"Exchange Rate Uncertainty and Strategic Export Behavior: An International Model of Duopoly", by Robert F. OWEN and Stylianos PERRAKIS

In addition to a certain number of expository changes and typographic errors which were rectified in the July version of the revised paper (available on request from the authors), please note the following more substantive modifications of the May draft:

1. Page 17; The paragraph continued from p. 16 should read as follows from "In the specific case of .....": "In the specific case of a linear demand and cost functions it is shown in appendix B that the strategic output of $X_2$ is strictly greater under uncertainty than that obtained from equation 6. This implies a lower export volume, as well as a lessened foreign currency value of exports, since $P_2$ decreases in this instance. Hence, exchange rate............"

2. Page 19; Graph 5a should be eliminated from the text; figure 5b should be relabeled as figure 5; and the top of page 19 should read as follows: "This pair of ex ante production strategies, symbolized by $N_1$ an $N_2$ for the two firms, is illustrated in figure 5. For the case shown in this figure, there is now the familiar Cournot-Nash interdependence between both the firms' production decision, which is quite unlike the initial uncertainty scenario where the international corporation's exports were never constrained. In figures 5 and 6, $Q_m$ is the solution of equations (16) or (19) for $X_2 = 0$. From equations 17 and 18 it is ..........

3. Page 22, (three lines before the end of the text); There should also be included a reference to Spulber (1981).

4. In appendix A the nature of the conditions for existence of a duopoly equilibrium is explored.

5. In appendix B the specific case of a linear demand and cost functions, referred to on page 17, is analyzed in detail.
EXCHANGE RATE UNCERTAINTY AND STRATEGIC EXPORT BEHAVIOR: AN INTERNATIONAL MODEL OF DUOPOLY*

Robert F. OWEN,
Ecole Nationale d'Exportation
and l'Université de Rennes I
Stylianos PERRAKIS,
University of Ottawa

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ABSTRACT

The paper offers the first international model of duopoly in which the strategic interactions between the firms are examined in the presence of uncertainty. This framework of imperfect competition is used to explore certain implications of exchange rate uncertainty for the pricing, output, and export decisions of the firms, and the calculation of their exposure to exchange risk. In the proposed two-country setting the duopoly in the foreign country consists of the sales of a consumption good by a single home firm and the exports to that market by another firm, which also sells competitively in its domestic market. Assuming Cournot behavior, risk neutrality, and fixed capital investments, so that the firms' capacity constraints and cost functions are given in the short run, a static one-period analysis of the firms' ex ante production and ex post sales decisions in the two countries' markets is offered, based on dynamic programming techniques. In the ex ante period when production and hiring decisions are made, firms maximize expected profits while only knowing the overall exchange rate distribution. However, at the end-of-period an asymmetry arises between the two competitors since the international firm has the possibility of allocating its given output between domestic sales and exports, as a function of the observed value of the exchange rate.

The analysis yields several new results which contrast with the conclusions of previous research, frequently based on models which assume perfect competition and perfect goods arbitrage. Specifically, it is

* The authors should like to express their thanks for the comments of seminar participants at the H.E.C.-C.E.S.A. (France), Institute of International Economic Studies of the University of Stockholm, The Johns Hopkins University, and the University of Ottawa.
demonstrated that even under risk neutrality exchange rate uncertainty can influence the strategic production and export decisions, as well as the realized profits of both firms. The exact nature of the changes depend on the relative cost and price conditions in the two markets. In particular, exchange rate uncertainty, coupled with an ex post asymmetry in the allocative responses of the two firms, can change fundamentally the production behavior of the foreign firm, and thereby the nature of the game theoretic solutions which emerge between the two competitors. Moreover, as the variance of the uncertain exchange rate distribution decreases uniformly to zero, the uncertainty duopoly model only collapses to the certainty case when the international firm is constrained by its domestic market sales objectives. Although the foreign firm only sells in its own market, it is obliged to take into account the ex post export behavior of its international competitor, and in so doing behaves similarly to a Stackelberg leader. Thus, it is possible for a firm, whose activities are limited to its local market, to be exposed to exchange risk because of strategic import penetration. In contrast to the findings of other research, the exposure of the exporting firm to exchange risk involves a non-linear relation in the exchange rate. Furthermore, the variability in foreign currency earnings arises from uncertainty in foreign prices and export quantities which is generated endogenously in the model. The analysis also suggests scenarios in which the price volatility in the duopoly market will be less than that of the exchange rate. Such a divergence from purchasing power parity appears in accord with recent empirical experience.
I - INTRODUCTION

One of the most distinctive features of the post-Brettonwoods period of floating exchange rates has been the substantial variations in both nominal and real exchange rates across a wide range of currencies. Although there has been a marked development of international financial instruments and markets for covering exchange risk, the uncertainties associated with significant deviations of exchange rates from their purchasing power parity levels would appear to have presented a number of difficulties for specific corporations. Yet, until recently much of both the pertinent empirical and theoretical literature, as surveyed by Canzoneri et al (1984), Farrell (1983), and in an International Monetary Fund Study (1984), has minimized any potential biases floating exchange rates may have engendered for the pattern of international production, investment, trade flows, and corporate profitability. Clark and Haulk (1972), and Hooper and Kohlhagen (1978), for example, found little evidence that floating exchange rates have reduced the volume of international trade, while Cushman's research (1983) indicated some negative effects of real exchange rate changes for a minority of the fourteen bilateral trade flows which were examined.

At a theoretical level, Kenen (1965) and Kawai (1981) have analyzed the optimal financial behavior of a risk averse firm with international transactions, to show how complete hedging for future foreign exchange receipts can eliminate the trade, employment, and investment effects of real exchange risk. However, these authors did not consider the potential endogeneity of firms' foreign exchange earnings to changes in exchange rates. A number of other papers, including those by Clark (1973), Ethier (1973), Baron (1976b), Dumas (1978), and Hodder (1982), have been concerned with the determinants of a firm's exposure to exchange risk. In contrast to the present research these studies assume perfect competition so

1. Certain quite representative cases are discussed by Owen (1985). In the public works industry, for example, where many contracts are undertaken in developing countries, exchange rate movements have often dwarfed the profit margins on even medium term contracts, which are frequently as low as six to twelve percent. The latter can be contrasted with, for instance, the hundred and twenty one percent change between 1980 and 1982 in the real exchange rate of the Mexican peso against the U.S. dollar (See Owen (1985) for other examples.).
that the exchange rate enters linearly in the calculation of a firm's exposure. Clark recognized that exchange risk may be inevitable when either forward markets are incomplete, or when the foreign currency price of a traded good is random, so that the value of foreign exchange to be covered is uncertain. On the other hand, Baron proposed a model in which "uncertainty regarding exchange rates has no effect on trade" (p. 253) and investors are indifferent to the covering decisions of firms. In a generalization of this latter analysis Dumas also concluded that the trading decision of the firm is independent of exchange rate and inflation risks. On the basis of a simplified 2-country financial model, where firms' production decisions are not explicitly considered, Hodder has shown that exposure depends on correlations between exchange rate movements and unanticipated changes in asset and goods prices. 2

Unlike the previously mentioned work, both Baron (1976) and Hu (1975) investigate certain of the effects of exchange rate uncertainty in models which assume imperfect competition. However, unlike the present research these authors are not concerned with the game-theoretic interactions arising between firms. In the context of a general model of oligopolistic behavior Baron demonstrated that the firms pricing strategies depend on risk aversion and the currency of invoicing. Hu considered the effects of export price uncertainty on the output and intermarket allocations of a discriminating monopolist selling in both a domestic and foreign country. Depending again on the attitude toward risk, and whether the firm has ex post flexibility to reallocate its international sales in light of revealed exchange rate changes, the monopolist's output and export levels can be reduced. More recently, Batra and Hadar (1979), Itagaki (1981), and Katz et al (1982) have also proposed models of monopolistic price discriminating firms or multinational corporations which are subject to exchange risk exposure through their foreign market sales. 3 Yet, unlike the present research, a feature common to all these studies is their specification of foreign demand (expressed in terms of foreign currency units) as an exogenous function which is independent of the exchange rate uncertainty. 4

2. For quite different reasons from those which are identified in the present paper, Hodder also concludes that purely domestic firms with no foreign assets or liabilities will typically be exposed to exchange risk.
4. Moreover, certain of these authors' conclusions differ from those deduced here. For example, both the Batra and Hadar, and Itagaki models suggest that optimal forward exchange cover can be used to eliminate the firms' risk exposure. Itagaki also maintains that exchange rate uncertainty may actually encourage international trade and production.
In the current paper a specific model is proposed that highlights the extent to which quantity and price uncertainty can be generated endogenously by an uncertain exchange rate, thereby leading to endogenous market shares which can only be partially hedged, independent of the completeness of forward exchange markets. As shown here, an uncertain exchange rate can generate uncertainty not only in the receipts of a firm which has international sales, but also for one that has no exporting activity. This may happen because such a firm faces import penetration which is influenced by the revealed value of the exchange rate. The two firms compete with each other in a model of duopoly under uncertainty, in which both their production decisions are made in advance of knowledge about the prevailing exchange rate. However, there remains an important structural asymmetry between the two competitors, since the international firm can use such subsequent information about exchange rate movements in its allocative strategy between exports to the foreign market and domestic sales. The equilibrium of the duopoly game in the foreign market and domestic sales. The equilibrium of the duopoly game in the foreign market determines both the necessary conditions for, and the nature of, the competition which will arise between the firms, as well as their relative exposure to foreign exchange risk.

Models of imperfect competition in a partial equilibrium context are relatively new in the international trade and finance literature, although they have a long history in industrial organization. In particular, duopoly models under uncertainty were presented by Perrakis and Warskett (1983, 1986). In these models the uncertainty was of the demand type, and it was shown to have major effects upon the production and investment decisions of the firms, even under risk neutrality. Similar effects are also shown in this paper with respect to production and export behavior, as well as exposure to exchange risk.

5. The general observation that there can be a number of different scenarios in which exchange risk is only partially hedgeable is convincingly argued by Plaut (1984) in a recent unpublished paper. In addition to the cases where forward exchange rates are biased predictors of future spot rates or no forward contract is feasible, Plaut points out that exchange risk is only hedgeable when there is no other source of uncovered risk or uncertainty. The latter include cases where the timing of a foreign exchange earning is variable, and where there is exogenous uncertainty regarding either the quantities sold or internal market prices. He also contends that forward exchange may be preferable to forward contracts when there are uncovered risks or other sources of uncertainty. A similar point is made in this paper.

The plan of this paper is as follows. In the next section the general structure of the model is presented. This is followed by a more detailed analysis of the sales and production strategies of the firms when the exchange rate is known with certainty. The case of an uncertain exchange rate is subsequently studied on the basis of a dynamic programming framework which considers both the ex ante and ex post optimizing behavior of the two firms. The paper's fourth section focuses on the ex post, international marketing decisions of the domestic firm, once information regarding the prevailing value of the exchange rate has been revealed. In contrast, both firm's production decisions at the beginning of period must be formulated while only knowing the general distribution of the uncertain exchange rate. In the fifth section the nature of this optimal ex ante behavior is characterized. The uncertainty scenario is also contrasted with that for a fixed exchange rate, and the exchange risk exposure for both the firms is examined. Finally, a concluding section summarizes the basic contributions of the present research, suggests several specific directions for extending the analysis, and identifies some more general questions that merit further investigation.

II - THE BASIC MODEL

Let the subscripts \( i = 1, 2 \) be used to denote respectively a domestic and a foreign country. In the former of these there is an international firm, 1, which sells a single non-storable commodity both at home and abroad. In contrast, firm 2 produces this same good uniquely for sales in the foreign market in which it is the only such producer, but competes with the imports by firm 1. It is assumed that the international firm operates under perfect competition in its domestic market which has a given price of \( p_1 \). Let also \( e \) denote the exchange rate between the two countries, in domestic currency units per unit of foreign currency. The tariff and transportation costs between the two markets are denoted by \( T \), and are expressed in terms of the foreign currency. If \( x_1 \) and \( x_2 \) denote the sales of output of firms 1 and 2 in the foreign country, the inverse demand curve in that market is given by \( p_2(x_1 + x_2) \). The cost functions of the two firms are represented by \( c_1(...), \) for \( i = \)

7. In a variant of the proposed model which is being analyzed in ongoing research, the domestic firm may decide to set up a subsidiary in the foreign country, producing an output \( x_1^* \) in addition to the exports \( x_1 \). The total sales in country 2 are then equal to \( x_1 + x_1^* + x_2 \), and \( p_2(x_1 + x_1^* + x_2) \) is the corresponding foreign country price. The subsidiary's cost function is \( c_1^*(x_1^*) \), where, in general, it is expected that \( c_1^*(x) < c_2(x) \forall x \), so that the multinational firm 1 is more efficient than the purely domestic firm 2.
1, 2, and are assumed to be convex within the relevant range of outputs. Since the production of firm 2 equals its sales, its total costs are equal to $c_2(x_2)$. However, firm 1 has total costs of $c_1(q_1)$ where its total output of $q_1$ will in general exceed its exports by the amount of the domestic sales, $q_1 - x_1$.

In order to capture the implications of market constraints and interaction effects between the domestic and foreign countries for the allocative decisions of the international firm, it is assumed that these domestic sales will not be allowed to fall below a certain limit, which signifies a minimum share which firm 1 seeks to preserve in its own market.  

In the uncertainty version of the model a single period is considered, in which the production decisions of both firms are taken ex ante, i.e. before the exact value of the uncertain exchange rate, $e$, has been observed. By contrast, the sales decisions, regarding the amounts in which the total production of the international firm is divided between its home market and exports, are taken ex post, after the value of $e$ has been revealed. The ex ante distribution of the exchange rate, $F(e)$, which has a range $[e_1, e_2]$ within which $dF$ is positive, is assumed to be common knowledge. In addition, prices, the demand and cost functions, and the quantities produced are common knowledge and non-random. Risk-neutrality is initially assumed for both firms.

The revenue functions for the duopolists in the foreign market, $x_i[p_2(x_1 + x_2)]$ for $i = 1, 2$, are assumed to have the following properties:

$p_2 < 0, x_1p_2' + p_2 > 0, 2p_2' + x_1p_2'' < 0, x_1p_2'' + p_2 < 0$. These inequalities embody the standard assumptions of a downward-sloping demand curve, positive and declining marginal revenues, and a negative effect of an increase in the output of one firm upon its competitor's marginal revenue. They are automatically satisfied if $p_2(x_1 + x_2)$ is concave, but are more general than concavity.

8. An alternative formulation of the constraints the domestic market conditions could impose upon the behavior of the exporting firm involves a limit-entry pricing argument. Under this specification firm 1 could export as large a share of its production as desired, provided the price differential between the two countries' markets does not exceed a certain limit, $L$, so that: $ep_2(x_1 + x_2) - p_1 < L$. The rationale for such a formulation is that in light of the fixed non-recoverable costs involved in penetrating a foreign market the exporting firm would have an incentive to voluntarily limit prices in the foreign market in order to deter entry by rivals.
III - THE EXPORT AND PRODUCTION STRATEGIES UNDER CERTAINTY

In order to provide a benchmark for the evaluation of the effect of exchange rate uncertainty, we first begin our analysis with the certainty case. Under certainty, the production and export decisions are taken simultaneously. Cournot assumptions are adopted for these decisions, which means essentially, that each firm maximizes profits given the opponent's production strategies. The Cournot-Nash equilibrium is the pair \((Q_1, x_2)\) that satisfies each firm's conjectures about the opponent's strategy while simultaneously maximizing the firm's expected profits given that strategy.

Since the cost and demand functions are assumed common knowledge, the production and export decisions for both firms are found for an given value of the exchange rate, \(\bar{e}\), by the simultaneous solution of the maximization problems (1) and (2) below:

\[
(1) \quad \text{Max} \{ p_1 \text{Max}[q, Q_1 - x_1] + e \hat{x}_1 [p_2(\hat{x}_1 + x_2) - T] - c_1(Q_1) \}
\]

where \(\hat{x}_1 = \text{Min}(x_1, Q_1 - q)\)

\[
(2) \quad \text{Max} \{x_2, p_2(\hat{x}_1 + x_2) - c_2(x_2)\}
\]

The solution of the first problem for a given \(x_2\) defines a reaction function in the foreign market sales of the domestic firm, \(\hat{x}_1 = R_1(x_2)\), which is decreasing according to our assumptions. For a sufficiently large value of production (sales) by firm 2, \(\hat{x}_2, R_1(x_2)\) equals zero, and exports are blocked from the foreign market. On the other hand, if the output of the foreign firm declines to zero, the exporting firm may or may not be constrained in its foreign sales by its domestic market share objective \(q\). Firm 1's overall production level for both the unconstrained and constrained cases is the solution, \(\hat{Q}_1\), to the equation:

\[
(3) \quad p_1 = c_1'(Q_1)
\]

Thus, in light of the competitive market conditions domestically, the exporting firm's output decision is independent of both the exchange rate and the foreign production strategy. If the firm is unconstrained, so that \(R_1(0) <\)
\( Q_1 - q \), the level of firm 1's exports when its foreign competitor is out of the market is given by the more general solution for its reaction function, \( R_1(x_2) \). The latter is obtained from:

\[
(4) \quad x_1 \ p_2'(x_1 + x_2) + p_2(x_1 + x_2) = \left( p_1/e \right) + T
\]

However, when the domestic firm is constrained in its export behavior, so that \( Q_1 - q = R_1(x_2) \) for some \( x_2 \), equation 4 only characterizes firm 1's reaction function for large values of \( x_2 \). Otherwise, the export reaction function for firm 1 equals those values of \( Q_1 - q \) which solve the following equation.

\[
(5) \quad (Q_1 - q) \ p_2'(Q_1 - q + x_2) + p_2(Q_1 - q + x_2) = \frac{c_1'(Q_1)}{e} + T
\]

The reaction function for the foreign firm, \( R_2(\hat{x}_1) \), involves values of \( x_2 \) which are solutions to:

\[
(6) \quad x_2 \ p_2'(x_1 + x_2) + p_2(x_1 + x_2) = c_2'(x_2)
\]

where \( \hat{x}_1 = \min (x_1, Q_1 - q) \). Note that \( R_2(\hat{x}_1) \) equals zero for those levels of exports that deter home production in the foreign country. In contrast, when domestic exports equal zero, one obtains the level of monopoly production, \( M_2 \), in country 2. In comparison to the subsequent analysis under uncertainty, it is worth remarking that the foreign firm's output and sales decision is not directly related to the exchange rate.

For a Cournot-Nash duopoly equilibrium to exist the reaction functions for the foreign market sales of each firm must intersect as depicted in one of the three Figures 1a, 1b or 1c. Note that in the equilibria
depicted in the latter two graphs the domestic firm's optimal export levels are constrained by its home market sales objectives. In order for there to be existence of the joint equilibria given by these three cases, which involve both home production and exports, the following two inequalities must hold:

\[(7) \quad R_1(0) < \{ x_1 \mid R_2(x_1) = 0 \} \]

\[M_2 < \{ x_2 \mid R_1(x_2) = 0 \} \]

In light of equations 4 through 6, a more explicit form of these necessary and sufficient conditions for existence of a Cournot-Nash equilibrium is specified respectively by either equations 8a or 8b (depending on whether \( R_1(x_2) > \hat{Q}_1 - q \) and equation 9 below:

\[(8a) \quad \langle x_1 \mid x_1 \cdot p'_2(x_1) + p_2(x_2) = \frac{c_1'(Q_1)}{\bar{e}} + T \rangle < \langle x_1 \mid p_2(x_1) = c'_2(0) \rangle \]

\[(8b) \quad \langle Q_1 - q \mid (Q_1 - q) \cdot p'_2(Q_1 - q) + p_2(Q_1 - q) = \frac{c_1'(Q_1)}{\bar{e}} + T \rangle < \langle x_1 \mid p_2(x_1) = c'_2(0) \rangle \]

\[(9) \quad \langle x_2 \mid x_2 \cdot p'_2(x_2) + p_2(x_2) = c'_2(x_2) \rangle < \langle x_2 \mid p_2(x_2) = \frac{c_1'(Q_1)}{\bar{e}} + T \rangle \]

On the basis of equations 4, 5, and 6, it is apparent that the equilibrium export quantity and export revenues are in general both non-linear functions of the exchange rate. A final observation is that although the foreign corporation has no international activities, its output also depends on the exchange rate, by way of a strategic interaction with the import penetration it faces.
IV - THE EX POST EXPORT AND SALES STRATEGIES UNDER EXCHANGE RATE UNCERTAINTY

The implications of exchange rate uncertainty for the production and sales decisions are now assessed by using dynamic programming techniques to analyze the firms' ex ante and ex post decisions during a single period. In the ex ante period when the production and hiring decisions are made, firms maximize expected profits while only knowing the overall exchange rate distribution. However, at the end-of-period an asymmetry arises between the two firms since the exporting firm has the option of allocating its given output between the domestic and foreign markets as a function of the observed value of the exchange rate. In contrast, the foreign firm sells its entire output in its local market.

Cournot behavior is again assumed, so that each firm determines its ex ante production strategies, $Q_1$ or $x_2$, by maximizing its expected profits given the opponent's production level. In contrast to the approach, adopted for example, by Hu (1975), the firms are taken to be risk neutral. This assumption is used to highlight the extent to which exchange rate uncertainty per se, rather than attitudes towards risk, can entail real effects for corporate decisions and performance. Finally, at the beginning of the period, the firms' cost functions are given. Hence, the investment decisions leading to the determination of their production capacities are not considered.

The determination of equilibrium is solved by working backwards from the end of the period. At that time, the total amounts produced, $Q_1$ and $x_2$, as well as the exchange rate, $e$, are given, and the only decision is how firm 1 should split its output between domestic consumption and exports. Since the production costs have already been incurred, that ex post decision is taken by maximizing net revenue. Therefore, the domestic firm's ex post maximization problem is:

$$\begin{align*}
(10) \quad \text{Max} \quad & (p_1 (Q_1 - x_1) + e x_1 [p_2 (x_1 + x_2) - T]) \\
\text{w.r.t.} \quad & x_1
\end{align*}$$

This yields the first-order condition:

$$\begin{align*}
(11) \quad & e [p_2 (x_1 + x_2) + x_1 p'_2 (x_1 + x_2) - T] = p_1
\end{align*}$$

9. Modifying this assumption is a worthwhile exercise which as shown in certain of the previously discussed studies can yield further insights concerning the effects of exchange rate uncertainty.
From equation (11) the reduced form of the exports function, \( x_1 = \gamma(x_2, \bar{e}, p_1, T) \), can be obtained. This function is homogeneous of degree zero in the domestic country's price and the exchange rate. Its derivatives have the following signs, which are the consequences of the assumed properties of the demand and revenue functions:

\[
\frac{\partial Y}{\partial x_2} < 0, \quad \frac{\partial Y}{\partial p_1} < 0, \quad \frac{\partial Y}{\partial e} > 0, \quad \frac{\partial Y}{\partial T} < 0
\]

For any given value of \( \bar{e} \), it is apparent from the first-order condition that firm 1 will export, if and only if the resulting price adjusted for the tariff and exchange rate, is higher in the foreign country than in the home market. Consequently, for given values of the domestic price and tariff, the equation, \( p(x_2) = (p_1/\bar{e}) + T \), defines an interval of production levels for the foreign firm which depends on the spectrum of exchange rate values \( \bar{e} \in [\underline{e}, \bar{e}] \). Each of these production levels represents an export-blocking level of firm 2, denoted by \( \hat{x}_2(\bar{e}) \), which is an increasing function of \( \bar{e} \). Hence, for the range of export-blocking levels, \( \hat{x}_2(\bar{e}) \in (\hat{x}_2(\underline{e}), \hat{x}_2(\bar{e})) \), there exists a partition for the interval of exchange rate values such that there are no exports (\( Y = 0 \)) in the first subinterval, and positive levels (\( Y > 0 \)) in the second. This partition is represented by \( [\underline{e} \geq \hat{e}(x_2)], [\hat{e}(x_2), \bar{e}] \).

10. Such a price differential between the two markets, and/or a difference in the cost functions of the two firms, could be used to motivate the fact that the foreign firm does not have any international activity.
For those levels of foreign firm production which are below the minimal export blocking level, $\hat{x}_2(e)$, the international firm will always sell in the foreign market, regardless of the value of the exchange rate. The specific amount of its exports is given by $\gamma(x_2, e, P_1, T)$ up till the point at which the domestic firm's international sales are constrained by its home market share objective. The share of residual output available for exports then equals $Q_1 - q$. The level of foreign firm output such that the domestic firm's optimal export volume just equals this residual share is the solution (if any) to the equation $\gamma(x_2, \bar{e}, P_1, T) = Q_1 - q$, and can be denoted by $x_2(\bar{e})$. Since $\gamma$ is an increasing function of the realized value for the exchange rate, there exists a range of exchange rates for levels of foreign production less than $x_2(\bar{e})$, such that the international firm's export level is constrained by its home market sales. Indeed, for $x_2 < x_2(\bar{e})$ the domestic firm's exports are always constrained, and thus independent of the exchange rate. A representative figure summarizing the functional relation between the firm's exports, the exchange rate, and its competitor's production strategies is given below. The ex post foreign market sales equilibrium will be determined.

![Figure 2](image-url)
by the intersection of one of the set of possible reaction functions for the domestic firm with a vertical reaction function for the foreign firm, where the latter's position is predetermined by that firm's ex ante production decision.

From the ex post export functions it is possible to derive several properties of the foreign product price, as a function of the exchange rate. This product price, expressed in terms of the domestic country's currency, is equal to $\bar{e}p_2(x_2 + \gamma(x_2, \bar{e}))$, which is clearly a nonlinear function of $\bar{e}$. By analyzing the elasticity of $\bar{e}p_2$ with respect to the exchange rate, it is straightforward demonstrating that the price volatility of the consumption good in the duopoly market will be less than that of the exchange rate for the important cases where the domestic firm's (non-zero) exports are unconstrained by the home market sales objective. Such a relation, which could be used to offer a partial explanation for both divergences from the law of one price and real exchange rate undershooting or overshooting, accords well with recent empirical evidence regarding the relative higher volatility of exchange rates compared to the prices of traded goods.\(^{11}\)

Specifically, for $x_2 \in (\bar{x}_2, \bar{e}, \bar{x}_2(\bar{e}))$ the ratio $(1/p_2) \frac{\partial(\bar{e}p_2)}{\partial\bar{e}}$, which equals $1+(p'/p_2) \frac{\partial\gamma}{\partial\bar{e}}$, is clearly less than one for all values of $\bar{e}$ above the level where exports are blocked; whereas it only equals one when the domestic exchange rate is sufficiently appreciated so that exports are actually blocked. Similarly the foreign price expressed in domestic currency units adjusts less than proportionately to the exchange rate changes for all $\bar{e}$ if $x_2 \in (\bar{x}_2, \bar{e}, \bar{x}_2(\bar{e}))$. Moreover, the elasticity of $\bar{e}p$ with respect to $\bar{e}$ only equals one for all $\bar{e}$ if $x_2 \leq \bar{x}_2(\bar{e})$, because the volume of exports is insensitive to exchange rate variations given the need to preserve the domestic market share.

\(^{11}\) See, for example, the comparison, discussed by Frenkel and Mussa (1980), between the greater variability of the $$/DM exchange rate relative to that of the consumption price indices in the United States and West Germany.
V - EX ANTE PRODUCTION DECISIONS UNDER EXCHANGE RATE UNCERTAINTY

In order to complete the analysis of the uncertainty scenario, the ex ante, beginning-of-period production decisions of the two firms, \( Q_1 \) and \( x_2 \), need to be determined under the same assumptions of Cournot behavior. As will become subsequently apparent, several different solutions emerge depending not only on the constraints the international firm faces in its home market, but also on whether both firms compete in the foreign market. The principal focus will be on the case where the necessary and sufficient conditions for the existence of a duopoly equilibrium on the foreign market are satisfied. The initial solution considered is for the case where the international firm's exports are unconstrained by its home market sales objectives. This scenario is then compared to the constrained cases which arise when the exporting firm's foreign earnings are suboptimal for either some or all of the potential ex post values of the exchange rate. Of particular interest is the contrast between these alternative solutions under uncertainty with those which were obtained in Section III for a fixed exchange rate.

The ex ante output decision of the international firm under the Cournot-Nash assumptions is determined by maximizing its expected profits, given the reduced form for its ex post exports function:

\[
(12) \quad \text{Max} \quad E \left\{ p_1 \text{Max}[q, Q_1 - \gamma(x_2, \tilde{e})] + \right. \\
\quad \quad \quad \quad \left. \tilde{e} \text{Min}[Q_1 - q, \gamma(x_2, \tilde{e})] [p_2(x_2 + \text{Min}[Q_1 - q, \gamma(x_2, \tilde{e})] - T] - c_1(Q_1) \right\}
\]

Similarly, the optimal production level for the foreign firm consists of the solution to:

\[
(13) \quad \text{Max} \quad E \{ x_2 p_2(x_2 + \text{Min}[Q_1 - q, \gamma(x_2, \tilde{e})]) - c_2(x_2) \}
\]

When the international firm is unconstrained by its home market sales target, the first order condition for equation 12 involves a particularly simple form which is the same as the earlier certainty solution:

\[
(14) \quad P_1 = c_1'(Q_1)
\]

Thus, in this instance it is only the competitive price in the home market which determines the exporting firm's output level, while the foreign market price, the exchange rate, and the output of its competitors are irrelevant. Hence, the domestic firm's output reaction

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12. The nature of these conditions for existence of a duopoly equilibrium is explored in Appendix A. Alternative outcomes, where one of the firms alters its behavior in a strategic manner in order to drive its competitor out of the foreign market, are also briefly considered there.
function, \( R_1(x_2) \), consists of an horizontal line. Note that the expression for the exchange risk exposure of the domestic firm is given by \( E \left( \gamma(x_2, \tilde{e}) [e(p_2(\gamma(x_2, \tilde{e}) - T) - p_1] \right) \). This is a non-linear function of the uncertain exchange rate, and reflects variable foreign currency earnings. In this model the uncertainty in foreign prices and export quantities is generated endogenously, because of their dependence on the ex post realized value of the exchange rate.

Unlike its international competitor, the optimal production decision of the foreign firm in this unconstrained case is dependent on the uncertainty in the exchange rate, due to the strategic import penetration it faces. This is apparent from the firm's production reaction function, \( R_2(Q_1) \) which can be derived from the first order condition for equation 13:

\[
(15) \ E \left( x_2 p_2(x_2 + \gamma(x_2, \tilde{e})) [1 + \frac{\partial \gamma}{\partial x_2}] + p_2(x_2 + \gamma(x_2, \tilde{e})) \right) = c_2'(x_2)
\]

Note that although this reaction function is independent of \( Q_1 \), its position is affected by the specific form of the ex post export function, \( \gamma(x_2, \tilde{e}) \). The pair of \textit{ex ante} equilibrium strategies, \( \hat{Q}_1 \) such that equation 14 holds, and \( x_2 \) equals a constant, \( S_2 \), is illustrated in Figure 3 by the intersection of \( R_1(x_2) \) and the vertical part of \( R_2(Q_2) \). Thus, the foreign firm's reaction function involves a necessary condition such that it acts

13. The non-vertical part of \( R_2(Q_1) \) reflects sufficiently low levels of production by the international firm, such that its export strategy is constrained by the domestic sales objective and \( \gamma(S_2, \tilde{e}) < Q_1 - q \).
as an ex post Stackelberg leader in its own market, since its sales decision is made prior to that of its import competition. Alternatively, the foreign firm can be viewed in this instance as a principal agent acting in a beginning-of-period Cournot-Nash framework. The payoff to the foreign firm depends both on its production commitment in advance of sales and the subsequent state of nature represented by the uncertain exchange rate. Of course, the underlying reason, explaining this principal agent setting, or alternatively the nested Cournot-Nash-Stackelberg game, involves the basic asymmetry between the duopolists in the timing of their foreign market sales commitments.

A further remark concerns the exchange risk exposure of the foreign firm, which is specified by \( E \langle x_2 p_2(x_2 + \gamma(x_2, \tilde{u})) \rangle \). Depending on the form of the demand function, this may also be non-linear in the exchange rate. Thus, even a corporation whose sales are limited to its home market can be exposed to exchange risk by means of the impact of strategic import penetration on the prevailing home market price. Nonetheless, one rarely hears of such firms using financial instruments to cover their exchange risk. A final insight concerns the effect of

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14. A concrete example of such exchange risk exposure occurred in the United States cement industry during the 1981 to mid-1985 period. With the pronounced depreciation of the Japanese and European currencies relative to the dollar, there was a marked increase in the market shares of foreign competitors.
uncertainty per se on the production and sales decision of the foreign firm. This can be ascertained by comparing the firm's reaction functions alternatively for the certainty and uncertainty scenarios, given the assumption that the value of the certain exchange rate under consideration equals the expected value of the uncertain rate. In the specific case of a linear demand function, the first term in equation 15, which is in general negative equals zero, so that the strategic output of $x_2$ is strictly greater under uncertainty than that obtained from equation 6. This implies a lower export volume, as well as a lessened foreign currency value of exports, since $p_2$ decreases in this instance. Hence, exchange rate uncertainty can engender a reduction in international trade even when agents are risk neutral, while affecting the production decisions of firms without any international sales more than those of international corporations.

A different uncertainty scenario to the one which has just been analyzed applies when for relatively depreciated values of the home currency —above a critical level, $e'$— the international firm's export strategy is constrained by its domestic sales objective, $q$. The consideration of such a case permits a focus on the role of market interdependence in defining a corporation's optimal production strategies in an uncertain international environment. For $\gamma(x_2, e')$ less than $Q_1 - q$, the domestic firm's ex post revenues are given by $p_1 [Q_1 - \gamma(x_2, e')] + \bar{e} \gamma(x_2, \bar{e}) [p_2 (\gamma(x_2, \bar{e}) + x_2) - T]$, whereas for more depreciated values of the exchange rate above $e'$ the revenues equal $p_1 q + e (Q_1 - q) [p_2 (Q_1 - q + x_2) - T]$. In light of the difference in these ex post formulas, it is apparent that the exchange risk exposure for both firms is a weighted average, depending on the probability distribution function $F(e')$, of their respective exposures for the unconstrained and constrained solutions. Figure 4 depicts the set of ex post outcomes for this intermediate scenario (identified as ii), which corresponds to a truncated subset of those shown in Figure 3 (represented here by i). Note that the size of this subset is determined by the strategic production play of the foreign firm, which also identifies the critical value of $e'$ at which the international firm's exports become constrained.
In light of the foregoing characterization of the ex post solutions for this intermediate scenario, the ex ante production decision of the domestic firm can be obtained from the maximization of the objective function for its expected profits:

\[
(16) \quad \max \mathbb{E} \left\{ \int_{Q_1}^{q_1} p_1(Q_1 - \gamma(x_2, \tilde{e})) + \tilde{e} \gamma(x_2, \tilde{e}) \left[ p_2(\gamma(x_2, \tilde{e})+x_2) - T \right] dF(\tilde{e}) \right. \\
\left. + \int_{e'}^{e} \left[ p_1 q + \tilde{e}(Q_1 - q) \right] \left[ p_2(Q_1 - q + x_2) - T \right] dF(\tilde{e}) - c_1(Q_1) \right\}
\]

This yields the following first order condition:

\[
(17) \quad p_1 f(e') + \int_{e'}^{e} \left[ (Q_1 - q) p_2^2 (Q_1 - q + x_2) + p_2 (Q_1 - q + x_2) - T \right] dF(\tilde{e}) = c_1'(Q_1)
\]

Similar reasoning can be used to derive the first order condition which determines the output level of the foreign firm:

\[
(18) \quad \int_{e'}^{e} \left[ x_2 p_2^2 \left( x_2 + \gamma(x_2, \tilde{e}) \left( 1 + \frac{\partial \gamma(x_2, \tilde{e})}{\partial x_2} \right) + p_2 (x_2 + \gamma(x_2, \tilde{e})) \right) dF(\tilde{e}) \right. \\
\left. + [x_2 p_2 (Q_1 - q + x_2) + p_2 (Q_1 - q + x_2)] (1 - f(e')) \right) = c_2'(x_2)
\]
(19) \( (Q_1 - q)p_2'(Q_1 - q + x_2) + p_2(Q_1 - q + x_2) = \frac{1}{E(\hat{u})}c_1'(Q_1) + T \)

(20) \( x_2p_2'(Q_1 - q + x_2) + p_2(Q_1 - q + x_2) = c_2'(x_2) \)

Figure 6

relative to 19 and 20 that this solution, in which the minimum home market share objective prevents the equalization of marginal revenues (adjusted for tariffs and the exchange rate) in the two markets, yields an equivalent Cournot-Nash interdependence to that under a certain exchange rate (which equals the expected value of \( \hat{u} \)). By contrast, there is no correspondence of the solutions in the other scenarios of the model; cases that cover the empirically most meaningful behavior of firm 1 in which its exports are dependent on at least some of the revealed values of the uncertain exchange rate.

As previously remarked, when the international firm produces at \( Q_1 \) which is determined by the competitive price in its home market, the certainty and ex post export functions are the same. However, the production decisions of the foreign firm are fundamentally different under certainty and uncertainty. Whereas the output is given by equation 6 in the former case, \( x_2 \) is determined by equation 15 under uncertainty. These correspond respectively to Cournot and Stackelberg-type solutions. The solution expressed in equation 18, on the other hand, consists
of a combination of the constrained and unconstrained analyses.

Hence as the range of the exchange rate distribution, \([e, \bar{e}]\),
decreases uniformly to \(E(\bar{e})\), the uncertainty model collapses to the
certainty case if and only if the international firm is constrained by
its home markets sales objective and \(\bar{e}\) equals its expectation with
probability one. Otherwise, uncertainty in the exchange rate, coupled
with an asymmetry in the ex post marketing options of the two firms
alters fundamentally the structure of the productive behavior of the
foreign firm by obliging it to behave like a Stackelberg leader. The
essential role of this asymmetric, ex post flexibility is suggested by a
consideration of the duopoly solution in which the domestic firm is
exclusively an exporter.

VI - CONCLUSION

The basic contribution of this paper has been to establish how
under imperfect competition exchange rate uncertainty can engender
changes in the strategic interactions between firms, thereby altering in
certain cases production levels, exports, prices, and corporate
profitability. In doing so, the analysis of duopoly in an international
environment has been extended to consider certain implications of
incorporating uncertainty. The model highlights the ex post variability
of foreign sales earnings due to quantity and price changes which are
generated endogenously as a result of the exchange rate uncertainty. It
is worth reemphasizing that the demonstrated real effects of such
uncertainty arise even under assumptions of risk neutrality, but are
essentially related to the asymmetry between the greater ex post
marketing flexibility of an international corporation relative to its
purely local competitor in a given foreign market. More specifically, it
it shown that exchange rate uncertainty changes fundamentally the game-
theoretic solutions by obliging the latter firm to behave like an ex
post Stackelberg leader when it formulates its output decision. Thus,
even a firm without any international activities can be exposed to
exchange risk because of strategic import penetration. In contrast to
certain other analyses, the exchange risk exposure of the firms in the
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