Seminar Paper No. 379

MONETARY POLICY AND BANK REGULATIONS
IN AN ECONOMY WITH FINANCIAL INNOVATIONS

by

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ISSN 0347-8769

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Financial support from Jan Wallanders stiftelse för samhälls-
vetenskaplig forskning and the Bank of Sweden Tercentenary
Foundation is gratefully acknowledged. I also wish to thank
Lars Svensson for helpful discussions and comments.

Seminar Papers are preliminary material circulated to
stimulate discussion and critical comment.

April, 1987

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Abstract

This is a study of financial innovations and moves towards "the cashless society" in a general equilibrium cash-in-advance model. It is assumed that a subset of goods - cash goods and check goods - can only be purchased with tangible means of payment, i.e. cash and checks drawn on interest-bearing bank accounts. Financial innovations are modelled as a decrease in the fraction of such goods.

In this world monetary policy and bank regulations have welfare effects. A main result is that Friedman's (1969) optimum quantity of money rule continues to hold in this setting. It does so because a non-zero interest rate distorts the composition of consumption.

This general result is translated into results on the optimum rate of expansion of the supply of base money under different assumptions. We also study optimum reserve requirements on banks.

1. Introduction

In recent years we have witnessed a number of innovations in the payments system. Most striking is perhaps the increased use of credit cards and similar systems, where payments are effected by bookkeeping entries in an accounting system handled by banks or other institutions rather than by the exchange of tangible means of payment.

Parallel to this development in the payments system there has been an increased interest among economists in the viability of systems that operate without any tangible medium of exchange at all. This literature, which contains papers by Black (1970), Fama (1980,1983), Hall (1982,1983) and Greenfield and Yeager (1983) is usefully surveyed by McCallum (1985). These authors in general strongly endorse the abolition of government regulations of the banking industry such as reserve requirements, interest ceilings etc, a conclusion which is generally reached by showing that the price level would indeed be determinate even in the absence of any regulations of the bank sector. The literature is, however, notable by its absence of any formal model within which to make welfare evaluations of this claim. Also, it appears that most of the contributors, implicitly at least, presume that there is no need for any systematic monetary policy in the sense of a rule for the rate of change of the supply of reserves.

This paper attempts to fill a gap in the literature by analyzing these issues in a stochastic general equilibrium model, within which welfare evaluations can be made. The demand for means of payment arises from cash-in-advance constraints along the lines of Lucas (1982), Svensson (1985) and others. The payments system is characterized by assuming that there are three types of goods in the economy: cash goods,

which can only be purchased with coins and notes issued by the central bank (we are not concerned with the question of privately issued bank notes), check goods which can also be bought with checks drawn on interest bearing bank accounts, and credit goods where no particular medium of exchange is needed and transactions may be settled simply by debits and credits in accounts. By a financial innovation we will mean an increase in the share of credit goods at the expense of any of the other types of goods, but the model of course allows consideration of any specified change in the relative proportions of the three types of goods. The world envisaged in the studies cited above would be one where credit goods made up the large majority of all goods, but with - see in particular Fama (1983) - some proportion of cash goods.

The division of goods into three different categories, the proportions of which change stochastically over time should be seen as an analytical short-cut for specifying the more fundamental issues of economy in the exchange of information that determine the evolution of the payments system. There are some interesting attempts at formulating models that motivate the existence of tangible means of payment as a mechanism to reduce the informational requirements of exchange [see Brunner and Meltzer (1971) and King and Plosser (1986)]. While such a framework would allow welfare evaluations at a deeper level than does the setup used here, it would be much beyond the scope of this paper. There is also the route of analysis, pursued by e.g. Fischer (1983), King and Plosser (1984), and Kimbrough (1986a, b), which postulates transactions costs associated with the use of different payments system. It is not clear, however, that this is a more fundamental way of specifying the payments structure than the cash-in-advance approach adopted here.

Monetary models where the distinction between cash and credit goods

plays an essential role have been analyzed by Lucas (1984) and Lucas and Stokey (1983, 1984), and the framework of the present paper is similar to theirs. In including a third type of goods, check goods, we follow Englund and Svensson (1985). [See also Hartley (1985) for a micro-economic analysis of cash and check goods]. This introduces banks into the model. These are modelled simply as mutual funds with checking facilities, and are subjected to reserve requirements by the central bank. There is nothing in the model that distinguishes the banks' investment opportunities from those of the consumer. This is in contrast to much of the recent literature on the role of financial intermediaries [e.g. Diamond (1984), Boyd and Prescott (1986), Bernanke and Gertler (1986), and Williamson (1986)] which stresses the role of the bank system of monitoring debt contracts in a world of asymmetric information.

We are interested in evaluating two types of policies with regard to two types of criteria. The policies are bank regulation in the form of reserve requirements, and monetary policy, i.e. rules for the (helicopter) transfer of cash. The criterion is the welfare of the representative consumer.

The welfare effects that arise in the model are due to inefficiencies in the composition of consumption; policies will affect the interest rates and thereby the rates of substitution but not rates of transformation between the three types of goods.

The model is set up in section 2. The properties of the equilibrium are analyzed in section 3. Particular emphasis is put on the determination of the composition of consumption, which is what ultimately matters from a welfare point of view. It depends on the payments structure both directly and indirectly via effects from the payments structure on the interest rates. For the following analysis it is

convenient to consider cases where the interest rate is state independent. Two such cases are analyzed in section 4. In section 5 monetary policies, rules for the expansion of the money supply, and bank regulations in the form of reserve requirements are evaluated from a welfare point of view. It is shown that they basically follow from the zero interest rate rule associated with Friedman (1969). Section 6 summarizes the main findings and discusses implications for future research.

2. The Model

We consider an economy where there is one basic production process, which is treated as exogenous. Total production, Y , follows a first-order Markov process; $Y_t = (1+y_t)Y_{t-1}$. The rates of growth, y_t , are serially uncorrelated. There is a linear technology which allows transforming y into any of a continuum of differentiated goods. Assuming competitive behaviour all goods will command the same producer price in equilibrium, which will be normalized at unity. The payment structure is represented by the vector $s_t = \{s_{1t}, s_{2t}\}$, where s_{it} represents the fraction of all goods made up of cash and check goods respectively. It is assumed that the realizations of s_t are serially uncorrelated. One may think of different goods as being sold through different stores where the idiosyncracies of the shop-keepers determine whether checks and credit cards are accepted as means of payment. Financial innovations are taken to mean that more shop-keepers accept credit cards, i.e. a decrease in s_1 and/or s_2 .

The representative consumer maximizes expected discounted utility

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U_{\tau},$$

where his utility depends on his consumption of all differentiated goods according to the instantaneous utility function

$$U_t = u(v_t), \text{ where}$$

$$v_t = \left[\sum_{j=1}^J c_{jt}^{\rho} \right]^{1/\rho} \quad \rho < 1.$$

This expresses the assumption that all goods are equally close substitutes. From the consumer's point of view they only differ with regard to the means of payment that can be used. For that reason, and since the producer price of all goods is equal, the equilibrium will be such that all goods in the same category are consumed in equal quantity. Anticipating this we may rewrite the utility function as

$$U(C_1, C_2, C_3, s) = u(v) = u \left[[s_1 C_1^{\rho} + s_2 C_2^{\rho} + (1-s_1-s_2) C_3^{\rho}]^{1/\rho} \right],$$

where C_i ($i = 1, 2, 3$) is the per good consumption of cash goods, check goods and credit goods respectively. This means that the total consumption of type i goods is $s_i C_i$.

Let us now specify the timing of events, which is analogous with that in, e.g., Lucas (1982) but differs from that of Svensson (1985) and Englund and Svensson (1985). In the beginning of each period consumers and producers learn about the current state vector $\Omega_t = \{y_t, s_t, w_t\}$, where w_t (see below) is the rate of monetary expansion, which is assumed to be serially uncorrelated. Based on this and on their knowledge of consumer preferences producers decide how to divide total output between all different goods.

The consumer first trades in the asset market and divides his wealth between cash, checking account holdings, bonds and other assets, i.e. he faces the wealth constraint

$$(2.1) \quad W_t = \pi_t (H_{ht} + D_{ht} + B_{ht}) + r_t X_{ht} + q_t Z_{ht},$$

where

H = cash holdings,

D = checking account holdings,

B = bond holdings,

X = firm share holdings,

Z = bank share holdings,

π = price of cash in terms of goods,

r = price of firm shares in terms of goods,

q = price of bank shares in terms of goods, and

subscript h denotes household holdings of the asset in question. The

wealth constraint, W, is the result of choices in the previous period and

is given by

$$(2.2) \quad W_t = (\pi_{t-1} H_{ht-1} - s_{1t-1} C_{1t-1}) \pi_t / \pi_{t-1} + \\ [(1+i_{t-1}^d) \pi_{t-1} D_{ht-1} - s_{2t-1} C_{2t-1}] \pi_t / \pi_{t-1} + \\ + (1+i_{t-1}) \pi_t B_{ht-1} + (r_t + a_{t-1}) X_{ht-1} + (q_t + Y_{t-1}) Z_{ht-1} \\ - (1-s_{1t-1} - s_{2t-1}) C_{3t-1} \pi_t / \pi_{t-1} + \pi_t w_t \bar{H}_{t-1},$$

where

i = interest rate on bonds,

i^d = interest rate on checking accounts,

a = bank share dividends,

y = firm share dividends,

w = the rate of expansion of the supply of base money, and

$\bar{H}$ = the supply of base money.

This expression states that the wealth carried over into period t is the value of cash acquired in t-1 and not spent on cash goods purchases in that period, plus the value of unspent bank deposits including interest receipts, plus returns on other assets including dividends, minus

purchases of credit goods, plus a transfer of cash. We note that the return on firm shares equals the market value of the production in the previous period, which is distributed after the markets have closed. We also note that monetary policy takes the form of helicopter transfers to the household, which are proportional to the previous period's stock of base money.

After the asset market has closed the goods market opens. Here the consumer obeys the liquidity constraints:

$$(2.3) \quad s_{1t} C_{1t} \leq \pi_t H_{ht}, \text{ and}$$

$$(2.4) \quad s_{2t} C_{2t} \leq \pi_t D_{ht}.$$

The consumer's decision problem can now be expressed in the form of a dynamic programming problem with the value function $J(W, \Omega)$ defined by

$$(2.5) \quad J(W, \Omega) = \max \{ U(C_1, C_2, C_3, s) + \beta E[J(W', \Omega')] + \mu_1 (\pi H_h - s_1 C_1) + \mu_2 (\pi D_h - s_2 C_2) \} + \lambda [W - \pi (H_h + D_h + B_h) - r X_h - q Z_h],$$

where primes denote variables in period $t+1$ and variables without a prime refer to period t . The Lagrange multipliers μ_1 , μ_2 and λ are associated with the constraints (2.3), (2.4) and (2.1) respectively and W' is given from (2.2). This problem gives rise to the following first-order conditions

$$(2.6) \quad U_1 = \beta E\{\lambda' \pi'\} s_1 / \pi + s_1 \mu_1$$

$$(2.7) \quad U_2 = \beta E\{\lambda' \pi'\} s_2 / \pi + s_2 \mu_2$$

$$(2.8) \quad U_3 = \beta E\{\lambda' \pi'\} (1 - s_1 - s_2) / \pi$$

$$(2.9) \quad \lambda = \beta E\{\lambda' \pi'\} / \pi + \mu_1$$

$$(2.10) \quad \lambda = \beta E\{\lambda' \pi'\} (1 + i^d) / \pi + \mu_2$$

$$(2.11) \quad \lambda = \beta E\{\lambda' \pi'\} (1 + i) / \pi$$

$$(2.12) \quad \lambda r = \beta E\{\lambda' (r' + a)\}$$

$$(2.13) \quad \lambda q = \beta E\{\lambda' (q' + y)\}$$

We assume the constraints to be binding in all states. This implies

that the multipliers are strictly positive and is tantamount to assuming $i > i^d > 0$ in all states.

Condition (2.8) states that the marginal utility of one dollar of current consumption of credit goods equals the discounted marginal utility of one dollar of future wealth accounting for the expected change in the purchasing power of a dollar. Corresponding conditions show that the marginal utilities of current consumption of cash goods and check goods exceed the marginal utility of credit goods by the corresponding shadow prices of liquidity. Equation (2.9) is an asset pricing equation giving the price of cash in terms of goods, i.e. the inverse of the consumption goods price level as usually defined. It can be used to rewrite (2.6) yielding $U_1/s_1 = \lambda$, i.e. the marginal utility of cash goods equals the marginal utility of current wealth. Equations (2.10) - (2.13) are asset pricing equations for the other assets.

An important feature of the model, which is directly seen from the first-order conditions, is that the marginal rates of substitution differ from the marginal rates of transformation which by assumption are unity; substituting away the Lagrange multipliers yields

$$(2.14) \quad \frac{U_1/s_1}{U_3/(1-s_1-s_2)} = 1 + i, \text{ and}$$

$$(2.15) \quad \frac{U_2/s_2}{U_3/(1-s_1-s_2)} = 1 + i - i^d.$$

This shows that relative consumer prices reflect the interest income foregone in holding cash relative to checking accounts or bonds. This leads to a suboptimal consumption of cash goods relative to the other types of goods for positive interest rates.

We will now go on to consider the operations of the bank. When the asset market is open it receives deposits from the consumer. It has to obey a reserve requirement forcing it to invest a fraction α of the

deposits in currency reserves. The remainder of the bank's assets are invested in bonds. As discussed in Englund and Svensson (1985) it does not, in a model with a representative consumer, affect any results that the bank's portfolio choice is restricted in this way. We can then state the bank's asset demands as

$$(2.16) \quad H_{bt} = \alpha D_{bt}, \text{ and}$$

$$(2.17) \quad B_{bt} = (1-\alpha)D_{bt}.$$

In period $t+1$ the bank redeems its bonds and repays its deposits, including interest. Any surplus is paid as dividends to the owners of the bank. The dividends equal

$$(2.18) \quad a_t = \pi_{t+1} H_{bt} + (1+i_t) \pi_{t+1} B_{bt} - (1+i_t^d) \pi_{t+1} D_{bt}.$$

We assume the representative bank to be a price taker in the asset market and to take as given all interest rates and asset prices. It then faces the linear constraints (2.16) and (2.17) and maximizes its stock market value r . This must be zero in equilibrium; if it was positive the bank could double its profits simply by doubling its activities. From this and the first order condition (2.12) it follows that the expected utility value of the bank's dividends are zero;

$$(2.19) \quad E[\lambda' a'] = 0.$$

This zero-market value condition is analogous to the zero-profit condition for a competitive firm under constant returns to scale. Since $\lambda > 0$ for all values of Ω this implies $a = 0$ in all states, and by the definition of a

$$(2.20) \quad i^d = (1-\alpha)i.$$

This simply says that, since the bank is required to hold a fraction α of its assets in non interest-bearing reserves, the deposit interest rate it can afford to pay is a correspondingly weighted average of i and zero.

This means, as previously observed by many authors, e.g. Black (1970) and

Fama (1980), that the reserve requirement can be seen as a tax on banks. In this context it is a tax on bank deposits, but as noted by Fama (1985) it is in a more general model not clear whether it is a tax on loans or deposits.

3. Equilibrium

In equilibrium the markets for goods, currency, deposits, bonds, firm shares and bank shares will all clear, i.e.

$$(3.1) \quad s_1 C_1 + s_2 C_2 + (1-s_1-s_2)C_3 = Y$$

$$(3.2) \quad H_h + H_b = \bar{H}$$

$$(3.3) \quad D_h = D_b$$

$$(3.4) \quad B_h + B_b = 0$$

$$(3.5) \quad Z_h = 1$$

$$(3.6) \quad X_h = 1.$$

Combining these conditions with the necessary conditions for the consumer's maximization problem we can express the equilibrium values of the endogenous variables in period t as functions of the state vector, Ω_t , and the parameters of the model.

Let us first look at price determination. Given our assumptions that the liquidity constraints will always bind and that the reserve requirement is always fulfilled with equality we can substitute from (2.3), (2.4) and (2.16) into (3.2) to yield

$$(3.7) \quad P \cdot (s_1 C_1 + \alpha s_2 C_2) = \bar{H},$$

where $P \equiv \pi^{-1}$. We note that the price level tends towards infinity as we approach a world where all goods are credit goods ($s_1=s_2=0$). In such a world cash is useless and its value in equilibrium zero.

Equation (3.7) determines the price level as a function of the supply of reserves, the determinants of consumption and the reserve requirement. It can be regarded as a refinement of the simple textbook equation, according to which the money stock is determined as the product of the monetary base and the credit multiplier, and the price level is given by the quantity equation. By the homotheticity of the utility function we can rewrite (3.7) as

$$(3.8) \quad P \cdot Y = \bar{H} \cdot \frac{1 + \theta_1 + \theta_3}{\theta_1 + \alpha} = M + \bar{H} \cdot \frac{\theta_3}{\theta_1 + \alpha},$$

where $\theta_1 = s_1 C_1 / s_2 C_2 = H_h / D_h$, $\theta_3 = (1 - s_1 - s_2) C_3 / s_2 C_2$, and $M = H_h + D_h$.

From this we see that if there are no credit goods, $\theta_3 = 0$, then we are back to the textbook case with the important difference that we have a theory linking the credit multiplier to preferences and the payments structure. In general, however, the construct M plays no useful role in this model and price determination is more fruitfully analyzed by looking directly at the multiplier that links $\bar{H}$ to $P \cdot Y$. We note that the elasticity of the price level with regard to income is unity. This property, which may be regarded as restrictive, is due to the assumption that the liquidity constraints always bind and it does not obtain in general.

We have seen that the composition of consumption depends on the shadow prices of cash vs. check goods and credit vs. check goods which in turn depend on the bond and deposit interest rates. Substituting from the CES utility function in (2.14), (2.15) and (3.1) gives the following demand functions:

$$(3.9) \quad C_1 = C_3 (1+i)^{1/(\rho-1)}$$

$$(3.10) \quad C_2 = C_3 (1+\alpha i)^{1/(\rho-1)}$$

$$(3.11) \quad C_3 = Y \{ s_1 (1+i)^{1/(\rho-1)} + s_2 (1+\alpha i)^{1/(\rho-1)} + 1 - s_1 - s_2 \}^{-1}.$$

In these expressions we have also made use of (2.20), which states that the bond and deposit interest rates are related by the zero-profits condition for banks. We note that an increase in the share of cash and/or check goods will, ceteris paribus, lead to an increase in the per unit demand for credit goods.

To say anything further about the determinants of resource allocation, i.e. the composition of consumption, we must look at what determines the bond interest rate. To study this we rewrite (2.11), defining $\beta = (1+\delta)^{-1}$, as

$$(3.12) \quad 1 + i = (1+\delta) \cdot \lambda \pi / E\{\lambda' \pi'\}.$$

Recalling that $\lambda = U_1/s_1$ this equation states that the nominal bond interest rate reflects the subjective rate of discount and the utility of one dollar today relative to one dollar tomorrow. Substituting from (3.7) this may be written more explicitly as

$$1 + i = (1 + \delta) E \left[\frac{\bar{H}' \cdot (s_1 C_1 + \alpha s_2 C_2) \cdot s_1' \cdot U_1}{\bar{H} \cdot (s_1' C_1' + \alpha s_2' C_2') \cdot s_1 \cdot U_1'} \right],$$

and by the properties of the utility and demand functions we then have

$$(3.13) \quad \frac{1 + i(s)}{\varphi(s, i(s))} = (1 + \delta) E \left[\frac{1 + w'}{(1+y')^{(1-\gamma)} \cdot \varphi(s', i(s'))} \right],$$

where

$$\begin{aligned} \varphi = & (1-\gamma)(1+i) \cdot [s_1(1+i)^{1/(\rho-1)} + \alpha s_2(1+\alpha i)^{1/(\rho-1)}] \\ & \cdot [s_1(1+i)^{1/(\rho-1)} + s_2(1+\alpha i)^{1/(\rho-1)} + 1 - s_1 - s_2]^{(\gamma-1)} \\ & \cdot [s_1(1+i)^{\rho/(\rho-1)} + s_2(1+\alpha i)^{\rho/(\rho-1)} + 1 - s_1 - s_2]^{((1-\gamma)/\rho-1)}. \end{aligned}$$

The coefficient γ is the Pratt-Arrow measure of relative risk aversion $(-u'' \cdot v/u')$ which in the case of an intertemporally additive utility function equals the inverse of the intertemporal elasticity of substitution. It is assumed to be a constant over all relevant intervals defined by the domain of y .

Equation (3.13) is a functional equation giving i as a function of s and the parameters of the process Ω . It is immediately seen that the interest rate is independent of y and w , since the assumption that these are serially uncorrelated implies that the expectational term on the right hand side of (3.13) is independent of Ω .

The relationship between the interest rate and s_1 and s_2 is in general quite complex. With a Cobb-Douglas utility function it may be shown (see appendix) that there is always a γ^* such that $di/ds_1 > 0$ for $0 < \gamma \leq \gamma^*$. If s_1 and/or s_2 are sufficiently large there will also be a γ^{**} such that $di/ds_1 < 0$ for $\gamma^* < \gamma < \gamma^{**}$, and $di/ds_1 > 0$ for $\gamma > \gamma^{**}$.

This says that with sufficiently large intertemporal substitutability (γ small enough) a high realization of s_1 leads to a high interest rate. The basic intuition behind this result is easy to understand. With a high value of s_1 , the consumer expects to be able to buy fewer goods with credit cards today relative to next period. Hence, the price of money [eq. (3.7)] will be high and hence the interest rate [eq. (3.12)]. This intuition is strictly valid only if the marginal utility of consumption is a constant. If that is not the case a higher interest rate, which leads to a more suboptimal composition of consumption since it will bring MRS further away from unity, will increase the marginal utility of consumption. For a sufficiently concave u ($\gamma > \gamma^*$) this effect will be so strong that it takes a lower instead of a higher interest rate to equilibrate a higher s_1 .

4. Two Special Cases

For the following analysis of optimal monetary policies it will be

convenient to restrict attention to cases where i is independent of the state. We will consider two such cases.

The first case is that of a non-stochastic payments system. With $s=s'$ it is immediate that if i were to depend on Ω , then so would the left-hand side expression of (3.13). But this contradicts that the right-hand side expression is independent of Ω . Hence, i is non-stochastic;

$$(4.1) \quad 1 + i = (1+\delta) \cdot E[(1+w') \cdot (1+y')^{(\gamma-1)}].$$

This is exactly analogous with, e.g., Lucas (1982). The nominal interest rate increases with the expected rate of monetary expansion, at least to the extent that this is not strongly correlated with the growth of output. The relationship between the interest rate and expected output growth is ambiguous. The expectation of higher consumption tomorrow will on the one hand imply a higher interest rate since it lowers the marginal utility of future consumption, but on the other hand it implies a lower interest rate since it leads to a higher purchasing power of nominal balances. The former effect will dominate if the elasticity of intertemporal substitution is less than unity, i.e. if the degree of relative risk aversion, γ , is above unity.

The second special case is that of Cobb-Douglas utility and risk neutrality. In this case

$$(4.2) \quad U_1 \cdot (s_1 C_1 + \alpha s_2 C_2) / s_1 = s_1 + \alpha s_2 (1+i) / (1+(1-\alpha)i).$$

Assume further a fixed proportion between s_1 and s_2 , i.e. regard financial innovations as a move between credit goods and the aggregate of other goods. Define $s'_1 = \sigma' s_1$ and assume σ to be serially uncorrelated.

Then

$$(4.3) \quad \frac{1+i}{\left[1+\alpha \frac{s_2(1+i)}{s_1(1+(1-\alpha)i)}\right]} = (1+\delta) E \left[\frac{(1+w')}{\sigma'} \left[1+\alpha \frac{s_2(1+i')}{s_1(1+(1-\alpha)i')} \right]^{-1} \right],$$

from which it is immediate that i is independent of Ω . This gives

$$(4.4) \quad 1+i = (1+\delta) \cdot E[(1+w')/\sigma'],$$

which says that a higher expected rate of monetary innovation (lower σ') leads, at least to the degree that it is uncorrelated with monetary expansion, to a higher interest rate. This is so because it leads to a lower expected value of future nominal balances, since financial innovations means that there are fewer goods that the consumer wants to purchase using high-powered money.

The analysis of optimal policies in the next section will be based on these two cases.

5. Some Welfare Economics of Monetary Policy and Bank Regulation

In this model monetary innovations and economic policies have welfare implications. From (2.14) and (2.15) we see that equilibrium is in general not Pareto efficient, since the marginal rates of substitution differ from the marginal rate of transformation (unity); even though the cost in production is the same for all goods, the opportunity cost faced by the consumer differs due to the payment structure. What matters is the difference in opportunity cost across types of goods. This depends on foregone interest income and we see immediately that $i = 0$ is a sufficient condition for Pareto efficiency. This is the rule associated with Friedman (1969) based on the insight that if currency can be provided costlessly it should be produced at such a rate that there be no opportunity cost for the consumer in using it. This first-best result has

recently been extended to a second best world by Kimbrough (1986 a,b). In his models money yields the service of reducing the time needed for the purchase of goods, thereby acting as an intermediate input. It then follows that a zero tax rate on money balances is always a part of the optimum tax structure.

Let us now look at the conditions for $i = 0$ in our model. Under either of the simplifying assumptions made above to ensure that i is state independent this is in principle straightforward. In a world of no monetary innovations we see from (4.1) that any monetary policy characterized by

$$(5.1) \quad (1+\delta) \cdot E[(1+w') \cdot (1+y')^{(\gamma-1)}] = 1$$

yields $i = 0$. The simplest policy that fulfills this condition is a constant rate of monetary expansion $\bar{w}$;

$$(5.2) \quad 1+\bar{w} = \{(1+\delta) \cdot E[(1+y')^{(\gamma-1)}]\}^{-1}.$$

In analyzing this rule we first consider the case of a deterministic growth rate $\bar{y}$. If $\bar{y} = 0$, then the optimum is achieved at a rate of monetary contraction that equals the rate of discount; $\bar{w} = \delta/(1+\delta)$. This result is found in Friedman (1969). If the optimal rate of monetary expansion is increasing or decreasing in $\bar{y}$ depends on the elasticity of intertemporal substitution. If this exceeds unity, $\gamma < 1$, then $\bar{w}$ is an increasing function of $\bar{y}$, otherwise it is decreasing in $\bar{y}$. This ambiguity reflects the ambiguity in the relation between expected growth and the interest rate discussed above.

We may also investigate the effects of an increased uncertainty with regard to y . Consider a mean-preserving increase in risk in the sense of Rothschild and Stiglitz (1970). This is known to increase (decrease) the expected value of a function that is everywhere convex (concave) in the random variable. Hence, a mean-preserving increase in the uncertainty of

y will increase $\bar{w}$ if $1 < \gamma < 2$ and decrease it otherwise.

To see the impact of financial innovations on monetary policy we note that (4.4) implies, for the case of Cobb-Douglas utility and risk neutrality, that the optimal deterministic monetary policy is given by

$$(5.3) \quad 1 + \bar{w} = \{(1+\delta) \cdot E[1/\sigma']\}^{-1}.$$

We see that the faster the expected rate of monetary innovation, i.e. the higher is $E[1/\sigma']$, the higher is the optimal rate of monetary contraction. We also see that a mean-preserving increase in the spread of the distribution of σ will decrease optimal monetary expansion.

Note that the policy rules considered above are quite simple both in the sense of only requiring summary knowledge of the distributions of the relevant variables and in the sense of achieving the first-best optimum by a fixed contraction rate of the money supply, independently of other policy parameters. These properties derive from the simplicity of the economic environment which ensures a state-independent interest rate. In general optimum policy requires knowledge not only of the function $i(s)$, i.e. the solution to (3.13), but also of the current realization of s .

Let us now consider the possibility that there are institutional constraints on the choice of monetary policy. Under such circumstances one may want to take the w -process as exogenously given and study the optimal choice of reserve requirement. The policy maker chooses α so as to maximize the expected utility of the representative consumer. This gives rise to the following first-order condition

$$(5.4) \quad \begin{aligned} dE[U]/d\alpha &= E[U_1 dC_1/d\alpha + U_2 dC_2/d\alpha + U_3 dC_3/d\alpha] = \\ &= E[U_3 \cdot \{(1+i)S_1 dC_1/d\alpha + (1+\alpha i)S_2 dC_2/d\alpha + dC_3/d\alpha\}] = 0, \end{aligned}$$

where $S_i = s_i / (1 - s_1 - s_2)$. Substituting from (3.9)-(3.11) gives

$$(5.5) \quad dE[U]/d\alpha = E \left[U_3 C_3 i^{(\rho-1)^{-1}} S_2 (1+(1-\alpha)i)^{(1/(\rho-1)-1)} \cdot \left[1 + \alpha i - \{s_1(1+i)^{1/(\rho-1)} + s_2(1+\alpha i)^{1/(\rho-1)} + 1-s_1-s_2\}^{-1} \cdot \{(1+i)^{\rho/(\rho-1)} s_1 + (1+\alpha i)^{\rho/(\rho-1)} s_2 + 1-s_1-s_2\} \right] \right] = 0.$$

Under the assumption that s is constant this has a simple and intuitive solution. In that case the value of the parenthesis in square brackets is state independent, and a stationary point is found where this parenthesis is zero. Denoting the optimal value by an asterisk, this gives

$$(5.6) \quad \frac{\alpha^*}{1 - \alpha^*} = S_1 (1+i)^{1/(\rho-1)} = S_1 \cdot \frac{C_1}{C_3};$$

the optimal choice of reserve requirement depends on the proportion of cash goods to credit goods in consumption, which in turn depends on the elasticity of substitution between the three types of goods, the structure of the payments system represented by S_1 , and the determinants of the interest rate.

We first note that what matters for the degree of regulation of banks is the relative proportion of cash goods to credit goods not the proportion of check goods per se. This may appear conterintuitive, since it is in the provision of checking accounts that banks play a role in the model, but it can be understood by noting that there are two distortions that should be balanced against each other: (i) check goods vs cash goods which decreases as α goes from 0 to 1, and (ii) check goods vs credit goods which increases as α goes from 0 to 1. If cash goods are relatively unimportant, S_1 small, then it is most important to have MRS_{23} close to $MRT = 1$, i.e. α close to zero. On the other hand if credit goods are relatively rare MRS_{12} should be close to one, i.e. α close to one. This means that financial innovations, moves towards "the cashless society",

should give rise to a decrease in reserve requirements.

From (4.6) it also follows that α is a decreasing function of the interest rate, i.e. the closer monetary policy is to being optimal the higher the reserve requirement. In the limit as the interest rate approaches zero, the value of α of course loses its impact on welfare.

6. Concluding Comments

We have studied financial innovations and moves towards "the cashless society" in a general equilibrium cash-in-advance framework. A main result is that Friedman's optimum quantity of money rule continues to hold in this setting. It does so because a non-zero interest rate distorts the composition of consumption as long as not all goods may be purchased without tangible means of payment.

This general result was translated into results on the optimum rate of expansion of the supply of base money under different assumptions. It was shown that a higher rate of financial innovation calls for a more rapid reduction of the stock of base money. Also an increased uncertainty about the the future rate of financial innovation implies slower optimal monetary expansion. The rate of growth of output has an ambiguous effect depending on whether the intertemporal elasticity of substitution is below or above unity.

We have also studied optimum reserve requirements on banks. This is a second-best problem, since it is possible to obtain a zero interest rate independently of the reserve ratio. Given an arbitrary monetary policy giving rise to a certain interest rate we found that the optimum reserve requirement reflects the share of cash goods to credit goods. The

closer we are to a "cashless" world dominated by credit goods the lower the optimal reserve ratio.

These results were reached in environments so simple as to yield state independent interest rates. In general the interest rate depends in a complicated way on the current state of the payments structure. Even with a Cobb-Douglas utility function it is only for sufficiently high rates of intertemporal substitution that the direction of this dependence is unambiguous. Under such circumstances first-best policies would require knowledge about the current state of the payments structure. A possible further development would be to assume that the central bank could observe a signal of the current state, e.g. current cash demand at a certain interest rate. It would then face a signal extraction problem in having to react to this without knowing whether a change in demand derives from a change in y or s .

A limitation of the model is that monetary innovations are not welfare enhancing per se. Indeed, the utility level is the same in a world with only cash goods as in one with only credit goods; in both worlds all goods are consumed in equal quantities. One way to overcome this restrictive feature would be to assume that transactions take time, but that the time requirement for the purchase of some goods may be diminished by the use of cash and checks instead of credit cards. In equilibrium a consumer maximizing utility over leisure and goods would use cash and checks for some goods and credit cards for others. Financial innovations could be modelled as a decrease of the time saved by the use of tangible means of payment.

REFERENCES

- Bernanke, B., and M. Gertler (1986), "Banking and General Equilibrium",
mimeo, Princeton University.
- Black, F., 1970, "Banking and Interest Rates in a World Without Money",
Journal of Bank Research 1, 8-20.
- Boyd, J.H. and E.C. Prescott, 1986, "Financial Intermediary Coalitions",
Journal of Economic Theory 38, 211-232.
- Brunner, K. and A.H. Meltzer, 1970, "The Uses of Money: Money in the
Theory of an Exchange Economy", American Economic Review 61,
784-805.
- Diamond, D.W., 1984, "Financial Intermediation and Delegated Monitoring",
Review of Economic Studies 51, 393-414.
- Englund, P. and L.E.O. Svensson, 1985, "Money and Banking in a
Cash-in-Advance Economy", revised version of Seminar Paper 332,
Institute for International Economic Studies, University of
Stockholm.
- Fama, E.F., 1980, "Banking in the Theory of Finance", Journal of Monetary
Economics 6, 39-57.
- _____, 1983, "Financial Intermediation and Price Level Control",
Journal of Monetary Economics 12, 7-28.
- _____, 1985, "What's Different about Banks?" Journal of Monetary
Economics 15, 29-39.
- Fischer, S., 1983, "A Framework for Monetary and Banking Analysis",
Economic Journal 93, supplement 1-16.
- Friedman, M., 1969, The Optimum Quantity of Money, Macmillan.

-
- Greenfield, R.L. and L.B. Yeager, 1983, "A Laissez-Faire Approach to Monetary Stability" Journal of Money, Credit, and Banking 15, 302-315.
- Hall, R.E., 1983, "Optimal Fiduciary Monetary Systems", Journal of Monetary Economics 12, 33-50.
- Hartley, P., 1985, "The Liquidity Services of Money", mimeo Princeton University.
- Kimbrough, K.P., 1986a, "The Optimum Quantity of Money Rule in the Theory of Public Finance", Journal of Monetary Economics 18, 277-284.
- _____, 1986b, "Optimal Taxation in a Monetary Economy with Financial Intermediation", mimeo Duke University.
- King, R. and C. Plosser, 1984, "Money, Credit and Prices in a Real Business Cycle", American Economic Review 74, 363-380.
- _____, 1986, "Money as the Mechanism of Exchange", Journal of Monetary Economics 17, 93-115.
- Lucas, R.E. Jr., 1982, "Interest and Currency Prices in a Two-Country World", Journal of Monetary Economics 10, 335-359.
- _____, 1984, "Money in a Theory of Finance", Carnegie-Rochester Conference Series on Public Policy 21, 9-46.
- _____ and N.L. Stokey, 1983, "Optimal Fiscal and Monetary Policy in an Economy without Capital", Journal of Monetary Economics 12, 55-93.
- _____, 1984, "Money and Interest in a Cash-in-Advance Economy", Working Paper No 1618, National Bureau of Economic Research.

-
- McCallum, B.T., 1985, "Bank Deregulation, Accounting Systems of Exchange, and the Unit of Account: a Critical Review", Carnegie-Rochester Conference Series on Public Policy 23, 13-46.
- Rothschild, M., and J.E. Stiglitz, 1970, "Increasing Risk: a Definition", Journal of Economic Theory 2, 225-243.
- Svensson, L.E.O., 1985, "Money and Asset Prices in a Cash-in-Advance Economy", Journal of Political Economy 93, 919-944.
- Williamson, S.D., 1986, "Costly Monitoring, Financial Intermediation, and Equilibrium Credit Rationing", Journal of Monetary Economics 18, 159-179.
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APPENDIX

With utility given by the Cobb-Douglas function

$$U = \left[\frac{s_1}{c_1} \frac{s_2}{c_2} \frac{(1-s_1-s_2)}{c_3} \right]^{(1-\gamma)}$$

we have

$$U_1(s_1 c_1 + \alpha s_2 c_2)/(1+i) = \Psi = (1-\gamma) Y^{-\gamma} [s_1/(1+i) + s_2/(1+\alpha i) + 1-s_1-s_2]^\gamma \cdot (1+i)^{(s_1 \gamma - 1)} (1+\alpha i)^{s_2 \gamma} [s_1 + \alpha s_2 (1+i)/(1+\alpha i)].$$

$$d \ln \Psi / ds_1 = \gamma A + \frac{1+\alpha i}{(1+\alpha i)s_1 + \alpha s_2 (1+i)}, \text{ where}$$

$$A = \ln(1+i) - \frac{i}{s_1 + s_2 \frac{1+i}{1+\alpha i} + (1-s_1-s_2)(1+i)}.$$

If s_1 is sufficiently high A is negative; $A(s_1=1) = \ln(1+i) - i$, which is unambiguously negative for positive values of i . In that case there is always a γ^* such that $d \ln \Psi / ds_1 < 0$ for $\gamma > \gamma^*$. Below we list such values for different i with $s_1=1$.

i	γ^*
.01	20133
.1	213
.5	10.6
1	3.3
2	1.1

For low values of s_1 and s_2 , on the other hand, A is unambiguously positive, and $d \ln \Psi / ds_1 > 0$ for all values of γ .

Further, we have

$$d \ln \Psi / di = \gamma \left[s_1/(1+i) + \alpha s_2/(1+\alpha i) - \frac{[s_1/(1+i)^2 + \alpha s_2/(1+\alpha i)^2]}{s_1/(1+i) + s_2/(1+\alpha i) + s_3} \right] - \frac{\alpha^2 s_2 (1+i)^2 + s_1 (1+\alpha i)^2}{(1+\alpha i)(1+i)[s_1(1+\alpha i) + \alpha s_2(1+i)]}.$$

It may be shown that the expression in brackets is positive if

$$s_3[s_1(1+\alpha i)^2 + \alpha^2 s_2(1+i)^2] + s_1 s_2 (1-\alpha)^2 > 0,$$

which holds if $s_3 < 1$. Hence, there will always be a value γ^{**} such
 $d\ln\Psi/di < 0$ for $\gamma < \gamma^{**}$ and $d\ln\Psi/di > 0$ for $\gamma > \gamma^{**}$.

From this it follows that $di/ds_1 > 0$ for sufficiently low values of
 γ and s_1 , but there is always a γ^* such that $di/ds_1 > 0$ for $\gamma > \gamma^*$. If A
 < 0 the sign of di/ds_1 will change twice as γ goes from 0 to ∞ .