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WALRASIAN AND MARSHALLIAN STABILITY

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1. Introduction

In the discussion of stability of competitive markets, two different adjustment processes have been considered. One is when prices adjust depending upon excess demand; the other is when quantities supplied adjust, either depending upon differences between prices and marginal costs, or via exit or entry of firms depending upon profit levels. The former adjustment process has been associated with Walras (*tâtonnement*), the latter with Marshall, although both Walras and Marshall were aware of the two processes and both used them in the proper context (see Takayama [7] for a general discussion and references to the literature). Partial equilibrium discussions of these processes are standard in intermediate micro textbooks. A standard result is that a market may very well be Walrasian stable and Marshallian unstable, or the other way around. Hence, there is no direct relation between the stability properties of the two processes.

The modern discussion of stability in general equilibrium has mostly been within the Walrasian (*tâtonnement*) process.¹ Very recently, however, the Marshallian adjustment process has been examined in this Journal by Mas-Colell [3] and Novshek and Sonnenschein [4], both of whom study quantity setting noncooperative equilibria of finite economies approximating a limit economy with infinitesimal firms, the latter being the usual standard competitive equilibrium model (although with infinitesimal firms). In particular, Novshek and Sonnenschein [4, Example 2] emphasize the possibility

of instability of the Marshallian adjustment process for very simple economies and they for this reason call into question the significance of the standard Walrasian competitive equilibrium, at least to the extent that the Walrasian equilibrium concept is considered as the limit of noncooperative quantity setting equilibria. Indeed, they give an interesting example of an economy with a unique long run Walrasian equilibrium and an unstable Marshallian adjustment process.

The purpose of this paper is to examine more closely the relation between the Marshallian long run adjustment and the Walrasian short run adjustment. I limit the discussion to competitive economies with infinitesimal firms. In particular, I want to emphasize the relevance of the short run Walrasian adjustment process for given number of firms, as opposed to the relatively irrelevant "long run" Walrasian tâtonnement process when the number of firms is variable. Indeed, I will show the former short run Walrasian process is very closely related to the long run Marshallian adjustment process.

We shall see that local stability, for all speeds of adjustment, of the Walrasian tâtonnement process in a short run equilibrium with a given number of firms (local Walrasian stability) implies local stability, for all speeds of adjustment, of the long run adjustment where firms enter or exit according to the level of profits (local Marshallian stability). Conversely, local Marshallian stability, for all speeds of adjustment, of the long run adjustment process implies short run Walrasian stability, for all speeds of adjustment, for the given long run equilibrium number of firms. (However, local stability, for given nonuniform speeds of adjustment, of the Walrasian process does not imply local stability of the Marshallian process. Neither does local stability, for given nonuniform speeds of adjustment, of the Marshallian process imply local stability of the Walrasian process. Since little is known about different speeds of adjustment, though, this latter concept of stability seems less interesting.)

These results, implying a close relation between Walrasian and Marshallian stability, may at first appear inconsistent with Novshek and Sonnenschein's example of a unique long run Walrasian equilibrium that is Marshallian unstable. For we know that a unique Walrasian equilibrium is always stable with a Walrasian tâtonnement process. However, it is the relatively irrelevant "long run" tâtonnement process with variable number of firms that would be stable, not the short run tâtonnement process for given number of firms. Indeed, in Novshek and Sonnenschein's example, the short run Walrasian process turns out to be unstable and their example is hence consistent with my results.

For a single-consumer economy, it is well known that the Walrasian adjustment process is globally stable, for all speeds of adjustment. In a forthcoming paper Novshek and Sonnenschein show under great generality that the Marshallian adjustment process is also globally stable for a single-consumer economy. Below I shall give a simple proof of the latter proposition in the present framework.

Section 2 presents the model and discusses local stability. Section 3 discusses global stability. Section 4 presents some conclusions. Appendix 1 presents some simple technical results on stability. Appendix 2 shows that the results for local stability holds also for short run variability of output for each firm, but not in general for joint production. Appendix 3 discusses Novshek and Sonnenschein's example in some detail.

2. Local Stability

We consider the following technology: There are n goods/industries, indexed $i = 1, \dots, n$. Each good is produced with a homogeneous input, labor. In industry i , all firms are identical and each firm produces one unit of good i , using up $a_i > 0$ units of labor (where a_i is a constant). Hence, industry i output $y_i > 0$ is equal to the number of firms in the industry, and industry input of labor is $\ell_i = a_i y_i$. In the long run the number of firms in each

industry is variable and there is constant returns to scale. In the short run the number of firms, and hence industry input and output, is fixed. (In Appendix 2 I deal with short run variability of output and joint production.)

Let p_i denote the price of good i relative to labor (we shall use labor as numeraire throughout). In the short run profits in industry i are

$$(1) \quad \pi_i = (p_i - a_i)y_i,$$

which can be positive, zero, or negative. In the long run, profits are zero and the long run equilibrium price fulfills

$$(2) \quad p_i = \bar{p}_i = a_i.$$

We assume that the adjustment of the number of firms in industry i follows the differential equation

$$(3) \quad \dot{y}_i = \lambda_i(p_i - \bar{p}_i), \quad \lambda_i > 0.$$

That is, for given goods prices the rate of change of the number of firms depends on profits per firm, with a constant speed of adjustment λ_i .

Let there be m consumers, indexed $h = 1, \dots, m$. Each consumer h has a utility function $U_h(c_h, l_h)$ where the n -vector $c_h = (c_{hi})$ denotes consumption of goods and l_h is supply of labor. Let the n -vector $\alpha_h = (\alpha_{hi})$ (with $0 < \alpha_{hi} < 1$, $\sum_h \alpha_{hi} = 1$) denote the shares in industries' profits of consumer h . Then the budget constraint for consumer h is

$$(4) \quad p'c_h = l_h + \alpha_h' \pi(p, y),$$

where $p'c_h$ denotes the inner product $\sum p_i c_{hi}$, etc., $\pi(p, y)$ is the n -vector of profits given by (1), and p and y are the price and output n -vectors.

Maximizing the utility function subject to (4) (for given p and y) gives rise to the goods (n -vector) demand function $C_h(p, \pi(p, y))$ of consumer h . Then we can define the aggregate demand function as

$$(5) \quad C(p, y) = \sum_h C_h(p, \pi(p, y)).$$

We define the (short run) goods excess demand (n-vector) function as

$$(6) \quad X(p, y) = C(p, y) - y.$$

(By Walras' Law we can disregard the labor market.) In a short run equilibrium for given y we have

$$(7) \quad X(p, y) = 0,$$

the solution to which gives rise to the (demand) price (n-vector) function $P(y)$, giving equilibrium goods prices as a function of the given industry output vector. The short run adjustment of prices is assumed to follow a tâtonnement process $\dot{p}_i(\tau) = k_i X_i(p(\tau), y)$ for each good i , and for given industry output y , with a constant positive speed of adjustment. If K is the positive diagonal matrix whose diagonal is the vector of speeds of adjustment, the short run Walrasian process can be written

$$(8) \quad \dot{p}(\tau) = KX(p(\tau), y),$$

for given y . This process is assumed to be very fast relative to the long run adjustment of the number of firms, resulting in the short run equilibrium price $P(y)$ for any given y . (Indeed, the process occurs in "instantaneous time," denoted by τ , which is distinct from "real time," denoted by t .)

The long run adjustment process (in real time) then follows (3), and we can hence write

$$(9) \quad \dot{y}(t) = \Lambda(P(y(t)) - \bar{p}),$$

where Λ is the diagonal matrix whose diagonal elements are the speeds of adjustment λ_i . In the long run equilibrium we have $\dot{y}(t) = 0$ and $P(y(t)) = \bar{p}$, which results in the long run output vector $\bar{y}$.²

We shall now see how local stability of (8) for given y (local Walrasian stability) is related to local stability of (9) (local Marshallian stability). To

examine this, we recall that we have local stability of (8) if and only if the $(n \times n)$ -matrix $KX_p = [k_i \partial X_i / \partial p_j]$ has negative real parts of all its eigenvalues for $p = P(y)$. (This is a simplification: the condition is a sufficient condition for local stability of the non-linear system (8), but a necessary condition for local stability of the linearized system derived from (8), only. We here disregard that distinction.) Similarly, (9) is locally stable if and only if the $(n \times n)$ -matrix $\Lambda P_y = [\lambda_i \partial P_i / \partial y_j]$ has negative real parts of all its eigenvalues for $y = \bar{y}$. Now, by differentiating (7) we get

$$(10) \quad P_y = -X_p^{-1} X_y,$$

where $X_y = [\partial X_i / \partial y_j]$. By (5) and (6) we have

$$(11) \quad X_y = C_y - I = \Sigma C_{im} \pi_y - I,$$

but $\pi_y = D(p - \bar{p})$ where $D(p - \bar{p})$ denotes the diagonal matrix with the vector $p - \bar{p}$ as its diagonal. For $y = \bar{y}$ we have $p = \bar{p}$, hence $\pi_y = 0$, $C_y = 0$ and $X_y = -I$. It follows that for $y = \bar{y}$,

$$(12) \quad P_y = X_p^{-1}.$$

Now, local stability of (8) implies that KX_p , and hence its inverse $X_p^{-1} K^{-1}$ has negative real parts of all its eigenvalues, but this does not imply that this is also the case for $\Lambda P_y = \Lambda X_p^{-1}$.³ However, assume that (8) is locally stable for all speeds of adjustment (this concept of stability, "D-stability," is further discussed in Quirk and Saposnik [5] and Arrow [1]).⁴ This is equivalent to X_p being a D-stable matrix, which by (P4) in Appendix 1 is equivalent to its inverse $X_p^{-1} = P_y$ being a D-stable matrix, which is equivalent to ΛP_y having negative real parts of all its eigenvalues, for all speeds of adjustment matrices Λ . Hence, local stability of (8) for all speeds of adjustment implies local stability of (9) for all speeds of adjustment. Similarly, local stability of (9) for all speeds of adjustment implies local stability of (8) for all speeds of adjustment, at the long run equilibrium $y = \bar{y}$, but not necessarily for other y .

To understand this result intuitively, consider the case with only one good/industry. In the neighborhood of the long run equilibrium profits are to a first-order approximation zero. Hence, at constant prices an increase in the number of firms and hence industry output has no (first-order) effect on profits and on demand for goods, which results in an excess supply for goods. If the Walrasian short run process is stable, the price will fall at this increased output, to restore short run equilibrium. The fall in price leads to negative profits, and by the Marshallian long run adjustment process firms exit and the long run equilibrium is restored. Hence the close relation between stability of the two processes.

3. Global Stability

It is well known that the Walrasian process is globally stable for a single consumer economy. To see the analogy with the Marshallian process below, let us quickly repeat the argument. We take $V(p, l'\pi(p, y))$ to be the utility level for given p and y , where $V(p, \Pi)$ is the indirect utility function and $\Pi = l'\pi$ denotes total profits $\sum_i \pi_i$. Taking the (instantaneous-) time derivative, we have, for given y ,

$$(13) \quad \dot{V} = V_p \dot{p} + V_{\Pi} l' \pi_p \dot{p} = -V_{\Pi} X(p, y) \cdot KX(p, y) < 0$$

for $X(p, y) \neq 0$, where we have used Roy's Identity ($C(p, \Pi) = -V_p/V_{\Pi}$), $\pi_p = D(y)$, $l'\pi_p = y'$, (6) and (8). Since V has a minimum for $p = P(y)$, global stability follows by the usual Liapunov argument. Intuitively, price is lowered for a good in net supply, which lowers utility (a terms-of-trade deterioration in international trade terminology).

For the Marshallian process, as Novshek and Sonnenschein have shown, the utility function again provides the appropriate Liapunov function. In our

framework we can write the utility level in a short-run equilibrium as $U(y, a'y)$ where we use that total labor input is $a'y = \sum a_i y_i$.

Taking the (real-) time derivative, we get

$$(14) \quad \dot{U} = U_c \dot{y} + U_\ell a' \dot{y} = (-U_\ell)(P(y) - \bar{p}) \dot{y} = (-U_\ell)(P(y) - \bar{p}) \Lambda(P(y) - \bar{p}) > 0$$

for $y = \bar{y}$, where we have used $U_\ell < 0$, $P(y) = -U_c/U_\ell$ and (9). Since $U(y, a'y)$ has a maximum for $y = \bar{y}$, global stability follows again by the usual Liapunov arguments. Intuitively, goods output and labor input are increased in the proportion $1:a_i$ for goods with a marginal rate of substitution for leisure $1/p_i$ that is less than $1:a_i$, which increases utility.

4. Conclusions

It is a standard result that there is no direct relation between stability of the Marshallian adjustment process and stability of the Walrasian (long run) tâtonnement process. When we make a distinction between long run Walrasian tâtonnement with variable number of firms, which process I find rather irrelevant, and short run Walrasian tâtonnement for given number of firms, a process that I find much more relevant, we get very different results. Thus, we have seen demonstrated a close relation between stability of short run Walrasian adjustment and stability of long run Marshallian adjustment. Indeed, with regard to local stability (for all speeds of adjustment) around the long run equilibrium, we could show a direct equivalence between the two. We also restated results according to which both are globally stable for a one consumer economy.

In a way the results are rather striking. It makes a lot of sense to consider a long run adjustment process with exit and entry of firms depending upon profit levels. It does not make much sense, though,

if the short run adjustment process which establishes those levels of profits is unstable. So it seems reasonable to restrict attention to stable short run adjustment. If, however, that stable short run adjustment is Walrasian tâtonnement, it implies, really without any further restriction, (local) stability of the long run Marshallian adjustment, and hence that the long run competitive equilibrium is attained.

The conclusion seems opposite to that of Novshek and Sonnenschein [4]. Rather than to call into question the significance of the (long run) Walrasian competitive equilibrium, these results seem to, if anything, increase its significance, since the Walrasian equilibrium can indeed - under the conditions discussed above - in general be approached by a rather reasonable Marshallian long run adjustment of the number of firms.

I would like to emphasize two qualifications, though. First, the above results have only been shown for the case with no joint production. They may or may not hold with joint production. Second, the stability concept used is stability for all speeds of adjustment, which in the absence of information about speeds of adjustment seems appropriate. With some information about speeds of adjustment of the long run Marshallian process, say, stability of this may be assured for short run Walrasian adjustment processes that fulfill less restrictive conditions than stability for all speeds of adjustment. These two qualifications may be suitable for further research.

Appendix 1: Stability and D-stability of inverses and transposes

An $n \times n$ -matrix A is said to be stable if and only if the real parts of all its eigenvalues are negative. It is said to be D-stable (Quirk and Saposnik [5] and Arrow [1]) if and only if the matrix product DA is stable for all positive diagonal matrices D . The following propositions are easy to show:

(P1) A is stable if and only if its inverse A^{-1} is stable.

(P2) A is stable if and only if its transpose A^T is stable.

(P3) A is D-stable if and only if A^T is D-stable.

(P4) A is D-stable if and only if A^{-1} is D-stable.

(P1) follows since the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A . (P2) follows since A and A^T have the same eigenvalues. (P3) follows by the following argument: Let D be a positive diagonal matrix, and let DA be stable. The Routh-Hurwitz Theorem gives necessary and sufficient conditions for stability of DA in terms of the principal minors of DA (Quirk and Saposnik (23)). But the principal minors of DA are identical to those of DA^T . Hence DA^T is stable. This holds for any D . Hence (P3) follows. To show (P4), finally, we let DA be stable. Then by (P1) this is equivalent to $(DA)^{-1} = A^{-1}D^{-1}$ being stable, and by (P2) equivalent to $(A^{-1}D^{-1})^T = D^{-1}(A^{-1})^T$ being stable. This sequence holds for all positive diagonal matrices D . Hence A is D-stable if and only if $(A^{-1})^T$ is D-stable. But by (P3) $(A^{-1})^T$ is D-stable if and only if A^{-1} is D-stable, and (P4) holds.

Appendix 2: Short-run output variability and joint production

Here we show that our results concerning local stability hold in full if we allow short run variability of output as long as we continue to assume no joint production. With joint production, it seems that the local results can no longer be shown, at least not with the methods used here.

Assume that each firm in industry i has a smooth technology, using labor as input and goods as inputs/outputs. This can be represented by a standard profit function $\tilde{\pi}_i(p, w) = \tilde{\pi}(p, 1) = \tilde{\pi}(p)$ (recall that labor is the numeraire: we drop the wage argument for convenience). The profit function has the standard property that the $(n+1)$ -vector $(\partial \tilde{\pi}_i(p, 1)/\partial p, \partial \tilde{\pi}_i(p, 1)/\partial w)$ is the vector of net outputs of goods and labor for a firm in industry i (positive components for outputs, negative for inputs). Let the column n -vector $\tilde{\pi}(p) = (\tilde{\pi}_i(p))$ be the vector of firms' profits in the different industries. We define the vector of industry profits as

$$(A1) \quad \pi(p, y) = D(\tilde{\pi}(p))y = (\tilde{\pi}_i(p)y_i),$$

where $y = (y_i)$ denote the vector of numbers of firms in each industry. Then the aggregate consumption function can be defined as in (5). The short run goods excess demand function is

$$(A2) \quad X(p, y) = C(p, y) - \tilde{\pi}_p(p)^{\sim} y,$$

where $\tilde{\pi}_p(p)$ is the $n \times n$ matrix with firms' net output vectors as rows, and $\tilde{\pi}_p(p)^{\sim}$ is its transpose. Corresponding to (8) we have as before

$$(A3) \quad \dot{p}(\tau) = KX(p(\tau), y),$$

and (9) is replaced by

$$(A4) \quad \dot{y}(t) = \Lambda \tilde{\pi}(P(y(t))),$$

where $P(y)$ is defined as before, that is from $X(p,y) = 0$. Local stability of (A3) and (A4) is, with the same simplification as before, equivalent to KX_p and $\tilde{\Lambda}\tilde{\pi}_p^P$ having negative real parts of all their eigenvalues. We note that by (A2),

$$(A5) \quad P_y = -X_p^{-1} X_y = X_p^{-1} \tilde{\pi}_p^P$$

at $y = \bar{y}$.

Assume that there is no joint production. Then π_p is a diagonal matrix. D-stability of X_p is then equivalent to D-stability of $\tilde{\pi}_p^{-1} X_p$, which by (P4) is equivalent to D-stability of $X_p^{-1} \tilde{\pi}_p^P = P_y$ at $y = \bar{y}$. Hence, the results about local stability holds with no joint production.

Assume now that there is joint production, so that $\tilde{\pi}_p$ is no longer a diagonal matrix. If X_p is D-stable, so is X_p^{-1} , and by (A5) $(\tilde{\pi}_p^P)^{-1} P_y$. But it does not follow that $\tilde{\pi}_p^P$ is D-stable, since $\tilde{\pi}_p$ is not a diagonal matrix. It seems that the local results do not hold for joint production or at least that the same methods cannot be used to show them.

Appendix 3: Novshek and Sonnenschein's Example [4, Example 2].

There are two consumers, one good/industry and labor. Each firm in the industry produces one unit of output, using up one unit of labor input. Hence the long run equilibrium price is unity, $\bar{p} = 1$, and profits are given by the scalar

$$(A6) \quad \pi(p, y) = (p-1)y.$$

Consumer A does not receive any profits. His/her preferences are

$U_A(c_A, \ell_A) = \min(c_A, 1-\ell_A)$ for $c_A \geq 0$, $0 \leq \ell_A \leq 1$ (he/she has one unit of leisure), and the budget constraint is $pc_A = \ell_A$. The resulting consumption demand function is

$$(A7) \quad C_A(p) = 1/(p+1).$$

Consumer B receives all profits. His/her preferences are $U_B(c_B, \ell_B) = \min(c_B/2, 1-\ell_B)$ for $c_B \geq 0$, $0 \leq \ell_B \leq 1$, and the budget constraint is $pc_B = \ell_B + (p-1)y$. This gives the consumption demand function

$$(A8) \quad C_B(p, y) = 2 [1+(p-1)y] / (2p+1).$$

Then the excess demand function is

$$(A9) \quad X(p, y) = C_A(p) + C_B(p, y) - y.$$

We know that the long run equilibrium price fulfills $\bar{p}=1$. Hence, the long run equilibrium output is given by $X(1, \bar{y}) = 0$ which gives $\bar{y} = 7/6$.

By differentiating (A9) we get

$$(A10) \quad X_p(p, y) = -1/(p+1)^2 + 2y/(2p+1) - 4[1 + (p-1)y]/(2p+1)^2,$$

$$(A11) \quad X_p(\bar{p}, y) = X_p(1, y) = 2y/3 - 25/36, \text{ and}$$

$$(A12) \quad X_p(\bar{p}, \bar{y}) = X_p(1, 7/6) = 1/12 > 0.$$

Hence, excess demand is increasing in price for given $y = \bar{y}$, and at the long run equilibrium the short run Walrasian process is unstable. Equivalently, the (demand) price function $P(y)$ is upward sloping at $y = \bar{y}$, and the long run Marshallian process is unstable. The intuitive reason the excess demand function is upward sloping for given number of firms is that an increase in the price via increased profits increases the demand of consumer B. (We recall that Leontief demand functions have zero substitution effects, and income effects only). In the language of international trade theory, consumer B, owning the firm and hence being a net exporter of the good, experiences a terms of trade improvement; whereas consumer A, being a net importer of the good, suffers a terms of trade deterioration of the same magnitude. Since the marginal propensity to consume the good is higher for consumer B, the net result is an increase in the demand for the good.

What does the long run excess demand function look like? We note that in the long run there is constant returns to scale, which gives rise to a supply correspondence defined by

$$(A13) \quad p \leq 1, \quad y \geq 0 \quad \text{and} \quad (p-1)y = 0,$$

Let $y(p)$ denote any y that fulfills (A13) for given p . Then, the long run excess demand function is defined as $X(p, y(p))$. It follows from (A7)-(A9) and (A13) that it will be given by

$$(A14) \quad \begin{aligned} X(p, y(p)) &= X(p, 0) = 1/(p+1) + 2/(2p+1) \text{ for } p < 1, \text{ and} \\ X(p, y(p)) &= X(p, y) = 7/6 - y, \quad 0 \leq y < \infty, \text{ for } p = 1. \end{aligned}$$

The long run excess demand function $X(p, y(p))$ is plotted in Fig. A1, as well as the short run excess demand function $X(p, \bar{y})$. We see that the long run Walrasian competitive equilibrium is unique, as observed by Novshek and Sonnenschein. What about long run Walrasian tâtonnement? This is a bit difficult to consider here, since the long run excess demand is not single-valued for $p = 1$ and not defined for $p > 1$. We can however consider a decreasing single-valued arbitrarily close approximation to the long run excess demand function, and with that approximation the long run Walrasian [^]tâtonnement process will be stable. The short run Walrasian process, and the long run Marshallian process, are both unstable at $y = \bar{y}$.

There are altogether three short run Walrasian equilibria for $y = \bar{y}$. One is the unstable one for $\bar{p} = 1$ that we have already noticed. The other two can be found by observing that for $y = \bar{y} = 7/6$, the economy can be taken to be a pure exchange economy, where consumer A has endowments of leisure and goods given by $(1, 0)$. Consumer B's endowments is the sum of his/her initial leisure/goods endowment $(1, 0)$ and the labor input/goods output of the firm, $(-7/6, 7/6)$, giving a total endowment of $(-1/6, 7/6)$. Hence, the leisure endowment is negative, a circumstance that must be handled with some care. The situation is illustrated in the Edgeworth Box in Fig. A2. The horizontal distance from A (B) measures consumer A's (B's) consumption of leisure. The vertical distance from A (B) measures consumer A's (B's) consumption of goods. The length of AD is $y = \bar{y} = 7/6$, the total endowment of goods and the length of AF is $5/6$, the total endowment of leisure. The line AG (BH) with slope 1 (2) is A's (B's) consumption expansion line. E is the endowment point. Point J shows the short run Walrasian equilibrium where $p = \bar{p} = 1$, $C_A = 1 - \ell_A = 1/2$, $C_B = 2/3$, $1 - \ell_B = 1/3$ and hence with $\ell_A = 1/2$ and $\ell_B = 2/3$. Another equilibrium is when $p = \infty$, corresponding to a horizontal budget line and a zero wage. Consumer A is at H with $C_A = 0$ and $1 - \ell_A = 1/4$ and $\ell_A = 3/4$. Consumer B is at H with $C_B = 7/6$, $1 - \ell_B = 7/12$ and $\ell_B = 5/12$. This

equilibrium requires consumer A to supply labor at a zero wage and zero consumption of goods, which he/she does not mind given his/her Leontief utility function. There is excess supply of leisure.

With negative leisure endowments of consumer B there does not exist any equilibrium for $p < 1$, where consumer A supplies enough labor. Suppose then that in the short run 1/6 "number" of firms closes down, such that B's endowments is $(1,0) + (-1,1) = (0,1)$.

This gives rise to an Edgeworth Box whose sides are unity, as in Fig. A3. The endowment point is N. There exists an equilibrium where $p = 0$, and $C_A = 1$, $\ell_A = 0$, $C_B = 0$ and $\ell_B = 1$. B works full time, has no leisure and does not consume goods, whereas A enjoys full consumption of goods and complete leisure. At all other non-zero prices there is excess supply of goods.

Footnotes

* The idea to this paper occurred during a seminar at Harvard University given by Hugo Sonnenschein, where results from Sonnenschein [6], Novshek and Sonnenschein [4] and a forthcoming paper by Novshek and Sonnenschein were presented. I thank Hugo Sonnenschein, Andreu Mas-Colell and Wilfred Ethier for very useful discussions of previous notes, which discussions helped to correct some crucial errors of mine. I have benefited from comments by Elhanan Helpman on the present version of the paper. Remaining errors and obscurities are solely my own responsibility. I am grateful to NBER for secretarial assistance during the early stage of this work.

1. There has also been considerable work on non-tâtonnement adjustment processes, which however are not relevant for the present problem. See Takayama [7] for references to earlier work on non-tâtonnement.
2. Mas-Colell [2] discusses stability of a system like (8) and (9) when the Walrasian and Marshallian processes both occur at the same time. The article is in Spanish and I regret therefore that I am unable to read it.
3. Except for the special case when Λ equals K^{-1} . ($X_p^{-1}K^{-1}$ is stable (has negative real parts of all its eigenvalues) if and only if its principal minors fulfill the Routh-Hurwitz conditions. But the principal minors of $X_p^{-1}K^{-1}$ are identical to the principal minors of $K^{-1}X_p^{-1}$. Hence, $X_p^{-1}K^{-1}$ is stable if and only if $K^{-1}X_p^{-1}$ is stable.)
4. The complications due to non-uniform speeds of adjustment, as well as the crucial distinction between stability for given and for all speeds of adjustment, were pointed out to me by Andreu Mas-Colell.

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Figure A1 Figure A2 Figure A3

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