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TIME CONSISTENCY OF FISCAL AND MONETARY POLICY: A COMMENT

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A REPLY

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In an important recent contribution Persson, Persson and Svensson (1987) (hereafter PPS) suggest that through careful restructuring of its nominal and real debt obligations, a government may be able to induce future governments to follow the monetary and fiscal policies that it regards as optimal today. The PPS argument builds on Lucas and Stokey’s (1983) demonstration that in a special nonmonetary setting, the time inconsistency of optimal fiscal policy can be avoided through managing the term structure of real government obligations to the public. The basic idea of the PPS scheme for monetary economies is disarmingly intuitive: in addition to continually restructuring nominal and real debt obligations a la Lucas–Stokey, each government must ensure that future governments inherit a stream of nominal claims on the public whose present discounted value equals the stock of money. This equality, PPS argue, removes the incentive for surprise inflation or deflation, because such surprises would not affect the real net worth of the government.

This note shows that the PPS prescription for avoiding time inconsistency, appealing as it is, is not generally sufficient. Even under the debt restructuring they recommend, optimal policy is likely to be time inconsistent. The main reason why their scheme fails is that the restrictions it imposes on government asset stocks satisfy first–order but not second–order conditions for an optimum. Because of the complex interactions between the current price level and future interest rates, a government can

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1 We are very grateful to Lars Svensson for his useful comments.
raise its objective function by moving several variables at once away from the levels planned by the previous government, even though price–level changes alone would not affect government net worth.

We develop our argument using the model, notation and equation numbers of PPS, to which the reader is referred for details. The maximization problem associated with equation (4.1) of PPS can be written as follows:

(1) \[ \text{Maximize } \sum_{t=\theta}^{\infty} \beta^t U(c_t, x_t, m_t) \]

with respect to \( c(\cdot), x(\cdot), m(\cdot), \pi_\theta \), subject to

(2) \[ \sum_{t=\theta}^{\infty} p_t (\theta^2 t + \frac{i_{t+1}}{1+i_{t+1}} m_t) - p_t \pi_\theta \theta^2 B_\theta + \sum_{t=\theta+1}^{\infty} (\theta^2 t \frac{1}{1+i_s}) + M_{\theta-1} \geq 0 \]

and

(3) \[ y_t - x_t - c_t - g_t \geq 0. \]

Our proof will show that under certain conditions, there exists a variation of \( c_t, x_t, m_t \) and \( \pi_\theta \) from the PPS solution that leaves utility unchanged, satisfies (3), and raises government net worth. This implies that government \( \theta \) could actually increase its objective function without violating constraints (2) and (3).

Consider a small variation in period \( t_0 \geq \theta \), \( (\Delta c_{t_0}, \Delta x_{t_0}, \Delta m_{t_0}) \), such that
\( \Delta c_{t_0} = -\Delta x_{t_0} \), which ensures (3), and such that the representative consumer remains on the indifference curve passing through the PPS optimum (i.e., \( \Delta U = 0 \) at time \( t_0 \)). Associated with this variation is the total derivative

\[
\frac{d}{dc_{t_0}} \sum_{t = \theta + 1}^{\infty} \left( \theta \mathcal{B}_t \prod_{s = \theta + 1}^{t} \frac{1}{1 + s} \right),
\]

which we assume, momentarily, to be different from zero. Since the PPS solution satisfies their equation (4.6), and since (4) is nonzero for variations \( \Delta c \) of the type just described, there exists some variation such that at the new allocation,

\[
\theta \mathcal{B}_\theta + \sum_{t = \theta + 1}^{\infty} \left( \theta \mathcal{B}_t \prod_{s = \theta + 1}^{t} \frac{1}{1 + s} \right) + M_{\theta - 1} < 0.
\]

Hence, there is a choice of \( \pi_{\theta} \) large enough that the government budget exhibits a surplus (i.e., (2) holds as a strict inequality). Given \( m_\theta \), which is one of the variables under control of government \( \theta \), changes in \( \pi_{\theta} \) do not directly affect utility. It is therefore trivial to show that the government could utilize the additional resources to increase its objective function. Notice, incidentally, that in our proof government \( \theta \) can increase its objective function by resorting, among other things, to a surprise deflation (i.e., by raising \( \pi_{\theta} \) over its value in the PPS solution), and not by a surprise inflation.

The above argument relies on the assumption that expression (4) is nonzero for some \( t_0 \). Suppose, to the contrary, that (4) is zero for all \( t_0 \). On the assumption that the total derivative \( d_{t_0} / dc_t \) is nonzero, which is true as a general rule, it follows that the present discounted value of the nominal debt starting from any \( t > \theta \) equals zero; hence,

\[
\theta \mathcal{B}_t = 0, \text{ for all } t > \theta.
\]
In other words, our argument that utility could be increased by deviating from the PPS solution fails only when all the nominal debt obligations mature in period $t$. As the PPS analysis shows, however, this nominal debt structure would arise in their scheme only under very special conditions.

The above argument has been an intuitive one. It should be noted, however, that at a more technical level, a clue to why the PPS solution does not describe an optimum is that the second-order conditions fail to hold there. One can show that the determinant of the bordered Hessian associated with (4.1), and involving variables $\pi^t$ and $m_t$, is identically equal to zero; whereas that involving variables $\pi^t$, $m_t$, and $c_t$ (along the path where (3) holds with equality) is positive when (4) above is different from zero.

We were not able to find obvious normal instruments that could be superimposed on the PPS scheme in order to ensure time consistency in a monetary economy. Floating-interest-rate bonds, for example, would seem to be natural candidates, because if government debt were entirely of that form, the present discounted value of nominal government debt would be invariant to future policy changes (implying that expression (4) above is identically equal to zero). However, this alternative would be equivalent to government debt being exclusively composed of one-period maturity bonds, which, as argued by PPS, ensures time consistency only in very special cases.

It should be noted that even in those special cases in which the PPS scheme is potentially capable of sustaining the precommitment equilibrium, the government, in each period, is indifferent about the current price level. This is so because, as observed earlier, changes in the current price level, given the path of real monetary balances, do not affect utility or the government's budget constraint directly. Therefore, in order that the precommitment equilibrium be sustained as the economy evolves over time, it is essential that the public believes in the path of the price level initially announced. The government does not have an incentive to deviate from the money supply announcement, but neither does it have a disincentive to modify its policy so as to change the initial price level at the
time of replanning.

One promising avenue for salvaging the basic PPS approach and avoiding the indeterminacy problem is to make assumptions that introduce costs of unanticipated inflation into the social welfare function. It is clear, however, that such costs would not lead to the PPS prescription of balancing nominal government assets and liabilities. Furthermore, the success of this approach will depend on the nature and parameters of the model, as well as on the initial conditions faced by the government (such as the time structure and magnitude of initial debt commitments).

Further research on these questions seems warranted in view of the intrinsic importance of public debt management. Lucas and Stokey and PPS have drawn the profession's attention to the possibility that government portfolio shifts between maturities and degrees of indexation—shifts that in the past have been considered essentially irrelevant—have important incentive effects. It is to be hoped, therefore, that future progress will lead to a better understanding of how actual government debt management practices affect policy credibility.

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TIME CONSISTENCY OF FISCAL AND MONETARY POLICY:

A Reply

Mats Perssson, Torsten Persson and Lars E. O. Svensson

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I. Introduction

In an insightful and clarifying note Calvo and Obstfeld (1988) have alerted us to an error in Persson, Persson and Svensson (1987) (hereafter PPS). The error arises when we demonstrate how the incentives for a surprise (that is, unanticipated) inflation can be eliminated. This note shows how the error can be corrected by modeling the demand for money differently and how our previous result is maintained albeit in a somewhat modified form.

A main result in PPS is that the (public finance) incentive to a surprise inflation for a government in a given period can be eliminated: The government in the previous period should buy nominal bonds in an amount equal to the stock of money and do any desired borrowing by issuing indexed bonds. This makes net nominal assets of the private sector (and of the government) zero so that a surprise inflation does not raise any revenue. This intuitive result appears from the first-order conditions of the government's optimal tax/seignorage problem. But in PPS, as is usual in the optimal-taxation tradition, we wave our hands with regard to second-order conditions. That is, we just assume that they hold and do not cause any problems. What Calvo and Obstfeld show is that the second-order conditions are generally violated for the specific problem discussed in PPS.

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We are grateful to Guillermo Calvo and Maurice Obstfeld for helpful discussions. We are of course solely responsible for any remaining errors and obscurities.
In this note we show that the second-order conditions are violated because there is no distortion whatsoever from a surprise inflation. In the realistic case where such a distortion is present, the second-order conditions are no longer necessarily violated. The PPS result is then modified: The government should buy nominal bonds in an amount less than the money stock, leaving the the private sector with positive net nominal assets instead of zero. This creates a gain from surprise inflation just large enough to balance the distortionary cost of a surprise inflation.

In the following, we clarify the error, present the correction, and conclude with some discussion. We refer to PPS for details of the model and the notation.

II. The Error

The error and the correction can be clarified by highlighting the structure of the optimal taxation/seignorage problem for the government in a given period $\theta$. In simplified notation, the problem can be rewritten as

\begin{align}
\max F(X) \quad & \text{subject to} \\
X, Y \quad & G(X) - H(X) Y = 0.
\end{align}

Here $X$ is the vector $\{c_{\theta}, m_{\theta}\}^\infty_\theta$ of current (period $\theta$) and future (period $t > \theta$) consumption and real balances. The objective function $F(X)$ is the private utility function

\begin{equation}
F(X) = \sum_\theta^\infty \beta^t U(c_t, y_t - g_t - c_t, m_t),
\end{equation}

where we have substituted $y_t - g_t - c_t$ from the resource constraint (2.1) in PPS for leisure, $x_t$, in the instantaneous utility function $U(c_t, x_t, m_t)$.\(^1\) The variable $Y$ is the period-$\theta$ price of money in terms of goods, $\pi_{\theta}$ in PPS ($\pi_t = 1/P_t$ is the inverse of the price level). The constraint (1b) is the government's budget constraint as it appears in (4.1). That is,

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\(^1\) Equation numbers of the form (a.b) refer to equations in PPS.
(2b) \[ G(X) = \sum_{\theta} p_t^\infty (\theta_t + \frac{i_t}{1+i_t} m_t) = \sum_{\theta} p_t^\infty (\tau_t (y_t - x_t) - g_t - \phi_t + \frac{i_t}{1+i_t} m_t) \]

is the present value of the government's tax revenues including seignorage less government expenditure and real debt repayments, and

(2c) \[ H(X) = -p_t^\infty \{ \sum_{\theta} p_t^\infty b_t \Pi_t^\infty (1+i_t)^{-1} \} + M_{\theta-1} \]

is the negative of the private sector's net nominal assets, the value of nominal bonds plus the stock of money. (The present-value price \( p_t \) is the price of goods in period \( t \) in terms of util--PPS uses util as numeraire but this of no importance here.) What is special about problem (1) is that \( Y \), the value of money, does not enter the objective function and enters the constraint only linearly.

The first-order conditions for problem (1) with respect to \( X \) and \( Y \) are,

(3a) \[ F_X + \lambda \phi (G_X - H_X Y) = 0, \quad \text{and} \]

(3b) \[ H = 0, \]

where \( \lambda \phi \) is the Lagrange multiplier for the constraint (1b). Condition (3b) implies the PPS result (4.2) that private net nominal assets should be set equal to zero by the period \( \theta-1 \) government in order to remove the incentive to a surprise inflation for the period \( \theta \) government.

In PPS we did not check the second-order conditions for problem (1) but assumed that they were fulfilled. However, it turns out that the second-order conditions are always violated, except in irrelevant special cases.\(^2\) Actually, the

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\(^2\) This can be seen by formulating the bordered Hessian for the special case when \( X \) is only two-dimensional. A necessary condition for a maximum is that the determinant of the 4×4 bordered Hessian is nonnegative. But the Hessian is actually positive, provided \( G_1/G_2 \neq H_1/H_2 \) (where \( G_1 \) denotes the partial of \( G \) with respect to the first component of \( X \), etc.).

Calvo and Obstfeld give a related but more intuitive argument why the PPS solution cannot be an optimum. They consider a small deviation \( \Delta X \) that leaves the objective function unchanged but makes \( H(X+\Delta X) \) differ from zero. Then one can change \( Y \) so that the left-hand side in constraint (1b) is positive, which means that there is slack in the government's budget constraint. Finally, \( X \) one can change again to take up the slack in the budget constraint and increase the value of the objective function. Hence, the initial PPS solution cannot be an optimum.
second-order conditions are violated in all problems on the general form (1).

III. The Correction

As mentioned, problem (1) is special in that the variable \( Y \), the current value of money, does not enter the objective function. The underlying assumption in PPS is that it is the real value \( m_t \equiv \pi_t M_t \) of the money balances \( M_t \) chosen in the current period that enter the current instantaneous utility function. This corresponds to an assumption that money balances obtained in the current period are used for transactions in the current period.

Suppose instead that money balances obtained in previous period are used for transactions in the current period. That is, what enters the instantaneous utility function are real balances \( m_t \) redefined as

\[
(4) \quad m_t = M_{t-1}/P_t = \pi_t M_{t-1}.
\]

This is the formulation used by for instance Danthine and Donaldson (1986). In this case the objective function can be written

\[
(5a) \quad F(X, Y) = U(c_{\theta} y_{\theta} - g_{\theta} - c_{\theta} \pi_{\theta}^{M_{\theta-1}}) + \sum_{\theta+1}^{\infty} \beta^\theta U(c_{\theta} y_{\theta} - g_{\theta} - c_{\theta} m_{\theta}),
\]

where now the vector \( X \) is identified with the vector \( (c_{\theta}, \{ m_{\theta} \}_{\theta+1}^{\infty}) \) and \( Y \) remains identical to \( \pi_{\theta} \). With the new formulation, \( G(X) \) in the constraint (1b) is

\[
(5b) \quad G(X) = \sum_{\theta}^{\infty} p_{\theta} \phi_{\theta} + \sum_{\theta+1}^{\infty} p_{\theta} m_{\theta},
\]

whereas \( H(X) \) remains as in (2c).

The first-order conditions for maximizing (5a) subject to (1b) are

\[
(6a) \quad F_X + \lambda \phi (G_X - H_X Y) = 0, \quad \text{and}
\]

\[
(6b) \quad F_Y - \lambda H = 0,
\]

where (6b) clearly differs from (3b).

What about the second-order conditions? With (6b) the second-order conditions are no longer necessarily violated. However, we are not sure that they are fulfilled. We can only say that it need not be inconsistent to assume that they are.
This is obviously not very satisfactory, but it is the usual way of proceeding in optimum taxation problems that are too complex for detailed examination of the second-order conditions.

With this modification of the model, the argument of the PPS goes through with appropriate modifications. Condition (6b), which replaces (3b) and (4.2) in PPS, can be written

\[ \lambda \theta p_\theta \left\{ \sum_{t=0}^{\infty} B_t \prod_{t} (1+i_t)^{-1} \right\} + M_{\theta-1} = \beta \theta U_{m\theta} M_{\theta-1}. \]

(7)

With the previous formulation in PPS the right-hand side is zero, and private nominal assets should be set equal to zero. In the new formulation private net nominal assets should be positive, so that the value of the gain in revenue from a surprise inflation (the left-hand side of (7)) equals the consumer's utility loss from lower real balances (the right-hand side of (7)). In equilibrium the incentive to inflate caused by positive private net nominal assets is balanced by the direct utility cost of inflation due to the predetermined money balances held for transactions purposes.

Condition (7) can also be written as

\[ - \sum_{t=0}^{\infty} B_t \prod_{t} (1+i_t)^{-1} = M_{\theta-1} (1 - i_\theta / \lambda_\theta), \]

where we have used that, with the new formulation, the equilibrium nominal interest rate between period \( t-1 \) and \( t \), \( i_t \) fulfills\(^3\) \( i_t = U_{m\theta} U_{ct} \) and that \( p_t = \beta \theta U_{ct} \) since utills are used as numeraire. According to (8), government \( \theta \) should be given positive nominal bond holdings equal to the outstanding money stock less a "correction factor". Notice that the total nominal assets are closer to zero: (i) the lower are the costs of unanticipated inflation, as measured by \( i_\theta \) (the marginal cost of unanticipated inflation in (7) is proportional to \( U_{m\theta} \)), which in turn is proportional to \( i_\theta \), and (ii) the higher are the gains from unanticipated inflation, as measured by \( \lambda_\theta \) (\( \lambda_\theta \) is "the

\[^3\] In the previous formulation the nominal interest rate fulfills \( i_t / (1+i_t) = U_{m\theta} / U_{ct} \) (see (2.7c) in PPS).
cost of public funds, which appropriately measures the overall distortions of taxation).

In addition to the optimal growth of money and of the value of nominal debt, PPS also derives the optimal maturity structure of the nominal and the indexed debt. With the new formulation of the demand for money these derivations go through with appropriate modifications.

The maturity structure of the nominal debt is now determined as follows. For each \( t \geq \theta + 1 \) the following equation should hold (in place of (4.7))

\[
(9a) \quad p_{\theta \theta} \bigg[ \sum_{t=0}^{\infty} \beta_b B_s \Pi_{\theta+1} (1 + i_{\theta})^{-1} \bigg] = E_t / \lambda_{\theta} + F_t
\]

\[
(9b) \quad E_t = - \beta^t U_{mt}(1 + i_t)/(\partial i_t / \partial m_t)
\]

\[
(9c) \quad F_t = - (1 + i_t)[p_t + i_t(\partial i_t / \partial m_t) + p_t m_t],
\]

where \( \partial i_t / \partial m_t \) denotes a derivative of the function defined by the first order condition

\[
i_t = U_m(c_{t}, y_{t}, g_{t} - c_{t}, m_{t})/U_c(c_{t}, y_{t} - g_{t} - c_{t}, m_{t}).
\]

The maturity structure of the indexed debt follows from the the definition of the real cash-flow \( \theta z_t \) in (2.10a) and from the following equations (in place of (4.8)).

\[
(10a) \quad \theta z_t = G_t / \lambda_{\theta} - i_t m_t + H_t
\]

\[
(10b) \quad G_t = - \beta^t (U_{ct} - U_{xt})/(\partial p_{\theta} / \partial c_{\theta})
\]

\[
(10c) \quad G_t = - \beta^t [(U_{ct} - U_{xt}) - U_{mt}(\partial i_t / \partial c_t)]/(\partial p_t / \partial c_t), \quad t \geq \theta + 1
\]

\[
H_t = - p_t (\tau_t + (c_t + g_t)(\partial \tau_t / \partial c_t) + 2m_t(\partial i_t / \partial c_t)
\]

\[
+ i_t(\partial i_t / \partial c_t)/(\partial i_t / \partial m_t)]/(\partial p_t / \partial c_t), \quad t \geq \theta + 1,
\]

where the derivatives \( \partial p_t / \partial c_t \) refer to the function \( p_t = \beta^t U_c(c_{t}, y_{t} - g_{t} - c_{t}, m_{t}) \).

Finally, the value of the cost of public funds for government \( \theta \), \( \lambda_{\theta} \) is

\[
(11) \quad \lambda_{\theta} = \frac{[(\sum_{t=0}^{\infty} p_t G_t) - \beta^t U_{mt}\theta m_{\theta}]/(\sum_{t=0}^{\infty} p_t H_t)}
\]

Thus, equations (8) - (11) determine the unique maturity structure of indexed as well as nominal debt which gives government \( \theta \) incentives to follow the optimal
policy under commitment. The implied restructuring scheme has basically the same form as in (4.12) - (4.15) in PPS.

IV. Discussion

In the pure case, when money demand is modeled such that there is no cost of a surprise inflation, we get first-order conditions which suggest a very simple rule for avoiding surprise inflation. Namely, the government should lend nominally up to the point where the private sector is left with zero net nominal assets. Calvo and Obstfeld have shown that the simple rule is too simple to be true, since the second-order conditions are violated.

The less pure case, when there is some direct cost of a surprise inflation, seems at least equally reasonable as the pure case. What is needed for a surprise inflation to have direct costs is that at least some agents will not be able to adjust their money balances before they do at least some of their transactions. Which way to model money demand is, of course, an empirical issue.

The less pure case leads to a less simple rule, namely that private net nominal assets should be left positive. Nevertheless, the rule still has an intuitive interpretation. It deserves to be emphasized that the second-order conditions are not automatically fulfilled in the less pure case. This is a problem common to most of the optimal taxation literature, however.

Direct costs of surprise inflation can be introduced in other ways than by assuming that nominal balances for transactions purposes are predetermined. It does not seem to matter where exactly the costs come from, as long as they are sufficiently large. Sufficiently small costs would not prevent the second-order condition from being violated.\(^4\) Whether the costs are sufficiently large is again an empirical issue.

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\(^4\) This follows from a continuity argument. It is also easy to see in the example mentioned in footnote 2, if costs of unanticipated inflation are just introduced as a term
References


additive to the private utility function. When that term is sufficiently small, the 4×4 bordered Hessian is again positive when it should be nonnegative.