Seminar Paper No. 420

ASSET MARKETS, TAX ARBITRAGE; AND THE REDISTRIBUTIVE PROPERTIES OF PROGRESSIVE INCOME TAXATION

by

Jonas Agell and Mats Persson

INSTITUTE FOR INTERNATIONAL ECONOMIC STUDIES

University of Stockholm

ISSN 0347-8769

Seminar Paper No. 420

ASSET MARKETS, TAX ARBITRAGE, AND THE REDISTRIBUTIVE PROPERTIES OF PROGRESSIVE INCOME TAXATION

by

Jonas Agell and Mats Persson

October, 1988

Institute for International Economic Studies
S-106 91 Stockholm
Sweden

ASSET MARKETS, TAX ARBITRAGE, AND THE REDISTRIBUTIVE PROPERTIES OF PROGRESSIVE INCOME TAXATION

by

Jonas Agell,
Trade Union Institute for Economic Research (FIEF), Stockholm, Sweden,
Uppsala University, Uppsala, Sweden

and

Mats Persson
Institute for International Economic Studies
Stockholm, Sweden

Abstract

It is commonly believed that tax arbitrage is anti-egalitarian. The present paper shows that this is not necessarily true; tax arbitrage might actually reduce inequality as well as increase efficiency. It is also shown that the introduction of tax arbitrage will linearize the tax system. Thus complicated, non-linear tax schedules in the spirit of Mirrlees (1971) cannot be sustained.

Mailing address: Mats Persson
Institute for International Economic Studies
S-106 91 Stockholm
Sweden

ASSET MARKETS, TAX ARBITRAGE, AND THE REDISTRIBUTIVE PROPERTIES OF PROGRESSIVE INCOME TAXATION

1. Introduction

When marginal tax rates differ between individuals, there is an incentive for tax arbitrage in various forms. The most simple form is perhaps the one involving fictituous interest payments: a person with a high marginal tax rate reports that he has borrowed money from a person with a lower marginal tax rate and is therefore allowed to deduct the interest payments. The interest payment constitutes taxable income for the other person, but is then taxed at a lower marginal tax rate, and the two parties split the resulting profit.[1]

The way the profit is to be divided - whether both should have 50 percent, or one should have e.g. 75 percent - is the result of market forces. One can represent the above transaction as the outcome of the following, more traditional market arrangement. The person with the high marginal tax rate borrows the money against a taxable bond from the person with the low marginal tax rate. The latter borrows the same amount from the former against a tax exempt bond. This situation can be analyzed within the framework of a model of asset market equilibrium in connection with the tax clientele model of Miller (1977), and the way the two parties divide the profits of their transaction can be thought of as the relative yields on taxable and tax exempt bonds in such an equilibrium.

In reality, tax arbitrage in the form just described is limited in magnitude by several factors. The most important one is perhaps the fact that the private costs of arranging, monitoring and enforcing such, perhaps outright illegal, contracts can in many cases be

---

[1] For other types of tax arbitrage, see Stiglitz (1983).

prohibitive. Instead there are other assets that are perfectly appropriate for tax arbitrage by being favourably taxed and perfectly legal, such as land and tax-exempt government bonds. The only difference between such assets and the tax-exempt bonds described in the previous paragraph is that the latter can be freely issued by the agents in the economy (i.e., they are "inside" assets) while the former can be provided only by some outside agency, e.g. the government.

In the present paper we will analyze only the basic case, where there are no restrictions on debt issue. The other setup, where only an outside agency can issue tax-exempt bonds, will in principle be similar to our setup. Due to the non-negativity constraint on tax-exempt bond holdings, however, corner solutions will occur and the resulting expressions will therefore be technically more complicated.

One should bear in mind that actual arbitrage behavior is not limited to the financial markets. Whenever marginal taxes differ, successful arbitrage can be carried out in the labor market, in the choice of home versus market production, etc. The results of this paper therefore have wider implications than is immediately evident from its setup. The full analysis of various forms of tax arbitrage must however take several complications, such as productivity differences, into account, which calls for a much more complicated model. The limitation to capital markets has the advantage of being pedagogically simple and is sufficient to elucidate the main points.

In the popular debate, tax arbitrage is often criticized on egalitarian grounds. It is a common belief that transactions like the ones described above will favor the high-income earners and thereby counteract the redistributive properties of the progressive income tax system. In Section 2 of this paper we show that this is not necessarily the case: in an economy with exogenous income, the distribution of disposable income after tax arbitrage has taken place might very well be more egalitarian than the distribution that would obtain if tax arbitrage could be effectively prohibited. We also examine the incidence of tax arbitrage; in particular, we find that individuals with an income close to the average income in the economy will lose from tax arbitrage, whereas individuals situated at the tails of the income distribution will typically gain.

The incidence of tax arbitrage is analyzed further in Section 3, where we model an economy where individuals' labor supply is endogenous. In this economy, the incidence effects of introducing asset trade is less clear-cut and will depend on the form of individual utility functions and on the type of tax changes used to ensure government budget balance when introducing tax arbitrage.

These sections of the paper contain only positive analysis, but our results also have implications for normative tax analysis. The paper concludes by relating our results to those of the theory of optimal income taxation. One of our conclusions is that tax arbitrage will make the effective, as opposed to the official, tax schedule linear. Thus the sophisticated non-linear schedules advocated in the literature following Mirrlees (1971) cannot be sustained.

2. Tax Arbitrage in an Economy with Exogenous Income

2.1 Redistribution Without Asset Trade

Assume an economy with an initial distribution of non-tradable endowments, say human capital, across individuals. A given individual then experiences an exogenous income y. In the absence of taxes the distribution of y is thus equal to the distribution of disposable income.

The government now introduces an income tax such that a gross taxable income y yields a net disposable income $\psi(y)$, where $\psi(.)$ is assumed to be twice continously differentiable. Marginal tax rates are assumed not to exceed 100 percent,[2] i.e. $\psi'(y) > 0$. The tax schedule is assumed to be progressive in the traditional sense that the ratio of disposable income to taxable income is a decreasing function of taxable income, i.e. $d(\psi(y)/y)/dy < 0$. Since the setup of our model relies on the tax clientele hypothesis, we also need marginal tax rates to increase with income; thus we assume that $\psi(.)$ is concave, i.e. that $\psi''(y) < 0$. Since we only consider purely redistributive tax systems, we require that

$$(1) \qquad \overline{\psi(y)} \equiv \int \psi(y)\, dF(y) = \int y\, dF(y) \equiv \bar{y}$$

From our knowledge of the distribution of gross income y, we can calculate statistics on the distribution of disposable income $\psi(y)$. By a second order Taylor expansion around the average income $\bar{y}$ we have

$$y^d \approx \psi(\bar{y}) + \psi'(\bar{y})(y - \bar{y}) + \frac{1}{2}\psi''(\bar{y})(y - \bar{y})^2$$

and thus the average disposable income is

$$(2) \qquad \bar{y}^d \approx \psi(\bar{y}) + \frac{1}{2}\psi''(\bar{y})\mathrm{Var}(y).$$

Inserting these formulae in the expression for the variance of disposable income, we obtain

---

[2] The tax paid is equal to $y - \psi(y)$, and thus the marginal tax rate is $1 - \psi'(y)$.

$$(3) \quad \mathrm{Var}(y^d) \approx (\psi')^2\, m_2 + \frac{1}{4}(\psi'')^2\, m_4 - \frac{1}{4}(\psi'')^2\, m_2^2 + \psi'\psi''\, m_3$$

where we have dropped the argument in the $\psi(.)$ function, and where m_k denotes the k-th central moment of the distribution of y, i.e. $m_k \equiv E[(y - \bar{y})^k]$.

In the following we will take a simplistic view of the problem of income redistribution, and we will identify the degree of inequality with the variance of disposable income.[3] In particular, we will compare the variance of gross income Var(y) and the variance of disposable income given by (3) to the variance of disposable income that results from a system with unrestricted trade in financial assets, i.e. with unrestricted tax arbitrage.

2.2 Redistribution With No Restrictions on Asset Trade

Assume now that two kinds of assets are introduced, namely taxable and tax exempt claims.[4] The existence of the latter does not necessarily mean that the government has issued them, or even endorses the existence of such claims. It can rather be that the individuals make personal loans in the informal sector of the economy, without ever reporting them to the authorities. Still, it is a well-known fact that there are many assets the tax favors of which are very well endorsed by the authorities (for example land) and that there are even tax-favored

---

[3] This obviously implies a very special social welfare function, namely a function with only the mean and the variance of the income distribution as arguments. Allowing for more general social welfare functions would not yield any additional insights to the basic functioning of the model.

[4] The analysis of market equilibrium with taxable and tax exempt bonds was pioneered by Miller (1977). The mechanism analysed in that paper, and in the ensuing papers by e.g. Auerbach and King (1983) and McDonald (1983) is however not the same as the one studied in the present paper.

assets that are issued by the authorities (for example tax exempt bonds).

Denote by r the (risk-free) yield on taxable bonds and denote by X the holdings of such bonds by a particular individual. The taxable income of the individual is then $y + rX$, and the disposable income is

$$(4) \quad \tilde{y}^d = \rho x + \psi(y + r X)$$

where ρ denotes the (risk-free) yield on tax exempt bonds and x denotes the holdings of such bonds by the individual. With no restrictions on bond issue, x and X can be negative as well as positive. If no individual owns any initial wealth apart from his human capital, and if that capital is non-tradeable, a negative x must be balanced by a positive X of the same absolute value, and vice versa.

Maximizing (4) with respect to X (= -x) yields the first-order condition

$$-\rho + r \psi'(y + r X) = 0$$

or

$$(5) \quad y + rX = \psi'^{-1}(\rho/r)$$

This is intuitively reasonable: with unlimited possibilities for tax arbitrage, high-income earners will issue taxable debt and hold tax exempt bonds, while low-income earners will hold taxable debt and issue non-taxed debt, until all taxable incomes and all marginal tax rates are equalized. From the first-order condition we get the optimal value of X for the individual:

$$(6) \quad X = \frac{1}{r} \cdot [\psi'^{-1}(\rho/r) - y].$$

With no outside debt the average bond holding $\bar{X}$ must be equal to zero, which by (6) can be written as

$$(7) \quad \psi'^{-1}(\rho/r) = \bar{y}.$$

This is the market equilibrium condition which gives us the relative asset yield $\rho/r = \psi'(\bar{y})$.

Plugging (7) into (6) we can express individual demand in market equilibrium as

$$(8) \quad X = \frac{1}{r}(\bar{y} - y).$$

The disposable income of the individual is then, by (4),

$$(9) \quad \tilde{y}^d = \psi'(\bar{y})(y - \bar{y}) + \psi(\bar{y}).$$

This means that allowing for tax arbitrage will make the tax system linear with a slope $\psi'(\bar{y})$ and an intercept $\psi(\bar{y}) - \psi'(\bar{y})\bar{y}$; instead of a non-linear function for disposable income $\psi(y)$, the actual function will now be linear in y, as shown by (9).

Here we note that if the tax system satisfies (1), i.e. it is purely redistributive in the absence of asset trade, the introduction of asset trade will affect the government's budget balance. The average disposable income with trade in assets is, by (9),

$$E(\tilde{y}^d) = \psi(\bar{y}).$$

Since $\psi(.)$ is a concave function, Jensen's Inequality implies that

$$E(\tilde{y}^d) > \overline{\psi(y)} \equiv \bar{y}$$

We can thus state

Proposition 1 : If the government's budget is initially balanced in an economy with exogenous income, the introduction of trade in assets will cause a government budget deficit.

This is hardly surprising; the very definition of tax arbitrage is that people try to avoid taxes, and Proposition 1 merely says that they succeed in that aim, even when general equilibrium effects are taken into consideration. To establish budget balance again, the tax system has to be changed in such a way that $E(\tilde{y}^d) = \bar{y}$. This can be done in an infinite number of ways by changing the $\psi(.)$ function. We will here assume that the new tax system $\tilde{\psi}(y)$ is such that

(10) $\tilde{\psi}(y) \leq \psi(y)$ with strict inequality for at least some y

and

(11) $\tilde{\psi}'(y) \leq \psi'(y)$.

These assumptions are fairly general and imply that the tax increases necessary to restore budget balance do not imply a lower tax rate for anybody (10) and do not imply a lower marginal tax rate for anybody (11). The simplest change to think of is that of a constant shift of the $\psi(\cdot)$ function, i.e. a lump-sum tax which is the same for everybody. Thus a new tax system $\tilde{\psi}(y)$ such that $\tilde{\psi}(y) = \psi(y) - t$, where $t = \psi(\bar{y}) - \bar{y}$, would restore budget balance. Such a tax system would satisfy (10) with strict inequality for all y, and would satisfy (11) with equality for all y. In the following, however, we will not assume any more specific tax changes than the very general ones stated in (10) and (11).

With the new tax system and a balanced budget, we rewrite the expression for disposable income (9) as

(12) $\tilde{y}^d = \tilde{\psi}'(\bar{y})(y - \bar{y}) + \tilde{\psi}(\bar{y})$,

i.e. $\tilde{y}^d$ is a linear function of y with a slope $\tilde{\psi}'(\bar{y})$ and with an intercept $\tilde{\psi}(\bar{y}) - \tilde{\psi}'(\bar{y})\bar{y}$. By the strict concavity of the $\tilde{\psi}$ function, we know that the intercept must be strictly positive.

To analyze what happens to the income distribution, we note from (12) that

(13) $\mathrm{Var}(\tilde{y}^d) = (\tilde{\psi}')^2 \mathrm{Var}(y)$.

where $\tilde{\psi}'$ is evaluated at $\bar{y}$. From the condition for budget balance we have, by (12), that $\tilde{\psi}(\bar{y}) = \bar{y}$. Combining this observation with the assumption of progressivity, which says that the ratio $\tilde{\psi}(y)/y$ should be decreasing in y, we have that[5] $\tilde{\psi}'(\bar{y}) < 1$. Equation (13) therefore implies that the introduction of tax arbitrage in the form of trade in assets does not eliminate the redistributive effects of the progressive income tax. We can state

Proposition 2: In an economy with asset trade and progressive income taxation, the distribution of disposable income is more egalitarian than the distribution of exogenous gross income.

More interesting, however, is whether the introduction of asset trade *per se* makes the distribution of disposable income more or less egalitarian than it would be in an economy with progressive income

---

[5] The assumption of progressivity implies that

$$\frac{d(\tilde{\psi}(y)/y)}{dy} = (\tilde{\psi}'(y)y - \tilde{\psi}(y))/y^2 < 0.$$

At the point $y = \bar{y}$, $\tilde{\psi}(\bar{y}) = \bar{y}$ and the assumption of progressivity reduces to $\tilde{\psi}'(\bar{y}) - 1 < 0$.

taxation but without tax arbitrage. The variance of disposable income in the absence of tax arbitrage is given by (3) and should be compared to the variance of disposable income in the presence of tax arbitrage as given by (13). Since $Var(y) \equiv m_2$, we have that

$$Var(y^d) - Var(\tilde{y}^d) = [(\psi')^2 - (\tilde{\psi}')^2]\, m_2 - \\ + \frac{1}{4}(\psi'')^2 (m_4 - m_2^2) + \psi'\psi''\, m_3.$$

Let us first look at the term within square brackets. By (11) this is non-negative. Assume now that the distribution of y is symmetric, so that $m_3 = 0$. Since, for any distribution, $m_4 > m_2^2$ we can state[6]

Proposition 3: If the distribution of exogenous income y is symmetric, allowing for tax arbitrage in the form of trade in assets will make the distribution of disposable income more egalitarian.

To pursue the redistributional issues further than can be done by a summary statistic like the variance, we are now going to take a look at who actually gains and who looses if asset trade is introduced. The basic observation is that without asset trade, disposable income is a concave function $\psi(\cdot)$ of gross income y, while with asset trade, disposable income is given by (12) as a linear function of gross income.

6 For an arbitrary variable z we have $Var(z) \equiv E(z^2) - [E(z)]^2$. Define a new variable ξ such that $z \equiv \xi^k$. Thus $Var(z) \equiv E(\xi^{2k}) - [E(\xi^k)]^2 > 0$. Thus, for any variable ξ, $\alpha_{2k} > \alpha_k^2$, where α_k is the k-th non-central moment $E(\xi^k)$. Set $\xi \equiv y - \bar{y}$. For that variable it obviously also holds that $\alpha_{2k} > \alpha_k^2$, i.e. that

$$E\,[(y - \bar{y})^{2k}] > [E(y - \bar{y})^k]^2$$

which proves the desired property: $m_4 > m_2^2$.

For the individual with the average income[7] $\bar{y}$, disposable income with asset trade is, by (12),

$$\tilde{y}^d(\bar{y}) = \tilde{\psi}(\bar{y}) = \bar{y}$$

where the second equality stems from the fact that the new tax system $\tilde{\psi}$ must yield a balanced budget. But that was also the case with the old tax system $\psi(\cdot)$:

$$\bar{y} = \overline{\psi(y)} < \psi(\bar{y})$$

where the inequality is due to Jensen's Inequality. Combining these two expressions yields

$$\tilde{\psi}(\bar{y}) < \psi(\bar{y}),$$

i.e. the disposable income of the average individual is less under asset trade than under a regime with no asset trade. It is noteworthy that this is a very general property which follows from the assumption of budget balance and a concave $\tilde{\psi}(\cdot)$ function; thus, the assumptions (10) and (11) about the relation between the old tax function ψ and the new tax function $\tilde{\psi}$ are not necessary for this result. Further, by continuity this inequality will also hold for individuals with gross income $\bar{y} + \varepsilon$, where the absolute value of ε is sufficiently small. This proves

<u>Proposition 4</u>: In an economy with exogenous incomes, introducing tax arbitrage in the form of asset trade will harm the individuals with incomes close to the mean $\bar{y}$.

---

7 With a continuum of individuals, such an individual will always exist.

To identify the winners we note that for an individual who is indifferent between a regime without asset trade and a regime with asset trade, it holds that $\psi(y) = \tilde{y}^d(y)$, where $\tilde{y}^d(\cdot)$ is the linear function of y given by (12). Thus, for the indifferent individual,

$$(14) \qquad \psi(y) = \tilde{\psi}'(\bar{y})(y - \bar{y}) + \tilde{\psi}(\bar{y}) \ .$$

Equation (14) is a non-linear equation in y which may have zero, one or several roots. We see that the left hand side is a concave function of y, while the right hand side is a straight line with a positive intercept. Since a straight line can intersect a concave function at most twice, equation (14) can have at most two roots.

This is depicted in <u>Figure 1</u>. By <u>Proposition 4</u> above we know that $\bar{y}$ is not a solution to (14); for $y = \bar{y} + \varepsilon$, where ε is a small number, the left hand side is larger than the right hand side. For some y, however, the left hand side must be smaller than the right hand side; otherwise $\psi(y) > E(\tilde{y}^d)$ which is impossible since $\psi(y) = E(\tilde{y}^d) = \bar{y}$ by the requirement of a balanced government budget. Thus the straight line representing the right hand side of (14) must intersect the concave function representing the left hand side of (14) at least once; equation (14) has therefore at least one root, and in <u>Figure 1</u> it has been drawn as if it has two.

One can observe that if there are some very poor individuals in the economy, that is, y = 0 for at least one person, then the introduction of asset trade will make this person better off if the tax system is such that

$$(1 - \tilde{\psi}'(\bar{y}))\bar{y} > \psi(0).$$

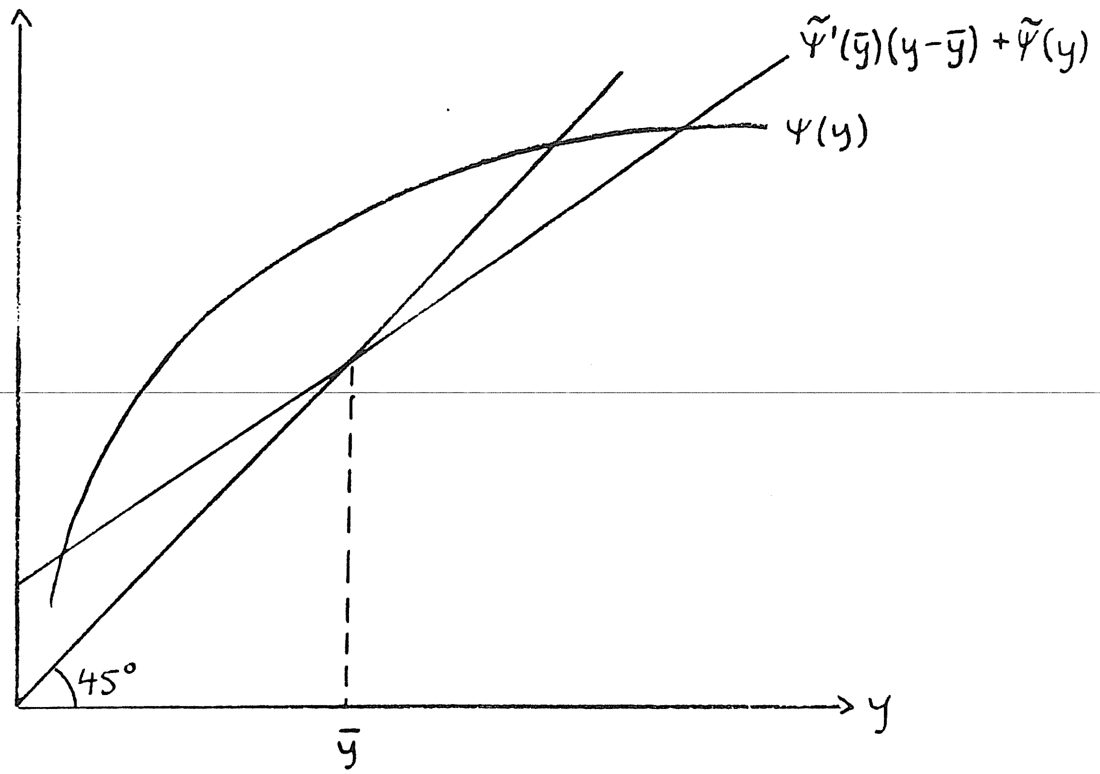

Figure 1.

In such a case, tax arbitrage would be justified from a Rawlsian (maximin) welfare point of view.

3. Incidence of Tax Arbitrage with Endogenous Labor Supply

3.1 Individual Optimization, Market Equilibrium and Budget Balance

In the preceeding section we have assumed income to be exogenous, and taxes have thus only redistributive effects. With endogenous labor supply, taxes will have effects on efficiency also. In this section we will examine the incidence of introducing tax arbitrage when individuals' utility depends on consumption of goods and leisure.

Consider an individual who derives utility from consumption and from leisure, and who has a marginal product w which depends positively on the ability of the individual. Assuming for simplicity that the production technology is linear, i.e. w is the marginal and the average product, and that the production sector is competitive, i.e. w is the individual's wage rate.

The individual maximizes a utility function $u(c, 1 - \ell)$ subject to an appropriate budget constraint.[8] The function u is continuously differentiable, increasing in c and $1 - \ell$, and strictly quasi-concave. In the absence of taxes, the budget constraint reads $c = w\ell$ and we have the first-order condition

$$(15) \quad \frac{u_2}{u_1} = w$$

where u_2 is the partial derivative of the utility function with respect to its second argument (i.e. leisure) and u_1 is the partial derivative with respect to its first argument (i.e. consumption). Equation (15)

---

[8] We have here assumed that the total time endowment is the same for everybody and equal to unity.

thus gives us the well-known requirement that the individual's marginal rate of substitution between consumption and leisure should equal the marginal rate of transformation in production.

In the presence of taxes, but without asset trade, the individual budget constraint reads $c = \psi(w\ell)$, and the corresponding first order condition becomes

$$(16) \quad \frac{u_2}{u_1} = w\ \psi'(w\ell).$$

Here the tax system drives a wedge between the consumption and the production decisions in the economy. Since $\psi'(.)$ varies over individuals, the wedge is different for different people.

With asset trade, the budget constraint becomes $c = -\rho X + \psi(w\ell + rX)$, where ρ, X and r have the same definition as in Section 2 above. Maximizing with respect to X and ℓ gives the two first-order conditions

$$(17) \quad w\ell + rX = \psi'^{-1}(\rho/r)$$

$$(18) \quad \frac{u_2}{u_1} = w\ \psi'(w\ell + rX).$$

Obviously, equation (17) is just the equivalent of equation (5); it says that in equilibrium, everybody will have the same taxable income $w\ell$ + rX, thereby equalizing the marginal tax rates. As before, equilibrium in the financial markets requires that $\bar{X} = 0$, which by (17) implies

$$(19) \quad \psi'(\overline{w\ell}) = \rho/r$$

where $\overline{w\ell}$ is the average labor income. Substituting this into (17) gives us the individual's bond holding in market equilibrium:

$$(20) \quad X = \frac{1}{r}(\overline{w\ell} - w\ell).$$

By this we get, from (18), that

$$(21) \quad \frac{u_2}{u_1} = w \frac{\rho}{r}.$$

Equation (21) can be compared to the corresponding expression for a first-best optimum (15) and to that for a situation without asset trade (16). We see that the situation under asset trade is more like the first-best situation in the sense that the individual marginal rates of substitution are proportional to the marginal rates of transformation. There is however still a wedge present, and we cannot a priori say whether this situation is better, from a welfare point of view, than the situation where the wedges differ over individuals, like in (16).

Let us denote the optimal labor supply in a situation without asset trade, as given by (16), by $\ell^* = \ell^*(w)$. The corresponding indirect utility is

$$V(w) \equiv u(\psi(w\ell^*), 1 - \ell^*).$$

For the case with asset trade, (17), (18) and (19) gives us the optimal labor supply

$$(22) \quad \ell = \ell(w, \bar{y})$$

where $\bar{y}$ is the average wage income in society, i.e. $\bar{y} \equiv \overline{w\ell} \equiv E(w\ell)$. Multiplying (22) by w and taking the average defines an implicit function for the endogenous variable $\bar{y}$:

$$\bar{y} = E[w\ell(w, \bar{y})].$$

This can in turn be substituted into (22) to yield the individual's labor supply as a function of the individual wage rate w and of the distribution of w across individuals.

Having derived the optimal labor supply and asset demand in market equilibrium, we can easily derive the disposable income of an individual with a wage rate w:

$$(23)\quad y^d = \psi'(\bar{y})(w\ell - \bar{y}) + \psi(\bar{y}).$$

This gives in turn the indirect utility function:

$$(24)\quad V(w, \bar{y}) \equiv u[\psi'(\bar{y})(w\ell - \bar{y}) + \psi(\bar{y}), 1 - \ell]$$

where $\ell \equiv \ell(w, \bar{y})$.

In deriving (24), we have assumed that the disposable income function $\psi(.)$ is identical to the one used before introducing asset trade. By assumption, the tax system is purely redistributive in the absence of asset trade, i.e. $\psi(.)$ satisfies

$$(25)\quad E[\psi(w\ell^*)] \equiv \int \psi(w\ell^*)dF(w) = \int w\ell^*dF(w) \equiv E(w\ell^*) \equiv \bar{y}^*.$$

With the $\psi(.)$ function being such that (25) is fulfilled, the introduction of asset trade will affect the government's budget balance. The resulting per capita budget deficit (surplus) is simply given as the difference between average disposable income and the average labor income:

$$(26)\quad D \equiv \int (-\rho X + \psi(\overline{w\ell}))dF(w) - \bar{y}$$

which can be rewritten as

$$(27)\quad D \equiv \psi(\bar{y}) - \bar{y}$$

since the integral over X is zero and since taxable income by (17) and (19) is equal to $\bar{y}$ for each individual.

Depending on the response of average labor income to the change in marginal tax rates induced when introducing asset trade, asset trade may lead to a budget deficit <u>or</u> a budget surplus. When the average labor income is unchanged, i.e. when $E(w\ell) \equiv \bar{y} = \bar{y}^* \equiv E(w\ell^*)$, allowing for asset trade leads to a budget deficit.[9] However, if average labor supply and average labor income increase when asset trade is introduced, there may be a budget surplus.[10] Consequently, there is with endogenous labor supply no simple analogue to <u>Proposition 1</u> in Section 2.2.

In the following, we reestablish budget balance by changing the $\psi(.)$ function. For simplicity we assume that there are no "Laffer effects", i.e. we assume that a budget deficit can be alleviated by a tax increase and not by a tax decrease. Thus, whenever D is positive we increase tax revenue as in (10) and (11) by replacing $\psi(.)$ by a new disposable income function $\tilde{\psi}(.)$ satisfying

$$(28)\qquad \begin{aligned} &\tilde{\psi}(y) \leqq \psi(y) \qquad \text{with strict inequality for at least one } y \\ &\tilde{\psi}'(y) \leqq \psi'(y). \end{aligned}$$

When D is negative, meaning that introducing asset trade leads to a budget surplus, we instead specify $\tilde{\psi}(.)$ as

---

[9] The proof follows from a straightforward application of Jensen's inequality. Budget balance without asset trade requires that $E[\psi(w\ell^*)] = E(w\ell^*)$. If $E(w\ell) = E(w\ell^*)$, this implies that $D = \psi[E(w\ell^*)] - E(w\ell^*)$. Since $\psi(.)$ is concave, we have by Jensen's inequality that $\psi[E(w\ell^*)] > E[\psi(w\ell^*)]$, which proves the result.

[10] Introducing asset trade implies a fall in the marginal tax rates of the high-income earners and an increase in the marginal tax rates of the low-income earners. Somewhat loosely, the case of $D < 0$ occurs if the induced realignment of marginal tax rates when introducing asset trade leads to a "large" increase in average labor supply.

$$(29)\quad \begin{aligned}&\tilde{\psi}(y) \geqq \psi(y) \qquad \text{with strict inequality for at least one } y\\ &\tilde{\psi}'(y) \geqq \psi'(y).\end{aligned}$$

When the government restores budget balance according to (28) or (29), the function for disposable income (23) becomes[11]

$$(30)\quad \tilde{y}^d = \tilde{\psi}'(\bar{\bar{y}})(w\tilde{\ell} - \bar{\bar{y}}) + \tilde{\psi}(\bar{\bar{y}}).$$

Thus the property of Section 2 above carries over to the case with endogenous labor supply: allowing for tax arbitrage will make the effective tax system linear in income $w\tilde{\ell}$. Since the general equilibrium solution for individual and average labor supply with a balanced budget and a new disposable income function $\tilde{\psi}(.)$ will differ from the general equilibrium solution with an unbalanced budget and the old function $\psi(.)$, we denote the equilibrium labor supply of the individual in the presence of $\tilde{\psi}(.)$ by $\tilde{\ell} = \tilde{\ell}(w, \bar{\bar{y}})$. We have analogously defined the average labor income as $\bar{\bar{y}} \equiv E(w\tilde{\ell})$.

The indirect utility function of an individual in the presence of asset trade, and a balanced government budget, becomes

$$(31)\quad \tilde{V}(w, \bar{\bar{y}}) \equiv u[\tilde{\psi}'(\bar{\bar{y}})(w\tilde{\ell} - \bar{\bar{y}}) + \tilde{\psi}(\bar{\bar{y}})\ ,\ 1 - \tilde{\ell}]$$

3.2 Distribution of Gains and Losses of Introducing Asset Trade

What is the incidence of introducing asset trade? For any individual, depending on whether

$$(32)\quad \tilde{V}(w, \bar{\bar{y}}) \gtreqless V(w),$$

---

[11] The derivation of (30) parallels the derivation of (23).

the individual gains, is indifferent to, or loses from the introduction of asset trade.

In the case when asset trade leads to a budget surplus which is eliminated by using the transfer scheme in (29), it is straightforward to show

Proposition 5: In an economy with endogenous labor supply where government budget balance is restored by lowering tax rates according to (29), introducing tax arbitrage in the form of asset trade will increase welfare of individuals whose labor income $w\ell^*$ before asset trade is greater than or equal to the average labor income $\bar{\bar{y}}$ with asset trade. For all other individuals, introducing asset trade makes them at least as well off as without asset trade.

To prove this, we first define the constrained indirect utility function in an economy with asset trade as

$$(33)\quad \tilde{V}(w,\ \bar{\bar{y}},\ \ell^*(w)) \equiv u[\tilde{\psi}(\bar{\bar{y}})(w\ell^*(w)\ -\ \bar{\bar{y}})\ +\ \psi(\bar{\bar{y}}),\ 1\ -\ \ell^*(w)].$$

Here $\tilde{V}(w,\ \bar{\bar{y}},\ \ell^*(w))$ simply defines the indirect utility when we have constrained the individual with a wage rate w to supply the same amount of labor as he would do in the unconstrained case with no asset trade and the old tax system $\psi(.)$. Let us now look at the individual whose labor income $w\ell^*$ in the case of no asset trade equals the average labor income $\bar{\bar{y}}$ that would result if asset trade is introduced.[12] Constraining this individual by imposing $w\tilde{\ell} = w\ell^*(w)$, his constrained indirect utility (33) reduces to

$$(34)\quad \tilde{V}(\hat{w},\ \bar{\bar{y}},\ \ell^*(\hat{w})) \equiv u[\tilde{\psi}(\bar{\bar{y}}),\ 1\ -\ \ell^*(\hat{w})],$$

---

[12] This presumes that there is a continuous distribution of w across individuals; then there will be at least one individual who supplies $\ell^*(w)$ in such a fashion that $w\ell^*(w) = \bar{\bar{y}}$.

where $\hat{w}$ is the wage rate of this particular individual. By the property of the indirect utility function, we have that $\tilde{V}(\hat{w}, \bar{\bar{y}}) \geq \tilde{V}(\hat{w}, \bar{\bar{y}}, \ell^*(\hat{w}))$. Further,

$$(35) \quad \tilde{V}(\hat{w}, \bar{\bar{y}}, \ell^*(\hat{w})) \geq V(\hat{w})$$

if

$$(36) \quad \tilde{\psi}(\bar{\bar{y}}) \geq \psi(\bar{\bar{y}}).$$

With asset trade, everyone has the same taxable income $\bar{\bar{y}}$. Consequently, the only way to restore budget balance using the transfer scheme in (29) is to set $\tilde{\psi}(\bar{\bar{y}}) > \psi(\bar{\bar{y}})$. Thus the inequality in (36) holds <u>strictly</u>. Hence $\tilde{V}(\hat{w}, \bar{\bar{y}}) > V(\hat{w})$, which proves that <u>Proposition 5</u> holds at least for the individual with a labor income in the absence of asset trade $w\ell^*(w) = \bar{\bar{y}}$.

In general we know that indirect utility with asset trade is greater than indirect utility without asset trade if

$$(37) \quad \tilde{\psi}'(\bar{\bar{y}})(w\ell^* - \bar{\bar{y}}) + \tilde{\psi}(\bar{\bar{y}}) > \psi(w\ell^*).$$

We have just showed that this holds if $w\ell^* = \bar{\bar{y}}$. Since

$$\tilde{\psi}'(\bar{\bar{y}}) \geq \psi'(\bar{\bar{y}}) \geq \psi'(w\ell^*)$$

for all $w\ell^* \geq \bar{\bar{y}}$, the left-hand side of (37) will always be greater than the right-hand side. We have thus proved that the introduction of asset trade will benefit all individuals with a pre-asset-trade income $w\ell^* \geq \bar{\bar{y}}$.

To complete the proof of <u>Proposition 5</u>, we finally have to show that all individuals with income $w\ell^* < \bar{\bar{y}}$ are at least as well off with asset trade as without. For these individuals, it is clear that they could abstain from asset trade, supply the same labor $\ell^*(w)$ as before, and enjoy a consumption $\tilde{\psi}(w\ell^*(w))$ which by (29) is at least as high as the one they enjoyed before. This completes the proof.

By <u>Proposition 5</u>, it is clear that introducing asset trade represents a Pareto improvement whenever asset trade leads to a budget surplus in the sense of D in (26) taking a negative value. This result obviously holds for very general tax changes - by (29), we only require that the tax reductions necessary to restore budget balance do not imply a higher tax rate for anybody and do not imply a higher marginal tax rate for anybody. In the much less general case when budget balance is restored by a lump-sum transfer which is the same for everybody, we can easily show a stronger version of Proposition 5, namely that <u>all</u> individuals will gain from the introduction of tax arbitrage.[13]

Turning to the case when introducing asset trade creates a budget deficit, meaning that D in (26) takes a positive value, things become less clear-cut. In particular, there will be both gainers and losers from introducing asset trade, and the incidence will critically hinge on the type of change in the tax system used to ensure budget balance and on the form of the utility function. However, even when budget balance is restored by the very general tax changes in (28), we can state

<u>Proposition 6</u>: In an economy with endogenous labor supply where government budget balance is restored by <u>raising</u> tax rates according to

---

[13] Defining the constrained indirect utility function as in (33) with $\tilde{\psi}(.) \equiv \psi(.) + t$, we have that

$$\tilde{V}(w,\bar{y}) \geq \tilde{V}(w,\ \bar{y},\ \ell^*(w)) > V(w)$$

if $\psi'(\bar{y})(w\ell^*(w) - \bar{y}) + \psi(\bar{y}) + t > \psi(w\ell^*(w))$. By the concavity of $\psi(.)$, this will hold for any positive value of t.

(28), introducing tax arbitrage in the form of asset trade will reduce welfare for individuals whose labor income with asset trade is close to the average labor income $\bar{\bar{y}}$.

This can be proved in the following way. Assuming that there is a continuous distribution of w across individuals, there will be at least one individual who earns the average wage income $\bar{\bar{y}}$ and who consequently, by (20), does not engage in tax arbitrage in an economy with asset trade. We assume that w_o is the wage of this individual. The individual loses from asset trade if

$$(38) \quad \tilde{V}(w_o, \bar{\bar{y}}) \equiv u(\tilde{\psi}(\bar{\bar{y}}), 1 - \tilde{\ell}(w_o, \bar{\bar{y}})) < V(w_o)$$

We next define the constrained indirect utility function without asset trade as

$$(39) \quad V(w_o, \tilde{\ell}(w_o, \bar{\bar{y}})) \equiv u(\psi(w_o\tilde{\ell}), 1 - \tilde{\ell})$$

which has the property $V(w_o, \tilde{\ell}(w_o, \bar{\bar{y}})) \leqq V(w_o)$. Obviously, we have that

$$(40) \quad \tilde{V}(w_o, \bar{\bar{y}}) < V(w_o, \tilde{\ell})$$

if

$$(41) \quad \tilde{\psi}(w_o\tilde{\ell}) < \psi(w_o\tilde{\ell}).$$

With asset trade, everyone has the same taxable income $E(w\tilde{\ell}) \equiv \bar{\bar{y}}$. As a consequence, the only way to restore budget balance using the tax raising scheme in (28) is to set $\tilde{\psi}(\bar{\bar{y}}) < \psi(\bar{\bar{y}})$, which means that (41) must hold. Consequently, $\tilde{V}(w_o, \bar{\bar{y}}) < V(w_o)$ and thus the individual with a wage w_0 loses from the introduction of asset trade. By continuity of the indirect utility function with respect to w, individuals with a wage $w_0 + \varepsilon$ also loses from introducing asset trade, where ε is a small positive or negative number. This completes the proof of Proposition 6.

When tax arbitrage leads to a budget deficit, identifying the gainers from introducing asset trade is less straightforward than identifying the losers. However, just as in the case with exogenous income the gainers must be situated at the tails of the distribution of earned income. Thus, depending on the precise from of utility functions and disposable income functions the gainers are either low-income earners or high-income earners, or both.

4. Tax Arbitrage and Normative Tax Theory

We have so far emphasized the positive rather than the normative aspects of tax arbitrage. Our incidence analysis has, however, identified instances when allowing for tax arbitrage in the form of asset trade increases social welfare defined according to standard criteria.

Thus, asset trade is justified from a Rawlsian (maximin) point of view if the tax system is such that $\psi(0) = 0$ and if the distribution of abilities has some positive mass at very low levels of ability. In the absence of asset trade, there would then be some individuals with net incomes $\psi(y)$ very close to, or equal to, zero. With asset trade, the income of those would instead be $\bar{\psi}(\bar{\bar{y}}) - \bar{\psi}'(\bar{\bar{y}})\bar{\bar{y}}$ which we know is strictly positive. Thus the introduction of asset trade has in fact improved the situation for the individuals who are worst off.

From Proposition 5 above, we also see that there are instances when the introduction of asset trade leads to a Pareto improvement. The conditions for this are however rather strict, namely that tax arbitrage stimulates labor supply to such an extent that the tax rates actually can be lowered. A somewhat weaker condition is the following: If the average income in the presence of tax arbitrage ($\bar{\bar{y}}$) is greater than the average income in the absence of tax arbitrage ($\bar{y}^* \equiv E(w\ell^*)$ in the notation of Section 3.2 above) then the introduction of asset trade leads to a Pareto improvement according to the compensation criterion.[14]

It is, however, important to bear in mind the partial nature of these welfare statements. Thus, they are derived for a given initial tax system and for arbitrary assumptions concerning the tax changes used to restore government budget balance. A more complete welfare analysis of tax arbitrage would treat both the initial tax system and the deficit induced changes in the tax schedule as endogenous choice variables of the government. For instance, if the initial tax system is set by a

---

[14] The proof is straightforward. For each individual, we know that $\tilde{V}(w, \bar{\bar{y}}) \geq \tilde{V}(w, \bar{\bar{y}}, \ell^*)$, where the indirect utility functions are as defined in Section 3.2 above. Further, $\tilde{V}(w, \bar{\bar{y}}, \ell^*) > V(w)$ if $\tilde{\psi}'(\bar{\bar{y}})(w\ell^* - \bar{\bar{y}}) + \tilde{\psi}(w\ell^*) > \psi(w\ell^*)$. It also holds that the gainers of introducing tax arbitrage can always compensate the losers if

$$\int \{ \tilde{\psi}'(\bar{\bar{y}})(w\ell^* - \bar{\bar{y}}) + \tilde{\psi}(\bar{\bar{y}}) \} dF(w) > \int \psi(w\ell^*) dF(w).$$

Evaluating the integrals and using the conditions of government budget balance $(\overline{\psi(w\ell^*)} = \overline{w\ell^*} \equiv \bar{y}^*$ and $\tilde{\psi}(\bar{\bar{y}}) = \bar{\bar{y}})$, we see that introducing tax arbitrage satisfies the compensation principle if $\bar{\bar{y}} > y^*$.

government maximizing social welfare, allowing for tax arbitrage *per se* cannot enhance welfare.

Another aspect of our results have a more direct bearing on normative tax theory. An important insight of the influential literature on optimal income taxation following Mirrlees (1971) is that the optimal income tax schedule will typically be a complicated non-linear function of taxable labor income. A conclusion of our analysis is however that tax arbitrage will equalize the marginal tax rates over individuals, thereby making the effective tax schedule linear. This also seems to be in conformity with the empirical observation that actual tax schedules often are linear over large intervals, regardless of the curvature of the official schedules.[15]

This implies that the sophisticated non-linear schedules advocated in the literature on optimal income taxation cannot be sustained; tax arbitrage will tend to linearize all non-linear tax systems. If the private issue of tax exempt bonds is prohibited by law, the arbitrage will perhaps not be complete and thus the tax schedule will not be fully linear, but the tendency towards linearization still remains. Also, one should bear in mind that the assumption of privately issued tax-exempt bonds is just a methodological methaphor for other actions that are frequently used in reality, such as fictituous interest payments of the kind mentioned in the Introduction, arbitrage in other markets than the capital market, etc.

Consequently, when allowing for tax arbitrage the government is left with a choice between different linear tax schedules. Thus, since second-best non-linear Mirrlees taxes are unattainable, a government maximizing social welfare has to settle for a third-best solution which

---

15 See e.g. Pechman and Okner (1974).

involves finding an optimal linear income tax along the lines of Sheshinski (1972) and Dixit and Sandmo (1977).[16] A solution to this dilemma may be found in the fact that opportunities for tax arbitrage seem to be most easily exploited in connection with the taxation of capital income,[17] while existing arbitrage mechanisms relating to labor income are more difficult to exploit. As a consequence, one could therefore conceive of a two-part tax system, with a linear tax schedule applying to capital income and a non-linear Mirrlees type of tax schedule applying to labor income. Interestingly, this seems to be the trend in the current debate on tax reform in most Western countries.

---

[16] There is a parallel to this result in the literature on optimal incentive design in principal-agent models. Thus, as shown by Holmstrom and Milgrom (1987), linear incentive schemes emerge as optimal, the reason being that agents undermine and exploit complicated non-linear reward functions.

[17] Cf. Stiglitz (1983).

References

Auerbach, Alan J. and Mervyn A. King, 1983, Taxation, portfolio choice, and debt-equity ratios: A general equilibrium model, Quarterly Journal of Economics 98, 587-609.

Dixit, Avinash and Agnar Sandmo, 1977, Some simplified formulae for optimal income taxation, Scandinavian Journal of Economics 79, 417-423.

Holmstrom, Bengt and Paul Milgrom, Aggregation and linearity in the provision of intertemporal incentives, Econometrica 55, 303-328.

McDonald, Robert L., 1983, Government debt and private leverage, Journal of Public Economics 22, 303-325.

Miller, Merton, 1977, Debt and taxes, Journal of Finance 32, 261-275.

Mirrlees, James, 1971, An exploration in the theory of optimum income taxation, Review of Economic Studies 38, 175-208.

Pechman, Joseph A. and Benjamin A. Okner, 1974, Who bears the tax burden? (The Brookings Institution, Washington, D.C.).

Sheshinski, Eytan, 1972, The optimal linear income-tax, Review of Economic Studies 39, 297-302.

Stiglitz, Joseph E., 1983, Some aspects of the taxation of capital gains, Journal of Public Economics 21, 257-294.