Seminar Paper No. 345

A PERMANENT DEMAND THEORY OF PRICING

by

Nils Gottfries

INSTITUTE FOR INTERNATIONAL ECONOMIC STUDIES

University of Stockholm
Seminar Paper No. 345

A PERMANENT DEMAND THEORY OF PRICING

by

Nils Gottfries

Seminar Papers are preliminary material circulated to stimulate discussion and critical comment.

January, 1986

Institute for International Economic Studies
S-106 91 Stockholm
Sweden
A PERMANENT DEMAND THEORY OF PRICING*

by

Nils Gottfries

ABSTRACT

Based on standard static models of firm behavior one would expect the price of goods to increase with demand for a given level of costs. In empirical studies prices have generally been found to be unaffected by short run variations in demand, however. The model in this paper is consistent with this stylised fact. In the model, a demand shock that is perceived as temporary may lead to unchanged or even lower prices, while a permanent demand shock leads to higher prices.

* First version February 1983.
This is a revised version of Chapter 4 in my thesis, Stockholm University 1985. It was presented at the Econometric Society Meeting in Madrid, September 1984. I thank the participants and Ronald Findlay for valuable comments.
1. Introduction

According to standard models, if the marginal product of labor falls as output increases, the real wage will have to fall, for output and employment to increase. However, the evidence suggests that the contemporaneous correlation between the real wage and economic activity is, if anything, positive (see e.g., Bodkin (1969), Modigliani (1977), Neftci (1978), Bils (1985)).

The same stylized fact is reflected in empirical studies of the pricing behavior of firms. In most such studies a price index is regressed on some indexes measuring costs (i.e. mainly wages), demand and, in some cases, foreign competitors prices. Most authors have been unable to find the expected positive effect of demand on prices, for a given level of costs. In some studies, an effect of demand is found, but the wide variety of variables used to measure demand suggests that the results should be interpreted with caution (cf. Nordhaus, 1972, p. 45, Tobin, 1972, pp. 7-9). The observed insensitivity of goods market prices to demand has motivated the mark-up pricing assumption in Keynesian macroeconomic textbooks, but many authors, e.g. Gordon (1981, p. 503) and Tobin (1979, p. 35), have pointed out the lack of microfoundations for this assumption.

It is well known that setting prices with a constant mark-up over average cost maximizes profit in monopolistic competition if firms have a constant returns to scale technology and face isoelastic demand curves. However, these assumptions seem unsuitable for explaining short run pricing behavior, since all factors of production are required to be flexible. Within the time horizon discussed in business cycle theory, at least some factors of production appear to be fixed or costly to change, implying that the marginal

\[^1\] Surveys of such studies are found in Nordhaus (1972) and Tobin (1972) and other examples are Ringstad (1971), deMenil (1974), Gordon (1975), Coutts, Godley and Nordhaus (1978), Calmfors and Herin ((1979)).
cost should increase with the quantity produced. Therefore, one would expect a positive effect of demand on prices, for a given level of wages.\footnote{2}

One reason, which has been suggested, for short run price stickiness with respect to demand shocks, is that price changes are costly (e.g. Barro 1972, Rotemberg 1982a,b). In the author's view such costs may play a role in the very short run but their effects are likely to be very transient. Other models of the microfoundations of pricing emphasize the idea that monopolistic firms must set prices \textit{before} they know demand and costs (Bruno 1979, Gordon 1981, section VII). This, by assumption, gives short run price stickiness and resulting \textit{unplanned} output movements. But again the effects are likely to be of a very short run nature. As Okun (1975) argues: "...it costs something and takes some time to make and implement the firm's decisions on prices and then to print and distribute the new price lists. But these elements imply an inertia of prices that should last for days or weeks, not years." (See also Gordon 1981, p. 495.) There is therefore, in the author's view, reason to further analyse of the microfoundations of short run price dynamics.

In the model presented here, an increase in demand which is perceived as \textit{temporary} may lead to unchanged or even lower prices, while if the demand shock is perceived as \textit{permanent}, there will be a price increase. The analysis does not rely on the existence of costs for changing prices nor on firms being imperfectly informed about current demand or competitors' prices. The model is

\footnote{2}{The first order condition for profit maximisation in monopolistic competition can be written

\[ p = \left(1 + \frac{1}{\varepsilon} \right)^{-1} mc \]

where \( p \) is price, \( \varepsilon \) is price elasticity of demand and \( mc \) is marginal cost. If marginal cost increases with production, a positive demand shock could still leave the price unchanged if the price elasticity simultaneously increased. However, it is hard to explain why the price elasticity would be systematically related to the business cycle.}
a modified version of the one developed by Phelps and Winter (1970). An important element in the model is the idea that a firm which charges a relatively high price will gradually lose customers. The following customer flow equation is postulated:

\[ x_t - x_{t-1} = h (p_t - \bar{p}_t) x_{t-1} \quad h > 0 \]

where \( x_t \) is the customer stock of a particular firm, \( p_t \) is the firm's price, \( \bar{p}_t \) is the average price charged by the other firms in the market (the "market price") and \( h \) is a positive constant. This equation will not be formally motivated here. One reason why customers react slowly to price differences may be that they learn slowly about other firms' prices. Another may be that it is costly for them to change supplier.\(^3\) The customer flow equation implies that the pricing decision of a firm is a dynamic optimization problem. Lower prices may reduce current profits, but increase the customer stock and, therefore, future profits.

Another important assumption is that the firm prefers a smooth income flow to an uneven one. This assumption can be motivated in different ways. One reason, why firms may want to smooth income over time may be imperfections in the credit market.\(^4\) Suppose, for example, that there is one owner who is

---

\(^3\) Equation (1) was formally derived by Phelps and Winter (1970) assuming distribution of prices in the market and that customers make one random price comparison each period. The existence of a (nondegenerate) distribution of prices is left unexplained, however. The analysis in Gottfries (1985, ch.3) is more relevant for the present purposes. There, as in Phelps and Winter (1970), customers are assumed to make one price comparison per period, but they also face random adjustment costs, which they have to pay to change supplier. This analysis does not rely on the existence of a (nondegenerate) distribution of prices in the market.

\(^4\) Stiglitz and Weiss (1981) have argued that adverse selection effects of the interest rate may be a reason why banks set a low interest rate and ration credit. It may also be the case that some firms are only able to borrow (at a reasonable rate) as long as they can offer tangible assets as collateral. It is often argued that, because of credit market imperfections, fixed investment of firms depends on the availability of internally generated funds (see e.g. Steigum 1983). A similar argument is made here with respect to price setting behavior.
unable (or unwilling) to borrow and lend in the credit market and who uses the income from the firm for consumption. In this case, the owner's preference for a smooth flow of consumption implies preference for a smooth flow of income from the firm. More generally, if the uses of income generated in a given period yield decreasing marginal returns there will be preference for a smooth income flow relative to an uneven one. A somewhat different motivation is suggested by Okun (1981, p.150.): the managers of the firm may prefer to report a smooth income flow to the shareholders.

Okun suggested that in this setup firms will not raise their prices in the face of temporary demand shocks. This proposition is examined below. I specify a model of a goods market with the features described above. Cost and demand shocks are taken as exogenous, thus, this is a purely partial analysis of price determination. A rational expectations equilibrium for the market is computed, which allows analysis of the effects of shocks on the equilibrium price.

The results confirm Okun's suggestion. If the desire to smooth income is sufficiently strong, the market price may be unchanged or even fall when a temporary demand shock occurs. This can be explained as follows: when demand increases the firm's profit will increase if the price is kept unchanged. If the use of the profit yields diminishing marginal returns (e.g., because of diminishing marginal utility of income of the owner) the firm will attempt to smooth its profit flow by increasing its customer stock when demand is high. To do this it sets a lower price. When all firms try to increase their customer stocks the market price falls.

The implication that prices do not respond to temporary demand shocks, but do respond to permanent shocks, is consistent with the empirical results of Kawasaki, McMillan and Zimmermann (1983). The recent literature on the relationship between inventories and pricing behavior has produced conclusions
which are partly similar, partly conflicting with the ones obtained here. This
will be discussed in the final section. In the present analysis, I abstract
from inventories. The model also yields predictions about the cyclical
variability of prices in different markets. Prices of relatively simple
products should be expected to vary more with the business cycle. Further, a
reduction in the availability of credit will induce firms to raise their
prices.

In the next section, the decision rule for an individual firm is derived
and in Section 3 the rational expectations equilibrium market price is
computed. Effects of shocks are analysed in Section 4 and some possible
extensions are discussed in Section 5. Section 6 contains some concluding
comments.

2. Optimum for the Firm

Firms produce identical goods and have perfect information about current
demand and costs. The demand per customer depends on the price charged by the
firm, \( p_t \), and income (or expenditure), \( y_t \). Prices of other goods are assumed
to be constant and are left out. Total demand facing the firm in period \( t \) is

\[
Q_t = q(p_t) y_t x_t \quad \text{where} \quad q_1 = \frac{dq}{dp_t} < 0 \quad q_{11} = \frac{d^2q}{dp_t^2} = 0
\]

The multiplicative specification implies that a shock to demand leaves
the price elasticity unchanged (compare note 2). For simplicity the demand
curve is assumed to be linear in the price.\(^5\) Some factors of production are
taken as given and the marginal cost is assumed to increase with the quantity
produced. The cost \( (C_t) \) is given by

\(^5\) Nonlinearity of the demand function would complicate the calculations
but would not affect the qualitative results.
(3) \[ C_t = F(Q_t) w_t, \quad F_1 > 0, \quad F_{11} > 0 \]

where \( w_t \), is the wage, labor being the only flexible factor of production.

The revenue obtained in period \( t \) is then

(4) \[ \pi_t = p_t q(p_t) y_t x_t - F(q(p_t) y_t x_t) w_t \]

The firm has a preference for a smooth income flow. To represent this I assume that the firm wants to maximize

(5) \[ E_t \sum_{j=0}^{\infty} \delta^j U(\pi_{t+j}) \]

where \( E_t \) is the expectation conditional on information available at time \( t \), \( \delta \) is a subjective discount factor and \( U_1 > 0, \quad U_{11} < 0 \).

When the firm sets its price for a period it is assumed to know the demand and cost shocks and have perfect foresight about the market price in that period. Thus, according to the customer flow equation (1) the choice of price also implies a choice of customer stock and we may treat the latter as the choice variable. Using (1) to substitute for \( p_t \) in the objective function, the problem can be stated as

(6) \[ \max_{x_{t+1}} \sum_{j=0}^{\infty} E_t \delta^j f(x_{t+1}, x_{t+j}, \tilde{p}_{t+j}, y_{t+j}, w_{t+j}) \]

s.t. \( x_{t-1} = x_{t-1}^0 \)

where

\[ f(x_{t-1}, x_t, \tilde{p}_t, y_t, w_t) \]

= \[ U \left[ g(x_{t-1}, x_t, \tilde{p}_t) q(g(x_{t-1}, x_t, \tilde{p}_t) y_t x_t - 
- F(q(g(x_{t-1}, x_t, \tilde{p}_t) y_t x_t) w_t) \right] \]
the function \( g(\cdot) \) being defined by:

\[
g(x_{t-1}, x_t, \bar{p}_t) = \bar{p}_t - h^{-1}(x_t / x_{t-1} - 1),
\]

Since the function \( f(\cdot) \) is highly nonlinear this maximization problem is very difficult. To derive a closed form solution an approximation must be made. Thus, imagine that the variables \( \bar{p}_{t+j}, y_{t+j} \) and \( w_{t+j} \) are known with certainty to be constant for all \( j = 0, 1, 2 \ldots \) and assume that a stationary solution for \( x_t \) exists in this case. Then, take a quadratic approximation to the objective function around this stationary solution.

Now, suppose that we instead do the maximization in (6) with \( f(\cdot) \) replaced by the quadratic approximation. To this new problem one can find a closed form solution. If the values of the exogenous variables are close to the point, where the quadratic approximation was taken, the solutions to the two maximization problems are close to each other. Thus, one can obtain an approximate solution to the original problem in this way. This is done in Appendix B. The optimally chosen price is

\[
p_t = \bar{p}_t - h^{-1}(x_t / x_{t-1} - 1)
\]

where

\[
x_t' = v_1 x_{t-1} + v_2 \delta f_{12} \sum_{j=0}^{\infty} v_2^{-j} (f_{23} + \delta f_{13} L^{-1}) \bar{p}_{t+j} + (f_{24} + \delta f_{14} L^{-1}) t y_{t+j} + (f_{25} + \delta f_{15} L^{-1}) t w_{t+j}
\]

\( x_t' \) stands for the deviation of \( x_t \) from the stationary solution and similarly for other variables. The signs of the second order derivatives \( f_{12} \) etc. are analysed in Appendix A. The notation \( t y_{t+j} \) stands for the expectation at time \( t \) about \( y_{t+j} \) and \( L \) is the lag operator. It is possible to show that \( v_1 < 1 \) and \( v_2 > 1/\delta \). This decision rule gives the optimal price
set by a firm, given its initial customer stock and expectations about current and future exogeneous shocks and market prices. Expectations about exogeneous shocks will be taken as given in the analysis while expectations about market prices are endogenously determined. The next step is to solve for these expectations and for market prices.

3. **Market Equilibrium**

All firms are assumed to have identical cost functions and normal costs. Thus the derivatives of $f(-)$ at the stationary solution ($f_{12}$ etc.) are the same for all firms. Firms may, however, be subject to different (small) cost shocks and have different customer stocks. The linearity of the decision rule (8) allows easy aggregation to an average decision rule

$$
\bar{x}'_t = v_{1} \bar{x}'_{t-1} + \frac{1}{v_{2}\delta f_{12}} \bar{x}_{L} \bar{v}^{-j} \left( (f_{23} + \delta f_{13}L^{-1}) \bar{p}'_{t+j} \\
\left( (f_{24} + \delta f_{14}L^{-1}) \bar{v}'_{t+j} + (f_{25} + \delta f_{15}L^{-1}) \bar{w}'_{t+j} \right) \right)
$$

where $\bar{x}_t$ is the average customer stock and $\bar{w}_{t+j}$ denotes the expected average cost shock. I assume that all firms have the same information and they therefore have identical expectations about future market prices. The decision rule also applies to planned future customer stocks. Therefore, the decision rule implies the following relation between the average planned future customer stock $\bar{t}x'_{t+1}$, and expected average shocks and market prices:

$$
\bar{x}'_{t+1} = v_{1} \bar{x}'_{t} + \frac{1}{v_{2}\delta f_{12}(1 - v_{2}L^{-1})} \left( (f_{23} + \delta f_{13}L^{-1}) \bar{p}'_{t+1} \\
\left( (f_{24} + \delta f_{14}L^{-1}) \bar{v}'_{t+1} + (f_{25} + \delta f_{15}L^{-1}) \bar{w}'_{t+1} \right) \right)
$$
\[ i = 0, 1, 2, \ldots \]

Now, by definition, \( \bar{x}_t' = 0 \) for all \( t \), i.e., on average firms cannot gain customers. Thus, for firms' plans to be consistent, expectations about the market price must be such that \( \bar{x}_t' = 0 \) for all \( i > 0 \). Imposing this condition in (10) we get a difference equation in terms of \( \bar{p}_{t+i} \), \( \bar{y}_{t+i} \) and \( \bar{w}_{t+i} \). Solving this difference equation forward and picking the solution that satisfies the transversality condition one obtains the solution for the expected market price at \( t+i \) (c.f. Sargent (1979) pp. 173-4):

\[
\bar{p}_{t+i} = - \frac{1}{f_{23}} \left[ (f_{24} + \delta f_{14} L^{-1}) \bar{y}_{t+i} + \right.
\]

\[
+ (f_{25} + \delta f_{15} L^{-1}) \bar{w}_{t+i} \right] =
\]

\[
= - \sum_{j=0}^{\infty} s \left[ (f_{24} \bar{y}_{t+i+j} + \delta f_{14} \bar{y}_{t+i+j+1}) + \\
+ (f_{25} \bar{w}_{t+i+j} + \delta f_{15} \bar{w}_{t+i+j+1}) \right]
\]

where

\[
0 < s = - \frac{\delta f_{13}}{f_{23}} < 1
\]

Setting \( i=0 \) and collecting terms we get the equilibrium market price for period \( t \).

\[
\bar{p}_t = \frac{1}{f_{23}} \left[ f_{24} \bar{y}_t + (s f_{24} + \delta f_{14}) \sum_{j=0}^{\infty} s^j \bar{y}_{t+j+1} + \\
+ f_{25} \bar{w}_t + (s f_{25} + \delta f_{15}) \sum_{j=0}^{\infty} s^j \bar{w}_{t+j+1} \right]
\]
4. Effects of Shocks

Provided that the marginal cost curve is upward sloping \((F_{11} > 0)\), the effect on the price of a permanent demand shock is positive:

\[
\frac{dp_t}{dy} = -\frac{f_{24} + \delta f_{14}}{f_{23}(1 - s)} > 0 \quad (dy_{t+j} = dy, \ j = 0, 1, 2, \ldots)
\]

since

\[
f_{24} + \delta f_{14} = \frac{U_{11} f_{4}}{U_{1}^2} (f_{2} + \delta f_{1}) + \\
\frac{f_{2} + \delta f_{1}}{y} - U_{1} F_{11} q_{1} g_{2}(1-\delta) + U_{1} F_{11} > 0
\]

where \(f_{2} + \delta f_{1} = 0\) by the first order condition at the stationary solution. If the marginal cost curve was flat \((F_{11} = 0)\) a permanent demand shock would not have any effect on the price. This result is the same as in a static model with monopolistic competition. Further, the effect of a permanent demand shock is independent of the utility function. What about a temporary demand shock?

\[
\frac{dp_t}{dy} = \frac{-f_{24}}{f_{23}} > 0 \quad (dy_{t+j} = 0, \ j = 1, 2, 3, \ldots)
\]

The effect of a temporary demand shock is ambiguous (cf. Appendix A). However, it is shown in the appendix that if the utility function is sufficiently concave, i.e., if the desire to smooth income is sufficiently strong, \(f_{24}\) is greater than zero so that a temporary increase in demand leads to a reduction in the price. The reason is that if the firm raised its price, to take advantage of current high demand, this would imply a loss of
customers. Thus, current profits would increase while future profits (when
demand falls back) would fall. This conflicts with the preference for a smooth
income flow.

Another way to explain this result is to note that the higher demand
today implies higher profits at a given price. The owner wants to invest some
of his increased wealth to smooth his consumption flow. By assumption, he is
unable or unwilling to put the money in the bank or to pay back loans, but
prefers to invest in an expansion of the customer stock, i.e. to charge a
lower price.

It should be noted that this result depends critically on the preference
for a smooth income flow. If the firm/owner had a linear utility function (or
access to a perfect credit market) the result would be the opposite: a
temporary increase in demand would lead to a larger price increase than a
permanent one because the incentive to invest in customer stock is stronger
when demand is permanently higher.

Of particular interest is the role of the parameter $h$, which measures
the speed with which customers react to a price difference. One can show that,
given the utility function, if $h$ is sufficiently large, the effect of a
temporary demand shock on the price will be positive. This is not surprising,
since if $h$ is large there is close to perfect competition in the market.

In which markets would one expect $h$ to be large? To analyse this, note
that in the model products of different firms are perfect substitutes.
According to the analysis in Gottfries (1986) the size of $h$ has to do with
how easy it is to compare prices and how costly it is to change supplier.
These costs should be lower, the simpler the product is. For example, it
should be easier to compare prices for a simple quality of steel, than to
compare prices for some particular kind of machine. In the latter case, a
detailed analysis may be necessary before one can conclude that a machine,
which is offered, really fulfills ones requirements. Further, the cost for changing supplier is likely to be smaller for relatively simple products where the setup costs for the buyer-seller relationship are minimal. Thus the theory is capable of explaining why prices of relatively simple products vary more with the business cycle than other prices.

With respect to cost shocks, it can be shown that both permanent and temporary cost shocks have a positive effect on the price. It is not possible to say, in general, which type of shock has the strongest effect on the price. To understand this, it is useful to examine the effect of an expected future increase in costs. This is given by

\[
\begin{align*}
\frac{dp_t}{dw_{t+k}} &= (sf_{25} + \delta f_{15}) s^k \\
(dw_{t+j} &= 0, j \neq k)
\end{align*}
\]

where

\[
sf_{25} + \delta f_{15} = \frac{11}{u_1} (sf_2 + \delta f_1) - sU_1F_1 - \frac{u_2}{u_1} U_1F_1g_1(s-\delta) > 0
\]

The sign, again, depends on the concavity of the utility function. If the utility function is linear, higher future costs lead to a higher price today, since the incentive to invest in the customer stock is reduced. If the firm desires to smooth its income flow, there is also another effect: high future costs imply low future incomes and a stronger incentive to invest in customers so as to smooth income. A permanent cost shock is equivalent to a temporary cost shock plus a number of expected future cost shocks. Thus, if the utility function is linear a permanent cost shock has the stronger effect, and the converse holds if the utility function is very concave.
5. **Extensions**

If specific stochastic processes are assumed for the cost and demand shocks, one can integrate the present analysis with the analysis of how firms form expectations. For example, suppose one assumes the following stochastic process for demand (c.f. Muth 1960)

\[
y_t' = y_{p,t}' + u_t
\]

\[
y_{p,t}' = r y_{p,t-1}' + u_{p,t}
\]

where \( r \) is close to, but less than one. In this case, there is a temporary and a persistent element in demand and the firm's problem is to establish how much of the current shock is due to the persistent element. The analysis would be exactly analogous to that of Muth (see also Sargent, 1979, pp. 308-313). In this framework, the adjustment of the market price to demand shocks would be **gradual** since a persistent demand shock would initially be perceived as (partly) temporary. The idea that prices adjust gradually to demand shocks is emphasized by e.g., Gordon (1981).

As discussed above, one reason, why firms want to smooth their income, may be that they are unable (or unwilling) to borrow or lend in the credit market. More generally, one could assume that firms are **rationed** in the credit market. Credit rationing may occur, either because of e.g. adverse selection problems which are inherent in the market, or because the government imposes quantitative regulations on the banks. Given that the firm is rationed in the current period the issue arises whether it also expects to be rationed in future periods. If this is assumed to be the case, one can show that a reduction in the availability of credit leads to a higher market price. When the firm is more severely rationed in the credit market, it can afford to

---

\[6\] A more satisfactory assumption would be that the cost of borrowing increases with the amount that is borrowed (c.f. Steigum 1983). This would make the firm's decision problem more complicated, however, since there would be two state variables, the customer stock and the debt. It seems difficult to derive any analytical results for such a model.
invest less in its customer stock, and therefore raises its prices. One can also show that this effect is stronger the more temporary the reduction in the availability of credit is. This suggests that a credit crunch may be a quite inefficient way of "cutting inflationary excess demand" in boom periods. Firms may reduce not only fixed and inventory investment (as desired), but also intended investment in customer stocks by charging higher prices.

6. Final comments

The present analysis has abstracted completely from the role of inventories for pricing decisions. Much recent research on pricing behavior has focused on this issue (see e.g. Blinder (1982)). Models with inventories also yield the conclusion that temporary demand shocks lead to smaller changes in price than permanent ones. However, there is at least one important difference between the conclusions obtained in this literature and those derived in the present paper: in the models with inventories, both prices and output responses become smaller as demand shocks become less persistent. In the model analysed here, the stickiness of the price with respect to temporary shocks makes the quantity response larger than if the demand shock had been perceived as permanent. This seems more in line with what we want to explain, namely, that prices change so little (relative to costs) in spite of substantial variations in output and employment.

A similarity between this model and the models with inventories is that it is a real and partial model of pricing behavior in the goods market. Therefore, it does not, in itself, imply nominal price stickiness (c.f. Blinder 1982, p. 346). Still, it may be used as one element in a macroeconomic analysis of the determination of nominal prices and wages.

Finally, as mentioned in the introduction, many models of price dynamics are based on the assumption that price changes are costly. The difference
between such models and the one that is analysed in this paper is, in some ways, parallel to the difference, in consumption theory, between the relative income hypothesis, on the one hand, and permanent income, or life cycle theories, on the other. The former ones emphasize that adjustment to new conditions is costly and slow and therefore the previous level of prices/consumption affects the choice in a given period. In the model in this paper, as in the permanent income/life cycle theory of consumption, history plays a role for current pricing decisions - not because price changes are costly, but because history affects expectations about the future.
APPENDIX A: Derivatives of \( g(-) \) and \( f(-) \).

Derivatives of \( g( x_{t-1}, x_t, \bar{p}_t ) \) at the stationary solution are:

\[
\begin{align*}
g_1 &= h^{-1} x_t x_{t-1}^{-2} , & g_2 &= h^{-1} x_{t-1} , & g_3 &= 1 \\
g_{11} &= -2 h^{-1} x_t x_{t-1}^{-3} , & g_{12} &= h^{-1} x_{t-1}^{-2} , & g_{22} &= g_{31} = g_{32} = g_{33} = 0
\end{align*}
\]

First order derivatives of \( f( x_{t-1}, x_t, \bar{p}_t, y_t, w_t ) \) at the stationary solution are:

\[
\begin{align*}
f_1 &= U_1( - ) \left[ q + ( g - F_1 w_t ) q_1 \right] g_1 x_t y_t \\
f_2 &= U_1( - ) \left[ \left[ q + ( g - F_1 w_t ) q_1 \right] g_2 x_t + ( g - F_1 w ) q \right] y_t
\end{align*}
\]

Note that in the stationary solution \( g_1 = -g_2 \) and \( f_2 + df_1 = 0 \) which implies

\[
(1 - d) \left( q + ( g - F_1 w ) q_1 \right) g_2 x = - ( g - F_1 w ) q
\]

so that \( q + ( g - F_1 w ) q_1 > 0 \) and, therefore, \( f_1 > 0 \) and \( f_2 < 0 \).

Define \( K_1 = q + ( p - F_1 w ) q_1 > 0 \) and \( K_2 = (2 - F_{11} x q_1 y w ) q_1 < 0 \)

and choose a normalisation so that \( x = y = w = q \) (p) = 1 at the stationary solution, then

\[
\begin{align*}
f_{11} &= U_{11} \frac{f_1^2}{U_1^2} + U_1 \left( g_{11} K_1 + g_{12} K_2 \right) < 0 \\
f_{12} &= U_{11} \frac{f_2 f_1}{U_1^2} + U_1 \left( g_{12} K_1 + g_1 K_1 + g_2 K_2 - g_1 F_{11} q_1 \right) > 0
\end{align*}
\]
\[ f_{13} = \frac{U_{11} f_3 f_1}{u_2} + U_1 K_2 g_1 < 0 \]

\[ f_{14} = \frac{U_{11} f_4 f_1}{u_2} + \frac{f_1}{y} - U_1 g_1 F_{11} q_1 \geq 0? \]

Notice that the stationary solution is independent of the utility function so the other terms in the expression are independent of the value of \( U_{11} \). If \( U_{11} \) is large in absolute value \( f_{14} \) is negative, while the opposite holds if \( U_{11} \) is close to zero.

\[ f_{15} = \frac{U_{11} f_5 f_1}{u_2} - U_1 g_1 F_{11} q_1 > 0 \]

\[ f_{21} = \frac{U_{11} f_1 f_2}{u_2} + U_1 \left[ g_{21} K_1 + g_2 g_1 K_2 + g_1 g K_1 - F_{11} q_1 g_1 \right] > 0 \]

\[ f_{22} = \frac{U_{11} f_2^2}{u_1} + U_1 \left[ 2g_2 K_1 + g_2^2 K_2 - 2g_2 F_{11} q_1 - F_{11} \right] < 0 \]

\[ f_{23} = \frac{U_{11} f_3 f_2}{u_2} + U_1 \left( g_2 K_2^2 + K_1 - F_{11} q_1 \right) > 0 \]

\[ f_{24} = \frac{U_{11} f_4 f_2}{u_2} + U_1 \left( - F_{11} \left( q_1 g_2 + q \right) + f_2 \right) \geq 0? \]

If \( U_{11} \) is large in absolute value, \( f_{24} \) is positive and converseley.

\[ f_{25} = \frac{U_{11} f_5 f_2}{u_2} - U_1 F_1 \left( g_2 q_1 + q \right) < 0 \]
APPENDIX B:

Assume that \( \bar{p}_t \), \( y_t \) and \( w_t \) are known to be constant \( (p^0, y^0, w^0) \) for all future periods and that the stationary solution for the customer stock is then \( x^0 \). A quadratic approximation to \( f(-) \) at this point is

\[
\begin{align*}
\phi(x_{t-1}', x_t', \bar{p}_t', y_t', w_t') & \\
= f(x^0, x^0, p^0, y^0, w^0) & \\
+ f_1 x_{t-1}' + f_2 x_t' + f_3 \bar{p}_t' + f_4 y_t' + f_5 w_t' & \\
+ \frac{1}{2} (x_{t-1}', x_t', \bar{p}_t', y_t', w_t') Q (x_{t-1}', x_t', \bar{p}_t', y_t', w_t')^T
\end{align*}
\]

where \( x_t' = x_t - x^0 \) and similarly for other variables, \( Q \) is the matrix of second order derivatives and \( T \) denotes the transpose. All derivatives are evaluated at the stationary solution. Now, replace \( f(-) \) by \( \phi(-) \) in the objective function in (6). Since the objective function is now quadratic the certainty equivalence principle (Bertsekas 1976, Chapter 3.1) implies that we can replace the uncertain future values of variables with their expectations. Differentiating with respect to \( x_{t+j} \) one gets a first order condition (an Euler equation) which must hold for each \( j \geq 0 \):

\[
\begin{align*}
(\text{C3}) & \\
f_2 + f_{21} t x_{t+j-1}' + f_{22} t x_{t+j}' + f_{23} t \bar{p}_{t+j}' + f_{24} t y_{t+j}' + \\
& + f_{25} t w_{t+j}' + \delta (f_1 + f_{11} t x_{t+j}' + f_{12} t x_{t+j+1}' + f_{13} t \bar{p}_{t+j+1}' \\
& f_{14} t y_{t+j+1}' + f_{15} t w_{t+j+1}') = 0
\end{align*}
\]

where e.g. \( y_{t+j}' \) denotes the expectation at \( t \) about \( y_{t+j}' \). Since the first order condition at the stationary solution is \( f_2 + \delta f_1 = 0 \) this can be rewritten
\[(C4) \quad (1 + \frac{c}{\delta} L + \frac{1}{\delta} L^2) x_t^{t+j+1} =
\]
\[- \frac{1}{\delta f_{12}} \left[ (f_{23} + \delta f_{13} L^{-1}) p_t^{t+j} + (f_{24} + \delta f_{14} L^{-1}) v_t^{t+j} + (f_{25} + \delta f_{15} L^{-1}) w_t^{t+j} \right] \]

where
\[(C5) \quad c = \frac{f_{22} + \delta f_{11}}{f_{12}} = -(1 + \delta) \frac{1}{f_{12}} \left[ (1 - \delta) h^{-1} F_{11}q_t - F_{11} \right] < -(1 + \delta) \]

The methods used to solve the Euler equation are described in Sargent (1979 pp. 171-177, 195-200). The polynomial in the lag operator on the left hand side in (C4) can be factorized: \(1 + (c/\delta) L + (1/\delta) L^2 = (1 - v_1 L) (1 - v_2 L)\) where \(0 < v_1 < 1\) and \(1/\delta < v_2\) (c.f. Sargent 1979, pp. 197-98).

Operating on both sides of (C4) with the forward inverse of \((1 - v_2 L)\) we get
\[(C6) \quad (1 - v_1 L) x_t^{t+j+1} =
\]
\[- \frac{v_2^{-1} L^{-1}}{\delta f_{12}} \left[ (f_{23} + \delta f_{13} L^{-1}) p_t^{t+j} \right.
\]
\[\left. + (f_{24} + \delta f_{14} L^{-1}) v_t^{t+j} + (f_{25} + \delta f_{15} L^{-1}) w_t^{t+j} \right] + C v_t^t \]

where \(C\) is set to zero to satisfy the transversality condition. Expanding this forward we get a decision rule that expresses the planned \(x_{t+j+1}\) (\(j = 0, 1, 2, \ldots\)) as a function of expected future market price, demand and costs.

Substituting the solution for \(x_{t+j+1}\) into the Euler equation for \(j = 0\) one finds that (C6) holds for \(j = -1\) as well. Thus the decision rule for the customer stock \((6)\) has been derived.
REFERENCES


