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ESCALATION AND COOPERATION IN INTERNATIONAL CONFLICTS

- THE DOLLAR AUCTION REVISITED

by

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Escalation and Cooperation in International Conflicts

- The Dollar Auction Revisited

by

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0. Introduction

This paper reexamines a simple and well-known auction game that represents important aspects of escalation as they are relevant to many social situations of strategic interaction. The dollar auction game introduced by Shubik (1971) casts two players against each other who can alternately bid for a prize of one dollar offered by a third non-strategic player (the auctioneer). The dollar will be awarded to the highest bidder, but both bidders will have to pay their bids to the auctioneer. If both players make a bid they find themselves trapped in a situation of mutually reinforcing behavior that is likely to escalate into socially and individually damaging - yet self-maintaining - patterns. Because as soon as each player is committed to a bid the lower bidder will find it in his best interest to defend his prior 'investment' (first bid) by making another bid that puts him ahead of the other player. But this draws - for the same reason - an even higher bid as response from the other player. Active bidding gets even reinforced as players gradually commit themselves to higher bids and may well reach bidding levels far in excess of the one dollar prize offered. Features of this 'paradigm for escalation' (Shubik) are present in many instances in which competitors commit resources irreversibly in order to win an indivisible prize; e.g. nations get locked in wasteful arms races, firms commit huge funds to R&D to win patent races, and governments engage in disastrous 'competition' by giving subsidies to weak or non-viable domestic industries that would be threatened by market exit otherwise.

In a recent contribution O'Neill (1986) has analyzed this conflict situation from a rigorous game theoretic point of view. His analysis suggests that rational players would never bid against each other (in equilibrium). Consequently, the observation of escalation must be

attributed to 'some irrationality in the bidders' behavior' (O'Neill, p. 33). The present contribution takes issue with this proposition. We show that this conclusion which is derived from a finite (discrete) extensive game model does not hold if the conflict is modelled as an infinite (continuous) extensive game. With this alternative modelling device certain patterns of escalation (bidding against each other) are compatible with equilibrium of the game; i.e. no 'irrationality' on behalf of players is involved. We give a detailed formal and intuitive account of this phenomenon and use the results to explain the occurrence of arms races.

The paper is organized as follows: section I reconsiders O'Neill's (1986) analysis of the discrete game. By heavily drawing on his work it reveals some new strategic aspects of the game and focuses on the behavior of the equilibrium correspondence as the discretization gets finer and finer in order to approach a 'continuous' limit game. Section II contains a formal definition and detailed equilibrium analysis of the continuous game. The concluding section points to the implications of the choice of a continuous resp. discrete game structure in a more general context.

I. The Discrete Dollar Auction

Our starting point is O'Neill's (1986) rigorous and ingenious analysis of Shubik's game. By the simple but effectful modelling device of introducing a (fixed and known) budget constraint for each player he is able to solve the noncooperative game in a backwards inductive fashion and to establish existence and characterization of subgame-perfect equilibria.

The rules of the game are as follows:

Two players bid for a prize worth s dollars on the condition that the winner and the loser must pay their final bids (to the auctioneer) but only the winner will receive the s dollars being at stake. Both players have a budget of b dollars each (and we assume that $b > s$).¹ Bidding proceeds in strictly alternating order and a bidding player, at each stage, is perfectly informed about all the previous bids made. Bids have to be made in full dollars (or another specified unit) and - when making a bid - each player has to overbid the last bid by at least one dollar (unit). (Who gets the first move in the game is decided by a mechanism external to the game.) The game is over if a player when it is his turn either declines to bid or is prevented from further bidding by the depletion of his budget. The stakes s are then awarded to the other player and both players have to pay the amount of their respective last bid.

The dollar auction game thus differs from standard auctions in that commitment to a bid can be binding even in the case of not winning the stakes, i.e. there is an extreme form of "loser's curse": the winner takes it all and the loser loses all he bid. Although the rules of this abstract game might seem strange at first sight there are many real life situations in which 'competitive bidding' takes place under precisely those strategic conditions: arms races (O'Neill 1986), in which governments commit non-returnable resources, patent races (Harris and

¹The analysis would not change with u players and the rule that the two highest bidders must pay their bids and the highest bidder wins the prize.

Vickers 1985), in which firms compete by R&D expenses to win the patent for a new technology, the practice of governments to subsidize home industries to maintain their existence and competitiveness on world markets are important examples. In those situations one frequently observes bizarre escalating of bids as each player - once engaged in the game - tries to avoid the "loser's curse". Arms races are a case in point, the combined superpower arsenals are sufficient to destruct Mother Earth a thousand times. On a much less grandiose, simple experimental level it was observed that the escalating forces triggered off by repeated bidding often lead to bids higher than the value of the stakes (Teger et al. 1980). That "rational" players should not engage in such behavior seems obvious, but what exactly should a rational player of the dollar auction do? O'Neill (1986) gives the following answer.

The dollar auction is a finite game of perfect information since no player can bid more than b units and has to increase his bid at each stage by at least 2 units over his own last bid. Consequently, it must admit a subgame-perfect equilibrium point. Such a point results from a pair of bidding strategies that form a Nash equilibrium which has the property that any bid specified by a player's strategy in any branch of the game tree is part of an optimal strategy for the game played from that branch onwards. O'Neill observes that the game admits many subgame perfect equilibria and postulates - a priori - the following bidding rule that effectively selects a unique equilibrium from the set of all subgame-perfect equilibria: whenever a player is indifferent between two (or more) optimal bids in any subgame he will choose the move that involves the smallest bid, including the possibility of not bidding at all. O'Neill (p.36) justifies this rule as most reasonable on rationality grounds: a player will not venture money without some positive reason for

doing so. With this rule O'Neill computes the following equilibrium strategies:

Let the current bids by players A and B be x_A and x_B .

Then if player A has the move, A should bid

$$(b - x_B - 1) \cdot \text{mod}(s-1) + x_B + 1 \text{ units}$$

if this quantity is less than $x_A + s$ and drop out otherwise.

This strategy prescription determines the following equilibrium path: the first player bids $(b-1) \cdot \text{mod}(s-1) + 1$ units and the other player drops out immediately. (For details see O'Neill (1986)).

In other words, if the non-cooperative solution concept subgame-perfect equilibrium and the additional bidding rule for breaking ties is used to determine what behavior is "rational" in this strategic 2-person decision problem escalating bids never occur. The second moving player foresees the trouble resulting from a response bid and player 1 exploits this foresight.

Our own analysis begins with a look at the entire set of subgame-perfect equilibria of the dollar auction that will cast some doubt on the appropriateness of the equilibrium selection rule chosen by O'Neill. Consider the following example.

Example: Let $b=4$ and $s=3$ (i.e. each player has 4 dollars

to bid for stakes of 3 dollars.) Denote the first moving player by A and the other by B, then the following game tree results:

Figure 1

Analyzing the game tree in a backwards inductive fashion by using O'Neill's rule for breaking ties determines an optimal decision (branch) at each node of the tree that is marked with a rectangular dash. In this equilibrium player A bids $2 = (4-1) \cdot \text{mod}(3-1) + 1$ and player B drops out (bids 0). The equilibrium pay-offs are (1,0); player A wins 1 unit because he has to pay 2 units for his winning bid and gets 3 units for winning, player B neither pays nor receives anything.

Interestingly, reverting O'Neill's rule to "whenever indifferent between optimal bids choose the one involving the highest bid" gives the following result: the unique optimal decision at each node of the tree is now marked by a small circle (see Figure 1), they determine the following path: A bids 1 unit and wins and B drops out; so the net pay-off to A is $2 = 3-1$ and player B still gets nothing. Obviously, the winner (player A) does better in this equilibrium than in the one chosen in O'Neill (1986). Why then should one regard that rule as "part of the concept of sequential rationality for the dollar auction" (O'Neill, p.36)? (O'Neill briefly mentions that with this opposite tie-breaking rule the equilibrium pay-offs are $(b-1) \cdot \text{mod}(s) + 1$ for the winner and 0 for the loser but does not further comment on it.)

The problem is the following: in the subgame that is reached if B would bid 2 after A had bid 1, A is indifferent between dropping out (with a net pay-off of -1 because of his first bid of 1) and bid of 4 and win (with an equal net pay-off of $3-4 = -1$). With O'Neill's rule A quits which provokes the bid of 2 by B (in the subgame resulting if A had bid 1) as an optimal one. That move, in turn, prevents A from bidding 1 in the first move (because B would respond with a bid of 2). So A finds the best he can do is make a bid of 2. But with the opposite rule A threatens to bid 4 should B dare to bid 2 in the subgame resulting from a first bid by A of 1. This threat is credible (because, if carried out, it would realize the best possible outcome for A given circumstances) and makes B to withdraw. As a consequence, A finds it optimal to bid 1 ($=(4-1) \cdot \text{mod}(3) + 1$) and win.

So in the game depicted in Figure 1 it is 'rational' (in any subgame) to threaten "toughness": "whenever you dare to escalate by a further bid I'll escalate further". (Recall that all threats made in a subgame-perfect equilibrium are credible in the sense that a player behaves optimally by carrying them out if the game moves to a 'threat position'.) Dropping out at a node at which an equally good further bid is available signals "weakness": such a player is not prepared to hurt himself further (by making another bid) and thereby invites a move by the opponent to this node. O'Neill's justification for the first tie-breaking rule makes sense given the node at which it is applied has already been reached (i.e. player B has bid 2). But it does not take into account that the fact of this node (subgame) being reached is only a consequence of the anticipation of that rule being used at this node. If the tie is anticipated (by B) to be broken in favour of a further bid the node (subgame) is not reached. In other words, A is only indifferent between

the two solutions of the subgame resulting from (prospective) bids of 1 by A and 2 by B in the subgame but not in the full game: the two solutions of the subgame correspond to two different solutions of the full game and A is not indifferent between those two solutions of the full game. A net (equilibrium) pay-off of 2 is better than a net (equilibrium) pay-off of 1. This illustrates that even very intuitive behavioral rules for what a "rational" player should do can be misleading if stated without considering the entire strategic context. In fact, not considering the entire context might not be "rational" (even by O'Neill's rule!): from a strategic point of view there is, after all, a positive reason for venturing further money for player A in the subgame considered above!

Strategic considerations of the above type form the basis for Leininger's (1986) notion of 'strategic equilibrium' which is a refinement of the concept of sequential (resp. subgame-perfect) equilibrium. Before we apply this solution concept we mention that the game shown in Figure 1 has further equilibria: any mixture of the two tie-breaking rules (i.e. apply one at some nodes and the other at others), too, yields a (different) subgame perfect equilibrium. However, it is easily seen that all equilibria determined in this way must produce one of the two already known equilibrium paths; i.e. no other equilibrium pay-offs apart from (1,0) and (2,0) are possible.

Since the concept of 'strategic equilibrium' (Leininger 1986) is defined w.r.t. the set of all sequential equilibria of a game we first study the set of all subgame-perfect equilibria of the dollar auction.²

²The concept of sequential equilibrium for games with (possibly) imperfect information coincides with the notion of subgame-perfect equilibrium for games of perfect information.

The following result is not difficult to prove:

Proposition 1: In any subgame-perfect equilibrium of the dollar auction the player who has the first move will win with his first bid; the other player never bids. The winning bid lies in the set

$$\{\min[(b-1) \cdot \text{mod}(s) + 1, (b-1) \cdot \text{mod}(s-1) + 1], \min[\dots] + 1, \min[\dots] + 2, \dots, \max [(b-1) \cdot \text{mod}(s) + 1, (b-1) \cdot \text{mod}(s-1) + 1]\}.$$

Proposition 1 says that the structure of the equilibria of the game shown in Figure 1 generalizes to all equilibria of all (discrete) dollar auctions. The two "extreme" tie-breaking rules also yield 'extreme'; i.e. the highest and lowest, equilibrium pay-offs to the winner. But the equilibrium yielding the highest possible equilibrium pay-off is not always determined by the same tie-breaking rule: in the example discussed above with $b=4$ and $s=3$ it is best for A to play it "tough". But with $b=5$ and $s=3$ 'weakness' pays: $(b-1) \cdot \text{mod}(s-1) + 1 = 0+1 = 1$ while $(b-1) \cdot \text{mod}(s) + 1 = 1+1 = 2$; i.e. dropping out when indifferent yields a net pay-off of $2-1 = 1$ while further bidding pays $2-2 = 0$. This observation explains the boundaries of the set of equilibrium opening bids in Proposition 1. Also equilibrium pay-offs resulting from 'mixtures' of the two selection rules may lie strictly inside the boundaries.

The notion of strategic equilibrium focuses on the reasonableness of a sequential equilibrium in relation to other sequential equilibria from an eductive (Binmore (1985)) point of view. For example in Figure 1 the

equilibrium resulting from O'Neill's tie-breaking rule is unreasonable because player A fails to take into account the strategic consequences for the full game of his choice of strategy in a subgame. There exists another choice which is equally good in the subgame (should it be reached) and better in the full game (it leads to the subgame not being reached). Note that player A must think about his choice in the subgame (in fact, in any subgame) before he can decide on his best first move. The same, of course, applies for player B's choice. I.e. both players - by ex ante reasoning - determine (from their respective point of views) the best equilibrium path of the full game and try to 'enforce' it in actual play. To keep things simple we only give an informal definition of strategic equilibrium that will suffice to apply the concept to the 2-person perfect information game "dollar auction" (For a general definition, existence proof and detailed discussion see Leininger (1986)).

A player is said to have strategic power in an equilibrium sustained by the strategy combination $e=(e_A, e_B)$, say, if he can profitably and credibly "deviate" from it to another equilibrium strategy; i.e. only deviations that are compatible with another equilibrium are allowed. "Profitably" means that the alternative equilibrium strategy is part of a strategy combination that supports an equilibrium path that yields a (strictly) higher pay-off to the deviant player; "credibly" means that to play according to the alternative strategy is optimal even if the previous player has played according to the originally proposed equilibrium e . An equilibrium is called strategic if no player has strategic power in it. The test whether a player has strategic power in an equilibrium is applied sequentially over the set of players such that the equilibria in which the last moving player has strategic power are

eliminated first, then those in which the last but one player has strategic power etc.. For finite extensive games (with perfect recall) there always exists a strategic equilibrium (Leininger (1986), Theorem 1).

For the dollar auction it is apparent that the player moving second cannot have strategic power in any equilibrium. His pay-off over all equilibria is zero (Proposition 1) and hence a contemplated deviation from one equilibrium to another can never be profitable. But the first moving player has strategic power in some equilibria. E.g. in the game of Figure 1 he can deviate from the equilibrium proposed by O'Neill to an equilibrium with an opening bid of 1 instead - as required by this equilibrium - of bidding 2. This is credible (as it constitutes equilibrium play) and clearly profitable. Note also that by bidding 1 unit player A clearly signals to player B that he is not playing O'Neill's equilibrium and so B must figure out the strategy that supports a first move of 1 by A in equilibrium. He then discovers that his best reply is to drop out; i.e. A "enforces" play of the equilibrium path (1,0) yielding net pay-offs of (2,0).³ In general, A will always choose the equilibrium path (and hence opening bid) that gives him the highest net pay-off. By Proposition 1 this is either the bid $(b-1) \cdot \text{mod}(s) + 1$ or the bid $(b-1) \cdot \text{mod}(s-1) + 1$. The next Proposition shows that a strategic equilibrium almost always requires an opening bid of $(b-1) \cdot \text{mod}(s)$; i.e. player A should always signal "toughness". The result depends on how one chooses the basic unit in which bidding is conducted that defines the discrete structure of the game. It can only happen with a "large" unit

³There are also games in which the second moving player can enforce play of a certain equilibrium path. For examples see Leininger (1986).

(and hence simple structure of the game) that O'Neill's equilibrium is strategic. Recall the example given above with bidding in full dollars and $b=5$ (dollars) and $s=3$ (dollars), in this case O'Neill's equilibrium is strategic. Now conduct the same auction with the basic unit being half a dollar. Then O'Neill's rule yields a win-ning bid of $(10-1) \cdot \text{mod}(6-1) + 1 = 4 + 1 = 5$ half-dollars = 2.50 dollars whereas the strategic equilibrium is given by an opening bid of $(10-1) \cdot \text{mod}(6) + 1 = 3+1 = 4$ half dollars = 2.00 dollars. It is instructive to see what happens if one further diminishes the basic unit: If the basic unit is a dime the two equilibria have winning bids of $(50-1) \cdot \text{mod}(30-1) + 1 = 20 + 1$ dimes = 2.10 dollars and $(50-1) \cdot \text{mod}(30) + 1 = 19 + 1$ dimes = 2.00 dollars, respectively. If bidding is in cents we have $(500-1) \cdot \text{mod}(300-1) + 1 = 201$ cents = 2.01 dollars with O'Neill's rule and $(500-1) \cdot \text{mod}(300) + 1 = 200$ cents = 2.00 dollars in the strategic equilibrium. This shows that the equilibrium selected by O'Neill strongly depends on the chosen unit or discretization of the dollar auction: if the unit is a dollar the first moving player wins with a bid of 1 dollar; if it is half a dollar he must bid 2.50 dollars, if dimes are used 2.10 dollars etc... In contrast, the equilibrium bid of the equilibrium selected by our alternative rule does not depend on the discretization: it is 2.00 dollars for all the discretizations considered.

One can prove:

Proposition 2: Assume that $b > s$ and $\frac{b}{s} \notin \mathbb{N}$.

- a) There exists $n_0 \in \mathbb{N}$ such that, for all $n \geq n_0$,

$$(n \cdot b - 1) \cdot \text{mod}(n \cdot s - 1) > (n \cdot b - 1) \cdot \text{mod}(n \cdot s)$$
- b)
$$\frac{1}{n} \cdot [(n \cdot b - 1) \cdot \text{mod}(n \cdot s) + 1] = (b - 1) \cdot \text{mod}(s) + 1$$
- c)
$$\lim_{n \rightarrow \infty} \frac{1}{n} \cdot [(n \cdot b - 1) \cdot \text{mod}(n \cdot s)] = \lim_{n \rightarrow \infty} \frac{1}{n} [(n \cdot b - 1) \cdot \text{mod}(n \cdot s - 1)]$$

$$= b \text{ mod}(s).$$

Multiplication of b and s by $n \in \mathbb{N}$ in the formulas of Proposition 2 is equivalent to the choice of unit equal to $\frac{1}{n}$ dollars, the greater n the smaller the unit chosen. Hence the first statement says that for a small enough unit the equilibrium selected by O'Neill requires a higher opening bid than the one based on the opposite selection rule; in particular, it is not strategic.

Corollary: Under the conditions of Proposition 2a) any strategic equilibrium of the dollar auction requires an opening bid of $(b - 1) \cdot \text{mod}(s) + 1$ units which leads to the withdrawal of the other player.

Statement b) of Proposition 2 says that the equilibrium path that is sustained by our alternative tie-breaking rule does not depend on the unit chosen. This implies that for $n \geq n_0$ (n_0 given from part a)) the strategic equilibrium path does not depend on the discretization. Given the ad hoc nature of the discretization this is a very desirable property. Thirdly, if we let n go to infinity (i.e. the basic unit approaches zero) both equilibria - and hence by Proposition 1 all equilibria - approach the same limiting path. Of course, a basic 'unit' of zero corresponds to a continuous (i.e. infinite) model of the dollar

auction. Such a model is analyzed in the next section.

II. The Continuous Dollar Auction

The above analysis suggests that equilibria of a discrete version of the dollar auction might strongly depend on the chosen discretization (i.e. basic unit in which bidding is conducted). As there appears to be no a priori most natural choice of the basic unit an examination of the limiting case of a continuous bidding domain is of interest.

So let b and s denote the (common) bankroll of players resp. the stakes, as before. Again, players make bids alternatingly under the requirement that every bidder has (strictly) to overbid the other bidder's bid. But now - in contrast to the previous section - bidders may choose any number from $(b_{-1}, b]$, where b_{-1} stands for the last bid made and b for the bankroll, at their respective turns of bidding. The first difference to note is that this change of rules of the game introduces an ambiguity not present in the discrete case: bidding could now go on indefinitely without any violation of the bidders' budget constraints, b .

E.g. if each bidder would always bid $\frac{b_{-1}+b}{2}$, if the last bid was b_{-1} , bidding could go on forever as long as the first bid was less than b . It is therefore necessary to amend the rules of the game by assigning pay-offs to the cases in which bidding does not stop after finitely many bids. So let us simply adopt the rule that - if this happens - no one is declared the winner of s , yet each player still has to pay the amount of his "last" bid (that will now be defined uniquely through an appropriate limit operation). Of course, each player is still free to drop out at any stage giving the next bid to him. To simplify notation we will identify the move 'drop out' of a player by a bid of b_{-1} ; i.e. if a player repeats the other player's bid he signals giving up.

More formally the game is described as follows:

Let $[0, b]$ denote the set of feasible bids for each player. A bidding strategy is a (possibly infinite) sequence of functions mapping finite sequences of previous bids into a new bid at each instance it is a player's turn to bid; to be feasible any bid determined by a strategy at each stage must exceed the last bid of the sequence of previous bids. If 1 denotes the first moving player and 2 the player moving second, a pair of feasible strategies $(g, h) = ((g_1, g_2, \dots, g_t, \dots), (h_1, h_2, \dots, h_t, \dots))$ is given by

$$\begin{aligned} g_1 &\in [0, b] \\ g_t &: [0, b]^{2t-2} \rightarrow [0, b] \quad (t \geq 1) \\ h_t &: [0, b]^{2t-1} \rightarrow [0, b] \quad (t \geq 1) \end{aligned}$$

such that $g_1 = b_1 \in [0, b]$ and $g_t(b_1, \dots, b_{2t-2}) \geq b_{2t-2}$ for all $(b_1, \dots, b_{2t-2}) \in [0, b]^{2t-2}$ and all $t \geq 1$; and $h_t(b_1, \dots, b_{2t-1}) \geq b_{2t-1}$ for all $(b_1, \dots, b_{2t-1}) \in [0, b]^{2t-1}$ and all $t \geq 1$. Recall the convention that bidding exactly b_{2t-2} (resp. b_{2t-1}) has the meaning of dropping out at ones t -th move.

The pay-off accruing to player 1 from the use of a strategy pair (g, h) is then given by

$$\begin{aligned} s - b_{2t_o-1} & \quad \text{if} \quad \begin{aligned} & g_t(b_1, \dots, b_{2t-2}) > b_{2t-2} \quad t \leq t_o \\ & h_t(b_1, \dots, b_{2t-1}) > b_{2t-1} \quad t < t_o \\ & \text{and } h_{t_o}(b_1, \dots, b_{2t_o-1}) = b_{2t_o-1} \end{aligned} \end{aligned}$$

$$\begin{aligned}
 & - b_{2t_o-3} && \text{if} && g_t(b_1, \dots, b_{2t-2}) > b_{2t-2} && t < t_o \\
 & && && g_{t_o}(b_1, \dots, b_{2t_o-2}) = b_{2t_o-2} && \text{and} \\
 & && && h_t(b_1, \dots, b_{2t-1}) > b_{2t-1} && t \leq t_o \\
 & - (\lim_{t \rightarrow \infty} b_{2t-1}) && \text{otherwise}
 \end{aligned}$$

and, similarly, for player 2. I.e. if 2 drops out first, 1 wins and gets s minus what he bid last. If 1 drops out first, 2 wins, gets s and pays the amount of his last bid. If neither player drops out he pays the amount of the supremum of all his bids. This number is well-defined because the sequence of his bids is strictly increasing. Note that the joint sequence of bids $(b_1, b_2, \dots, b_{2t-1}, b_{2t}, \dots)$, too, is strictly increasing and hence

$$\lim_{t \rightarrow \infty} b_{2t-1} = \lim_{t \rightarrow \infty} b_{2t} = \lim_{t \rightarrow \infty} b_t = c \leq b$$

holds. For this reason we also refer to c as a parity level.

A pair of feasible strategies (g, h) is a (subgame-) perfect equilibrium if it induces a Nash-equilibrium in any subgame.

A subgame can be identified with a finite sequence of bids such that the last bid is less than b . In fact, it suffices to take the last two elements of such a sequence which give the relevant data (the last bid of each player) for the continuation. They completely determine the strategic 'status quo', in particular the pay-offs that result from immediate termination of the game. The bids before the last bid of each player are irrelevant in this respect. It is then natural to simplify the notion of strategy to a sequence $(f_1, \dots, f_t, \dots)$ of functions that are

defined on the last two bids observed (and not all previous bids) for $t \geq 2$. We call such strategies simple.

We can now prove the following result:

Theorem 1: The continuous dollar auction (in simple strategies) has a unique subgame-perfect equilibrium path. This path has the first moving player bidding $b \bmod s$ if $\frac{b}{s} \notin \mathbb{N}$ and s if $\frac{b}{s} \in \mathbb{N}$ and the second player dropping out.

Proof: Following O'Neill the game can be thought of as being played on the square of side length b , $[0, b]^2$. Any point of this square not lying on the diagonal can be interpreted as a combination of current bids of the two players and hence as a subgame of the full game. Any point (b_1, b_2) above the diagonal gives the right to bid to player 2 (as $b_1 > b_2$ means that 1 was the last bidder) and any point below the diagonal gives the right to bid to player 1. If a player moves to a point on the diagonal the game is over and the other player wins the stakes, s . Adapting the techniques employed by O'Neill in a discrete setting to the given continuous game one can proceed in the following backwards inductive fashion.

Figure 2

Let ABC denote the triangle with nodes labelled A, B and C and denote by $\text{int}(ABC)$ the interior of that region. Suppose $(b_1, b_2) \in \text{int}(ABC)$. Then it is 2's turn to bid and his unique optimal bid is given by b . This will make sure that 2 wins s by an increase of his last bid, b_2 , by less than s since $b_2 > b - s$. Hence the pay-off $s - b$ (although negative) exceeds $-b_2$, the pay-off associated with dropping out. Similarly, a draw is worse than bidding b as it could only be achieved at a parity level $c > b_1$ and would yield an even higher loss of at least $-b_1$. This means that a draw cannot be reached because any bid smaller than b would give 1 the right to bid and he would then bid b and win. Hence such a bid could only further increase 2's payment in the loss. Consequently, any $(b_1, b_2) \in \text{int}(ABC)$ represents a win for 2. Analogously, any $(b_1, b_2) \in \text{int}(ABD)$ represents a win for 1. What if (b_1, b_2) lies on the edge AC , in particular on $\text{int}(AC)$? Now 2 is indifferent between dropping out (and getting $-(b-s)$) and bidding b (and receiving $s-b$ as the winner) while indefinite bidding is still worse. It is now important to observe that O'Neill's rule of dropping out if indifferent is incompatible with perfect equilibrium: Suppose 2 elects to drop out which means that any point $(b_1, b_2) \in \text{int}(AC)$ is a win for 1. Now consider a subgame $(\bar{b}_1, \bar{b}_2) = (b_1 - \Delta, b_2)$ such that $b_1 - \Delta < b - s$; i.e. the point lies below the diagonal on the line $A(b-s)$ and it is 1's turn to move. 1's optimal decision, if $\Delta < s$, would then be to make the lowest bid that realizes a subgame $(\tilde{b}, \bar{b}_2)$ that represents a win for 1. But this bid does not exist because the region of wins for 1 on edge AC is not closed: in order to secure a win 1 has to bid strictly more than b_2 ; in order to maximize his pay-off from a win he has to bid "as little as possible" above b_2 . This problem has no solution in the

continuous case (it does, of course, have a solution in any discrete case!). Hence the subgame $(\bar{b}_1, \bar{b}_2)$ has no equilibrium. But then the entire game can have no equilibrium, because if it had it would induce one in subgame $(\bar{b}_1, \bar{b}_2)$.

It is not difficult to see that any other rule that does not resolve indifference by always stipulating "make another bid" runs into the same problem of non-compact 'winning' sets that give rise to a subgame having no equilibrium. Hence the only candidate for an equilibrium is the rule that resolves a tie between 'dropping out' and 'bid' in favor of

continued bidding (recall from Propositions 1 and 2a of the previous section that this rule is already prominent for fine enough discretizations). Here it produces closed 'winning' sets in the backwards induction procedure and unique solutions to all choice problems: With this rule we know that ABC represents wins for 2 and ABD represents wins for 1 (exclusive of the diagonal, of course). It follows immediately that $ACb(b-s)$ contains only wins for 1 because 2, who has the move can only move to an area that represents wins for 1 (namely, ABD). Bidding up to b is not worthwhile because it requires an increase of the last bid of more than s (in order to win s). Similarly, $ADb(b-s)$ only contains wins for 2.

Now one can repeat the analysis on the square $[0, b-s]^2$ and label all points off the diagonal in the strips $[b-2s, b-s] \times [0, b-s]$ and $[0, b-s] \times [b-2s, b-s]$ wins for either one of the two players, etc., until one is left with a square of side length less than s , i.e. of length $b \bmod s$. Note that it is crucial for this continuation of the argument that the points on $(b-s)A$ (horizontal) are wins for 1 and those on $(b-s)A$ (vertical) are wins for 2. So if $b \bmod s > 0$ then the optimal decision for player 1 is to bid $b \bmod s$ reaching a subgame that is a win for him, and player 2 must drop out. If $b \bmod s = 0$ then 1 bids s and 2 drops out.

The unique strategy pair supporting this equilibrium path is given by

$$g_1 = \begin{cases} b \bmod s & \text{if } \frac{b}{s} \notin \mathbb{N} \\ s & \text{otherwise} \end{cases}$$

and

$$g_t(b_1, \dots, b_{2t-2}) = \begin{cases} (b - b_{2t-2}) \bmod s + b_{2t-2} & \\ \text{if this number is } \leq b_{2t-3} + s & \\ b_{2t-2} & \text{otherwise} \end{cases} \quad \text{for } t > 1$$

respectively,

$$h_t(b_1, \dots, b_{2t-1}) = \begin{cases} (b - b_{2t-1}) \bmod s + b_{2t-1} & \\ \text{if this number is } \leq b_{2t-2} + s & \\ b_{2t-1} & \text{otherwise} \end{cases} \quad \text{for } t \geq 1.$$

This proves the Theorem. □

Remark: The above analysis and conclusion is similar to the main theorem in Harris and Vickers (1985) who prove existence of (subgame-) perfect equilibrium in a strategically much more complex model of a race between two players for an indivisible prize. The continuous dollar auction could be considered a limiting case (with no discounting) of games contained in the class of games (races) with discounting analysed by Harris and Vickers. However, the uniqueness part of our theorem does not seem to relate to their theorem by a similar limit process.

Theorem 1 indicates that non-uniqueness of the equilibrium path in Proposition 1 might be a consequence of the imposition of a discretization. It also shows that the unique equilibrium of the continuous auction can be approached in the limit in terms of equilibrium paths by any sequence of equilibria of finer and finer discretizations (Proposition 1 and 2c),

but not necessarily in terms of equilibrium strategies. A sequence of equilibria based on O'Neill's (1986) selection rule converges w.r.t. the equilibrium paths to the path of the unique equilibrium of the continuous game but not w.r.t. strategies: as the proof of Theorem 1 shows the limit strategies resulting from O'Neill's construction cannot be equilibrium strategies in the continuous game. The dollar auction, hence, neatly illustrates a theorem proven by Hellwig and Leininger (1986) that asserts for a class of sequential games of perfect information that the limit process discretized games/continuous game is continuous in equilibrium paths and discontinuous in equilibrium strategies. However, the continuous dollar auction itself is not contained in that class because its pay-off functions might be discontinuous (at diagonal points).

On the other hand, the solution of the continuous game has the same qualitative features as the solutions of the discrete games that were discovered by O'Neill: escalating "bidding wars" are not compatible with perfect equilibrium; backwards induction rationality should lead the second-moving player to abstain from bidding. Likewise, in Harris and Vickers' (1985) model of a race players never race each other in equilibrium. As O'Neill observes this is in conflict with real observations of international conflicts or the dollar auction played by real people and he offers several informal explanations for this perceived difference (O'Neill (1986, pp. 43-49)). We would like to take the purely theoretical analysis one step further. Recall that we have justified consideration of the continuous version of the dollar auction on the grounds that it removes the -in our view artificial- dependence of equilibria on the arbitrary discretization. We now observe that the above specification of the continuous dollar auction is arbitrary, too, to the extent that we imposed a certain pay-off to the possibility of indefinite

bidding; in particular, one that gives nothing of s to either of the players. (In Harris and Vickers (1985) this rule is invoked in the form that if no one reaches the finishing line of the race course, no one will win the prize. The following analysis should therefore also be relevant for their model.) One could also assign any portion of s (or part of s) to the players while maintaining the rule of payment amounting to the last (limiting) bid. E.g. instead of assigning pay-off $-\lim_{t \rightarrow \infty} b_t = -c$ to both players in case of indefinite bidding up to parity level c , one could assign to them pay-off $-\lim_{t \rightarrow \infty} b_t + \frac{s}{2} = -c + \frac{s}{2}$. One could interpret such a rule as players getting a reward (pay-off) for 'not having lost'. Alternatively, players might attach a subjective value to this possibility, which clearly separates it from a loss. In fact, O'Neill mentions that experiments run by Teger (1980) were interpreted as showing that bidding of both players (even beyond s) was due to the desire of both players not to lose the game. He also notes that this feature is presumably present in many situations of international conflict as expressed by attitudes like "Our dead shall not have died in vain" (O'Neill, p. 46). E.g. the United States and the Soviet Union might each attach a certain (even common) "value" s to sole world supremacy and at the same time value "being on equal terms as superpowers" at $p \cdot s$ with $p \in (0,1)$ and being clearly inferior to the other at 0. The particular value of "being on equal terms" is seen in implying "not being second to the other". "Being on equal terms" is possible in the continuous dollar auction if both players choose bidding strategies that approach a parity level, but it is not a feasible option in any discrete version of the dollar auction. Rationality then dictates an outright winner. Hence discrete modelling of conflict situations that reduce to the strategic content of the continuous dollar auction might be considered inappro-

priate as it precludes a priori considerations of important behavioral aspects. Taking the argument one step further, one might judge the above continuous specification inadequate because it treats "being on equal terms" for the individual players like a loss ("being second to the other"). Moreover, any continuous model of the dollar auction that attaches positive values to 'not losing' in the above sense has the same right to be considered a 'limit' game of any sequence of discrete games converging to its continuous strategy spaces. The rules and pay-offs that apply to behavior in the discrete games identically apply to the same behavior in the continuous game. The games only differ insofar as rules and pay-offs have to be specified anew for a course of action in the continuous game that cannot arise in any discrete game.

So let us describe in more general terms the preferences of players by a pay-off function that takes into account whether a certain bid and return on that bid represents a loss, draw or win for the players. Define, for $t \in \{\text{loss, draw, win}\}$ and $p_i \in (0,1)$, $i=1,2$

$$u_i(\tilde{b}, h | t) = \begin{cases} -\tilde{b} & \text{if } t = \text{loss} \\ p_i h - \tilde{b} & \text{if } t = \text{draw} \\ h - \tilde{b} & \text{if } t = \text{win} \end{cases}$$

u gives the final pay-off (in utility terms) to a player from a (last) bid $\tilde{b}$ for stakes h conditioned on whether it was made in a loss, draw or win. I.e. if $p_i < \frac{1}{2}$ one might think of a player as being "aggressive", because he prefers bidding $\tilde{b}$ for a lottery that would make him 'win' or 'lose' with equal probability $\frac{1}{2}$ (and yielding expected pay-off $\frac{1}{2} h - \tilde{b}$) to a bid of $\tilde{b}$ on a sure draw (with pay-off $p_i h - \tilde{b}$). A player with $p_i > \frac{1}{2}$

It is 2's turn to bid and he can bid b and win s by an increase of his last bid of less than s ; i.e. dropping out is dominated by bidding b and win. But bidding into the region IHA (and ultimately reaching a draw at a parity level on IA) is even better: This secures a win of $\frac{3}{4}s$ by a final overall increase over the present bid that is more than $\frac{s}{4}$ less than the bid b that would secure s (i.e. $\frac{s}{4}$ more); hence the net pay-off to 2 is higher from a draw. Moreover, 2 can count on 1 to reach a draw, because after 2 has moved the game into IHA 1 will find it (by the same argument) in his best interest to move it back into IFA and so on, until bidding converges onto the diagonal IA. Hence IFAH represents draws which we label 'd'. In fact, the argument given extends to regions KFI and LHI (the horizontal resp. vertical distance from KF to IA resp. LH to IA being equal to $\frac{3}{4}s$). The lines $\text{int}(\text{KF})$ and $\text{int}(\text{LH})$ themselves do not represent draws: dropping out for 2 on KF is better than a draw, because a draw requires an increase of his bid of (slightly) more than $\frac{3}{4}s$ (in order to win $\frac{3}{4}s$) and, analogously, for 1 on LH. Hence KFEM represents wins for 1 (2 drops out) and LHGN represents wins for 2 (1 drops out). Points on $\text{int}(\text{KI})$ and $\text{int}(\text{LI})$ do represent draws because FC represents wins for 2 and HD wins for 1. The little arrows indicate the nature of the edges. Before we carry the analysis to the square ONIM recall that in order to avoid nonexistence of equilibrium we have used the tie-breaking rule identified in Theorem 1: indifference between dropping out and a further bid is resolved in favour of the bid.

Moving into ONIM we see, that whenever the difference between the last two bids is $\geq \frac{3}{4}s$ a player should drop out; i.e. any point outside the $\frac{3}{4}s$ - strip around the diagonal OI are losses (including the lines TK and UL). On the other hand, at a point inside the strip a draw is obtainable at a parity level less than $\frac{3}{4}s$ above the last bid of the

bidding player which yields a positive increase of his net pay-off, while dropping out leaves it unchanged and bidding into a region representing a win must lower it because it requires an increase in the bid of almost $\frac{3}{4}s$ more than the increase for the draw, but it can only yield $\frac{1}{4}s$ more in pay-off. Hence OULIKT represents draws. The existence of "optimal" draws over a wide region of the square points to the exciting prospect of explaining the occurrence of "bidding wars" (i.e. step-wise escalation) that eventually recede to approach a parity level (i.e. implicit cooperation) as the outcome of perfect equilibrium play of the game. Theorem 2 below verifies this assertion. We do not attempt a complete description of the set of perfect equilibria but concentrate on the possibility that continued bidding can occur in perfect equilibrium. Let us index the dollar auction by the parameter $p=p_1=p_2$ defining players' preferences. We then have

Theorem 2: Let $p > \frac{1}{2}$ and $\bar{b} < (2p-1)s$. Then the continuous dollar auction has a perfect equilibrium that implements a draw at parity level $\bar{b}$. This equilibrium involves continued bidding of both players.

Proof: It is not difficult to see that the qualitative pattern of Fig. 3 generalizes to all dollar auctions with parameter $p > \frac{1}{2}$; in particular, $p > \frac{1}{2}$ ensures the existence of a strip around the diagonal containing only draws. It is also straight forward to see that (for the first player, say) a draw at parity level $\bar{b} < (2p-1)s$, which yields a net pay-off greater than $p \cdot s - \bar{b}$, is better than making the minimal winning bid of $p \cdot s$, which yields a pay-off of $(1-p)s$. The former pay-off exceeds the latter precisely when $\bar{b} < (2p-1)s$ holds. That a bidding path approaching $\bar{b}$ is

sustainable as a perfect equilibrium path is seen as follows; one has to "patch up" optimal draws and wins or losses in a way that is consistent in the full game.

Consider the following (symmetric) strategy prescription for the two players in which we denote by (b_{-2}, b_{-1}) the last two bids of any given finite sequence $(b_1, \dots, b_t)$ of bids (i.e. any (b_{-2}, b_{-1}) represents a unique subgame):

$$f(b_{-2}, b_{-1}) = \begin{cases} \frac{1}{2} (b_{-1} + \bar{b}) & \text{if } 0 \leq b_{-1} < \bar{b} \\ \frac{1}{2} (b_{-1} + (2p-1)s) & \text{if } \bar{b} \leq b_{-1} < (2p-1)s \\ \frac{1}{2} (b_{-1} + \min\{n \cdot (2p-1)s, b - (1-p)s\}) & \text{if } (n-1) \cdot (2p-1)s \leq b_{-1} < \min\{n \cdot (2p-1)s, b - (1-p)s\} \\ & \text{for } n \geq 2 \\ b_{-1} & \text{if } b_{-1} - b_{-2} \geq ps \text{ and } \\ & b_{-1} < b - (1-p)s \text{ or } \\ & b_{-1} \geq b - (1-p)s \text{ and } \\ & b_{-2} < b - s \\ b & \text{if } b_{-1} \geq b - (1-p)s \text{ and } \\ & b_{-2} \geq b - s \end{cases}$$

The idea underlying f is rather simple: if the last bid made is less than $\bar{b}$ continue with a bid of $\frac{1}{2} (b_{-1} + \bar{b})$ which is the average of the last bid and the target parity level, $\bar{b}$. Note that this bid does not depend on b_{-2} ; $f(b_{-2}, b_{-1})$ will be closer to $\bar{b}$ than b_{-1} was and finally converge onto this parity level from below. Given the other player's adherence to f it is a best reply for a player to follow f 's prescription, too. If b_{-1} lies in $[\bar{b}, (2p-1)s)$ f averages between b_{-1} and $(2p-1)s$ to approach the

parity level $(2p-1)s$ (which realizes a net pay-off of $ps - (2p-1)s = (1-p)s$ and is at least as good as making a winning bid that requires an increase over the last bid of the moving player by at least ps and hence yields less than $(1-p)s$ as net pay-off). For values of b_{-1} greater than $(2p-1)s$ and points (b_{-2}, b_{-1}) such that $b_{-1} - b_{-2} < ps$ we proceed by setting target parity levels at $(2p-1)s$ above the last one. This ensures that an increase of less than (or equal to) $(2p-1)s$ over the present last bid wins $p \cdot s$ yielding an increase in net pay-off of at least $(1-p)s$, whereas a winning bid requires an increase over the present bid of more than ps and can only yield an increase in net pay-off of less than $(1-p)s$. One can continue in this way until one hits the 'ceiling' $b - (1-p)s$; beyond $b - (1-p)s$ no further draws are possible and a player either has to withdraw (bid b_{-1}) if a winning bid would require an increase of more than s or make a winning bid of b if that bid implies an increase of less than s (see Figure 3). A player should always withdraw if his lowest feasible bid is more than ps - the pay-off in a draw - above his last bid. Because a draw is clearly not profitable and - given the other player's response according to f - neither is a win: it requires a raise above the last bid of the bidding player of more than $2ps > s$. This explains f .

The equilibrium path determined by f is not unique: it only determines that the first moving player must make a bid of less than $\bar{b}$; any such bid will "trigger off" a round of bidding that recedes at $\bar{b}$. The associated net pay-off is always $ps - \bar{b} > (1-p)s > 0$ for both players. Any higher first bid can only result in a lower net pay-off for the first player: because it either leads to a higher parity level and no additional winnings or results in a win with an associated net pay-off of at most $(1-p)s$. These arguments apply for all $\bar{b} < (2p-1)s$. q.e.d.

Theorem 2 shows that O'Neill's dissatisfaction with his solution of the discrete dollar auction ("Bids cannot be rescinded in the dollar auction, and by the perverse rules of the game the players cannot bid up to equal levels and then quit. But equal withdrawal is an excellent solution in the international context.") has its cause in the decision to analyze a discrete structure. If players are restrained it is compatible with perfect equilibrium play of the continuous bidding game that players bid up to equal levels and then quit. Note that a perfect equilibrium with (realized) parity level $\bar{b}_1$, pareto-dominates one with parity level $\bar{b}_2$ if $\bar{b}_1 < \bar{b}_2$. Also, the waste of resources through continued bidding in a perfect equilibrium is limited to at most $(1-p)s$ per player. So the more restrained players are (i.e. the higher their common p_i 's) the less waste of resources is to be expected from equilibrium play of the game. Similarly, and independently of the value of p ($> \frac{1}{2}$), players should have an interest to agree on a low rather than a high parity level, $\bar{b}$, because a deviation from f by a player after some bidding by making a winning bid hurts the other player less if the envisaged parity level (and hence his last bid) was low. A parity level $\bar{b}$ too close to $(2p-1)s$ might be considered "dangerous" because it imposes only a small loss on the deviant player who steps up his bid to $p \cdot s$ with the expectation of a withdrawal by the other player (who then would incur a high loss). This kind of stability argument is quite different from the point that both players also stand to gain from a reduction of the parity level.

International arms limitation negotiations or the 'dislocation' of troops and weaponry can be interpreted as measures facilitating the prevention of too high parity levels between the parties involved. Similarly, the signing of treaties banning the testing and development of certain weapons (like the treaty between the United States and the Soviet

Union banning atomic tests in the atmosphere) can be interpreted as equilibrium play of the auction game at a very low parity level: foreseeing the harmful escalating process of reinforcements triggered off by mutual bids both players try to waste as little resources as possible and reach an agreement that is sustainable as a non-cooperative equilibrium. A slightly more speculative argument suggests that repeated play of the continuous dollar auction can explain the occurrence of sporadic "rounds" of arms races (with sharp stepping up of bids) interrupted by periods of perseverance around (low) parity levels. In particular, arms races need not reach "dangerous" levels, which means that disputes that are preceded by an arms race need not escalate. Wallace (1979) has pointed out that an arms race is not sufficient for a dispute to escalate into full war, but disputes preceded by an arms race do escalate much more frequently than those not preceded by an arms race. A basic problem for players of the continuous dollar auction remains: they have to reach (implicit or explicit) agreement on which f-strategy to pursue, in particular, which parity level they should choose (or, in a more applied context, consider 'safe'). So-called 'confidence building measures' known from arms limitations talks seem to be aimed at such agreements. Those measures may -but need not- involve a third party mediator.

Our next result shows that "aggressive" players cannot reach a draw by equilibrium play of the game.

Theorem 3: Let $0 < p \leq \frac{1}{2}$.

- a) If $(2-p)s \leq b$ then there is no perfect equilibrium of the continuous dollar auction (in simple strategies).
- b) If $(2-p)s > b$ then perfect equilibrium exists and any equilibrium path is characterized by a winning bid of the first moving player and the immediate withdrawal of the second player.

The proof of Theorem 3 is omitted. For case a) it shows that 'aggressive' players would only agree on a draw within reach of the budget constraint b , but whenever both players' last bids are less than $b-s$ they never draw, because a better winning bid is always available. Intuitively, the difference to the case of 'restrained' players is that it now does not pay to stay within a ps -strip around the diagonal because a draw is valued so lowly (p is too small). 'Leaping' over the ps -strip with ones bid ensures a win and an additional increase in net pay-off of at least $(1-p)s - ps > 0$ as $p < \frac{1}{2}$. This quantity is negative for 'restrained' players with $p > \frac{1}{2}$. As a consequence, the region of draws does not extend backwards to the first round of bidding. Unfortunately, this eagerness of both players to win results in non-compact winning sets and subgames that do not have an equilibrium. If $(2-p)s > b$ (case b)) non-compact winning sets do not occur and the structure of the solution is as before: the first player bids and the second drops out immediately. Theorems 2 and 3 suggest that only 'restrained' players can rationally engage in rivalrous bidding for the stakes s . This result is in close correspondence to real world situations in which escalation is observed. Governments participating in an ongoing arms race usually stress the totally

'defensive' (i.e. non-aggressive) nature of their last bid which is aimed at maintaining approximate parity in order to continue 'peaceful coexistence'. According to this view it is precisely the difference between the pay-off associated with a loss of the race (i.e. being dominated or subjugated by another government) and the pay-off associated with a draw (sovereign coexistence) that makes bidding necessary (and worthwhile) without any intention to win involved. Similar arguments are put forward by governments to defend payment of subsidies to declining domestic industries in retaliation to similar payments abroad. Those subsidies are not aimed at winning entire world markets but at the 'protection' of the market share of domestic industries that are threatened by market exit. Once the process has been started further and further subsidies are given, one argument for further escalation being that let the industry die now would forfeit all subsidies given previously. The European steel industry is a good example. A more formal application of our results to this topic is the subject of a sequel to this paper.

III. Conclusion

The preceding analysis shows that the continuous dollar auction game differs qualitatively from the discrete auction game analyzed by O'Neill (1986). The main result (Theorem 2) shows that players of the dollar auction can rationally bid against each other in order to reach a final parity level; i.e. both players show their determination to prevent the other from winning - which is their prime motive for reactive bids - without threatening the other with a win of their own. Rationalization of this realistic behavioral pattern ($\rightarrow$ deterrence!) is made possible through the choice of a continuous rather than discrete game model.

A continuous model changes communication (or 'signalling') conditions for players in an important way: in the absence of any formal mechanism for joint agreements or unilateral precommitments (non-cooperative) players can only communicate with each other through their sequentially made - actions. In the present game "the only communication is the bid, and the only signals are the history of bidding in the auction" (Shubik, 1971). Choosing a discrete game structure thus restricts - a priori - the scope for communication between players. A discretization acts like a communication channel that can transport certain messages but not others. Specifically, the regular discretization chosen by Shubik and O'Neill makes it impossible for restrained players to signal their restrainedness through their bidding behavior. Each one has to overbid the other by at least one unit which - because of the limited scope for actions and communication - can only be read by the other player as a serious threat and attempt to win the game. In contrast, a continuous choice of bidding variables not only allows for more messages and communication but also for more interpretations of messages (bids). Theorem 2 shows that 'benevolent' interpretation of bids by restrained players is credible and mutually self-enforcing. To obtain a similar result with a discrete structure is not impossible but would require to force a very odd discrete structure upon players: its (countably infinite) action sets must admit a sequence with a cluster point (that is to be interpreted as the desired parity level). Shubik (1971) is aware that the formal (regularly discrete) game structure does not adequately reflect communication conditions that are important in interorganization or international escalation and concludes that "a game theory analysis alone will probably never be adequate to explain such a process, it can (only) serve to delimit the threat and enforcement

possibilities" (Shubik, p.111). We agree with this statement but maintain that the present analysis shows that pure 'game theory analysis' can - by simply delimiting threat and enforcement possibilities - produce a much more realistic model of such a process as thought previously.

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