Seminar Paper No. 197

ON VARIABLE CAPITAL UTILIZATION AND
INTERNATIONAL TRADE THEORY

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1. Introduction

The degree of capital utilization, appearing for instance as the number of shifts working, varies considerably between sectors and countries.¹ Recently, there has been some work on how different degrees of capital utilization would modify the standard international trade theory.²

Winston (1979) argues that the usual way of measuring factor intensities in Leontief-type tests of the Heckscher-Ohlin theory is wrong. Instead of just taking capital stock per man-year, or capital stock per employee, one should adjust the factor intensities for the degree of capital utilization. Then, given the considerable variance in capital utilization between sectors, the errors in the usual measurements might be large, and Winston argues that they may indeed explain the Leontief Paradox.

In this paper, I wish to make two points as to how standard international trade theory is modified when an endogenous degree of capital utilization is introduced. The two points refer to two polar extreme assumptions concerning the source of the endogenous degree of capital utilization. The first point refers to the extreme situation when the degree of capital utilization during a period may be less than full, not because wages or other factor prices are higher during part of the period, but because there is, for some reasons,

a time-dependent variation in a firm's (or an industry's) technical productivity during the period. As an example, we can think of a fishing industry where the productivity of a given number of fishermen and boats are subject to seasonal fluctuations during the period, a year say, because the availability of fish varies exogenously during the period.

Under these circumstances we show that the firm's production per period can be considered to be a function $f(K, L, u)$, of its capital stock, K , labor input during the period, L , and the degree of capital utilization, u . It follows that the behaviour of a profit maximizing firm can then be described as a two-stage procedure. In the first stage, the firm, for each capital stock and labor input, simply chooses the output-per-period maximizing degree of capital utilization, $u^*(K, L)$. This results in an "indirect" production function $F(K, L) = f(K, L, u^*(K, L))$. In the second stage, for given factor prices, i.e. capital rental and wage rate, the firm chooses the optimal level of capital stock K and labor input L so as to maximize profits, subject to the indirect production function. To the extent the indirect production function is a well-behaved neoclassical production function, it is clear that the whole standard Heckscher-Ohlin trade theory applies. In particular, it is clear that the relevant factor intensity is K/L , capital stock over labor input, and not u^*K/L , Winton's adjusted capital intensity.

The second point refers to the other polar extreme situation, when different capital utilization is due, not to any time-dependent variation in productivity during the period, but to a variation in wages within the period. For concreteness, let the period be a day and night, i.e. 24 hours, and let the night wage be higher than the day wage. The wage differential between day and night time labor then reflects workers' relative preferences between working day and

working night. Many examples of ordinary manufacturing industries would seem to correspond to this case.

It is argued below that day and night labor should then be regarded as different factors. Hence, this situation differs from the standard 2 x 2 Heckscher-Ohlin case in that there are three factors rather than two, namely capital, day labor, and night labor. Furthermore, to make endogenous the total capital utilization in an economy, night labor (or both kinds of labor) should be in variable supply, which is formally equivalent to introducing production and consumption of non-traded goods.

Thus, the modifications of the trade theory which follow from introducing variable capital utilization with a wage premium on overtime are thus exactly those that follow from the introduction of more than two factors of production and of non-traded goods. Once this is realized, the existing results on these modifications in the trade literature can hence, suitably interpreted, be directly applied to the variable capital utilization case.

The first point is developed in Section 2, and the second in Section 3. Section 4 presents some conclusions. The Appendix shows an example of a simple model of a small open economy with day and night labor being distinct.

2. Constant wages throughout period

Let the period be denoted by the unit interval, $0 \leq t \leq 1$. Consider a firm, whose technology is described by the instantaneous production function

$$(2.1) \quad g(k(t), \ell(t), t),$$

which gives the instantaneous flow of output at time t as a function of the instantaneous flow of capital services, $k(t)$, and labor services, $l(t)$, at time t . Let the instantaneous production function be a well-behaved neoclassical production function of $k(t)$ and $l(t)$,³ and assume that the productivity of capital and labor services is decreasing within the period, i.e. $g_t \leq 0$.

Let the firm own the capital stock K , and let one unit of capital stock, when used, give rise to one unit of instantaneous capital services. Assume that the flow of labor services is constant when the firm produces.⁴ The firm will produce only during the subperiod $0 \leq t \leq u$, where u , $0 \leq u \leq 1$, denotes the degree of capital utilization. Let $L = \int_0^1 l(t)dt$ denote labor (services) input per period. It follows that the instantaneous flows of capital and labor services fulfill

$$(2.2) \quad \begin{aligned} k(t) &= \begin{cases} K & \text{for } 0 \leq t \leq u, \\ 0 & \text{for } u < t \leq 1, \end{cases} \text{ and} \\ l(t) &= \begin{cases} L/u & \text{for } 0 \leq t \leq u, \\ 0 & \text{for } u < t \leq 1. \end{cases} \end{aligned}$$

Production per period will be a function, $f(K, L, u)$, of the capital stock, the labor input per period, and the degree of capital utilization, defined by

$$(2.3) \quad f(K, L, u) = \int_0^u g(K, L/u, t)dt.$$

The indirect production function, $F(K, L)$, will be given by

$$(2.4) \quad F(K, L) = \max \{f(K, L, u) : 0 \leq u \leq 1\} = f(K, L, u^*(K, L)),$$

where $u^*(K, L)$ denotes the optimal degree of capital utilization.

Let p be the price of output and r the rental per capital good per period. In particular, assume that the wage rate is the same during the period. Let w denote the wage rate, measured per unit of labor services and per period. Then profits per period are

$$(2.5) \quad \pi = pF(K, L) - rK - wL.$$

It is clear that this leads to the standard neoclassical profit maximization problem. In particular, the relevant factor intensity is K/L , the unadjusted capital stock over labor services per period. It is not Winston's adjusted u^*K/L , accumulated capital services per period over labor input per period, which equals instantaneous capital intensity $k(t)/\ell(t)$ during production.

In general, the indirect production function defined in (2.4) need not be a well-behaved neoclassical production function. Then the standard Heckscher-Ohlin theory would not apply.⁵ Let us however consider a special case when the indirect production function is well-behaved: Let the instantaneous production function be

$$(2.6) \quad g(k(t), \ell(t), t) = (1 - t)(k(t))^\alpha (\ell(t))^{1-\alpha}, \quad 0 < \alpha < 1.$$

That is, it is a Cobb-Douglas function in $k(t)$ and $\ell(t)$, and productivity decreases linearly. It follows from (2.3) and (2.5) that

$$(2.7) \quad f(K, L, u) = \int_0^u (1 - t) dt K^\alpha (L/u)^{1-\alpha} = A(u) K^\alpha L^{1-\alpha},$$

where $A(u) = (u^\alpha - u^{\alpha+1}/2)$. The optimal degree of capital utilization, u^* , maximizes $A(u)$ and hence fulfills the first order condition

$$(2.8) \quad \alpha(u^*)^{\alpha-1} - (\alpha + 1)(u^*)^\alpha/2 = (u^*)^{\alpha-1}(\alpha - (\alpha + 1)u^*/2) = 0,$$

which implies⁶

$$(2.9) \quad u^* = 2\alpha/(1 + \alpha).$$

In this case the optimal degree of capital utilization is independent of the capital stock and labor input. The indirect production function is

$$(2.10) \quad F(K, L) = A(u^*)K^\alpha L^{1-\alpha},$$

which is Cobb-Douglas in capital stock and labor input.

3. Higher wages during the night

For concreteness, let the period be a day and night, i.e. 24 hours. Consider a firm that can operate with the same technology during the day as during the night. That is, the firm is not subject to any variation in technical productivity during the day and night. It however faces one wage rate during the day, and another (higher) wage rate during the night. Let it own a capital stock K , and let L and N denote labor input during the day and the night, respectively. Production per day will be given by $f(K, L)$, a wellbehaved neoclassical production function. Production per night is $f(K, N)$. Hence, production per day and night is given by

$$(3.1) \quad F(K, L, N) = f(K, L) + f(K, N).$$

Let w denote the day wage per day, and let s denote the night wage per night. Profits per day and night will be given by

$$(3.2) \quad \pi = pF(K, L, N) - rK - wL - sN.$$

Assume that the firm can choose one factor intensity during the day and another during the night. Then, for a given capital stock, the profit maximizing day labor input and night labor input, $L(w/p, K)$ and $N(s/p, K)$, are given by the equality of marginal product and the product wage,

$$(3.3) \quad \begin{aligned} f_L(K, L) &= w/p \quad \text{and} \\ f_N(K, N) &= s/p. \end{aligned}$$

The two labor input levels can be illustrated in a familiar way in Fig. 1.

It is obvious that from the point of view of the firm, as long as day and night wages differ, day and night labor are different factors. It is also clear that, if a higher night wage reflects workers' relative preference against working nights, day and night labor cannot be perfect

substitutes from the point of view of workers, and are hence different factors also from the supply side. Furthermore, to make endogenous the total capital utilization in an economy, day and night labor cannot both be fixed in supply. We conclude that a proper treatment of these issues requires the introduction of variable supply of labor.

It is clear that a model of an open economy incorporating these aspects would differ from the standard 2×2 Heckscher-Ohlin model of a small country in two respects (see the Appendix for details). First, it has two different kinds of labor, in addition to capital. Hence, there are three primary factors instead of the standard two. Second, the supply of one of the labors is variable rather than fixed. Formally, introducing variable supply of labor in a Heckscher-Ohlin model is identical to introducing non-traded goods. This is so, since leisure can be interpreted as a non-traded good.

4. Conclusions

First we have looked at the extreme situation when a variable degree of capital utilization results solely from a technical variation of productivity within the period and not from variations in wages. We have found that the behaviour of the firm can then be represented by an indirect production function of capital stock and labor input per period, where the degree of capital utilization is chosen optimally and hence does not appear in the indirect production function. For given factor prices the firm then chooses capital stock and labor input per period so as to maximize profit per period.

The conclusions are straightforward: If the indirect production function is well-behaved, the standard Heckscher-Ohlin theory applies directly. If it is not, the standard theory does not apply. With regard to empirical work, in the former case the relevant factor intensity is capital stock over labor input per period, without any adjustment according to the degree of capital utilization. In the latter case, the question of which factor intensity is relevant is not meaningful.

In the other extreme situation, the variable capital utilization is the result of different day and night wages, and not of any technical variation of productivity between day and night. The difference in day and night wages is due to workers' relative preference for working day rather than night. Both for firms and workers, day and night labor are not perfect substitutes but different factors. The required modification of the standard trade theory is that of introducing more than one kind of labor, at least one of which is in variable supply and hence enters workers' utility functions. As is well known, this is exactly analogous to introducing production of non-traded goods in trade models. The results in the existing literature on problems with non-traded goods can then, suitably interpreted, be applied to the case of variable capital utilization.

In this other extreme situation, what are the conclusions relevant to empirical work? Rigorously, day and night labor should be regarded as different factors. In most empirical work in the trade literature there is aggregation of heterogeneous kinds of labor into one labor input (or a few labor inputs). In that context, the error in aggregating day and night labor may be relatively small. Hence, in less rigorous empirical work, the relevant factor intensity may again be capital stock over total labor input per period, the latter aggregate including both day and night labor. Again, there is no

adjustment of the capital stock according to its degree of utilization.⁷ The other alternative is to explicitly disaggregate labor into day and night labor.

Appendix: A trade model with a premium on over-time

Consider an open economy with two firms/industries, each producing a distinct traded good, and one representative consumer. The goods and industries are indexed 1 and 2, and the two industries' production functions are $F^1(K_1, L_1, N_1)$ and $F^2(K_2, L_2, N_2)$. Assume that capital is mobile between the industries, and consider the industries' demand for day and night labor, conditional upon their equilibrium capital stocks K_1^* and K_2^* , and the given (world) prices p_1 and p_2 of the goods. One possibility is illustrated in Fig. 2.

Industry 1 is more capital-intensive and has the steeper value-of-marginal-productivity-of-labor schedule A_1A_1 , given by $p_1 F_L^1(K_1^*, L_1)$, in the diagram to the left. Industry 2's schedule is A_2A_2 , given by

Fig. 2

$p_2 F^2(K_2^*, L_2)$. Horizontal addition gives the aggregate labor demand schedule AA in the diagram to the right.

Let us then consider the consumer. He/she owns the total capital stock $\bar{K}$, and earns the rental r per day and night and unit of capital stock. Assume also that he/she has a given endowment $\bar{L}$ per day of day labor, the whole of which is supplied to the market. Assume further that he/she has a fixed endowment $\bar{N}$ per night of night labor, part of which, N , is supplied to the market, part of which, $\bar{N} - N$, is consumed as leisure. Let the consumer's preferences be given by the utility function $U(c_1, c_2, \bar{N} - N)$, where c_1 and c_2 denote consumption per day and night of the two goods. Maximization of this subject to the budget constraint $p_1 c_1 + p_2 c_2 = w\bar{L} + sN + r\bar{K}$ gives rise to the supply function $N(s, w, r, p_1, p_2)$ of night labor per night. (We suppress the endowments $\bar{K}$, $\bar{L}$, $\bar{N}$ in the supply function.)

A possible equilibrium is shown in the diagram to the right in Fig. 2. The equilibrium capital stocks, K_1^* and K_2^* , fulfilling $K_1^* + K_2^* = \bar{K}$, and the equilibrium capital rental per day and night, r^* , are simultaneously determined in the capital market. (The capital market equilibrium is given by $p_1^1 F_K^1(K_1^*, L_1^*, N_1^*) = p_2^2 F_K^2(K_2^*, L_2^*, N_2^*) = r^*$.) Given these capital stocks, the given labor supply of day labor $\bar{L}$ determines the equilibrium day wage, w^* , and the input of labor in the two industries, L_1^* and L_2^* . For given world prices p_1 and p_2 , and equilibrium day wage w^* and capital rental r^* , the supply schedule for night labor BB determines the equilibrium night wage s^* , and the night labor input in the two industries, N_1^* and N_2^* . As the curves are drawn, the night wage exceeds the day wage. Industry 1 uses both day and night labor, industry 2 only day labor.

With this construction of the model, there is a discreteness in the degree of utilization of the capital stock, in the sense that the firm/industry either does, or does not, use night labor. There is continuity in the sense that night labor input can vary continuously from zero up.

For simplicity, we do not model the timing of production within the day or within the night. Behind the production functions $f^d(K, L)$ and $f^n(K, N)$ for the day and night there may be a construction like in Section 2, giving optimal day and night utilization rates $u^{d*}(K, L)$ and $u^{n*}(K, N)$. Another possibility is to consider a continuum of different night labors where $0 \leq t \leq 1$ denotes the night, $N(t)$ denotes input of labor services of type t (and at time t), and $w(t)$ denotes the wage at time t . An assumption that the ratio between the instantaneous flow of capital services and that of labor services cannot differ between day and night can then easily be added. Our conclusions would not be affected, except that we would not have three factors but a continuum of factors to deal with.

Footnotes

* Comments on a previous version by Harry Flam, Carl Hamilton, Henrik Horn, John Hutton, Gordon Winston, and an anonymous referee are gratefully acknowledged. The usual caveat applies.

1. See Winston for references to empirical studies.

2. Winston, and several recent unpublished papers by R. Betancourt, C. Clague, and A. Panagariya, University of Maryland.

3. That is having constant returns to scale, and with $g_k, g_l > 0$ and $g_{kk}, g_{ll} < 0$. Subindices will denote partials throughout the paper.

4. We thus assume that the firm chooses the instantaneous capital services/labor services ratio $k(t)/l(t)$, subject to the constraint that the ratio must remain the same during the period of production.

5. It follows that the question of which factor intensity is relevant would then not be meaningful.

6. We note that we have an inner solution, $0 < u^* < 1$.

7. Suppose that workers' marginal rate of substitution of day labor for night labor is constant and equal to $1 + \beta$, where β is a positive constant. In an equilibrium the day and night wages would fulfill $s = (1 + \beta)w$, that is, there is a constant (proportional) premium β on night labor. From the point of view of firms, the relative price between day and night labor is constant. Hence, a composite Hicks aggregate labor $M = L + (1 + \beta)N$ can be constructed with the indirect production function $G(K, M) = \max \{F(K, L, N) : M = L + (1 + \beta)N\}$.

The wage rate corresponding to the labor composite is w , and the firm

will maximize $pG(K, M) - rK - wM$. It is clear that the relevant factor intensity is K/M , capital stock over the labor composite.

We note that from the point of view of workers, we can also aggregate labor into a labor composite, with the same weights. The model essentially collapses into a one-labor model.

References

Winston, G. C., 1979, On measuring factor proportions in industries with different seasonal and shift patterns, or did the Leontief Paradox ever exist?, *Economic Journal* 89, 897-904.