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WELFARE EFFECTS OF ALTERNATIVE FORMS OF
PUBLIC SPENDING

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Assar Lindbeck and Jörgen W. Weibull*

Abstract: The paper analyzes welfare effects, in a second-best world, of public spending of three different types: transfer payments, public consumption and public investment. It is shown how these effects depend not only on the type of spending but also on the initial level of spending, as well as on anticipated consequences for future taxes and transfer payments. The analysis is performed in an overlapping generations model with (i) individuals who in each period endogenously allocate their time between (taxed) market work and (tax free) home production, (ii) a publicly provided good ("public consumption") that enters as a second factor in home production, and (iii) productivity-enhancing public investment in the private (market) production sector. The consequences for the current account of the balance of payments of the different spending alternatives are also studied.

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1. Introduction

This paper is concerned with the welfare effects on representative households in two consecutive generations of three types of debt-financed public spending - transfer payments, public consumption and public investment. The analysis, performed within the framework of a two-period overlapping generations model with a constant population, covers both the case when the incurred public debt is repaid within the life time of the currently young generation, and when it is passed on to future generations.

In contrast to other overlapping generations models, individuals allocate their time in each period not between work and leisure, but between (tax free) home production and (taxed) working time supplied in the labour market. Another characteristic feature of the present model is that a publicly (and freely) provided good, "public consumption", enters as a second factor in home production; indeed this is how public consumption is modelled. The public sector also makes productivity-enhancing investment in the private (market) production sector (also free of charge). Since individuals supply labour in both periods of their lives, such investment has an effect on the wage earnings not only of the next generation, but also on those of the currently young generation.

Disregarding the overlapping-generations structure, the household sector of the present model has some resemblance with the household production models of Becker (1965) and Lancaster (1966), though these lack public consumption. However, our approach differs from theirs mainly in that here the household produces a good which is a perfect substitute to the one available in the market - rather than assuming, as do Lancaster and Becker, that households produce a collection of "utility-enhancing attributes". A closer analogue to our household model can be found in Gronau (1977), the main differences being that in his analysis labour is the only input in home-production, and "pure leisure" is a third use of

time. Our rationale for excluding "pure leisure" is to highlight that part of "non-working" time which is devoted to the production of goods and services that are substitutes to market goods. We believe that this aspect of "non-working" time is particularly relevant for evaluations of government policies.²

The present model has also some similarity with the macro models of Barro (1981) and Aschauer and Greenwood (1985), where the utility and productivity enhancing effects of public spending on goods and services are explicitly considered. However, in their models no household production takes place.

The formal model structure is presented in Section 2. A first analysis of the welfare effects of the three types of public spending is made in Section 3, which focuses on the simplest case of "pure" debt financing (i.e. with next-period transfers and tax rates being fixed). Section 4 treats situations in which debt is repaid by way of a reduction in the next-period transfer payment to the currently young generation. Section 5 considers debt repayment by means of an increase in the next-period tax rate on wage earnings. These two forms of debt repayment represent polar cases in the sense that the first is non-distortionary and born by the first generation alone, while the second is distortionary but shared with the succeeding generation. Derivations of relevant equations are relegated to an appendix at the end of the paper.

The whole analysis refers to a second-best situation; there are distortionary taxes on both saving and labour supply already at the outset. As will be seen, the mere presence of such taxes - even when they are not varied - has important qualitative implications for the effects of various policy experiments. However, due to the complexities already introduced, we have abstained from many other relevant complications. In particular, private inheritance between generations is excluded, the

length of life is assumed to be known with certainty, and we restrict the analysis to a small open economy, exposed to a fixed world market interest rate.

2. The model

The model consists of three aggregate sectors: households, firms and a public sector. There is only one type of output, a composite good which can be used both for consumption and as capital in production.

The *representative household* of generation t lives in periods t and $t+1$, and maximizes an intertemporally separable utility function with current and future consumption, $c^t = (c_t^t, c_{t+1}^t)$, as arguments: $u^t(c^t) = u_1(c_t^t) + u_2(c_{t+1}^t)$. (Superscripts refer to generations and subscripts to time periods.) Every household is endowed with one unit of labour in each period, and this unit can be used either for home production or sold at the going wage in the labour market. From the viewpoint of consumption, output from home production is assumed to be a perfect substitute for the market good:

$$(2.1a) \quad c_t^t = x_t^t + g(e_t^t, G_t^t)$$

$$(2.1b) \quad c_{t+1}^t = x_{t+1}^t + h(e_{t+1}^t, G_{t+1}^t),$$

where $x^t = (x_t^t, x_{t+1}^t)$ denotes purchases in the market, and g and h are the home production functions. Home production uses *home labour* $e^t = (e_t^t, e_{t+1}^t)$ and the publicly provided good $G^t = (G_t^t, G_{t+1}^t)$ as input factors. (We hence neglect private capital in home production.) The publicly provided good could be either a collective or a private good, though the subsequent analysis is focused on the latter case, since this seems to be the predominant type of public consumption today.¹ We think about, for instance, child-care or the care of disabled or old relatives -

services that can be provided by various combinations of family-labour and public institutions. Both the publicly provided good and the output from home production are assumed to be non-marketable. We assume that the home-production functions g and h are continuously differentiable with positive marginal returns to both factors and a decreasing marginal return to labour. The publicly provided good G_t^t will be called a technological substitute (complement) to home labour if $g_{eG} < 0$ ($g_{eG} > 0$).²

Assuming a perfect capital market and using the current product-market good as numeraire, the intertemporal budget constraint for the household may be expressed in terms of its life-time wealth ω^t (a two-period analogue of Becker's (1965) "full-time income"), and the (intertemporal) price p_{t+1} of the next-period market good:

$$(2.2) \quad x_t^t + \bar{w}_t e_t^t + (x_{t+1}^t + \bar{w}_{t+1} e_{t+1}^t) p_{t+1} = \omega^t,$$

where

$$(2.3) \quad \omega^t = \bar{w}_t + T_t^t + [\bar{w}_{t+1} + T_{t+1}^t] p_{t+1}$$

$$(2.4) \quad p_{t+1} = 1/(1+\bar{r}_{t+1}).$$

Here $\tau_t^W \in (0,1)$ denotes the tax-rate on (market) labour-income in period t , and $\bar{w}_t = (1-\tau_t^W)w_t$ the net real wage-rate in the same period.

Likewise, $\tau_t^R \in (0,1)$ is the tax-rate on capital income in period t , and $\bar{r}_{t+1} = (1-\tau_{t+1}^R)r_{t+1}$ the net real interest rate from period t to period $t+1$. Moreover, $T_t^t > 0$ and $T_{t+1}^t > 0$ are net lump-sum transfers to each member of generation t in periods t and $t+1$, respectively. The publicly provided good is given free of charge, and hence does not enter the budget equation (2.2). We assume *perfect foresight* in the sense that the next-period tax rates τ_{t+1}^W and τ_{t+1}^R and transfer T_{t+1}^t are correctly anticipated by generation t already in period t .

The representative household is supposed to maximize utility $u^t(c^t)$ subject to the life-time budget constraint (2.2) (for $c^t \geq 0$ and $0 \leq e^t \leq 1$). We

~~assume the utility function to be continuously differentiable with~~

positive and decreasing marginal utilities. Hence, at positive prices and wealth, there exists a unique solution to the household's decision problem - a solution which is a function of private wealth (w^t), prices ($\bar{w}_t, \bar{w}_{t+1}, p_{t+1}$) and the supply of the publicly provided good (G_t^t, G_{t+1}^t).

Intertemporal separability and perfect substitutability between the home-made good and the market good together imply the following first-order conditions for an interior solution:

$$(2.5a) \quad g_e(e_t^t, G_t^t) = \bar{w}_t$$

$$(2.5b) \quad h_e(e_{t+1}^t, G_{t+1}^t) = \bar{w}_{t+1}.$$

In other words, the households' optimal allocation of time between home production and market work in each period is dictated solely by the efficiency condition that *marginal returns should be equal* - the marginal return to market work simply being the net wage rate, and the marginal return to home production being dependent not only on home labour, but also on the provision of the public good. If the latter is a technological substitute (complement), then the marginal conditions above tell us that household work declines (rises) as the good is more amply provided.³ As a real-world illustration: an expansion of public day-care services for children would reduce the input of time in home-production of such services, and hence raise labour supply in the market. Moreover, conditions (2.5 a,b) imply that market labour supply in any period is independent both of private wealth and of the wage rate in the other period. (These special features would of course disappear if "pure leisure" was introduced as a third use of time.) We write *aggregate (market) labour supply* in any period t as $L_t = L_t^{t-1} + L_t^t$, where $L_t^{t-1} = 1 - e_t^{t-1}$ and $L_t^t = 1 - e_t^t$.

Formally, the present household model has the same general structure as the traditional trade model, with a "domestic" production sector for a good that may alternatively be bought "from the outside" at a given price; albeit with labour mobility in our model. For the households' consumption-labour decision can be partitioned into a two-step production-consumption algorithm: first it determines how to allocate its (productive) time between market production and home production, on the basis of wage rates and the provision of the publicly provided good only, then it decides how much to buy in the product market and how to divide consumption between the two periods of life.

The production sector employs two factors, labour and private capital, both of which are augmented (in the sense of Harrod's concept of "factor augmentation") by public investment - typical examples being professional education, R&D and infrastructure. Both private capital and public investment are, without essential consequences for the analysis, assumed to have a life-time of one period. Labour and private capital services are bought at (real) prices w and r , respectively, while public investment, like public consumption, is provided free of charge. Production is assumed to exhibit constant returns-to-scale with respect to augmented labour and capital inputs. Hence, with I_t for public investment in period t , and $a(I_t)$ and $b(I_t)$ denoting the "multipliers" by which capital and labour are augmented, gross output in period $t+1$ is written $F(aK_{t+1}, bL_{t+1})$, where F is homogenous of degree one. We assume the functions a and b to be differentiable and non-decreasing. It can be shown that, even in the presence of such factor augmentation, labour and (private) capital expenditures together exhaust net output at profit maximum: $F(aK_{t+1}, bL_{t+1}) - K_{t+1} = r_{t+1}K_{t+1} + w_{t+1}L_{t+1}$. Hence, as is usual in constant returns-to-scale models, we set aggregate employment, L_{t+1} , equal to aggregate labour supply.⁴

Denoting augmented variables by $*$, we make the conventional assumptions that the function f , defined by $f(k^*) = F(K^*, L^*)/L^*$, is twice continuously differentiable with $f' > 0$, $f'' < 0$, and f' ranging from plus infinity to zero as the effective capital intensity $k^* = K^*/L^*$ increases from zero to infinity. The first-order conditions for profit maximum requires factor prices to lie on the frontier ϕ , in the present model defined by

$$(2.6) \quad w_{t+1} = \phi(r_{t+1}, I_t) \quad \text{iff for some } k^*: \quad r_{t+1} = af'(k^*) - 1 \quad \text{and} \\ w_{t+1} = b[f(k^*) - k^*f'(k^*)] ,$$

where r_{t+1} is exogenously given by the world capital market. The corresponding demand schedule for private capital (intensity), Ψ , is defined by the marginal productivity condition above: $k_{t+1} = \Psi(r_{t+1}, I_t)$ iff $af'(k_{t+1}^*) = 1 + r_{t+1}$, where $k_{t+1} = K_{t+1}/L_{t+1}$.

Both schedules ϕ and Ψ are decreasing in r_{t+1} , and they shift with public investment as follows:

$$(2.7a) \quad \phi_I = (f - k^*f')b' + ka'f'$$

$$(2.7b) \quad \Psi_I = (b'/b - a'/a)k - (a'b/a^2)f'/f''$$

(with f and its derivatives evaluated at k^*). It follows that $\phi_I > 0$ by concavity of f , while the sign of Ψ_I is indeterminate. In other words: at a fixed interest rate, public investment unambiguously raises the real wage rate (in the next period), while its effect on private capital formation is ambiguous. The unambiguous effect on the wage rate follows directly from the definition of the factor price frontier. For as seen in (2.6), public investment induces an upward shift of the whole factor price

frontier. By the small country assumption, the interest rate is fixed, so the wage rate must rise.

As for the ambiguous effect on private capital formation, first note that if public investment is purely labour-augmenting ($a'=0 < b'$), then, as expected, the private capital intensity necessarily increases, see eq.(2.7b). If public investment instead is purely capital-augmenting ($a' > 0 = b'$), then the same equation tells us that the private capital intensity falls if and only if $f' + k^* f'' < 0$, i.e. iff f is sufficiently concave at the current effective capital intensity. This inequality is equivalent to the condition that the elasticity of the marginal product with respect to the effective capital intensity, $-k^* f''/f'$, be greater than unity.

The reason for this ambiguity can be seen directly in eq.(2.6): on the one hand capital-augmenting public investment induces a rise in the effective capital intensity k^* , in order to maintain the equality between the augmented marginal product (af') and the interest rate factor $(1+r)$. However, at a given private capital intensity, public investment itself directly raises this intensity (as $k^* = ak/b$). If this "automatic" rise is insufficient for preservation of the efficiency condition (2.6), then also the private capital intensity, k , has to increase, while in the opposite case it has to decline. We will say that public investment is *effectively* a *complement* to (substitute for) private capital if $\Psi_I > 0$ ($\Psi_I < 0$).

The *public sector*, finally, in addition to supplying public consumption and making real investment, collects taxes from labour and capital incomes, pays lump-sum transfers, and borrows (or lends) in the capital market. Its budget equation in period t can hence be written

$$(2.8) \quad D_t = D_{t-1} + r_t D_{t-1} + T_t^{t-1} + T_t^t + G_t^t + I_t - \tau_t^w w_t L_t - \tau_t^r r_t s_{t-1}^t,$$

where D_t is public debt and $D_t - D_{t-1}$ the current budget deficit, while

$$(2.9) \quad s_{t-1}^{t-1} = \bar{w}_{t-1}(1-e_{t-1}^{t-1}) + T_{t-1}^{t-1} - x_{t-1}^{t-1}$$

is the households' supply in the capital market, on which they earn taxable interest in period t . (Foreigners' capital incomes are not taxed in the country under consideration.) The taxation of labour and capital income makes tax revenues endogenous and creates tax wedges. Finally note that the public sector does not produce G_t^t or I_t itself; it buys these quantities in the product market, transforms them without costs and provides them free of charge to the households and firms, respectively.

3. Pure debt financing

Let us first study the special case when transfers and tax rates during the life span of generation t are fixed - "pure" debt financing. The three policy experiments considered here are: $dT_t^t > 0$, $dG_t^t > 0$ and $dI_t > 0$, respectively. Note that, by the small open economy assumption, wage rates are unaffected in the first two experiments, while in the third experiment $dw_{t+1} = \phi_I dI_t > 0$.

It turns out that the welfare consequences of these three experiments are quite clear-cut if one assumes, as we do, an interior solution to the households' optimization program, along with differentiability of the indirect utility function. By the Envelope Theorem:

$$(3.1a) \quad dv^t/dT_t^t = u_1'$$

$$(3.1b) \quad dv^t/dG_t^t = g_G u_1'$$

$$(3.1c) \quad dv^t/dI_t = p_{t+1}(1-\tau_{t+1}^w)\phi_{I L_{t+1}}^t u_1' ,$$

all welfare effects being positive. The first two effects are straightforward: if the public good is provided at the "bench-mark" level where

its marginal productivity is unity, then the welfare effects are the same for transfers and public goods, while if the public good is under-supplied (over-supplied), in the sense $g_G > 1$ (< 1), then welfare rises more (less) in the second experiment than in the first.⁵ The welfare effect of public investment hinges on four factors: the intertemporal price, the wage-tax rate, (initial) labour supply of generation t , and the wage-raising effect of public investment. The product of these factors is the present value of the induced rise in after-tax wage earnings of generation t .

Still neglecting repayment of public debt, the ranking of the three experiments from the viewpoint of *generation $t+1$* is also simple: no welfare effect of increased transfers and public consumption, and a positive welfare effect of public investment via the rising wage rate in period $t+1$. However, the conditions for the second generation are also *potentially* influenced by the extent to which it has to contribute to the repayment of this debt - an issue which will be discussed to some extent below, and which we have analyzed in some detail in a previous paper (Lindbeck and Weibull (1985)). (If there were also private inheritance in the model, then the next generation could be compensated, more or less, for "inheriting" public debt by way of a larger bequest.)

The next step in our analysis is to examine the welfare effects of the three types of public spending when this spending is anticipated to induce shifts in taxes and transfers during the life time of generation t . Two types of such shifts will be examined: a reduced lump-sum transfer for generation t in period $t+1$, and an increase in the labour-income tax rate in the same period, respectively. Before embarking on this analysis, however, let us first briefly consider the effects of *isolated* shifts in these two policy parameters.

By the budget equation (2.2) and the Envelope Theorem, the welfare loss for generation t of an isolated infinitesimal shift $dT_{t+1}^t < 0$ is

$-u'(c_{t+1}^t) dT_{t+1}^t$; a non-distortionary form of financing, fully born by generation t . Likewise, the welfare loss for the same generation of a tax rise $d\tau_{t+1}^w > 0$ is $w_{t+1}^t L_{t+1}^t u_1'(c_{t+1}^t) d\tau_{t+1}^w$. While domestic private supply in the capital market is affected by both types of financing, labour supply is influenced only in the second case, since the households' allocation of time between home production and market work is independent of private wealth.

4. Anticipated future tax financing: "reduced pension"

We are now in a position to analyze the welfare effects of the three types of public spending when the next-period transfer T_{t+1}^t to generation t is reduced by the amount needed to precisely offset the induced effect on the public debt in period $t+1$. More exactly, we require the pension cut dT_{t+1}^t in each of the three experiments to be such that the corresponding total differential dD_{t+1} be zero. It is readily verified that this amounts to

$$(4.1a) \quad dT_{t+1}^t / dT_t^t = -\alpha_T = -(\partial D_{t+1} / \partial T_t^t) / (\partial D_{t+1} / \partial T_{t+1}^t)$$

$$(4.1b) \quad dT_{t+1}^t / dG_t^t = -\alpha_G = -(\partial D_{t+1} / \partial G_t^t) / (\partial D_{t+1} / \partial T_{t+1}^t)$$

$$(4.1c) \quad dT_{t+1}^t / dI_t = -\alpha_I = -(\partial D_{t+1} / \partial I_t) / (\partial D_{t+1} / \partial T_{t+1}^t).$$

The intuition behind these definitions is straight-forward. The quantities $\partial D_{t+1} / \partial T_t^t$, $\partial D_{t+1} / \partial G_t^t$ and $\partial D_{t+1} / \partial I_t$ are the "multipliers" by which (marginal) current public spending affects public debt in the following period. Likewise, $\partial D_{t+1} / \partial T_{t+1}^t$ is the multiplier by which a (marginal) shift in the pension to generation t affects the same debt position. (We hence view D_{t+1} as a reduced-form function of these policy parameters.) Accordingly, eqs.(4.1a-c) simply state that the "pension cut" needed to offset the effect on public debt equals the amount spent

multiplied by the ratio between the debt multiplier associated with the spending and the debt multiplier associated with the pension cut.

The welfare effects of the three policy experiments are obtained as the difference between the welfare gain from the spending in question and the welfare loss due to the accompanying lump-sum tax, or, in other words, as the difference between marginal benefits and marginal costs of public funds, for generation t :

$$(4.2a) \quad dv^t/dT_t^t = (1-p_{t+1}\alpha_T)u_1'$$

$$(4.2b) \quad dv^t/dG_t^t = (g_G - p_{t+1}\alpha_G)u_1'$$

$$(4.2c) \quad dv^t/dI_t^t = p_{t+1}[(1-\tau_{t+1}^w)\phi_{IL_{t+1}}^t - \alpha_I]u_1'.$$

It can easily be shown that the welfare effect of a transfer vanishes when financed by a future lump-sum tax:

$$(4.3) \quad dv^t/dT_t^t = 0.$$

In the absence of capital-income taxation one would intuitively expect this "debt neutrality" to hold, since the reduction in transfers is non-distortionary, and by assumption the capital market is perfect and the households' original solution interior. Eq.(4.3) tells us that this neutrality still holds when capital incomes are taxed.

To clarify further the effects of increased public consumption, eq. (4.2b) may be developed to yield

$$(4.4) \quad dv^t/dG_t^t = \frac{(1+r_{t+1})[g_G - 1 - \tau_{t+1}^w \partial e_t^t / \partial G_t^t] - \gamma}{1 + r_{t+1} + \tau_{t+1}^r r_{t+1} (\partial x_t^t / \partial \omega^{t-1})} u_1',$$

where γ reflects the effect on the next-period transfer via tax revenues from the induced shift in private saving:

$$(4.5) \quad \gamma = \tau_{t+1}^r r_{t+1} [(\partial x_t^t / \partial \omega^t - 1) g_G + \partial x_t^t / \partial G_t^t + \bar{w}_t \partial e_t^t / \partial G_t^t].$$

In a non-distorted situation (i.e. where $\tau_t^w = \tau_{t+1}^r = 0$), welfare improves if and only if the public good is under-supplied at the outset: then $\gamma = 0$ and $dv^t/dG_t^t > 0$ iff $g_G > 1$, by eq.(4.4). If only labour incomes are taxed (i.e. $\tau_t^w > 0 = \tau_{t+1}^r$), then still $\gamma = 0$, so by the same equation $dv^t/dG_t^t > 0$ iff $g_G > 1 + \tau_t^w w_t \partial e_t^t / \partial G_t^t$. The latter inequality clearly holds whenever G_t^t is an under-supplied ($g_G > 1$) substitute for home labour, while the reversed inequality holds when it is an over-supplied ($g_G < 1$) complement or a neutral good.

Hence, when the public good is a substitute, which we believe it typically is, there is a case for increasing public spending above the bench-mark level at which its marginal productivity, g_G , is one. In fact, Pareto efficiency requires g_G to equal $1 + \tau_t^w w_t \partial e_t^t / \partial G_t^t$, since otherwise G_t^t and T_{t+1}^t can be adjusted so as to improve the welfare of generation t without affecting the public debt left for future generations.

The reason for the positive welfare effect of increased public consumption at, or somewhat above, the bench-mark level $g_G = 1$ (still assuming G_t^t to be a substitute and $\tau_t^w > 0 = \tau_{t+1}^r$) is that the households' allocation of labour, dictated by the marginal efficiency condition (2.5a), is distorted by the tax wedge on labour income. For, at the outset, the marginal social product is higher in the market sector than in the household sector. Thus, a government action which induces labour to move into the former sector can be welfare improving, provided the increased social return to labour at least to some extent accrues to the individual. This is certainly the case in the present policy experiment, in which the next-period transfer is tied to the development of the tax base via the government budget equation. Indeed, due to

increased tax revenues from labour moving "out" to the market, the lump-sum tax needed to repay increased public consumption is lower than the corresponding lump-sum tax in the case of a transfer. (Had there also been a negative income effect on labour supply from transfers, as in a more general household model, then transfers would be even less favourable.) More generally, with an initial distortion in favour of home production, welfare can be improved by subsidizing substitutes for home production - either market goods or goods provided by the government.

Concerning the welfare effect of increased public consumption in the presence of capital-income taxation, finally, it can be shown that in any (second-best) situation where $\tau_t^w > 0$ and $\tau_{t+1}^r > 0$:

$$(4.6) \quad dv^t/dG_t^t > 0 \quad \text{if } G_t^t \text{ substitute and } g_G > 1$$

$$< 0 \quad \text{if } G_t^t \text{ complement and } g_G \leq 1.$$

For instance, assume that G_t^t is an under-supplied substitute ($g_G > 1$).

Then, in addition to the positive welfare effect discussed above, there will be another positive such effect via private saving: tax revenues from capital income can be shown to rise more than in the case of a transfer (i.e. $\gamma > 0$), so the required lump-sum tax is even lower.

Turning to public investment, we note

$$(4.7) \quad dv^t/dI_t = \frac{[(1-\tau_{t+1}^w)L_{t+1}^t/L_{t+1} + (1+\varepsilon)\tau_{t+1}^w]\phi_I L_{t+1} - (1+r_{t+1})}{1 + r_{t+1} + \tau_{t+1}^r r_{t+1} (\partial x_t^t / \partial \omega^t - 1)} u_1'.$$

Here ε denotes the (uncompensated) elasticity of aggregate labour supply in period $t+1$ with respect to the net wage rate in the same period, i.e. $\varepsilon = \bar{w}_{t+1} (dL_{t+1}/d\bar{w}_{t+1})/L_{t+1}$. By eqs. (2.5a,b), this elasticity is nonnegative. As expected, the welfare effect on generation t of public investment is more favourable (or less unfavourable) the higher is the marginal productivity effect ϕ_I accruing to labour of such an investment.

This wage-raising effect acts via two channels. It increases both the after-tax wage earnings of the generation (the first term in square brackets in eq.(4.7)), and tax revenues from aggregate labour supply in period $t+1$ (the second term in square brackets). Due to increased tax revenues, the required pension cut becomes smaller and hence welfare higher. These two positive effects, materializing in period $t+1$, are counteracted by the public sector's costs of temporarily borrowing the amount needed for the investment in the international capital market; this is the term $1+r_{t+1}$ in the numerator. The latter cost can be meaningfully compared with $\phi_I L_{t+1}$, the direct rise in national income due to the investment itself. In the bench-mark case when these two quantities are equal, i.e. when $\phi_I L_{t+1} = 1+r_{t+1}$, eq.(4.7) implies

$$(4.8) \quad dv^t/dI_t > 0 \quad \text{iff} \quad (1-\tau_{t+1}^W)L_{t+1}^t/L_{t+1} + (1+\varepsilon)\tau_{t+1}^W > 1.$$

Hence, in this bench-mark case, public investment, financed by a lump-sum tax on generation t alone, can improve welfare only when there is an initial tax wedge that keeps down labour supply. In fact, (4.8) implies $dv^t/dI_t < 0$ whenever $\tau_{t+1}^W = 0$, while, at any given elasticity $\varepsilon > 0$, $dv^t/dI_t > 0$ for τ_{t+1}^W sufficiently close to one. Interestingly enough, this qualitative conclusion is true even when generation t has no share at all in aggregate labour supply, and hence does not benefit directly from the investment. For (4.8) tells us that in this limiting case, welfare increases if and only if $\tau_{t+1}^W > 1/(1+\varepsilon)$, i.e. iff the tax rate on labour income is sufficiently high at the outset.⁶ The explanation must then be sought in the accompanying shift in the next-period transfer T_{t+1}^t : in fact, when $\tau_{t+1}^W > 1/(1+\varepsilon)$, then $\partial D_{t+1}/dI_t < 0$, and hence $dT_{t+1}^t > 0$. In such situations, the initial distortion of the allocation of labour is so strong that the mitigating effect of increased public investment induces

tax revenues from wage earnings to rise more than the incurred capital cost of the investment.

As in the case with unchanged taxes and transfers (Section 3), the welfare of generation $t+1$ is unaffected in the first two experiments, while it is favored in the third experiment by way of a higher wage rate in period $t+1$. Moreover, as compared to the "pure debt financing" case, generation $t+1$ is potentially favoured by not having to take over increased public debt. (The situation would have been more complicated if the model had also included private inheritance.)

5. Anticipated future tax financing: "wage tax rise"

Suppose instead that the induced public debt is repaid in period $t+1$ by means of an increase in the tax-rate τ_{t+1}^W on labour income in that period.⁷ This type of financing differs from the preceding one in two respects: it is distortionary, and it is shared with the members of the next generation. By the same argument as in the case of lump-sum taxation, the next-period balance condition $dD_{t+1}=0$ requires:

$$(5.1a) \quad d\tau_{t+1}^W/dT_t^t = \beta_T = -(\partial D_{t+1}/\partial T_t^t)/(\lambda D_{t+1}/\partial \tau_{t+1}^W)$$

$$(5.1b) \quad d\tau_{t+1}^W/dG_t^t = \beta_G = -(\partial D_{t+1}/\partial G_t^t)/(\partial D_{t+1}/\partial \tau_{t+1}^W)$$

$$(5.1c) \quad d\tau_{t+1}^W/dI_t = \beta_I = -(\partial D_{t+1}/\partial I_t)/(\partial D_{t+1}/\partial \tau_{t+1}^W) .$$

The marginal benefits for generation t of various types of public spending have now to be reduced by the marginal welfare costs of the corresponding next-period tax rise:

$$(5.2a) \quad dv^t/dT_t^t = [1 - w_{t+1}p_{t+1}L_{t+1}^t\beta_T]u_1'$$

$$(5.2b) \quad dv^t/dG_t^t = [g_G - w_{t+1}p_{t+1}L_{t+1}^t\beta_G]u_1'$$

$$(5.2c) \quad dv^t/dI_t = p_{t+1}L_{t+1}^t[(1-\tau_{t+1}^W)\phi_I - w_{t+1}\beta_I]u_1'.$$

Of crucial importance for subsequent comparisons is, as we shall see, the dependency of public debt in period $t+1$ on the labour-income tax rate in the same period, *ceteris paribus*:

$$(5.3) \quad \partial D_{t+1} / \partial \tau_{t+1}^W = -w_{t+1} [(1-\mu)L_{t+1} + \tau_{t+1}^r r_{t+1} p_{t+1} L_{t+1}^t \partial x_t^t / \partial \omega^t],$$

where $\mu = \varepsilon \tau_{t+1}^W / (1 - \tau_{t+1}^W)$. Note that $1 - \mu$ can be identified as the elasticity of labour-income tax revenues with respect to the labour-income tax rate: $d(\tau_{t+1}^W w_{t+1} L_{t+1}) / d\tau_{t+1}^W = (1 - \mu) w_{t+1} L_{t+1}$. Hence, the parameter

- here accounts for the "tax-base drain" caused by the households' adjustment to the increased tax rate on labour income. Note in particular that the "debt multiplier" $\partial D_{t+1} / \partial \tau_{t+1}^W$ is an increasing function of the "tax-base drain" coefficient μ ; indeed, at a sufficiently high elasticity of labour supply ($\mu \gg 1$), this multiplier is positive, i.e. the "tax-revenue function" is downward sloping (expressing a "Laffer effect"). However, $\partial D_{t+1} / \partial \tau_{t+1}^W$ is negative, as one would expect, at "empirically reasonable" elasticities: with a tax rate on labour earnings of 40-50%, and an elasticity of labour supply in the range 0.1-0.5, one has approximately $0.07 < \mu < 0.5$ (cf. figures in various OECD publications for tax rates, and in Hansson and Stuart (1984) for labour supply elasticities).

Equipped with these preliminary observations, we are now in a position to analyze the welfare effects of the three policy experiments. In the case of transfers one can show that the total effect on the welfare of generation t (as defined in eq.(5.2)) is given by

$$(5.4) \quad dv^t / dT_t^t = \frac{1 - \mu - L_{t+1}^t / L_{t+1}}{1 - \mu + \tau_{t+1}^r r_{t+1} p_{t+1} (\partial x_t^t / \partial \omega^t) L_{t+1}^t / L_{t+1}} u_1'.$$

(Recall that the denominator is positive in the empirically relevant case of $0 < \mu < 1$.) Hence $0 < dv^t / dT_t^t < u_1'$, i.e. in contrast to the case of debt repayment by way of a "reduced pension", welfare increases, though

less, of course, than under "pure" debt repayment. As expected, the welfare effect for generation t is a decreasing function both of its share of aggregate labour supply in period $t+1$, and of the aggregate labour supply elasticity in that period. Moreover, the welfare effect is positive if and only if $\mu + L_{t+1}^t / L_{t+1} < 1$. Comparing with the earlier analyzed case of a non-distortionary tax financing born by generation t alone, we conclude that the benefit of *sharing* the tax burden with the next generation outweighs the cost of the *distortion* caused by the financing. Of course, the members of the next generation make the opposite ranking of forms of financing, since they are unaffected by lump-sum taxation of generation t , and negatively affected by an increased tax rate on wage earnings in period $t+1$.

In the case of increased public consumption, one can show

$$(5.5) \quad dv^t/dG_t^t = \frac{(1-\mu)g_G - \theta L_{t+1}^t / L_{t+1}}{1 - \mu + \tau_{t+1}^r r_{t+1} p_{t+1} (\partial x_t^t / \partial \omega^t) L_{t+1}^t / L_{t+1}} u_1',$$

where

$$(5.6) \quad \theta = 1 + p_{t+1} (1+r_{t+1}) \tau_{t+1}^w w_t \partial e_t^t / \partial G_t^t - \tau_{t+1}^r r_{t+1} p_{t+1} [g_G \partial x_t^t / \partial \omega^t - 1 - \partial x_t^t / \partial G_t^t - \bar{w}_t \partial e_t^t / \partial G_t^t].$$

Let us first consider the bench-mark case $g_G=1$. Just as in the corresponding case of eq.(4.4), it can be shown that the private saving term in eq.(5.6) then vanishes, so $\theta < 1$ if G_t^t is a substitute to home labour, and $\theta > 1$ if it is a complement. Thus, spending on public consumption then is more (less) favourable to generation t than an equally large spending on transfers, granted the public good is a substitute (complement). As in the case of lump-sum taxation, increased public consumption mitigates the tax distortion favouring home production, even when using a distortionary tax to repay debt.

Secondly, in situations outside the bench-mark case, it can be shown that $\theta < 1$ holds when the public good is an under-supplied substitute to home labour, while $\theta > 1$ holds when it is an over-supplied complement. In other words, the qualitative welfare result (4.6), originally referring to repayment by means of lump-sum taxation, is valid also for the present type of (distortionary) tax financing. It is evident from eq.(5.5) that the welfare effect on generation t of increased public consumption is positive if and only if $1 - \mu > \theta L_{t+1}^t / L_{t+1}$, i.e. iff both the "tax-base drain" and the second-period labour supply of that generation are small, at a given initial level of public consumption. In a non-distorted situation ($\tau_t^W = \tau_{t+1}^R = 0$), finally, increased public consumption increases welfare if and only if $g_G > (1 - \mu)^{-1} L_{t+1}^t / L_{t+1}$. (In the same situation, but with lump-sum financing, we noted earlier that welfare increases if and only if $g_G > 1$.)

Turning to public investment, finally, one can show

$$(5.7) \quad dv^t / dI_t = \frac{[\phi_I L_{t+1} - (1 + r_{t+1})] L_{t+1}^t / L_{t+1}}{1 - \mu + \tau_{t+1}^R r_{t+1} p_{t+1} (\partial x_t^t / \partial \omega^t) L_{t+1}^t / L_{t+1}} p_{t+1} u_1'.$$

Hence, if and only if public investment initially is at the earlier discussed bench-mark level, given by $\phi_I L_{t+1} = 1 + r_{t+1}$, the (positive) welfare effect of the investment per se is exactly neutralized by the (negative) welfare effect of the accompanying tax rise. (An explanation of this neutrality will be given below.) Below (above) this level, the net welfare effect is positive (negative).

This result contrasts sharply with the corresponding result in the case of lump-sum taxation: there the welfare effect generally did not vanish in the bench-mark case - its sign and magnitude was seen to depend in a particular way on the elasticity of labour supply and on the generations' share of that supply. In the present case, on the other hand,

the welfare effect of public investment pivots on the bench-mark level, irrespective of this elasticity and share. Despite the fact that the present form of financing is distortionary, and that a high elasticity means costly distortions, the only role of the elasticity here is to magnify the (positive or negative) welfare effect outside the bench-mark case.

By contrast to the situation in Section 4, when the increased debt was repaid by way of a reduced pension for generation t , generation $t+1$ is negatively influenced by the financing in the present three policy experiments. This means, of course, that its welfare will fall when spending goes to generation t in the form of higher transfers or increased public consumption. However, when the spending goes to public investment, then the negative effect on the welfare of generation $t+1$ from the tax rise is counteracted by the investment, which raises the gross wage rate in the first period of life of that generation. The net effect is given by

$$(5.8) \quad dv^{t+1}/dI_t = \frac{[\phi_1 L_{t+1} - (1+r_{t+1})] L_{t+1}^{t+1}/L_{t+1}}{1 - \mu + \tau_{t+1}^r r_{t+1} p_{t+1} (\partial x_t^t / \partial \omega^t) L_{t+1}^t / L_{t+1}} p_{t+1} u_1'(c_{t+1}^{t+1}) .$$

Note the similarity with the corresponding expression for generation t : the effect on each generation is proportional to its share of aggregate labour supply in period $t+1$, as well as to its own marginal utility of consumption in that period (recall $p_{t+1} u_1'(c_t^t) = u_2'(c_{t+1}^t)$). Moreover, both welfare effects are positive if and only if $\phi_1 L_{t+1} > 1+r_{t+1}$, i.e. iff the national income from the investment is higher than its capital cost. This unanimity is after all natural, since the two generations share both the benefits of the investment, in terms of a higher gross wage rate in period $t+1$, and the cost of its financing, in

terms of a higher tax-rate on wage earnings in the same period. (Had the depreciation rate been less than unity, then the welfare effect on generation $t+1$ would have been even more favourable.)

A heuristic explanation of the somewhat surprising "neutrality result" that the welfare of all generations is unaffected in the benchmark case, can be given in terms of national income. For clearly the public investment per se tends to raise the net wage rate in period $t+1$, while the increase in the wage tax-rate has the opposite effect. Hence, one may conceive of a combination of dI_t and $d\tau_{t+1}^W$ such that $d\bar{w}_{t+1}=0$. At such a combination, the households' allocations of labour and consumption are clearly unaffected. Consequently, also national income in period $t+1$, Y_{t+1} , viewed as the sum of consumption and investment, is unaffected. However, if instead national income is expressed in terms of factor payments, then $dY_{t+1} = d(w_{t+1}L_{t+1} + r_{t+1}K_{t+1}) - (1+r_{t+1})dI_t = [\phi_I L_{t+1} - (1+r_{t+1})]dI_t$, since both aggregate labour supply and the private capital stock are unchanged. We are hence led to the conclusion that if the tax rise accompanying the increase in public investment is such that the net wage-rate is unaffected, then public investment must have initially been at its benchmark level. It is also clear that public debt in period $t+1$ is constant under these circumstances. This demonstrates the intuition behind our "neutrality result", which, by the way, is valid independently of the degree of initial distortions of the allocation of labour and private capital.⁸

Equations (5.7) and (5.8) also provide a criterion for (second best) optimality of public investment. If namely the public sector were constrained to finance public investment by the distortionary tax τ_{t+1}^W , then the condition $\phi_I L_{t+1} = 1+r_{t+1}$ clearly is necessary for a pair (I_t, τ_{t+1}^W) to be (constrained) Pareto efficient, since for other combinations of public investment, I_t , and taxes, τ_{t+1}^W , both

generations' welfare could be improved without increasing the public debt left for later periods.

6. Consequences for foreign debt

The analysis has so far focused on the welfare effects on two consecutive generations. However, as the model depicts an "open economy", it is also of interest to look at the consequences for the foreign debt "passed on" to the future, after the first generation has left the scene, i.e. foreign debt in period $t+1$.

From the definition

$$(6.1) \quad B_{t+1} = K_{t+2} + D_{t+1} - s_{t+1}^{t+1},$$

and from the observation that $dK_{t+2}=0$ in all the policy experiments under consideration (since $dL_{t+2}=0$ and $dr_{t+2}=0$), it follows that

$$(6.2) \quad dB_{t+1} = dD_{t+1} - ds_{t+1}^{t+1},$$

where $D_{t+1} \neq 0$ only in the pure debt case, and $ds_{t+1}^{t+1} \neq 0$ only when $dI_t \neq 0$ and/or $d\tau_{t+1}^w \neq 0$. The direction of the effects on foreign debt in the various experiments is summarized in Table 1 below.

Table 1: The sign of dB_{t+1} in the nine policy experiments

	Pure debt financing	Lump-sum taxation	Labour-income taxation
transfers	+	0	+
publ.cons.	amb.	0	amb.
publ.inv.	amb.	-	amb.

The simplest cases emerge when the initial increase in public debt is repaid by a future lump-sum tax (middle column). Foreign debt then turns out to be unchanged not only when public spending goes to transfers, as may be expected, but also when it goes to public consumption, the reason being that there is no effect on private saving in period $t+1$. By contrast, foreign debt necessarily falls in the case of increased public investment, due to increased private saving from the rise in the wage rate and in labour supply.

Moreover, in the case of pure debt financing, an increase in transfers unambiguously raises foreign debt, as expected from normality of consumption. The same qualitative result emerges for increased public consumption when this is *not* a strong and initially under-supplied substitute for home labour (and hence a complement to the market good). In the opposite case, i.e. when the public good is an over-supplied complement to home labour (and hence a substitute to the market good), foreign debt instead falls. More exactly:

$$(6.3) \quad dB_{t+1}/dG_t^t \begin{cases} < \\ = \\ > \end{cases} dB_{t+1}/dT_t^t \quad \begin{cases} \text{if } g_G > 1 \text{ and } G_t^t \text{ substitute} \\ \text{if } g_G = 1 \text{ and } G_t^t \text{ neutral} \\ \text{if } g_G \leq 1 \text{ and } G_t^t \text{ complement} \end{cases}$$

The effect of purely debt financed public investment is ambiguous, though for a sufficiently large elasticity of labour supply it reduces foreign debt, as then aggregate labour supply in period $t+1$ increase substantially, and hence public savings increases by way of larger tax revenues.

Let us finally consider debt repayment by way of a higher labour-income tax rate. As in the case of pure debt financing, foreign debt unambiguously increases in the case of transfers, and the effects of increased public consumption still are as in (6.3). While the effect of

increased public investment is ambiguous in general, foreign debt will necessarily fall (increase) if the initial level of public investment is below (above) its bench-mark level.

$$(6.4) \quad dB_{t+1}/dI_t \begin{matrix} < \\ > \end{matrix} 0 \quad \text{iff} \quad \phi_{I,t+1} L_{t+1} \begin{matrix} > \\ < \end{matrix} 1+r_{t+1} .$$

Note the similarity with the welfare effects for generations t and $t+1$ (eqs.(5.7) and (5.8)): foreign debt falls in precisely the same circumstances as both generations' welfare increases. In particular, in the bench-mark case neither welfare nor foreign debt are affected.

In models with private bequests between generations it does matter, from a welfare point of view, whether "inherited" public debt is held domestically or by foreigners, since in the first case the ownership of government bonds are inherited while in the second case they are not (see e.g. Lindbeck and Weibull (1986)). However, in models without private inheritance, such as the present, the composition of "inherited" public debt does not matter from a welfare point of view - hence foreign debt is only of secondary interest in the present welfare analysis.

7. Summary and concluding remarks

Our conclusions concerning the welfare effects on generation t of the three policy experiments are summarized in a general form in Table 2. (The effects are expressed with the marginal utility of consumption as the unit.)

Table 2

In general, the signs of the welfare effects are unambiguous only in the case of "pure" debt financing. However, it is possible to reach rather clear-cut conclusions of the *ranking* of the effects in the various policy

Table 2: Welfare effects on generation t (modulo u'_1)

	Pure debt finance	Future lump-sum taxation	Future labour-income taxation
transfers	1	0	$1 - p_{t+1}^w L_{t+1}^t \beta_T$
publ.cons.	g_G (>0)	$g_G^{-p_{t+1} \alpha_G}$	$g_G^{-p_{t+1}^w L_{t+1}^t \beta_G}$
publ.inv.	$p_{t+1} (1 - \tau_{t+1}^w) \phi_{I,t+1}^t$	$p_{t+1} [(1 - \tau_{t+1}^w) \phi_{I,t+1}^t L_{t+1}^{-\alpha_I}]$	$p_{t+1} L_{t+1}^t [(1 - \tau_{t+1}^w) \phi_{I,t+1}^{-w} \beta_I]$

experiments in the "bench-mark situation", defined by $g_G=1$ (the marginal return of the public good in home production being unity), and $\phi_I L_{t+1} = 1+r_{t+1}$ (the marginal return to public investment in market production, (the marginal return to public investment in market production, in terms of national income, being equal to the return of lending in the international capital market)). The general expressions in Table 2 then can be specified as follows:

Table 3

The simplest, and indeed rather trivial conclusion for this "bench-mark" case is that, in the "pure debt financing" case, the welfare effects of transfers and public consumption are both equal to the marginal utility of current consumption. Furthermore, the welfare effects of public investment is now "typically" less favourable than a transfer by the same amount.

Turning to the second column, we note "debt neutrality" of transfers which are anticipated to be "repaid" by a lump-sum tax in the next period. In contrast, debt-financed public consumption, if anticipated to be repaid in the next period by way of a lump-sum tax, has a positive welfare effect when the public good is a substitute to home labour (which we take to be the typical case). The reason is that the initial distortion caused by the tax wedge between the marginal social return to home production and market work, respectively, favouring (tax free) home production, is mitigated by an increased public provision of a good that is a substitute to home labour. The welfare effects of increased public investment, with debt repayment by way of a lump-sum tax, is "typically" negative. Note also that, *ceteris paribus*, this welfare effect is an *increasing* function of τ_{t+1}^W .

Table 3: Welfare effects on generation t (modulo u'_1), in bench-mark case

Pure debt finance	Future lump-sum taxation	Future labor-income taxation
1	0	$\frac{(1-\mu)L_{t+1}^t/L_{t+1}^t - 1}{(1-\mu)L_{t+1}^t/L_{t+1}^t + \tau_{t+1}^r P_{t+1}^r \partial x_t^t / \partial \omega^t}$ (>0 for small μ) (<0 for large μ)
1	$-\frac{P_{t+1}(1+r_{t+1})\tau_{t+1}^w \partial e_t^t / \partial G_t^t}{1 + \tau_{t+1}^r P_{t+1}^r \partial x_t^t / \partial \omega^t}$ (>0 when G subst.)	$\frac{(1-\mu)L_{t+1}^t/L_{t+1}^t - 1 - P_{t+1}^r (1+r_{t+1})\tau_{t+1}^w \partial e_t^t / \partial G_t^t}{(1-\mu)L_{t+1}^t/L_{t+1}^t + \tau_{t+1}^r P_{t+1}^r \partial x_t^t / \partial \omega^t}$ (>0 when μ small and G subst.)
$P_{t+1}(1+r_{t+1})\frac{L_{t+1}^t}{(1-\tau_{t+1}^w)L_{t+1}^t}$	$\frac{P_{t+1}(1+r_{t+1})[(1-\tau_{t+1}^w)L_{t+1}^t/L_{t+1}^t + (1+\epsilon)\tau_{t+1}^w - 1]}{1 + \tau_{t+1}^r P_{t+1}^r \partial x_t^t / \partial \omega^t}$	0

transfers

publ.cons.

publ.inv.

~~Some general conclusions can be drawn for the bench-mark case also~~

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when debt is repaid by way of an increased labour-income tax rate. For instance, the welfare effect of a transfer is positive if and only if the elasticity $(1-\mu)$ of labour-income tax revenues with respect to the tax rate exceeds the generation's share of aggregate labour supply in the second period. In contrast, the welfare effect of increased public consumption is positive for any positive value of the above mentioned elasticity when the public good is a substitute to home labour (i.e. the same conclusion as in the case of lump-sum taxation). While the welfare effect of increased public investment was positive in the case of pure debt financing, and ambiguous (though typically negative) under lump-sum taxation, this effect vanishes when public debt is repaid by way of a labour-income tax. As explained in the main text, the reason for the latter, rather surprising, outcome is that national income is unchanged in the bench-mark case, with the benefits and costs shared in proportion to each generation's labour supply in that period.

As a further illustration of the bench-mark case, Table 4 below is based on "stylized empirical estimates" of some parameters: $\tau^w = \tau^r = 0.4$, $r = 0.6$ (a period being some 25 years), $\varepsilon = 0.3$ and $L_{t+1}^t / L_{t+1} = 0.35$. These numbers seem to be within realistic bounds for developed OECD countries, which have a large share of married women in the labour force; (the substitution effects on labour supply of changes in the after-tax wage rate is quite high for married women).

Table 4

Assuming the publicly provided good to be a substitute to home labour, public consumption now emerges as a better option than public investment under all types of debt repayment. Transfers now give a clearly

Table 4: Welfare effects on generation t (modulo u'_t), in numerical example

	Pure debt finance	Future lump-sum taxation	Future labour-income taxation
transfers	1	0	$\frac{1.3}{2.3 + 0.18\partial x_t^t / \partial \omega^t}$ (>0)
publ.cons.	1	$-\frac{0.47w_t \partial e_t^t / \partial G_t^t}{1 + 0.18\partial x_t^t / \partial \omega^t}$ (>0 when G subst.)	$\frac{1.3 - 0.47w_t \partial e_t^t / \partial G_t^t}{2.3 + 0.18\partial x_t^t / \partial \omega^t}$ (>0 when G subst.)
publ.inv.	0.25	$\frac{-0.32}{1 + 0.18\partial x_t^t / \partial \omega^t}$ (<0)	0

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positive welfare contribution when increased debt is repaid by a wage tax.

It is interesting to note that this ranking is robust for (reasonable) variations in both the labor supply elasticity and the tax rate.

A priori it is not obvious whether it is better or worse for a given generation to have to finance the repayment of the public debt all by itself in a non-distortionary fashion (such as by a reduced pension for itself), or to share the financing with another generation in a distortionary fashion (such as by a general wage tax) - an illustration of the old fable of choosing between a small share of a large cake, and a large share of a small cake. We have tried to shed some light on this trade-off between "tax-sharing" and "tax-distortions" for a given generation, by specifying under which conditions one or the other alternative form of debt repayment is favourable for each of the two generations. It has also been shown how this trade-off, and associated potential conflicts of interests between generations, depends on the initial tax wedge between the marginal social product of labour in the two sectors (household and market).

FOOTNOTES

¹If G_t^t is a collective (public) good, then the quantity available to each member of generation t should be nG_t^t , where n is the number of member of a generation and G_t^t the spending per member.

²A more general specification of the utility function would be $u^t = u(x^t, e^t, G^t)$. However, the effects of subsequent policy experiments would become too complex for an analytical study. In Aschauer and Greenwood (1985), utility is additively separable in consumption and leisure, and the composite consumption good is the sum of market purchases and (a monotone transform of) the quantity of the publicly provided good, i.e. in our notation, $c_t^t = x_t^t + g(G_t^t)$. In a general sense their approach harks back to Bailey (1971, ch.9) and Barro 1981).

³This topic is analyzed in Lindbeck (1982).

⁴Aschauer and Greenwood (1985) uses the specification $F(K, L, I_1, I_2) = F^1(L, I_1) + F^2(K, I_2)$, where I_1 and I_2 denote public services and investment. The more general specification $F(K, L, I)$ is used by H. Grossman and R.F. Lucas (1974), who study the short-term multiplier effects of public spending on aggregate demand and employment in a Keynesian macro model.

⁵If G_t^t is a collective good, then the right-hand side of (3.1b) should be divided by n , as each individual then gets nG_t^t (see footnote 1).

⁶The same condition, $\tau_{t+1}^w > 1/(1+\varepsilon)$, was shown to be necessary for constrained Pareto efficiency in steady state in another overlapping generations model in Svensson and Weibull (1985).

⁷The reason why we keep τ_{t+1}^r constant is simply to avoid additional complications via variations in the intertemporal price p_{t+1} .

⁸A rigorous proof to this informal line of reasoning is given by the observation that if $d\tau_{t+1}^w = \beta_I dI_t$, then $d\bar{w}_{t+1}/dI_t = w_{t+1}(1+r_{t+1} - \phi_I L_{t+1}) \partial D_{t+1} / \partial \tau_{t+1}^w$.

REFERENCES

- Aschauer, David A. and Jeremy Greenwood, "Macroeconomic Effects of Fiscal Policy", *Carnegie-Rochester Conference Series on Public Policy* 23 (1985): 91-138.
- Barro, Robert, "Output Effects of Government Purchases", *J.P.E.* 89 (1981): 1086-1121.
- Becker, Gary, "A Theory of the Allocation of Time", *Econ. Journ.* 75 (1965): 493-517.
- Gronau, Reuben, "Leisure, Home Production, and Work - the Theory of the Allocation of Time Revisited", *J.P.E.* 85 (February-December 1977): 1099-1123.
- Grossman, Herschel I. and Robert F. Lucas, "The Macro-economic Effects of Productive Public Expenditures", *Manchester School*, (June 1974): 162-170.
- Hansson, Ingemar and Charles Stuart, "Tax Revenues and the Marginal Costs of Public Funds in Sweden", *Journal of Public Economics* 27 (August 1985): 331-353.
- Lancaster, Kelvin, J. "A New Approach to Consumer Theory", *J.P.E.* 74 (April 1966): 132-157.
- Lindbeck, Assar, "Tax Effects versus Budget Effects on Labor Supply", *Economic Inquiry* XX (October 1982): 473-489.
- Lindbeck, Assar and Jörgen W. Weibull, "Debt-financed transfers, public consumption and public investment in an open economy", Institute for International Economic Studies, Seminar Paper No.330, 1985.
- Lindbeck, Assar and Jörgen W. Weibull, "Intergenerational Aspects of Public Transfers, Borrowing and Debt", *Scandinavian J. Econ.* 88 (1986): 239-267.
- Svensson L.-G. and J. W. Weibull, "Efficient income taxation in steady state", Institute for International Economic Studies, Seminar Paper No.329, 1985.

APPENDIX

The household sector:

Consider the program

$$\begin{aligned} \max \quad & u_1(x_t^t + g(e_t^t, G_t^t)) + u_2(x_{t+1}^t + h(e_{t+1}^t, G_{t+1}^t)) \\ \text{s.t.} \quad & x_t^t, x_{t+1}^t \geq 0, e_t^t, e_{t+1}^t \in [0, 1] \quad \text{and} \\ (A1) \quad & x_t^t + \bar{w}_t e_t^t + (x_{t+1}^t + \bar{w}_{t+1} e_{t+1}^t) p_{t+1} = \omega^t, \end{aligned}$$

where we assume $\bar{w}_t, \bar{w}_{t+1}, p_{t+1}, \omega^t > 0$. Necessary first-order conditions for an interior solution are:

$$(A2) \quad u_1' = \lambda$$

$$(A3) \quad u_1' g_e = \bar{w}_t \lambda$$

$$(A4) \quad u_2' = p_{t+1} \lambda$$

$$(A5) \quad u_2' h_e = \bar{w}_{t+1} p_{t+1} \lambda$$

for some $\lambda > 0$. Eqs. (A2)-(A3) imply (2.5a), and (A4)-(A5) imply (2.5b). Thus e_t^t is a function of $\bar{w}_t$ and G_t^t , and e_{t+1}^t of $\bar{w}_{t+1}$ and G_{t+1}^t .

Differentiation of (A1) gives

$$(A6) \quad \partial x_t^t / \partial \omega^t + p_{t+1} \partial x_{t+1}^t / \partial \omega^t = 1$$

$$(A7) \quad \partial x_t^t / \partial \bar{w}_t + e_t^t + \bar{w}_t \partial e_t^t / \partial \bar{w}_t + p_{t+1} \partial x_{t+1}^t / \partial \bar{w}_t = 0$$

$$(A8) \quad \partial x_t^t / \partial \bar{w}_{t+1} + p_{t+1} [\partial x_{t+1}^t / \partial \bar{w}_{t+1} + e_{t+1}^t + \bar{w}_{t+1} \partial e_{t+1}^t / \partial \bar{w}_{t+1}] = 0$$

$$(A9) \quad \partial x_t^t / \partial p_{t+1} + x_{t+1}^t + \bar{w}_{t+1} e_{t+1}^t + p_{t+1} \partial x_{t+1}^t / \partial p_{t+1} = 0$$

$$(A10) \quad \partial x_t^t / \partial G_t^t + \bar{w}_t \partial e_t^t / \partial G_t^t + p_{t+1} \partial x_{t+1}^t / \partial G_t^t = 0.$$

Eqs. (A2) and (A4) give $u_2' = p_{t+1} u_1'$. Differentiation of this equation w.r.t. ω^t shows that $\partial x_t^t / \partial \omega^t$ and $\partial x_{t+1}^t / \partial \omega^t$ must have the same sign, by concavity of u_1 and u_2 . Hence, by (A6):

$$(A11) \quad \partial x_t^t / \partial \omega^t, \partial x_{t+1}^t / \partial \omega^t > 0.$$

Combining (A1), (A2) and (A4) one obtains

$$(A12) \quad p_{t+1} u_1' [x_t^t + g(e_t^t, G_t^t)] =$$

$$= u_2' [\omega^t/p_{t+1} - x_t^t/p_{t+1} - \bar{w}_t e_t^t/p_{t+1} - \bar{w}_{t+1} e_{t+1}^t + h(e_{t+1}^t, G_{t+1}^t)] .$$

Differentiation of (A12) w.r.t. ω^t gives

$$(A13) \quad (p_{t+1} u_1'' + u_2''/p_{t+1}) \partial x_t^t / \partial \omega^t = u_2''/p_{t+1} .$$

Likewise, differentiation of (A12) w.r.t. $\bar{w}_{t+1}$ gives

$$(A14) \quad (p_{t+1} u_1'' + u_2''/p_{t+1}) \partial x_t^t / \partial \bar{w}_{t+1} = -e_{t+1}^t u_2''$$

where (2.5b) has been used. A comparison with (A13) gives

$$(A15) \quad \partial x_t^t / \partial \bar{w}_{t+1} = -p_{t+1} e_{t+1}^t \partial x_t^t / \partial \omega^t .$$

Differentiation of (A12) w.r.t. G_t^t , finally, gives

$$(A16) \quad (p_{t+1} u_1'' + u_2''/p_{t+1}) \partial x_t^t / \partial G_t^t + \\ + (g_e p_{t+1} u_1'' + \bar{w}_t u_2''/p_{t+1}) \partial e_t^t / \partial G_t^t = -g_G p_{t+1} u_1'' .$$

By eq.(2.5a) this implies

$$(A17) \quad (p_{t+1} u_1'' + u_2''/p_{t+1}) (\partial x_t^t / \partial G_t^t + \bar{w}_t \partial e_t^t / \partial G_t^t) = -g_G p_{t+1} u_1'' .$$

Subtraction of (A17) from (A13) gives

$$(A18) \quad (p_{t+1} u_1'' + u_2''/p_{t+1}) [\partial x_t^t / \partial \omega^t - \partial x_t^t / \partial G_t^t - \bar{w}_t \partial e_t^t / \partial G_t^t] = \\ = (g_G p_{t+1} u_1'' + u_2''/p_{t+1}) ,$$

from which follows

$$(A19) \quad \begin{aligned} & < \partial x_t^t / \partial \omega^t - 1 & \text{if } g_G > 1 \\ \partial x_t^t / \partial G_t^t + \bar{w}_t \partial e_t^t / \partial G_t^t & = \partial x_t^t / \partial \omega^t - 1 & \text{if } g_G = 1, \\ & > \partial x_t^t / \partial \omega^t - 1 & \text{if } g_G < 1 \end{aligned}$$

implying that (4.6) follows from (4.4).

The partial derivatives of the indirect utility function v^t can be derived from eqs.(A2)-(A10) as follows:

$$(A20) \quad \partial v^t / \partial \omega^t = (\partial x_t^t / \partial \omega^t) u_1' + (\partial x_{t+1}^t / \partial \omega^t) u_2' \\ = (\partial x_t^t / \partial \omega^t + p_{t+1} \partial x_{t+1}^t / \partial \omega^t) u_1' = /(A6)/ = u_1'$$

$$(A21) \quad \partial v^t / \partial \bar{w}_t = (\partial x_t^t / \partial \bar{w}_t + g_e \partial e_t^t / \partial \bar{w}_t) u_1' + (\partial x_{t+1}^t / \partial \bar{w}_t) u_2' \\ = (\partial x_t^t / \partial \bar{w}_t + \bar{w}_t \partial e_t^t / \partial \bar{w}_t + p_{t+1} \partial x_{t+1}^t / \partial \bar{w}_t) u_1' \\ = /(A7)/ = -e_t^t u_1'$$

$$\begin{aligned}
(A22) \quad \partial v^t / \partial \bar{w}_{t+1} &= \\
&= (\partial x_t^t / \partial \bar{w}_{t+1}) u_1' + (\partial x_{t+1}^t / \partial \bar{w}_{t+1} + h_e \partial e_{t+1}^t / \partial \bar{w}_{t+1}) u_2' \\
&= [\partial x_t^t / \partial \bar{w}_{t+1} + p_{t+1} (\partial x_{t+1}^t / \partial \bar{w}_{t+1} + \bar{w}_{t+1} \partial e_{t+1}^t / \partial \bar{w}_{t+1})] u_1' \\
&= /(A8)/ = -p_{t+1} e_{t+1}^t u_1' \\
(A23) \quad \partial v^t / \partial p_{t+1} &= (\partial x_t^t / \partial p_{t+1}) u_1' + (\partial x_{t+1}^t / \partial p_{t+1}) u_2' \\
&= (\partial x_t^t / \partial p_{t+1} + p_{t+1} \partial x_{t+1}^t / \partial p_{t+1}) u_1' \\
&= /(A9)/ = -(x_{t+1}^t + \bar{w}_{t+1} e_{t+1}^t) u_1' \\
(A24) \quad \partial v^t / \partial G_t^t &= \\
&= (\partial x_t^t / \partial G_t^t + g_e \partial e_t^t / \partial G_t^t + g_G) u_1' + (\partial x_{t+1}^t / \partial G_t^t) u_2' \\
&= (\partial x_t^t / \partial G_t^t + \bar{w}_t \partial e_t^t / \partial G_t^t + g_G + p_{t+1} \partial x_{t+1}^t / \partial G_t^t) u_1' \\
&= /(A10)/ = g_G u_1'
\end{aligned}$$

The production sector:

Let $K^* = \alpha K$ and $L^* = bL$. Gross output is $F(K^*, L^*)$, where F is homogeneous of degree one, so $F(K^*, L^*) = K^* F_1(K^*, L^*) + L^* F_2(K^*, L^*)$. At the depreciation rate $\delta=1$ profits are

$$(A25) \quad \pi = F(K^*, L^*) - (1+r)K - wL$$

$$(A26) \quad \pi_K = aF_1 - (1+r)$$

$$(A27) \quad \pi_L = bF_2 - w$$

Consequently, at an interior profit maximum:

$$(A28) \quad \pi = F - K^* F_1 - L^* F_2 = 0.$$

Let $k^* = K^*/L^*$, and define f by $f(k^*) = F(K^*, L^*)/L^*$. Then $f' = F_1$, and we may write the first-order conditions $\pi_K = \pi_L = 0$ on the following form:

$$(A29) \quad af'(k^*) = 1+r$$

$$(A30) \quad w = L^* F_2 / L = (F - K^* F_1) / L = b[f(k^*) - k^* f'(k^*)].$$

These two equations define the factor-price frontier ϕ and capital demand schedule Ψ as in eq.(2.6). To obtain (2.7a), first differentiate (A29) and (A30) w.r.t. I (r is exogenously fixed by assumption):

$$(A31) \quad 0 = a'f' + af''k_I^*$$

$$(A32) \quad \phi_I = b'(f - k^*f') + b(f' - f' - k^*f'')k_I^*$$

Substitution of k_I^* from (A31) gives (2.7a). As for (2.7b), note that

$$(A33) \quad k_I^* = d[ak/b]/dI = [(a'k + a\Psi_I)b - akb']/b^2 = [(a'b - ab')k + ab\Psi_I]/b^2$$

Hence, by (A31):

$$(A34) \quad (a'b - ab')k + ab\Psi_I + b^2(a'/a)f'/f'' = 0,$$

which gives (2.7b).

The public sector:

Viewing D_{t+1} as a function of $(T_t^t, G_t^t, I_t, T_{t+1}^t, \tau_{t+1}^w)$, we find:

$$(A35) \quad \partial D_{t+1} / \partial T_t^t = 1 + r_{t+1} + \tau_{t+1}^r r_{t+1} (\partial x_t^t / \partial \omega^t - 1)$$

$$(A36) \quad \partial D_{t+1} / \partial G_t^t = (1 + r_{t+1}) (1 + \tau_{t+1}^w w_t \partial e_t^t / \partial G_t^t) + \tau_{t+1}^r r_{t+1} (\partial x_t^t / \partial G_t^t + \bar{w}_t \partial e_t^t / \partial G_t^t)$$

$$(A37) \quad \begin{aligned} \partial D_{t+1} / \partial I_t &= 1 + r_{t+1} - \tau_{t+1}^w \phi_{I L_{t+1}} - \tau_{t+1}^w w_{t+1} (1 - \tau_{t+1}^w) \phi_{I dL_{t+1}} / d\bar{w}_{t+1} \\ &\quad + \tau_{t+1}^r r_{t+1} (1 - \tau_{t+1}^w) \phi_I [p_{t+1} \partial x_t^t / \partial \omega^t + \partial x_t^t / d\bar{w}_{t+1}] \\ &= 1 + r_{t+1} - \tau_{t+1}^w \phi_{I L_{t+1}} (1 + \varepsilon) + \\ &\quad + \tau_{t+1}^r r_{t+1} p_{t+1} (1 - \tau_{t+1}^w) \phi_{I L_{t+1}}^t \partial x_t^t / \partial \omega^t, \end{aligned}$$

where (A15) has been used in (A37). Moreover,

$$(A38) \quad \partial D_{t+1} / \partial T_{t+1}^t = 1 + \tau_{t+1}^r r_{t+1} p_{t+1} \partial x_{t+1}^t / \partial \omega^t = p_{t+1} \partial D_{t+1} / \partial T_t^t.$$

Insertion of these equations into (4.1-2) gives (4.3-5) and (4.7), while

(4.6) uses (A18). Finally,

$$(A39) \quad \begin{aligned} \partial D_{t+1} / \partial \tau_{t+1}^w &= \\ &= -w_{t+1} L_{t+1} - \tau_{t+1}^w w_{t+1} dL_{t+1} / d\tau_{t+1}^w - \tau_{t+1}^r r_{t+1} ds_t^t / d\tau_{t+1}^w \\ &= -w_{t+1} L_{t+1} + \tau_{t+1}^w w_{t+1}^2 dL_{t+1} / d\bar{w}_{t+1} \\ &\quad - \tau_{t+1}^r r_{t+1} w_{t+1} (p_{t+1} \partial x_t^t / \partial \omega^t + \partial x_t^t / \partial \bar{w}_{t+1}) \\ &= -w_{t+1} [L_{t+1} - \tau_{t+1}^w \varepsilon L_{t+1} / (1 - \tau_{t+1}^w)] + \tau_{t+1}^r r_{t+1} p_{t+1} L_{t+1}^t \partial x_t^t / \partial \omega^t \\ &= -w_{t+1} [(1 - \mu) L_{t+1} + \tau_{t+1}^r r_{t+1} p_{t+1} L_{t+1}^t \partial x_t^t / \partial \omega^t], \end{aligned}$$

((A15) has been used), which amounts to eq.(5.3). Eqs.(5.4-7) may then be

obtained from eqs.(5.1-2), (A35)-(A37) and (A39) as follows. By (5.2a):

$$(A40) \quad (dv^t/dT_t^t)/u_1' = 1 + w_{t+1}p_{t+1}L_{t+1}^t (\partial D_{t+1}/\partial T_t^t)/(\partial D_{t+1}/\partial \tau_{t+1}^w)$$

$$= 1 - \frac{w_{t+1}L_{t+1}^t [1 + \tau_{t+1}^r r_{t+1} p_{t+1} \partial x_t^t / \partial \omega^t]}{w_{t+1}[(1-\mu)L_{t+1} + \tau_{t+1}^r r_{t+1} p_{t+1} (\partial x_t^t / \partial \omega^t)L_{t+1}^t]}$$

which gives (5.4). Likewise, by (5.2b):

$$(A41) \quad [(1-\mu)L_{t+1} + \tau_{t+1}^r r_{t+1} p_{t+1} (\partial x_t^t / \partial \omega^t)L_{t+1}^t] (dv^t/dG_t^t)/u_1' =$$

$$= [(1-\mu)L_{t+1} + \tau_{t+1}^r r_{t+1} p_{t+1} (\partial x_t^t / \partial \omega^t)L_{t+1}^t] g_G -$$

$$- [p_{t+1}(1+r_{t+1})(1+\tau_t^w w_t \partial e_t^t / \partial G_t^t) +$$

$$+ \tau_{t+1}^r r_{t+1} p_{t+1} (\partial x_t^t / \partial G_t^t + \bar{w}_t \partial e_t^t / \partial G_t^t)] L_{t+1}^t$$

$$= (1-\mu)g_G L_{t+1} +$$

$$+ \tau_{t+1}^r r_{t+1} p_{t+1} [g_G \partial x_t^t / \partial \omega^t - 1 - \partial x_t^t / \partial G_t^t - \bar{w}_t \partial e_t^t / \partial e_t^t] L_{t+1}^t -$$

$$- [p_{t+1}(1+r_{t+1} - \tau_{t+1}^r r_{t+1}) + p_{t+1}(1+r_{t+1})\tau_t^w w_t \partial e_t^t / \partial G_t^t] L_{t+1}^t$$

$$+ [L_{t+1}^t / L_{t+1}],$$

which gives (5.6). By (5.2c), finally,

$$(A42) \quad w_{t+1}[(1-\mu)L_{t+1} + \tau_{t+1}^r r_{t+1} p_{t+1} (\partial x_t^t / \partial \omega^t)L_{t+1}^t] (dv^t/dI_t^t)/u_1' =$$

$$= p_{t+1}L_{t+1}^t (1-\tau_{t+1}^w) \phi_I w_{t+1} [(1-\mu)L_{t+1} + \tau_{t+1}^r r_{t+1} p_{t+1} (\partial x_t^t / \partial \omega^t)L_{t+1}^t]$$

$$- w_{t+1}p_{t+1}L_{t+1}^t [1+r_{t+1} - \tau_{t+1}^w \phi_I L_{t+1} (1+\varepsilon) +$$

$$+ \tau_{t+1}^r r_{t+1} p_{t+1} (1-\tau_{t+1}^w) \phi_I (\partial x_t^t / \partial \omega^t)L_{t+1}^t]$$

$$= w_{t+1}p_{t+1}L_{t+1}^t [(1-\tau_{t+1}^w)(1-\mu)\phi_I L_{t+1} + \tau_{t+1}^w (1+\varepsilon)\phi_I L_{t+1} - (1+r_{t+1})],$$

which gives (5.7), since $(1-\tau_{t+1}^w)(1-\mu) + \tau_{t+1}^w (1+\varepsilon) = 1$.

For (5.8), finally, note that $dv^{t+1}/dI_t^t = L_{t+1}^{t+1} [(1-\tau_{t+1}^w) \phi_I -$
 $- w_{t+1} \beta_I] u_1' (c_{t+1}^{t+1})$, cf. (5.2c).