THE FOREIGN EXCHANGE RISK PREMIUM IN
A TARGET ZONE WITH DEVALUATION RISK

by

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Abstract

The foreign exchange risk premium in an exchange rate target zone regime with devaluation/realignment risks is derived. In contrast to previous literature, the exchange rate's heteroscedasticity within the band, as well as a separate devaluation/realignment risk, is taken into account. The risk premium is then the sum of two separate risk premia, arising from stochastic exchange rate movements within the band and from stochastic devaluations/realignments when the band is shifted. Both real and nominal exchange rate premia are considered. The real and nominal risk premia from movements within the band are very small for narrow target zones and can therefore be disregarded. The real and nominal risk premia from devaluations/realignments are larger but still relatively small proportions of the expected rate of devaluation/realignment.

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1. Introduction

The foreign exchange risk premium can be defined as the expected rate of return differential between similar bonds or deposits denominated in home and foreign currencies. *(Real rate of return differentials give rise to real risk premia; nominal rate of return differentials in a particular currency give rise to nominal risk premia expressed in that currency.)* This paper examines the foreign exchange risk premium in an exchange rate target zone, with devaluation/realignment risks. It argues that both real and nominal risk premia are likely to be small for narrow target zones. This is not surprising in the absence of devaluation risks: The previous theoretical and empirical literature on risk premia in floating exchange rate regimes have indeed generally concluded that the risk premia are likely to be small. In target zone regimes the risk premia should be even smaller, since exchange rate uncertainty should be less. With devaluation risks, however, it is not obvious what the magnitude of the risk premia are. This paper argues that devaluation risk premia are larger than non-devaluation risk premia, but that the devaluation risk premia are still relatively small proportions of expected rates of devaluation.

The foreign exchange risk premium has been much discussed in the international finance literature. It has in particular been discussed in the context of whether central banks' sterilized foreign exchange interventions have any noticeable effect on exchange rates. *(Sterilized foreign exchange interventions change the relative outstanding stocks of domestic and foreign currency denominated assets but do not affect the domestic monetary base, whereas nonsterilized foreign exchange interventions do affect the domestic monetary base.)*

With significant non-constant risk premia bonds denominated in home and foreign currencies are imperfect substitutes. If risk premia do depend on the relative supply of home and foreign currency bonds (and if agents are non-Ricardian), sterilized interventions may have effects on exchange rates. This channel for the effect of foreign
exchange interventions has been called the *portfolio* effect. Another possible way for sterilized interventions to affect exchange rates is as indicators of the intentions of central banks and of forthcoming non-sterilized interventions or generally changes in monetary policies, which would affect expectations of future exchange rates. This channel has been called the expectations effect, or the *signaling* effect.

The empirical literature generally rejects uncovered interest arbitrage, that is, it rejects the hypothesis of zero risk premia. On the other hand, the empirical findings seem to indicate rather small risk premia. The empirical literature hardly finds any effect on risk premia of relative asset supplies. Different specific models of the determination of risk premia are generally rejected. The dominating view seems to be that the portfolio effect of sterilized intervention is insignificant, whereas there is some empirical support for a significant signaling effect.¹

The discussion about risk premia and sterilized interventions has mostly concerned floating exchange rate regimes. For a credible, *completely fixed*, exchange rate regime with free capital mobility, the foreign exchange risk premium should be zero. That is, bonds denominated in home and foreign currency of the same maturity should (absent default risk) be perfect substitutes since there is no exchange rate risk, and domestic and foreign interest rates should be equal.

However, real world fixed exchange rate regimes typically do not have completely

¹ Obstfeld (1988) provides an extensive survey of theory, recent experience, and empirical results regarding the effect of sterilized interventions. Dominguez and Frankel (1990) distinguish between the portfolio and signaling channels. They find preliminary evidence of small portfolio effects and seizable signaling effects. Edison (1990) provides an annotated bibliography of research on foreign currency interventions.

Hodrick (1987, chapter 5) and Hörgren and Vredin (1988b) survey different models of foreign exchange risk premia and corresponding empirical tests. Dooley and Isard (1983), using a portfolio-balance approach, report a risk premium of about 2.5 percent per year, but warn that their method may overestimate the risk premium. Frankel (1982, 1988), using a mean-variance approach, estimates variance-covariance matrices which, with relative risk aversion equal to 2, imply risk premia of about 1 percent per year for six major currencies.

Sibert (1989) and Engel (1990) emphasize the distinction between real and nominal foreign exchange risk premia. Empirical estimates have generally been of the nominal risk premium, whereas the real risk premium is more relevant.
fixed exchange rates, but are better described as narrow target zones. That is, there is a narrow band within which the exchange rate is allowed to fluctuate. The exchange rate is prevented from moving outside the band by foreign exchange interventions. This is so for the Exchange Rate Mechanism within the European Monetary System, where the exchange rate bands are ±2.25 percent (except ±6 percent for Spain), and for the Nordic countries outside EMS. Sweden, for instance, now has a target zone of ±1.5 percent.

Such target zones imply some remaining exchange rate uncertainty because of movements inside the band and because of devaluation/realignment risks. They are also characterized by non-zero and fluctuating interest rate differentials. That interest rate differentials in a target zone are nonzero is not surprising, since the expected rate of depreciation of a currency varies both with the exchange rate's position in the band and with the probability and size of a devaluation/realignment. For instance, with a credible band a currency which is at the strong edge of its band can only depreciate, which contributes to a positive interest rate differential.

The issue arises, however, whether the exchange rate uncertainty due to movements within the band and to devaluations/realignments is sufficient to create a significant foreign exchange risk premium. This issue is important for two reasons. First, some target zones, for instance Sweden's and Norway's, are reported in official statements to be defended mostly by sterilized interventions.² If these sterilized interventions indeed have effects, we may wonder through which channel the effects operate, in particular against the discussion of sterilized interventions reported above. If it is through the portfolio effect, significant risk premia are required. Second, it has frequently been argued that a

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² The Swedish target zone is to a large extent defended by forward market interventions. To purchase foreign currency on the forward foreign exchange market is equivalent to buying a claim on future foreign currency and simultaneously selling a claim on future domestic currency. This has no effect on current domestic liquidity and is hence a sterilized intervention. For further discussion of this, see Frantzén and Sardelis (1988). The Norwegian target zone is defended mostly by spot market interventions. These interventions have in practice been sterilized at the end of each month via domestic liquidity measures (see Norges Bank (1989)).
target zone like the Swedish one allows the central bank some monetary autonomy, in the sense that Swedish interest rates may differ from foreign interest rates, also with abolished capital controls and high international capital mobility. This argument also seems to presume a significant risk premium (see Vredin (1988, chapter 2) for a detailed discussion of monetary autonomy and capital mobility).

Understanding the risk premium in a target zone requires an explicit model of the target zone. The frequently used target zone model first presented by Krugman (1990) is formulated under the simplifying assumption of uncovered interest parity and hence a zero foreign exchange risk premium. An alternative assumption is that the risk premium is exogenous and follows a Brownian motion. This simplification allows a very neat closed form solution for the exchange rate. The assumption of a zero or at least exogenous risk premium is necessary also for a closed form solution to the interest rate differentials (Svensson (1989, 1990)).

The rigorous derivation of an endogenous risk premium in a target zone is a difficult task, since the underlying exchange rate is a complicated nonlinear heteroscedastic stochastic process. For the purpose of deriving an upper bound on the risk premium, some simplifying approximations can be made, as we shall see. However, exchange rate uncertainty in a target zone originates not only from exchange rate movements inside the band but also from discrete shifts of the band, that is from devaluations/realignments. Devaluations and realignments have indeed been a common experience of real world target zones, in particular of the Exchange Rate Mechanism of the European Monetary System. The effect on the risk premium of devaluations/realignments should therefore be incorporated. A few papers present target zone models with realignments, for instance Miller and Weller (1988, 1989a,b) and Bertola and Caballero (1989). The related problem

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3 Dumas (1989) specifies a two-country general equilibrium model with physical capital movements where the real exchange rate behavior is similar to the nominal exchange rate behavior in the Krugman model. Dumas manages to derive a simple closed form expression for a real risk premium, defined as the difference between the two countries' own rates of interest less the expected relative price change.
of a collapse of a target zone exchange rate regime to a free float, as well as other kinds of regime switches, have been discussed by Krugman (1990), Froot and Obstfeld (1989), Delgado and Dumas (1990) and Krugman and Rotemberg (1990). In all these papers, realignments and regime switches by assumption occur only at the edges of the exchange rate band. When the exchange rate is in the interior of the band, the probability of a realignment or regime switch in the next instant is zero. Then there is no separate effect of realignments and regime switches on interest rate differentials and risk premia in the interior of the exchange rate band. Hence, within those models it is reasonable to continue to disregard the risk premium, at least for narrow target zones, except possibly at the edges of the band.\textsuperscript{4}

In the real world, devaluations and realignments seem to occur also when exchange rates are in the interior of their bands.\textsuperscript{5} In addition, interest rate differentials seem to reflect possible devaluation risks in the interior of exchange rate bands.\textsuperscript{6} Therefore it indeed seems relevant to consider target zone models where devaluations/realignments can occur inside the exchange rate band. Svensson (1990) develops a simple model of devaluations along these lines, where devaluations occur according to a Poisson process regardless of where in the band the exchange rate is. It remains to be seen whether such devaluation risks can cause a significant risk premium.

Consequently, in this paper we shall establish an upper bound on the foreign exchange risk premium in a target zone model with devaluation risk, where the devaluations are modeled as a Poisson process. The analysis of the risk premium differs from that of the

\textsuperscript{4} Even though there is no risk premium in the interior of the band in those models, the actual determination of the exchange rate is affected by possible devaluations at the edge of the band (see for instance Bertola and Caballero (1989)).

\textsuperscript{5} For instance, when Sweden devalued in September 1981 and October 1982 the Krona's value was above previous minimum values and away from the edges of its band. Most of the realignments of the Lira against the Mark during the EMS period have occurred when the Lira has been away from the edges of its band (see Bertola and Caballero (1989, Figure 3b)).

\textsuperscript{6} For an examination of Swedish data, see Svensson (1990).
previous literature in two respects: (1) the exchange rate's variable standard deviation (heteroscedasticity) inside the band is explicitly taken into account in the portfolio problem, and (2) the devaluation risk's separate contribution to the risk premium is specified.

Section 2 lays out the model of a small open economy with a target zone. Section 3 specifies the portfolio choice of a representative investor, and section 4 derives the real and nominal risk premia. Section 5 presents some conclusions. An appendix contains some technical details.

2. The Model

The purpose of this paper is to derive generous upper bounds on the real and nominal foreign exchange risk premia. This purpose influences the approach used, and allows some considerable shortcuts to a very complicated problem. The approach used in the paper is to specify the portfolio problem for an investor in an open economy in a particular way. The investor consumes home and foreign goods and has access to bonds denominated in home and foreign currency. Nominal goods prices, interest rates and exchange rates are exogenous stochastic processes (some variables are even assumed constant) that are functions of one single state variable, the exchange rate. Some known properties of the stochastic process for the exchange rates in theoretical target zone models with devaluation risks are exploited. Then a relation between the risk premia and the portfolio shares of the investor are derived and examined.

The simplifications of this approach are obvious. There could be other assets, and there could be other state variables in addition to the exchange rate, like money supplies, government bond supplies, etc. In particular, the stochastic processes for the domestic interest rate, the exchange rate and domestic nominal goods prices should preferably be part of a full general equilibrium.
Nevertheless, given the limited purpose of the paper, the simplifications seem warranted. I cannot see that the simplifications would bias downwards the upper bounds on the risk premia to be derived.

The analysis builds on Merton's (1971) model of continuous-time portfolio choice with state variables affecting assets' rates of return and rates of return being mixed Brownian and Poisson processes. Nominal bonds, exchange rates and two consumption goods are introduced as in Kouri's (1976) model of the determinants of the forward exchange premium. Kouri's model and the recent model of a collapse of a fixed exchange rate regime by Penati and Pennachi (1989) include rates of return being mixed Brownian and Poisson processes but no state variables.\(^7\)

We consider a small open economy with free capital mobility. There are two goods, home and foreign. The home currency price of the home good, \(P_h\), and the foreign currency price of the foreign good, \(P_f\), are sticky, and for simplicity set constant and equal to unity, \(P_h = P_f = 1\). The home currency price of foreign goods, \(P_f\), is then given by \(P_f = S P_f^* = S\), where \(S\) is the exchange rate measured in home currency per unit foreign currency. (The case with flexible and stochastic nominal goods prices is examined in the appendix. The results are the same as with sticky prices.)

There is a target zone exchange rate regime. Foreign exchange interventions keep the exchange rate in a band of \(\pm 100b\) percent around a central parity. The central parity is now and then shifted \(100g\) percent by devaluations/realignments. These devaluations occur according to a Poisson process \(N(t)\) with intensity \(\nu > 0\). Here the integer \(N(t)\) denotes the number of devaluations up to and including time \(t\). The probability of a unit jump in \(N(t)\), \(dN(t) = 1\), during a short interval \(dt\) is equal to \(\nu dt + o(dt)\), whereas the

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\(^7\) Adler and Dumas (1983) and Branson and Henderson (1985) provide useful surveys of international portfolio-choice models, although without considering devaluation risks and Poisson processes. Closed economy asset pricing and portfolio choice models with jump processes have been developed by Ahn and Thompson (1988) and Jarrow and Rosenfeld (1984).
probability of no jump in $N(t)$, $dN(t) = 0$, is equal to $1 - \nu dt + o(dt)$.

After $N(t)$ devaluations, the central parity $a(N(t)) = a(1 + g)^{N(t) - N(0)}$, where $a$ is the central parity at time 0. The exchange rate $S(t; N(t))$ is then restricted to the band

$$a(N(t))(1-b) \leq S(t; N(t)) \leq a(N(t))(1+b).$$

Inside the band the exchange rate is a stochastic process which follows the stochastic differential equation

$$(2.1a) \quad dS/S = \mu_S(S,N)dt + \sigma_S(S,N)dz + gdN,$$

where the drift $\mu_S(S,N)$ is the home currency's expected rate of depreciation within the band, $dz$ is the increment of a Wiener process (that is, $E[dz] = 0$ and $\text{Var}[dz] = dt$), and $\sigma_S(S,N)$ is the instantaneous standard deviation of the rate of exchange rate depreciation within the band. The last term is the jump of 100$g$ percent when a devaluation occurs.

The exchange rate's drift and instantaneous standard deviation depend only on where in the band the exchange rate is, that is, on $s = S/a(N)$. Let us call $s$ the normalized exchange rate. The normalized exchange rate will obey $ds/s = dS/S - gdN$, hence

$$(2.1b) \quad ds/s = \mu_S(s)dt + \sigma_S(s)dz.$$

There is a representative investor with preferences given by the expected discounted utility

$$(2.2) \quad \mathbb{E}_t \int_{\tau=t}^{\infty} u(c(\tau)) \exp[-\delta(\tau-t)]d\tau, \quad \delta > 0,$$

where $u(c)$ is a standard instantaneous utility function and $c$ is real consumption. Real consumption is in turn given by a Cobb–Douglas utility function of consumption of home and foreign goods, $c_h$ and $c_f$,

$$(2.3) \quad c = c_h^{1-\beta} c_f^\beta,$$

where $\beta$, $0 < \beta < 1$, is the consumption share of foreign goods.

The Cobb–Douglas utility function results in the corresponding exact price index,

$$(2.4) \quad P = P_h^{1-\beta}(S^f)\beta = S^\beta,$$

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8 We let $o(dt)$ denote terms of order higher than $dt$, that is, $\lim_{dt\to0} o(dt)/dt = 0$. 
which will be used to deflate nominal returns. Application of Ito’s lemma and (2.1a), with special consideration of the Poisson component, gives the stochastic differential equation for the price index,

\[ dP/P = \left[ \beta \mu_S(s) - \beta(1-\beta)\sigma^2_S(s)/2 \right] dt + \beta \sigma_S(s)dz + [(1+g)^{\beta} - 1]dN. \]

The last term in (2.5), the Poisson component, is the relative change in the price index when a devaluation occurs.

There are two assets, home and foreign currency short-term bonds. Let \( B_h \) and \( B_f^* \) denote the own-currency value of the two bonds. The own-currency nominal rates of return on the two bonds are then, respectively,

\[ dB_h/B_h = i(s)dt \quad \text{and} \quad dB_f^*/B_f^* = r^*dt \]

where the home interest rate \( i(s) \) in equilibrium will depend on where in the band the exchange rate is, that is on the normalized exchange rate. The foreign currency interest rate \( r^* \) is taken to be constant.

The real values of the two bonds are

\[ b_h = B_h/P = B_h S^{-\beta} \quad \text{and} \quad b_f = S B_f^*/P = B_f^* S^{1-\beta}. \]

By Ito's lemma the real rates of return on the two bonds can then be written

\[ db_h/b_h = \mu_h(s)dt - \beta \sigma_S(s)dz + [(1+g)^{1-\beta} - 1]dN, \quad \text{where} \]

\[ \mu_h(s) = i(s) - \beta \mu_S(s) + \beta(\beta + 1)\sigma^2_S(s)/2; \]

and

\[ db_f/b_f = \mu_f(s)dt + (1 - \beta)\sigma_S(s)dz + [(1+g)^{1-\beta} - 1]dN, \quad \text{where} \]

\[ \mu_f(s) = r^* + (1 - \beta)\mu_S(s) - \beta(1 - \beta)\sigma^2_S(s)/2. \]

Let \( w_f \) be the share of wealth the investor holds in foreign bonds, and let \( w_h = 1 - w_f \) be the share of wealth held in home bonds. Real wealth \( W \) then follows the stochastic

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9 With stochastic processes that are mixed Ito and Poisson processes, Ito's lemma needs to be modified. Let \( dx/x = \mu(x)dt + \sigma(x)dz + g dN \) be a mixed Ito and Poisson process. Then Ito's lemma can be written

\[ df(x) = \left[ f_x'(x) \mu(x) + f_{xx}(x) \sigma(x)^2/2 \right] dt + f_x(x) \sigma(x)dz + [f((1+g)x) - f(x)]dN. \]
differential equation

\[
(2.9) \quad dW = \{W[\mu_h(s) + w_f(\mu_f(s) - \mu_h(s))] - c\} dt + W(w_f - \beta)\sigma_s(s)dz
\]

\[
+ W[(1+w_f g)/(1+g^\beta) - 1] dN.
\]

The first term on the right hand side follows since the expected change of real wealth absent any devaluation can be written \(dW = [W(w_h \mu_h + w_f \mu_f) - c] dt = \{W[\mu_h + w_f(\mu_f - \mu_h)] - c\} dt\), where we have used \(w_h = 1 - w_f\). The second term follows since the Brownian component of the change in wealth can be written \(dW = W[w_h(-\beta \sigma_s) + w_f(1-\beta)\sigma_s]dz = W(w_f - \beta)\sigma_s dz\). The third term is the relative change in real wealth from a devaluation. This term can be understood as follows. The nominal wealth after a devaluation is \(PW[w_h + w_f(1+g)] = PW(1 + w_f g)\). The real wealth is \(PW(1 + w_f g)/P(1+g)^\beta\), since the price index jumps from \(P\) to \(P(1+g)^\beta\). The relative change in real wealth is therefore \((1 + w_f g)/(1+g)^\beta - 1\).

3. Portfolio choice

The portfolio problem of the investor is then to choose the portfolio share of foreign bonds \(w_f\) and consumption \(c\) so as to maximize (2.2a) subject to the wealth equation (2.9), taking into account the dependence of \(\mu_h(s), \mu_f(s),\) and \(\sigma_s(s)\) on the normalized exchange rate. This is a standard portfolio problem, except that the normalized exchange rate \(s\) is a state variable that affects the expectation and the instantaneous standard deviation of the assets' real rates of return. The resulting value function will then be a function of both wealth and the state variable, \(I(W,s)\exp(-\delta t)\) (cf. Merton (1971)).

From the Bellman equation for this problems follows a first-order condition for the share of foreign bonds (see appendix for details). This first-order condition can be rewritten as the following equation for the equilibrium share of foreign bonds,

\[
(3.1) \quad w_f = \beta + \frac{\mu_f - \mu_h}{\gamma(W,s)\sigma_s^2} + \frac{I_W s(W,s)}{I_W s(W,s) \gamma(W,s)}
\]
\[
+ \frac{I_W \{ W(1 + w_f g) / (1 + g)^\beta, s \}}{I_W(W, s)} \frac{\nu g / (1 + g)^\beta}{\gamma(W, s) \sigma_S^2},
\]
where \( \gamma(W, s) = -I_{WW}(W, s) W / I_W(W, s) \) is the relative aversion to wealth risk.

The equilibrium share of foreign bonds consists of the sum of four terms. Let us write these four terms \( w_f = w_f^G + w_f^T + w_f^H + w_f^D \). Accordingly, the equilibrium portfolio of foreign and home bonds can be separated into four different portfolios. The first term on the right hand side of (3.1), \( w_f^G = \beta \), corresponds to the share of foreign bonds in a global minimum-variance portfolio, with the share of domestic bonds \( w_h^G = 1 - \beta \). This is the portfolio an infinitely risk-averse investor would choose (when \( \gamma \) approaches infinity).

With the portfolio shares of foreign and home bonds equal to the consumption shares of foreign and home goods, the variance of real wealth is minimized and equal to zero. The other three terms correspond to "speculative" portfolios of zero value. The second term is the foreign bonds' share of wealth \( w_f^T \) in a standard so-called tangency portfolio, with domestic bonds' share of wealth \( w_h^T = -w_f^T \).

The third term is the foreign bonds' share of wealth \( w_f^H \) in a so-called hedge-portfolio against movements in the state variable \( s \), with the domestic bonds' share of wealth \( w_h^H = -w_f^H \). The fourth and last term is the foreign bonds' share in wealth \( w_f^D \) in a portfolio resulting from the devaluation risk, with the domestic bonds' share of wealth \( w_h^D = -w_f^D \).\(^{10}\)

Next, we shall rewrite (3.1) in order to find an expression for the risk premium.

4. The Foreign Exchange Risk Premium

Let us define the real foreign exchange risk premium, \( \rho \), as the expected real rate of return differential between home and foreign currency bonds. Then we have from (2.8)

\(^{10}\) If the utility function \( u(c) \) is logarithmic, \( u(c) = \ln c \), we may guess (as in Merton (1971)) that the value function is of the form \( I(W, s) = A \ln W + B(s) \). Then \( I_{WS} = 0 \) and the hedge portfolio is zero. Furthermore, the relative aversion to wealth risk is unity, \( \gamma(W, s) = 1 \), and the portfolio in (3.1) has a very simple form.
\[ (4.1) \quad \rho = \mathbb{E}[d_{h}/b_{h}] - \mathbb{E}[d_{f}/b_{f}] = \{\mu_{h} + \nu[(1+g)^{-\beta} - 1]\} - \{\mu_{f} + \nu[(1+g)^{1-\beta} - 1]\} = \mu_{h} - \mu_{f} + \nu g/(1+g)^{\beta}. \]

Here the first two terms in the last line of (4.1) give the expected real rate of return differential between home and foreign currency bonds due to exchange rate movements inside the band in the absence of devaluations. The third term in the last line is the expected real rate of return differential between home and foreign currency bonds due to devaluations.

It follows from (4.1) and (3.1) that the real risk premium can be written as the sum of two terms,
\[ (4.2a) \quad \rho = \rho_{b} + \rho_{d}, \]

where the two terms are given by
\[ (4.2b) \quad \rho_{b} = [\beta + w_{f}^{H}(W,s) - w_{f}] \gamma(W,s) \sigma_{\delta}^{2}(s) \quad \text{and} \]
\[ (4.2c) \quad \rho_{d} = \frac{I_{W}[W(1+w_{f}g)/(1+g)^{\beta},s] - I_{W}(W,s)}{I_{W}(W,s)} \frac{\nu g/(1+g)^{\beta}}. \]

Here \( w_{f}^{H}(W,s) \) in (4.2b) is the foreign bonds' share of wealth in the state-variable hedge portfolio, the third term in the right-hand side of (3.1).

The real risk premium is hence the sum of two separate risk premia, \( \rho_{b} \) and \( \rho_{d} \). The risk premium \( \rho_{b} \) is due to exchange rate uncertainty within the band. It is the product of three factors. The first factor is the sum of the consumption share of foreign goods and the share of foreign bonds in the hedge portfolio, the total portfolio share of foreign bonds. The second factor is the relative aversion to wealth risk, and the third is the instantaneous variability of exchange rate depreciation within the band.

The risk premium \( \rho_{d} \) is due to the exchange rate uncertainty caused by devaluations. It is the product of two factors. The first factor is the relative jump in the marginal utility of real wealth if a devaluation occurs. The second factor is the expected real rate of return differential between home and foreign currency bonds due to devaluations.
Let us next define the nominal (home currency) foreign exchange risk premium, \( \tilde{\rho} \), as the expected nominal (home currency) rate of return differential between home and foreign currency bonds. That is, the nominal risk premium equals the interest rate differential less the expected rate of depreciation of the home currency,

\[
\tilde{\rho} = i(s) - i^* - \mu_S(s) - \nu g,
\]

where \( \mu_S(s) \) is the expected rate of depreciation of the home currency within the band, and \( \nu g \) is the expected rate of depreciation due to devaluations (the expected devaluation per unit time).

It follows from (2.8), (3.1) and (4.2) that the nominal risk premium can also be written as the sum of two terms,

\[
\tilde{\rho} = \tilde{\rho}_b + \tilde{\rho}_d,
\]

where the two terms are given by

\[
\begin{align*}
\tilde{\rho}_b &= \rho_b - \beta \sigma_S^2(s) \quad \text{and} \\
\tilde{\rho}_d &= \rho_d + \nu g[1 - 1/(1+g)] \\
&= \frac{I_W(W_1 w_f g)/(1+g) - I_W(W_2, s)}{I_W(W_2, s)} \nu g.
\end{align*}
\]

The nominal risk premium also consists of two separate risk premia, one due to exchange rate movements inside the band, and one due to exchange rate movements from devaluations. The nominal risk premium \( \tilde{\rho}_b \) is in general the corresponding real risk premium less the covariance between the rate of depreciation and the rate of inflation (see appendix). The latter term, the famous "convexity term" due to Jensen's inequality, has the simple form in (4.4b) since nominal home goods are assumed to be constant in the price index (2.4). The nominal risk premium \( \tilde{\rho}_d \) due to devaluations can be interpreted as the product of the jump in the marginal utility of nominal wealth from a devaluation and the expected nominal rate of return differential due to devaluations.\(^{11}\)\(^{12}\)

\(^{11}\) The nominal risk premium in (4.3) is, because of Siegel's paradox and the convexity term, not invariant to the currency denomination. This is so since, because of Jensen's
Let us first look at the term $\rho_b$, the real risk premium due exchange rate uncertainty within the band. We shall see that this risk premium should be very small in narrow target zones. Since the risk premium is proportional to the instantaneous variance of the rate of depreciation, let us estimate this variance, both theoretically and empirically. To get a theoretical estimate, we use the Krugman (1990) model. There, the log of the exchange rate is typically given by the function

$$\ln s(f) = f - \sinh(\lambda f)/[\cosh(\lambda f)],$$

where $\lambda = \sqrt{2/\alpha}/\sigma$, $f$ (the "market fundamental") is a regulated Brownian motion with zero drift, instantaneous standard deviation $\sigma$ and lower and upper bounds $\pm f$, and $\alpha > 0$ can be interpreted as the semi-elasticity of money demand with respect to the nominal interest rate. The instantaneous standard deviation of the rate of exchange rate depreciation is then by Ito's lemma given by

$$\sigma_S(f) = \sigma \partial \ln s/\partial f = \sigma[1 - \cosh(\lambda f)/\cosh(\lambda f)].$$

The instantaneous standard deviation is shown in Figure 1, plotted against $\ln s$. The maximum of $\sigma_S(f)$ results for the middle of the band and is given by $\sigma_S(0) = \sigma/\cosh(\lambda f)$. With the reasonable parameters $\alpha = 3$ and $\sigma = 0.1$ per year, an upper

\[\text{inequality, the expected rate of depreciation of the foreign currency is not equal to the negative of the expected rate of depreciation of the home currency: E}[d(1/S)/(1/S)] = E[2S'/S] + \text{Var}[dS'/S]. \text{ The real risk premium in (4.2) is invariant to the currency denomination. See for instance Adler and Dumas (1983, p. 955), Engel (1990), Frankel (1982, Appendix) and Sibert (1989) for further discussion of this point.}\]

\[\text{In the absence of a state variable s affecting the rates of return of the state variable, the hedge portfolio in (4.4b) is zero and the instantaneous standard deviation of the rate of exchange rate depreciation is constant. If the utility function } u(c) \text{ in (2.2) is assumed to have constant relative risk aversion } \gamma, \text{ the value function } l(W) \text{ has the same constant relative risk aversion, and expression (4.4b) simplifies to } \hat{\rho}_b = (\beta - w_f)\gamma\sigma^2_S - \beta\sigma^2_S. \text{ Similar expressions for the foreign exchange risk premium has been derived in a mean-variance framework by Dornbusch (1983) and Frankel (1982).}\]

\[\text{We recall that the hyperbolic sine and cosine fulfill } \sinh(x) = [\exp(x) - \exp(-x)]/2 \text{ and } \cosh(x) = [\exp(x) + \exp(-x)]/2.\]

\text{The Krugman model is solved under the assumption that the foreign exchange risk premium is either zero or exogenous. We only need the model to get a numerical estimate of the instantaneous standard deviation of the rate of exchange rate depreciation, and there is no reason to believe that that estimate would be very different if the assumption was not fulfilled.}\]
bound $f = 0.094$ results in an exchange rate band of $\pm 1.5$ percent, the Swedish target zone. The corresponding theoretical standard deviation $\sigma_s(0)$ is 2.4 percent per year, hence the variance $\sigma_s^2(0)$ is about $5.8 \cdot 10^{-4}$, that is 5.8 basis points per year (0.058 percent per year). Even with a relatively high risk aversion $\gamma(W, s) = 8$ and with a relatively large expression $\beta + w_f^H(W, s) - w_f = .5$, the real risk premium $\rho_b$ would be bounded by 23 basis points per year.$^{14}$

To get the nominal risk premium $\tilde{\rho}_b$ due to exchange rate uncertainty inside the band, the second term in the right-hand side of (4.4b) should be subtracted from the real risk premium. With $\beta$ less than 0.5, this second term is less than 2.9 basis points. This term may be of same order of magnitude as a small real risk premium, making the nominal risk premium very close to zero indeed.$^{15}$

The empirical standard deviation for the Swedish exchange rate index is actually smaller than the theoretical 5.8 basis points per year, which makes for an even smaller risk premium $\rho_b$. Hörgren and Vredin (1988b) report a standard deviation of monthly changes in the Swedish index for the period October 1982-May 1987 of 0.42 percent per month, that is, a variance of about $0.17 \cdot 10^{-4}$ per month and $2.12 \cdot 10^{-4}$ per year, or 2.1 basis points per year. I have computed the variance of daily exchange rate changes in the Swedish index for the years 1986-88 and found it equal to 1.6 basis points per year. For narrow target zones like the Swedish one, it seems that we can safely disregard both the real and the nominal risk premium due to exchange rate movements inside the band.

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$^{14}$ Note that additional state variables would have an effect on the risk premium only to the extent that the size of the hedge portfolio share $w_f^H$ is affected. This seems unlikely to change the upper bound on the risk premium much.

Even if the hedge portfolio share would be very large and above unity, there is obviously a considerable margin before the risk premium would be sizeable.

$^{15}$ Engel (1990) emphasizes that many models used as theoretical frameworks for empirical studies of the foreign exchange risk premium include assumptions that (absent devaluation risks) imply that the real risk premium is zero. This makes the nominal risk premium simply identical to the negative of the covariance between the rate of depreciation and the rate of inflation and completely unrelated to any risk aversion.
Let us next look at the term $\rho_d$, the real risk premium due to the devaluation risk. Let us assume that the elasticity of the marginal utility of wealth, that is, the relative aversion to wealth risk, is fairly stable, and that it can be approximated by a constant $\gamma > 0$, that is, $\gamma(W,s) = \gamma$. Then $I_W(W,s)$ can be approximated by $A(s)W^{-\gamma}$ and $\rho_d$ can be approximated by

$$\rho_d = \frac{(1 + g)^{\beta\gamma}}{(1 + w_f g)^{\gamma}} - 1 \nu g/(1+g)^{\beta}. \tag{4.6}$$

Let us look at the ratio of this risk premium to the expected rate of depreciation due to devaluations,

$$\rho_d / \nu g = \frac{(1 + g)^{\beta\gamma}}{(1 + w_f g)^{\gamma}} - 1)/(1+g)^{\beta}. \tag{4.7}$$

This ratio does not depend on $\nu$, the probability per unit time of a devaluation. It depends on size of the devaluation $g$, the relative risk aversion $\gamma$, the consumption share of foreign goods $\beta$ and the total share of foreign bonds $w_f$. The ratio is increasing in $\beta$ (for positive $g$ and $\gamma$ larger than unity) and decreasing in the share of foreign bonds (for positive $g$). In order to get a generous upper bound for the ratio, let us use $\beta = .5$ as a very high consumption share of foreign goods, and $w_f = .25$ and 0 as a low and very low share of foreign bonds. The ratio $\rho_d / \nu g$ is plotted in Figure 2 (for $w_f = .25$) and Figure 3 (for $w_f = 0$), for devaluation sizes $g$ between plus and minus 20 percent, and for relative risk aversion $\gamma$ equal to 2, 4, and 8.

Consider a devaluation of 10 percent. We see in Figure 2 (with the more reasonable $w_f = .25$) that then the ratio of the risk premium to the expected depreciation is between 0.05 for relative risk aversion equal to 2 and about 0.2 for relative risk aversion equal to 8. Suppose the probability of a devaluation is 100 percent per year, so the expected rate of depreciation is 10 percent per year. Then the risk premium is between 0.5 and 2 percent per year.

Consider the more extreme case $w_f = 0$ in Figure 3. For a 10 percent devaluation the
ratio of the risk premium to the expected depreciation is between 0.1 for risk aversion equal to 2 and 0.45 for risk aversion equal to 8. With an expected depreciation equal to 10 percent per year, the risk premium is between 1 and 4.5 percent per year.

The ratio of the nominal risk premium due to devaluation risks to the expected rate of devaluation is given by

\[ \tilde{\rho}_d / \nu g = \frac{(1 + g)^{\beta(\gamma-1)}}{(1 + w_f g)^\gamma} - 1. \]  

(4.8)

This ratio is plotted in Figure 4, for the case with \( w_f = 0.25 \). From a comparison with Figure 2 it appears that the nominal risk premium due to devaluations is less in magnitude than the corresponding real risk premium.

Except for unlikely extreme cases, it seems that both real and nominal devaluation risk premia should still be relatively small. For Sweden, with observed interest rate differentials between 1 and 5 percent (possibly due to an expected positive rate of devaluation), the devaluation risk premia are in all likelihood rather small, and it seems that they could hardly exceed 1 percent.

What about sudden exchange rate jumps within the band? Could expectations of such jumps generate significant risk premia? Consider the Swedish band, \( \pm 1.5 \) percent. If the Krona is at its strong edge, there is room for a sudden one-time depreciation of 3 percent within the band. Expectations of such an imminent depreciation could generate considerable short term interest rate differentials: Suppose such a depreciation is expected with 100 percent probability within one month. This corresponds to an interest rate differential of \( 1.03^{12} - 1 = 43 \) percent per year for one-month interest rates, without any risk premium. (The contribution to the twelve-month interest rate differential from this expected one-time depreciation would only be 3 percent.) What would the risk premium be? In order to use our framework, let us modify the experiment to involve repeated Poisson devaluations of 3 percent size, with a probability of 100 percent per month, that
is, a probability of 1200 percent per year. Then the expected depreciation is 36 percent per year. In Figures 2 and 3 we can find the ratio of the risk premium to this expected depreciation: For instance, in Figure 3 with the extreme case \( w_f = 0 \), for \( \gamma \) equal to 8 and \( g \) equal to 3 percent, the ratio is about 0.12. Hence, of this large expected depreciation of 36 percent per year, only about 4.3 percent per year would constitute the risk premium. I conclude that the risk premium should be small in relation to observed interest rate differentials also for expected sudden depreciations within the band.\(^{17}\)

5. Conclusions

Using Merton's (1971) model of portfolio choice with rates of return depending upon state variables and being mixed Brownian and Poisson processes, we have shown that risk premia arising from exchange rate movements inside narrow bands are insignificant. Risk premia arising from devaluation risks may be considerably larger, but are still relatively small in comparison with the expected rate of devaluation.

We immediately conclude that the practice in the new literature on target zones to rely on uncovered interest rate arbitrage and disregard the risk premium seems warranted, at least for narrow target zones.

We also conclude that if sterilized interventions have effects in narrow target zones, it cannot be through the portfolio effect as modeled here. Any effects must be signaling effects on expectations, for instance as implicit threats of future nonsterilized interventions, or possibly portfolio effects modeled in some other way.

We finally conclude that any monetary independence in narrow target zones cannot be explained by risk premia as modeled here, except possibly in connection with large

\(^{16}\) Note that the jump probability per unit of time is a *rate*, hence nothing prevents it from being above 100 percent per unit of time.

\(^{17}\) Perraudin (1990) develops a model of a completely credible target zone, where the fundamental is a mixed reflected Brownian and jump process.
expected rates of devaluation. Observed interest rate differentials must be interpreted as normally arising wholly from expected currency depreciation, from exchange rate movements inside the band and from devaluations. Alternatively, there may be risk premia that must be modeled in some other way than here.

Could there be risk premia for other reasons than those modeled here? Market imperfections, regulations, institutional practice, transactions costs and costs of adjustment are of course possible candidates. A related possibility is that the instantaneous portfolio stock equilibrium underlying the Merton (1971) model is a misleading description of the foreign exchange market, and that actual foreign exchange markets are better described as thinner markets in flows (cf. Kouri (1984)), where sterilized interventions may have at least short run effects. The existence of specific agents in foreign exchange markets with large positive foreign exchange positions would contribute to larger risk premia. Note that large negative foreign exchange positions would contribute to negative risk premia, though. Non-standard preferences, for instance giving large negative weights to home-currency measured losses on foreign investments, have also been suggested as reasons for larger risk premia. Further both theoretical and empirical research seems necessary in order to clarify these issues.

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18 For instance, specifics in corporations' and banks' accounting practices may reduce the amount of interest arbitrage. See Hedman (1986) for a report on some special aspects of foreign exchange management by Swedish corporations.

19 It is interesting that non-standard choice theories can give rise to "first-order risk aversion" rather than the standard "second-order risk aversion" (cf. Duffie and Epstein (1990) and Epstein and Zin (1989)). In our context, first-order risk aversion would presumably imply that the risk premium from exchange rate movements inside the band would be proportional to the instantaneous standard deviation rather than the instantaneous variance of the rate of depreciation, as in (4.2b). Recall that one empirical estimate of the instantaneous variance of the Krona rate of depreciation was 1.6 basis points per year. The corresponding standard deviation is about 1.3 percent per year, a very different order of magnitude. As far as I know the treatment of portfolio choice under these new non-standard choice theories has not yet advanced to the stage where the foreign exchange risk premium can be rigorously derived.
Appendix

A.1 The Bellman Equation

The Bellman equation for the portfolio problem is

\[
0 = \max_{(c, w_f)} \left\{ U(c) - \delta l(W, s) + I_W(W, s) \left\{ W[\mu_h + w_f(\mu_f - \mu_h)] - c \right\} 
+ I_{WW}(W, s) W^2 \sigma_S^2 (\beta - w_f)^2 / 2 + I_s(W, s) s \mu_S + I_{ss}(W, s) s^2 \sigma_S^2 
- I_{Ws}(W, s) W(\beta - w_f) s \sigma_S^2 + \nu I(W(1 + w_f)/(1 + g)^{\beta}, s) \right\},
\]

where \(\mu_h, \mu_f, \mu_S\) and \(\sigma_S\) are functions of \(s\).

The first-order condition for the share of foreign bonds is

\[
0 = I_W(W, s) W(\mu_f - \mu_h) - I_{WW}(W, s) W^2 \sigma_S^2 (\beta - w_f) 
+ I_{Ws}(W, s) W s \sigma_S^2 + I_W(W(1 + w_f)/(1 + g)^\beta, s) W v_g/(1 + g)^\beta.
\]

A.2 Stochastic Own-Currency Goods Prices

Suppose the own-currency prices of home and foreign goods follow geometric Brownian motions,

\[
dP_h / P_h = \mu_P dt + \sigma_P dz_P,
\]

and

\[
dP_f / P^*_f = \mu_{P^*} dt + \sigma_{P^*} dz_{P^*}.
\]

From Ito's lemma follows that the price index (2.4) obeys

\[
dP / P = \mu_P(s) dt + \sigma_P(s) dz_P + [(1 + g)^\beta - 1] dN, \quad \text{where}
\]

\[
\mu_P(s) = \beta \mu_S(s) + (1 - \beta) \mu_{P h} + \beta \mu_{P^*} + \beta(1 - \beta) [\sigma_S^2(s) + \sigma_P^2(s) + \sigma_{P^*}^2(s)]
+ \beta(1 - \beta) \sigma_{SP h}(s) + \beta^2 \sigma_{SP^*}(s) + \beta(1 - \beta) \sigma_{P h} \sigma_{P^*} \quad \text{and}
\]

\[
\sigma_P dz_P = \beta \sigma_S dz + (1 - \beta) \sigma_{P h} dz_{P h} + \beta \sigma_{P^*} dz_{P^*}.
\]

Here \(\sigma_{SP h}(s)\) denotes the instantaneous covariance between the rate of depreciation and the rate of change of the home currency price of home goods, etc.
The real rates of return on home and foreign bonds will be given by

(A.5a) \[ \frac{db_h}{b_h} = \mu_h(s) dt + \sigma_{h}(s) dz_h + [(1+g)^{-\beta} - 1]dN \]
and

(A.5b) \[ \frac{db_f}{b_f} = \mu_f(s) dt + \sigma_f(s) dz_f + [(1+g)^{1-\beta} - 1]dN; \text{ where} \]

(A.6a) \[ \mu_h(s) = \bar{\mu}(s) - \mu_f(s) + \sigma_f^2(s), \]

(A.6b) \[ \mu_f(s) = \bar{\mu}(s) - \mu_f(s) + \sigma_f^2(s), \]

(A.6c) \[ \sigma_{h}(s) dz_h = - \sigma_f(s) dz_P \text{ and} \]

(A.6d) \[ \sigma_f(s) dz_f = \sigma_s(s) dz - \sigma_f(s) dz_P. \]

It follows that the fourth term on the right-hand side of the Bellman equation (A.1) will be replaced by

(A.7) \[ I_{W W}(W,s) W^2 \left[ \sigma_P^2 - 2w_f \sigma_{SP} + w_f^2 \sigma_S^2 \right]. \]

Here \( \sigma_P^2 \) is the instantaneous variance of the rate of inflation and \( \sigma_{SP} \) is the instantaneous covariance between the rate of depreciation and the rate of inflation. They are given by

(A.8a) \[ \sigma_P^2 = \beta^2 \sigma_S^2(s) + (1-\beta)^2 \sigma_{P h}^2 + \beta^2 \sigma_{P f}^2 + 2\beta(1-\beta) \sigma_{SP h}(s) \]

\[ + 2\beta^2 \sigma_{SP f}(s) + 2\beta(1-\beta) \sigma_{P f P f}(s) \text{ and} \]

(A.8b) \[ \sigma_{SP} = \beta \sigma_S^2(s) + (1-\beta) \sigma_{SP h}(s) + \beta \sigma_{SP f}(s). \]

The second term in the first-order condition (A.2) will be replaced by

(A.9) \[ - I_{W W}(W,s) W^2(\sigma_{SP} - w_f \sigma_S^2). \]

The only change in (3.1) is that the first term on the right-hand side, the consumption share of foreign goods \( \beta \), is replaced by the share of foreign bonds in the new global minimum-variance portfolio,

(A.10) \[ w_{f}(s) = \sigma_{SP}(s)/\sigma_S^2(s). \]

With this replacement, the real risk premium is still given by (4.2). The nominal risk premium is still given by (4.4), except that the term \( \beta \sigma_S^2(s) \) in (4.4b) is replaced by \( \sigma_{NP}(s) \), the instantaneous covariance between the rate of depreciation and the rate of inflation.
References


Dumas, Bernard (1989), "Pricing Physical Assets Internationally: A Non Linear


