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THE SPECIFICATION OF DISEQUILIBRIUM
IN FLOW OF FUNDS MODELS

by

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The flow of funds data are a quarterly multi-sector, multi-instrument tableau of financial transactions. The estimation of models explaining these data on the basis of interest rates and real sector inputs has by now become quite refined and almost standardized when equilibrium is assumed to hold over time in all markets. The possibility of allowing for disequilibrium in such models, however, has been considered only for special cases and specific markets.² The general neglect of disequilibrium in flow of funds models is disturbing not only as a matter of principle, but also as a matter of practical importance. It is generally agreed that disequilibrium, or credit rationing, occurs in a number of the negotiated loan markets---specifically business, mortgage, and consumer loans---and it seems likely that, over the time of a quarter year, interest rates may not clear other financial markets as well. The purpose of this note, therefore is to suggest a *general* method for specifying and estimating disequilibrium in a flow of funds model.

Basically, four criteria have been used in settling on a specific method:

(i) The specification should take into account the two major constraints that restrict a flow of funds model: the budget constraints holding for each sector, and the market clearing conditions holding (even in disequilibrium) for each instrument. The budget constraints are a complication because the disequilibrium

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2) The two most carefully studied cases are the business loan and mortgage loan markets. Jaffee [6] and [7] provide examples of how disequilibrium can be treated in these specific markets and survey the literature.

in one market can "spillover", indeed it must spillover, into other markets, and these effects must be consistent with the balance sheet constraint. The market clearing conditions are a complication because the possibility of disequilibrium necessarily implies that interest rates may not be clearing the markets. It thus becomes necessary to specify in exactly what way the markets for each instrument are cleared.

(ii) The equation system with its disequilibrium specification should be reasonably easy to estimate. "Reasonably" certainly means that we rule out specifications that would require non-linear maximization techniques. In addition, it is clearly desirable that the constraints holding across equations be kept to a minimum (some such as the budget constraints are obviously unavoidable), and that consistent estimates be obtained with two stage least squares (TSLS). Finally, it is useful if the specification is such that preliminary testing can be reasonably carried out on a single equation basis using ordinary least squares (OLS) as the estimation technique.

(iii) The specification should allow for a *test* for disequilibrium in the various markets. This is important because disequilibrium may not be a valid assumption for *all* financial markets. Consequently, it is desirable that the specification be such that it can be applied to all markets, and that it reduce to an equilibrium specification for a particular market when parameters take on special values. The method proposed, in fact, allows for degrees of disequilibrium in each market in terms of a single parameter, and thus simple statistical tests are available to determine whether the parameter is different from zero.

(iv) The specification should have a natural deterministic simulation mode. This is important because the main current use of flow of funds models involves simulating them, whether for the evaluation of policy proposals or for prediction. In this context, it is also important that the model converge in some systematic way to an equilibrium, and that this equilibrium be identified directly in terms of the model's parameters.

This, then, sets up the problem. We proceed as follows.

In Section 1, we present as the starting point the standard equilibrium specification for a flow of funds model, and then outline the various possibilities for introducing disequilibrium into the model. In Section 2, the proposed disequilibrium specification is discussed in detail. In Section 3, some alternative procedures are discussed further.

1. THE EQUILIBRIUM SPECIFICATION OF THE FLOW OF FUNDS MODEL

The standard general form for instrument demand and supply functions is given by,

$$(1) \quad X_i^j = \left(\sum_{k=0}^n a_{ik}^j r_k \right) W^j, \quad j = 1, \dots, J, \quad i = 1, \dots, N.$$

The notation is as follows. X_i^j is the net demand of instrument i by sector j . We adopt the convention that X_i^j is positive when the instrument is held as an asset and is negative when the instrument is held as a liability. A general case with J sectors and N instruments is assumed. r_k is the interest rate in market k , except that, to simplify the notation, r_0 is a constant of value one. W^j is the fixed net financial wealth of sector j .

The a_{ik}^j are the behavioral coefficients of the net demand functions.³ The variables X_i^j , r_k , and W^j are all functions of time, but a time subscript is omitted except where it is essential below. Also, there can be a stochastic error term U_i^j in (1) and this will be introduced explicitly when needed below.

Two further sets of conditions are required to complete the model. First, there is the budget constraint that,

3) The specification could be changed in two ways without creating any problems for the analysis that follows. First, the specification could be fully linear in both r_k and W^j following, for example Parkin [14], rather than the stated multiplicative form which follows Brainard and Tobin [2]. Second, r_k could be replaced by $1/r_k$, presumably with a corresponding change in the signs of the coefficients. Also note the assumption that W^j be fixed is important and is discussed further below.

$$(2) \quad \sum_{i=1}^N X_i^j = W^j, \quad j = 1, \dots, J.$$

For this to be met, it is sufficient that the coefficients satisfy the following conditions:⁴

$$(3) \quad \sum_{i=1}^N a_{i0}^j = 1, \quad j = 1, \dots, J,$$

and

$$(4) \quad \sum_{i=1}^N a_{ik}^j = 0, \quad k = 1, \dots, N, \quad j = 1, \dots, J.$$

Second, there is the equilibrium condition that,

$$(5) \quad \sum_{j=1}^J X_i^j = 0, \quad i = 1, \dots, N.$$

It is assumed, in particular, that interest rates adjust to clear all markets in each period. One can thus define an equilibrium vector of interest rate $[r_k^*]$, which depends on the W^j values and the a_{ik}^j coefficients, and an equilibrium matrix of net demands $[(X_i^j)^*]$ such that (5) holds identically at these values. It is also clear, from Walras' law, that one of the $N + J$ equations in (2) and (5) is not independent. This is generally used in the form that if the budget constraints of (2) are all met (or equivalently that the restrictions of (3) and (4) are met) and if $N - 1$ of the markets meet condition (5), then the last market identically meets condition (5).

The model of (1), (2), and (5) is, of course, often extended to incorporate a variety of dynamic considerations. For one thing, it is frequently assumed that the net demands respond only partially to changes in either the W^j or r_k , so that the lagged

4) If there is an additive stochastic error in (1), it is also required that $\sum_{i=1}^N U_i^j = 0$ for each j . For this reason, the error terms *within* a sector are necessarily correlated, and different estimating techniques will have different efficiency characteristics depending on the exact form of the correlation.

values of the X_1^j enter the specification (implying a Koyck lag). Another common assumption is that changes in W^j have an impact effect that differs from the first period coefficient of the Koyck lag on W^j ; in this case ΔW^j enter the specification.⁵ For these two cases, and in fact quite generally, dynamic extensions thus have the effect of adding variables to the X_1^j net demands of (1). It is always assumed, however, that the budget constraints of (2) and the equilibrium conditions of (5) are met, whatever way the actual net demands of (1) are augmented with dynamic elements. Moreover, our discussion of disequilibrium models below uses only the information of (2) and (5) with respect to the equilibrium model. Consequently, the proposed disequilibrium specification can incorporate dynamic elements in (1) without any complications. This will become even more apparent as we proceed. Similar considerations also apply to the possibility that the r_k variables may enter the net demand equation (1) directly with a time lag.

We now introduce the case of disequilibrium. By disequilibrium we mean that interest rates can no longer be assumed to clear all markets period by period. Thus equation (5) is eliminated from the system. Now, one strategy for specifying a disequilibrium model is to replace (5) directly with some alternative. For example, a suggestion of Fair and Jaffee [5] in a single market context was that the quantity actually transacted in disequilibrium markets is the minimum of the demand and supply present at the ruling market price. Other possibilities are also available. As indicated in the survey of Jaffee [8], however, even in a single market context, specifications in this form (and where there is no additional information) can be estimated only with complicated nonlinear techniques. In a flow of funds context, the problems would be even worse. So we do not follow this type of model here, although some relationships to it will be noted below.

An alternative strategy is to make adjustments for disequilibrium in the net demand functions of (1). It is useful in this

5) See Clinton [3] and Ladenson [11], [12] for further discussion of such specifications.

context to refer to the X_i^j net demands of (1) as the *notional demands*, that is, X_i^j are the net demands when market participants assume, and correctly so, that they can buy or sell all they wish at the ruling interest rates. In contrast, we define as *effective (net) demands*, which may be denoted as $\bar{X}_i^j$, those behavioral relationships that determine the quantities actually transacted when markets are in disequilibrium. Furthermore, we require that the effective demand functions meet three basic conditions:

$$(6) \quad \sum_{i=1}^N \bar{X}_i^j = W^j, \quad j = 1, \dots, J,$$

$$(7) \quad \sum_{j=1}^J \bar{X}_i^j = 0, \quad i = 1, \dots, N.$$

$$(8) \quad \bar{X}_i^j = X_i^j \text{ whenever } [r_i] = [r_i^*], \text{ for all } i \text{ and } j.$$

Equation (6) requires the effective demand functions to meet the individual sector budget constraints in analogy with (2). Equation (7) is a market clearing condition that requires, in essence, that the amount actually sold in a market must equal the amount actually purchased. Since interest rates are no longer assumed to clear markets, the correspondence between (7) and (5) is only one of form, and indeed the economic implications of (7) will be determined only with further specification, as will be done below. Equation (8) is used here to insure that the disequilibrium model produces exactly the same results as the equilibrium model when the equilibrium interest rate vector $[r_i^*]$ is actually observed. This is not a condition that must necessarily be met by a disequilibrium system, see Jaffee [8] and Leijonhufvud [13], but it is an implication of the stability conditions generally accepted in economic theory, and it is an important condition if the model is to be used to simulate equilibrium results in any meaningful way.

The next step is to specify the precise form for the effective demand function $\bar{X}_i^j$, and to indicate the method for estimating the

system. The proposed case is developed fully in the following Section 2. Then, in Section 3, the relationships of the model to some other specifications are noted.

2. A DISEQUILIBRIUM SPECIFICATION OF A FLOW OF FUNDS MODEL

2.1 The Basic Model

The effective demand functions are specified as,

$$(9) \quad \bar{X}_i^j = X_i^j + \sum_{k=1}^N b_{ik}^j Z_k, \quad i = 1, \dots, N, \quad j = 1, \dots, N,$$

where

$$(10) \quad Z_k = \sum_{j=1}^J X_k^j \quad k = 1, \dots, N.$$

and X_i^j are the notional demand functions defined in (1). Equation (10) defines Z_k as the *notional* excess demand existing in the market at any point in time. Below we refer to Z_k simply as the excess demand of market k . Equation (9) specifies the effective demand of a sector for an instrument as the sum of the notional demand and a weighted sum of the excess demands in all markets.

The idea is that when there is excess demand in some market ($Z_i > 0$), then the effective demand must be made less than the notional demand (suggesting the b_{ii} coefficients should be negative). In addition, the specification allows for spillover effects in that the excess demand in some market k will influence the effective demand in some other market i through the b_{ik} ($i \neq k$) coefficients.

It is perhaps helpful for the moment to think of the Z_k excess demands as if they were observed variables. Moreover, the Z_k are linearly dependent, in that,

$$(11) \quad \sum_{k=1}^N Z_k = 0.$$

This is verified in Appendix Note 1. Condition (11) is sometimes

termed the Say's Law relationship that holds in disequilibrium in analogy with the Walras' Law equilibrium relationship; see Clower [4]. The significance of (11) for present purposes is that (9) can not be estimated as it stands because of the collinearity of the Z_k . Thus, we rewrite (9) to eliminate Z_N from the equation:

$$(12) \quad \bar{X}_i^j = X_i^j + \sum_{k=1}^{N-1} c_{ik}^j Z_k, \quad i = 1, \dots, N, \quad j = 1, \dots, J.$$

It is easily verified that (9) implies (12) when (11) holds and we define,

$$c_{ik}^j = b_{ik}^j - b_{iN}^j, \quad \text{for all } i \text{ and } j \text{ and } k = 1, \dots, N-1.$$

We now consider how the specification of (12) meets the conditions (6), (7), and (8) developed at the end of the previous section. Condition (8) is met directly by the specification since, from (10) and the definition of the equilibrium interest rate vector $[r_k^*]$, it is apparent that the Z_k are each zero when $[r_k] = [r_k^*]$. For condition (6) to be met, it is easily verified that it is sufficient that the a_{ik} coefficients of the notional demands X_i^j meet the budget restrictions (3) and (4) as already indicated, and that the c_{ik}^j coefficients of (12) satisfy:

$$(13) \quad \sum_{i=1}^N c_{ik}^j = 0, \quad k = 1, \dots, N-1; \quad j = 1, \dots, J.$$

This will be called the budget constraint condition on the c_{ik}^j . For condition (7) to be met it is sufficient that we have,

$$(14) \quad \sum_{j=1}^J c_{ik}^j = 0, \quad i = 1, \dots, N-1; \quad k=1, \dots, i-1, i+1, \dots, N-1,$$

and

$$(15) \quad \sum_{j=1}^J c_{iN}^j = -1, \quad i = 1, \dots, N-1.$$

6) There is a further condition on the c_{ik}^j that $\sum_{j=1}^J c_{N,k}^j = 1$,

• $k = 1, \dots, N-1$. This can be deduced, however from (13), (14), and (15), and therefore it is not an independent constraint. See Appendix Note 2 for the proof.

The sufficiency of (14) and (15) is confirmed in Appendix Note 3. The implication of (15) is that the excess demand Z_i in some market i must be fully distributed across the individual sector effective demand functions for that market. The implication of (14) is that the aggregate (over sectors) spillover effects from some market k into some other market i must be zero. The conditions of (14) and (15) may be termed respectively the diagonal and off-diagonal adding across constraints.

Equation (12) is fully general in that it allows all sectors to deal in all instruments and in that it allows disequilibrium in all markets. If a sector does not deal in an instrument, then all the a_{ik}^j and c_{ik}^j coefficients for that sector (j) and instrument (i) are set to zero. If some market k is in equilibrium, then Z_k will be zero, and the terms involving Z_k will vanish from the specification. Note, however, that the effective demand functions for an equilibrium market k will still involve the disequilibrium spillover effects from other (disequilibrium) markets.

Also note that at least two markets must always be specified in disequilibrium: from (11) only one market in disequilibrium is impossible. This, in fact, has a more fundamental implication in terms of our assumption that the W^j are fixed. In particular, without this assumption it is possible that the W^j respond to disequilibrium in the financial sector, in which case the "second" disequilibrium market could be in the real sector. Moreover, the basis for the fixed W^j assumption in equilibrium flow of funds models, that interest rates affect real sector decisions with a lag, would appear much less convincing when the financial market effects include actual credit rationing in disequilibrium. And more generally, this raises the basic question of how real sector specifications should be adjusted to take into account financial market disequilibrium. Consequently, the approach suggested here with the W^j fixed should be interpreted as only a temporary format for gaining experience with disequilibrium flow of funds models and for arriving at preliminary estimates of the disequilibrium. If financial market disequilibrium is, in fact,

important, then it must be fully integrated with real sector specifications.⁷

Turning now to the estimation of (12), if data observations were available for the Z_k variables, then we could easily proceed subject, of course, to the coefficient constraints of (3), (4), (13), (14), and (15). Since the Z_k are the *notional* excess demands, however, they will not be directly observed unless the market participants are actually polled as to their notional intentions. Lacking this, there are then basically two ways to proceed. The first is to consider proxy measures for the Z_k , that is, variables that are observed and are known to be correlated with the Z_k . This, in fact, is the strategy adopted in the following sections of the paper. It should be noted that the use of proxies effectively requires that additional information be added to the system. Another possibility, however, is to use (10) to rewrite (12) in terms of the X_i^j and then to carry out the estimation directly. The problem with this strategy is that key coefficients are not identified in the system, and therefore the simulation uses of the model would be severely limited. Or to put the matter differently, the information added with the use of the proxy approach is exactly the set of information needed to identify the full system. The lack of identification with the second approach is detailed in Appendix Note 4.

7) This is a two-way street, however. Studies that have stressed the inter-connection of labor market and consumer goods market disequilibrium, such as Clower [4] and Barro and Grossman [1], have been equally guilty of ignoring the implications of real sector disequilibrium for financial sector behavior. In particular, in these studies, if a permanent income or life-cycle theory of consumption demand is included, then perfect borrowing opportunities for the unemployed laborers would preclude or minimize the multiplier contractions that are generated in goods markets. What these studies implicitly assume, instead, is that the supply of credit is fixed, which corresponds directly to our assumption that the W^j are fixed.

2.2 Identifying the System with Interest Rate Change Information

We now assume that,

$$(16) \quad \Delta r_k = \gamma_k Z_k + \varepsilon_k, \quad k = 1, \dots, N.$$

That is, we assume that in each market the interest rate changes in a fixed proportion to the excess demand in the market. In (2.3) just below we will discuss the details of alternative forms and timing lags that can easily be incorporated in the same way as (16). (16) can be rewritten as:

$$(17) \quad Z_k = \frac{\Delta r_k}{\gamma_k} - \frac{\varepsilon_k}{\gamma_k}, \quad k = 1, \dots, N.$$

Substituting (17) into (12) then yields:

$$(18) \quad \bar{X}_i^j = X_i^j + \sum_{k=1}^{N-1} c_{ik}^j \left(\frac{\Delta r_k}{\gamma_k} - \frac{\varepsilon_k}{\gamma_k} \right), \quad i = 1, \dots, N; \quad j = 1, \dots, J.$$

Letting,

$$(19) \quad d_{ik}^j = \frac{c_{ik}^j}{\gamma_k}, \quad \text{for all } i, j, k,$$

we then have,

$$(20) \quad \bar{X}_i^j = X_i^j + \sum_{k=1}^{N-1} d_{ik}^j (\Delta r_k), \quad i = 1, \dots, N; \quad j = 1, \dots, J,$$

where we omit the error term. From the coefficient restrictions on the c_{ik}^j , (19) yields the following restrictions on the d_{ik}^j :

From the budget constraint (13) we have:

$$(21) \quad \sum_{i=1}^N d_{ik}^j = \frac{1}{\gamma_k} \left(\sum_{i=1}^N c_{ik}^j \right) = 0, \quad k = 1, \dots, N-1; \quad j = 1, \dots, J.$$

From the diagonal adding across constraint (15), we have:

$$(22) \quad \sum_{j=1}^J d_{ii}^j = \frac{1}{\gamma_i} \left(\sum_{j=1}^J c_{ii}^j \right) = -\frac{1}{\gamma_i}, \quad i = 1, \dots, N-1.$$

From the off-diagonal adding across constraint (14), we have:

$$(23) \quad \sum_{j=1}^J d_{ik}^j = \frac{1}{\gamma_k} \sum_{j=1}^J c_{ik}^j = 0, \quad i=1, \dots, N-1; \quad k=1, \dots, i-1, i+1, \dots, N-1.$$

The estimated d_{ik}^j in (20), of course, should meet the restrictions of (21) and (23). It is noteworthy, however, that the diagonal adding across constraint (22) does not restrict the estimated coefficients; instead, (22) implies that the sum of the diagonal elements for any market i ($i \neq N$) will serve to identify directly the adjustment coefficient γ_i for that market. When γ_i is infinite, or $1/\gamma_i$ is zero, then the market i will always be in equilibrium (see also footnote 9 below). Thus from (22) a direct test can be applied to determine whether any specific market is characterized by disequilibrium. Also, once the γ_i are recovered, then the full set of c_{ik}^j can be deduced from the estimated d_{ik}^j by reversing (19). A minor complication in this context is that (22) does not determine γ_N . Instead, from (17) and (11), we can write:

$$(24) \quad \Delta r_N = \frac{1}{\gamma_N} \left(-\sum_{k=1}^{N-1} \frac{\Delta r_k}{\gamma_k} \right) + \epsilon.$$

This can be estimated as an additional relationship and the γ_N deduced in a consistent fashion when the γ_k ($k = 1, \dots, N-1$) are derived from (22).

The main estimation problem then centers on (20) with the budget constraints of (3), (4), and (21), and the off-diagonal adding across constraints of (23). Also, the simultaneity of the interest rate variables, as they enter into the X_1^j of (1) and into Δr_k , should be taken into account. Both the budget constraints and the simultaneity, however, are present even in the equilibrium model, so, in fact, the off-diagonal adding across restrictions are the only new element introduced into the estimation problem.

For the purposes of consistent (but not efficient) estimation, however, even these adding across restrictions could be ignored. And finally, it would not appear unreasonable to do preliminary testing using OLS on each equation alone; in particular, in any equation if the diagonal coefficient d_{ii}^j were estimated not different from zero, then it would appear that the market is an equilibrium one (for a sufficient condition one must test all sectors). So, in summary, it would appear the conditions for a "reasonable" estimation technique have been met.

In terms of simulating the system in a deterministic model, the main difference relative to an equilibrium model is the determination of the interest rates. The interest rates in each period would have to be generated from equation (16) in terms of the Z_k , and the Z_k can be determined in turn from the estimated coefficients using (10) and (1). An iterative procedure could then be used to find the interest rates that satisfy (16). This would appear no more complicated in practice than solving an equilibrium system for the equilibrium set of interest rates. In addition, of course, "equilibrium" simulations could be carried out by setting all the $1/\gamma_k$ coefficients to zero, and then proceeding with normal equilibrium model methods. Finally, it should be noted that the simulated system will have all the specified properties of the estimated system including, of course, the properties specified in (6), (7), and (8). This requires, however, that the coefficients used in the simulation meet the off-diagonal adding across restrictions; otherwise certain discrepancies will arise in the simulation.

2.3 *Extensions with Alternative Interest Rate Change Equations*

The Addition of Extraneous Variables. In some cases, it may be important to specify additional variables (Y_k) in the rate change equation (16):

$$(25) \quad \Delta r_k = \gamma_k Z_k + e_{1k} Y_k.$$

One possibility is if the dynamic behavior of interest rates is

affected by government policy; Jaffee [6], indicates, for example, that changes in the commercial loan rate may be affected by changes in the Federal Reserve discount rate. Another possibility concerns markets in which specific institutions do the rate-setting, and in which profit maximization factors other than simple excess demand enter the decision; savings deposit markets are a good example of this.⁸ Such extraneous variables can be incorporated along the lines of (25) as long as the excess demand Z_k continues to enter the equation. The procedure then is to invert (25) as we did with (16) to obtain:

$$(26) \quad Z_k = \frac{\Delta r_k - e_{1k} Y_k}{\gamma_k} .$$

If the coefficient e_{1k} is known exactly from prior information, then the right hand side of (26) is effectively one variable, and we can proceed just as in section 22. If e_{1k} has to be estimated, then whenever Z_k appears in the system, it has to be replaced by the two variables on the right hand side of (26). This creates no basic problems, but there then is an implicit coefficient constraint that the e_{1k} be the same wherever it appears. Consistent estimates, however, can be obtained without enforcing the constraint. In simulating a system with a rate change equation like (25), we will of course need the data on Y_k to generate the simulated values for the rate change; also, certain discrepancies will arise if the e_{1k} coefficient takes on different values in different parts of the system.

It is thus clear that, as a technical matter, even quite complicated rate change equations can be handled. In practice, however, the

8) It is worth noting that savings deposit markets are a special case in another sense. For most savings institutions, their chartering laws require that they accept deposits at the quoted rates from all individuals. In particular, even when the deposit rate is set too high, they are not free to limit the amount of savings deposits. In such a situation, it would appear the disequilibrium coefficients should be set on prior grounds such that the savings institutions absorb the total excess demand. This also guarantees that the excess demand should appear in the rate-setting equations in these markets.

the situation is not quite this sanguine. First, if a number of variables enter the rate change equations, and if complicated expressions are required in many of the markets, then the actual number of variables entering the estimation of each equation will be very large, and the problems of multicollinearity could be severe. Second, it is clear that the informational content of rate changes becomes less, in terms of proxying for excess demand values, the more extraneous factors there are that determine the course of the rate change. In this sense, the efficiency of the procedure falls as a disequilibrium method the more complicated are the required rate change equations.

Alternative Scalings. One possibly disconcerting feature of (16) is that the adjustment coefficient γ_k relates a dollar magnitude (excess demand) to a pure number (the interest rate). It thus might be considered desirable to scale the variables so that both sides of the equation are pure numbers. On the interest rate side, one obvious scaling is simply the level of the interest rate. On the excess demand side, a natural scaling is the level of demand or supply (or perhaps an average). In a multi-sector setting, the summed or averaged values for the W^j variables could be used since these are the basic driving forces of the level of demand and supply. In this case the relationship would have the form:

$$(27) \quad \frac{\Delta r_k}{r_k} = \gamma_k \left(\frac{Z_k}{\sum_{j=1}^J W^j} \right)$$

Such a formulation can be handled in the system in the manner proposed just above. An even more appealing specification for the rate change equation is

$$(28) \quad \Delta r_k = \gamma_k \left(\frac{Z_k}{\sum_{j=1}^J a_{ii}^j W^j} \right)$$

This has the property that it can be directly derived from the

assumption that

$$(29) \quad \Delta r_k = \gamma_k (r_k^* - r_k)$$

where r_k^* is the equilibrium value for the interest rate. It has the drawback, however, that the equations would then become non-linear in the parameters specified in (28).

Alternative Timing Relationships. In many markets it could be expected not only that the interest rate responds slowly, but also that it responds with a distinct period lag to excess demand. In this case, (16) could be specified with a lagged value for Z_k . When this is inverted, as in (17), there then is the implication that the current value of Z_k will be a function of the future value (a lead) of the rate change. This may appear a bit strange, but it introduces no real problems. In the estimation, one would simply use the lead values of the interest rate for the proxy. And in simulation, the assumed nature of the rate change equation would indeed insure that the required lead value could be generated by the data available in each current period.

In this context, it is also perhaps useful to raise a more basic question concerning the timing of all variables in a flow of funds model. Timing is usually chosen in a rather *ad hoc* fashion in most models, and the disequilibrium context provides an opportunity to pin down at least some issues. If data were available on an instantaneous basis, then the rate change equation (16) could be specified as

$$(30) \quad \frac{dr_\tau}{dt} = \gamma Z_\tau$$

where γ is the instantaneous rate of adjustment, τ is the time index, and we omit market subscripts. Integrating between two time periods, say t and $t-1$, then yields,

$$\begin{aligned}
 (31) \quad r_t - r_{t-1} &= \Delta r_t = \int_{t-1}^t \gamma Z_\tau d\tau \\
 &= \gamma \bar{Z}_t
 \end{aligned}$$

where $\bar{Z}_t$ is the average value of Z_t over the period $t-1$ to t . Consequently, if we measure interest rates as end of period values, and the demand and supply variables (the X_i^j , W^j , etc.) as average of period values, then equation (16) is accurate for the timing of the rate change (that is, it is identical to (31)).⁹ Of course, this arrangement of timing may or may not be consistent with the behavior of the demand and supply sectors in responding to interest rates. In particular, if X_t were related to r_t , then this would build in a 1/2 period lead in the response of X to r (which could be rationalized only on the basis of expectations). Similarly, if X_t were related to r_{t-1} , then this would build in an automatic 1/2 period lag (which might be quite reasonable). On the other hand, if we measure r_t and X_t at the same points in time, either as period averages or as end of period values, then we get into certain complications with the rate change equation. To take one example, if X_t and r_t are measured as end of period values, then the rate change equation of (16) is not consistent with (31). A reasonable adjustment would be to specify in this case that

$$(32) \quad \Delta r_t = \frac{\gamma}{2} (Z_t + Z_{t-1})$$

This can be handled in the model in the ways described above, but, of course, it is still only an approximation to (31). The point therefore is not that there is a simple right or wrong answer to the problem of timing, but rather that the problem deserves serious

9) The question is sometimes raised how a system of this form can distinguish between interest rate changes that are generated by disequilibrium, and the rate changes that are generated by simple shifts in the demand or supply functions in an equilibrium setting. The answer can be seen quite clearly in (31). If the markets are always in equilibrium, then $\bar{Z}_t$ will be zero and γ will be estimated as infinity; but this does not preclude that Δr_t can still be a nonzero number. On the other hand, when markets are in disequilibrium, then both $\bar{Z}_t$ and Δr_t will be nonzero, and γ will be estimated at some finite value. And, of course, if the rates r_t never change, then γ will be estimated as zero.

consideration when disequilibrium rate change equations are involved.

A Demand-Supply Minimum Interpretation. As was noted above, Fair and Jaffee [5] have considered in a single market context the possibility that the transacted quantity is determined as the minimum of the notational demand and supply amounts. This type of hypothesis can be implemented in a flow of funds framework as well, given the additional information from the interest rate changes. The key step is to define the following two variables:

$$\Delta r_k^d = \begin{cases} \Delta r_k & \text{when } \Delta r_k \geq 0 \\ 0 & \text{otherwise} \end{cases},$$

$$\Delta r_k^s = \begin{cases} -\Delta r_k & \text{when } \Delta r_k \leq 0 \\ 0 & \text{otherwise} \end{cases}.$$

Then, wherever Δr_k appears in any of the equations of (20), it is replaced with Δr_k^s if the sector supplies the instrument, and it is replaced with Δr_k^d if the sector demands the instrument. This, of course, assumes that sectors are unambiguously suppliers or demanders throughout the sample period; otherwise the method becomes much more complicated. In terms of the d_{ik}^j coefficients, the budget constraints are enforced just as before. Also, the adding across constraints (both diagonal and off-diagonal) have the same basic form, but the constraints hold separately (for each market) for the demand and the supply sectors. This does double the number of adding across constraints, but the number of elements in each constraint is now essentially half. The system can be simulated essentially as before. The reader is referred to Fair and Jaffee [5] to see that this technique will in fact enforce the constraint that transacted quantity be the minimum of notional demand and supply.

3. ALTERNATIVE DISEQUILIBRIUM SPECIFICATIONS

The discussion in Section 2.1 above set up a general disequilibrium model in terms of the excess demand variables Z_k , while

the discussion in Sections 2.2 and 2.3 indicated how such a model can actually be estimated when information on interest rates is used to identify the system. In fact, of course, the disequilibrium model of Section 2.1 can be identified and estimated whenever any proxy variable is observed that correlates with the Z_k . Such proxy variables could arise, for example, when waiting queues for the demand or supply of financial instruments are observed, when broker or dealer inventories (or bid-ask spreads) change systematically with Z_k , or, of course, when market participants are polled directly concerning market excess demand.¹⁰ The advantages of the interest rate change proxy in this context are that the required hypothesis is reasonable and that the required data are available for all markets. This is not to deny, however, that some other proxy may be more efficient (that is, have a higher correlation with Z_k) in specific cases.

Changes in the nonprice terms on financial contracts are also sometimes suggested as good proxies for excess demand in financial markets (see Jaffee [9] and Jaffee and Russell [10] and cited references). The idea is that if interest rates adjust slowly, then the task falls upon nonprice terms to clear the markets during the disequilibrium. This is not a fully satisfying hypothesis, in that it is not clear why nonprice terms should adjust so easily, while interest rates do not. Still, compared to the model in Section 2.1, the nonprice term theory has the seeming advantage that the market is cleared by an adjustment in some variable; the theory of 2.1 required, in contrast, that the adding across constraints be met in order that markets be cleared. In fact, however, the two approaches are closely related, and the method of Section 2 is all the more appealing when this relationship is made clear. We thus now turn to a brief development of a nonprice term adjustment model.

10) With respect to waiting queues, such data may be available, for example, in European countries where the government explicitly manages capital markets by restricting new issues. With respect to polling, the Federal Reserve *Bulletin* regularly publishes survey data relating to the nonprice terms of business loans.

3.1 A Nonprice Adjustment Model

The effective demand functions of the model are now written

$$(33) \quad \bar{X}_i^j = X_i^j + \sum_{k=1}^N f_{ik}^j v_k^j W^j, \quad i = 1, \dots, N; \quad j = 1, \dots, N;$$

where v_k is the nonprice term on the instrument traded in market k . By comparison with (1), it can be seen that the nonprice variables are incorporated in the model in exactly the same way that the interest rates are included in the notional demand functions. Indeed (33) can be interpreted as a notional schedule, in that individuals are free to demand or supply all they wish at the quoted market interest rates and nonprice terms. Furthermore, the f_{ik}^j are subject to budget constraints in exactly the form of (4) (thus meeting condition (6)), and they are not subject to any adding across constraints. The model of (33) differs from the equilibrium model of (1), however, in that the interest rates are assumed to adjust in the disequilibrium form of equation (16); and the disequilibrium market clearing condition (7) is met by the assumption that the nonprice terms v_k adjust appropriately. Also, note that when the interest rates are at the equilibrium values of r_k^* , so that the Z_k are zero by definition, then the market clearing condition (7) is met with all v_k equal to zero. In other words, the full equilibrium of this system has the interest rates at r_k^* and the v_k^* all zero.¹¹

11) There is actually more behind this result than is apparent. The equilibrium condition for the v_k^* follows directly from the assumption that interest rates adjust in proportion to excess demand as given by the summed X_i^j terms in (16). If the equilibrium for the v_k^* is different from zero, however, it is then appropriate for the interest rates to adjust in response to notional excess demand including terms with v_k^* . In this sense, the use of (16) is self-fulfilling, but not necessarily right. When the nonprice terms enter the specification linearly, as they do in (33), however, then the equilibrium values for v_k^* are independent of any parameters of the problem; in fact, there is a corner solution with zero a likely, but not necessary case. In the current example, a nonzero but constant solution for v_k^* would not affect the main points. In actual application, however, care should be taken in this matter.

We now wish to demonstrate that equation (20) of the excess demand disequilibrium model can be written in a form that corresponds closely to equation (33). Summing (33) over j , using (7) and (10), switching the order of summation, and solving for Z_i yields:

$$(34) \quad Z_i = -\sum_{k=1}^N F_{ik} v_k, \quad i = 1, \dots, N,$$

$$\text{where } F_{ik} = \sum_{j=1}^J f_{ik}^j W^j, \quad k = 1, \dots, N; \quad i = 1, \dots, N.$$

Equation (34) indicates that the Z_i can be written as linear combinations of the v_k nonprice terms. Thus, if the Z_i of (34) are substituted into equations (20), the results are effective demand functions which depend on linear combinations of the non-price terms in the same basic form as (33), except for the adding across constraints on the d_{ik}^j as they enter (20). In this sense, the excess demand disequilibrium model is a special case of a nonprice term adjustment model.

3.2 Concluding Comments. The purpose of this note has been to develop a specification for disequilibrium in flow of funds that can be estimated in quite a reasonable fashion. It is now hoped, of course, that the suggested approach is sufficiently attractive to induce builders or possessors of full-scale flow of funds models to implement it. The result, of course, would be a simulation system capable of generating intriguing patterns in terms of the interactions of disequilibrium markets and the spillover effects within sectors.

Note 1: To verify that (11), $\sum_{k=1}^N Z_k = 0$, is valid.

$$\begin{aligned}
 \sum_{k=1}^N Z_k &= \sum_{k=1}^N \sum_{j=1}^J x_{kj}^j && \text{(from 10)} \\
 &= \sum_{j=1}^J \left(\sum_{k=1}^N x_{kj}^j \right) && \text{(switching summation)} \\
 &= \sum_{j=1}^J (W^j) && \text{(from budget constraint 2)} \\
 &= 0 && \text{(from flow of funds accounting identity).}
 \end{aligned}$$

The accounting identity is that net financial wealth summed over all sectors must be zero. This assumes that all demanders and suppliers of financial instruments, and in particular that the fiscal and monetary authorities, are included in the accounting. The portfolio behavior of the government policy sectors can, of course, still be treated as exogenous by setting the appropriate coefficients to zero. There is also a question of economic theory suggesting that the household sector may not count the liabilities of the government as net real wealth assets; in this case the accounting framework stands (or is easily adjusted) but the economic implications for real sector behavior may be different.

Note 2: To show that $\sum_{j=1}^J c_{Nk}^j = 1$, $k = 1, \dots, N-1$, when (13), (14), and (15) hold.

$$c_{Nk}^j = -\sum_{i=1}^{N-1} c_{ik}^j, \quad k = 1, \dots, N-1, \quad J = 1, \dots, J, \quad (\text{from 13});$$

$$\sum_{j=1}^J c_{Nk}^j = -\sum_{i=1}^{N-1} \sum_{j=1}^J c_{ik}^j, \quad k = 1, \dots, N-1, \quad (\text{summing over } j \text{ and switching summation}),$$

$$= -\sum_{\substack{i=1 \\ i \neq k}}^{N-1} \sum_{j=1}^J c_{ik}^j - \sum_{j=1}^J c_{kk}^j, \quad k = 1, \dots, N-1, \quad (\text{splitting the summation}),$$

$$= 0 - (-1) = 1, \quad k = 1, \dots, N-1, \quad (\text{from 14 and 15}).$$

Note 3: To show that (14) and (15) are sufficient for (7).

$$\sum_{j=1}^J \bar{x}_i^j = \sum_{j=1}^J x_i^j + \sum_{j=1}^J \sum_{k=1}^{N-1} c_{ik}^j z_k \quad (\text{from 12}),$$

$$= z_i + \sum_{k=1}^{N-1} \sum_{j=1}^J c_{ik}^j z_k \quad (\text{using 10 and switching the summation}),$$

$$= (1 + \sum_{j=1}^J c_{ii}^j) z_i + \sum_{\substack{k=1 \\ k \neq i}}^{N-1} \sum_{j=1}^J c_{ik}^j z_k \quad (\text{splitting the summation}),$$

$$= 0 + 0 \quad (\text{using 14 and 15}).$$

Note 4: To show that (12) is not identified when the Z_k are given by (10).

We demonstrate the result explicitly for the 2 sector, 2 instrument case. Using (12) and (10), the effective demand for instrument 1 by sector 1 can be written:

$$\bar{X}_1^1 = X_1^1 + c_{11}^1 (X_1^1 + X_1^2) = (1 + c_{11}^1) X_1^1 + c_{11}^1 X_1^2.$$

Now consider the following transformation of the coefficients:

The "a" coefficients of X_1^1 (see equation 1) are all multiplied by $(1 + \rho)$;

The "a" coefficients of X_1^2 are all multiplied by $\frac{(1 + \rho)c_{11}^1}{c_{11}^1 - \rho}$;

The c_{11}^1 coefficient is transformed to $\frac{c_{11}^1 - \rho}{(1 + \rho)}$.

The effective demand $\bar{X}_1^1$ is then,

$$\begin{aligned} & \left(1 + \frac{c_{11}^1 - \rho}{1 + \rho}\right) (1 + \rho) X_1^1 + \left(\frac{c_{11}^1 - \rho}{1 + \rho}\right) (c_{11}^1) \left(\frac{1 + \rho}{c_{11}^1 - \rho}\right) X_1^2 \\ &= 1 + c_{11}^1 X_1^1 + c_{11}^1 X_1^2 \\ &= \bar{X}_1^1 \end{aligned}$$

Thus the coefficients of the first equation are not identified. Moreover, using the coefficient restrictions, the other three equations of the system, $\bar{X}_1^2$, $\bar{X}_2^1$, and $\bar{X}_2^2$, can all be written in exactly the form of $\bar{X}_1^1$. Thus none of the equations are identified. Similar considerations apply to the general $N \times J$ case.

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