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DOES DEBT MANAGEMENT MATTER?

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I Introduction

The persistent deficits in government budgets over the last two decades have caused a substantial accumulation of public debt throughout the Western world. At the beginning of the 1970s, the average ratio of gross public debt to GNP in the OECD was 38 per cent, while in 1986 it had increased to 56 per cent.[1] Behind these aggregate figures there are substantial differences both across time and across individual countries. For example, Japan experienced a dramatic increase from a very low level (17 percent in 1972) to a very high level (67 percent in 1986), the United Kingdom had a fall from a very high (75 percent) to an intermediate (55 percent) level, while the United States experienced a decline from 44 percent in 1972 to 37 percent in 1981, followed by a fairly rapid increase to 48 percent in 1986.[2] In Figure 1 we show the historical development of the public debt/GNP ratio for a sample of OECD countries. Although these figures are not fully comparable, neither across countries nor over time for a single country, they indicate roughly the actual development.

These developments have led to a growing interest in the economics of deficit financing and the controllability of an increasingly complex and sophisticated financial system. The purpose of this paper is to discuss the theory and evidence of one particular economic policy towards financial markets, namely government debt management. In general terms, the question is to what extent changing the

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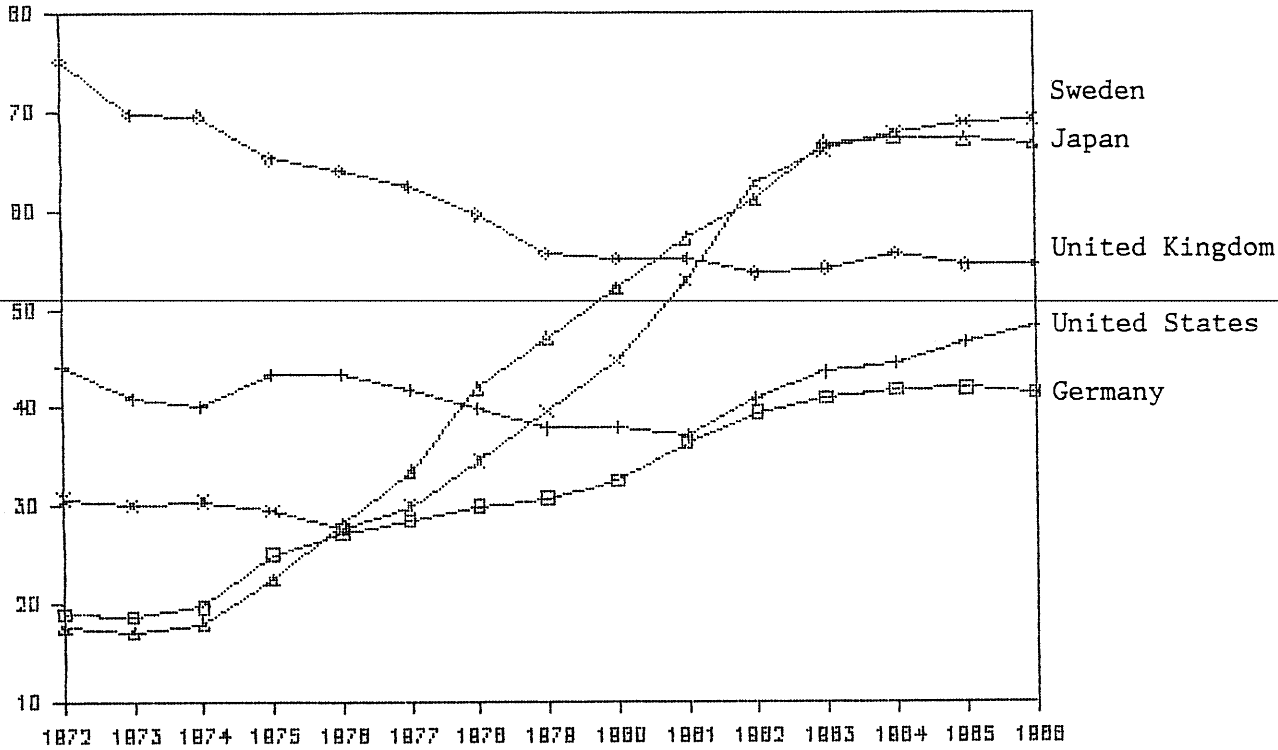
[1] All figures on debt/GNP ratios in this section are taken from Chouraqui, Jones and Montador (1986, p. 108).

[2] Of all OECD countries, only the United Kingdom, Australia and Norway experienced a declining debt/GNP ratio during the period.

Figure 1: Gross public debt as a percentage of GNP for five OECD countries, 1972-1986.

Source: Chouraqui, Jones and Montador (1986).



composition of public debt, rather than its size, can facilitate macroeconomic policy-making. The main channel for debt-management policy is the portfolio choice of investors and the pricing of assets in financial markets. To the extent that different debt instruments are considered as less than perfect substitutes, changing the pattern of government borrowing will alter the structure of relative asset yields. This will in turn affect the "real" side of the economy, since the consumption and investment decisions of households and firms will typically depend on the relative costs of financing the spending in question.

The composition of public debt can obviously be classified along various lines, each one raising its own particular policy issues. Government debt instruments thus differ with respect to tax treatment, to what extent they are traded in well-functioning second-hand markets, and whether they are subject to various regulative restrictions. However, the dimension most commonly associated with debt management policy is the maturity structure of government debt. In Table 1 we report the development of the average time to maturity of outstanding public debt for a few OECD countries.[3] We see that there is no uniform development; while the average time to maturity has fallen sharply for some countries (e.g. Great Britain and Sweden) it has remained fairly stable, or even increased somewhat, for others (e.g. the United States).

[3] The data has been obtained from the central banks of the respective countries.

Table 1 Average of remaining years to maturity of public debt outstanding, 1980–1987.

	USA	Japan	Germany	United Kingdom	Sweden
1980	3.7	5.2	4.5	12.2	4.4
1981	4.0	4.5	4.2	12.4	n.a.
1982	3.9	3.8	4.0	12.1	n.a.
1983	4.0	3.8	3.9	11.9	n.a.
1984	4.5	3.7	4.0	10.7	n.a.
1985	4.9	3.9	3.8	10.4	3.5
1986	5.2	4.1	4.1	10.4	3.6
1987	5.7	4.8	4.1	10.9	3.3

Sources: The central banks of the respective countries.

The traditional aim of debt management has been to minimize the government's costs of borrowing. In a series of papers in the late 1950s and early 1960s, several researchers[4] – with James Tobin as the leading proponent – showed that debt management could also be used as an instrument for stabilization policy. The rapidly growing volumes of public debt have more recently led to a renewed academic interest in the economics of debt management. The insights in the older literature have thus been "rediscovered" and spurred a growing body of research. The aim of the present paper is to provide a presentation and a critical discussion of this research. We have however not aimed at writing a survey article in its ordinary sense – for that purpose, our discussion is far too selective – but rather at providing a review of some topics in the literature that we find particularly interesting and intriguing. In particular, it should be stressed at the outset that the empirical analysis of the present paper is of an explicit partial equilibrium nature. A comprehensive discussion requires analyzing not only the effects of debt management on relative asset yields, but also how these yield adjustments

[4] See e.g. Rolph (1957), Musgrave (1959), Brownlee and Scott (1963), Okun (1963) and Tobin (1963).

ultimately affect the macroeconomy.[5]

The second section of this paper examines some of the basic conceptual issues involved. A distinction is made between a "pure" debt management policy and monetary and fiscal policy. We review some of the neutrality theorems of government finance and discuss under what set of conditions debt management operations are of no economic significance. Also, our discussion digresses on the potential targets and instruments of debt management, concluding that there is no simple single–dimensional characterization of government debt policy; any particular debt management operation involves choosing between a continuum of different "debt attributes" to attain equally multi–dimensional goals.

In section III we turn to the basic portfolio balance approach to debt management, which in one form or another constitutes the backbone of the empirical studies in the area. Section IV deals with some of the problems involved when implementing the basic model. As the key asset substitutabilities and risk premia governing the effects of debt management depend on the subjective risk perceptions of investors, we typically expect estimates of the effects of debt management to show little stability over time. After comparing the results obtained using times series methods for inferring agents risk perceptions we find that this anticipation is indeed supported by our data. We also examine the robustness of results to the choice of data interval. After comparing the results obtained using quarterly and monthly data, we conclude that the frequently neglected problem of temporal aggregation bias should be of more concern in future work.

A problem associated with the vector autoregression approach to investors' risk perceptions used in section IV is that it is inherently backward looking. In any

[5] For an example of such a "general equilibrium" treatment, see Friedman (this volume).

period, the elements of the return covariance matrix entering investors' asset demand functions are estimated using historical data. More realistically, investors rather use whatever information they have available when assessing the riskiness of different assets. In addition to past return data, the information set may thus include "noise" in the form of nonfundamental market rumours and "news" concerning market fundamentals (e.g. changes in monetary and fiscal policy). Section V suggests a direct method to deal with these issues. Using options data and standard models of option pricing, we compare the "implicit" standard deviations of underlying asset returns with those derived from a conventional time series approach. As the "options" variances in any given period mirror whatever information investors find relevant, we typically expect them to exhibit little resemblence with those implied by the vectorautoregression procedures.

Section VI turns to one of the analytical short–cuts underlying the basic "work–house" model used in previous sections. Most of the literature examines the effects of debt management on expected asset returns while (implicity) taking current asset prices as given. As the return adjustments on most long–term assets occur via changes in current asset prices, this procedure is far from innocent. For instance, as current asset prices change, so does aggregate private wealth, which in turn implies wealth effects which affect the pattern of equilibrium returns. In section VI we thus drop the standard assumption of exogenous current asset prices and examine the effects of debt management when <u>all</u> yield adjustments occur trough changes in current prices. Section VIII concludes the paper by presenting some policy conclusions and some suggestions for future research.

II Some General Concepts

II.1 What is Debt Management?

To study debt management, we first have to define it. The concept of government debt is in principle straightforward; it should include all assets issued by the government. The most obvious examples of such assets are treasury bills and long–term bonds, but governments have other liabilities too, some of which are being discussed below. Government debt instruments are held by households, by the corporate sector, and by financial institutions. Some of these assets are also held by the central bank, perhaps as a result of open market operations. We will in this paper include the central bank in the debt–managing authorities and thus consider as government debt only those assets held by the private sector. As a consequence, various kinds of privately held government bonds should, of course, be included in the definition of governement debt as well as non–marketable securities like deposits of the private sector in the central bank or in other government agencies. Money is also an asset issued by the government and held by the private sector, and should be regarded as government debt. At the very general level, the vast and vague array of government commitments to pay out money in the future in the form of social security benefits, veteran's pensions, etc, should also be included.

Public debt management can be defined as the government's (including the central bank) choice regarding the composition of the outstanding stock of all the securities entering the liability side of its balance sheet. However, what should count is not the government's gross liabilities, but its net position vis à vis the private sector. This means that the government's position with regard to securities appearing on the asset side of its balance sheet, such as land, or stock in private companies, could also for all practical purposes be regarded as debt management; by trading in such assets, the government might affect the equilibrium in asset markets and thereby the pattern of asset yields.

When we want to study debt management empirically, we have to exclude many of the kinds of actions that in principle ought to be considered as debt management. We will thus disregard all government trading in the markets for land and common stock. Also, the "privatization" of government–owned companies, which has recently attracted so much attention,[6] will not be treated here. The reason for this is mainly that the selling of government–owned firms still tend to be of a rather minor magnitude. It does not therefore have any noticeable impact on financial markets, but seem to be mainly motivated by ideological considerations and by attempts to increase the productivity and efficiency of these firms, rather than by concern about the equilibrium configuration in asset markets.

The question of how to treat money is another difficult problem. In principle money should be included in the concept of public debt, and there is no practical reason to exclude it because of a lack of data. On the other hand there is a distinction between money and other assets with respect to the theoretical tools used to study them; the transactions approach typically used for studying monetary phenomena is quite different from the portfolio approach commonly used for studying the markets for other financial assets. And while there is a fairly well-developed and widely accepted theory of the demand for bonds, no such generally accepted theory of the demand for money exists. In this paper we will therefore limit our attention to "pure" debt management operations, which we regard as being separate from monetary policy. We will also try to treat debt management policy as separate from fiscal policy, although that distinction sometimes seems as difficult as that from monetary policy. Unless one possesses a full–scale model of

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[6] See for example Yarrow (1986).

the macroeconomy, within the framework of which all three types of policy can be studied simultaneously, specialization on one type of policy requires that it can be reasonably isolated from the others. We will therefore develop in some detail how such an isolation can be conceived.

In any time period, the government's spending can be financed in three ways: by taxation, by money creation (i e by the inflation tax) and by borrowing. The proper mix of these three ways of financing is an intricate but nevertheless well-defined optimization problem which was first formulated by Phelps (1971). In this formulation, fiscal policy (i e the choice of the amount of government spending and of tax financing), monetary policy (i e the choice of the amount of money creation) and government borrowing all have to be determined simultaneously as integrated parts of the "the general macroeconomic problem of public finance".

Solving this general problem in practice would require a full–scale model of the entire economy. On a less ambitious scale, it is possible to study the effects of changing the composition of government debt while taking fiscal and monetary policy as given. In a given period t, we can write the government's budget constraint as

$$(1) \qquad G_t + rb_{t-1} = T_t + M_t - M_{t-1} + b_t - b_{t-1}$$

where the variables on the left–hand side refer to the government's expenses (consumption G_t and interest on previous period's debt rb_{t-1}) while the variables on the right hand side refer to the three sources of revenue (tax revenues T_t, money creation $M_t - M_{t-1}$, and net borrowing $b_t - b_{t-1}$). One would then like to identify changes in G_t and/or T_t with fiscal policy, and changes in $(M_t - M_{t-1})$ with monetary policy. For given values of G_t, T_t and $(M_t - M_{t-1})$, i e for a given value of net borrowing $(b_t - b_{t-1})$, we say that there is scope for debt management if changes in the composition of the debt b_t can affect the real side of the economy.

Equation (1) is in many respects incomplete. For example, we have not distinguished between debt instruments of different maturities, with different tax treatments, etc, but aggregated all interest rates into a single rate r and all instruments into a single number b_t. Depending on the composition of the aggregate b_t, a variety of future cash flows for the government is conceivable. For example, if b_t consists only of short–term debt, the cash flow at time t+1 in the form of interest payments and amortization will be large. On the other hand, if b_t only consists of long–term debt, the cash flow at time t+1 and t+2 will be relatively moderate; if all bonds are discount bonds, there will not even be any interest payments until the date of redemption, and the cash flow at say date t+10 or t+20 will be correspondingly large.

While equation (1) only treats what happens at time t, a full theory of debt management must also take into account the fact that different borrowing strategies will have different implications for the government's finances in the future. Since such intertemporal aspects have to be taken into account, it is difficult to keep debt management distinct from fiscal and/or monetary policy.

In the short run, things are simple. We can always conceive of a change in the composition of the aggregate b_t which does not affect its size, i e which does not affect G_t, M_t or T_t at time t. This is the standard way of analyzing debt management, and it is the approach to be taken in this paper. In the longer run, however, such a change might have implications for the size of future values of total debt, implying that future taxes or expenditures may have to be changed. In principle one would like to define long run debt management in the following way. Consider given sequences $\{G_t\}_0^\infty$, $\{T_t\}_0^\infty$ and $\{M_t\}_0^\infty$. These sequences have to satisfy the government's solvency constraint, which means that the sum of the present discounted values of these expenditure and income streams has to be non–negative.

Given such sequences, we can consider changes in the composition of the sequence $\{b_t\}_0^\infty$ that leave the entire sequences in G_t, T_t and M_t unaffected. In that way debt management will be distinct from fiscal and monetary policy even in the long run.

The question naturally arises whether such a strict definition of debt management is meaningful, i e whether not all changes in the composition of b_t will have to be countered by changes in some future G_{t+s}, T_{t+s} or M_{t+s}. The answer is that this depends on the model used; at this general level, where we have not yet specified any model to which to apply our definition, one can easily conceive of an infinite number of changes in b_t that leave all future fiscal and monetary variables unaffected. For example, if the economy is such that the Ricardian equivalence theorems of debt management[7] hold, changes in the composition of public debt have no effects on the economy whatsoever, and thus an infinite number of policies satisfying our definition is conceivable – whether or not any of these policies are warranted is another matter.[8] On the other hand, if the economy is of a Keynesian variety, where debt management does affect macroeconomic variables, future fiscal and monetary instruments generally have to adjust, thus rendering it impossible to isolate the effects of debt management per se.

In the present paper, however, these explicitly dynamic aspects will be disregarded, and we will follow the standard approach to debt management by considering a simple, atemporal model of the economy. It turns out that many of the issues that make this approach problematic have to do with the difficulties of

[7] This is being discussed in section II.2 below.

[8] In the models of Lucas and Stokey (1983) and Persson, Persson and Svensson (1987) the government at time t can choose the optimal values of the fiscal and monetary variables, and then has enough degrees of freedom left to choose any maturity structure of the public debt. Thus debt management (which in this case means choosing a particular maturity structure) does not affect the government's objective function, but could be used to achieve something else, for example to impose time consistency on future governments.

trying to mimic a multi–period reality by an atemporal model.[9] Still there does not seem to exist any practical alternative to this approach yet; existing intertemporal models of capital market equilibrium[10] are still at a stage of development where empirical implementation for the purpose of analyzing government debt management seems extremely difficult.

The definition of debt management in an atemporal (or one–period) setting is now straightforward. Debt management means that the government changes the composition of outstanding debt by operations in financial market, i.e. by selling as much (in value terms) of one type of instrument as it simultaneously buys of another. If sales were not matched by purchases of exactly the same value, this would have to be met by changes in the fiscal and/or monetary variables, and the analysis of such changes requires quite different tools.

From this definition it follows that e.g. open–market operations, where the central bank buys or sells bonds against money, have to be considered as a mixture between monetary policy (in the sense that they change the money stock) and debt management (in the sense that the government has to decide upon the composition of the bonds sold or purchased). To concentrate on the latter, we will therefore consider the monetary parts of such mixed policies as given. It also follows that we have to disregard the implications of foreign borrowing. If the government changes the mixture of domestic and foreign borrowing, this will by definition affect the money stock; increased borrowing abroad and a corresponding reduction in the borrowing at home will increase the money stock, and vice versa. To concentrate on pure debt management, we will therefore only consider domestic borrowing.

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9 See for example section VI below.

10 For example, the recent theoretical work of Cox, Ingersoll and Ross (1985a), integrating intertemporal capital asset pricing models and linear stochastic production theory, still goes a long way from providing a truly macroeconomic framework for empirical policy anlalysis.

Our definition of debt management draws an equally sharp borderline against fiscal policy. By studying regular market operations involving, say, the sale of long-term bonds in exchange for short–term bonds, we limit ourselves to letting fiscal policy be a given parameter determined outside the analysis. There is, however, also another approach to debt management analysis, emphasizing the effects of introducing new debt used to finance an increase in government spending, that warrants a brief digression.

This latter approach – involving a mixture of debt management and fiscal policy – is intimately related to the crowding in – crowding out issue in macroeconomics. The question addressed is to what extent extra government spending financed by bonds "crowds out" private investment and reduces the potency of fiscal policy. As shown by Tobin (1963) and Friedman (1978), the answer to this question is highly dependent on the assumptions made concerning the asset menu available to investors. In a portfolio balance model incorporating different types of bonds, Friedman showed how bond–financed deficits could actually lead to "crowding in". The mechanism behind this counterintuitive result is that if the new government spending is financed by the issue of bonds that are distant, rather than close, substitutes to corporate equity, portfolio readjustments of investors will lead to increased demand for equity. Thus equity returns go down and private investment increases.

The "market operation" approach to be adopted in the following does obviously not deal with the effects of an increase in government spending. The question of crowding in or crowding out in its ordinary sense is not relevant here. What matters is rather the portfolio crowding out associated with regular market operations. The policy issues involved are thus somewhat less exciting when one limits oneself to this approach – or at least somewhat further away from the question discussed in the popular debate.

This narrow-mindedness can be defended on two grounds. First, to properly study the effects of mixed debt management – fiscal policy experiments would require a model of the entire macroeconomy. In the absence of a coherent view on a variety of theoretical and empirical issues, ranging from the signs and size of key elasticities to the proper integration of the monetary and real sectors of the economy, this is obviously too demanding a task.

For example, when examining the effects of new debt issues the question of Ricardian equivalence becomes immediately relevant. If the government increases its borrowing, we have to consider whether the public regards these bonds as net wealth or not. In the debt management literature dealing with the effects of new debt issues, rather than regular open–market operations, one typically assumes that all new government debt is regarded as net wealth by investors.[11] The plausibility of this assumption is of course an open question; it could well be argued that private agents at least to some extent take into account the future tax increases required to service the debt.[12] However, for the time being it seems appropriate to leave this particular form of Ricardian equivalence for the future research agenda.[13]

Second, examining the effects of debt management in the form of open market operations is an important intermediate step when analyzing the traditional crowding out issues. Only by first understanding the effects of debt management per se can we hope to make progress in understanding its interaction with fiscal policy.

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[11] See e.g. Friedman (1985) and Frankel (1985).

[12] See Werin (1988) for a model incorporating partial discounting of future tax payments.

[13] It should be stressed right–away that we cannot as easily circumvent the Modigliani-Miller theorems of debt management, which extend the basic Ricardian irrelevance principle to other financing decisions than the choice between debt and taxes. See the ensuing section for further discussion.

II.2 When Debt Management Does Not Matter

It is a well–known result in macroeconomics that under certain conditions the way the government chooses to finance its expenditures is immaterial to the real allocation in the economy. Financing by taxation or financing by borrowing will result in exactly the same budget sets for the agents, who will therefore follow the same intertemporal consumption plans. This is the so called Ricardian Equivalence Theorem.[14]

In this section we will discuss under what set of assumptions debt management operations, rather than the choice between borrowing and taxation, have no effects on real magnitudes. We will review some of the recent "Modigliani–Miller" theorems of government finance, which – using arguments similar to those of Modigliani and Miller (1958) – extend the basic Ricardian irrelevance principle to other government financing decisions than the choice between debt and taxes. In models where agents correctly take account of the intertemporal government budget constraint and the stochastic processes governing asset returns, irrelevance propositions have thus been derived concerning, among other things, index bonds, open market operations involving money and bonds, and the maturity composition of government debt.

These theoretical developments are obviously in glaring contrast to most of the empirical work on debt management. In the latter literature – following Tobin (1963) and Friedman (1978) – the presumption is that debt management at least in principle ought to matter: by altering relative asset supplies the government can affect relative asset yields and, hence, the rest of the economy. Failure to identify empirically significant effects of debt management on relative asset yields is

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[14] Barro (1974).

typically interpreted as a sign of assets being close substitutes in investors' portfolios rather than as an indication of the empirical plausibility of various irrelevance theorems.

This discrepancy between recent theoretical developments and current empirical practice is obviously discomforting. Thus, depending on where one wants to lay the burden of evidence the question seems to be either what confidence we can have in established empirical work with somewhat ambiguous theoretical foundations or what credibility we can assign to theoretical work which is at odds with much of the empirical findings indicating the potential importance of debt management operations. In this section we will highlight some of the issues involved by examining the assumptions that must hold for neutrality of debt management to prevail. In particular, we will identify the kind of assumptions that we need to relax for debt management to regain its potency.

To pin-point the key considerations involved we first need a bit of more formal analysis. Assume an economy with a single good, which is consumed by a household sector and a government sector. Each household is infinitely lived and supplies an exogenous amount of labor in each time period. The choice problem of the typical consumer consists of finding a sequence of asset holdings which maximizes expected utility of life–time consumption.

In every period t the government finances its expenditures by tax collections and by borrowing.[15] For notational simplicity, we assume that all government borrowing takes place in the form of discount bonds. At time t the outstanding quantity of government bonds maturing at time τ is $B(t,\tau)$. Each of these bonds promises its owner the receipt of one unit of the consumption good at τ. Denoting the current market price of a bond maturing at time τ by $p(t,\tau)$ the government

[15] We abstract from financing by money creation.

budget constraint at time t is given as[16]

$$(2)\qquad G(t) + B(t-1,t) = T(t) + \sum_{\tau=t+1}^{\infty} p(t,\tau)[B(t,\tau) - B(t-1,\tau)],$$

where all values are measured in terms of the underlying consumption good. The left-hand side variables refer to the government's expenditures (consumption G(t) and debt service) and the right–hand side variables refer to the sources of revenue (tax revenue T(t) and net borrowing).

Assuming that also all private securities are discount bonds promising to pay their owners one unit of the consumption good at the time of maturity, the i:th household's budget constraint can be written as

$$\begin{aligned}(3)\qquad & X_i(t) + B_i(t-1,t) + D_i(t-1,t) - T_i(t) = C_i(t) + \\ & + \sum_{\tau=t+1}^{\infty} p(t,\tau)[B_i(t,\tau) - B_i(t-1,\tau)] + \\ & + \sum_{\tau=t+1}^{\infty} \pi(t,\tau)[D_i(t,\tau) - D_i(t-1,\tau)].\end{aligned}$$

The left–hand side of (3) is disposable income after payment of taxes $T_i(t)$. The gross income consists of three parts: exogenous income from, say, human capital $X_i(t)$, income from government bonds held by the household and maturing at time t, $B_i(t-1,t)$, and income from private discount bonds held by the household and maturing at time t, $D_i(t-1,t)$. Since we allow for short–selling of private assets $D_i(t-1,t)$ can be positive as well as negative.[17] The right–hand side of (3) defines

[16] Our specification of government and household budget constraints in terms of period–by–period income and consumption flows – rather than in terms of present values of life-time income and consumption – is due to Chan (1983).

[17] For our purposes it suffices to note that the private bonds can be viewed either as claims against different households or as debt instruments issued by private firms.

the uses of disposable income: consumption $C_i(t)$ and savings, possibly negative, in government bonds and private securities.

The variable $\pi(t,\tau)$ is the market price at time t of a privately issued security maturing at time τ. In the absence of uncertainty, it must obviously be the case that $\pi(t,\tau) = p(t,\tau)$. In what follows, we will however allow for uncertainty. Assuming that there for every period t are some states of the world where either the government or the private security issuers default on the bonds falling due (meaning that $B_i(\tau,\tau)$ or $D_i(\tau,\tau)$ are worthless for some of the states realized at time τ), $\pi(t,\tau)$ and $p(t,\tau)$ are to be interpreted as the current market prices of risky consumption claims due at time τ. When the state-dependent pay–off structure on a government bond differs from the pay–off structure on a private bond of the same maturity, $\pi(t,\tau)$ will typically differ from $p(t,\tau)$. This corresponds to the case when government and private bonds are less than perfect substitutes.

The polar case is when government and private bonds are perfect substitutes. Then the pay–off structure on a government bond is identical to the pay–off structure on a private bond of the same maturity. With competitive financial markets $p(t,\tau)$ must then equal $\pi(t,\tau)$.

The existence of a perfect private substitute for public debt is one of the key assumptions needed to establish the irrelevance of government debt management. That this must be the case is hardly surprising. Whenever government and private bonds are less than perfect substitutes – meaning that the pay–off structure on government bonds can not be replicated by a portfolio of private bonds – and when there are binding non-negativity constraints on private holdings of government bonds, government borrowing in general and debt management in particular will alter the risk-return opportunity set confronting investors. Hence, market-clearing asset prices and resource allocation will be affected by government financial operations.

All taxes are assumed to be of the lump–sum variety. In every period t the household pays a known fraction θ_i of total government tax revenue T(t). Thus, we have

$$(4) \qquad T_i(t) = \theta_i T(t),$$

where $\Sigma_i \theta_i = 1$.

We next assume that we are in an initial equilibrium at time t_0 where all asset markets clear at prices $p^*(t_0,\tau)$ and $\pi^*(t_0,\tau)$. This equilibrium is consistent with each consumer having chosen optimal consumption–portfolio strategies given the household budget constraint (3), the government budget constraint (2) and the tax sharing rule (4).

One of the key issues in the economics of government debt management is whether changes in the maturity composition of government debt affect the economy. This question is easily addressed using the simple framework just presented.[18] As a matter of definition we first say that debt management is irrelevant if a change in the maturity composition of government debt at time t_0 leaves the sequences of equilibrium prices $\{p^*(t,\tau)\}_{t=t_0}^{\infty}$ and $\{\pi^*(t,\tau)\}_{t=t_0}^{\infty}$ unchanged.

Assume that the government at time t_0 sells one unit of long–term bonds at a price $p(t_0,t_L)$ and uses the proceeds to buy $p(t_0,t_L)/p(t_0,t_S)$ units of short–term bonds ($t_L>t_S$). This financial perturbation policy represents a "pure" debt management operation in the static sense defined in the previous section: For given

[18] See Chan (1983) and Stiglitz (1983) for earlier derivations of irrelevance propositions concerning the maturity structure of government debt.

values of $G(t_0)$ and $T(t_0)$ the government changes its borrowing mixture at time t_0 in a manner compatible with the financing constraint (2). The financing constraint must, however, also hold at time t_S and t_L. At time t_S aggregate tax revenue $T(t_S)$ is reduced by $p(t_0, t_S)/p(t_0,t_S)$ reflecting the reduced amount of government debt coming due. At time t_L taxes are raised by one unit in order to finance the amortization of the long–term bonds issued at time t_0.

One possible portfolio response of households to the financial perturbation policy is as follows. At time t_0 the i:th household buys a fraction θ_i of the newly issued long–term government bonds and finances this by selling $\theta_i p(t_0,t_L)/p(t_0,t_S)$ units of short–term government debt. As this operation simply involves adjusting the maturity composition of a given wealth portfolio it leaves feasible consumption at time t_0 unchanged.

It is also easily established that this particular portfolio strategy leaves feasible consumption at time t_S and t_L unchanged. At time t_S the household experiences a reduction in state dependent capital income corresponding to the quantity $\theta_i p(t_0,t_L)/p(t_0,t_S)$ of short–term debt sold at time t_0. This income loss is, however, balanced by a state dependent tax cut of the same magnitude; as aggregate taxes are cut by an amount reflecting the reduced quantity $p(t_0,t_L)/p(t_0,t_S)$ of short-term government debt coming due at time t_S, the tax sharing rule (4) implies that household-specific taxes are cut by an amount corresponding to the quantity $\theta_i p(t_0,t_L)/p(t_0,t_S)$ of short–term government debt sold by the household at time t_0. Similarly, at time t_L the household obtains additional state-dependent capital income, corresponding to the quantity θ_i of long–term government debt bought at time t_0, which is exactly matched by a state-dependent tax hike used to finance the increased quantity of government debt coming due at t_L.

We have thus seen that there for each household exists a particular portfolio strategy which implies that debt management operations of the government have no effects on the intertemporal opportunity sets of households. It is also immediately clear that this portfolio strategy satisfies our irrelevance condition of government debt management, meaning that changes in the maturity composition of government debt leave the sequences $\{p^*(t,\tau)\}_{t=t_0}^{\infty}$ and $\{\pi^*(t,\tau)\}_{t=t_0}^{\infty}$ of equilibrium asset prices unchanged. At time t_0 the supply of long-term government debt increases by one unit and the supply of short–term government debt decreases by $p(t_0,t_L)/p(t_0,t_S)$ units. Asset market equilibrium at time t_0 is, however, unaffected: Aggregate household demand for long–term government bonds increases by $\Sigma\theta_i = 1$ units and aggregate household demand for short-term government bonds decreases by $\Sigma\theta_i p(t_0,t_L)/p(t_0,t_S) = p(t_0,t_L)/p(t_0,t_S)$ units. Also, since household budget constraints are unchanged at time t_S and t_L, it is trivially true that asset markets clear at the old set of equilibrium prices at t_S and t_L. Consequently, the maturity structure of government debt is immaterial to the price sequences $\{p^*(t,\tau)\}_{t=t_0}^{\infty}$ and $\{\pi^*(t,\tau)\}_{t=t_0}^{\infty}$ and to the real allocation in the economy.

The next important question is under what set of assumptions the presumed household portfolio strategy, implying neutrality of government debt management, will actually be the one chosen by households maximizing expected utility. This question is obviously crucial. If there is no optimizing portfolio strategy corresponding to the one postulated above the irrelevance proposition of government debt management is devoid of economic content.

Irrespective of the form of household utility functions the following assumptions are sufficient for optimizing households to actually choose the postulated portfolio strategy. First and foremost is the assumption that price-taking households do "pierce the government veil"; households correctly infer the interrelations between private and government budget constraints and fully realize any change in future state–dependent tax liabilities, induced by current-period debt management. Second, all taxes must be lump-sum, thus excluding deadweight losses related to the consumption-saving decision of households. Third, the tax sharing rule (4) does not change over time, implying the absence of redistributional effects. Fourth, there must be a perfect private substitute for every type of government bonds, or no binding non-negativity constraints on households' holdings of government bonds.[19]

When asset prices remain unchanged, these assumptions ensure that the intertemporal budget constraints of households are unaffected by government debt management operations. As a consequence, the typical household can not increase welfare by changing its original intertemporal consumption plan in response to the financial perturbation policy of the government. But the household portfolio strategy consistent with both unchanged consumption and equilibrium in asset markets at the old set of asset prices is the one already suggested. By buying and selling government bonds in proportion to its fixed tax share θ_i the typical household thus maintains its original welfare.

The Modigliani–Miller flavor of the above irrelevance argument is obvious. In the Modigliani–Miller economy rational investors, seeing through the corporate veil and operating in perfect capital markets, undo the effects of corporate financial decisions by adjusting their portfolios. In the economy underlying the irrelevance propositions of government debt management investors pierce the government veil and reverse the effects of changes in the maturity composition of governement debt by appropriate adjustments of private portfolios. The same type of reasoning can also be applied to other debt management operations. Parallel irrelevance propositions have thus been derived concerning open market operations involving

[19] See e.g. Chan (1983) and Stiglitz (1983).

money and indexed bonds[20] and money and real assets[21].

What are then the main insights to be gained from the Ricardian debt management results? First, casual empiricism suggests that their underlying assumptions are far from descriptive. In the real world, capital markets are less than perfect; taxes are redistributive and distortionary; empirical evidence indicates that agents not fully discount future tax payments;[22] at least some government debt instruments (e.g. tax exempt government bonds) have no private counterparts; etc. In most situations relevant to the real world we would thus expect debt management to affect the real economy. At the most practical level, we may therefore safely conclude that the Modigliani–Miller analysis of debt management is of little direct significance for the day–to–day business of the authorities managing the government debt.

However, viewed as an exercise in modelling design, the Ricardian irrelevance literature raises a crucial empirical issue. By constructing models of hypothetical economies, incorporating carefully worked out intertemporal budget constraints, specification of underlying return–generating processes, and behavioral functions consistent with basic microeconomic principles, it points to the need of specifying structural models of the economy. This argument gains particular force in an empirical context. Quantitative evaluations of the effects of debt management using various "reduced form" models of portfolio choice and asset markets are thus susceptible to the well–known Lucas critique of econometric policy evaluation[23] – when one cannot econometrically identify "deep" structural parameters relating to

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[20] See Peled (1985).

[21] Chamley and Polemarchakis (1984) and Wallace (1981).

[22] See Boskin (1987) and Barro (1989) for reviews of the evidence.

[23] See Lucas (1976).

preferences and basic asset technology, the estimated response of asset returns to a shift in government borrowing will not be policy–invariant.

In view of the current state of financial economics, the idea of estimating truly structural models – which among other things consistently integrate real and financial markets – seems very difficult to honor in practice. It does, however, serve as a necessary reminder of some of the shortcomings of the traditional Tobin–type portfolio balance approach to debt management adopted in subsequent sections. By suppressing intertemporal linkages between private and government budget constraints, and by taking observed asset return series as a given datum rather than as the outcome of some underlying stochastic return–generating processes, this pragmatic approach obviously represents a second–best solution to empirical debt management analysis. Thus, the Lucas critique applies and all conclusions must be carefully guarded.

II.3 Objectives and Instruments of Government Debt Management

To the extent that changes in the composition of government debt does affect the real economy, one may ask to which objective one should assign debt management policy. At a very general level, the answer is straightforward. Debt management should be combined with other monetary and fiscal policy instruments so as to maximize a properly defined social welfare function. At a somewhat lower level of abstraction the issue becomes more involved. In the real world the government welfare function is thus a rather vacuous concept; typically, policy instruments are assigned to various intermediate targets which only in a very loose sense relate to some underlying basic objective function.

The literature following Tobin (1963) has typically treated government debt policy as a component part of stabilization policies aimed at controlling aggregate demand. Thus, the effectiveness of debt management policy is typically discussed only in terms of the ability to influence corporate investment activity by controlling Tobin's q of the corporate sector. This is obviously a rather narrow view of the scope of debt management – we can easily list some other goals where government debt policy may make a difference.

First, the size of government interest payments is without economic significance whenever the government can use lump-sum taxes to finance current expenditures. In the real world, however, all tax instruments (including the inflation tax on the real cash balances of investors) have negative side effects on resource allocation and economic efficiency. Recent empirical work indicate that these effects may be substantial.[24] Consequently, the size of the government's interest bill has real consequences – each dollar's worth of reduction in interest cost imply that less revenue has to be raised through distortionary taxes. This suggests that a second, particularly simple, policy rule for the national debt office would be to promote allocational efficiency by choosing a pattern of government borrowing which minimizes interest cost. This is in fact the traditional, pre–Tobin, goal of debt management.

Second, to the extent that the pattern of asset holdings differs across households, any change in the structure of asset yields (whether caused by debt management policy or some other exogenous disturbance) will have distributional implications. Since existing data typically show that the structure of portfolio holdings differs widely across different types of investors,[25] we cannot in any way exclude the possibility that debt management policy may have first–order effects on the distributional goals of the government.

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[24] See e.g. Jorgenson and Yun (1986).

[25] See e.g. King and Leape (1984).

Third, in the menu of assets facing the investors, foreign assets may play a large role. Changes in the composition of public debt, affecting the equilibrium asset yields, also affects the relative yields of foreign and domestic assets. This gives rise to changes in the capital account (if the exchange rate is fixed) or in the exchange rate (if it is flexible) which in turn has consequences for various macroeconomic variables.

To which, if any, of these goals should we assign debt management policy? An important insight in the literature on optimal policy design is that very restrictive assumptions are required before we can assign any particular policy instrument to one particular policy objective. In the general case with at least as many objectives as instruments, the optimal use of different instruments will depend on all the economic objectives of the government. This also implies that it is less meaningful to analyze the effects of debt management in terms of only one single policy objective, like controlling q or minimizing government borrowing cost. In the special case of a small open economy, where all asset yields are internationally determined, the only conceivable goal of debt management is to minimize government borrowing costs. However, in the case when at least some asset yields depend on relative domestic asset supplies, optimal policy design requires that debt management is combined with other policy instruments in such a manner that some overall government loss function is minimized.

We have so far discussed debt management policy as if it was a homogeneous and clearly defined concept. This is, however, an oversimplification. As already noted in the Introduction, in formulating its debt management policy the government has a wide variety of choices concerning the type of debt instrument used and to what extent regulative elements are warranted.

A first important distinction is that between market oriented and regulative debt management policies. A regulative debt policy works through the use of interest ceilings and reserve requirements forcing private investors to acquire government debt instruments at interest rates below those that would exist in competitive financial markets. A market–oriented debt policy presumes reciprocity between lenders and borrowers. Here, the government must adjust its borrowing conditions in such a manner that investors willingly hold the outstanding volume of government debt instruments.

Which of these debt management regimes is preferable? In principle, the advantage of a regulative debt policy is that the government by defining its own advantageous borrowing conditions can reduce the need for alternative financing through distortionary tax instruments. The main disadvantage is, of course, that a regulated pattern of interest rates provides misleading information of the social costs involved when evaluating investment projects, which in turn leads to an inefficient allocation of capital across sectors. We cannot a priori decide which of these effects is the dominating one. Less surprisingly, the choice between a regulative and a market oriented debt management policy must therefore be based on empirical considerations, which are far beyond the scope of the present article.[26] Consequently, the fact that we in the following lay stress upon various aspects of a market-oriented debt management policy should not be interpreted as the outcome of an implicit cost–benefit analysis of the efficiency of a regulative debt policy. Instead, it is to be seen as a reflection of the rapid institutional developments in financial markets in the last decade, which make it likely that government debt policy henceforth will be shaped within a broader context of market oriented credit policies.

The government has also within the confines of a market–oriented debt management policy much latitude concerning its choice of borrowing instruments.

[26] See e.g. Tobin (1963) for a general discussion.

Thus, in the real world, the government debt can be classified along a number of different – but not mutually exclusive – dimensions, each one having its own particular implications for the performance of debt management policy. The first one is simply the time to maturity at the date of issue. This is in most of the literature considered as the central choice variable of debt–management policy; the task of the national debt office is to choose a maturity composition of government debt which is optimal in the sense of achieving a certain impact on the relevant economic target variables. The main channel for this kind of maturity oriented debt policy is the yield curve. From a policy perspective, the usefulness of changes in debt composition along the maturity dimension is therefore highly dependent on the ability of the debt office to affect the yield curve in a systematic and predictable manner.

A section distinction is whether government borrowing instruments are fully or partially exempted from taxes. Thus, in many countries, governments issue both tax-exempt and taxable debt. For instance, in the United States, a large fraction of the debt issued by state an local governments is tax exempt; in Sweden, debt instruments intended mainly for household investors are given a tax–sheltered treatment. There are no clear–cut empirical or theoretical conclusions concerning the implications for asset markets and the portfolio behaviour of investors of the government's choice between borrowing in tax–exempt and taxable forms. In general terms, the effects, if any, will depend on the institutional characteristics of the economy such as the overall structure of the tax system and the precise form of any constraints on borrowing and short–sales facing private investors.[27]

A third distinction is that between issues of marketable and nonmarketable debt. While a marketable debt instrument may be freely traded among investors, a

[27] See for instance Auerbach and King (1983) and McDonald (1983).

nonmarketable debt instrument is a personal and nontransferable contract between the government and a particular lender. It should be noted that this distinction is independent of the distinction between regulatory and market–oriented borrowing as discussed above. Regulatory borrowing could well be performed by means of marketable debt instruments; in that case, the investors are forced to buy newly issued bonds at an interest rate below the market rate, but as soon as they have bought them they are free to sell them in the market at a corresponding capital loss.[28] As argued in Musgrave (1959), issuing non–marketable rather than marketable debt makes it possible for the government to earn the profits of discriminating monopolist: whenever the financial system is segmented, with different types of investors operating in different markets, the government can reduce its overall interest bill by offering different returns to different investors.

A final dimension of debt–management policy is whether debt instruments are denominated in real or nominal terms. Existing debt instruments are, with very few exceptions,[29] defined nominally, thereby exposing lenders to uncertainty concerning the future purchasing power of their debt holdings. To the extent that a market failure prevents a private agents from issuing indexed bonds, government debt instruments defined in real terms might benefit financial markets by offering an investment opportunity that would otherwise not exist.[30] Also, if index bonds are closer substitutes for equity than nominally defined government debt, debt

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[28] This was the way e.g. the Swedish life insurance companies were regulated between 1984–1987.

[29] Historically, governments in high–inflation countries have issued indexed bonds. Among the more recent examples are the United Kingdom, Argentina, Brazil, Chile and Israel.

[30] Fischer (1983) gives a discussion of the advantages and disadvantages of index bonds.

management operations involving index bonds would increase the possibility of controlling q and corporate investment activity (e.g. Tobin (1963)).

In sum, the above considerations underline that there is no simple single-dimensional characterization of debt management policy. In the real world, the design of debt policy involves choosing between a continuum of different packages of "debt attributes" to attain equally multi–dimensional and possibly mutually inconsistent goals. As a consequence, any description of a particular debt policy in terms of only one variable, such as the maturity composition of government debt, conveys only partial information concerning its economic effects.

III The Portfolio Balance Approach to Debt Management

This section turns to the question of to what extent changes in the composition of government debt can systematically shift the structure of asset yields. As emphasized in section II, this is in reality a multidimensional problem involving choices between debt instruments with different tax treatment, varying degree of marketability and different times to maturity. We will however in the following limit ourselves to discussing the effects of changing the maturity composition of government debt. Since the work of Rolph (1957), Musgrave (1959), Brownlee and Scott (1963), Okun (1963) and Tobin (1963), and more recently Friedman (1978), this is considered as <u>the</u> problem of debt management. As the spending decisions of firms and households typically depend on the relative costs of financing, steering relative asset yields by altering the maturity structure of government debt is viewed as a potentially important policy target.

Since the theoretical work of Brownlee and Scott (1963) and Roley (1979), the effects of debt management has been analyzed using the mean–variance approach to portfolio choice originating in the work of Markowitz (1959) and Tobin (1958). By incorporating this basic optimizing theory into a Brainard–Tobin[31] capital accounting framework, one thus obtains a model linking asset supplies to the structure of relative asset yields. This basic "work–horse" model has served as a vehicle for a number of studies trying to empirically isolate the effects of government debt management on relative asset yields. This section presents a simple reference model which illustrates the basic mechanisms involved.

We consider a representative investor who can invest his wealth W in n assets. In the empirical applications below we will set n = 3 and we will identify asset No. 1 with corporate shares, asset No. 2 with long–term bonds, and asset No. 3 with short–term bonds. For the time being, however, the names of the assets are of no importance.

The investor is concerned about the real yield of his portfolio. Since there is always some inflation risk, all assets will thus be risky – even the short–term bonds. In general, we may think of the investor as choosing those optimal portfolio and consumption rules which maximize a properly defined intertemporal utility function. If we further assume that the investment opportunity set changes in an either completely random or non-stochastic manner over time (cf. Merton (1982)), the intertemporal optimization problem can be represented by a sequence of single-period portfolio models. Denote the end–of–period wealth by $\tilde{W}$. The investor chooses to invest a fraction α_i of his initial wealth W in asset i, where i = 1, ... , n, so as to maximize expected utility of end-of-period wealth. He thus solves the problem

$$\underset{\alpha_1,\ldots,\alpha_n}{\text{Max}} \quad E[U(\tilde{W})] \equiv E[U(W(1+r))]$$

[31] See Brainard and Tobin (1968) and Tobin (1969).

subject to $r = \alpha_1 r_1 + \alpha_2 r_2 + ... + \alpha_n r_n$,

$\Sigma\alpha_i = 1$

where r is the (random) yield on the investor's portfolio, and where r_i is the individual (random) yield on asset i.

When analyzing this problem, it is common to assume that the utility function displays constant relative risk aversion, i.e. that $-xU''(x)/U'(x) = c$ for all x, where c is a positive constant[32]. Making this assumption, it is a standard exercise in financial economics to show[33] that the demand for assets is given by the system

$$(5) \qquad \alpha = \frac{1}{c} B\, r^e + \Pi.$$

Here α is the n–dimensional vector of asset demands in terms of portfolio shares, i.e. $\alpha \equiv (\alpha_1, \alpha_2, ..., \alpha_n)'$, where α_i is the share of total wealth that the investor will invest in assets i. Further, r^e is the vector of expected real returns on the assets, $r^e \equiv (r_1^e, r_2^e, ..., r_n^e)'$, where $r_i^e \equiv E(r_i)$. The n x n matrix B contains information about the variance–covariance properties of the assets and is given by

$$B \equiv [\Omega^{-1} - (\underline{1}'\Omega^{-1}\underline{1})^{-1}\Omega^{-1}\underline{1}\,\underline{1}'\Omega^{-1}] ,$$

where Ω is the covariance matrix of the asset returns, i.e. the typical element of Ω is $\sigma_{ij} \equiv \mathrm{Cov}\,(r_i, r_j)$. The n–dimensional vector $\underline{1}$ is the unit vector $(1, 1, ..., 1)'$, while the n–dimensional vector Π is given by[34]

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32 For empirical evidence supporting this assumption, see Friend and Blume (1975).

33 See for instance Friedman and Roley (1987).

34 A special case occurs when one of the assets, say the n:th one, is riskfree, i.e. $\sigma_{in} = 0$ for all i. The demand system for the n–1 risky assets then simplifies to

$$\Pi \equiv (\underline{1}'\Omega^{-1}\underline{1})^{-1}\Omega^{-1}\underline{1}.$$

Equation (5) gives us asset demand in terms of portfolio shares. If we want to express it in value terms instead, we simply multiply (5) by the scalar W:

$$(6) \qquad d = (\frac{1}{c} B\, r^e + \Pi)\, W$$

where the elements d_i of the vector d tells us how many dollars the agent will invest in asset i.

Let us for expository reasons take a look at the three–asset case, and let us denote the individual elements in the B matrix and the Π vector by

$$B \equiv \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \quad \text{and} \quad \Pi \equiv \begin{bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{bmatrix}.$$

Since we know that the individual's budget constraint implies $\alpha_1+\alpha_2+\alpha_3 = 1$, the system (5) is actually linearly dependent. We can therefore drop one of the demand equations, for example the last one, obtaining

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$$\alpha = \frac{1}{c}\,\Omega^{-1}(r^e - r),$$

where α now is the (n–1)–dimensional vector of wealth shares, where r is the (n–1)–dimensional vector all elements of which are equal to the (riskfree) return on asset n, and where r^e is the vector of expected returns on the n–1 risky assets. Finally, Ω is the (n–1)x(n–1) covariance matrix of the risky asset returns. The demand for the riskfree asset is then given residually as

$$\alpha_n = 1 - \sum_{i=1}^{n-1} \alpha_i.$$

$$(5') \qquad \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \frac{1}{c}\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} \begin{bmatrix} r_1^e \\ r_2^e \\ r_3^e \end{bmatrix} + \begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix}.$$

Denoting the (exogenous) supply of the assets by $(\alpha_1^s, \alpha_2^s, \alpha_3^s)'$, we can use (5') to obtain the market equilibrium conditions

$$(7) \qquad \begin{pmatrix} \alpha_1^s \\ \alpha_2^s \end{pmatrix} = \frac{1}{c}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} r_1^e \\ r_2^e \end{pmatrix} + \begin{pmatrix} b_{13}(r_3^e+1+\Pi_1) \\ b_{23}(r_3^e+1+\Pi_2) \end{pmatrix}.$$

Since this system contains three unknowns (r_1^e, r_2^e, r_3^e) and two equations, only two of the unknowns can be determined. Treating r_3^e as exogenous (which has been indicated in (7) by grouping the terms containing r_3^e together with the parameters on the right–hand side) we can write (7) as

$$(8) \qquad \alpha^s = \frac{1}{c}(br + k),$$

where α^s, r and k are two–dimensional vectors and b is a 2 x 2 matrix consisting of the first 2 rows and columns of the three–dimensional matrix B in equation (5) above. Solving for the endogenous r, we have

$$(9) \qquad r = b^{-1}\ (c\alpha^s - k).$$

This equation gives us the equilibrium asset yields r_1^e and r_2^e as functions of the exogenous supplies α_1^s and α_2^s, the coefficient of relative risk aversion, and the covariance matrix Ω.

Denote the elements of b^{-1} by β_{ij}. Thus

$$b^{-1} \equiv \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix},$$

and we see from (9) that

$$(10) \qquad \frac{\partial r_i^e}{\partial \alpha_j^s} = c\beta_{ij}.$$

Equation (10) does not have any immediately intuitive interpretation, since the coefficient β_{ij} is a rather complicated function of the elements in the covariance matrix. However, for the special case when short–term debt is riskless ($\sigma_{i3} = 0$, $i = 1, 2, 3$), equation (10) reduces to

$$(10') \qquad \frac{\partial r_i^e}{\partial \alpha_j^s} = c\sigma_{ij} \qquad\qquad i, j = 1, 2.$$

Here σ_{ij} is the covariance between the two risky assets; a positive covariance is thus equivalent to a positive derivative. For example, if the i:th asset is corporate equity and the j:th is long–term bonds, and if these assets are positively correlated, we see that lengthening the maturity structure of public debt increases the equity return and thereby "crowds out" private investment. Expression (10) thus gives us the effects on asset yields of government debt management. Note that this is debt management according to the definition in section II.1 above; the derivative $\partial r_1^e / \partial \alpha_2^s$ shows how the return on asset 1 changes if the supply of asset 2 is changed at the same time as α_1^s remains constant and α_3^s is changed so as to leave $\alpha_1^s + \alpha_2^s + \alpha_3^s = 1$.

Finally a few words about taxes. What matters to the investor is of course the after–tax yield, and thus the expected return vector r^e and the covariance matrix Ω should be interpreted as referring to after–tax yields. Now, there is a simple way to incorporate this into the analysis. Denote by Ω and r^e the covariance matrix and the expected return vector of gross (before-tax) yields, and by $\hat{\Omega}$ and $\hat{r}^e$ the corresponding matrix and vector of net (after-tax) yields. Let t_i indicate the tax rate applied to asset i. Then the typical element in the $\hat{\Omega}$ matrix is

$$\begin{aligned}\hat{\sigma}_{ij} &\equiv \text{Cov}\left((1-t_i)r_i,\ (1-t_j)r_j\right) \equiv (1-t_i)(1-t_j)\ \text{Cov}\ (r_i,\ r_j) \equiv \\ &\equiv (1-t_i)(1-t_j)\ \sigma_{ij}\ ,\end{aligned}$$

where σ_{ij} is the element in the net matrix Ω. Thus $\hat{\Omega}$ can be simply expressed in terms of Ω as

$$\hat{\Omega} = T\Omega T\ ,$$

where T is an n x n diagonal matrix with elements $(1-t_i)$ in the diagonal and zeros elsewhere. Similarly we have

$$\hat{r}^e = T\ r^e.$$

From now on, we will assume that the tax rates are the same for all assets, i.e. that $t_i = t_j = t$ for all i, j. Then we obtain

$$\hat{\Omega} = (1-t)^2\ \Omega \quad \text{and} \quad \hat{\Omega}^{-1} = \frac{1}{(1-t)^2}\ \Omega^{-1}.$$

Similarly, the return vector becomes

$$\hat{r}^e = (1-t)r^e.$$

Inserting this into the demand system (5) yields

$$\alpha = \frac{1}{c}\frac{1}{(1-t)} B\, r^e + \Pi\ ,$$

where B and Π are defined as before in terms of the covariance matrix Ω of gross returns. Thus the demand system will look exactly as it did without taxes, except for the fact that the degree of risk aversion has been multiplied by (1–t). Unless one has exogenous information on one of them, it will therefore be impossible to identify c and t separately.

From now on we will use the formulation (5), or its corresponding expression (6) for demand in value terms, but bear in mind that "c" stands for risk aversion cum the tax rate. In the empirical analysis below we will assume that c is equal to 4, a value which seems like a compromise between various estimates of relative risk aversion[35] and a reasonably realistic tax rate for the average investor. If however some reader has a strong opinion about the correct value of c, one should note that c occurs in the policy derivative (10) only as a scale factor. In the following sections, where we have computed numerical values of $\partial r_i^e / \partial \alpha_j^s$, the reader who does not agree with our assumption of c = 4 could just shift the curves up or down, applying any other scale factor.

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[35] The numerical value of Arrow–Pratt's measure of relative risk aversion is an unsettled empirical question. The analysis in Pindyck (1984) suggests a value of R_j around 5 or 6, indirect evidence in Grossman and Shiller (1981) indicate a value in the neighborhood of 4, whereas the cross-sectional household estimates in Friend and Blume (1975) imply a value of at least 2.

IV Implementing the Basic Model by Using Historical Data

The basic empirical question involved when assessing the potency of government debt management is how close substitutes different assets are in investors' portfolios. There are several different approaches used in the literature to estimate the relevant asset demand parameters, not all of which impose the constraints of mean-variance optimization discussed above.[36] In the following, however, we will make the admittedly strong assumption of taking the mean-variance model as a given datum. In a mean-variance context, the substitutability of different assets ultimately depends on the elements of the covariance matrix Ω. Rather than testing the basic mean-variance model, the purpose of the present section is to discuss different procedures for estimating the covariance elements to be fed into the model. Since the covariance matrix typically depends on the subjective risk perceptions of investors as well as underlying "objective" return probabilities, estimating its elements is far from a trivial task.

We will start in section IV.1 by using a very simple estimate of historical covariances. In section IV.2 and IV.3 we will make the analysis somewhat more sophisticated, employing the vector autoregression methods used by Friedman (1985) to estimate covariance structures. In particular, we will examine the stability over time of the implied debt management derivatives calculated according to equation (10). From a policy point of view the stability issue is obviously crucial, as successful debt management rules require systematic and stable policy responses. The stability issue is further analyzed in section IV.4, where we examine the robustness of results to the choice of data interval. After comparing the results obtained using quarterly and monthly data, we conclude that the frequently

[36] See Frankel (1985) for an overview of the literature.

neglected problem of temporal aggregation bias should be of more concern in future work.

IV.1 A Simple Estimate of the Covariance Matrix

The total yield of any asset consists of a dividend yield (d) and capital gain or loss (k), adjusted for inflation ($\dot{p}$):

$$\text{Total real yield} = d + k - \dot{p}.$$

Of these three components, the dividend can be regarded as certain[37] and the main uncertainty is therefore associated with the capital gain (or loss) and the rate of inflation. Thus d should be included in the r^e terms of the previous section, but not in the covariance matrix Ω.

For the sake of illustration, we have computed the covariance matrix of the real capital gain ($k-\dot{p}$) for three types of assets, namely 1) corporate stock, 2) long-term government bonds and 3) three-months treasury bills, using U.S. quarterly data from 1960.1 to 1988.2.[38] When calculating the capital gains of the long-term bonds from the published interest rate series, we have treated them as consols, which seems like a reasonable approximation. The (nominal) capital gain on stock is simply the change in the Standard and Poor's 500 share price index. The treasury bills display no capital gains or losses: for them $k = 0$ in all periods, and the only risk is the one associated with $\dot{p}$.

The covariance matrix of these three yield series is given in Table 2a. To

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[37] We disregard the possibility of bankruptcy.

[38] Source: IMF, International Financial Statistics.

facilitate comparison, the coefficients of correlation[39] are given in Table 2b. We see first that the variance of the real yield on short–term bonds is very low; since the only source of uncertainty here is the inflation rate, which in turn varies very little on a quarterly basis, these bonds could almost be regarded as a safe asset. Second, we see that all three assets are positively correlated.

If we instead compute the correlations of the nominal yields, this pattern is changed. This is shown in Table 3a. Here we have excluded the short–term bonds, since they do not have any nominal risk. Instead we have included a real asset, say land or consumer durables, and assumed that the price of that asset behaves according to the consumer price index. In Table 3a we thus report the covariances between the nominal yield on corporate stock, the nominal yield on long–term government bonds, and the consumer price index. The corresponding coefficients of correlation are reported in Table 3b. Here we see that both long–term bonds and corporate stock have a negative correlation with the real asset, while they are positively correlated with each other.[40]

Using the covariances of Table 2 we can now apply formula (10) to calculate the policy effect. Assuming that the degree of relative risk aversion c is equal to 4, we have

$$(11) \quad \frac{\partial r_1^e}{\partial \alpha_2^s} = 0.00352 \qquad \text{and} \qquad \frac{\partial r_2^e}{\partial \alpha_2^s} = 0.01444.$$

[39] The covariances σ_{ij} in Table 2a are related to the coefficients of correlation ρ_{ij} in Table 1b according to $\rho_{ij} = \sigma_{ij}/\sqrt{\sigma_{ii}\sigma_{jj}}$.

[40] Cf. Bodie (1982) and Fama and Schwert (1977).

Table 2: Covariations between the quarterly real yields of (1) corporate equity, (2) long–term government bonds, and (3) short–term government bonds. U.S. data 1960.1 – 1988.2.

Table 2a: Covariance matrix

	corporate stock	long–term bonds	short–term bonds
corporate stock	0.00457	0.00127	0.00023
long–term bonds		0.00401	0.00024
short–term bonds			0.00008

~~Table 2b: Coefficients of correlation~~

	corporate stock	long–term bonds	short–term bonds
corporate stock	1	0.30	0.38
long–term bonds		1	0.43
short–term bonds			1

Source: IMF, International Financial Statistics.

Table 3: Covariations between the quarterly nominal yields of (1) corporate equity, (2), long–term government bonds, and (3) the consumer price index. U.S. data 1960.1 – 1988.2.

Table 3a: Covariance matrix

	corporate stock	long–term bonds	CPI
corporate stock	0.00418	0.00088	–0.00015
long–term bonds		0.00361	–0.00016
CPI			0.00008

Table 3b: Coefficients of correlation

	corporate stock	long–term bonds	CPI
corporate stock	1	0.23	–0.26
long–term bonds		1	–0.31
CPI			1

Source: IMF, International Financial Statistics.

This means that marginally increasing the stock of long–term bonds (α_2^s) would increase the equity yield (r_1^e), thereby "crowding out" equity financed investments. The figure 0.00352 might seem rather small, but compared to similar figures in the literature, it is actually quite large. Our figure indicates that if we increase the share of long–term bonds in the investor's portfolio by one percentage point (say, from 15 to 16 per cent, which corresponds roughly to the actual U.S. portfolio share in 1987) in exchange for short-term debt, this would raise the quarterly yield on equity by 0.00352 percentage points. The increase in the yearly yield would then be four times as large, i.e. 0.0141 percentage points. For the same experiment within the framework of a quite different model and data set, Frankel (1985, p. 1057) obtains an increase in the yearly yield on corporate equity of only 0.0005 percentage points. Our figure is thus almost thirty times as large as that of Frankel. On the other hand, Roley (1982, p. 662) obtains a figure which is for some periods even larger than ours. Although these studies differ with respect to estimation period as well as model construction, they still indicate the wide range of possible policy responses.

Turning to the own-yield effect, the impact of debt management is more substantial. The policy derivative for long-term debt is thus 0.01444, or more than four times as large as the figure for equity. This means that a one percentage point increase in the portfolio share of long-term debt raises the expected yearly yield on long-term debt by 0.05776 percentage points. This figure is almost seven times as large as that reported by Frankel, who performs the same experiment.

IV.2 A Vector Autoregression Approach

The figures in Table 2 and Table 3 indicate the degree of covariation between corporate stock, long–term bonds and short–term bonds (and some real asset represented by the consumer price index). One should however be careful not to mistake variability for uncertainty. For example, assume that a variable develops over time as the sum of two components: a completely predictable cycle (for example the business cycle) and a random disturbance from day to day (say the impact of the weather). Such a variable is shown in Figure 2, where the predictable cycle is assumed to have a low frequency and a high amplitude, while the random disturbance is assumed to have a high frequency and a low amplitude.

Now, such a series would yield a rather large variance, due to the high amplitude of the business cycle. But since this is completely predictable, the large variance does not correspond to a high degree of uncertainty. The only source of uncertainty is the small random disturbance, and thus the variance of the series itself is an incorrect measure of the degree of uncertainty involved.

A way to cope with this is to estimate the coefficients of the ARMA process[41] describing the time series of Figure 2. Writing the time series as

$$X_t = k_0 + \sum_{i=1}^{\infty} a_i X_{t-i} + \epsilon_t ,$$

we have decomposed it into a predictable component ($k_0 + \Sigma a_i X_{t-i}$) and a random component (ϵ_t). More generally, if we have three different series X_t, Y_t and Z_t, like the yields of our three assets above, we can run a vector autoregression according to

$$(12) \quad X_t = k_0 + \Sigma a_i X_{t-i} + \Sigma b_i Y_{t-i} + \Sigma c_i Z_{t-i} + \epsilon_t$$

$$(13) \quad Y_t = f_0 + \Sigma g_i X_{t-i} + \Sigma h_i Y_{t-i} + \Sigma m_i Z_{t-i} + e_t$$

$$(14) \quad Z_t = n_0 + \Sigma p_i X_{t-i} + \Sigma q_i Y_{t-i} + \Sigma r_i Z_{t-i} + \eta_t .$$

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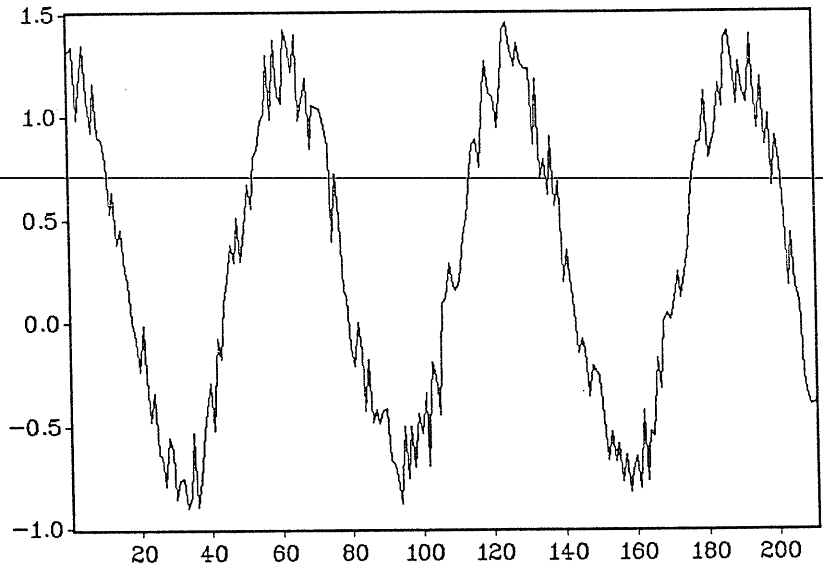
[41] See Box and Jenkins (1970).

Figure 2 A deterministic cycle with random disturbances



Having estimated the coefficients (k_0, a_i, b_i, c_i, f_0, g_i, h_i, m_i, n_0, p_i, q_i, r_i) one could say that all of the predictable variation has been removed, and that the true uncertainty that should be taken into account in the investor's portfolio decision is captured in the estimated covariance matrix of the residuals (ϵ_t, e_t, η_t).[42] This also means that the expected yields r^e can be more accurately computed than by using only the coupons of the three assets, since the system (12) – (14) provides us with optimal forecasts

$$
\begin{aligned}
X_t^e &= \hat{k}_0 + \Sigma\hat{a}_i X_{t-i} + \Sigma\hat{b}_i Y_{t-i} + \Sigma\hat{c}_i Z_{t-i} \\
Y_t^e &= \hat{f}_0 + \Sigma\hat{g}_i X_{t-i} + \Sigma\hat{h}_i Y_{t-i} + \Sigma\hat{m}_i Z_{t-i} \\
Z_t^e &= \hat{n}_0 + \Sigma\hat{p}_i X_{t-i} + \Sigma\hat{q}_i Y_{t-i} + \Sigma\hat{r}_i Z_{t-i}.
\end{aligned}
$$

Estimating the system (12) – (14) on our data for real capital gains ($k-\dot{p}$) on shares, long-term bonds and short–term bonds, and computing the covariances of the residuals gives us the covariance matrix displayed in Table 4.[43] We see that these "conditional" variances are much smaller than the "unconditional" variances reported in Table 2. This is a consequence of the fact that we have eliminated the predictable variation and only preserved the "genuine" uncertainty in the time series. In particular, the uncertainty about future inflation is virtually zero on a quarterly basis. The general features of Table 2 and Table 4 remain however the

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42 This is not entirely true, since there is also some uncertainty regarding whether the estimated parameter values are equal to the actual parameter values. Since we can never know a priori whether there has recently been a major change in the stochastic process governing the development of the yields, i.e. since we can never know a priori whether there has recently been a shift in the parameters, it is not self–evident that the covariance of the residuals of the system (12) – (14), estimated from historical data, gives a better picture of the "true" covariance matrix than the simple, unconditional matrix reported in Table 2. Cf. the discussion of the GARCH approach at the end of section IV.3 below.

43 In our vector autoregression, we have used a four–period lag throughout.

Table 4 — Conditional covariances between the quarterly real yields of (1) corporate equity, (2) long–term government bonds, and (3) short-term government bonds. U.S. data 1960.1 – 1988.2.

Table 4a: Covariance matrix

	corporate stock	long–term bonds	short–term bonds
corporate stock	0.00315	0.00071	0.00008
long–term bonds		0.00315	0.00009
short–term bonds			0.00002

Table 4b: Coefficients of correlation

	corporate stock	long–term bonds	short–term bonds
corporate stock	1	0.22	0.31
long–term bonds		1	0.41
short–term bonds			1

Source: IMF, International Financial Statistics.

same; the unconditional covariances of the former table are positive, like the conditional covariances of the latter. And the coefficients of correlation are of the same order of magnitude in both tables, displaying the same pattern: the correlation between long-term bonds and short-term bonds is the highest, while the correlation between shares and short-term bonds in both tables takes an intermediate value.

IV.3 Changes in the Agents' Information Set

Eliminating predictable variation from the data makes it possible for us to concentrate on the uncertainty underlying the investor's portfolio choice. We have not yet, however, clearly defined his information set. To be able to make the estimates underlying the residual covariances reported in Table 4, the agent would have to use quarterly data from the entire sample period, i.e. from 1960.1 to 1988.2. At some intermediate date, say 1975.3, all this information is not available to him; the best he can do then is to make estimates using the data from 1960.1 to 1975.2. This means that the parameter estimates of equations (12) – (14) made at date 1975.3 will be different from those underlying Table 4, and thus the agent's perception of the covariance matrix at date 1975.3 will also be different. The agent's perception of the covariance matrix will hence vary over time, as more and more data points become available to him. We have taken this into account in the same fashion as Friedman (1985, 1986) has done, i.e. in the following way.

Our data series begins in 1960.1, and we assume that the agent's data series does the same. At 1970.1 enough observations are available to permit reasonably reliable vector autoregressions of the system (12) – (14). For each date following 1970.1 we have re–estimated the system, re–computed the residuals, and re-calculated the corresponding covariance matrix. Thus there will be a new covariance matrix for each quarter following 1970.1. One would expect that these covariance matrices do not change much from one quarter to another, since the parameter estimates of the system (12) – (14) will not change very much when only one more observation is added. Still, the changes can sometimes be quite substantial, as is shown in Figure 3. In particular, the sharp jump in the variance of long-term bonds in 1982 is surprising; it depends on the sharp decline in the interest rate between June and October that year, when the long-term interest rate fell by approximately three percentage points.[44] We also see that while there seems to be no clear trend in the variance of corporate stock, the variance of long-term debt rises almost monotonically since 1979. The covariances and the variance σ_{33} (real yield on short-term bonds, i.e. inflation risk) seem however to be fairly stationary. The numerically low levels of the covariances including short–term bonds are also striking; the maximum values of σ_{13}, σ_{23} and σ_{33} are substantially lower than the minimum value of σ_{12}. This indicates that the vector autoregression model (12) – (14) is rather efficient in explaining inflation, and it would not be unreasonable to treat short-term bonds as a safe asset also in real terms. Finally, the coefficients of correlation are shown in Figure 4. Their development is rather stationary, and they are of approximately the same order of magnitude as the unconditional ones reported in Table 2b above.

The sequence of covariance matrices obtained by the adaptive vector autoregression method has been plugged into the expression for the policy derivatives (10). This results in a sequence of policy derivatives $\partial r_1^e/\partial \alpha_2^s$ and $\partial r_2^e/\partial \alpha_2^s$ as shown in Figure 5. We see first that both derivatives are positive, thus indicating that an increase in long-term financing of government debt will increase the cost of capital for the private sector, thereby reducing industry investment. Or

[44] The decline in the short–term interest rate was even more dramatic. As a result of the easing up of the monetary policies of the Federal Reserve in the summer of 1982, the treasury bill rate fell by 5.5 percentage points between June and August.

<u>Figure 3</u> Conditional quarterly covariances between the real yields on (1) corporate stock, (2) long-term bonds, and (3) short-term bonds 1970.1-1982.2.

Figure 3a

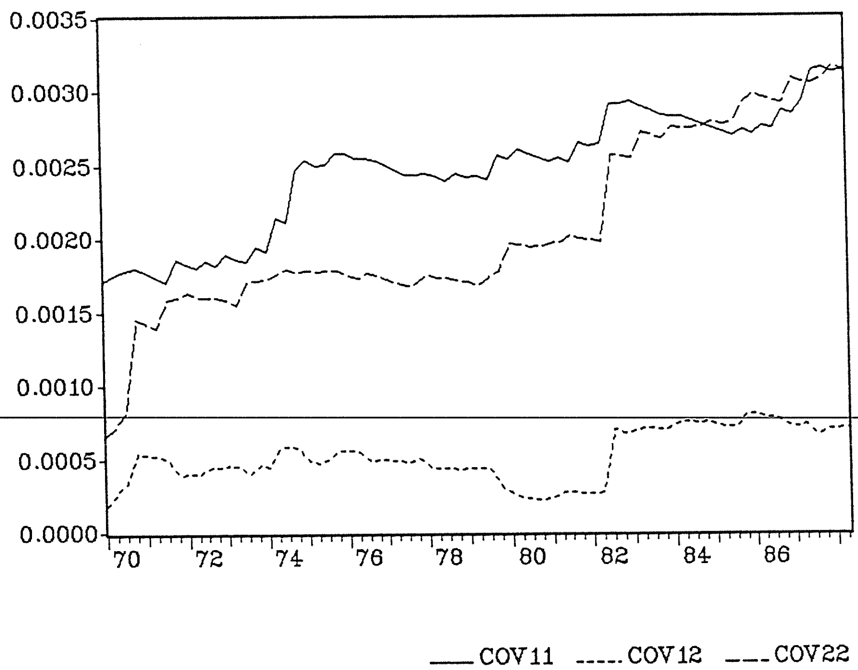


Figure 3b

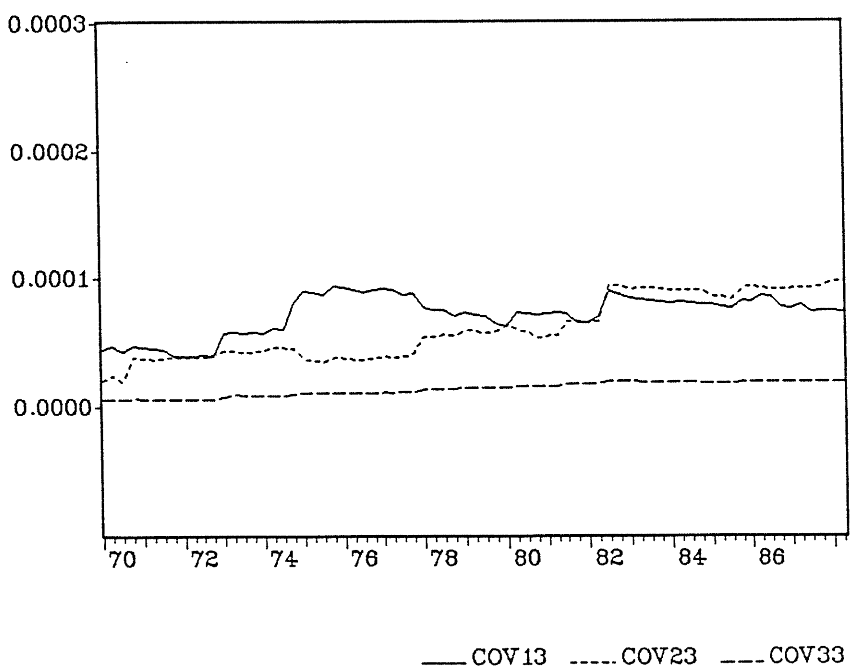


Figure 4 Conditional coefficients of correlation between the quarterly yields on (1) corporate stock, (2) long-term bonds, and (3) short-term bonds 1970.1-1988.2.

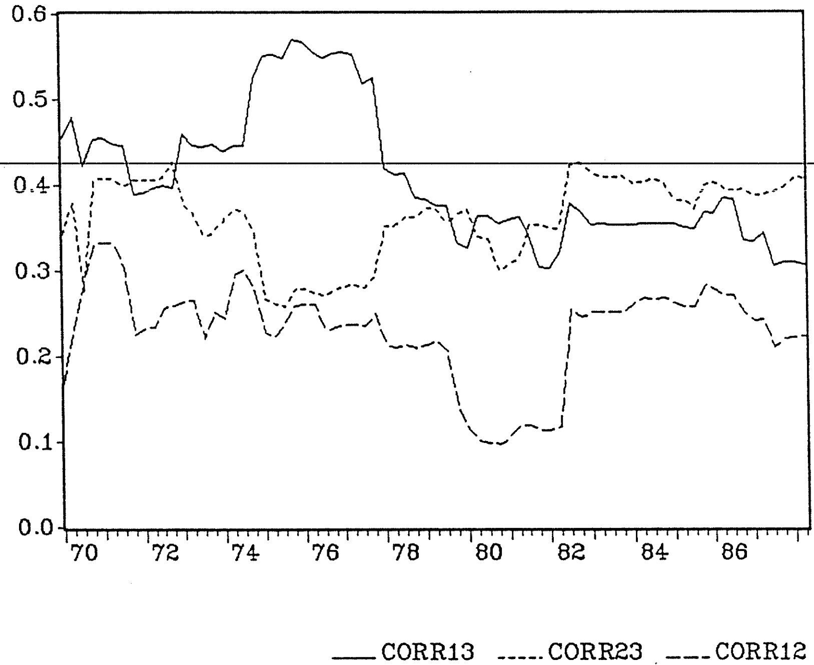


Figure 5 The effects on asset yields of increasing the supply of long-term bonds, quarterly data 1970.1-1988.2.

Figure 5 a: Both policy derivatives

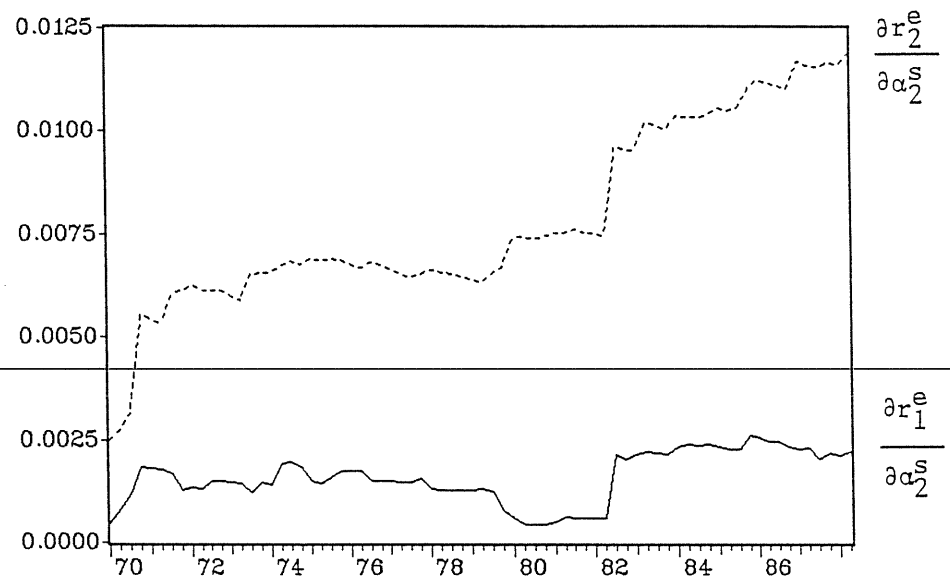
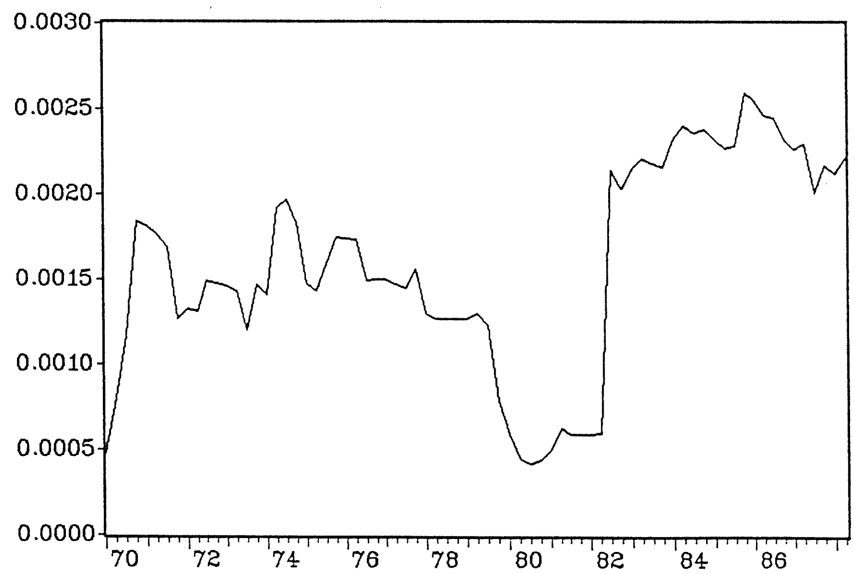


Figure 5b: The derivative $\partial r_1^e/\partial \alpha_2^s$ on on a magnified scale



equivalently, a change to more short-term financing, will tend to stimulate industry investment. It is worth noting that the signs of the two derivatives are the same over the entire period, which could be regarded as a sign of the robustness of the result. If $\partial r_1^e/\partial \alpha_2^s$ had changed in sign frequently over the period, one would be less inclined to believe firmly in the qualitative result that more short–term financing of government debt stimulates investment.

Second, we see that while there is no obvious trend in the development of $\partial r_1^e/\partial \alpha_2^s$, there is a marked upward trend in $\partial r_2^e/\partial \alpha_2^s$. In fact, however, the path of $\partial r_1^e/\partial \alpha_2^s$ shows violent variations over time, from a minimum of 0.00042 in 1980.3 to a maximum of 0.00260 in 1985.3; i.e. an increase by 500 per cent. To emphasize this, we have plotted $\partial r_1^e/\partial \alpha_2^s$ separately, using another scale, in Figure 5b. One could therefore say that although the qualitative result (i.e. the positive sign of the derivative) is robust, quantitative results should be regarded with caution.

Translating the quarterly yield effects into yearly figures, we have that an increase in long-term debt by one percentage point (from 15 to 16 per cent of total wealth) increases the yearly yield of corporate stock by a minimum of 0.00168 and a maximum of 0.0104 percentage points. Of these, the latter is perhaps the more interesting, since it refers to a later date in the estimation period. Our figures are considerably lower than the corresponding ones obtained using unconditional covariances reported in (11) above. The maximum figure (obtained for 1985.4) is about thirty percent smaller than the unconditional derivative, while the minimum figure (1980.3) is eight times smaller than the unconditional one. Also, the minimum figure is about three times as large as that of Frankel (1985 p. 1057) while the maximum is about twenty times as large, thereby emphasizing the violent changes in policy responses over time.

The own-yield effect $\partial r_2^e/\partial \alpha_2^s$ increases in a trend-like manner, from a minimum of 0.0025 in 1970.1 to a maximum of 0.0119 in 1988.2. The last figure is about twenty percent smaller than the unconditional derivative reported in (11). In yearly terms, increasing the share of long-term bonds by one percentage point will, using the conditional covariances for 1988.2, raise the expected bond yield by merely 0.0476 percentage points – a figure which is almost six times larger than that reported by Frankel, but still substantially smaller than some of the simulation results reported by Roley (1982) and Friedman (1989).[45]

What policy conclusions can we draw from the preceding analysis? In qualitative terms, the results conform with standard presumptions. Lengthening the maturity composition of government debt increases the expected yields on corporate equity and long-term bonds relative to the expected return on short-term debt. For a given expected return on short-term debt, this means that an increase in the supply of long-term government bonds (in exchange for Treasury bills) increases the costs of corporate financing and "crowds out" corporate investment. A full discussion of whether the calculated relative return adjustments are large enough to matter economically would of course require bringing in additional macroeconomic structure concerning aggregate supply and demand relationships. A more casual inspection, however, suggests that the effects are small. Using the conditional covariances computed for 1988.2, we find that increasing the share of long-term bonds in investors' portfolios by (an unprecedently large) ten percentage points will raise the expected yearly yields on corporate equity and long-term bonds by about 0.09 and 0.48 percentage points, respectively.

Also, in assuming that the government can determine relative asset supplies at its own will, we have implicitly assumed that the capital structure decisions of

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[45] Also, using a vector autoregression model similar to ours, Friedman (1985) concludes that debt management may affect relative asset yields in an economically significant way. Friedman's results are, however, not directly comparable to ours, as they by pertaining to new debt issues – rather than pure open market operations – also reflect wealth effects induced by an increase in net private wealth. (Cf. the discussion in section II.1 above.)

firms are exogenously given. More realistically, the ultimate effects of debt management operations depend on to what extent firms respond by adjusting their liability mix. When lengthening the maturity composition of government debt drives up the relative yields on corporate equity and long-term bonds, we would thus expect firms to rely less on financing by long-term bonds and corporate equity and more on short-term bonds. As these private supply adjustments at least partly neutralize the debt management operations of the government, the net impact of debt management on asset yields will typically be even less significant than the yield effects reported above.

As repeatedly stressed, systematic use of debt management also requires stable and predictable yield responses in asset markets. As indicated by our results, this requirement creates additional difficulties for the authorities managing the government debt. In particular, we found that the policy-derivative for the return on corporate equity (i.e. the target-variable most commonly associated with debt management in the literature) exhibited sharp fluctuations over time, thus underlining the difficulty of using debt management for fine-tuning puposes.

Implicit in the approach described above is a particular view of the world. It is assumed that the stochastic processes governing asset yields are stationary, i.e. that the elements of the underlying "true" covariance matrix are constant over time. The agent's perception of the covariance matrix changes as more data points become available, but the objective probability distribution actually generating the data is assumed to be time-invariant. An alternative view would be to allow for non-stationarity in the time series in the sense that not only does the agent's perception of the covariance matrix change over time as more and more data is collected, but also does the actual covariance matrix change. This could be motivated by e.g. changing monetary regimes; as governments, central bankers and doctrines of stabilization policy come and go, so does the nature of surprises confronting investors in asset markets. It could also be motivated by shifts in the underlying asset technology; such shifts (e.g. the development of new financial instruments) may occur at random intervals and thereby affect the return pattern until the next shift occurs. Of course, it could be the case that the basic stochastic process is still stationary, and is defined as the joint outcome of random changes in returns, in monetary regimes, and in technology. Describing the asset yields by a stationary autoregressive system like (12) – (14) would then still be appropriate. Nevertheless, it could sometimes be suitable to estimate the asset return structure using methods that explicitly allow for non-stationarity.

The simplest way to do this is to use the "depreciation method" developed by Friedman and Kuttner (1988). Since older observations are more likely to have been generated by a different process than more recent ones, old data points are given less weight in the estimation of the system (12) – (14). Another approach is to use the so-called Generalized AutoRegressive Conditional Heteroskedastic (GARCH) estimation technique developed by Engle (1982) and applied to asset markets by e.g. Bollerslev, Engle and Wooldridge (1988). With this approach, the elements of the "true" covariance matrix are themselves regarded as being generated by some autoregressive process, and the technique allows for estimating the parameters of that process.

There is no a priori reason to prefer one of these approaches to the others. The proper choice between them has to be based on practical considerations and on extensive empirical experience.[46] We have here chosen to work with the simplest assumption, i.e. that of a stationary underlying stochastic process, but we want to point out the vast opportunities for further empirical research to shed light on this issue.

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[46] Cf. Friedman and Kuttner (1988).

IV.4 The Time Aggregation Problem

The policy conclusion of the previous section is straightforward, at least qualitatively. It relies however on the data material used. We, as well as many other students of debt management referred to above[47], have used quarterly data. The question is whether the choice of the data interval affects the conclusion.

To illustrate the problem, we have constructed a hypothetical example. Assume that two time series X and Y move as depicted in Figure 6. Computing their covariance yields Cov (X,Y) = –0.689. Let us now however use a different periodization, taking only every second observation into account. The two time series thus obtained look rather different, and their covariance is 0.124. Taking instead every third observation into account yields Cov (X,Y) = –0.434. In Figure 7 we show the values of the covariance between X and Y for different choices of the data interval. We see that not even the sign of the covariance is robust to the choice of data interval, not to speak of the magnitude. Particularly striking are the very large values obtained for a period length of 11 and 13, emphasizing the high sensitivity of the estimates of a covariance matrix Ω.

The econometric problems involved in choosing the data interval has been a neglected issue in most of the literature. Some studies have been made in the field of marketing research,[48] showing how the quantitative conclusions regarding the impact of various marketing actions depend on whether monthly, quarterly or yearly data is used.

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[47] E.g. Friedman (1985, 1986) and Roley (1982). Frankel (1985) used yearly data for his study.

[48] See the references given in Berndt (1988, chapter 8). In the financial economics literature, Grossman, Melino and Shiller (1985) have discussed some of the problems involved when estimating continuous time asset pricing models using data which is time averaged. For a discussion of the problem from the point of view of statistical theory, see Bergstrom (1984).

<u>Figure 6</u> The realization of two hypothetical stochastic processes.

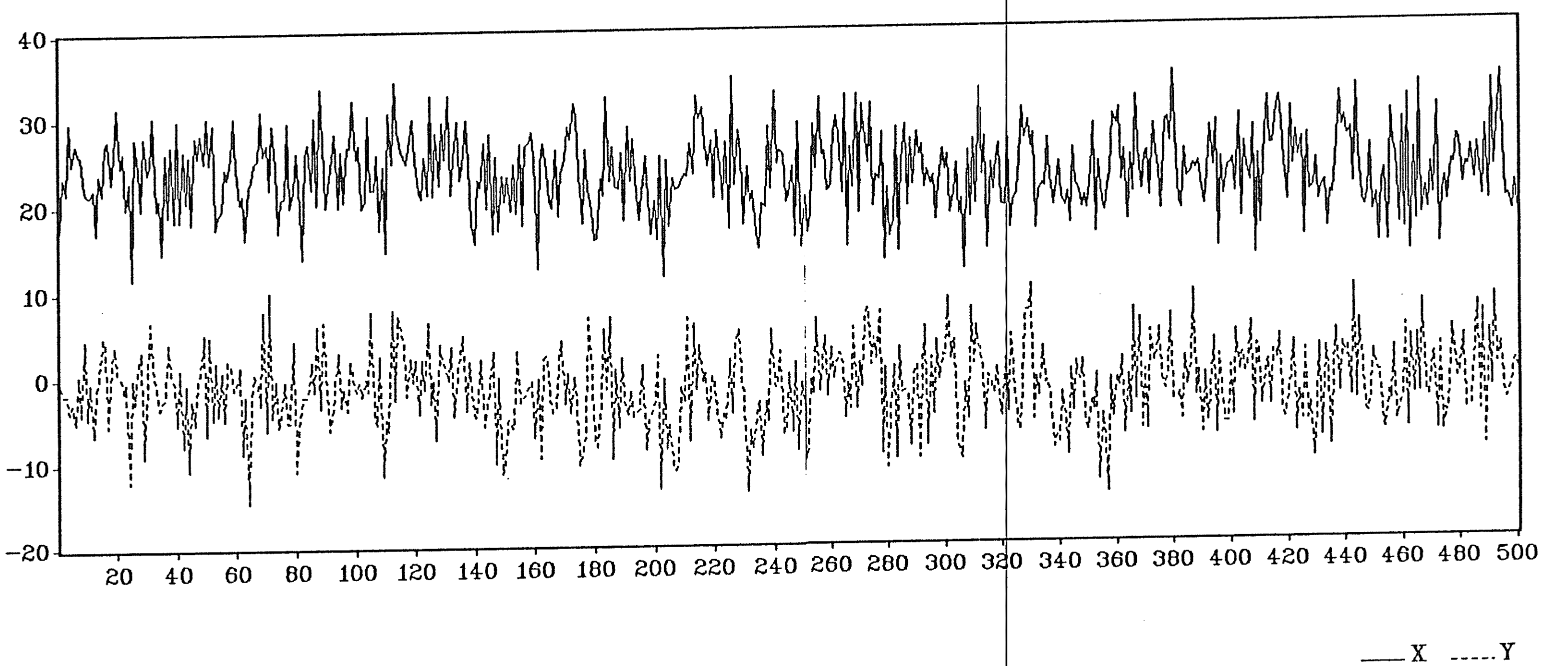
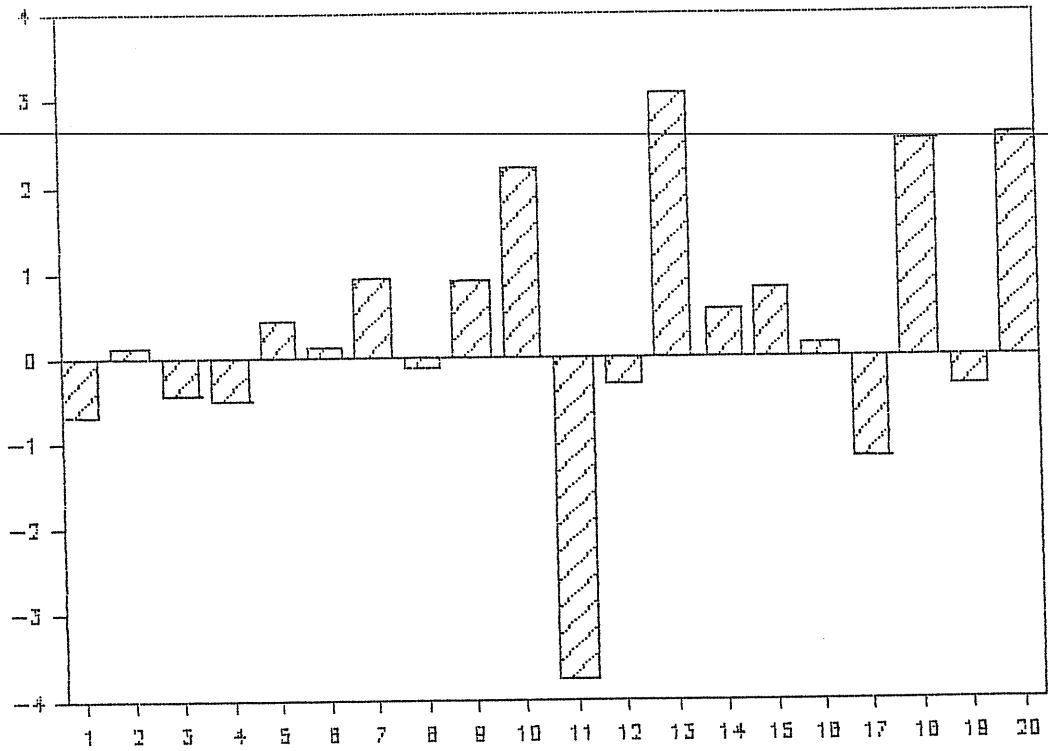


Figure 7 : The covariances of the two time series in Figure 6 for different data intervals.



What is then the "proper" data interval? That of course depends on the nature of the agent's decision problem. In our single period portfolio balance model, the data interval should be equal to the investor's holding period, which is the time interval between successive portfolio reallocations. But the holding period varies over assets and agents. If we had included real estate among our assets, the planning horizon for e.g. owner-occupied homes should perhaps be measured in years if not in decades, due to the considerable transaction costs involved, while real estate bought by professional investors might be associated with a shorter holding period. For bonds and shares traded by tax-exempt institutional investors, the holding period could be as short as a few minutes. For bonds and shares bought by households, the transactions costs and the tax rules involved may imply a holding period of say a few months or perhaps a year or two.[49]

A complete model would determine the optimal holding period together with the optimal portfolio demands. However, when we try to represent a multiperiod world with transactions costs and heterogeneous investors by a simple model of the type used in the present paper, it is impossible to say a priori what is the correct periodization of the data series. What one can do is to replicate the analysis for different data intervals and see whether the policy conclusions are robust with respect to the choice of data interval.

In Figure 8 we therefore report the covariances, and in Figure 9 the coefficients of correlation, using monthly data, that result from the vector autoregression model of equations (12) – (14) above. When comparing those covariances to their quarterly counterparts in Figure 3 above, we see that the general shapes of the curves are quite similar.[50] The important difference is in the

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[49] Cf. Fischer (1983) for a discussion of how the dynamics and uncertainty of asset returns depend on the holding period of investors.

[50] To make the one–month interest rates comparable to the quarterly interest rates, we have multiplied the former by a factor of three. Further, when estimating the model (12) – (14) in Section IV.2 above using quarterly data, we assumed a four-period lag. When using monthly data, we have consequently assumed a twelve-period lag.

Figure 8 Conditional monthly covariances between the real yields on (1) corporate stock, (2) long-term bonds, and (3) short-term bonds 1970.1-1988.4.

Figure 8a

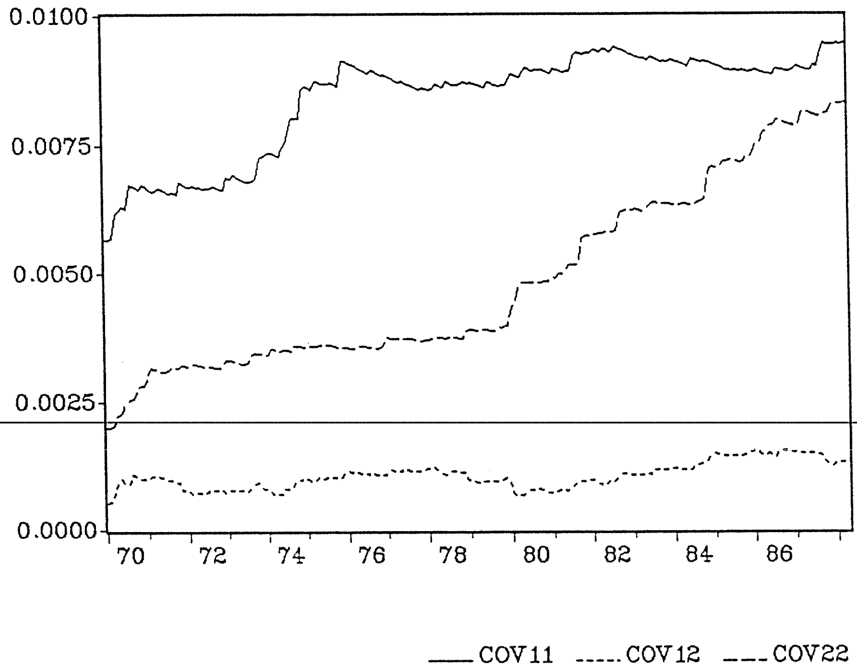
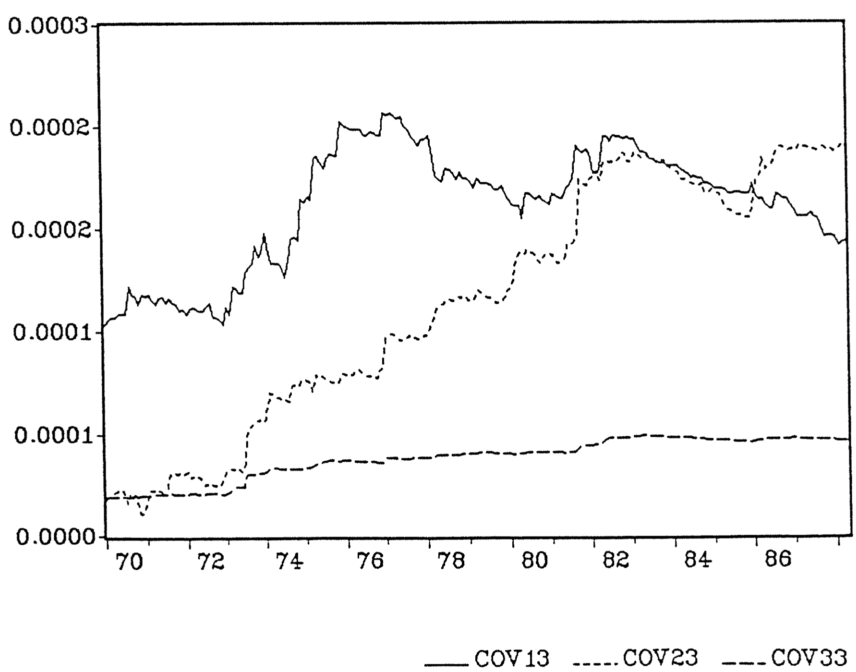


Figure 8b



<u>Figure 9</u> Conditional coefficients of correlation between the real yields on (1) corporate stock, (2) long-term bonds, and (3) short-term bonds, monthly data 1970.1-1988.4.

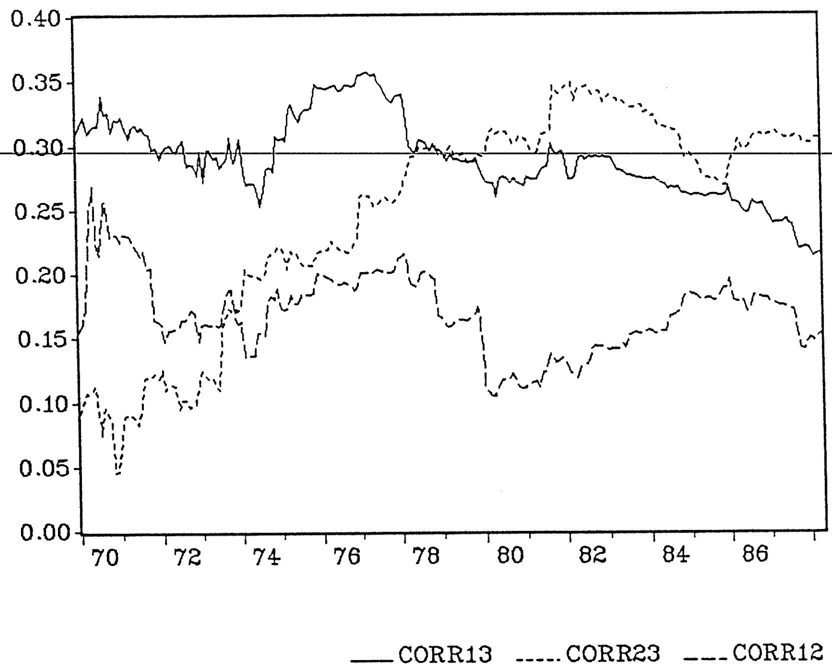


Figure 10a The derivative $\partial r_1^e / \partial \alpha_2^s$ for monthly and quarterly data, 1970-1988.

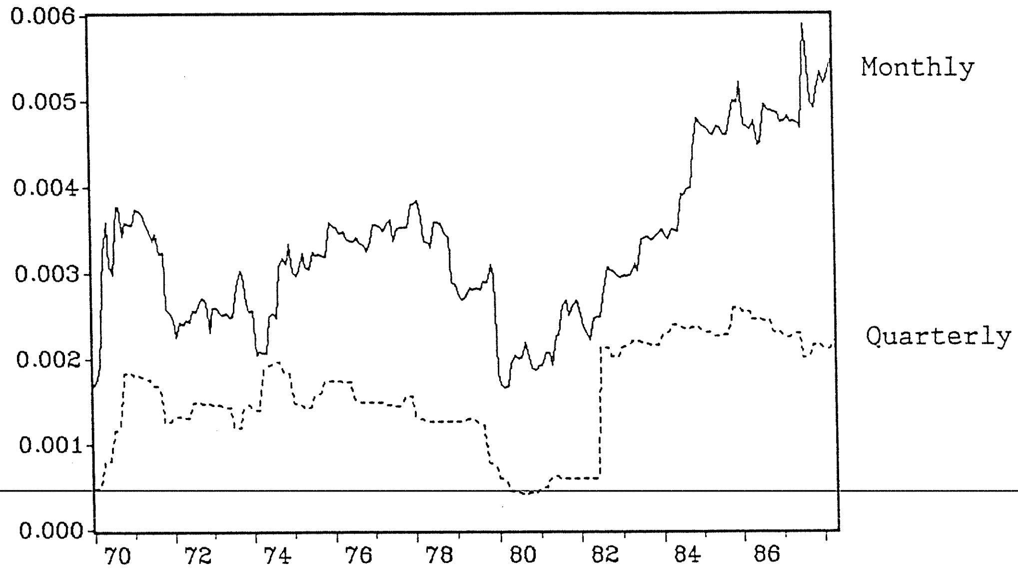
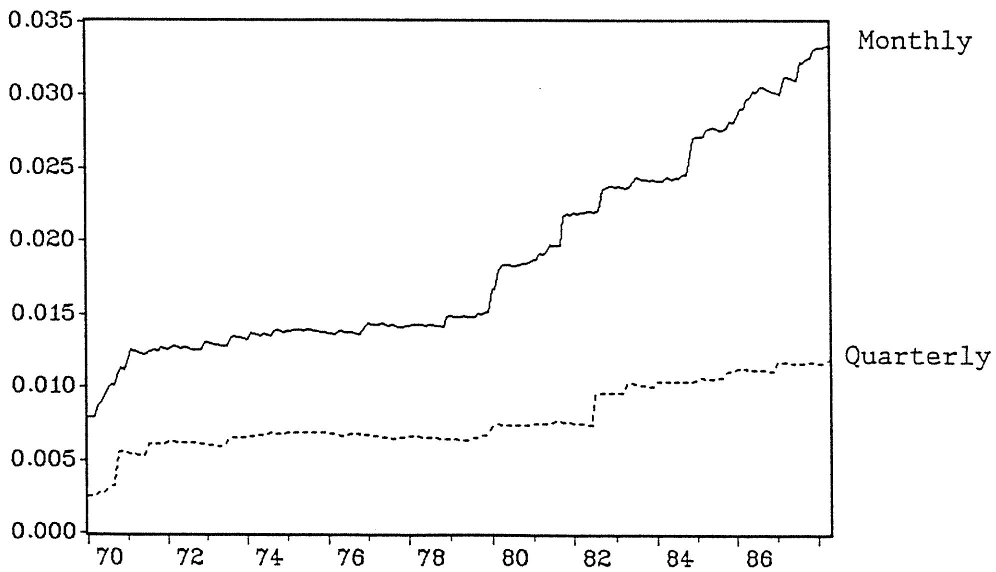


Figure 10b The derivative $\partial r_2^e / \partial \alpha_2^s$ for monthly and quarterly data, 1970-1988.



order of magnitude; the monthly variances are about three times as large as the quarterly ones.[51] The coefficients of correlation reported in Figure 9 differ somewhat from their quarterly counterparts of Figure 4. Apart from the order–of–magnitude problem, we see that ρ_{12} is less than both ρ_{13} and ρ_{23} over the entire sample period when we use quarterly data, but that ρ_{12} is larger than ρ_{23} at the beginning of the period when we use monthly data.

It is hard to judge from a visual inspection whether these deviations in pattern are important or not. We have therefore re-computed the "policy derivatives" of equation (10) above using monthly data. In Figure 10a we have plotted $\partial r_1^e / \partial \alpha_2^s$, i.e. the impact on the equilibrium yield on corporate stock of an increase in the supply of long–term bonds (the solid curve) using monthly data. As a comparison we have also plotted the same derivative using quarterly data (the dotted curve; this is the same curve as that in Figure 5 above). We see that the qualitative properties are the same, indicating crowding out. The peak in 1985–1986, the through in 1980–81 and the varying pattern in the 1970s are also the same. The orders of magnitude are however different; the crowding–out effect on the equity yield seems much stronger if we use monthly data. As is evident from Figure 10b, similar considerations apply to the derivative $\partial r_2^e / \partial \alpha_2^s$.

In sum, although the qualitative conclusions seem robust with respect to the choice of data interval, the quantitative conclusions seem much more sensitive. Any empirical conclusion concerning the potency of debt management may thus depend

[51] This might depend on the construction of the basic data. They refer to period averages, and – depending on the stochastic properties of the time series – the averaging procedure might have the effect that the variance becomes smaller for long periods (e.g. a quarter) than for short periods (e.g. a month).

crucially on the time length between the data observations available to the researcher.

V An Alternative Approach to the Covariance Matrix

In the previous analysis we have considered the covariance structure as something objectively measurable that can be constructed out of historical data. Either it is simply the unconditional covariance matrix of Table 2 above, or the conditional covariance matrix of Table 4. On a more sophisticated level, it is the "adaptive" structure reported in Figure 3, which allows risk perceptions to vary over time.

In reality, however, the problem is much more difficult. As the risk perceptions of investors are inherently subjective, it is far from self–evident that the "true" elements of the covariance matrix Ω coincide with the ones we have estimated from historical data. In the real world we would thus expect investors to use whatever information they have available when forming their risk–return beliefs. In addition to the backward–looking information implicit in historical return data, their information sets may include "news" in the form of recent announcements by the government of changes in monetary policy, tax rules or debt management policies, as well as "noise" in the form of market rumours unrelated to changes in economic fundamentals.

This problem is also inherent in the GARCH approach of Engle (1982) and Bolerslev et al. (1988) referred to above. Such an approach would employ more sophisticated estimation techniques than the ones used in the previous sections in the sense that it also includes estimation of how the elements of the actual (as opposed to the perceived) covariance matrix change over time. However, it nevertheless uses a generalized vector autoregression methodology, thereby confining itself to the use of historical data for computing the covariances.

There are in principle several escapes from the adaptive expectations straightjacket implicitly underlying the vector autoregression procedure used in the previous sections. A first possible solution is the rational expectations method developed by Frankel (1985). In every period t, he assumes that the expected returns entering the asset demands of investors equal the realized ex post returns plus an error term uncorrelated with any information available to investors at time t–1. Thus, the expected returns can vary freely over time and are not restricted to any particular backward-looking expectations mechanism. Finally, by imposing the constraint that the subjective covariance matrix Ω equals the covariance matrix of the residuals associated with estimating an equilibrium asset market model of the form (9), Frankel obtains estimates of the relevant asset demand parameters.

A second method is to use the information contained in opinion surveys. This approach is represented by Friedman (1986), who infers investors' risk perceptions from expectational survey data concerning inflation, stock prices and long–term interest rates. Such a survey data methodology is of course subject to the standard criticism that we do not know a priori whether the people interviewed really are identical, or even remotely similar, to the representative individual(s) in the market. Still the survey approach is warranted; the elusiveness of the very concept of expectations calls for considerable eclecticism in empirical work. We simply have to try all possible approaches in order to get some view of the robustness of the empirical results.

A third procedure is suggested in this section. For the diagonal elements σ_{ii} in the covariance matrix, we will use the subjective variances of stock and bond yields that can be inferred from options data and standard models of options valuation not.[52] According to the Black and Scholes (1973) options pricing formula, the price of an option is a rather complicated function of today's price of the underlying asset, the variance of that price, the time till the option can be exercised, the strike price, and the risk-free interest rate. That formula has been mostly used to calculate the theoretical option values, but of course it could also be used the other way around: By knowledge of the market price actually paid for the option, and by knowledge of all the other variables except the variance, one can compute the variance implicit in the observed market price.

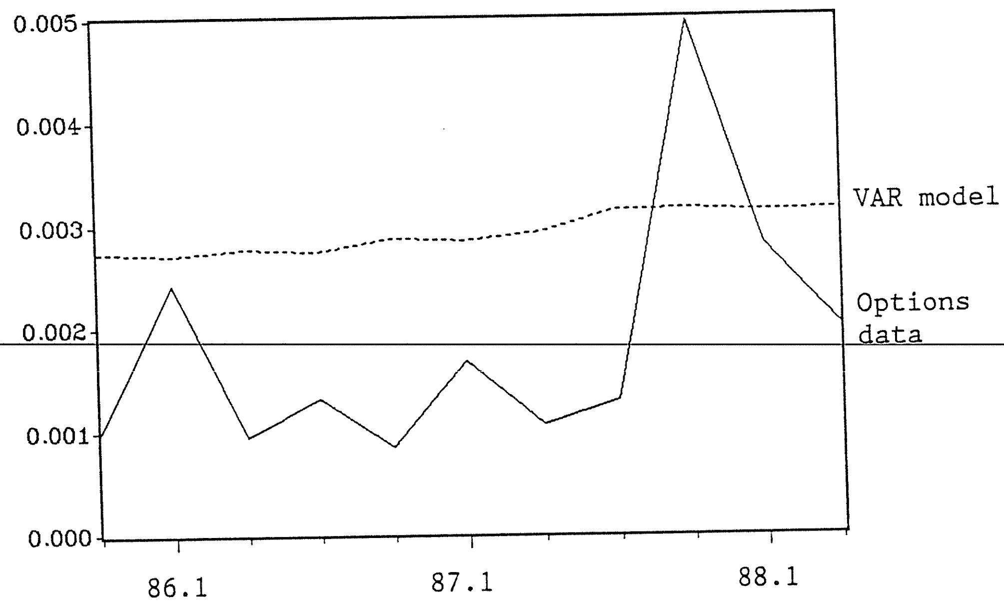
For the stock market, we have used the Standard & Poor's 500 Index, of which an option is traded at the Chicago Mercantile Exchange. For the long-term government bonds, we have used the option traded at the Chicago Board of Trade, for which the underlying asset is a futures contract on a 15–20 years treasury bond. There are also options for which the underlying asset is the treasury bond itself, but these markets are rather thin, and it is not self-evident that the prices quoted are the "correct" ones. The options on futures contracts, however, are very actively traded, and it seems fairly reasonable to use a theoretical option pricing formula when studying this market. Such an option is most appropriately evaluated using the Black (1976) formula, which is also the one we have employed in this context. Since the underlying bond is of such a long duration, and since we deal only with options with a very short time duration (i.e. a quarter), the problems of compound interest, and the approaching date of redemption can be disregarded.

We have used quarterly data for the period 1985.4–1988.2, and we have used the price quotations reported in The Wall Street Journal at the end of each quarter. The options studied have been those with a strike price closest to the current market price of the underlying asset, and with three months left to the strike date.

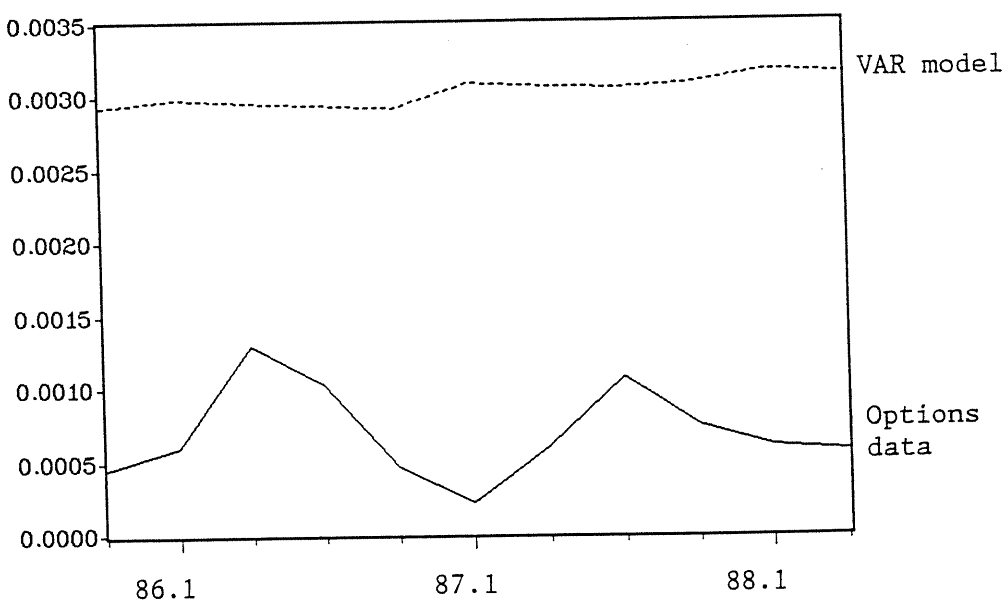
52This possibility was first suggested to us by Jeffrey Frankel.

<u>Figure 11</u> A comparison of the quarterly variances of nominal asset yields using options data versus autoregression procedures, 1985.4-1988.2.

<u>Figure 11a</u> Variance of corporate equity.



<u>Figure 11b</u> Variance of long-term bonds.



As a measure of the risk-free interest rate, we have used the current yield on three-months Treasury Bills. The implicit variances thus obtained are reported in Figure 11a for the stock market and Figure 11b for the bond market.

These series of variances have several interesting features, in particular if we compare them to the series based on historical yield data as reported in Figure 3 above. For comparison, these series for the period 1985.4–1988.2 are also depicted in Figure 11 in the form of dashed curves. We see that the variances computed from options data (the implicit variances) are considerably lower than the variances based on vector autoregressions on historical data (the VAR variances). This is what one would expect. If historical yield data are available to the agents in the options market — which they presumably are — then the information set underlying the implicit variances is larger than that underlying the VAR variances. And thus the latter should be larger than the former.

An exception is provided by the stock market crash in the fourth quarter of 1987. Here we have an event which was absent in the historical data, and due to the confusion and uncertainty during these hectic days, the implicit variance increases drastically while the VAR variance hardly reacts at all. Soon, however, the implicit variance resumes its old level, while the VAR variance slowly increases far into 1988. Finally we see that, interestingly enough, the variance of bond yields is totally unaffected by the dramatic events in the stock market, both when we use the implicit approach and when we use the VAR approach.

To translate the variances of Figure 11 into numerical values of the policy coefficients $\partial r_i^e / \partial \alpha_j^s$, we recall from section III above that in the case of three assets, where one is riskless, the derivative can be written

$$(10') \qquad \frac{\partial r_i^e}{\partial \alpha_j^s} = c\sigma_{ij} \qquad i,j = 1,2.$$

From the options data, we have values of σ_{11} and σ_{22}, and since

$$(15) \quad \sigma_{12} = \rho_{12} \sqrt{\sigma_{11}\sigma_{22}}$$

we can compute the covariance σ_{12} if we know the correlation coefficient ρ_{12}. Here we have to rely on historical data. In Figure 4 above we reported the conditional quarterly correlation coefficients based on the vector autoregression technique. For the period 1985.4–1988.2, these numbers have been plugged into formula (15), together with the implicit variances σ_{11} and σ_{22} obtained from options data. The resulting values of the policy parameters $\partial r_1^e / \partial \alpha_2^s$ for that period are shown in Figure 12a.

For the policy derivative $\partial r_2^e / \partial \alpha_2^s$ we do not need any correlation coefficient ρ_{12}. Thus we do not have to resort to any figures obtained by VAR techniques, but can compute $\partial r_2^e / \partial \alpha_2^s$ directly from the variances implicit in bond option prices. This policy derivative is shown in Figure 12b. For comparison, the policy derivatives obtained from historical data are also shown in Figure 12 by the dashed curves.[53]

Two features should be noted. First we see that the policy derivate based on options data is much more volatile than the derivative based on historical data. This is what one would expect. The very concept of an autoregressive process implies a large degree of inertia in the time series, while the options data approach allows the variance to respond immediately to new information in the form of new

[53] These are in principle identical to those shown in Figure 5 above, for the period 1985.4–1988.2. In practice, however, there is a slight and hardly observable difference: the numbers in Figure 5 were computed on basis of the assumption that all three assets (including treasury bills) are risky, while when computing the numbers in Figure 12 we assumed that two assets are risky while treasury bills are riskless. We have thus here disregarded the inflation risk, which in practice is negligible since the VAR technique reduces the uncertainty about inflation to almost zero (cf. Table 4 above).

Figure 12a The derivative $\partial r_1^e/\partial \alpha_2^s$ for options and VAR approaches, quarterly data 1985.4-1988.2.

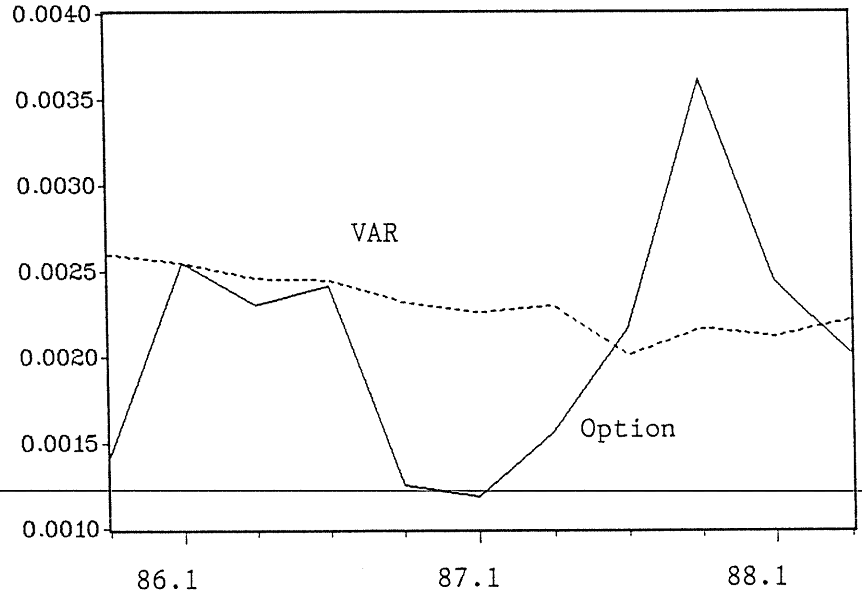
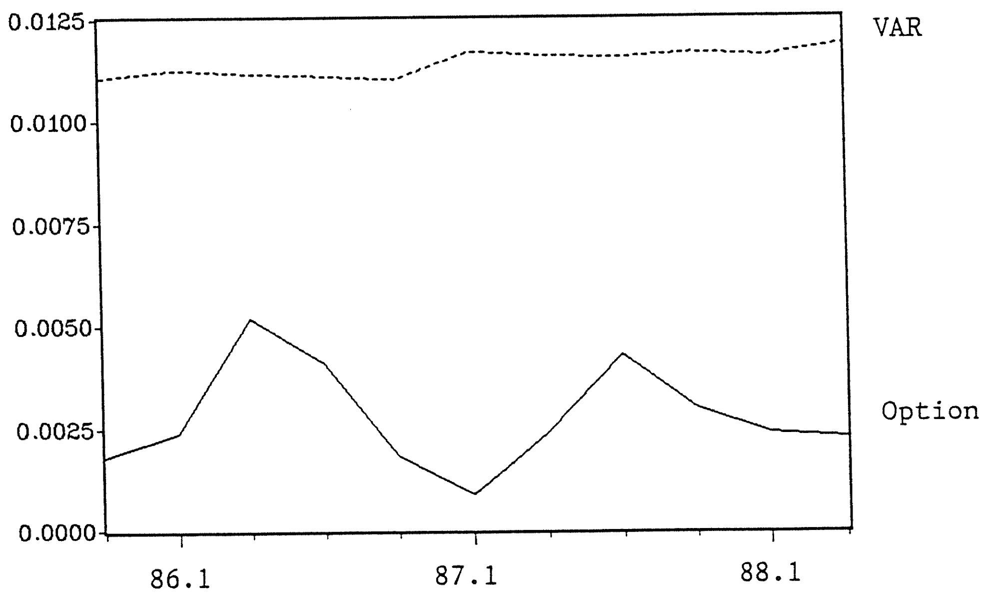


Figure 12b The derivative $\partial r_2^e/\partial \alpha_2^s$ for options- and VAR-approaches, quarterly data 1985.4-1988.2.



policy announcements, more or less substantiated market rumors, etc. Second, except for the stock market crash in the fourth quarter of 1987, the derivatives based on options data are much lower than the ones based on historical data.

VI How Returns Adjust: The Effects of Endogenous Prices

In the preceding section we discussed the effects of government debt management using a portfolio balance model widely used in the literature. Like all economic models, this "work–horse" model can be characterized as a compromise between analytical tractability and economic plausibility. Some of the well–known simplifying assumptions include the suppression of intertemporal dependencies and taking the supply of financial assets as exogenously given. This section focuses on another, perhaps less obvious, analytical short–cut. Most of the literature examines the effects of changes in the composition of government debt on expected asset returns while taking current asset prices as exogenously given. As noted by Friedman and Kuttner (1987 p. 26), this " ...embodies the contradiction of implicitly taking as given the prices of the assets whose expected returns the model is supposed to determine, even though for most assets it is primarily variation in price that delivers variation in expected return".

We will in the following incorporate endogenous adjustments of asset prices into the basic model. This entails introducing three distinct adjustment mechanisms, with potentially important implications for the effects of government debt management: 1) allowing for valuation changes implies introducing wealth effects – with endogenous prices initial portfolio wealth W becomes an endogenous variable; 2) the elements of the covariance matrix Ω now become endogenous; 3) with given asset prices it is immaterial to distinguish between asset supplies defined in quantitative terms and in value terms, while with endogenous prices this distinction may be important. The third point says that if we perform debt management in the sense of changing the supply of some asset in quantity terms, then the supplies of other assets may change in value terms – even if we have not changed the supplies of those assets in quantity terms.

This section provides qualitative and quantitative assessments of the effects of debt management when allowing for endogenous prices. Subsections VI.1 and VI.2 describe the model and derive the relevant comparative static results. Subsection VI.3 turns to the empirical evidence and compares the results of the "work-horse" model used in Section IV with those implied by the model incorporating valuation changes.

VI.1 The Basic Model with Endogenous Asset Prices

The mechanism behind the change in some expected yield r_i^e is of course that the price of asset i changes. In our simple atemporal model, there is only one time period. We denote the price of the asset at the beginning of the period by P_i and at the end of the period by $\tilde{P}_i$, and let the tilde indicate that $\tilde{P}_i$ is uncertain. Disregarding the coupon (we could for example treat all assets as discount bonds[54]), the yield r_i is defined by

$$r_i \equiv \frac{\tilde{P}_i - P_i}{P_i}$$

and the expected yield r_i^e is the corresponding mathematical expectation with respect to the probability distribution of $\tilde{P}_i$.

[54] See e.g. Roley (1979).

A change in the expected yield r_i^e then means that "today's" price P_i changes, or that "tomorrow's" expected price $\tilde{P}_i^e$ changes, or both. Without specifying the institutional setup of the model further, we can not say which one applies; we can only say that the prices P_i and $\tilde{P}_i$ have to change in such a fashion that equation (7) is satisfied. In the case of short–term discount bonds, where the time to maturity is equal to our period of analysis, the future price $\tilde{P}_i$ is legally fixed in nominal terms[55], and all changes in r_i^e have to be channeled via changes in today's price P_i.

However, in the case of long–term assets, with a time to maturity going beyond our period of analysis, it is natural to regard both P_i and $\tilde{P}_i$ as endogenously determined market prices. The exact relationship between the two prices is then ambiguous. A policy $d\alpha_j^s$ which, via the formula (10), leads to an increase in r_i^e could have two effects on the price P_i. Either the increase in r_i^e is accomplished by an <u>increase</u> in today's price P_i together with a sufficiently large increase in tomorrow's price $\tilde{P}_i$ so as to make the ratio $(\tilde{P}_i - P_i)/P_i$ grow, or by a <u>decrease</u> in P_i together with some (perhaps minor) change in $\tilde{P}_i$. The question of which alternative to apply can be settled only by the analysis of an explicitly intertemporal model. In practice however, it is always implicitly assumed that r_i^e and P_i move in opposite directions, that is, an increase in the yield is achieved by a fall in today's price of the asset.[56]

Let us now start with the demand system (6) in value terms, with all asset

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55 For the time being we disregard the problem of nominal versus real yields.

56 This question is crucial if we want to use debt management for the purpose of stabilization policy. If a policy action $d\alpha_j$ results in an increase in the equilibrium yield of common stock ($dr_i^e > 0$) we say that the policy leads to reduction of industry investment demand. But if $dr_i^e > 0$ goes hand in hand with an <u>increase</u> in today's share prices ($dP_i > 0$) it means that Tobin's q has actually increased, which should stimulate investment demand.

prices explicit. Setting demand equal to supply, also in value terms, we have the equilibrium system

$$(16)\qquad \begin{bmatrix} P_1\ q_1^s \\ P_2\ q_2^s \\ \vdots \\ P_n\ q_n^s \end{bmatrix} = \left[\frac{1}{c}B \begin{bmatrix} (\tilde{P}_1^e-P_1)/P_1 \\ (\tilde{P}_2^e-P_2)/P_2 \\ \vdots \\ (\tilde{P}_n-P_n)/P_n \end{bmatrix} + \Pi\right](P_1\bar{q}_1+P_2\bar{q}_2+\ldots+P_n\bar{q}_n)$$

where $\tilde{P}_i^e \equiv E[\tilde{P}_i]$, and where q_i^s is the supply of asset i and $\bar{q}_i$ is the corresponding initial endowment. The matrix B and the vector Π are defined as before in terms of the covariance matrix Ω. The latter is given by

$$(17)\qquad \Omega \equiv [\mathrm{Cov}\,(r_i, r_j)] \equiv \{\mathrm{Cov}\,[(\tilde{P}_i-P_i)/P_i, (\tilde{P}_j-P_j)/P_j]\} \equiv$$

$$\equiv \left[\frac{1}{P_iP_j}\mathrm{Cov}\,(\tilde{P}_i, \tilde{P}_j)\right] \equiv \left[\frac{1}{P_iP_j}\tilde{\Omega}\right],$$

where $\tilde{\Omega}$ is the covariance matrix of the end prices $\tilde{P}_i$. More compactly, (17) can be written as

$$\Omega \equiv \Gamma\,\tilde{\Omega}\,\Gamma$$

where Γ is the n x n diagonal matrix with elements $1/P_i$ in the diagonal and zeros elsewhere.

Several things should be noted here. First, the left–hand side of (16) is the supply vector. Since it is written in value terms, it includes the market prices P_i

and the physical supplies (i.e. the face values, the number of shares, or equivalent) of the assets. Second, the covariance matrix Ω is endogenously determined either if we let the covariance matrix $\tilde{\Omega}$ of the end prices $\tilde{P}_i$ be endogenously affected by debt management, or if we let today's prices P_i be endogenous, or both. The decomposition of Ω into the two parts $1/P_iP_j$ and $\tilde{\Omega}$ spells out these two approaches clearly, and in the analysis below we will assume $\tilde{\Omega}$ to be constant while letting P_i and P_j be endogenously determined.

Third, the expression on the right–hand side is the investor's wealth $W \equiv P_1\bar{q}_1+P_2\bar{q}_2+...+P_n\bar{q}_n$, defined in value terms. The investor's initial endowments $\bar{q}_i$ are necessarily identical to supplies q_i^S at an initial equilibrium. When the government engages in debt management it becomes however important to distinguish between $\bar{q}_i$ and q_i^S. A debt management operation can be done either in the form of helicopter drops of assets, or in the form of regular open market operations. In the former case $dq_i^S = d\bar{q}_i$ and the comparative static derivative $\partial r_i^e/\partial\bar{q}_i$ tells us how asset yields are affected by that kind of debt management. We will in the following however use the latter approach and assume that the government buys and sells bonds in the market. Thus the q_i^S will change, but the $\bar{q}_i$ will remain constant, and the derivative $\partial r_i^e/\partial q_i^S$ will in general be different from $\partial r_i^e/\partial\bar{q}_i$.

Taking the system (16) – (17) as a point of departure, we can see that the analysis in previous sections implicitly assumed that

(i) changes in yields r_i^e took the form of changes in "tomorrow's" prices $\tilde{P}_i^e$, while today's prices P_i were assumed to remain constant,

(ii) the covariance matrix Ω was assumed to be constant, With this set of assumptions, which is the standard one implicitly underlying all empirical work on debt management, B, Π and W in (16) will remain constant throughout the analysis, and the only equilibrating changes will take place in the r^e vector. Also, the supply vector α^S will remain constant apart from the j:th and i:th element in which the debt management operation $dq_j^S = -dq_i^S$ takes place.

When current price changes are accounted for there are three induced effects in the system (16):

(i) Wealth effects. A debt management experiment will make asset prices change so that wealth W is changed.[57]

(ii) Covariance effects. The covariance matrix Ω is endogenously determined, either because both (P_i,P_j) and $\tilde{\Omega}$ are affected by the experiment, or in the simpler case because at least (P_i,P_j) are affected.

(iii) Supply effects. Writing the supply in value terms like in (16) decomposes it into a physical supply and a price. A change in the physical supplies dq_i^S and dq_j^S will change all prices[58] $P_1, ..., P_n$,

[57] One would like to think that since the assumption of constant relative risk aversion makes the asset demand functions (16) linearly homogeneous in wealth, possible wealth effects would not matter for the equilibrium solution. This is however not true. First, if investors are heterogeneous (for example in terms of risk aversion, subjective perception of the covariance matrix, or differential taxation) wealth effects will affect people differently, thereby affecting the equilibrium solution. Second, even with one representative investor, wealth effects will matter. Since one asset is treated as a numeraire, its price will not increase if wealth effects make demand increase. The equilibrium prices of all other assets will however go up, which implies that the composition of asset supplies in value terms must change and that the expected equilibrium returns adjust.

[58] Apart from the numéraire.

thereby affecting all elements in the supply vector in value terms and not just two elements of it.

VI.2 Debt Management when Asset Prices are Endogenous

To highlight the basic mechanisms involved we will in the following use a special version of the three–asset model used in section IV. Abstracting from inflation uncertainty, short–term debt instruments become riskless.[59] Then the equilibrium system (16) reduces to

$$(18) \quad \begin{pmatrix} P_1 & q_1^s \\ P_2 & q_2^s \end{pmatrix} = \frac{1}{c} \begin{bmatrix} \frac{\tilde{\sigma}_{11}}{P_1^2} & \frac{\tilde{\sigma}_{12}}{P_1 P_2} \\ \frac{\tilde{\sigma}_{12}}{P_1 P_2} & \frac{\tilde{\sigma}_{22}}{P_2^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\tilde{P}_1^e - P_1}{P_1} - r_3 \\ \frac{\tilde{P}_2^e - P_2}{P_2} - r_3 \end{bmatrix} (P_1\bar{q}_1 + P_2\bar{q}_2 + P_3\bar{q}_3),$$

where P_1, P_2 and P_3 are the current prices of corporate equity, long–term debt and short-term debt, and $\tilde{\sigma}_{12}$ is the covariance of the end prices $\tilde{P}_1$ and $\tilde{P}_2$. As before, our asset market model is only capable of determining a set of relative rates of return. Consequently, we take the return on short–term debt r_3 as given and treat the expected absolute returns $(\tilde{P}_i^e - P_i)/P_i$ on equity and long–term debt as endogenous. Also, since short–term debt is our numeraire asset we can without loss of generalization set P_3 equal to unity.

Now, (18) is the basic equation system used to infer the effects of

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59 This assumption is perhaps not too unrealistic; as noted in section III.2 the estimated conditional variance of the real yield on short–term debt is a very small number.

government debt management. To provide a clear–cut benchmark for the ensuing analysis we first examine the case when valuation changes do not matter. This corresponds to the standard procedure used in the literature, implying the determination of expected asset returns given the assumption that current asset prices are fixed. As already noted, this case is formally equivalent to viewing (18) as an equilibrium system determining the expected end prices $\tilde{P}_1^e$ and $\tilde{P}_2^e$ for given values of P_1 and P_2. We assume that the government performs an open market operation of the form $P_2 dq_2^s = -dq_3^s$, implying the substitution of dq_2^s units of long-term bonds for $-P_2 dq_2^s$ units of short–term bonds. This experiment, implying that the dollar value of long–term bonds sold equals the dollar value of short–term bonds bought, is carried out by totally differentiating (18) with respect to q_2^s, $\tilde{P}_1^e$ and $\tilde{P}_2^e$.[60] The solution turns out to be

$$\frac{\partial \tilde{P}_1^e}{\partial q_2^s} = \frac{c}{W} \tilde{\sigma}_{12}$$

$$\frac{\partial \tilde{P}_2^e}{\partial q_2^s} = \frac{c}{W} \tilde{\sigma}_{22} \cdot$$

Normalizing by setting $P_i = 1$ and using the definition $r_i^e \equiv (\tilde{P}_i^e - 1)$, these comparative static results obviously imply that

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[60] Since we assume that the initial endowments $\bar{q}_i$ are unchanged, this comparative static experiment represents a regular open market operation where the increased supply of long–term debt is balanced by a reduced supply of $P_2 dq_2^s$ bond units in the (redundant) market for short–term debt.

$$(19)\qquad \frac{\partial r_1^e}{\partial q_2^s} = \frac{c}{W}\tilde{\sigma}_{12}$$

$$(20)\qquad \frac{\partial r_2^e}{\partial q_2^s} = \frac{c}{W}\tilde{\sigma}_{22}.$$

Equations (19) and (20) conform with well–established intuition. An increase in the supply of long–term debt in exchange for short–term debt drives up the expected long-term bond yield. The response of the expected equity yield depends on the sign of $\tilde{\sigma}_{12}$. When $\tilde{\sigma}_{12}$ is positive long–term debt and equity are substitutes[61] and lengthening the maturity composition of outstanding debt increases the expected equity yield. In the case of complementarity $\tilde{\sigma}_{12}$ is negative and our debt management experiment lowers the expected equity yield, i.e. we obtain a "crowding in" effect.

We will next see how these results are affected when we drop the assumption that today's prices P_i are fixed. We thus examine the polar case when the probability distribution of end prices $\tilde{P}_i$ is given and current asset prices are endogenous. As a by–product of this, apart from introducing wealth effects and making the supply vector endogenous in value terms, we will also be able to treat the covariance of asset yields Cov $(r_i, r_j) \equiv \tilde{\sigma}_{ij}/P_iP_j$ as endogenous, although we still assume that the covariance of future prices $\tilde{\sigma}_{ij}$ is exogenously given.

As before, we examine a regular open market sale of long–term bonds, satisfying $P_2 dq_2^s = -dq_3^s$. With exogenous current prices this flow based definition of debt management is identical to the stock based definition saying that debt

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[61] Only in the case of only two risky assets there is equivalence between the sign of $\tilde{\sigma}_{ij}$ and propositions concerning whether assets i and j are complements or substitutes. See Blanchard and Plantes (1977).

$$\mathrm{var}(r_m) \equiv \bar{\alpha}_1^2\tilde{\sigma}_{11} + \bar{\alpha}_2^2\tilde{\sigma}_{22} + 2\bar{\alpha}_1\bar{\alpha}_2\tilde{\sigma}_{12}$$

is the return variance of the market portfolio at the initial equilibrium.

To make some headway in interpreting (21) and (22) we first need some additional information. With constant relative risk aversion the first–order conditions for individual portfolio optimum in the initial equilibrium are[63]

$$(23)\quad \tilde{P}_1^e - 1 - r_3 = c(\bar{\alpha}_1\tilde{\sigma}_{11} + \bar{\alpha}_2\tilde{\sigma}_{12})$$
$$(24)\quad \tilde{P}_2^e - 1 - r_3 = c(\bar{\alpha}_1\tilde{\sigma}_{12} + \bar{\alpha}_2\tilde{\sigma}_{22}).$$

Using the expressions for $\tilde{P}_i^e$ implicit in (23) and (24), it is easily seen that in an economy where the initial supplies of equity and long–term debt are zero ($\bar{q}_1 = \bar{q}_2 = 0$), price effects do not matter and (21) and (22) reduces to (19) and (20). Also, local stability of the equilibrium system (18) around some equilibrium point (P_1^*, P_2^*) requires that the denominator in (21) and (22) is positive.[64]

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[63] Equations (23) and (24) follow from the optimization problem

$$\max_{\alpha_1,\alpha_2,\alpha_3} U = \sum_{i=1}^{3} \alpha_i r_i^e - \frac{c}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}\alpha_i\alpha_j\tilde{\sigma}_{ij}$$

subject to $\alpha_1+\alpha_2+\alpha_3 = 1$.

[64] Invoking a Walrasian price adjustment rule and taking a first order Taylor expansion of (18) around an equilibrium point (P_1^*, P_2^*) = (1, 1) yields the system

$$(\mathrm{i})\quad \begin{bmatrix} dP_1/dt \\ dP_2/dt \end{bmatrix} = h \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} P_1-1 \\ P_2-1 \end{bmatrix}$$

where $h \equiv$ a positive adjustment rate coefficient

$a_{11} \equiv \bar{\alpha}_1(\tilde{\sigma}_{22}\hat{r}_1 - \tilde{\sigma}_{12}\hat{r}_2) - \tilde{\sigma}_{22}(1+r_3)$

$a_{12} \equiv \bar{\alpha}_2(\tilde{\sigma}_{22}\hat{r}_1 - \tilde{\sigma}_{12}\hat{r}_2) + \tilde{\sigma}_{12}(1+r_3)$

management operations are experiments leaving the total value of government debt unchanged. The equivalence between the stock and flow definitions of debt management, where one implies the other, does not hold with endogenous current asset prices. When P_2 is endogenous our flow constraint $P_2 dq_2^s = -dq_3^s$ will thus generally go hand in hand with revaluations of the outstanding stock of government debt.[62]

Totally differentiating (14) with respect to q_2^s, P_1 and P_2 and evaluating all partial derivatives at an initial equilibrium where $P_1 = P_2 = 1$, $q_2^s = \bar{q}_2$ and $q_3^s = \bar{q}_3$, we obtain the general equilibrium partial derivatives

$$(21) \qquad \frac{\partial r_1^e}{\partial q_2^s} = \frac{c\tilde{P}_1^e[\tilde{\sigma}_{12} + c\bar{a}_1\bar{a}_2(\tilde{\sigma}_{11}\tilde{\sigma}_{22} - \tilde{\sigma}_{12}^2)/(1+r_3)]}{W[1+r_3-c\ \mathrm{var}(r_m)]}$$

$$(22) \qquad \frac{\partial r_2^e}{\partial q_2^s} = \frac{c\tilde{P}_2^e[\tilde{\sigma}_{22} - c\bar{a}_1^2(\tilde{\sigma}_{11}\tilde{\sigma}_{22} - \tilde{\sigma}_{12}^2)/(1+r_3)]}{W[1+r_3-c\ \mathrm{var}(r_m)]},$$

where we have used the definition $r_i^e = (\tilde{P}_i^e - P_i)/P_i$, where W is the initial equilibrium value of initial wealth, where $\bar{a}_i$ is the initial portfolio share $\bar{q}_i/(\bar{q}_1+\bar{q}_2+\bar{q}_3)$ of asset i, and where

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62 In sections III and IV above, where today's prices were constant, the flow definition of debt management $P_2 dq_2^s = -dq_3^s$ also implies that the market value of outstanding government debt, $B \equiv P_2 q_2^s + q_3^s$, is constant. In this section, where prices are endogenous, keeping B constant would require a trading rule according to

$$(i) \qquad P_2 dq_2^s = -q_2^s dP_2 - dq_3^s,$$

which differs by the term $-q_2^s dP_2$ from our flow definition of debt management. The stock trading rule (i) has previously been used by Roley (1979) and by Agell and Persson (1987) as <u>the</u> definition of debt management in models with endogenous prices. However, as shown by Jungenfelt (1988) this procedure leads to mutually inconsistent definitions of public and private wealth.

Adopting this sign convention several observations are in order. First, it is trivially true that the quantitative effects with endogenous current asset prices will differ from those obtained with exogenous prices. Thus, the induced wealth, supply, and covariance effects will generally make both the numerator and denominator of (21) and (22) differ from those of (19) and (20). However, with one important exception, the qualitative results prove to be robust. Turning to the basic crowding-out issue in the economics of debt management, we see that the second term in the numerator in (21) is always non–negative.[65] Thus, whenever $\tilde{\sigma}_{12}$ is positive, signalling that long–term debt and corporate equity are substitutes, we still obtain the result that lengthening the maturity composition of government debt leads to crowding out by increasing the expected equity yield. In the case when $\tilde{\sigma}_{12}$ is negative we previously derived an unambiguous crowding–in effect. With endogenous prices this is no longer true. Depending on the configuration of initial portfolio holdings, relative risk aversion and covariances, the numerator in (21) may well be positive, implying a crowding out effect also in the case when long–term debt and equity are complements.

In sum, we have thus found that allowing for endogenous price changes also

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$$
\begin{aligned}
a_{21} &\equiv \bar{\alpha}_1(\tilde{\sigma}_{11}\hat{r}_2 - \tilde{\sigma}_{12}\hat{r}_1) + \tilde{\sigma}_{12}(1+r_3) \\
a_{22} &\equiv \bar{\alpha}_2(\tilde{\sigma}_{11}\hat{r}_2 - \tilde{\sigma}_{12}\hat{r}_1) - \tilde{\sigma}_{11}(1+r_3) \\
\hat{r}_i &\equiv \tilde{P}_i^e - 1 - r_3.
\end{aligned}
$$

The system (i) is locally stable if

$$
\begin{aligned}
&a_{11} + a_{22} < 0, \text{ and} \\
&a_{11}a_{22} - a_{12}a_{21} > 0.
\end{aligned}
$$

It is straightforward to show that the latter inequality reduces to the condition that the denominator in (21) and (22) is positive.

[65] This is so because $\tilde{\sigma}_{11}\tilde{\sigma}_{22} - \tilde{\sigma}_{12}^2 \geq 0$.

serves to strengthen the crowding–out case associated with long–term debt financing in much of the literature. Whether the effects will be quantitatively larger or smaller than those represented by (19) is still an open question. Depending on the precise values of the additional terms entering the numerator and denominator of (21), compared to (19), allowing for valuation changes may thus serve to magnify or diminish the effects obtained in previous literature.

Finally, turning to the own–yield effect of an open market sale of long–term debt, it may at a first glance appear as if the numerator in (22) can become negative, thus implying a decrease in the expected yield on long–term debt. However, by the sign restriction imposed on the denominator we can always rule out this paradoxical result;[66] also with endogenous prices the own–yield effect must be positive. As before, we can not a priori determine whether allowing for price changes magnifies the original results.

VI.3 The Empirical Evidence

In principle, it is easy to construct hypothetical numerical examples where allowing for return adjustments through variations in current asset prices may make a significant difference to the results. From an empirical point of view, it is of more interest to examine whether the policy derivatives in (21) and (22) differ from those in (19) and (20) when using real-world data on asset returns and portfolio holdings.

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[66] The numerator in (22) is positive whenever

$$1 + r_3 - c\bar{\alpha}_1^2\tilde{\sigma}_{11}(1-\rho_{12}^2) > 0.$$

However, since $\mathrm{var}(r_m) > \bar{\alpha}_1^2\tilde{\sigma}_{11}(1-\rho_{12}^2)$, the sign restriction on the denominator implies that (this inequality) holds.

For this reason, we have computed the policy derivatives using U.S. quarterly data for 1970.1 to 1988.2.

As the policy derivatives in (21) and (22) contain some parameters not included in previous sections, we have made some adjustments of the original data set. In accordance with the derivation of (21) and (22), we assume an initial equilibrium in each quarter where $P_1 = P_2 = 1$. The expected end-of-quarter prices $\tilde{P}_1^e$ and $\tilde{P}_2^e$ are then calculated as

$$\tilde{P}_i^e = 1 + d_i + \hat{k}_i,$$

where d_i is the known dividend yield (i.e. dividends on common stock and coupon on long-term bonds) and $\hat{k}_i$ is the predicted nominal capital gain obtained from the moving-sample vector autoregression procedures discussed above.

Since the initial equilibrium prices are normalized to unity, the covariances of end-of-period asset prices are assumed to be identical to the conditional covariances of nominal asset yields implied by the VAR model.[67] The initial portfolio shares represent the composition of the aggregate financial wealth of U.S. households for each quarter.[68] The empirical counterpart of $\bar{\alpha}_1$ is an aggregate of corporate equity and equity-like assets (e.g. holdings of mutual fund shares). The portfolio share $\bar{\alpha}_2$ corresponds to all long-term assets (corporate and government

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[67] By definition

$$\text{(i)} \qquad \text{cov}\,(r_1, r_2) \equiv \text{cov}\,[(\tilde{P}_1 - P_1)/P_1, (\tilde{P}_2 - P_2)/P_2] \,.$$

When $P_1 = P_2 = 1$, (i) reduces to

$$\text{(ii)} \qquad \text{cov}\,(r_1, r_2) \equiv \text{cov}\,(\tilde{P}_1, \tilde{P}_2) \,.$$

[68] This data set was provided by Benjamin Friedman.

bonds, tax-exempt state and local bonds) held by households. The residual asset, finally, includes all other liquid and interest-bearing assets. We also normalize the initial equilibrium value W of initial wealth to unity, which implies that the policy derivatives now show the effects of marginally increasing the fraction of long-term bonds held by investors.

The calculated policy derivatives for the period 1970.1 to 1988.2 are shown in Figure 13. For comparison, the dashed curve depicts the results obtained when using the policy derivatives (19) and (20), which apply for the standard model without endogenous price changes. Somewhat surprisingly, and in spite of the theoretical arguments suggesting the importance of allowing for the endogeneity of asset prices, the results obtained in the case of endogenous asset prices corresponds closely to those derived in the case of exogenous prices.

To conclude, these results suggests that allowing for valuation changes does not in a significant way alter the yield effects obtained using the standard set-up (discussed in section IV) where current asset prices are implicitly treated as exogenous. However, since the effects of debt management ultimately depend on the interaction of the real and financial sectors of the economy, incorporating endogenous asset prices may still be warranted. In macro models where current asset prices are taken as given, the transmission mechanism linking relative asset supplies to the spending decisions of firms and households is restricted to the relative costs of financing the spending in question. In models allowing for endogenous asset prices, there will be an additional transmission channel due to the wealth effects implied by valuation changes on outstanding assets.

Figure 13a The derivative $\partial r_1^e / \partial \alpha_2^s$ for models with and without revaluation changes, quarterly data 1970.1-1988.2. (Solid curve: model with revaluation changes.)

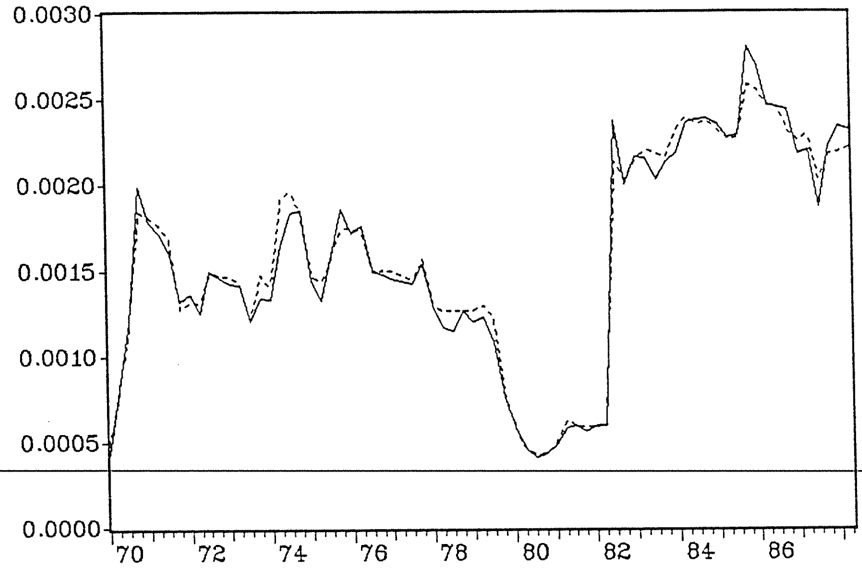
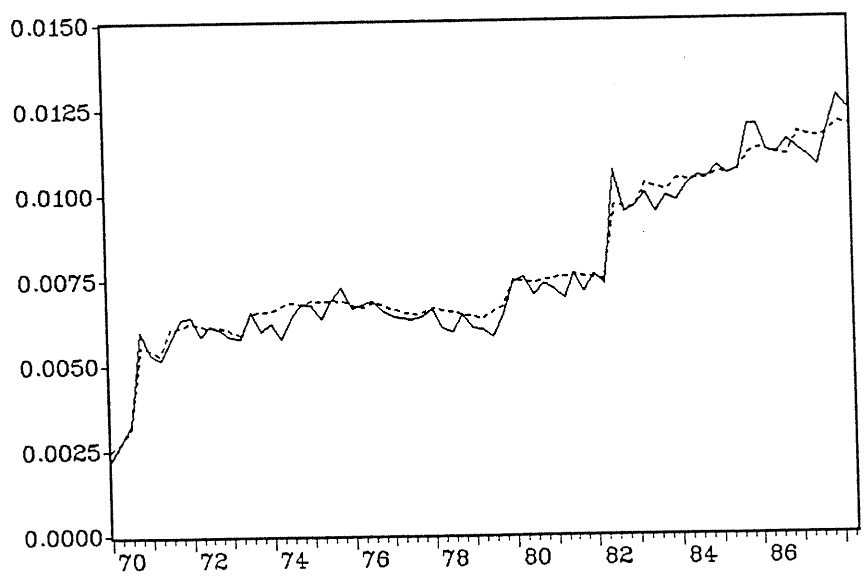


Figure 13b The derivative $\partial r_2^e / \partial \alpha_2^s$ for models with and without revaluation changes, quarterly data 1970.1-1988.2. (Solid curve: model with revaluation changes.)



VII Summary and Conclusions

This paper has analyzed the effects of government debt management. For analytical simplicity, we have been careful to define debt management as involving open-market operations only; i.e. for given government expenditures and taxes we have considered the substitution of, for example, short-term for long-term government bonds. After reviewing the literature and bringing in some evidence of our own, several points stand out.

First, most empirical work on debt management – including the empirical parts of the present paper – focuses on the effects of the maturity composition of government debt on relative asset yields in general, and on corporate equity yields in particular. In the real world, however, there is no such simple and single-dimensional characterization of debt management. Recognizing the wide array of debt instruments available to the government (e.g. tax-exempt versus taxable bonds, marketable assets versus nonmarketable assets, etc.) and the multitude of conceivable policy targets unrelated to the crowding in/crowding out issue, the design of debt policy entails a choice between a continuum of different debt attributes to attain many different goals. As a consequence, discussing the stance of debt management in terms of only the maturity composition of government debt is potentially misleading.

Second, using the mean-variance "work-horse" model of portfolio choice and asset market equilibrium, we turned to the empirical evidence. Upon invoking a moving-sample vector autoregression model to allow for time-varying risk perceptions, we examined the effects on relative asset yields of lengthening the maturity composition of government debt. It turned out that these effects were rather small in magnitude, and that their numerical values were highly volatile. Thus the policy conclusion to be drawn seems to be that there is not much scope for a debt management policy aimed at systematically affecting asset yields. This conclusion is further strengthened if we take into account the possibility of Ricardian equivalence discussed in Section II.2.

Third, we examined whether the results are robust with respect to the choice of time unit implicitly made when constructing the data. Rerunning the model with monthly instead of quarterly data, we found that the qualitative conclusions remain intact: increasing the supply of long-term bonds in exchange for short-term bonds still tends to raise the expected yields on equity and long-term bonds. In quantitative terms, however, the yield responses now seemed stronger. This suggests that the frequently neglected time aggregation problem should be of more concern in future empirical work. Fourth, since the above results were based on computing conditional covariance matrices based on the information contained in historical return data, we have tested the robustness of the policy conclusions by using an alternative data source, namely the implicit variances obtained from options data. It turned out that this data set produced results that differed substantially from those obtained using a more conventional vector autoregression procedure: the yield responses to changing relative asset supplies were both considerably smaller and more volatile over time than those obtained using historical return data. Fifth, we have dropped the standard maintained assumption of constant current asset prices, thereby allowing for more complex policy effects than in previous studies. It turns out that the model is surprisingly robust to this. Allowing for endogenous asset prices – which in turn implies various effects on wealth, on asset supplies (in value terms) and on covariances – hardly affects the results.

There are several problems associated with the present setup. For example, a satisfactory dynamic formulation of the model (allowing – among other things – for a consistent treatment of term-structure problems) is still non-existent. In previous empirical work, today's prices have been more or less implicitly assumed to be constant and unaffected by debt management, while asset yields have adjusted via changes in tomorrow's prices. And since the model is strictly atemporal, there is no date after tomorrow. In Section VI we have tried the alternative of assuming (the probability distribution of) tomorrow's prices remaining unaffected of debt management, while today's prices change instead. In reality, of course, both prices are affected – as are the prices at dates after tomorrow. This however calls for a fully dynamic model, which consistently integrate the real and financial sides of the economy. The problems associated with constructing such a model provide a promising research agenda for the future.[69] Another important extension of the basic model is due to the fact that the supplies of financial assets are treated in an overly simplistic manner. The portfolio model outlined in Section III assumed an exogenous net supply of assets, including both the government's and the private firms' supplies of bonds and shares. We know, however, that firms' financial decisions are more complex than that, and thus a more realistic representation of corporate financial decisions is warranted.

On a less ambitious level there are several interesting topics for further research. For example, we have here dealt with a very limited menu of assets. It would be straightforward to include a few more, e.g. real estate and consumer durables. This would illuminate how sensitive the policy conclusions are to the choice of assets included. Real estate and consumer durables would however place some restrictions on the choice of the data interval, since reliable yield figures are not available for short time intervals. In addition, the most important item in most household portfolios is still missing, namely human capital. The analysis of non-

[69] Cf. Cox, Ingersoll and Ross (1985).

marketable assets is fairly straightforward in principle,[70] but in practice we will suffer from the lack of yield data. As for other types of assets with a limited degree of marketability, for example pension savings, data are however available. Such assets also point at the need for studying a model with heterogeneous investors, for example insurance companies and pension funds as well as non-financial companies. The inclusion of such investors calls for taking into account the specific tax situation of each investor category, as well as the fact that such investors are ultimately owned by the households,[71] who constitute the basic agents in the markets.

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[70] Cf. Mayers (1972).

[71] See e.g. Agell and Persson (1986).

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