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MONEY AND BANKING IN A
CASH-IN-ADVANCE ECONOMY

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1. Introduction

Innovations in banking, like the introduction of credit cards and interest-bearing checking accounts, certainly affect the demand for money and hence price levels and interest rates. Indeed the observed instability of money demand functions in the 1970's and the corresponding difficulty in estimating money demand functions (see Judd and Scadding (1982)) has to a large extent been attributed to financial innovations. In an extreme case the current system based on fiat money may be replaced with one that wholly relies on inside money. The functioning of such a system in a general equilibrium setting has been the concern of Black (1970, 1975), Fama (1980, 1983), Fischer (1982) and others.

Recent advances in monetary theory include the development of monetary asset-pricing models, essentially variants of Lucas's (1978) barter asset-pricing model (see Lucas (1980, 1982, 1983), LeRoy (1984a, b), Townsend (1984) and Svensson (1984, 1985)). These papers have improved our understanding of monetary phenomena, in particular the relations between real and monetary aspects. They do, however, model

¹ We have received helpful comments by participants in seminars at Tel-Aviv University, IIES and the Stockholm School of Economics. We thank Karin Edénholm and Lotten Bergström for typing and editorial assistance.

the supply of money in the most rudimentary way, by abstracting from a banking sector and by considering an exogenous stock of money and lump-sum monetary transfers. They are therefore in their present form not suitable to study financial innovations.

In this paper we take a step towards the study of banking and financial innovations in a rigorous general equilibrium framework by introducing a banking sector in a monetary asset-pricing model of the cash-in-advance variety previously examined in Svensson (1984). We allow both cash and checks (bank money) as means of payment, but some goods can only be purchased with cash.² ~~We include a banking sector which~~ accepts checking account deposits and invests these partly in bonds, partly in currency reserves. The central bank cannot directly determine the sum of these two items, i.e. the money stock as conventionally defined. It can only determine the total supply of reserves, i.e. the monetary base, which will be endogenously divided between banks and consumers.

We derive a full rational expectations equilibrium via utility and stock-market value maximization, and consider effects of changes in bank reserve ratios and in the share of goods where only cash can be used as means of payment. While the latter experiment has a clear interpretation as a financial innovation, the reserve ratio can be seen as a regulation. We examine the effects on checking and bond interest rates and on the level and variability of the price level. We find that changes in the credit multiplier have different effects than changes in

² A thorough micro-economic study of the liquidity services of money in a world with cash goods and credit goods is done by Hartley (1985).

the monetary base. The former have non-neutral, the latter neutral effects. An increase in the usage of checks as a means of payment and a decrease in the bank reserve ratio will in general both lead to a decrease in the variance of the price level.

Section 2 presents the model. The equilibrium is specified and solved in Section 3 for a special case with an exogenous currency ratio. Section 4 discusses the effect on price levels and interest rates of financial innovations, and Section 5 concludes the paper. An appendix deals with the more general case of an endogenously determined currency ratio.

2. The Model

We consider a monetary economy with three representative agents; a firm, a consumer and a bank.

The firm produces a nonstorable good. Production y_t in period t , $t = \dots, -1, 0, 1, \dots$, is stochastic, serially uncorrelated and characterized by the distribution function $F(y_t)$. There is a linear technology which allows transforming total output between two types of goods that are imperfect substitutes in consumption, $y_{1t} + y_{2t} = y_t$. Cash goods, y_1 , can only be purchased with currency, whereas check goods, y_2 , can be purchased with either currency or check.

The consumer has preferences according to the expected utility function

$$(2.1) \quad E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_{1\tau}, c_{2\tau}), \quad 0 < \beta < 1,$$

where $c_{1\tau}$ and $c_{2\tau}$ is consumption of cash and check goods in period τ and E_t is the expectations operator conditional upon information available at t . The consumer enters period t with pre-determined holdings of a

number of assets: cash, H_{ht} , bank deposits in a checking account, D_{ht} , nominal bonds, B_{ht} , shares in the firm, Z_t , and shares in the bank, X_t . Both Z_t and X_t are expressed as fractions of the total number of shares of the firm and bank respectively. After the consumer has learnt about the current value of y_t , the goods market opens. Here the consumer can only pay with cash or checks, i.e. he must obey the liquidity

constraints

$$(2.2a) \quad c_{1t} \leq \pi_t H_{ht} \text{ and}$$

$$(2.2b) \quad c_{2t} \leq \pi_t D_{ht},$$

where π_t is the price of money (cash or checks) in terms of goods.³

That is, π_t is the inverse of the price level P_t as usually understood, $\pi_t = 1/P_t$. Due to the linear technology, the price of cash goods and check goods will always be the same. Our distinction between cash and check goods is different from Lucas and Stokey's (1983, 1984) distinction between cash and credit goods. In their models credit goods are not subject to any liquidity constraint, whereas in our model both cash and check goods are subject to liquidity constraints.

After the goods market has closed the consumer receives interest payments, $i_t^d D_{ht}$ and $i_t^b B_{ht}$, where i_t^d and i_t^b are the interest rates on deposits and bonds, respectively. Furthermore he receives dividends from the firm, $y_t Z_t$, and the bank's dividends, $a_t X_t$, to be specified below. Then the asset market opens, on which the consumer can trade bonds, shares and currency and make deposits in and withdrawals from his checking account. The consumer's budget constraint is

³ The liquidity constraint for check goods should be written $c_{2t} \leq \pi_t (H_{ht} + D_{ht})$ to explicitly allow check goods to be purchased with either cash or checks. In an equilibrium, deposits will pay positive interest rates and then cash will be used to purchase cash goods only. Anticipating this, we might as well use (2.2b).

$$\begin{aligned}
 & \pi_t H_{ht+1} + \pi_t D_{ht+1} + \pi_t B_{ht+1} + q_t Z_{t+1} + r_t X_{t+1} \leq \\
 (2.3) & \leq [\pi_t H_{ht} + \pi_t D_{ht} - c_{1t} - c_{2t}] + i_t^d \pi_t D_{ht} + \\
 & + (1 + i_t^b) \pi_t B_{ht} + (q_t + y_t) Z_t + (r_t + a_t) X_t.
 \end{aligned}$$

Here q_t and r_t are the prices in terms of goods of firm shares and bank shares.⁴ We use the convention that asset quantities purchased on the asset market in period t and carried into period $t+1$ are indexed by $t+1$. On the left-hand side of (2.3) are the consumer's new holdings of cash, deposits, bonds, firm shares and bank shares, to be carried into period $t+1$. The bracketed terms on the right-hand side is cash and deposits left over after the goods market purchases. The other terms are interest on deposits, the value of and interest on bonds, and the value of and dividends on firm shares and bank shares.

Let us next look at the bank. When the asset market in period t is open, it receives deposits, D_{bt+1} . It invests these in currency reserves, H_{bt+1} , and bonds, B_{bt+1} .⁵ We assume that the bank by the central bank is required to keep a fixed reserve ratio, α . This means that

$$(2.4a) \quad H_{bt+1} = \alpha D_{bt+1} \quad \text{and}$$

$$(2.4b) \quad B_{bt+1} = (1 - \alpha) D_{bt+1},$$

⁴ Let Q_t and R_t be the money price on the asset market of firm shares and bank shares, respectively. Then the prices in terms of goods of firm and bank shares are defined by $q_t = Q_t/P_t$ and $r_t = R_t/P_t$, where P_t is the money price of goods on the goods market. We find it practical to define and use these prices in terms of goods, although firm and bank shares are not directly exchanged for goods.

⁵ In a more general model one may want to allow the bank to invest in firm shares, with deposits working as shares in a mutual fund against which checks can be written as envisaged by e.g. Fama (1983). This would not affect any results in the present model with one representative consumer.

that is, the bank invests the proportion α of its deposits in currency and $(1-\alpha)$ in bonds.

Alternatively, we can think of (2.4) as being the result of a portfolio decision by the bank, under the assumption that a constant fraction α of bank customers will for some reason make cash withdrawals from their checking accounts. For simplicity we use the interpretation that the reserve ratio α is imposed upon the bank as a regulation. In period $t+1$ the bank redeems its bonds and repays deposits, including interest. Any surplus is paid as dividends to the owners of the bank shares. The dividends equal

$$(2.5) \quad a_{t+1} = \pi_{t+1} H_{bt+1} + (1 + i_{t+1}^b) \pi_{t+1} B_{bt+1} - (1 + i_{t+1}^d) \pi_{t+1} D_{bt+1}.$$

The bank is assumed to maximize its stock market value, r_t , to be specified below.

3. Equilibrium

In an equilibrium, the markets for goods, currency, deposits, bonds, firm shares and bank shares will all clear. Using the notation $x = x_t$, $x' = x_{t+1}$ for $x = c, y, H, D$, etc., we can write

$$(3.1a) \quad c_1 + c_2 = y,$$

$$(3.1b) \quad H_h' + H_b' = H,$$

$$(3.1c) \quad D_h' = D_b',$$

$$(3.1d) \quad B_h' + B_b' = 0,$$

$$(3.1e) \quad Z' = 1 \text{ and}$$

$$(3.1f) \quad X' = 1.$$

We assume that the supply of currency, H , the monetary base, on the right hand side of (3.1b), is fixed at a constant level determined by the central bank.

Let us define the money stock as the sum of the consumer's currency and deposits,

$$(3.2a) \quad M \equiv D_h + H_h,$$

and the currency ratio as,

$$(3.2b) \quad \theta \equiv H_h/D_h.$$

We then have, from (2.4a) and (3.1b), the usual textbook formula

~~according to which the money stock equals the monetary base times a~~
credit multiplier, that is,

$$(3.3a) \quad M = mH,$$

where the credit multiplier m is given by

$$(3.3b) \quad m = (\theta + 1)/(\theta + \alpha).$$

In our model the credit multiplier is determined endogenously as a function of α and the working of the financial system as reflected in the proportion of check and cash goods.

We now consider a stationary stochastic rational expectations equilibrium of the model. In order to specify the equilibrium, we first combine the consumer's first-order condition with the market equilibrium conditions (3.1) and the definition (3.2). This gives (see Appendix for details)

$$(3.4a) \quad c_1 + c_2 = y,$$

$$(3.4b) \quad c_1 \leq \pi H_h \quad [\mu_1 \geq 0],$$

$$(3.4c) \quad c_2 \leq \pi D_h \quad [\mu_2 \geq 0],$$

$$(3.4d) \quad u_1(c_1, c_2) = \lambda + \mu_1,$$

$$(3.4e) \quad u_2(c_1, c_2) = \lambda + \mu_2,$$

$$(3.4f) \quad \beta E[(\lambda' + \mu_1')\pi'] = \lambda\pi,$$

$$(3.4g) \quad \beta E[(\lambda' + \mu_2')\pi'] + i^d \beta E[\lambda'\pi'] = \lambda\pi,$$

$$(3.4h) \quad \beta E[(\lambda'(1 + i^b)\pi')], = \lambda\pi,$$

$$(3.4i) \quad \beta E[\lambda'(q' + y')] = \lambda q \text{ and}$$

$$(3.4j) \quad \beta E[\lambda'(r' + a')] = \lambda r.$$

In equilibrium the endogenous variables will be functions of output y and the parameters α , β , γ , H and the parameters of the utility function. Let us first interpret the equations. Expressions (3.4b,c) are the liquidity constraints, according to which output and consumption cannot exceed the real values of the means of payment. The brackets denote the complementary slackness conditions according to which the liquidity constraints are binding if the corresponding Lagrange multipliers μ_1 and μ_2 are positive, and the multiplier is zero if the liquidity constraint is slack. The multipliers can be interpreted as the marginal utilities of cash and deposits. From (3.4d,e) we see that the marginal utility of consumption equals the sum of the marginal utility of wealth, λ , the Lagrange multiplier of the budget constraint (2.3), and the marginal utility of real balances. Hence, if the liquidity constraint is binding, the marginal utility of consumption is larger than the marginal utility of wealth. Equations (3.4i,j) are standard asset-pricing equations for firm shares and bank shares. They give the utility-value of an asset as the discounted expected utility value of its value and dividends. Equation (3.4h) also gives the present value of a unit nominal bond as the discounted utility value of its value and its dividends, the interest rate i^b . Equation (3.4f) is

also an asset-pricing equation, for cash. Cash does not pay any interest, but gives liquidity services, the utility value of which is $\mu_1'\pi'$ per unit of cash. Deposits both pay interest and give liquidity services, $i^d\lambda'\pi' + \mu_2'\pi'$, as seen in equation (3.4g).

With y serially uncorrelated, it can be shown that the equilibrium is such that the right hand side of (3.4f), and hence $\lambda\pi$, is independent of y ; the marginal utility of wealth is inversely proportional to π , i.e. directly proportional to the price level of goods. Then equation (3.4h), the asset-pricing equation for nominal bonds, can be simplified to give

$$(3.5) \quad i^b = 1/\beta - 1 = \delta,$$

where δ is the rate of time preference, defined by $\beta = 1/(1 + \delta)$.

Hence, the interest rate on nominal bonds is constant and equal to the rate of time preference.

We also want to have an expression for the deposit interest rate, i^d . We get this from the bank's decision problem. We assume the representative bank to be a price taker in the asset market and to take as given all interest rates and other asset prices. The bank then faces the linear constraints (2.4) and attempts to maximize its stock market value, r . It follows that its equilibrium stock market value must equal zero, $r = 0$. If it was positive, the bank could double its activities and this way double its stock market value. From this and the asset-pricing equation (3.4j) it follows that the expected utility value of bank's dividends is zero,

$$(3.6) \quad E[\lambda'a'] = 0.$$

This zero-market-value condition is analogous to the zero-profit condition for a competitive firm with constant returns to scale. Since

we restrict the bank to hold cash and bonds only, the bank's dividends in our case are given by

$$(3.7) \quad a' = \pi' [\alpha + (1 + i^b)(1 - \alpha) - (1 + i^d)] D_b',$$

where we have exploited (2.4). Since $\lambda > 0$ it follows directly from (3.6) that the bracketed expression in (3.7) is zero. We hence get

$$(3.8) \quad i^d = (1 - \alpha) i^b.$$

This is very intuitive. The bank holds the proportion $(1 - \alpha)$ of its deposits as bonds which pay a positive interest rate, and the proportion α as zero-interest paying currency. Hence the deposit interest rate it can afford is simply the corresponding average of the positive bond interest rate and zero currency interest rate.

We have seen that the interest rates are independent of output y . They only depend on time preferences and the reserve ratio. The other endogenous variables - π , λ , μ_1 , μ_2 , c_1 and c_2 - will however be functions of output. In the Appendix we discuss the determination of these for the case with substitution possibilities between cash goods and check goods. Here we shall now make the simplifying assumption that there are fixed proportions between cash goods and check goods, and hence that the currency ratio is exogenous. More specifically, we assume the Leontief utility function

$$(3.9) \quad u(\min[(\theta + 1)c_1/\theta, (\theta + 1)c_2]), \quad 0 < \theta < 1,$$

where θ is a constant. Then it will be optimal to hold cash and check deposits in fixed proportions, such that

$$(3.10) \quad H_h = \theta D_h,$$

and we can interpret θ as the currency ratio. With

$$(3.11) \quad c = c_1 + c_2$$

being total consumption of goods, we can then for simplicity use the utility function $u(c)$ of total consumption in (2.1).⁶ The equilibrium conditions are then simplified such that (3.4a-f) are replaced by

$$(3.12a) \quad y \leq \pi M \quad [\mu \geq 0]$$

$$(3.12b) \quad u_c(y) = \lambda + \mu \text{ and}$$

$$(3.12c) \quad \lambda \pi = \beta E[\lambda' \pi' (1 + i^d/(\theta + 1)) + \mu' \pi'].$$

Here μ is the Lagrange multiplier of the combined liquidity constraint

$$(3.13) \quad c \leq \pi(H_h + D_h) = \pi(\theta + 1)D_h = \pi M_h,$$

where we use (3.2) and (3.10) and the fact that with fixed proportions between cash and check goods the liquidity constraints (3.4b,c) will either both bind or both be slack.

We note that by (3.12c) we can define the interest rate on money, i^m , as

$$(3.14) \quad i^m = i^d/(\theta + 1),$$

which reflects that money is an aggregate of currency with zero interest rate and deposits with the interest rate i^d .

We next consider the solution for the case when the utility function has a (not necessarily constant) relative risk aversion, γ , less than unity,

$$(3.15) \quad 0 < \gamma \equiv -cu_{cc}/u_c < 1.$$

That is, the elasticity of intertemporal substitution is larger than unity.⁷ Then the solution can be explained in terms of a critical level of output y^* such that for output levels falling short of y^* , the liquidity constraint (3.12a) is slack, whereas it is binding for output

⁶ Note that (3.9) implies $c_1 = \theta c_2$. Hence, $c = c_1 + c_2 = (\theta + 1)c_1/\theta = (\theta + 1)c_2$, which demonstrates the equivalence.

⁷ We recall that with an additively separable utility function like (2.1), risk aversion and intertemporal substitution are not separable.

levels exceeding y^* . More precisely, for the critical level of output, y^* , the marginal utility of wealth, λ , equals the marginal utility of consumption, $u_c(y^*)$, and the real price of money, π , equals y^*/M . This implies that the product $u_c(y^*)y^*$ equals the constant right hand side of (3.12c), and we may write

$$(3.16) \quad \lambda\pi = u_c(y^*)y^*.$$

Then for $y \leq y^*$,

$$(3.17a) \quad \pi = u_c(y^*)y^*/u_c(y)M < y/M,$$

$$\lambda = u_c(y) \text{ and}$$

$$\mu = 0;$$

whereas, for $y > y^*$,

$$(3.17b) \quad \pi = y/M,$$

$$\lambda = u_c(y^*)y^*/y \leq u_c(y) \text{ and}$$

$$\mu = u_c(y) - u_c(y^*)y^*/y \geq 0.$$

The solution (3.17) is such that for low values of output and consumption the liquidity constraint is slack, the marginal utility of real balances is zero, and the marginal utility of wealth is equal to the marginal utility of consumption. If output increases, the price of money and hence real balances increases, but less than in proportion. For the critical output level the liquidity constraint barely binds. If output increases further, the liquidity constraint strictly binds, the marginal utility of real balances increases above zero, and the marginal utility of wealth falls below the marginal utility of consumption.

Furthermore, (3.12c) and (3.16) give

$$(3.18) \quad u_c(y^*)y^* = (m/\delta)E[\mu'\pi'].$$

From the solution to μ and π in (3.17) follows that the critical level of output y^* is determined by the equation

$$(3.19) \quad u_c(y^*)y^* = (m/\delta) \int_{y^*}^{\infty} (u_c(y)y - u_c(y^*)y^*) dF(y).$$

The left hand side is an increasing function of y^* , and the integral on the right hand side is a decreasing function of y^* , by (3.15). Hence y^* will be a decreasing function of (δ/m) ,

$$(3.20) \quad y^* = y^*(\delta/m) \quad \text{and} \quad dy^*/d(\delta/m) < 0.$$

Finally, we have

$$(3.21) \quad \delta/m = i^b - i^m.$$

Therefore, the critical output level is a decreasing function of the difference between the bond interest rate i^b and the money interest rate i^m .

$$(3.22) \quad y^* = y^*(i^b - i^m), \quad dy^*/d(i^b - i^m) < 0.$$

This is intuitive. The interest rate differential $i^b - i^m$ is the opportunity cost of holding money. If it increases, consumers hold less real balances and become liquidity constrained for lower levels of output. Given the functions π , λ and μ , all the other endogenous functions can be solved, and the equilibrium is fully specified.

4. Interest Rates and Price Levels

We shall now study the effects on interest rates and the price level of changes in the parameters of the model. In the general version of the model, with an unspecified utility function, the key parameters are α and s . A decrease in s can be interpreted as a financial innovation in the sense that a larger proportion of all goods can be bought with checks (see Appendix). In the special case of a Leontief utility function the same interpretation holds for the currency ratio θ .

In general, however, this is endogenously determined as a function of s and α . We show in the Appendix that it in general is an increasing function in both arguments.

A change in α , the reserve ratio, can also be interpreted as a financial innovation if α is seen as reflecting the consumer's propensity to make cash withdrawals. This may be related to the consumer's belief in the stability of the financial system. As discussed previously α can also be interpreted as a regulation in which case $\alpha = 0$ would indicate an unregulated banking industry.

To simplify the presentation we will conduct the analysis in this section on the assumption of a Leontief utility function. Let us first, for easy reference, summarize the crucial relationships,

$$(4.1a) \quad H_h = \theta D_h, \quad H_b = \alpha D_b,$$

$$(4.1b) \quad M = D + H_h = mH,$$

$$(4.1c) \quad m = (\theta + 1)/(\theta + \alpha),$$

$$(4.1d) \quad i^b = \delta,$$

$$(4.1e) \quad i^d = (1 - \alpha)i^b = (1 - \alpha)\delta,$$

$$(4.1f) \quad i^m = i^d/(\theta + 1) = (1 - 1/m)\delta,$$

$$(4.1g) \quad i^b - i^m = \delta/m \text{ and}$$

$$(4.1h) \quad dy^*/d(i^b - i^m) < 0.$$

We also write the price level function in logarithmic form by transforming (3.17). For simplicity we assume a constant relative risk aversion, γ . Then we have

$$(4.2) \quad \ln P = \ln H + \ln m - \gamma \ln y - (1 - \gamma) \ln y^* \text{ for } y < y^*,$$

$$\ln P = \ln H + \ln m - \ln y \quad \text{for } y \geq y^*.$$

This is illustrated in Figure 1, with $\gamma < 1$ in accordance with (3.15).

For output levels less than the critical level, when the liquidity

constraint is slack, (the absolute level of) the elasticity of the price level is less than unity. For output levels exceeding the critical level, the liquidity constraint binds and (the absolute value of) the elasticity is unity.

First, we note that changes in the monetary base H lead to proportional changes in the price level. The kinked line in Figure 1 just shifts up and down. Neither any interest rates nor the critical output level are affected. The average real balances ($E[\ln M/P] = \ln M - E[\ln P]$) are constant. In this sense changes in the monetary base are neutral. Changes in the reserve or currency ratios are not neutral,

however. Let us consider a fall in the reserve ratio, α . This increases the multiplier (4.1c) and the money stock, and increases the price level in proportion. The kinked line in Figure 1 shifts up. The fall in the reserve ratio also increases the deposit interest rate (4.1e), since banks hold less cash and more interest-bearing bonds and hence can afford paying higher deposit interest rates. As a consequence the money interest (4.1f) increases, which decreases the opportunity cost of holding money, the interest rate differential (4.1g). This makes consumers want to hold higher real balances and decreases the likelihood that the liquidity constraint binds, which increases the critical output level (4.1h). The kinked line shifts from AA to A'A' in Figure 2.⁸ The average logarithmic price level $E[\ln P]$ increases less than in proportion to the multiplier and the money stock, and

⁸ The partial shift in the price level due to the shift in y^* is less than the (partial) shift due to the increase in M , so the price level increases for all y . It can be shown that $d \ln P / d \ln m = \int_{y^*}^{\infty} u_c(y^*) y^* dF(y) / \int_{y^*}^{\infty} u_c(y) y dF(y) < 1$.

average logarithmic real balances ($E[\ln M/P] = \ln M - E[\ln P]$) increase.

We also see that the variance of the logarithmic price level ($\text{Var} [\ln P]$) falls.

Now, consider a fall in the currency ratio, θ . This again increases the multiplier, and increases the money stock and price level in proportion. In contrast to a fall in the reserve ratio, it does not increase the deposit interest rate. It still increases the money interest rate (4.1f), because the share of interest-bearing deposits in money increases. The opportunity cost of money (4.1g) falls, the critical output level increases, and the kinked price line shifts as for a fall in the reserve ratio, and as in Figure 2.

Let us here briefly comment on the corresponding effects for a more general utility function. In this case the parameters are α and s , while θ is determined endogenously. Now s has an interpretation similar to θ in the Leontief case. We show in the Appendix that a decrease in s will in general lead to a decrease in θ and hence to a decrease in the variance of $\ln P$. A fall in α will now both have a direct effect and an indirect effect via the induced fall in θ . We have seen above for the Leontief case, when both of these were treated parametrically, that they have qualitatively the same effects. It is therefore not surprising that the signs of the effects of a fall in the reserve ratio in general are the same with an unspecified utility function as under the Leontief assumption.

Historically it is well known that variations in reserve and currency ratios in certain periods - e.g. during war years in the U.S., have contributed to large changes in the money stock. See Cagan (1965) for the U.S. and Jonung (1976) for Sweden. One interpretation of simple

monetary rules in the tradition of Milton Friedman would say that the central bank should adjust the growth rate of the monetary base for such a development so as to keep the money supply on a special growth path. Rigorous analysis of this case requires incorporation of a multiplier that changes over time in our model. Let us nevertheless conjecture that our analysis indicates that it is not sufficient to look at the money stock if one want to stabilize the (expected) price level. This is obvious from the fact that changes in the multiplier have a non-neutral effect on the price level for output levels below the critical level. Thus, we see from Figure 3 that an increase in the credit

multiplier with a corresponding decrease in the monetary base to keep the stock of money constant shifts the kinked line from AA to A'A. Hence, financial innovations, keeping the money supply constant, will lead to reductions of the price level in the region where the liquidity constraint is not binding, and hence to a fall in the average logarithmic price level.

We note in passing that the effect of a fall in the rate of time preference is the same as that of an increase in the multiplier and a fall in the base to keep money stock constant. For a fall in the discount factor decreases all interest rates in proportion, hence lowering the opportunity cost of holding money and increasing the critical output level, exactly as in Figure 3.

Let us finally consider some extreme cases. If either the currency ratio is infinite or if the reserve ratio is unity, the multiplier is unity and the economy is equivalent to the pure cash-in-advance economy analyzed in Svensson (1984a). If the currency ratio is infinite, the consumer holds no deposits and the money interest rate is

zero. If the reserve ratio is unity the banks hold reserves only and no bonds, and can only afford a zero deposit interest rate, making the money interest rate equal zero. Hence, the opportunity cost of money is at a maximum, and consumers hold minimal real balances, giving a low critical output level and the maximum likelihood of binding liquidity constraints.

If the currency ratio is zero and the reserve ratio is positive, only banks hold currency. Consumers pay all their consumption with checks. The money interest rate equals the deposit interest rate. This situation with only banks holding the monetary base is a perfectly viable equilibrium, as previously observed in a different setting by Fischer (1982).

If the currency ratio is zero and the reserve ratio approaches zero, the multiplier and the price level approaches infinity. The money interest rate approaches the bond interest rate, and the opportunity cost of holding money approaches zero. The critical output level approaches the largest possible output level, and the consumer is almost always not liquidity constrained. In the limit the monetary economy collapses, and the price level cannot be determined. If deposits are indexed in something else than currency units, say goods, the economy becomes an Arrow-Debreu economy like in the Lucas (1978) barter asset-pricing model.

5. Conclusions

In the concluding section we shall discuss some welfare aspects of our analysis, the issue of regulated versus unregulated banking, and mention some possible extensions.

Our use of an explicit utility function allows welfare evaluations of the equilibria discussed. Substitution between cash and check goods, due to a positive interest rate on deposits, creates an obvious distortion in that the marginal rate of substitution between cash and check goods will differ from the marginal rate of transformation between the two (the latter is unity in our model). Since check deposits pay interest and cash does not, cash goods become effectively expensive relative to check goods, and in equilibrium the share of cash goods in total consumption is lower than optimal. A removal of this distortion requires that the deposit interest rate is zero, which results if the reserve requirement is 100 %, regardless of what the nominal bond interest rate is. Since this is the only explicit distortion in our model, nothing further can be done to improve welfare.

However, at the cost of considerable technical difficulties, we can consider a second source of distortion in the following way. Suppose as in Lucas and Stokey (1983, 1984), that there is a third category of goods, called credit goods, which are received on credit which is settled on the asset market, without any cash or check liquidity constraints. Then a positive bond interest rate creates a distortion in that credit goods are effectively inexpensive relative to cash and check goods. Removing this distortion requires the (nominal) bond interest rate to be set equal to zero, the standard Friedman rule. This cannot be done in our model with a given stationary supply of currency, but requires a specific sequence of changes in the supply of currency. Note that such a policy also removes the distortion between cash and check goods, since the deposit interest rate is not above the bond interest rate.

If such a policy is infeasible, and the bond rate remains positive, the deposit interest rate cannot alone be manipulated to give full efficiency and only a second-best situation is feasible. Let us at this stage briefly comment on regulated versus unregulated banking. Fama (1980, 1983) has argued in favor of unregulated banking on efficiency grounds. In our model we may identify unregulated banking with a zero reserve requirement. It follows that the deposit interest rate equals the bond rate. The distortion between check goods and credit goods is eliminated, but the distortion between check goods and cash goods is maximized. ~~Clearly there is no reason to believe this to be the second-~~ best optimum. The polar extreme, a 100 % reserve requirement, removes the distortion between check goods and cash goods, but maximizes the distortion between check and credit goods.

We realize from these considerations that the welfare consequences of innovations in banking that show up as changes in currency and reserve ratios, as well as in the structure of different interest rates, may be complicated and ambiguous, in that some distortions may be decreased and others increased.

Let us finally mention some possible extensions of our analysis. An alternative way of modelling the demand for different means of transaction is to explicitly introduce transactions costs that differ between cash and checks, say. Innovations in banking can then be represented as changes in such costs. In equilibrium, the currency ratio will be endogenously determined and the modified asset pricing equations will imply that interest plus liquidity services minus transactions costs will be equalized for currency and checks.

A further extension is to explicitly allow checks to be cashed in the banks for currency. Whether the consumer holds currency or deposits will then depend on the size of the transactions cost relative to the deposit interest rate. In general the consumer will hold some currency and make cash withdrawals from the bank in periods with a large consumption of cash goods. In such a model the reserve ratio will be the outcome of the bank's portfolio decision, as distinct from a ratio imposed by regulation. In general the amount of cash withdrawals will be stochastic. This would require specifying rules for banks' bankruptcy - the simplest solution being that it must not occur with positive probability. An extension along these lines is farreaching and much beyond the scope of this paper, but should be a fruitful are for further research.

Appendix

Substitution between Cash Goods and Check Goods

In a stationary stochastic equilibrium the prices π_t , q_t , and r_t and the interest rates i_{t+1}^d and i_{t+1}^b will depend on output y_t (the interest rates will actually be constant). Then the consumers decision problem can be expressed in the dynamic-programming fashion by a value function $v(w, H_h, D_h, y)$ that fulfills (w is wealth)

$$(A.1) \quad v(w, H_h, D_h, y) \equiv \max \{u(c_1, c_2) + \beta E(w', H'_h, D'_h, y')\}$$

subject to

$$(A.2) \quad c_1 + c_2 + \pi D'_h + \pi H'_h + \pi B'_h + qZ' + rX' = w,$$

$$(A.3) \quad c_1 \leq \pi H_h,$$

$$(A.4) \quad c_2 \leq \pi D_h \text{ and}$$

$$(A.5) \quad w' = \pi' H'_h + (1+i^d)\pi' D'_h + (1+i^b)B'_h + (q'+y')Z' + (r'+a')X'.$$

Denoting the Lagrange-multipliers of the two liquidity constraints by μ_1 and μ_2 , and using $v_w = \lambda$, $v_H = \mu_1\pi$ and $v_D = \mu_2\pi$, we have the following first-order conditions,

$$(A.6) \quad u_1 = \lambda + \mu_1,$$

$$(A.7) \quad u_2 = \lambda + \mu_2,$$

$$(A.8) \quad \beta E[(\lambda' + \mu'_1)\pi'] = \lambda\pi,$$

$$(A.9) \quad \beta E[(\lambda' + \mu'_2)\pi'] + i^d \beta E[\lambda'\pi'] = \lambda\pi,$$

$$(A.10) \quad \beta E[(\lambda'\pi'(1 + i^b))], = \lambda\pi,$$

$$(A.11) \quad \beta E[\lambda'(q' + y')] = \lambda q \text{ and}$$

$$(A.12) \quad \beta E[\lambda'(r' + a')] = \lambda r,$$

which gives (3.4). Further, by combining (A.8) and (A.9), we have

$$(A.13) \quad E[\mu'_2\pi'] = \alpha E[\mu'_1\pi'].$$

In analyzing the solution of the model we will assume that the utility function is homogeneous of degree $(1 - \gamma)$, where $\gamma < 1$. It will be practical to express the solution in terms of the ratio $\eta(y) \equiv c_1(y)/c_2(y)$. That is, $c_1(y) = \eta(y)y/(\eta(y) + 1)$ and $c_2(y) = y/(\eta(y) + 1)$. As there are two liquidity constraints in this model, the solution will have three distinct regions with none, one or both constraints binding. Since $E[\pi_1'\pi'] > E[\pi_2'\pi]$, by (A.13), it follows that the cash goods constraint becomes binding before the check goods constraint. The solution can then be characterized as follows, where we denote the marginal rate of substitution u_1/u_2 by $h(\eta)$, and $\theta = H_h/D_h$ is the currency ratio, which is now endogenous.

The boundaries between the three regions are given by y^* and y^{**} , with $y^* \leq y^{**}$. In Region I we have $y \leq y^*$ and the solution is characterized by

$$(A.14a) \quad \mu_1 = \mu_2 = 0, \quad \eta(y) = \eta^* \text{ and } h(y^*) = 1.$$

In Region II we have $y^* < y \leq y^{**}$ and

$$(A.14b) \quad \mu_1 > 0, \quad \mu_2 = 0 \text{ and } h(\eta(y)) = (\lambda + \mu_1)/(\lambda + \mu_2) > 1$$

Finally, in Region III $y > y^{**}$, and

$$(A.14c) \quad \mu_1 > 0, \quad \mu_2 > 0 \text{ and } \eta(y) = \theta.$$

The solution to $\eta(y)$ is illustrated in Figure A.1.

It shows how η is constant at the value η^* , where the marginal rate of substitution equals unity (the marginal rate of transformation). In region II η falls gradually as the shadow price associated with the cash-goods liquidity-constraint becomes more binding. In region III finally both constraints are binding and η equals the currency ratio θ .

We shall now express this solution in greater detail. We note that in equilibrium the critical output levels, y^* and y^{**} , and the

currency ratio, θ , will be jointly determined. To express the equations for this more compactly we define

$$(A.15) \quad A \equiv \beta E[u_1' \pi'].$$

At y^* the cash goods liquidity constraint holds and simultaneously $\mu_1 = \mu_2 = 0$. Hence, since $c_1(y^*) = \eta^* y^* / (\eta^* + 1)$ and $H_h = \theta H / (\theta + \alpha)$,

$$(A.16) \quad \eta^* y^* / (\eta^* + 1) = \pi \theta H / (\theta + \alpha),$$

Further, since for $y = y^*$ $\lambda = u_1 = u_2$, $\pi u_2 = A$. Combining this with (A.16) gives

$$(A.17) \quad g(\eta^*) y^{*1-\gamma} = A \theta H / (\theta + \alpha)$$

where $g(\eta) \equiv \eta u_2(\eta / (\eta + 1), 1 / (\eta + 1)) / (\eta + 1)$. The upper critical

output level is given analogously by

$$(A.18) \quad g(\theta) y^{**1-\gamma} = A \theta H / (\theta + \alpha).$$

In the region $y^* < y < y^{**}$, $\eta(y)$ is determined by the equation

$$(A.19) \quad g(\eta(y)) y^{1-\gamma} = A \theta H / (\theta + \alpha).$$

Note that (A.19) yields $\eta(y^*) = \eta^*$ and $\eta(y^{**}) = \theta$.

Equations (A.14), (A.15) and (A.17)-(A.19) jointly determine y^* , y^{**} , θ , $\eta(y)$ and A . To analyze the qualitative properties of this solution we express (A.14) and (A.15) more explicitly as

$$(A.20) \quad [A \theta H / (\theta + \alpha)] [\alpha \int_{y^*}^{y^{**}} h(\eta(\tilde{y})) dF(\tilde{y}) - \alpha \int_{y^*}^{\infty} dF(\tilde{y}) + \int_{y^{**}}^{\infty} dF(\tilde{y})] = [1 - \alpha h(\theta)] g(\theta) \int_{y^{**}}^{\infty} \tilde{y}^{1-\gamma} dF(\tilde{y})$$

and

$$(A.21) \quad [A \theta H / (\theta + \alpha)] [1 - \beta \int_0^{y^*} dF(\tilde{y}) - \beta \int_{y^*}^{y^{**}} h(\eta(\tilde{y})) dF(\tilde{y})] = \beta g(\theta) h(\theta) \int_{y^{**}}^{\infty} \tilde{y}^{1-\gamma} dF(\tilde{y}).$$

After some manipulation (A.21) and (A.22) can be reduced to

$$(A.22a) \quad h(\theta) [\alpha(1 - \beta) + \beta \int_{y^{**}}^{\infty} dF(\tilde{y})] = 1 - \beta \int_0^{y^*} dF(\tilde{y}) - \beta \int_{y^*}^{y^{**}} h(\eta(\tilde{y})) dF(\tilde{y}).$$

Further, combining (A.21) and (A.22) with (A.19) yields

$$(A.22b) \quad g(\theta)\beta\int_{y^{**}}^{\infty}\tilde{y}^{1-\gamma}dF(\tilde{y})y^{\gamma-1}/[\alpha(1-\beta) + \beta\int_{y^{**}}^{\infty}dF(\tilde{y})] = g(\eta(y)).$$

To sum up, for given values of y^* and y^{**} equations (A.22a) and (A.22b) determine θ and the function $\eta(y)$ in the interval $y^* < y < y^{**}$. Then equation (A.15), (A.17) and (A.18) determine y^* , y^{**} and A . The rest of the solution - $\lambda(y)$, $\mu_1(y)$, $\mu_2(y)$ and $\pi(y)$ - follows from the first-order conditions.

A.1. Effects of a change in α

We shall now see how the solution depends on the reserve ratio α .

Hence, we regard the solution as a function of α , that is $y^*(\alpha)$, $y^{**}(\alpha)$, $\theta(\alpha)$ and $\eta(y, \alpha)$, and we would like to find the corresponding derivatives

y_{α}^* , y_{α}^{**} , θ_{α} and $\eta_{\alpha}(y, \alpha)$. Differentiating the equations (A.22a) and (A.22b), noting that terms multiplied by dy^* and dy^{**} cancel, we get

$$(A.23a) \quad \Phi_1(\theta, \alpha)d\theta + \Phi_2(\theta)d\alpha = \int_{y^*}^{y^{**}} \Phi_3(\eta(\tilde{y}))d\eta(\tilde{y})dF(\tilde{y})$$

and

$$(A.23b) \quad \Psi_1(\theta, \alpha)y^{\gamma-1}d\theta + \Psi_2(\theta, \alpha)y^{\gamma-1}d\alpha = \Psi_3(\eta(y))d\eta(y), \text{ where}$$

$$(A.24a) \quad \Phi_1(\theta, \alpha) \equiv h_{\eta}(\theta)[\alpha(1-\beta) + \beta\int_{y^{**}}^{\infty}dF] < 0,$$

$$(A.24b) \quad \Phi_2(\theta) \equiv h(\theta)(1-\beta) > 0,$$

$$(A.24c) \quad \Phi_3(\eta) \equiv -\beta h_{\eta}(\eta) > 0,$$

$$(A.24d) \quad \Psi_1(\theta, \alpha) = g_{\eta}(\theta)\beta\int_{y^{**}}^{\infty}\tilde{y}^{1-\gamma}dF/[\alpha(1-\beta) + \beta\int_{y^{**}}^{\infty}dF] > 0,$$

$$(A.24e) \quad \Psi_2(\theta, \alpha) = -g(\theta)(1-\beta)\beta/[\alpha(1-\beta) - \beta\int_{y^{**}}^{\infty}dF]^2 < 0$$

and

$$(A.24f) \quad \Psi_3(\eta) = g_{\eta}(\eta) > 0.$$

We have suppressed the arguments y^* and y^{**} . The signs of Ψ_1 and Ψ_3 follow from

$$(A.25) \quad g_\eta = [u_2 - (u_{22} - u_{21})\eta/(\eta + 1)]/(\eta + 1)^2 = \\ = [\eta(1 - 1/\gamma)u_{21} - (\eta + 1/\gamma)u_{22}]/(\eta + 1)^3 > 0,$$

where use is made of the fact that u_2 is homogeneous of degree $-\gamma$.

Substituting from (A.23b) into (A.23a) yields

$$(A.26) \quad \theta_\alpha(\alpha) = [\Psi_2 \int_{y^*}^{y^{**}} (\Phi_3 \tilde{y}^{\gamma-1}/\Psi_3) dF - \Phi_2]/ \\ [\Phi_1 - \Psi_1 \int_{y^*}^{y^{**}} (\Phi_3 \tilde{y}^{\gamma-1}/\Psi_3) dF] > 0.$$

Substituting this into (A.23b) then gives

$$(A.27) \quad \eta_\alpha(y) = (\Psi_1 \theta_\alpha(\alpha) + \Psi_2) y^{\gamma-1}/\Psi_3.$$

This can be shown to imply

$$\text{sign} [\eta_\alpha(y)] = \text{sign} [\varepsilon h(\theta)/\varepsilon \eta + \varepsilon g(\theta)/\varepsilon \eta],$$

where $\varepsilon h(\theta)/\varepsilon \eta$ denotes the elasticity of $h(\eta)$ with respect to η , at $\eta = \theta$, etc. Thus, the sign of the partial $\eta_\alpha(y)$ equals the sign of the sum of the elasticities of the marginal rate of substitution, $h(\theta)$, and of the function $g(\theta)$, the latter related to the marginal utility of check goods.

To interpret this we assume that the utility function has a constant elasticity of substitution, i.e.

$$(A.28) \quad u = [s c_1^\rho + (1-s) c_2^\rho]^{(1-\gamma)/\rho}, \quad 0 < s < 1.$$

With this specification

$$(A.29) \quad \varepsilon g/\varepsilon \eta = 1 - [(1-s)\eta(1-\gamma+\rho) + \rho s \eta^{\rho+1} - (1-\gamma)s \eta^\rho]/ \\ (1+\eta)(s \eta^\rho + 1-s) \text{ and} \\ \varepsilon h/\varepsilon \eta = \rho - 1 \equiv -1/\sigma,$$

where $\sigma > 0$ is the elasticity of substitution between cash and check goods. Combining this and simplifying yields

$$(A.30) \quad \text{sign}[\eta_\alpha(y)] = \text{sign}[\rho(\eta^\rho + \eta^{*\rho-1}) + (1-\gamma)\eta(\eta^{\rho-1} - \eta^{*\rho-1})]$$

where the first-order condition $(1 - s)/s = \eta^{\rho-1}$ is used.

We see that when the elasticity of substitution is larger than or equal to unity ($\rho \geq 0$), then $\eta_\alpha(y) > 0$. This sign is however reversed for sufficiently small values of the elasticity of substitution;

$$\lim_{\rho \rightarrow -\infty} \eta_\alpha(y) < 0$$

We now go on to look at the determinants of the critical values y^* and y^{**} . Combining (A.17) and (A.19) yields

$$(A.31) \quad g(\eta^*)y^{*1-\gamma} = g(\eta(y))y^{1-\gamma}.$$

Hence,

$$(A.32) \quad \text{sign } y_\alpha^*(\alpha) = \text{sign } \eta_\alpha(y), \quad y^* < y < y^{**}.$$

With regard to y^{**} , we combine (A.18) and (A.19) to get

$$(A.33) \quad g(\theta)y^{**1-\gamma} = g(\eta(y))y^{1-\gamma}.$$

Since $g_\eta > 0$ it follows that

$$(A.34) \quad \text{sign } y_\alpha^{**} = \text{sign } [\eta_\alpha(\theta) - \theta_\alpha].$$

It seems from (A.27) that y_α^{**} can be of either sign.

Combining (A.26) - (A.34) we can now illustrate the effects on $\eta(y)$ of a change in α in Figures A.2 and A.3. The dashed lines refer to the higher value of α . The case where $\eta_\alpha(y) < 0$, which occurs for sufficiently low elasticities of substitution between cash and check goods, is illustrated in Figure A.2. The case with $\eta_\alpha(y) > 0$, for sufficiently high elasticities of substitution, is illustrated in Figure A.3.

Let us also consider the price function, $\pi(y)$ and $P(y)$. We have

$$(A.35a) \quad \begin{aligned} \pi(y) &= A/u_2(\eta^*y/(\eta^* + 1), y/(\eta^* + 1)) = \\ &= Ay^\gamma \eta^*/g(\eta^*)(\eta^* + 1) \quad \text{for } y \leq y^*, \end{aligned}$$

$$(A.35b) \quad \pi(y) = Ay^\gamma \eta(y)/[g(\eta(y))(\eta(y) + 1)] \quad \text{for } y^* < y < y^{**} \text{ and}$$

$$(A.35c) \quad \pi(y) = y(\theta + \alpha)/(\theta + 1)H \quad \text{for } y \geq y^*.$$

Then we can write

$$(A.36a) \quad \ln P = -\ln A + \ln [g(\eta^*)(\eta^* + 1)/\eta^*] - \gamma \ln y \quad (y \leq y^*),$$

$$(A.36b) \quad \ln P = -\ln A + \ln [g(\eta(y))(\eta(y) + 1)/\eta(y)] - \gamma \ln y$$

$$(y^* < y < y^{**}) \quad \text{and}$$

$$(A.36c) \quad \ln P = \ln[(\theta + 1)H/(\theta + \alpha)] - \ln y \quad (y > y^{**}).$$

The shift of this function corresponding to a change in α is illustrated in Figure A.4, which can be compared with Figure 2 in the text. A fall in α leads to an upward shift from the solid to the dashed line (since $\partial A/\partial \alpha > 0$). ~~The effect on the variance of $\ln P$ is somewhat~~ ambiguous. However, for the case with high elasticity of substitution ($\eta_a(y) < 0$), we will have the situation depicted in the figure. Both y^* and y^{**} will decrease making the curve flatter over some intervals, and steeper nowhere. Hence, the variance of $\ln P$ will decrease.

A.2. Effects of a change in s

The parameter s in the utility function can be interpreted as the fraction of all goods that can be purchased with checks. Assume, as in Dixit and Stiglitz (1977), that there exists a continuum of differentiated goods with the utility function

$$(A.37) \quad u = [\int_0^1 c_j^\rho dj]^{(1-\gamma)/\rho}.$$

Further assume that all goods of type 1 can be purchased with cash and all goods of type 2 with checks. Interpreting s as the share of all goods that are cash goods, i.e.

$$c_j = c_1, \quad 0 \leq j \leq s \quad \text{and}$$

$$c_j = c_2 \quad s \leq j \leq 1,$$

will then give rise to the CES-function (A.28).

Looking at the effects of a change in s , we first note that

$$(A.38) \quad \eta_s^* = -h_s(\eta^*)/h_\eta(\eta^*) > 0.$$

Further we have from total differentiation of (A.22a) and (A.22b)

$$(A.39) \quad \theta_s = [\int_{y^*}^{y^{**}} \Phi_3 \Psi_4 / \Psi_3 dF - \Phi_4] / [\Phi_1 - \int_{y^*}^{y^{**}} (\Phi_3 \Psi_1 \tilde{y}^{\gamma-1} / \Psi_3) dF].$$

where

$$(A.40a) \quad \Phi_4 \equiv h_s(\theta) [\alpha(1-\beta) + \beta \int_{y^*}^{\infty} dF] + \beta \int_{y^*}^{y^{**}} h_s(\eta(\tilde{y})) dF > 0$$

and

$$(A.40b) \quad \Psi_4 \equiv g_s(\theta) \beta \int_{y^*}^{\infty} (\tilde{y}/y)^{1-\gamma} dF / [\alpha(1-\beta) + \beta \int_{y^*}^{\infty} dF] - \\ - g_s(\eta(y)) > 0.$$

We see that $\theta_s > 0$ iff the numerator of (A.39) is positive. A sufficient condition for this is $\Psi_4 < 0$. To express conditions for this to hold we first state

$$g_s(\eta) = (1-s)(1-\gamma)\eta(\eta+1)^{\gamma-1} [s\eta^\rho + 1-s]^{(1-\gamma-2\rho)/\rho} \cdot \\ \cdot [((1-\gamma-\rho)/\rho - \eta^{*1-\rho})\eta^\rho - (1-\gamma)/\rho].$$

It can be shown that $\rho < 0$ is a sufficient condition for this expression to be increasing in η , in which case $g_s(\eta(y)) > g_s(\theta)$. The sign of (A.40b) also depends on the expression

$$\Omega \equiv \beta \int_{y^*}^{\infty} (\tilde{y}/y)^{1-\gamma} dF / [\alpha(1-\beta) + \beta \int_{y^*}^{\infty} dF].$$

This is larger than unity for $\alpha = 0$, since $\tilde{y} > y$, but there exist combinations of α and the distribution function F that give $\Omega < 1$.

Conclining the above information we can state that a sufficient condition for $\theta_s > 0$ is $\rho < 0$ and $\Omega \leq 1$.

We have further

$$\eta_s(y) = [\Psi_1 y^{\gamma-1} \theta_s + \Phi_4] / \Phi_3.$$

This is unambiguously positive if $\theta_s > 0$ and $\Phi_4 > 0$. Note, however, that $\Psi_4 < 0$ is a sufficient condition for $\theta_s > 0$. Hence, for Ψ_4 sufficiently close to zero both θ_s and $\eta_s(y)$ are positive.

The effects on the critical values y^* and y^{**} follow from differentiating equations (A.31) and (A.33). Intuitively the effects depend on the relative magnitudes of η_s^* , $\eta_s(y)$ and θ_s . Very loosely we may conjecture - since the signs of $\eta_s(y)$ and θ_s are ambiguous - that $y_s^* < 0$ while y_s^{**} may have any sign. If that is the case, it follows, in analogy with previous argument, that an increase in the proportion of check goods ($ds < 0$) will reduce the variance of $\ln P$.

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