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NONCOOPERATIVE FLEXIBLE PRICING IN
A HOMOGENEOUS MARKET

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Ante Farm

and

Jörgen W. Weibull

INSTITUTE FOR INTERNATIONAL ECONOMIC STUDIES

University of Stockholm
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The Swedish Institute for Social Research
and
Institute for International Economic Studies
University of Stockholm

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Institute for International Economic Studies
S-106 91 Stockholm
Sweden
NONCOOPERATIVE FLEXIBLE PRICING IN A HOMOGENEOUS MARKET*

Ante Farm and Jörgen W. Weibull**


Abstract: A market is studied in which prices are set by the sellers, and where equilibrium is established through pure price adjustment. It is assumed that the sellers can observe each others' prices, and that each of them is free to instantly and costlessly change his price. In this setting it is shown that, when the number of competitors is large, a "cartel price", above the (Walrasian) competitive price, is an equilibrium - in fact the unique equilibrium price on the Pareto frontier.

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*The basic ideas underlying this note were independently developed by each of the two authors. By Farm in the context of "rational price-setting" under proportional rationing (Farm (1985,1986)), and by Weibull in the context of "price-cut matching strategies" of neo-Keynesian firms (Weibull (1984)). We are grateful for helpful comments from Henrik Horn, Lars E.O. Svensson and Hans Wijkander, as well as from the participants of a seminar at the Institute for International Economic Studies.

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1. Introduction

Economic theorists have in recent years devoted much attention to the foundations of the neoclassical doctrine of perfect competition. The motive and scope of this strand of research was well expressed by Andreu Mas-Colell in a special issue of the Journal of Economic Theory devoted to noncooperative approaches to this topic:

"Modern Walrasian economics ... provides an analysis of the decentralized economic coordination problem under the hypothesis that prices are publicly quoted and are viewed by economic agents as exogenously given. If we regard this as the Hypothesis of Perfect Competition, then modern Walrasian economics is a theory of perfect competition only in the sense of examining the consequences of this Hypothesis but not in that of giving a theoretical explanation of the Hypothesis itself." (Mas-Colell (1980, p.121))

Also the present note takes a noncooperative approach, but differs from other studies in that it is focused on the classical case of homogeneity, perfect information, and, in particular, the limiting case of perfectly "flexible" prices in which each seller is free to instantly and costlessly revise his price when observing the prices of his competitors. To discuss these basic principles of decentralized price setting in the simplest possible framework, we turn to Marshall's corn market example, in which stocks offered for sale are in existence already at the outset, and where equilibrium is established through pure price adjustment (Marshall (1920, book V, p.278)).

In this simple setting, sufficient conditions are given under which noncooperative equilibrium prices exist. Let $p^C$ and $p^o$ denote the competitive (Walrasian) price, and the price that maximizes the value of aggregate demand, respectively. If $p^o \leq p^C$ ("elastic demand"), then $p^C$ is the only possible equilibrium price, while in the opposite case ("inelastic demand") $p^o$ turns out to be an equilibrium price. In fact, it is then the unique equilibrium price on the Pareto frontier, and one can show that all
individual strategies supporting \( p^0 \) are equivalent - hence this price is an ideal candidate for tacit collusion (cf. Friedman (1983, p.132)). The result is shown to be valid for any fixed number \( n \) of sellers, as well as in the limit when \( n \) tends to infinity.

2. Preliminaries

Let \( q_i > 0 \) denote the (exogenous) supply of seller \( i \), and let \( p_i > 0 \) be his price. Let \( Q = \sum q_i \), and let the aggregate demand function \( D \) be continuous and strictly decreasing. Suppose \( D(p^C) = Q \) for some \( p^C > 0 \) - then \( p^C \) is the (unique) competitive price. We assume that the aggregate expenditure function \( E \), defined by \( E(p) = pD(p) \), has a unique maximum at a positive price \( p^0 \), the "optimal" price, to the left (right) of which it is strictly increasing (decreasing).

If all sellers' quote prices above \( p^C \), then some sellers are rationed. In particular, if they all quote the same price \( p > p^C \), then each seller's sales will be some share \( \alpha_i(p) \) of his supply \( q_i \). We assume \( \alpha_i = \alpha_j \) for every \( i \) and \( j \), i.e. every unit of the good has an equal "chance" of being sold. In this sense the good is absolutely homogeneous. It follows that, at any uniform price \( p \), \( \alpha(p)Q = D(p) \), so individual trades are then given by

\[
(1) \quad t_i = \min\{q_i, q_iD(p)/Q\} \quad (i=1,\ldots,n).
\]

We assume that unsold goods are worthless to the sellers, i.e. they simply seek to maximize their sales revenues, \( p_i q_i \). (This assumption can be somewhat relaxed, see discussion in Section 4 below.)

In the traditional game-theoretic underpinning of the Walrasian model, a seller's strategy is simply his price, and then it is evident
that a uniform price above the competitive price cannot be a Nash equilibrium (NE). For if \( p_j > p^c \) for all \( j \neq i \), then a slight price cut, \( p_i < p \), will yield a discontinuous increase in the revenues of seller \( i \). Note, however, that this conclusion relies on an implicit assumption of price rigidity (or commitment). For while the buyers are assumed to observe all prices before making purchases, the sellers are not permitted to change prices once announced. If instead each seller were free to revise his price after having observed his competitors' prices, then a seller wouldn't rationally expect competitors not to react to his deviation - the "no-response" hypothesis above would then at best be a special case to be deduced from anticipated rational behaviour on behalf of the competitors.

Of course, competitors (as well as buyers) react only with some delay and at certain costs in practice. However, in order to highlight the role of price flexibility for the character of equilibrium, we focus on the limiting case of instantaneous and costless price revisions before trade commences.

3. The model

More precisely, we consider a tatonnement process, without an auctioneer, in which each seller observes all prices and sets his own price. At the outset, all sellers simultaneously announce their initial prices, \( p(0) = (p_1(0), \ldots, p_n(0)) \), without cooperation. Immediately afterwards, they observe this price vector and are free to instantly and costlessly revise their prices. Hence, a new price vector \( p(1) \) immediately arises, where \( p_i(1) = f_i(p(0)) \) and each function \( f_i \) is selected by seller \( i \) himself. For simplicity, we now restrict his further price revisions to follow the scheme \( p_i(t) = f_i(p(t-1)) \) for all \( t > 0 \), where \( f_i \) is continuous. In other words, the sellers are allowed to react to each others' prices, but only
in terms of a stationary response function. Traditional pricing is obtained if one further restricts response functions to fulfill the requirement \( f_i(p) = p_i \), so the present extension seems to be the simplest possible representation of perfect price flexibility. 6

If the generated price sequence \( \{p(t)\} \) converges, then its limit is the associated trading-price vector \( \rho \). If the sequence diverges, then it is not evident how trading-prices should be determined. For definiteness, we let the competitive price then be the uniform trading price:

\[
\rho = \begin{cases} 
\lim_{t \to \infty} p(t) & \text{if } \{p(t)\} \text{ converges} \\
(p^C, p^C, \ldots, p^C) & \text{otherwise.}
\end{cases}
\]

Individual trades \( t_i(\rho) \) are supposed never to exceed the corresponding supplies, \( 0 \leq t_i \leq q_i \), and \( t_i(\rho) \) is assumed to satisfy eq.(1) whenever trading prices are identical at all sellers. Moreover, since the good is homogeneous, we suppose that the buyers turn in priority to the sellers with the lowest price, or more exactly, that \( t_i(\rho) = 0 \) if \( \rho_j = \rho_i \) for all \( j \neq i \) and \( q_j > D(\rho) \). Individual revenues are obtained by simply multiplying trades with trading prices: \( \pi_i(\sigma) = \rho_i t_i(\rho) \).

In sum: we have defined an infinite-dimensional non-zero-sum game on normal form, in which a (pure) strategy \( \sigma_i \) is a pair \( (p_i(0), f_i) \), where \( p_i(0) \in R^+ \) and \( f_i : R^+ \to R^+ \) is continuous. Combined strategies are written \( \sigma = (\sigma_1, \ldots, \sigma_n) \) and the combined response function \( f \) is defined by \( f(p) = (f_1(p), \ldots, f_n(p)) \). Individual payoffs associated with any combined strategy \( \sigma \) are \( (\pi_1(\sigma), \ldots, \pi_n(\sigma)) \).

By continuity, only fixed points of \( f \), i.e. price vectors at which no seller wants to change his price (under the restrictions of the game), can be limit points of the above tatonnement process. In particular, no
price revision actually takes place if the initial price vector \(p(0)\) is a fixed point of \(f\) - then \(p(0)\) is itself the trading-price vector. This observation suggests the following definition of equilibrium: a strategy vector \(\sigma=((p_1,f_1),\ldots,(p_n,f_n))\) is an equilibrium if \(p=f(p)\) and

\[
(3) \quad \pi_i(\sigma_1,\ldots,\sigma_i,\ldots,\sigma_n) \geq \pi_i(\sigma_1^*,\ldots,\sigma_i^*,\ldots,\sigma_n)
\]

for every seller \(i\) and strategy \(\sigma_i^*=(p_i^*,f_i^*)\). In other words, an equilibrium is a Nash equilibrium in strategies with the further property that initially quoted prices are also trading prices.

Of particular relevance for economic modelling are single-price equilibria (SPE), i.e. equilibria \(\sigma\) with \(p_i=p_j\) for all \(i\) and \(j\). The subsequent analysis will be confined to this type of equilibrium. Let \(P\) be the corresponding (possibly empty) set of equilibrium prices, i.e. \(P=\{p \in \mathbb{R}_+: ((p,f_1),\ldots,(p,f_n))\text{ is a SPE for some } (f_1,\ldots,f_n)\}\).

It follows immediately from our definition of equilibrium that the "no response" strategy \((f_i(p)=p_i)\) does not support the "optimal" price \(p^0\) as a SPE. For any seller could then obtain a discontinuous increase in revenues by deviating from this strategy by a slight price cut in the "inelastic" case \(p^c>p^0\), or a slight price rise in the opposite case. However, the price \(p^0\) will indeed be supported by certain other (noncooperative) strategies in many situations.

This result will be shown to hold under a simple but powerful condition. Let \(p^e\) be the effective optimal price, defined as the "optimal" price \(p^0\) multiplied by the corresponding rationing factor, i.e. \(p^e=p^0\alpha(p^0)=E(p^0)/Q\). It follows from our assumptions that \(p^c<p^e<p^0\) in the "inelastic" case.\(^7\) The condition requires sellers to be sufficiently small in the
sense that all individual supplies should be smaller than aggregate excess supply at \( p^e \):

\[
C: \max \{q_i\} < Q-D(p^e),
\]

**Proposition:** If \( p^o > p^c \) and \( C \) holds, then \( p^o \notin P \).

Our proof of this result (see below) is based on a modification of the kinked response function (cf. Sweezy (1939) and Marschak and Selten (1978)). More exactly, in the "inelastic" case \( p^c < p^o \), one can show that any strategy vector \( \sigma^o \), with identical components \((p^o, f^o)\), where

\[
(4) \quad f^0(p) = \max\{ p^e, \min\{p_1, \ldots, p_n\} \} \quad \text{for all } p,
\]

indeed is a SPE.\(^8\) For then no seller has an incentive to initiate a price cut, since competitors will retaliate (otherwise profitable) price cuts, and hence make every price cut unprofitable. The above proposition may therefore be viewed as a formalization and slight extension of the insight in the literature on industrial organization (see e.g. Scherer (1980, ch.5) that price competition may be prevented by threats to match price cuts, an idea originally due to Chamberlin (1962, ch.3).\(^9\)

**Proof of the proposition:** Let \( i \in \{1, \ldots, n\} \) be arbitrary, and let \( \sigma_j = (p^0, f^0) \) for all \( j \neq i \). For any strategy \( \sigma_i \), let \( \sigma \) be the corresponding strategy vector and \( \rho \) the trading-price vector. If \( \sigma_i = (p^0, f^0) \), then clearly \( \rho_i = p^0 \) and \( \pi_i = q_i E(p^0) / Q = p^e q_i \). If \( \sigma_i \) is such that \( \{p(t)\} \) diverges, then \( \pi_i = p^c q_i > p^e q_i \). In case of convergence, the necessary condition \( \rho = f(\rho) \) implies \( \rho_j = \rho_k = f^0(\rho) \) for all \( j, k \neq i \). If \( \rho_j = \rho_i \), then, by eq.(1), \( \pi_i = \)
\[ \min(\rho_i q_i, q_i E(\rho_i)/Q) \leq q_i E(p_0^e)/Q = p_0^e q_i. \] If \( p_0^e < p_0^e \), then clearly \( p_0^e < p_0^e q_i. \) It hence remains to check strategies \( \sigma_i \) such that \( \rho_i > \rho_j \) and \( \rho_i > p_0^e \). Then \( p_0^e < p_0^e < p_0 \) since \( p_0 = f(\rho) \) for all \( j \neq i \), so \( \pi_i = 0 \) by C. The index \( i \) being arbitrary, it follows that \( \sigma^0 \) satisfies (3). End of proof.

In fact, the difficulty here is not the existence of equilibria but their multiplicity. However, under the hypothesis of the proposition, the price \( p_0^o \) is actually the unique (uniform) equilibrium price that is not Pareto dominated, and all strategies supporting \( p_0^o \) are equivalent (in the sense of Luce and Raiffa (1957, p.106)). Hence \( p_0^o \) is an ideal candidate for tacit collusion (cf. Friedman (1983, p.132)). The point is that no seller has anything to loose by quoting \( p_0^o \) initially, since he can always match a price cut.

More exactly, suppose \( p^c < p_0^o \), and let \( P^* \) be the subset of Pareto efficient prices in \( P \), i.e. (uniform) prices supported by SPE strategy vectors that are not dominated, in terms of individual revenues, by other SPE vectors. Assuming, as we do, that \( p_0^o \) is the unique maximum of the aggregate expenditure function \( E \), it follows directly that \( P^* = \{ p_0^o \} \), i.e. the price \( p_0^o \) is indeed optimal. Moreover, if \( \sigma^1, \ldots, \sigma^n \) are SPE strategy vectors supporting \( p_0^o \), then any mixed strategy vector, e.g. \( (\sigma_1^1, \sigma_2^2, \ldots, \sigma_n^n) \), can easily be shown to also support \( p_0^o \). In other words: all Pareto efficient (single price) equilibria give the same maximal payoffs to all players, irrespective of which particular individual strategies in this set have been selected. Hence, individual rationality alone, in the sense of Johansen (1982) leads the sellers to choose (any) strategies supporting the optimal price \( p_0^o \).

This result is clearly valid for any fixed number \( n \) of competitors and any supply vector \( q = (q_1, \ldots, q_n) \) satisfying condition C. In particular,
if each seller becomes less and less significant in the sense that $n$ increases while aggregate supply $Q$ is constant and no individual supply increases, then condition C just becomes more and more easily satisfied. Furthermore, it is readily verified that $p^0$ remains an equilibrium price even when aggregate supply increases along with the number of competitors. For let the aggregate demand function $D$ be fixed and given, and let $q=(q_1, \ldots, q_n)$ be a fixed supply vector. For any positive integer $m$, let $mq$ be the $m$-fold replication of $q$. Since $D$ is fixed, so is $p^0$, while for $m$ sufficiently large, the corresponding competitive price $p^c(m)$ (defined by $D(p)=mq$, granted $D(0)=+\infty$ and $D(+\infty)=0$), falls below $p^0$. In other words, if we duplicate the supply side sufficiently many times, we sooner or later wind up in the "inelastic" case $p^c<p^0$. Moreover, the largest individual supply, $\max(q_i)$, does not increase in this process, while the corresponding effective price, $p^e(m) = E(p^0)/(mq)$, decreases monotonically towards zero—a natural effect of stiffening competition. Nevertheless, excess supply at this price rises monotonically. For

\begin{equation}
(5) \quad mQ-D(p^e(m)) = [1-E(p^e(m))/E(p^0)]mq,
\end{equation}

where the right-hand side, for $p^e(m)<p^0$, is increasing in $m$ (in fact toward infinity), since by assumption $E(p)$ is increasing for $0<p<p^0$. Hence condition C holds for all $m$ above a certain level, and $p^0$ remains an equilibrium price, also in the limit as the number of competitors increases towards infinity.

4. Extensions

Conspicuous restrictions in the present model are the assumptions (i) that supply is exogenously fixed, (ii) that pricing strategies are
stationary, and (iii) that unsold goods are worthless to the seller. As for the first two aspects, extensions will be presented in subsequent studies (see Farm (1986) and Weibull (1986) concerning (i), and Weibull (1986) for (ii)). Concerning the third restriction, on the other hand, a few words will be said here.

If the good is perishable, then assumption (iii) is natural, while if the good is storeable, then one can argue that the present model does not allow the agents to fully exploit potential gains of trade. However, suppose we extend the model by assigning an exogenously fixed and non-negative value $r$ to each unit of unsold goods ("the expected present value of net revenues from future sales"). Then individual profits are given by the equation $\pi_i = \rho_i t_i + (q_i - t_i)r = (\rho_i - r)t_i + q_i r$. The last term, $q_i r$, is exogenously fixed, so maximization of $(\rho_i - r)t_i$ is the relevant objective for seller $i$. Let $p^0(r) = \min \arg \max (p - r)D(p)$. (Note that $p^0(r)$ is formally identical with the monopoly price for an industry in which every firm produces according to the linear cost function $c_i(q_i) = rq_i$.) Then $p^0(r) > p^0$ for every $r$ in $(0, p^0)$ and $p^0(0) = p^0$. Moreover, it is readily verified that if we, in the original model, replace $p^0$ and $p^0$ by the more general $p^0(r)$ and $p^0(r) = E(p^0(r))/Q$, then the proof of our proposition remains valid for any $r$ in the interval $(0, p^0)$. Hence, if the model is closed with respect to trading possibilities in this fashion, then noncooperative pricing in fact leads to an even higher equilibrium price.
1 Extensions of the present model to include production will be discussed in subsequent studies, see Farm (1986) and Weibull (1984, 1986).

2 Stigler (1964) and Spence (1978) analyze the difficulties of tacit collusion in the presence of imperfect information (in particular, the possibility of "secret price cutting") and product heterogeneity.

3 See Green (1980) or Weinrich (1984) for rigorous derivations of proportional stochastic rationing from basic axioms concerning general properties of the underlying rationing scheme.

4 However, it does not follow that $p^c$ is a NE in all situations. For if some individual contingent demand function is inelastic at $p^c$, then a price rise is profitable. As emphasized by Shubik (1959, ch.5), the derivation of contingent demand curves is, in general, a complex problem though.

5 Also Marschak and Selten (1978) assume that reaction times are negligible, so that transitory profits during the price revision process can be ignored. However, in contrast to the present model, they use an equilibrium concept which "is not an equilibrium action n-tuple in the traditional Nash-Cournot ... sense, .." (op.cit. p.73).

6 A further extension is given in Weibull (1986), in which tatonnement pricing is modelled as an infinite game in extensive form with perfect recall.

7 If $p^0 > p^c$, then $p^e = p^0D(p^0)/Q < p^0D(p^c)/Q = p^0$. By assumption $p^0$ maximizes $pD(p)$, thus $p^e = p^0D(p^0)/Q > p^cD(p^c)/Q = p^c$.

8 It is not sufficient to use the simpler response function $f^m(p) = \min\{p_j\}$. For if all sellers but one would have such strategies, then it could be advantageous for the remaining seller $i$ to (1) quote a very low price,
e.g. $p_i(0)=0$, hence driving all other sellers down to that price, and 

(2) have a response function $f_i$ with the property $f_i(p_i(0),\ldots,p_i(0))>p_i(0)$. Such a strategy could lead to a favourable "contingent demand" for seller $i$, because of the small income loss incurred at purchases from other sellers.

A result similar in spirit, but different in its theoretical foundation, is given in Osborne (1976). He shows that cartels are internally stable once the contract surface (Pareto frontier) has been located and cheating can be detected. Green and Porter (1984) extend this analysis of cartel self-enforcement to the case of demand uncertainty.

If each vector $\sigma^i$ supports $p^o$ as a uniform equilibrium price, then $f_i(p^o,\ldots,p^o)=(p^o,\ldots,p^o)$ for every $i$. Hence, if $\sigma^{mix}=(\sigma_1^1,\ldots,\sigma^n_1)$, then $p^{mix}_i=p^o$ and $f^{mix}_i(p^o,\ldots,p^o)=p^o$ for every component $i$ of $\sigma^{mix}$.

As recently emphasized in the literature on oligopoly theory (cf. e.g. Bresnahan (1981), reaction functions should be consistent. Interpreting "consistent conjectures" concerning other agents' behaviour as rational expectations of their (complete) market strategies, one may note that the strategies singled out by the Nash equilibrium condition are consistent in precisely this way, cf. Johansen (1982) and Friedman (1983, p.110).
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