⁵ Hence, it is sufficient to consider the case of lump-sum taxation, $dT_t^{t-1} > 0$ and $dT_t^t = -dT_t^{t-1}$, ceteris paribus.

It is obvious that the (ex post) wealth, consumption, final asset holdings and utility of the old generation increase (cf. eqs. (a1)-(a3)). Moreover, the interest rate rises and capital formation is depressed. The reason is that while public saving is unaffected, private saving decreases at fixed factor prices since only part of the transfer to the old is bequeathed to the young who thus suffer a net loss in wealth (by (A2)), cf. eq. (a15).

The wealth of the young decreases as the negative direct effect is amplified by the factor price movement (cf. eq. (a16)). The effect on their current consumption, though ambiguous in general, is negative if the substitution effect of the rise in the interest rate dominates its income effect (eq. (a17)). The consequences for their utility, lastly, is in general ambiguous except in a small open economy, or when the marginal utility of current asset holdings is sufficiently high (since then $V' < 0$, cf. eq. (a18)).

Reversing signs, one may readily compare the effects for the younger generation of a debt-financed transfer to them with a balanced-budget redistribution from the old generation. As for utility, we note a more favorable effect for the young with debt financing than redistribution in the "typical" case $V' > 0$, irrespective of possible effects on private inheritance from the old. The reason being that while the direct effect is the same in both cases, the interest rate increases with debt financing but falls with redistribution (cf. eqs. (25), (a15) and (a18)). The effects on their wealth and consumption, however, depend on the old generation's marginal consumption propensity. For instance, if this propensity is maximal ($C_{\omega}^{t-1} = 1$), then the total effect on the wealth of the young is more favorable in the redistribution case as the indirect effect on their wealth then is positive instead of negative (cf. eqs. (23) and (a16)). The same quality conclusion holds with respect to current consumption if $C_r^t < 0$. On the other hand, in the opposite extreme case of the old having zero marginal propensity to consume ($C_{\omega}^{t-1} = 0$), all all effects on factor prices and the conditions of the young vanish under redistribution since the young generation receives an inheritance which has been reduced by exactly the same amount as the transfer they received.

IV. "Inherited" public debt

So far the analysis has, in addition to balanced budget expansion, dealt with the consequences of public borrowing in a period. Another, and in fact more controversial issue is whether the "inheritance" of public debt from a previous period is a "burden", and if it matters whether the debt is held domestically or abroad.

To highlight the role of the size of initial debt, we first examine the comparative statics effects of "inheriting" a larger stock of public debt D_{t-1} in a closed economy with a given initial productive capital stock k_t . Consequently, also r_t is fixed via the marginal productivity condition. Assuming that the capital market in period $t-1$ was in equilibrium, a marginal difference $dD_{t-1} > 0$ must have been matched by an equal difference in initial private savings: $ds_{t-1}^{t-1} = dD_{t-1}$. In other words, to "be born" in a closed economy with higher public debt and given productive capital means in this case that one is born in an economy with higher earlier private saving that matches the public dissaving. Consequently, in such an economy the older generation has more both to consume and bequeath. By the second-period budget equation (27), $p_t(dc_{t-1}^t + da_{t-1}^t) = ds_{t-1}^{t-1} = dD_{t-1} > 0$, ceteris paribus. Assuming normality in the sense $dc_{t-1}^t > 0$ and $da_{t-1}^t > 0$, we hence have $da_{t-1}^t = p_t^{-1} \alpha dD_{t-1}$ for some $\alpha \in [0, 1)$, i.e. the private inheritance received by generation t is not lower if the public debt is larger.

Since excess demand for capital in period t (at a given interest rate) is higher when D_{t-1} is higher, also the new equilibrium interest rate is higher,

$$(28) \quad dr_{t+1}/dD_{t-1} = p_t^{-1} [1 - \alpha(1 - C_\omega^t)]/\Psi' \quad (> 0 \text{ by A1}),$$

"crowding out" part of the real capital stock formed in period t , and, via the factor price frontier, pushing down the wage rate w_{t+1} . As

a consequence, the wealth of the young generation is reduced if the consumption propensity of the old is high (α small),

$$(29) \quad d\omega_t^t/dD_{t-1} = p_t^{-1} [\alpha(|K'| - C_r^t) + (1-\alpha)\Omega'] / \Psi',$$

where the first term is positive by A1 and the second, representing the indirect effect, negative as noted in Section II.2. Likewise, if the substitution effect dominates the income effect on the young generation's current consumption and the consumption propensity of the old is high, then also the current consumption of the young falls,

$$(30) \quad dc_t^t/dD_{t-1} = p_t^{-1} [\alpha|K'|C_\omega^t + (1-\alpha)(\Omega'C_\omega^t + C_r^t)] / \Psi',$$

where $C_\omega^t > 0$ by A1. The consequence for the utility level of generation t is ambiguous in general,

$$(31) \quad dv_t^t/dD_{t-1} = p_t^{-1} \alpha \partial u^t / \partial c_t^t + V' dr_{t+1} / dD_{t-1}.$$

However, in the "typical" case $V' > 0$, the younger generation clearly experiences an increase in utility, even if the old generation consumes all of their increase in savings themselves (i.e. even if $\alpha=0$). Thus, as long as generation t does not have to service higher "inherited" public debt, the debt is a "blessing" rather than a "burden" for the young generation when measured in terms of utility (assuming $V' > 0$) even though both their wealth and (under reasonable conditions) current consumption goes down.

If generation t not only inherits a larger public debt but also has to service and/or repay the increase (or part of it), by means of a higher tax rate τ_t or lower transfer T_t^t , then the (in most cases negative) effects of such tax/transfer shifts have to be added to (or rather deducted from) the effects discussed above (cf. Table 1), which then raises the possibility of a negative net effect on the utility of generation t .

Suppose now instead that the economy is small and open, so that interest rates (and hence also wage rates) are fixed and given from abroad. Also assume that D_{t-1} and k_t are composed of domestic and foreign claims as follows: $D_{t-1} = D_{t-1}^d + D_{t-1}^f$ and $k_t = k_t^d + k_t^f$. The sum $k_t^d + D_{t-1}^d$ is by definition equal to s_{t-1}^{t-1} , private domestic claims (savings in period $t-1$). Accordingly, the national foreign debt in period $t-1$, E_{t-1} , equals the private productive capital stock plus the public debt minus private domestic claims: $E_{t-1} \equiv k_t^f + D_{t-1}^f = k_t + D_{t-1} - s_{t-1}^{t-1}$.

To clarify the importance of the composition of the debt, let us study the effects of a marginal increase, $dE_{t-1} > 0$, at fixed values of k_t and D_{t-1} , so that $ds_{t-1}^{t-1} = -dE_{t-1} < 0$. We immediately see from the definition of E_{t-1} that to be born in an economy with a higher share of foreign claims out of given total claims means "being born" in an economy with lower earlier private savings. Consequently, in such an economy the older generation has less to consume and to bequeath, so the life-time wealth, current consumption and utility of the younger generation is correspondingly lower. Thus, the analysis supports the common belief that it is better to be born in an economy with a lower share of foreign debt out of given total public debt.

In contrast, Diamond (1965) concludes that it is better for the individual to live in a steady-state society with foreign rather than domestic public debt. The reason for Diamond's conclusion is (1) that in his model the individual leaves no final wealth, so it does not matter for a generation whether public bonds (with interest) are used for consumption by the parent generation or by foreigners, and (2) that domestic borrowing drives the interest rate up and the wage rate down, which in steady state hurts all generations. Thus, a basic

explanation for the divergence between his and our results is the existence of final assets in the present model: a new generation will inherit some public bond holdings when the debt is domestically held, but not if held abroad. Without inheritance, we would obtain the same conclusions as Diamond, viz. that it is immaterial whether public debt is domestic or foreign in a small open economy. The differential effects of domestic vs. foreign public debt in an open economy hence hinges on inheritance.

V. Anticipated shifts in future taxes, and debt neutrality

V.1 Future transfers and taxes

Let us first examine the roles of the expectations $\tilde{T}_{t+1}^t$ and $\tilde{\tau}_{t+1}^t$ in the model. As with a perfect credit market a shift $d\tilde{T}_{t+1}^t$ is equivalent to a shift $dT_t^t = p_{t+1} d\tilde{T}_{t+1}^t$ for the members of generation t , it readily follows that these two shifts have identical effects on r_{t+1} , ω_t^t , c_t^t and v_t^t . Thus, the case $d\tilde{T}_{t+1}^t \neq 0$ has in fact already been analyzed in sub-section III.1 (cf. also Table 2).

A more interesting experiment is $d\tilde{\tau}_{t+1}^t > 0$ (ceteris paribus) since such a shift has distortionary effects on private savings. First, it can be shown that the direct effect (i.e. at fixed factor prices) on the life-time wealth of a member of generation t is negative (positive) if $(1-\lambda)w_{t+1}(\geq) r_{t+1} \tilde{T}_{t+1}^t$, i.e. if the second-period labor supply $1-\lambda$ is sufficiently large (small) as compared to the second period lump-sum transfer $\tilde{T}_{t+1}^t$, the present value of which is increased by the direct reduction in the effective interest rate.

Secondly, the combined direct shifts in ω_t^t and p_{t+1} cause a factor price movement according to

$$(32) \quad dr_{t+1}/d\tilde{\tau}_{t+1} =$$

$$= - ([(1-\lambda)w_{t+1} - r_{t+1}\tilde{T}_{t+1}^t] p_{t+1}^2 C_{\omega}^t + r_{t+1} C_r^t / (1-\tilde{\tau}_{t+1})) / \Psi'$$

(cf. Appendix). The two terms in the large parenthesis account for the wealth effect and the intertemporal price effect, respectively, on private savings at fixed factor prices. (Recall that current public saving is unaffected in this experiment.) If $C_r^t < 0$ and $(1-\lambda)w_{t+1} < r_{t+1}\tilde{T}_{t+1}^t$, then the (gross) interest rate r_{t+1} rises and the wage rate w_{t+1} falls, as in Diamond (1970, eq. (25)), where in fact $\lambda=1$ and $\tilde{T}_{t+1}^t > 0$.¹⁶ Also as in Diamond (1970, Section 6), the effective interest rate $(1-\tilde{\tau}_{t+1})r_{t+1}$ may either rise or fall in the case

$$(1-\lambda)w_{t+1} < r_{t+1}\tilde{T}_{t+1}^t:$$

$$(33) \quad dp_{t+1}/d\tilde{\tau}_{t+1} = p_{t+1}^2 ([|\Omega'| C_{\omega}^t + |K'|] r_{t+1} + (1-\tilde{\tau}_{t+1}) [(1-\lambda)w_{t+1} - r_{t+1}\tilde{T}_{t+1}^t] C_{\omega}^t p_{t+1}^2) / \Psi'.$$

By contrast, if instead $(1-\lambda)w_{t+1} > r_{t+1}\tilde{T}_{t+1}^t$, then the effective interest rate necessarily falls (p_{t+1} increases) by the same equation. The total effects on wealth, current consumption and utility of generation t are summarized in Table 2 below (cf. also eqs. (a19)-(a21)).

In a closed economy, the only unambiguous effects are that wealth and utility of generation t fall in the case when $(1-\lambda)w_{t+1} > r_{t+1}\tilde{T}_{t+1}^t$ and $C_r^t < 0$. The reason is that the negative direct effect on wealth then is accentuated by the induced fall in the effective interest rate. In a small open economy, we note, in contrast, unambiguous increases in wealth and current consumption if $(1-\lambda)w_{t+1} < r_{t+1}\tilde{T}_{t+1}^t$ (and $C_r^t < 0$).

Table 2: Comparative-statics effects of expected policy shifts in period $t+1$

	dr_{t+1}	dp_{t+1}	$d\omega_t^t$	dc_t^t	dv_t^t
1. $d\tilde{T}_{t+1}^t < 0 :$	- (0)	+ (0)	-	-	-
2. $d\tilde{\tau}_{t+1}^t > 0 :*$	amb (0)	+	-	amb	-
3. $d\tilde{\tau}_{t+1}^t > 0 :**$	+ (0)	amb (+)	amb (+)	amb (+)	amb
* If $C_r^t < 0$ and $(1-\lambda)w_{t+1} > r_{t+1}\tilde{T}_{t+1}^t$					
** If $C_r^t < 0$ and $(1-\lambda)w_{t+1} < r_{t+1}\tilde{T}_{t+1}^t$					

V.2 Future tax liabilities due to current transfers

We finally consider the possibility that increased public spending today in favor of a certain generation entails higher future taxes for that same generation. To shed some light on the role of such tax/transfer shifts let us look at the case of increased current transfers to the young generation, resulting either in an anticipated reduction in future lump-sum transfers (case 1) or in an anticipated increase in the income tax rate (case 2).

Case 1 ($dT_t^t > 0$ and $d\tilde{T}_{t+1}^t < 0$). Let the ratio between the present value of the anticipated future transfer reduction and the present increase in transfers be $\sigma = -p_{t+1}d\tilde{T}_{t+1}^t/dT_t^t$. In other words, the direct effect on the wealth of generation t is $(1-\sigma)dT_t^t$. The only modification of the analysis in Section III.1 then is that the derived

effects on factor prices and the wealth, current consumption and utility of the young generation, as expressed in eqs. (22)-(25), will be multiplied by the factor $(1-\sigma)$. To see this, note that, at fixed factor prices, $ds_t^t = dT_t^t - (\partial c_t^t / \partial \omega_t^t)(1-\sigma)dT_t^t$ and $dD_t = dT_t^t$. Consequently, $dr_{t+1}/dT_t^t = (1-\sigma)C_{\omega}^t/\Psi'$, cf. eq. (22). In other words, also the indirect effects are multiplied by $(1-\sigma)$. In particular, the effects on all the variables considered are positive if $\sigma < 1$, zero if $\sigma = 1$, and otherwise negative. We will refer to the special case $\sigma = 0$, analyzed in Section III.1, as a benevolent fiscal policy towards generation t in its second period of life.

Another point of reference is $\sigma = 1$. Temporary debt neutrality then emerges since $d\omega_t^t = dp_{t+1} = 0$: hence all real private variables in period t are unchanged. The intuitive reason for this neutrality is straight-forward, as at given factor prices, the life-time wealth of generation t is unchanged: $dT_t^t + p_{t+1} \tilde{dT}_{t+1}^t = 0$. Since the original consumption plan was optimal and since there are no capital market imperfections the same plan is still optimal, so generation t saves all of the lump-sum transfer dT_t^t to cover the reduction in the next-period transfer. Hence, the supply of capital in period t is unaffected, so the interest rate does not change and temporary debt neutrality arises.¹⁷

In contrast to the Ricardian "equivalence theorem", as developed by Barro (1974) and others, our result is not based upon explicit altruism towards future generations. In Barro's model, the altruistic parents increase their bequest in order to compensate their children's future tax liabilities caused by increased current public spending. In the present model, the parents may anticipate increased tax liabilities for themselves when retired; only if they anticipate tax

liabilities with the same present value as the increase in current transfers will they increase their saving by the same amount while keeping their final assets unchanged. Thus, the same conclusion of debt neutrality can be reached either by assuming that one generation is perfectly benevolent to the next generation in their private behavior or that the next generation completely lacks benevolence to the first generation in their political behavior.

Finally, if $\sigma=1$ and the anticipated future government policies materialize, i.e. $\tau_{t+1} = \tilde{\tau}_{t+1}$ and $T_{t+1}^t = \tilde{T}_{t+1}^t$, then (cf. eq. (10))

$$(34) \quad dD_{t+1}/dT_t^t = (1+r_{t+1})dD_t/dT_t^t + dT_{t+1}^t/dT_t^t - r_{t+1}\tau_{t+1}ds_t^t/dT_t^t = 0.$$

Hence, in this special case the debt neutrality in period t becomes permanent rather than just temporary, in the sense that all real private variables are unchanged since $d\omega_s^s = dp_{s+1} = 0$ for all periods $s \geq t$.

Case 2. The consequences for the economy are much more complex in the case when the young generation anticipates a change in the future income tax rate as a result of currently increased transfers to itself ($dT_t^t > 0$, $d\tilde{\tau}_{t+1} \neq 0$). The reason is, of course, that the shift in $\tilde{\tau}_{t+1}$ has a direct influence not only on wealth but also on the intertemporal price (cf. sub-section V.1). In particular, debt neutrality cannot arise in this case. This is most easily seen in a small open economy. Then

$$(35) \quad d\omega_t^t = dT_t^t - [(1-\lambda)w_{t+1} - \tilde{T}_{t+1}^t r_{t+1}]p_{t+1}^2 d\tilde{\tau}_{t+1},$$

$$(36) \quad dp_{t+1} = r_{t+1} p_{t+1}^2 d\tilde{\tau}_{t+1}.$$

Hence, for every value of $d\tilde{\tau}_{t+1}$ either $d\omega_t^t \neq 0$ or $dp_{t+1} \neq 0$ (or both), so the original interior solution of the individuals' consumption/saving program (6) is no longer optimal.¹⁸

VI. Summary of results

Though most of the policy actions discussed in this paper at first glance might seem certain to favor, or disfavor, a specific generation, the issues often turn out to be quite complex. For instance, we have seen that there is what may be called "weak political consensus" between the young and the old generations about debt-financed transfers to the young, in the sense that while the young unambiguously gain, the old are unaffected — possibly with negative consequences for some future generations which may have to service and/or repay the additional debt (see below). It has also been shown that while debt financed transfers to the old unambiguously favor them, the consequences for the young are ambiguous, with the effects strongly dependent on the marginal propensity of the old to accumulate final assets (and hence in fact leave bequest). Of course, if this propensity is high enough, both wealth and consumption will increase also for the young. Though a high propensity to accumulate final wealth of the old by itself favors also the utility of the young, this variable may be unfavorably influenced by the induced factor price movement. However, in "typical cases", including standard overlapping-generations models (without assets in the utility function and with work only in the first period), the effect on the utility level of the young is unambiguously positive. Moreover, in a small open economy, wealth, consumption and utility of the young are favorably influenced in the cases of transfers both to the young and to the old.

It was also found that a balanced budget transfer to the old, financed by a lump-sum tax on the young in the same period, beside increasing wealth, consumption, asset holdings and utility of the old unambiguously reduces the wealth of the young. Moreover, their

current consumption falls if the substitution effect of a change in the effective interest rate dominates the income effect. The utility of the young will fall if the interest rate does not rise very much, as for example in the extreme case of a small open economy.

We have also compared the consequences for both generations of debt-financed transfers, on the one hand, with the consequences of explicit balanced budget redistributions, on the other hand. We then found, somewhat surprisingly, that in "the typical case" including standard overlapping-generations models, a transfer to the young has a more favorable utility effect if the transfer is financed by borrowing than by redistribution from the old. This means that both

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generations' utility in some cases is better promoted by the mentioned debt-financed transfers than by balanced budget redistribution. It is also worth noticing that in the case of a small open economy, with an exogenous interest rate, debt-financing is always more favorable to the young than a redistribution from the old in terms of all three indicators, the reason being that a redistribution lowers the bequest to the young.

A general conclusion of our analysis was that the indirect effects on wealth and utility typically go in opposite directions. This conclusion is valid also in the special case which we have referred to as "standard" overlapping-generation models. In the empirically relevant case of positive second-period incomes of generation t , its life-time wealth falls as the factor prices move along the factor-price frontier in the direction of a higher interest rate (and lower wage rate). If the public sector is in net debt then the utility of the same generation simultaneously increases, granted that the marginal utility of first-period asset holdings is sufficiently

low. This general property implies that evaluations in terms of wealth and utility can give radically different results.

As has been pointed out, several policy actions tend to leave higher public debts to future periods and future generations. This raises the issue of the consequences of "inheriting" a larger public debt from previous periods, i.e. of entering working life with a higher level of public debt. In our model, "inheriting" domestically held public debt has a negative effect on the wealth of the young unless the old have a high marginal propensity to build up final assets. The young also experience a negative effect on their consumption if the substitution effect of an interest rate change dominates the income effect. While the effect on their utility is ambiguous in general, it is positive in "the typical case".

We also find that it is better to "be born" in an economy with domestic rather than foreign debt, in contrast to the conclusion by Peter Diamond that it is better to live in a steady-state society with a foreign rather than a domestic public debt -- the main reason being that Diamond assumes no inheritance and that domestic borrowing drives up the interest rate and reduces the capital stock and the wage rate, which in steady state hurts all generations.

All these conclusions refer to the case when the two generations that "inherit" the increased public debt do not have to service and/or repay the higher debt by way of higher taxes or lower transfers. When the generation that "inherits" larger public debt has to repay the entire debt, or part thereof, then the effects on their wealth, current consumption and utility are of course more unfavorable.

All the experiments summarized above assume fixed future taxes and transfers. However, it was also shown that if higher transfers

"today" result in higher (anticipated) lump-sum taxes in the future, then the effects under fixed taxes and transfers simply have to be multiplied by a factor $(1-\sigma)$, where σ expresses the fraction of (the present value of) the transfer that has to be repaid in the next period. Thus, in the general case, we find that current lump-sum transfers to generation t have positive effects on their utility if and only if it experiences some degree of "fiscal benevolence" in the next period in the sense of not being hit by tax increases (transfer reductions) with the same or higher present value.

It is obvious that temporary "debt neutrality" occurs when $\sigma=1$, i.e. when the present value of future transfer reductions is the same as the value of the higher transfers today. Permanent "debt neutrality" will arise if in addition the anticipated transfer reduction materializes. Thus, while debt neutrality is a general property of Barro's model, assuming perfect private altruism between generations, it emerges in the present model only in the special case of complete absence of fiscal benevolence in the next period towards the generation that receives the transfer.

The effects are more complex if it is instead the future income tax rate that is raised as a consequence of the higher transfers today. In particular, debt neutrality cannot occur then, due to the combination of wealth effects and incentive effects, confirming the general proposition that debt neutrality presupposes, *inter alia*, nondistorting taxes.

APPENDIX

This appendix contains all equations that have been referred to, but not explicitly stated, in the text.

Sub-section III.2:

$$(a1) \quad dc_t^{t-1}/dT_t^{t-1} = C_\omega^{t-1}$$

$$(a2) \quad da_t^{t-1}/dT_t^{t-1} = 1 - C_\omega^{t-1}$$

$$(a3) \quad dv_t^{t-1}/dT_t^{t-1} = \partial u^{t-1}/\partial c_t^{t-1}$$

$$(a4) \quad dr_{t+1}/dT_t^{t-1} = [1 - (1-C_\omega^t)(1-C_\omega^{t-1})]/\Psi'$$

$$(a5) \quad d\omega_t^t/dT_t^{t-1} = 1 - C_\omega^{t-1} + \Omega' dr_{t+1}/dT_t^{t-1}$$

$$(a6) \quad dc_t^t/dT_t^{t-1} = C_\omega^t d\omega_t^t/dT_t^{t-1} + C_r^t dr_{t+1}/dT_t^{t-1}$$

$$(a7) \quad dv_t^t/dT_t^{t-1} = (1-C_\omega^{t-1})\partial u^t/\partial c_t^t + V' dr_{t+1}/dT_t^{t-1}$$

Sub-section III.3:

$$(a8) \quad dc_t^{t-1}/d\tau_t = - [(1-\lambda)w_t + r_t s_{t-1}^{t-1}] C_\omega^{t-1}$$

$$(a9) \quad da_t^{t-1}/d\tau_t = - [\quad - \quad - \quad] (1-C_\omega^{t-1})$$

$$(a10) \quad dv_t^{t-1}/d\tau_t = - [\quad - \quad - \quad] \partial u^{t-1}/\partial c_t^{t-1}$$

$$(a11) \quad dr_{t+1}/d\tau_t = -(w_t + r_t s_{t-1}^{t-1} - [\lambda w_t + ((1-\lambda)w_t + r_t s_{t-1}^{t-1})(1-C_\omega^{t-1})](1-C_\omega^t))/\Psi'$$

$$(a12) \quad d\omega_t^t/d\tau_t = -[\lambda w_t + ((1-\lambda)w_t + r_t s_{t-1}^{t-1})(1-C_\omega^{t-1})] + \Omega' dr_{t+1}/d\tau_t$$

$$(a13) \quad dc_t^t/d\tau_t = C_\omega^t d\omega_t^t/d\tau_t + C_r^t dr_{t+1}/d\tau_t$$

$$(a14) \quad dv_t^t/d\tau_t = -[\lambda w_t + ((1-\lambda)w_t + r_t s_{t-1}^{t-1})(1-C_\omega^{t-1})]\partial u_t^t/\partial c_t^t + v' dr_{t+1}/d\tau_t$$

$$(a15) \quad dr_{t+1}/dT_t^{t-1} = (1-C_\omega^t)C_\omega^{t-1}/\Psi'$$

$$(a16) \quad d\omega_t^t/dT_t^{t-1} = -[|K'| - C_r^t + |\Omega'|]C_\omega^{t-1}/\Psi'$$

$$(a17) \quad dc_t^t/dT_t^{t-1} = -[(|K'| + |\Omega'|)C_\omega^t - C_r^t]C_\omega^{t-1}/\Psi'$$

$$(a18) \quad dv_t^t/dT_t^{t-1} = -C_\omega^{t-1}\partial u_t^t/\partial c_t^t + v' dr_{t+1}/dT_t^{t-1}$$

Sub-section V.1:

$$\text{Note first that } \partial \omega_t^t/\partial \tilde{\tau}_{t+1} = -[(1-\lambda)w_{t+1} - r_{t+1}\tilde{T}_{t+1}^t]p_{t+1}^2; \quad \partial p_{t+1}/\partial \tilde{\tau}_{t+1} =$$

$$= r_{t+1}p_{t+1}^2; \quad \partial p_{t+1}/\partial r_{t+1} = -(1-\tilde{\tau}_{t+1})p_{t+1}^2 \quad \text{and}$$

$$C_r^t = (\partial c_t^t/\partial p_{t+1})(\partial p_{t+1}/\partial r_{t+1}) .$$

$$(a19) \quad d\omega_t^t/d\tilde{\tau}_{t+1} = -p_{t+1}^2(|K'| - C_r^t)[(1-\lambda)w_{t+1} - r_{t+1}\tilde{T}_{t+1}^t]/\Psi' +$$

$$+ r_{t+1}(1-\tilde{\tau}_{t+1})^{-1}|\Omega'|C_r^t/\Psi'$$

$$(a20) \quad dc_t^t/d\tilde{\tau}_{t+1} = C_{\omega}^t d\omega_t^t/d\tilde{\tau}_{t+1} - p_{t+1}^{-2} (1-\tilde{\tau}_{t+1})^{-1} C_r^t dp_{t+1}/d\tilde{\tau}_{t+1}$$

$$(a21) \quad dv_t^t/d\tilde{\tau}_{t+1} = (\partial u^t/\partial c_t^t) d\omega_t^t/d\tilde{\tau}_{t+1} -$$

$$- (\partial u^t/\partial c_t^t - \partial u^t/\partial a_t^t) (a_t^t/p_{t+1}) dp_{t+1}/d\tilde{\tau}_{t+1}$$

REFERENCES

- Abel, A.B., 1984, "Capital accumulation and uncertain life times with adverse selection", Harvard University (mimeo).
- Barro, R.J., 1974, "Are government bonds net wealth?", Journal of Political Economy, no. 6.
- Blanchard, O.J., 1983, "Debt, deficit and finite horizons", Massachusetts Institute of Technology (mimeo).
- Bowen, W.G., Davis, R.G., Kopf, D.H., 1960, "The public debt: A burden on future generations?", American Economic Review, September. (Also in Ferguson, J., op. cit.).
-
- Buchanan, J.M., 1958, Public principles of public debt, (Richard D. Irvin, Homewood, Illinois). (Also in Ferguson, J., op. cit.).
- Buiter, W.H., 1980, "'Crowding out' of private capital formation by government borrowing in the presence of intergenerational gifts and bequests", Greek Economic Review 2, Aug., pp. 112-142.
- Dasgupta, P., 1974, "On some alternative criteria for justice between generations", Journal of Public Economy, Nov.
- Diamond, P., 1965, "National debt in a neoclassical growth model", American Economic Review.
- Diamond, P., 1970, "Incidence of an interest income tax", Journal of Economic Theory 2, pp. 211-224.
- Ferguson, J.M. (ed.), 1964, Public debt and future generations, (The University of North Carolina Press, Chapel Hill).
- Gale, D., 1983, Money: in Disequilibrium, (Cambridge University Press).
- King, M., 1983, "Saving Behavior", Yrjö Jahnsson Symposium in Finland, June 1983 (mimeo).

- Koskela, E. and Virén, M., 1983, "National debt neutrality: some international evidence", Bank of Finland Research Department (mimeo).
- Kurz, M., 1982, "Capital accumulation and the characteristic of private intergenerational transfers", Institute for Mathematical Studies in the Social Sciences (mimeo).
- Leininger, W., 1983(a), "The existence of Nash equilibria in a model of growth with altruism between generations", International Centre for Economics and Related Disciplines, April (mimeo).
- Leininger, W., 1983(b), "Rawls's maximin criterion and time-consistency: some further results", Department of Economics, Bonn University, Discussion paper 131 (mimeo).
- Lindbeck, A., 1963, A Study in Monetary Analysis, (Almqvist och Wiksell, Stockholm).
- Modigliani, F., 1961, "Long-run implications of alternative fiscal policies and the burden of the national debt", Economic Journal, December.
- Persson, T., 1983, "Deficit and intergenerational welfare in open economies", National Bureau of Economic Research, Working Paper No. 1083.
- Phelps, E.S. and Pollak, R.A., 1968, "On second-best national saving and game-equilibrium growth", Review of Economic Studies, 35, April.
- Ray, D. and Bernheim, D., 1983, "Altruistic growth economies: I. Existence of bequest equilibria, II. Properties of bequest equilibria", Stanford University, Institute for Mathematical Studies in the Social Sciences, Technical Report 419 (mimeo).

- Samuelson, P., 1958, "An exact consumption-loan model of interest with or without the social contrivance of money", Journal of Political Economy, 66, pp. 467-482.
- Sheshinski, E. and Y. Weiss, 1981, "Uncertainty and optimal social security systems", Quarterly Journal of Economics 96, pp. 189-206.
- Stiglitz, J.E., 1983(a), "On the relevance or irrelevance of public financial policy", National Bureau of Economic Research, Working Paper No. 1057.
- Stiglitz, J.E., 1983(b), "On the relevance or irrelevance of public financial policy: indexation, price rigidities and optimal monetary policy", National Bureau of Economic Research, Working Paper No. 1106.
- Tobin, J., 1980, Asset Accumulation and Economic Activity (Basil Blackwell, Oxford).
- von Weizsäcker, C.C., 1974, "Intergenerationelle Einkommensverteilung: Einfache Beispiele für Wirkungen steuerlicher Massnahmen und für die optimale Steuerstruktur", in Bombach G., B.S. Frey, and B. Gahlen (eds.), Neue Aspekte der Verteilungstheorie, Schriftenreihe des Wirtschaftswissenschaftlichen Seminars Ottobeuren, Band 2 (J.C.P. Mohr (Paul Siebeck), Tübingen).

FOOTNOTES

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¹Uncertainty, in particular about the length of one's remaining life-time, creates a "precautionary motive" for positive net wealth holdings throughout the entire life-span, or for the purchase of insurances that provide annuities if appropriate markets for these exist (cf. Sheshinski and Weiss (1981), Blanchard (1983), and Abel (1984)). In a world with capital market imperfections, in particular for borrowing, asset holdings also provide for desirable liquidity.

²In a less known paper, von Weizsäcker (1974) independently develops the same argument as Barro (1974). For a critical analysis of Barro's model, see Buiter (1980), Tobin (1980) and Gale (1983, Section 1.8). A comprehensive empirical cross-section study for OECD countries by Koskela and Virén (1983) lends no support to the debt neutrality hypothesis.

³While by altruism we mean a concern for at least some other generation, "perfect" altruism assumes, directly or indirectly, concern for all future generations, cf. Leininger (1983,b). See Phelps and Pollak (1968), Barro (1974), Dasgupta (1974), Kurz (1982), Leininger (1983a,b), Ray and Bernheim (1983) for analyses of different types of altruism between generations.

⁴Torsten Persson (1983) compares public borrowing in a closed and an open economy, though without distinguishing between domestic and foreign debt.

⁵ $\tilde{T}_{t+1}^t$ and $\tilde{\tau}_{t+1}$ are the only expectations relevant for the consumption/saving decision of generation t . We will treat only the case of perfect foresight whenever a comparison is needed with the actual values of these two parameters, cf. subsection V.2. As for $\tilde{T}_t^{t-1}$, however, also the case of an unanticipated marginal shift will be considered, cf. subsection III.2.

⁶~~In the present simple model, future incomes are treated as certain in~~ the individuals' budget equations (1)-(2). Hence, in this context a_t^t is equivalent to the market value of resources available in the second period.

⁷ w being the price of a unit of labor, the model would, in principle, permit different wage rates for the two generations working in a period. Suppose hours worked by the younger (older) generation is h_1 (h_2) and the productivity per hour is λ_1 (λ_2). Then $\lambda = h_1\lambda_1$, if the aggregate labor supply $h_1\lambda_1 + h_2\lambda_2$ is normalized to unity. Consequently, the wage rate per hour of a young (old) worker $\lambda_1 w$ ($\lambda_2 w$).

⁸If, moreover, $\partial u^t / \partial a_t^t \equiv \partial u^t / \partial a_{t+1}^t \equiv 0$, $\lambda = 1$ and $\tau_t = T_{t+1}^t = 0$ for all t , then we wind up in the steady-state analysis in Diamond (1965).

⁹The reason why Diamond (1965) noted a negative indirect effect on utility is that he studied balanced-budget steady-states, in which also the wage earnings and lump-sum taxes of the young are affected by factor price movements.

¹⁰In retrospect, generation $t-1$ can be blamed for imprudence in having treated $\tilde{T}_t^{t-1}$ as a point estimate rather than as a genuine random variable. In this sense, the comparative-statics exercises in subsections III.2 and III.3 assume bounded rationality on behalf of generation $t-1$. For a treatment of uncertainty as to future taxes and transfers, cf. e.g. Stiglitz (1983a,b).

¹¹The "ex post" adjustment (26) implies that general equilibrium in period t is restored although generation $t-1$ held incorrect expectations when young, cf. eqs. (11) and (12).

¹²This is in line with our interpretation that it is the awareness of asset holdings that is directly utility creating (cf. Introduction). If we today learn that we actually were wealthier yesterday than we then thought, then we assume that this does not make us happier today (i.e. after the event of the holding of the assets).

¹³If expectations had been correct, then the ex post and ex ante solutions would coincide.

¹⁴Since the differential equations in subsequent experiments are either straight-forward or similar to the equations in the previous sub-section, most of them have been relegated to an Appendix.

¹⁵By eq. (27), $d\omega_t^{t-1} = dT_t^{t-1} - [(1-\lambda)w_t + r_t s_{t-1}^{t-1}]d\tau_t =$

$$= (1 - [(1-\lambda)w_t + r_t s_{t-1}^{t-1}]/(w_t + r_t s_{t-1}^{t-1}))dT_t^{t-1} = \beta dT_t^{t-1}.$$

¹⁶More exactly, the model in Diamond (1970) can, in our notation, be obtained from the present model by letting $\partial u^t / \partial a_t^t \equiv \partial u_t^t / \partial a_{t+1}^t \equiv 0$, $\lambda = 1$, $T_t^{t-1} = T_t^t = 0$, $\tau = 1 - \tilde{\tau}_{t+1}$, and $\tilde{T}_{t+1}^t = T$.

¹⁷Cf. Joe Stiglitz's proposition that non-distortionary fiscal policy has no real effects unless it results in intergenerational redistributions - in intertemporal models with perfect credit markets (Stiglitz 1983a,b).

¹⁸Thus, the analysis confirms the statement by James Tobin (1980) that distorting tax rates violate the possibility of debt neutrality.