Seminar Paper No. 129

INFLATION AND BALANCE OF PAYMENTS ADJUSTMENTS
WITH MAXIMIZING CONSUMERS

by

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1. INTRODUCTION

The hypothesis of maximizing behavior has a long tradition in economics. It has yielded powerful results in such diverse areas as general equilibrium theory, the theory of optimal economic growth, public finance, urban economics, international trade, and many more. Nevertheless, an explicit use of the maximizing hypothesis is virtually absent in the literature on international monetary problems. [For exceptions, see Helpman and Razin (1979), Kareken and Wallece (1978), Stockman (1978), and Helpman (1979).] True, specifiers of portfolio balance equations and expenditure functions have in mind functions that represent maximizing behavior. But are the assumptions made about these functions consistent with maximizing behavior? And even if they are, is it not possible that maximizing behavior imposes more restrictions, thus adding more structure than admitted by these writers?

Since the expenditure function and the demand for money function are central to the analysis of many problems in international economics, the paper starts with a derivation of an expenditure function and a money demand function from intertemporal optimization, using an explicit microeconomic theory of money (Sections 2 and 3). This exercise is interesting in its

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own right, because it provides a characterization of optimal spending behavior in the presence of liquidity constraints. Here, money is used for transaction purposes and as a store of value, but it does not provide direct utility. [Cf. Dornbusch and Mussa (1975).] A well-defined money demand function arises nevertheless.

Following the characterization of the expenditure function and the money demand function which arise from intertemporal maximization of utility, these functions are applied to the problem of international adjustment under fixed exchange rates (Section 4). Contrary to the existing view, it is shown that for stationary economies without capital markets, the behavior of the price level during the process of balance of payments adjustments is predictable: either it will be constant, or it will rise for a finite number of periods at the beginning of the adjustment process and remain constant thereafter. This is a general result; it is not based on any specific functional forms. It is proved first for countries populated by identical individuals, and then extended to economies populated by many different individuals. The concluding remarks explain why in the presence of capital markets, and independently of whether international capital flows are permitted, the price level remains constant during the process of adjustment.

The results derived in this paper are encouraging. They suggest that the hypothesis of maximizing behavior coupled with a microeconomic foundation for the role of money can be used instead of the current ad hoc models to explain the course of economic variables. There is, of course, room for much more work along these lines which, hopefully, will be done.
2. THE BASIC FRAMEWORK

The world consists of two countries; the home country, country H, and the foreign country, country F. Each country produces a single perishable good whose output level $y_{it}$, $i=H, F$, is given in every period: $y_{Ht} (> 0)$ in the home country, and $y_{Ft} (> 0)$ in the foreign country, $t = 1, 2, \ldots$. Outputs are perfect substitutes in consumption.

Every country has its own money. Purchases in the home country must be made in home money, while foreign goods must be purchased with foreign money. Monies are the only assets, so that apart from transaction purposes they also serve as the only stores of value.

At the beginning of period $t$ a representative consumer has certain quantities of home and foreign money. He can -- subject to his assets budget constraint -- reallocate his asset holdings via trading with the Exchange Rate Stabilizing Authority (ERSA) to desired holding levels of each currency. During period $t$ the consumer can spend part or all of his home currency holdings on home goods and part or all of his foreign currency holdings on foreign goods. Money not spent may be carried over to the next period, $t+1$.

At the end of period $t$ the consumer also receives dividends from firms, which we added to his initial money holdings at the beginning of period $t+1$; he cannot use the dividends distributed at the end of period $t$ to buy goods in period $t+1$. The amounts of money that have not been spent during period $t$ plus dividend payments at the end of period $t$ determine the individual's money holdings at the beginning of period $t+1$. Then the process repeats itself. It is assumed that home firms are owned by home residents while foreign firms are owned by foreign residents.

Firms sell output and pay out dividends. Otherwise they play a passive role in these economies. Monies accumulated from the sale of output are redistributed to stockholders at the end of each period.
The ERSA exchanges one currency for the other at a fixed rate of exchange that equals one. If it is short on, say, home currency, the home government supplies it with the required quantity of home money. Similarly, if it is short on foreign currency, the foreign government supplies it with the required quantity of foreign money. This is the cooperative peg regime described in Helpman (1979), and it conforms to common modelings of a fixed exchange rate regime. Thus, ERSA operations do not change the aggregate quantity of money in circulation in the world economy, only its composition. In what follows it is assumed that individuals do not expect to pay taxes or receive transfers from their government, so that the aggregate quantity of money in the world economy is not expected to change.

Now we are prepared to state the decision problem of a representative individual of country $i$, $i = H, F$. The following decision problem is for period $t$. Prices for periods $t + \tau$, $\tau = 1, 2, \ldots$, should be interpreted as expected prices (all expectations are point expectations). In order to avoid excessive notation, I do not introduce notations for country-specific expectations. The decision problem of a representative individual of country $i$ is:

(1) Choose $(c_{Ht+\tau}^i, c_{Ft+\tau}^i, m_{Ht+\tau}^i, m_{Ft+\tau}^i) \geq 0$, $\tau = 0, 1, \ldots$, to maximize

$$\sum_{\tau=0}^{\infty} (\delta^\tau)^i u^i(c_{Ht+\tau}^i + c_{Ft+\tau}^i)$$

subject to

(a) $m_{Ht}^i + m_{Ft}^i \leq m_{Ht}^i + m_{Ft}^i$

(b) $m_{Ht+\tau}^i + m_{Ft+\tau}^i \leq (M_{Ht+\tau-1}^i - p_{Ht+\tau-1}^i m_{Ht+\tau-1}^i) + (M_{Ft+\tau-1}^i - p_{Ft+\tau-1}^i c_{Ft+\tau-1}^i) + p_{it+\tau-1}^i y_{it+\tau-1}$, $\tau = 1, 2, \ldots$
(c1) \[ p_{Ht+\tau}^i c_{Ht+\tau}^i \leq M_{Ht+\tau}^i, \quad \tau = 0, 1, \ldots \]

(c2) \[ p_{Ft+\tau}^i c_{Ft+\tau}^i \leq M_{Ft+\tau}^i, \quad \tau = 0, 1, \ldots \]

where superscript \( i \) stands for the country, \( i = H, F \), and

\[ \delta_i = \text{discount factor, equal to one over one plus the rate of time preference, } 0 < \delta_i < 1 \]

\[ u_i = \text{strictly increasing, strictly concave, twice continuously differentiable utility function, defined on } [0, +\infty) \]

\[ c_{Ht+\tau}^i = \text{consumption of home goods in period } t+\tau \]

\[ c_{Ft+\tau}^i = \text{consumption of foreign goods in period } t+\tau \]

\[ M_{Ht+\tau}^i = \text{holdings of home money in the beginning of period } t+\tau \]

\[ M_{Ft+\tau}^i = \text{holdings of foreign money in the beginning of period } t+\tau \]

\[ M_{Ht}^i = \text{initial holdings of home money in period } t \]

\[ M_{Ft}^i = \text{initial holdings of foreign money in period } t \]

\[ p_{it+\tau} = \text{price of country } i\text{'s goods in terms of } i\text{'s money in period } t+\tau \]

Since in an equilibrium the demand for each output should equal its supply, it implies that in an equilibrium \( p_{Ht+\tau} = p_{Ft+\tau} \). For if \( p_{Ht+\tau} > p_{Ft+\tau} \), the demand for the home country's output in period \( t+\tau \) is zero, and if \( p_{Ht+\tau} < p_{Ft+\tau} \) the demand for the foreign country's output in period \( t+\tau \) is zero. [This is an implication of (1).] With equal prices in both countries, a consumer is indifferent to whether he buys home or foreign goods, as well as to whether he stores home or foreign money. Hence, letting

\[ p_{t+\tau} = p_{Ht+\tau} = p_{Ft+\tau}, \quad M_{t+\tau}^i = M_{Ht+\tau}^i + M_{Ft+\tau}^i, \quad c_{t+\tau}^i = c_{Ht+\tau}^i + c_{Ft+\tau}^i, \quad M_t^i = M_{Ht+\tau}^i + M_{Ft+\tau}^i, \]

problem (1) can be rewritten as:
(2) Choose \((c^i_{t+\tau}, M^i_{t+\tau}) > 0, \quad \tau = 0, 1, \ldots\) to maximize

\[
\sum_{\tau=0}^{\infty} (s^i)_{t+\tau} u^i(c^i_{t+\tau})
\]

subject to

(a) \(M^i_t \geq M_t^i\)

(b) \(M^i_{t+\tau} \leq M^i_{t+\tau-1} - p^i_{t+\tau-1} c^i_{t+\tau-1} + p^i_{t+\tau-1} y^i_{t+\tau-1}, \quad \tau = 1, 2, \ldots\)

(c) \(p^i_{t+\tau} c^i_{t+\tau} \leq M^i_{t+\tau}, \quad \tau = 0, 1, \ldots\)

In a temporary equilibrium in period \(t\) the demand for each output equals its supply and the demanded quantity of each currency for the beginning of period \(t+1\) equals its supply, where demands are derived from the consumers' decision problems. Since the ERSA can provide any desired composition of currencies, and since country-specific outputs are perfect substitutes in consumption, the equilibrium conditions in period \(t\) are:

\[
c^H_t(\cdot) + c^F_t(\cdot) = y^H_t + y^F_t
\]

\[
M^H_{t+1}(\cdot) + M^F_{t+1}(\cdot) = M^H_t + M^F_t
\]

where variables followed by brackets indicate demand functions derived from (2). We shall be interested in investigating the infinite sequence of temporary equilibria, which will be proved to exist for the specification adopted below.

As a temporary equilibrium depends on price expectations, these expectations must be defined prior to any analysis of the adjustment process. The decision problem presented above is rich enough to encompass a wide variety of expectational hypotheses. However, I have not been able to solve the problem for all interesting expectational hypothesis (like perfect foresight).
As a result of this constraint, and for the sake of comparison with existing
specifications of the adjustment process under fixed exchange rates in
which current spending is written as a function of current variables only
[see, for example, Dornbusch (1973)], I concentrate on the case of static
expectations, that is, the case in which the expected price of goods in all
future periods is the present price. Under these circumstances, \( p_{t+\tau} = p_t \)
for \( \tau = 1,2, \ldots \) in (2). It is also assumed that each country's output level
is constant over time, so \( y_{Ht+\tau} = y_H \), and \( y_{Pt+\tau} = y_P \), \( \tau = 0,1, \ldots \). Under
these assumptions, and taking into account the fact that it is never optimal
to have strict inequalities in constraints (a) and (b) in (2), problem (2)
can be rewritten as follows:

\[
(3) \quad \text{Choose } \left( c_{t+\tau}^i, m_{t+\tau}^i \right) \geq 0, \quad \tau = 0,1, \ldots, \text{ to maximize}
\]

\[
\sum_{\tau=0}^{\infty} (\delta^i)^\tau \pi^i(c_{t+\tau}^i)
\]

subject to

(a) \( m_t^i = m_t^i \)

(b) \( m_{t+\tau}^i = m_{t+\tau-1}^i - c_{t+\tau-1}^i + y_t^i, \quad \tau = 1,2, \ldots \)

(c) \( c_{t+\tau}^i \leq m_{t+\tau}^i, \quad \tau = 0,1, \ldots \)

where \( m_{t+\tau}^i \) equals real balance holdings in period \( t+\tau \) \((=M_{t+\tau}^i/p_t)\) and \( m_t^i \) equals
initial real balance holdings \((=M_t^i/p_t)\).

From (3) it is clear that, given \( \delta^i \) and \( u^i(\cdot) \), real spending in period \( t \)
\( (c_t^i) \) depends only on initial real balance holdings \((m_t^i)\) and permanent real
income \((y_t)\). Moreover, the functional relationship between current real
spending and the level of real balance holdings and permanent real income
is invariant to the time period \( t \). Hence, we can write:
3. PROPERTIES OF THE DEMAND FUNCTIONS

This section provides a complete characterization of the solution to (3) which is then used to derive the properties of the real spending functions $z^i(m^i, y^i_1)$ and the real balance demand functions $\lambda^i(m^i, y^i_1)$. Using the equilibrium conditions (6) and (7), I then derive the properties of the adjustment process. Since this section deals with a single country, the country-identifying index $i$ is dropped for notational convenience.

In order to characterize the solution to (3), we need to define a sequence of auxiliary numbers $\mu_0, \mu_1, \ldots$. These numbers have the following meaning: If the country under consideration has real balances in the interval $(\mu_{T-1}, \mu_T)$, then it plans to arrive at its steady-state consumption level and real balance holdings in $T$ periods (the sequence $\{u_T\}$ is, of course, country specific). This interpretation will become clear after the statement of the main theorem. At this stage I define the sequence $\{u_T\}$ (implicitly) in the following way:

\begin{align*}
(8a) & \quad u_0 = 0 \\
(8b) & \quad u'(\mu_1) = \delta u'(y) \\
(8c) & \quad u'(\mu_T - \mu_{T-1} + y) = \delta^T u'(y), \quad T = 2, 3, \ldots
\end{align*}

where a prime indicates a derivative.

Due to the strict concavity of $u(\cdot)$, (3) implies:

\begin{align*}
(9a) & \quad y < \mu_1 < \mu_2 < \ldots < \mu_T < \ldots
\end{align*}
\((\bar{m}, y)\) at which \(\bar{m} = \mu_T\), \(T = 1, 2, \ldots\). The reason that differentiability fails at these points is that on the left-hand side of \(\bar{m} = \mu_T\) convergence to the steady state takes \(T\) periods, while on the right-hand side it takes \(T + 1\) periods. However, for \(\mu_{T-1} < \bar{m} < \mu_T\) we can calculate the partial derivatives directly from conditions (i) and (ii) of the theorem, using the implicit function theorem. Denoting by \(\bar{c}(\bar{m}, y), \lambda(\bar{m}, y)\), the solution to system (i) and (ii), we have:\(^1\)

\[
(10a) \quad z_m(\bar{m}, y) = \begin{cases} 
1 \quad &\text{for } 0 < \bar{m} < \mu_1 \\
\frac{\delta^{T-1}/u''[\bar{c}_t(\bar{m}, y)]}{\sum_{T=1}^{T} \delta^{T-1}/u''[\bar{c}_t(\bar{m}, y)]} \quad &\text{for } \mu_{T-1} < \bar{m} < \mu_T, \quad T = 2, 3, \ldots
\end{cases}
\]

\[
(10b) \quad z_y(\bar{m}, y) = (T-1)z_m(\bar{m}, y) \quad \text{for } \mu_{T-1} < \bar{m} < \mu_T, \quad T = 1, 2, \ldots
\]

It is clear from (10) that the marginal propensity to spend out of real balance holdings is strictly between zero and one except for \(0 < \bar{m} < \mu_1\), for which it equals one.\(^2\) However, since the marginal propensity to spend out of real (permanent) income is \((T-1)\) times larger than the marginal propensity to spend out of real balances, the marginal propensity to spend out of real income is zero for \(0 < \bar{m} < \mu_1\), positive for \(\bar{m} > \mu_1\), and may be larger than one, as will be demonstrated in one of the following examples. Spending as a function of real balance holdings is presented graphically in Figure 1. The demand function for real balances

\[z(\bar{m}, y) = \bar{m} - z(\bar{m}, y) + y\]

is represented diagramatically in Figure 2.

I conclude this section with two examples.
Example 1

Let $u = \log c$. Then from (8), we have:

$$\mu_T = y \left( \sum_{\tau=1}^{T} \delta^{-\tau} - T + 1 \right), \quad T = 1, 2, \ldots$$

From conditions (i) and (ii) of the theorem, we calculate:

$$c_T^* = \left[ \bar{m} + (T-1)y \right] \sum_{\tau=1}^{T} \delta^{T-1}$$

Together, these calculations imply the following spending function:

$$z(\bar{m}, y) = \begin{cases} 
\bar{m} & \text{for } 0 < \bar{m} \leq y/\delta \\
[\bar{m} + (T-1)y] \sum_{\tau=1}^{T} \delta^{T-1} & \text{for } y \left( \sum_{\tau=1}^{T} \delta^{-\tau} - T \right) < \bar{m} \leq y \left( \sum_{\tau=1}^{T} \delta^{-\tau} - T + 1 \right), \\
T = 2, 3, \ldots
\end{cases}$$

This is a piecewise linear function with a declining marginal propensity to spend out of real balance holding. It admits also a marginal propensity to spend out of income which exceeds one. For example, if $\delta = .5$, then for $\mu_2 < \bar{m} < \mu_3$, the marginal propensity to spend out of income is $2/(1 + .5 + .25) > 1$.

Example 2

Let $u = -e^{-ac}$, $a > 0$. Then:

$$\mu_T = y - \frac{T(T+1)}{2a} \log \delta, \quad T = 1, 2, \ldots$$

and, using the theorem:

$$c_T^* = - \frac{(T-1)}{2} \log \delta + \frac{1}{T} \left[ \bar{m} + (T-1)y \right]$$
Hence,

\[
z(\bar{m}, y) = \begin{cases} 
\bar{m} & \text{for } 0 < \bar{m} < y - \frac{1}{a} \log \delta \\
- \frac{(T-1)}{2a} \log \delta + \frac{1}{T} \bar{m} + (T-1)y & \text{for } y - \frac{(T-1)T}{2a} \log \delta \leq \bar{m} \leq y - \frac{T(T+1)}{2a} \log \delta
\end{cases}
\]

This is also a piecewise linear expenditure function. It has a declining marginal propensity to spend out of real balance holdings. As real balances increase, the marginal propensity to spend out of permanent income also increases, but it is always smaller than one for final values of real balance holdings.

Usually, nominal spending is assumed to be a linear homogeneous function of two variables, nominal income and nominal money balances. This, however, is a misspecification; nominal spending should be specified as a linear homogeneous function of nominal income, nominal money balances and the price level. The relevance of the price level as an independent variable is apparent from Example 2.
4. **INFLATION AND THE ADJUSTMENT PROCESS**

In this section I investigate the behavior of the price level during the process of balance-of-payments adjustments. By the process of balance-of-payments adjustments I mean the process in which the temporary equilibria described in Section 2 shift as a result of the existence of deficits and surpluses in countries' balances of payments, but in the absence of other economic forces which may cause the temporary equilibria to shift (e.g., monetary injections). The main argument that follows is that the price level never falls during the process of balance-of-payments adjustments; either it is constant, or it rises for several periods and from then on remains constant. This is in contrast to common arguments that the price level may either rise or fall, and that there is no presumption as to which will occur.

First, let me observe that with constant prices a single country's adjustment path can be described by means of a diagram constructed around Figure 2, as shown in Figure 3. Superscript $i$ is reintroduced in order to identify countries.

In Figure 3 I have reproduced the real balance demand function from Figure 2. In addition I have added a straight line (C) parallel to the $45^\circ$ line that goes through the origin, but higher by $y_1$. For every real balance holding, the vertical difference between line C and $\lambda^i(\cdot)$ represents real spending.

Now, suppose $m^i_1$ is between $\mu^i_2$ and $\mu^i_3$, as in Figure 3. Then real spending in period 1 is $z^i(m^i_1, y_1)$ and second-period real balance holdings will be $m^i_2$, as indicated in the figure. Given real balances in period 2 we can read off real spending in period 2 and real balance holdings in period 3. In the fourth period the economy reaches its steady state with real spending equal to real income equal to real balance holdings. Thus,
\[ z^i(m^i_t, y_1) \]

\[ \tau = 4, 5, \ldots \]

**Figure 3**
the economy converges to its steady state in three steps. It is also clear that an economy with initial real balance holdings smaller than \( \mu_i^1 \) converges to its steady state in only one step.

The preceding discussion identifies a fundamental asymmetry between deficit and surplus countries. It is apparent from Figure 3 that a country runs a surplus in its balance of payments if real balance holdings fall short of real income; while if real balance holdings exceed real income it runs a deficit. Hence, a surplus country attempts to work out its adjustment process in one period, while a deficit country may attempt to work out its adjustment process in more than one period, depending on the extent to which its real balances exceed its real income. The two types of country generate asymmetric pressures on the price level, and this will be shown to produce inflation during the international process of adjustment.

Suppose that the distribution of money holdings in period 1 is given by \( (\bar{M}_1^H, \bar{M}_1^F) \). Then the first-period equilibrium price level is implicitly defined by the following equilibrium condition:

\[
\sum_i^2 (\bar{M}_1^i/p_1, y_1^i) = \Sigma y_i^1
\]

The equilibrium price level exists and it is unique.\(^3\) This price level also satisfies

\[
\sum_i^2 (\bar{M}_1^i/p_1, y_1^i) = \Sigma \bar{M}_1^i/p_1
\]

Now, either \( \bar{M}_1^i/p_1 = y_i \), \( i = H, F \), or it does not. Equilibrium real balances will equal real incomes only when initial money stocks are distributed proportionately to real income (when \( M_1^H/y_H = M_1^F/y_F \)). If this is not the case, there exists an \( i \) for which \( \bar{M}_1^i/p_1, y_1^i) > \bar{M}_1^i/p_1 \) with the opposite inequality holding for the other country. Suppose, for concreteness, that the home country satisfies \( \bar{M}_1^H/p_1, y_1^H) > \bar{M}_1^H/p_1 \); it therefore also satisfies
\[ z^H(M^H_1/p_1, y_H) < y_H^* \]. The home country then spends all its money balances and runs a surplus in its balance of payments while the foreign country runs a deficit.

The second-period distribution of monies is given by:

\[ (13) \quad M^H_2 = p_1 y_H \]

\[ (14) \quad M^F_2 = \sum_i M^F_i - p_1 y_H = \tilde{\mu}^F_1 - p_1 z^F(M^F_1/p_1, y_F) + p_1 y_F \]

Now there are two possibilities: either \( \tilde{\mu}^F_1/p_1 < \mu_1^F \) or \( \tilde{\mu}^F_1/p_1 > \mu_1^F \). In the first case the foreign country also spends all its money balances in the first period, which implies

\[ (15) \quad p_1 = \sum_i \tilde{\mu}^F_i / y_i \]

From (13)-(15) it is clear that \((M^H_2, M^F_2)\) is the steady-state distribution of money, and that \( p_1 \) is the steady-state equilibrium price level. Hence, in this case the world economy adjusts to its steady state in one period, and without inflation.

In the second case (when \( \tilde{\mu}^F_1/p_1 > \mu_1^F \)), using the price level \( p_1 \) in the second period we find that there is excess demand for goods. This can be shown as follows. Using (13) and (14), we have:

\[ \sum_i z^i(M^i_2/p_1, y_1) = z^H(y_H, y_H) + z^F(M^F_1/p_1 - z^F(M^F_1/p_1, y_F) + y_F, y_F) \]

\[ = y_H + \tilde{\mu}^F_1(p_1, y_F) \]

Since \( \tilde{\mu}^F_1/p_1 > \mu_1^F \), \( \tilde{\mu}^F_1(p_1, y_F) > y_F \). Hence, \( \sum_i z^i(M^i_2/p_1, y_1) > \sum_i y_1 \).
which means that with the price level $p_1$ in the second period there is an excess demand for goods. The second-period equilibrium price level satisfies:

\[(15) \sum_{i} y_i(z^{i}(H^i_p, y_1)) = \sum_{i} y_i\]

and from the preceding argument we have $p_2 > p_1$. As a result, using (13), $M_H^2 / p_2 < y_H$, so that the home country keeps running a surplus in its balance of payments while the foreign country keeps running a deficit. The argument that was advanced for the second-period equilibrium can also be advanced for the third-period equilibrium. Therefore, by induction, we arrive at the following conclusions:

A country that starts with a deficit in its balance of payments will run deficits during the entire adjustment process while a country which starts with a surplus in its balance of payments will run a surplus during the entire adjustment process. The adjustment process will work out in a finite number of periods, with the number of periods in which the deficit country works out its deficit at constant prices serving as an upper bound on the number of periods required for the international process of adjustment to work itself out. If the process of adjustment works itself out in $T > 1$ periods, the price level will rise during the first $T - 1$ periods.\(^4\)

Some of these conclusions depend on the assumption that each country is composed of identical individuals, but the main conclusion does not: the process of international adjustment reaches a steady state in a finite number of periods, say $T$, and for $T > 1$ the price level during the first $T - 1$ period rises. For $T = 1$, the price level is constant.

This can be seen from the following. Suppose that there are a finite number of individuals of the type described above in the world economy. Every individual has his own utility function, rate of time preference, share of
ownership in his country's output, and initial money holdings. Let \( z^j(m^j,y^j) \) be individual \( j \)'s real spending function, derivable from an optimization problem like (3). Let \( I^i \) be the set of individuals who reside in country \( i \), \( i = H,F \). Then \( \sum_{j \in I^i} y^j = y^i, \ i = H,F \).

Let \( \{M^j_1\} \) be the initial distribution of money; \( \sum_{j \in I^i} M^j_1 = \bar{M}^i_1, \ i = H,F \).

Then the first-period equilibrium price level is implicitly defined by:

\[
(17) \quad \sum_{j} z^j(M^j_1/p_1,y^j) = \sum_{j} y^j \quad (= y^H + y^F)
\]

The equilibrium price level \( p_1 \) exists and it is unique (see Note 3).

Now, for each \( \tau \), partition the set of individuals into two subsets;
\( A^\tau = \{ j | M^j_1/p_\tau \leq M^j_{1/2} \} \) and \( B^\tau = \{ j | M^j_1/p_\tau > M^j_{1/2} \} \). The set \( A^\tau \) is not empty, while \( B^\tau \) may be empty. Then, we have:

\[
(18a) \quad M^j_2 = p_\tau y^j, \text{ for } j \in A^\tau
\]

\[
(18b) \quad M^j_2 > p_\tau y^j, \text{ for } j \in B^\tau
\]

which implies

\[
\sum_{j} z^j(M^j_2/p_1,y^j) = \sum_{j \in A^\tau} z^j(y^j,y^j) + \sum_{j \in B^\tau} z^j(M^j_2/p_1,y^j) = \sum_{j \in A^\tau} y^j + \sum_{j \in B^\tau} z^j(M^j_2/p_1,y^j)
\]

However, \( M^j_2/p_1 > y^i \) implies \( z^j(M^j_2/p_1,y^j) > y^i \), which together with (18) yields:

\[
(19a) \quad \sum_{j} z^j(M^j_2/p_1,y^j) = \sum_{j} y^j \quad \text{if } B^\tau \text{ is empty}
\]

\[
(19b) \quad \sum_{j} z^j(M^j_2/p_1,y^j) > \sum_{j} y^j \quad \text{if } B^\tau \text{ is not empty}
\]
Hence, if \( B_1 \) is empty, the second-period equilibrium price level is equal to \( p_1 \) and a steady state is reached in one period. If \( B_1 \) is not empty, the second-period price level is larger than \( p_1 \), and the adjustment process takes more than one period.

By induction, we see that the price level cannot fall, and that as long as the set \( B_t \) is not empty, it has to rise. Once \( B_t \) becomes empty, there are no further adjustments in the price level.

In order to see that the steady state is reached in a finite number of periods, observe that once an individual is in set \( A \), he will remain in set \( A \) in all future periods, because the price level cannot decline. Let \( T(j) \) be implicitly defined by \( \mu_T(j) - 1 < \frac{\bar{r}_j}{p_1} \leq \mu_T(j) \). Then individual \( j \) can be in set \( B \) for at most \( T(j) \) periods. Hence, the process of adjustment can last at most \( \max_j T(j) \) periods.

The preceding argument deals with individuals and is independent of the distribution of individuals between countries. It shows clearly that the inflationary bias of the adjustment process is independent of the characteristics of the countries. The number of countries is also irrelevant for this result.

The common argument about the behavior of the price level during the adjustment process is based on the following reasoning [see Dornbusch (1973).] If, say, at time \( t \), the home country runs a surplus in its balance of payments and the foreign country runs a deficit, the home country's money stock is on the increase while the foreign country's money stock is on the decline. The money transferred from the foreign country to the home country increases spending at home and reduces spending abroad. Hence, if the home country's marginal propensity to spend out of money balances exceeds the foreign country's marginal propensity to spend out of money balances, there will be an excess of spending over income in the world economy if the price level does not change. As a result, the price level will increase.
However, if the surplus country has the smaller marginal propensity to spend out of money balances, the price level will decline. Since there is no presumption as to which country has the larger marginal propensity to spend, and generally these marginal propensities will change during the adjustment process, the course of the price level during the adjustment process is unpredictable.

One way in which one may reconcile this with my analysis is to say that in the model adopted in this paper, the surplus country's marginal propensity to spend out of money balances always exceeds that of the deficit country. This is certainly true in the case in which a country is populated by identical consumers, for then the surplus country's marginal propensity to spend out of money balances must equal one, while the deficit country's marginal propensity to spend out of money balances is less than or equal to one. However, we have seen that the inflationary bias also exists in a world with many different consumers, which shows that the analogy is not as simple as it might appear on first sight.

With nonidentical consumers a country's marginal propensity to spend out of money balances may not be well defined, for the effect of a marginal increase in the stock of money on aggregate spending depends generally on the way in which the additional money is distributed among consumers. In any case, during the process of adjustment, some consumers of a deficit country may be accumulating money balances while others may be decumulating them; the terms deficit country and surplus country refer only to the aggregate result. Therefore, at constant prices the surplus country need not increase spending by the amount of money accumulated, so that its effective marginal propensity to spend out of money balances may be smaller than one. However, what I have shown is that at constant prices, aggregate spending in the world economy has to increase (or remain the same) during the process of adjustment, thus forcing prices to rise (or remain constant).
5. CLOSING REMARKS

The hypothesis of maximizing behavior in conjunction with an explicit microeconomic theory of money have been shown to shed light on the behavior of the price level during the process of balance of payments adjustments in a fixed exchange rate regime. In particular, it has been shown that in the absence of financial assets other than money, a fixed exchange rate regime has an inherent inflationary bias, but that without autonomous monetary injections the process of inflation can only last for a finite time span.

The framework of this paper can readily be modified to take account of bonds. The interesting result that emerges in this case is that the price level remains constant during the adjustment process, independently of whether bonds are traded only within national borders or whether they are also internationally traded. This result contrasts sharply with the frequently expressed view that economies with nontraded bonds behave similarly to economies without bonds. I do not provide a formal derivation of this result because the reasoning is quite simple. In the presence of bonds, the equilibrium interest rate must be positive. For if the interest rate is zero, it can be shown that consumer spending will go to infinity as long as there is a positive rate of time preference. However, with a positive interest rate bonds dominate money for store-of-value purposes. Therefore consumers will not use money to store wealth, implying that all money balances will be spent during every period. As a result, the equilibrium price level will equal the ratio of aggregate world money balances to aggregate world output, and it will be the same in each period.

Coming back to the framework without bonds, it is illuminating to describe the adjustment of the world economy to an unexpected monetary shock. Suppose the world economy is in long-run equilibrium and that at time $t_0$ the home country's government increases unexpectedly the domestic money supply. If the monetary injection amounts to, say, $x$% of the world's initial stock
of money, we know that in the new long-run equilibrium the price level will increase by x %. We also know that the impact effect of the monetary expansion is to raise the price level, since at the initial price level the increase in the money supply generates an excess demand for goods. Now there are two possibilities; either the price level rises immediately by x % or it does not. Suppose it does not, which will be the case for sufficiently large x. Then, during the process of balance-of-payments adjustments the price level has to increase over time. As a result, the impact effect will result in an increase in the price level of less than x % and this will be followed by a gradual upward adjustment of the price level to the x % limit. The above described course of the price level is depicted in Figure 4.

![Figure 4](image)

Now consider a monetary contraction which reduces the world's stock of money by x %. Again, assume that x is sufficiently large so that the new long-run equilibrium is not reached in one period. We know that in the new long-run equilibrium the price level will be lower by x % and that the impact effect of the monetary contraction is to reduce the price level. However, since during the process of balance-of-payments adjustments the price level has to rise, the impact effect of the monetary contraction reduces the price level by more than x %. The course of the price level is described in Figure 5.
This analysis brings out again the inherent asymmetry in the fixed exchange rate regime. Monetary contractions do not trigger an adjustment process that is the mirror image of the adjustment process that results from a monetary expansion. Similar asymmetries exist in response to other types of shocks, and they all stem from the fact that at the margin, deficit countries are fundamentally different from surplus countries.
APPENDIX

Proof of the Theorem from Section 3

For $t = 1$, the constraints in (3) are (the constraint $m_t \geq 0$ is redundant):

(A.1a) $m_1 = \bar{m} \quad (0 < \bar{m} < +\infty)$

(A.1b) $m_t = m_{t-1} - c_{t-1} + y, \quad t = 2, 3, \ldots$

(A.1c) $c_t \leq m_t, \quad t = 1, 2, \ldots$

(A.1d) $c_t \geq 0, \quad t = 1, 2, \ldots$

However, using (A.1a) and (A.1b), we have:

(A.2) $m_t = \bar{m} - \sum_{n=1}^{t-1} c_n + (t-1)y, \quad t = 2, 3, \ldots$

Hence, an alternative way to represent the constraints (A.1) is:

(A.3a) $\sum_{n=1}^{\tau} c_n \leq \bar{m} + (\tau-1)y, \quad \tau = 1, 2, \ldots$

(A.3b) $c_\tau \geq 0, \quad \tau = 1, 2, \ldots$

A feasible consumption program is a sequence $\{c_\tau\}$ that satisfies (A.3).
Lemma A1: Let \( \{c_t\} \) be a feasible consumption program, then
\[
\sum_{\tau=1}^{\infty} \delta^{\tau-1} u(c_{\tau}) \leq \frac{1}{1-\delta} u[\delta m + (1-\delta)y].
\]

Proof: If \( \{c_t\} \) is feasible, then from (A.3a) it satisfies:
\[
\sum_{\tau=1}^{\infty} \delta^{\tau-1} \sum_{n=1}^{\infty} c_{\tau n} \leq \sum_{\tau=1}^{\infty} \delta^{\tau-1}[m + (\tau-1)y] = \frac{1}{1-\delta} m + \frac{\delta}{(1-\delta)^2} y.
\]
However,
\[
\sum_{\tau=1}^{\infty} \delta^{\tau-1} \sum_{n=1}^{\infty} c_{\tau n} = c_1 + \delta c_1 + \delta^2 c_1 + \delta^2 c_2 + \delta^2 c_3 + \ldots
\]
\[
= \frac{1}{1-\delta} c_1 + \frac{\delta}{1-\delta} c_2 + \frac{\delta^2}{1-\delta} c_3 + \ldots
\]
\[
= \frac{1}{1-\delta} \sum_{\tau=1}^{\infty} \delta^{\tau-1} c_{\tau}.
\]

Hence, if \( \{c_t\} \) is feasible, it satisfies

(A.4) \[
\sum_{\tau=1}^{\infty} \delta^{\tau-1} c_{\tau} \leq m + \frac{\delta}{1-\delta} y.
\]

Now, as is well known, the maximum of \( \sum_{\tau=1}^{\infty} \delta^{\tau-1} u(c_{\tau}) \) subject to (A.4) is attained at \( \bar{c}_\tau = \bar{c}_{\tau+1} = (1-\delta)m + \delta y, \tau = 1,2,\ldots \). Hence, for every feasible \( \{c_t\} \),
\[
\sum_{\tau=1}^{\infty} \delta^{\tau-1} u(c_{\tau}) \leq \sum_{\tau=1}^{\infty} \delta^{\tau-1} u[(1-\delta)m + \delta y] = \frac{1}{1-\delta} u[(1-\delta)m + \delta y].
\]

This completes the proof.
Lemma A.1 establishes the fact that for every feasible consumption program the objective function has an upper bound. This will be used in what follows to make direct comparisons between a feasible program \( \{c^*_\} \) and the program \( \{c^*_\} \) specified in the theorem.

**Lemma A2:** The program \( \{c^*_\} \) specified in the theorem is feasible.

**Proof:** For \( \bar{m} \leq u_1, \{c^*_\} \) satisfies (A.3), since \( \sum_{n=1}^{t} c^*_n = \bar{m} + (t-1)y \) and \( c^*_t > 0 \) for \( t = 1, 2, \ldots \). Also, for \( u_{T-1} < \bar{m} \leq u_T, T = 2, 3, \ldots \), \( \sum_{n=1}^{T} c^*_n = \bar{m} + (T-1)y \) and \( c^*_T > 0 \) for \( T, T+1, \ldots \). Hence, it remains to show that (A.3) is satisfied for \( u_{T-1} < \bar{m} \leq u_T, T = 2, 3, \ldots \), and \( t = 1, 2, \ldots, T-1 \).

From (i) of the theorem \( c^*_\tau = \phi(\delta^{T-1}_\tau) \), where \( \phi(\cdot) \) is the inverse of \( u'(\cdot) \).

The function \( \phi \) is well defined, continuous, and strictly decreasing, due to the strict concavity of \( u(\cdot) \) and its being twice continuously differentiable.

From (8), we have:

\[
\phi[\delta^{T-\tau}u'(y)] = \begin{cases} 
  y & \tau = T \\
  u_1 & \tau = T-1 \\
  u_{T-\tau} - u_{T-\tau-1} + y & \tau = 1, 2, \ldots, T-2
\end{cases}
\]

Hence,

\[
(A.5) \quad \sum_{\tau=1}^{T} \phi[\delta^{T-\tau}u'(y)] = y + u_1 + \sum_{\tau=1}^{T-2} (u_{T-\tau} - u_{T-\tau-1} + y)
\]

\[= u_{T-1} + (T-1)y\]
From (8), we also have,

\[(A.6) \sum_{\tau=1}^{T} \phi(\delta^{T-\tau}u'(\mu_1)) = \mu_1 + \sum_{\tau=1}^{T-1} (\mu_{T-\tau} - \mu_{T-\tau-1} + y) = \mu_T + (T-1)y\]

Hence, since \(\mu_{T-1} < m < \mu_{T}\), by continuity of \(\phi(\cdot)\) and its being strictly decreasing, there exists a unique \(\lambda^\sharp\), \(u'(u_1) \leq \lambda^\sharp < u'(y)\), such that

\[(A.7) \sum_{\tau=1}^{T} \phi(\delta^{T-\tau}\lambda^\sharp) = m + (T-1)y\]

The unique solution to the system of equations (i) and (ii) is:

\[\lambda = \lambda^\sharp\]

\[c^\sharp_\tau = \phi(\delta^{T-\tau}\lambda^\sharp), \quad \tau = 1, 2, \ldots, T\]

Clearly, \(c^\sharp_1 > c^\sharp_2 > \ldots > c^\sharp_T\). However, since \(u'(u_1) \leq \lambda^\sharp < u'(y)\), we also have \(u_1 > c^\sharp_T > y\). (It is also easy to show that \(\mu_{T+1-\tau} > c^\sharp_\tau > \mu_{T-\tau}\) for \(\tau = 1, 2, \ldots, T-1\).) Hence, for \(\tau = 1, 2, \ldots, T-1\), and since \(c^\sharp_n > y\) for \(n = 1, 2, \ldots, T\),

\[\sum_{n=1}^{T} c^\sharp_n = m + (T-1)y - \sum_{n=T+1}^{T} c^\sharp_n = m + (T-1)y - \sum_{n=T+1}^{T} (c^\sharp_n - y) < m + (T-1)y\]

which means that \(c^\sharp_\tau, \tau = 1, 2, \ldots, T-1,\) satisfy (3). This completes the proof.
It remains to be shown that the program \( \{ c_t^a \} \) is optimal. Compare it to a feasible program \( \{ c_t \} \) which differs from \( \{ c_t^a \} \). Due to strict concavity of \( u(\cdot) \) and Lemma A.1, we have:

\[
(A.8) \quad \Delta \overset{\text{def}}{=} \sum_{\tau=1}^{\infty} \delta^{T-\tau-1} [u(c_\tau^a) - u(c_\tau^a)] < \sum_{\tau=1}^{\infty} \delta^{T-\tau-1} u'(c_\tau^a)(c_\tau^a - c_\tau^a)
\]

If \( 0 < \bar{m} < u_1, u'(c_1^a) = u'(\bar{m}), u'(c_2^a) = u'(y), \tau = 2, 3, \ldots, \) and if \( u_{T-1} < \bar{m} < u_T \) for some \( T \geq 2 \), then \( u'(c_\tau^a) = \delta^{T-\tau} u'(c_1^a), \tau = 1, 2, \ldots, T, \) and

\[
u'(c_\tau^a) = u'(y) \text{ for } \tau = T+1, T+2, \ldots.
\]

Hence,

\[
(A.9) \quad \Delta < \sum_{\tau=1}^{T} \delta^{T-\tau} \delta^{T-\tau} u'(c_1^a)(c_\tau^a - c_\tau^a) + \sum_{\tau=T+1}^{\infty} \delta^{T-\tau} u'(y)(c_\tau^a - y)
\]

where \( c_1^a = \bar{m} \) and \( T = 1 \) if \( 0 < \bar{m} \leq u_1 \)

From (A.9)

\[
(A.10) \quad \Delta < \delta^{T-1} u'(c_1^a) \sum_{\tau=1}^{T} (c_\tau^a - c_1^a) + \delta^{T} u'(y) \sum_{\tau=1}^{\infty} \delta^{T-1} (c_{\tau+1}^a - y)
\]

From (A.3), since \( \{ c_\tau \} \) is feasible,

\[
(A.11) \quad \sum_{\tau=1}^{\infty} \frac{\delta^{T-1}}{\delta + T+1} c_n \leq \sum_{\tau=1}^{\infty} \frac{\delta^{T-1}}{\delta} (\bar{m} + (T+1)y) = \frac{1}{1-\delta} \bar{m} + \frac{T-1}{1-\delta} y + \frac{1}{(1-\delta)^2} y
\]

However,
(A.12) \[ \sum_{\tau=1}^{\infty} \delta^{\tau-1} \sum_{n=1}^{T} c_{n} = \frac{1}{1-\delta} \sum_{n=1}^{T} c_{n} + c_{T+1} + \delta c_{T+1} + \delta c_{T+2} + \delta^{2} c_{T+1} + \delta^{2} c_{T+2} + \delta^{2} c_{T+3} + \ldots \]

\[ = \frac{1}{1-\delta} \left( \frac{T}{\tau} c_{\tau} + \sum_{\tau=1}^{\infty} \delta^{\tau-1} c_{\tau+\tau} \right) \]

Combining (A.11) with (A.12) yields:

(A.13) \[ \sum_{\tau=1}^{\infty} \delta^{\tau-1} c_{\tau+\tau} - \frac{1}{1-\delta} y = \sum_{\tau=1}^{\infty} \delta^{\tau-1} c_{\tau+\tau} - y \leq \bar{m} + (T-1)y - \sum_{\tau=1}^{T} c_{\tau} \]

Now combining (A.13) with (A.10) and the fact that \[ \sum_{\tau=1}^{T} c_{\tau} = \bar{m} + (T-1)y, \]
yields:

(A.14) \[ \Delta < \delta^{T-1}[\delta u'(y) - u'(c_{T})] \left[ \bar{m} + (T-1)y - \sum_{\tau=1}^{T} c_{\tau} \right] \]

The expression in the second square bracket on the right hand side of (A.14) is nonnegative, since \( \{c_{\tau}\} \) is feasible and therefore satisfies (A.3). The expression in the first square brackets on the right hand side of (A.14) is nonpositive, since \( c_{T} \leq \mu_{1} \) and \( \delta u'(y) = u'(\mu_{1}) \) due to (8b), and \( u'(\cdot) \) is strictly decreasing. Hence,

\[ \Delta < 0 \]
which by (A.8) implies

$$\sum_{\tau=1}^{\infty} \delta^{\tau-1} u(c_{\tau}) < \sum_{\tau=1}^{\infty} \delta^{\tau-1} u(c_{\tau})$$

This completes the proof.
FOOTNOTES

1. Since the \( \mu \)'s depend on \( y \) (in a continuous way), the condition \( \mu_{T-1} < \bar{m} < \mu_T \) in (10b) should be interpreted as applying for the \( \mu \)'s calculated for the value of \( y \) at point \((\bar{m},y)\) on the left-hand side of (10b).

2. It is also easy to see that

\[
\lim_{\varepsilon \to 0} z m(\mu_T - \varepsilon, y) < \lim_{\varepsilon \to 0} z m(\mu_T + \varepsilon, y)
\]

for all \( T = 1,2,\ldots \).

3. To see this, remember that \( z_i(\cdot) \) in its first argument is strictly increasing [see (10)]. Now, for \( p_1 \) sufficiently high so that

\[
\frac{M_i^i}{p_1} < y_i, \quad i = H,F, \quad \sum_i \left( \frac{M_i^i}{p_1}, y_i \right) = \sum_i M_i^i / p_1 < \sum_i y_i, \quad \text{while for } p_1 \text{ sufficiently low so that } \frac{M_i^i}{p_1} > y_i, \quad i = H,F, \quad \sum_i \left( \frac{M_i^i}{p_1}, y_i \right) > \sum_i y_i. \]

Hence, there is a unique \( p_1 \) which satisfies (11).

4. The fact that the number of periods in which the deficit country works out its deficit at constant prices serves as an upper bound on the number of periods in which the international process of adjustment works itself out can be shown as follows. Suppose \( \mu_{T-1} < \frac{M_i^F}{p_1} < \mu_T \), for \( T > 1 \). Then one can see from Figure 3 that with rising prices, the real balances in each period will be lower than with constant prices. Thus, since prices can never rise so as to make the deficit country a surplus country, rising prices reduce the number of periods required for convergence to a steady state.
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