

Seminar Paper No. 492

DUAL LABOR MARKETS, EFFICIENCY  
WAGES, AND SEARCH

by

James W. Albrecht and Susan B. Vroman



INSTITUTE FOR INTERNATIONAL ECONOMIC STUDIES  
Stockholm University

ISSN 0347-8769

**Seminar Paper No. 492**

**Dual Labor Markets, Efficiency Wages, and Search**

**by**

**James W. Albrecht and Susan B. Vroman**

Seminar Papers are preliminary material circulated to stimulate discussion and critical comment.

June 1991

Institute for International Economic Studies  
S-106 91 Stockholm  
Sweden

Dual Labor Markets, Efficiency Wages, and Search\*

James W. Albrecht and Susan B. Vroman

Department of Economics  
Georgetown University  
Washington, DC 20057  
(202) 687-6105

Revised, June 1991

## 1. Introduction

In this paper we present an equilibrium model of a dual labor market in which firms endogenously separate into two sectors, even though the firms are identical *ex ante*. In the primary sector effort requirements are high, jobs are imperfectly monitored, and workers are commensurately rewarded, while in the secondary sector effort requirements are low, only minimal monitoring is necessary, and workers are poorly paid.

The dual labor market paradigm, ie, the idea that the labor market can be thought of, to a useful first approximation, as being divided into a primary and a secondary sector, enjoyed popularity in the late 1960's and early 1970's (eg, Doeringer and Piore [1971]). There has been a recent resurgence of empirical interest in dual labor markets, eg, Dickens and Lang [1985]. This empirical revival has its roots in theoretical developments; in particular, in the recognition that an efficiency wage model might provide a theoretical underpinning for dual labor markets.

Bulow and Summers [1986] constructed an efficiency wage theory of dual labor markets. In their model the technology associated with each job is specified exogenously. Assuming the existence of two types of jobs (secondary sector jobs, which are menial and require no supervision, and primary sector jobs in which supervision is required), they used efficiency wage theory to explain how an outcome in which equally skilled workers are paid different wages can persist as an equilibrium.

Our approach is complementary to that of Bulow and Summers. Rather than exploring the implications of an assumed technological duality, we instead generate that duality as an equilibrium outcome. This makes our model more in the spirit of the institutionalist dual labor market literature, which emphasizes the endogeneity of technology choice. In addition, our approach addresses an obvious question that is begged by Bulow and Summers, namely, why the labor market should separate into two as opposed to "many" or a continuum of sectors. That is, our approach provides a microfoundation for an assumption that has proven useful in applications; namely, that the labor market can be viewed as divided into two technologically distinct sectors.

The key assumption behind our dual labor market outcome is a nonconvexity in the monitoring technology. It is this nonconvexity that allows firms, all identical in the sense of having access to the same technology, to be indifferent in equilibrium between offering the two job types. Specifically, we assume that a firm can observe costlessly whether its workers are exerting effort at an exogenously specified minimum level,

but that effort above the minimum level can be monitored only imperfectly and at a cost. This is consistent with intuition: one can observe costlessly whether the receptionist is answering the telephone; observing whether he or she is helpful to those who call requires the expenditure of time and effort.

Like Bulow and Summers, we use the Shapiro and Stiglitz [1984] model as our starting point. However, our setup differs from Shapiro/Stiglitz and Bulow/Summers in two significant respects. First, we allow for heterogeneity among workers. Workers are not assumed to be heterogeneous with respect to their abilities since we want to preserve the Bulow and Summers outcome that in equilibrium equally skilled workers are paid different wages on different jobs. Instead, we assume workers differ according to the value placed on leisure. This assumed heterogeneity leads to an equilibrium with two realistic properties: not all workers will accept secondary sector jobs, and some primary sector workers will shirk. Second, the equilibrium concept used in our model owes more to the equilibrium search literature than it does to the efficiency wage literature. Unlike the standard efficiency wage model in which wages are determined by equating supply and demand in efficiency units, our equilibrium is explicitly Nash. Wages and effort requirements are set optimally by firms in conjunction with optimal job acceptance and effort decisions by workers.

The outline of the rest of the paper is as follows. The next section sets out the decision problem faced by workers. Workers differ according to the value of leisure, which is measured along a continuum. A worker has two choices to make. First, if unemployed and offered a job, he or she must decide whether to accept the job or remain unemployed. Second, if employed on a job with an effort requirement, the worker must decide whether to shirk or work.

Our analysis of worker decisions is used in Section 3 to characterize the probabilities entering the firm's decision problem, namely, the probability that its job offer will be accepted and the probability that a worker, having accepted the job, will meet its effort requirement. These probabilities depend on the distribution of wage/effort requirement packages extant in the market and on the distribution of worker types across the unemployed.

The firm's decision problem is discussed in Section 4. Firms are viewed as collections of independent jobs or "work stations." The revenue that a firm derives from a job is proportional to the effort of the worker

occupying it. A firm can observe costlessly whether its workers are exerting effort at an exogenously specified minimum level, but effort above this minimum cannot be monitored perfectly. The monitoring technology is exogenous: the cost of monitoring and the rate at which shirkers are detected are parameters of the firm's problem. Workers who are monitored and found to be putting forth less than the required level of effort are fired. The firm's problem is to decide what wage to offer, whether to incur monitoring costs or not, and if it does, what effort requirement to set. A firm that requires only the minimum effort level is a secondary sector firm, while a firm that chooses to incur monitoring costs is a primary sector firm.

In Section 5 we construct the equilibrium of our model. An equilibrium is a distribution of wages and effort requirements across vacant jobs together with a corresponding induced distribution of individual types across unemployed workers. We focus on symmetric equilibria in which primary sector firms offer a common wage/effort requirement package and secondary sector firms offer a common wage. Equilibrium is then characterized by four variables: the wage for secondary sector firms, the wage and effort requirement for primary sector firms, and the fraction of vacancies arising in the primary sector. Three types of symmetric equilibria can arise: (i) pure secondary sector equilibria in which all firms set the effort requirement at the minimum level, (ii) pure primary sector equilibria in which all firms incur monitoring costs, and (iii) dual labor market equilibria in which both types of behavior are optimal. We prove the existence of symmetric equilibria and show how dual labor market equilibria can arise.

Section 6 characterizes dual labor market equilibria. The occurrence and nature of dual labor market outcomes depend on the parameters of the production and monitoring technologies. For reasonable values of these parameters, primary sector workers will be paid more than secondary sector workers but will be subject to an effort requirement above the secondary sector minimum. There will be rationing in the sense that secondary sector workers would prefer to be in primary sector jobs. Our equilibria thus exhibit some of the key features described in the dual labor market literature.

In sum, we show that efficiency wage and search considerations can produce an economy in which firms having access to the same technology ex ante choose to produce output in two distinct sectors. Secondary sector

firms offer lower wages and require only minimal effort, while firms in the primary sector incur monitoring costs and pay a higher wage in order to elicit greater productivity. This result is discussed in the final section.

## 2. Workers

We begin with the decision problem faced by workers. This problem has two aspects. First, if unemployed and offered a job paying a wage of  $w$ , providing a nonpecuniary benefit of  $\xi$ ,<sup>1</sup> and requiring an effort of  $e$ , ie, a " $(w, \xi, e)$  job," does the worker accept or reject the offer? Second, if the offer is accepted, does the worker shirk or meet the effort requirement? The decision rules determining these choices vary with the worker's value of leisure, which is the worker's private information. The value of leisure,  $\theta$ , varies across workers according to the continuously differentiable density,  $f(\theta)$ . This density and  $g(\xi)$ , likewise a continuously differentiable density, are the fundamental exogenous elements of our model and are assumed to be common knowledge.

We examine worker behavior using three value functions: (i) the value of meeting the effort requirement on a  $(w, \xi, e)$  job, (ii) the value of shirking on a  $(w, \xi, e)$  job, and (iii) the value of unemployment. These value functions vary with the worker's type,  $\theta$ . We establish four results in this section. First, we prove that the worker's problem is well-posed, ie, we verify that the three value functions are defined by contractions. Second, we show that the decision to accept or reject a job offer is independent of the job's effort requirement; ie, the job acceptance decision depends only on  $w$  and  $\xi$  and on the worker's type,  $\theta$ . Third, for given  $w$  and  $\theta$ , there is a critical value of  $\xi$  such that jobs offering  $w$  with an associated  $\xi$  greater than or equal to this critical value are accepted by workers of type  $\theta$ ; jobs with a lower  $\xi$  are rejected. We show that this critical value is continuously differentiable, decreasing in  $w$ , and increasing in  $\theta$ . Finally, for given  $w$ ,  $e$ , and  $\theta$ , there is an analogous critical value of  $\xi$  for the work/shirk decision. A worker of type  $\theta$  meets the effort requirement on a  $(w, \xi, e)$  job if  $\xi$  is greater than or equal to this critical value; otherwise the worker shirks. We show that this critical value is continuously differentiable in its arguments, decreasing in  $w$ , and increasing in  $e$  and  $\theta$ .

---

<sup>1</sup>Upon receiving a job offer, the worker discovers the value of its nonpecuniary component,  $\xi$ . The random variable  $\xi$  is iid across matches, is independent of the worker's value of leisure, and its realization is the worker's private information.

The details are as follows. Workers live forever. Time is continuous, and the future is discounted at the rate  $r$ . Utility is derived from the rate at which income and nonpecuniary benefits are received and disutility from the rate at which effort is expended. The rate at which effort is supplied is a choice variable, bounded below by the minimum level, which we normalize to 1;<sup>2</sup> the effort level is zero for the unemployed. A worker employed on a  $(w, \xi, e)$  job and meeting the effort requirement enjoys an instantaneous utility of  $w + \xi - e$ ; ie, the worker's utility in an interval of time of length  $\Delta t$  is given by  $[w + \xi - e]\Delta t + o(\Delta t)$ . If the worker shirks on that job, his or her instantaneous utility is given by  $w + \xi - e^*$ , where  $e^* < e$ . Finally, unemployment generates an instantaneous utility of  $\theta$ .

A worker on a  $(w, \xi, e)$  job must decide what effort rate to supply. A worker not meeting the effort requirement faces a separation risk of  $\mu$ ; ie, the probability of a separation in an interval of time of length  $\Delta t$  equals  $\mu\Delta t + o(\Delta t)$ . This separation risk is independent of how far below the requirement the worker's effort falls; so, if a worker decides to shirk, he or she never exerts more than the minimum level of effort, ie,  $e^* = 1$ . A worker meeting the effort requirement suffers a separation risk of  $\delta < \mu$ .

The worker's effort choice is then simply one of whether or not to shirk. The value of not shirking on a  $(w, \xi, e)$  job is:

$$V_N(w, \xi, e; \theta) = \frac{1}{1+r\Delta t} [(w+\xi-e)\Delta t + \delta\Delta t U(\theta) + (1-\delta\Delta t)V_N(w, \xi, e; \theta) + o(\Delta t)].$$

The nonshirking worker gets an instantaneous utility of  $(w+\xi-e)\Delta t + o(\Delta t)$ . With probability  $\delta\Delta t + o(\Delta t)$  the worker loses the job, in which case he or she becomes unemployed with associated value  $U(\theta)$ ; otherwise the value  $V_N(w, \xi, e; \theta)$  is retained. Rearranging, dividing through by  $\Delta t$ , and taking the limit as  $\Delta t \rightarrow 0$  yields:

$$(1) \quad V_N(w, \xi, e; \theta) = \frac{w+\xi-e}{r+\delta} + \frac{\delta}{r+\delta} U(\theta).$$

The corresponding value of shirking is:

$$(2) \quad V_S(w, \xi; \theta) = \frac{w+\xi-1}{r+\mu} + \frac{\mu}{r+\mu} U(\theta).$$

The job's effort requirement does not enter into  $V_S(\cdot)$ . Note that if the firm sets  $e = 1$ , the shirk/no-shirk distinction disappears. The separation risk is  $\delta$ , and the value of having the job is given by (1) with  $e = 1$ .

Next, consider an unemployed worker. Suppose job offers arrive at the

---

<sup>2</sup>We also assume that there is an upper bound on effort. This natural assumption ensures that the set of policies open to the firm is compact, as will be required by our existence proof.



rate  $a$ . This arrival rate is exogenous and reflects the underlying matching technology. Then the value of unemployment to a worker of type  $\theta$  is:

$$(3) \quad U(\theta) = \frac{\theta}{r+a} + \frac{a}{r+a} \text{E}_{\max}[A(w, \xi, e; \theta), U(\theta)],$$

where

$$A(w, \xi, e; \theta) = \max[V_N(w, \xi, e; \theta), V_S(w, \xi; \theta)].$$

The expectation in (3) is taken with respect to the joint distribution of wages and effort requirements across all vacancies,  $H(w, e)$ , and with respect to the distribution of  $\xi$  across all matches,  $G(\xi)$ . The value of unemployment to a worker of type  $\theta$  thus depends on all the  $(w, e)$  offers extant in the market. Since  $V_N(\cdot)$  and  $V_S(\cdot)$  depend on  $U(\cdot)$ , they also depend on  $H(w, e)$  and  $G(\xi)$ .

We now verify that the worker's decision problem is well-defined.

**Proposition 1:** For any joint distribution  $H(w, e)$  across vacancies coupled with any  $G(\xi)$  and for any offer arrival rate  $a$ , there exist unique value functions  $V_N(w, \xi, e; \theta)$ ,  $V_S(w, \xi; \theta)$ , and  $U(\theta)$ .

**Proof:** Given in the Appendix.

The expression for  $U(\theta)$  given by (3) incorporates the worker's job acceptance decision: a  $(w, \xi, e)$  job is accepted iff  $A(w, \xi, e; \theta) \geq U(\theta)$ . We now show:

**Proposition 2:** The job acceptance decision is independent of the job's effort requirement.

**Proof:** Given in the Appendix.

A worker accepts a job if either  $V_N(w, \xi, e; \theta) \geq U(\theta)$  or  $V_S(w, \xi; \theta) \geq U(\theta)$ . Since the condition  $V_S(w, \xi; \theta) \geq U(\theta)$  is more easily satisfied than  $V_N(w, \xi, e; \theta) \geq U(\theta)$ , the acceptance decision is determined only by  $w$  and  $\xi$ .

The intuition for this result is as follows. Consider a worker on the accept/reject margin. If the worker accepts the job, it is a matter of indifference to the worker whether the job is retained or lost. He or she therefore has no incentive to put forth the required effort.<sup>3</sup>

The "acceptance condition" (AC) is thus:

$$(4) \quad \text{AC: } w + \xi \geq rU(\theta) + 1.$$

Let  $\xi_A(w, \theta)$  be defined by:

---

<sup>3</sup>This is similar to the "Dougal result," established in Burdett and Mortensen [1980]. They show that the layoff risk on a marginally acceptable job does not influence the chance that such a job is accepted.

$$(5) \quad \xi_A = rU(\theta) + 1 - w;$$

that is,  $\xi_A(w, \theta)$  is the acceptance value of  $\xi$  for a person of type  $\theta$  considering a  $(w, \xi, e)$  job. The job is accepted if  $\xi \geq \xi_A$  and rejected otherwise. The properties of  $\xi_A(w, \theta)$  are crucial for the acceptance probability entering the firm's problem.

Proposition 3: The critical value  $\xi_A(w, \theta)$  is continuously differentiable, decreasing in  $w$ , and increasing in  $\theta$ .

Proof: Given in the Appendix.

The second aspect of the worker's decision problem, the shirk/no-shirk decision, is characterized by an analogous critical value. The "no-shirk condition" (NSC) is  $V_N(w, \xi, e; \theta) \geq V_S(w, \xi; \theta)$ ; or,

$$(6) \quad \text{NSC: } w + \xi \geq rU(\theta) + 1 + \left(\frac{r+\mu}{\mu-\delta}\right)(e-1).$$

Let  $\xi_N(w, e, \theta)$  be defined by:

$$(7) \quad \xi_N = rU(\theta) + 1 - w + \left(\frac{r+\mu}{\mu-\delta}\right)(e-1).$$

The critical value,  $\xi_N(w, e, \theta)$ , is the value of  $\xi$  with the property that an individual of type  $\theta$  is indifferent between meeting the effort requirement on a  $(w, \xi, e)$  job and shirking.

We also need to examine the properties of this second critical value, as this value is key to the second probability entering the firm's decision, the probability that a worker who accepts the job will meet its effort requirement.<sup>4</sup>

Proposition 4: The critical value  $\xi_N(w, e, \theta)$  is continuously differentiable in its arguments, decreasing in  $w$ , and increasing in  $e$  and  $\theta$ .

Proof: Given in the Appendix.

### 3. The Acceptance and No-Shirk Probabilities

In the preceding section we developed two critical values to characterize worker decision rules. In this section we use these critical values to develop expressions for the probabilities entering the firm's

---

<sup>4</sup>Since  $\xi_N(w, e, \theta) > \xi_A(w, \theta)$  for all  $e > 1$ , the shirk/no-shirk and acceptance decisions are independent in the sense that the unconditional probability that a worker will meet the effort requirement on an offered job is the same as the probability that he or she will meet that requirement conditional on acceptance.

problem. Let  $q(w)$  denote the probability that an applicant accepts a job offering a wage of  $w$ , and let  $p(w,e)$  be the probability that an applicant who accepts the job meets its effort requirement. The acceptance probability is given by one minus the distribution function of  $\xi$  evaluated at  $\xi_A(w, \theta)$

and integrated against the distribution of  $\theta$  among the unemployed, ie,

$$(8) \quad q(w) = \int [1 - G(\xi_A)] f_U(\theta) d\theta = \int \{1 - G[rU(\theta) + 1 - w]\} f_U(\theta) d\theta.$$

The no-shirk probability is given by the same expression, evaluated at  $\xi_N(w, e, \theta)$  and similarly integrated against  $f_U(\theta)$ . That is,

$$(9) \quad p(w, e) = \int \{1 - G[rU(\theta) + 1 - w + (\frac{r+\mu}{\mu-\delta})(e-1)]\} f_U(\theta) d\theta.$$

Note that the acceptance probability is a function of  $w$  alone, while the no-shirk probability depends on both  $w$  and  $e$ . Proposition 2 established that the effort requirement attached to the job offer is irrelevant to the acceptance decision; thus  $e$  is not an argument of the acceptance probability. The applicant's two decisions depend on  $\xi$ . However,  $q(w)$  and  $p(w, e)$  are probabilities viewed from the firm's point of view, and  $\xi$  is not under the firm's control; indeed, the nonpecuniary component is the worker's private information.

A key point to note in deriving  $q(w)$  and  $p(w, e)$  is that the density function of  $\theta$  among the unemployed,  $f_U(\theta)$ , and the corresponding population density,  $f(\theta)$ , are not the same. Individuals with higher values of  $\theta$  are overrepresented among the unemployed since they are more likely to shirk (and be fired) and less likely to accept a given job. By definition:

$$(10) \quad f_U(\theta) \equiv P[\theta = \theta | \text{unemployed}].$$

By Bayes Rule:

$$(11) \quad f_U(\theta) = P[\text{unemployed} | \theta = \theta] \cdot P[\theta = \theta] / P[\text{unemployed}]$$

$$= u(\theta) f(\theta) / u,$$

where  $u(\theta)$  is the  $\theta$ -specific unemployment rate and  $u = \int u(\theta) f(\theta) d\theta$  is the overall unemployment rate.

The derivation of  $u(\theta)$  is given in the Appendix. We find that:

$$(12) \quad u(\theta) = \mu\delta / \{\mu\delta + a\mu p^*(\theta) + a\delta[q^*(\theta) - p^*(\theta)]\},$$

where

$$q^*(\theta) = \int \{1 - G[\xi_A(w, \theta)]\} dH(w, e) \quad \text{and} \quad p^*(\theta) = \int \{1 - G[\xi_N(w, e, \theta)]\} dH(w, e).$$

We now have all the elements of the contaminated distribution of  $\theta$  among the unemployed, and can show:

Proposition 5: The density  $f_U(\theta)$  is continuously differentiable in  $\theta$ .

Proof: Given in the Appendix.<sup>5</sup>

As a consequence of Propositions 3 and 5, we have the following result:

Proposition 6: The acceptance probability  $q(w)$  is continuously differentiable and increasing.

Likewise, as a consequence of Propositions 4 and 5, we have:

Proposition 7: The no-shirk probability  $p(w,e)$  is continuously differentiable in both its arguments, increasing in  $w$ , and decreasing in  $e$ .

Proposition 7 shows that our model has a standard efficiency wage property, namely, higher wages reduce shirking.

#### 4. Jobs

A firm must decide what wage to offer, whether to incur monitoring costs or not, and, if it does incur monitoring costs, what effort requirement to set. We first set up the maximization problem for a secondary sector firm; then we consider the analogous problem for a primary sector firm. The choice between the sectors is determined by comparing the values associated with the corresponding strategies. In this section we set out the firm's decision problem and verify that it is well-defined.

A firm consists of a large number of jobs. A job is either occupied or vacant and entails a fixed cost at the rate  $c$ , whether occupied or vacant. If occupied, output equals a constant  $\gamma$  times the effort of the worker in the job. Effort is the only input. There is independence across jobs in the sense that a firm's aggregate output is just the sum of the outputs of its jobs. This means that we can treat the job as the basic unit of analysis. All jobs are identical ex ante, but production may vary ex post across jobs because different  $(w,e)$  packages in conjunction with the randomly drawn  $\xi$  can elicit variations in worker effort. Entry and exit costs are assumed to be zero; thus, new jobs will be created if the value of a vacancy is positive and eliminated if this value is negative.

The minimum effort level, interpreted as "showing up for work," can be

---

<sup>5</sup>The nonpecuniary component plays an essential role in the proof of Proposition 5. Without a continuously distributed  $\xi$ ,  $f_U(\theta)$  would not be differentiable. Then the firms' payoff functions would fail to be concave in the wage and effort requirement, and a symmetric equilibrium would fail to exist. This point is discussed in the context of equilibrium search theory in Albrecht and Vroman [1991].

perfectly monitored in all occupied jobs. If a firm chooses to require only the minimum effort level, it need only decide upon a wage offer. A wage offer of  $w$  will be accepted with probability  $q(w)$ . If the wage offer is accepted, then the value to the firm of having a worker in that job is:

$$R(w) = \frac{1}{1+r\Delta t}[(\gamma-w-c)\Delta t + \delta\Delta t\Pi + (1-\delta\Delta t)R(w) + o(\Delta t)].$$

This value is the sum of the instantaneous return,  $\gamma-w-c$ , realized over the interval of time  $\Delta t$ , and the future value. With probability  $1-\delta\Delta t+o(\Delta t)$  the firm retains the worker and the associated value  $R(w)$ . Otherwise, the firm loses the worker, and the job becomes vacant with associated value  $\Pi$ .

Passing to the limit in the usual way gives:

$$(13) R(w) = \frac{\gamma-w-c}{r+\delta} + \frac{\delta}{r+\delta}\Pi.$$

With probability  $1-q(w)$  the firm's wage offer is rejected. In this case the firm retains the value of a vacancy,  $\Pi$ . Thus, a firm that requires only the minimum effort level chooses  $w$  to solve:

$$(14) \max_w q(w)R(w) + [1-q(w)]\Pi.$$

Alternatively, a firm can attempt to achieve a higher level of effort on its jobs. In order to elicit a level of effort above the minimum, a firm incurs a fixed monitoring cost at the rate  $m$  while the job is occupied. This enables the firm to detect shirking, albeit not necessarily immediately.<sup>6</sup> As discussed above, a shirker suffers a separation risk of  $\mu > \delta$ .

Consider a firm incurring monitoring costs, offering a wage of  $w$ , and imposing an effort requirement of  $e$ . The firm's wage offer is accepted and the effort requirement is met with probability  $p(w,e)$ . With probability  $q(w) - p(w,e)$  the job is accepted and the worker shirks. Finally,  $1 - q(w)$  is the probability that the firm's offer is rejected.

The value of having a nonshirker on a  $(w,e)$  job is:

$$(15) N(w,e) = \frac{\gamma e - w - c - m}{r+\delta} + \frac{\delta}{r+\delta}\Pi;$$

the value of having a shirker on the same job is:

$$(16) S(w) = \frac{\gamma - w - c - m}{r+\mu} + \frac{\mu}{r+\mu}\Pi.$$

The maximization problem faced by such a firm is thus:

$$(17) \max_{w,e} p(w,e)N(w,e) + [q(w)-p(w,e)]S(w) + [1-q(w)]\Pi.$$

Finally, consider a vacant job. Suppose applicants for this job arrive

---

<sup>6</sup>We assume that each firm is too large to be able to use its total output to infer whether a worker on a particular job is shirking or not.

at the rate  $\lambda$ .<sup>7</sup> Then the value of a vacancy is:

$$\Pi = \frac{1}{1+r\Delta t} [-c\Delta t + \lambda\Delta t B + (1-\lambda\Delta t)\Pi + o(\Delta t)],$$

where:

$$(18) B = \max_w \left[ \max \{ q(w)R(w) + (1-q(w))\Pi \}, \right. \\ \left. \max_{w,e} \{ p(w,e)N(w,e) + [q(w)-p(w,e)]S(w) + [1-q(w)]\Pi \} \right]$$

is the value to a firm of meeting a job applicant. The firm's decision of whether to incur monitoring costs or not is incorporated in the value B. In the limit:

$$(19) \Pi = \frac{-c}{r+\lambda} + \frac{\lambda}{r+\lambda} B.$$

We can now verify that the firm's decision problem is well-posed.

**Proposition 8:** There exist unique value functions  $R(w)$ ,  $N(w,e)$ ,  $S(w)$  and  $\Pi$ .

**Proof:** Given in the Appendix.

Thus far, our discussion of firm behavior has been limited to jobs "in the market." To complete the firm side of the model we invoke the free entry/exit condition. A firm creates jobs so long as the value of a vacancy,  $\Pi$ , is positive; a firm eliminates vacancies from the market if  $\Pi < 0$ . This free entry/exit condition implies that  $\Pi$  must be zero in equilibrium. From equation (19) this condition also determines the equilibrium applicant arrival rate, namely:

$$(20) \lambda = c/B.$$

## 5. Equilibrium

In the preceding sections we characterized optimal behavior for workers in the face of any arbitrary distribution  $H(w,e)$  and for firms in the face of any arbitrary distribution  $F_U(\theta)$ . An equilibrium is a distribution  $H(w,e)$  together with a corresponding distribution  $F_U(\theta)$  that is Nash in the sense that  $H(w,e)$  reflects the optimizing behavior of firms given  $F_U(\theta)$ , while at the same time  $F_U(\theta)$  reflects the optimizing behavior of workers given  $H(w,e)$ .

The most general equilibria to consider are those in which some firms require effort above the minimum level, while some do not. The possibility that a variety of wage/effort requirement packages might be offered by primary sector firms and that such dispersion might be self-supporting in

---

<sup>7</sup>The arrival rate,  $\lambda$ , is endogenous. Its determination is discussed below.

equilibrium is not ruled out a priori. However, we focus our attention on symmetric equilibria. A symmetric equilibrium is one in which primary sector firms offer a common wage/effort requirement package; likewise, secondary sector firms offer a common wage. Given suitable restrictions on the underlying exogenous distributions, we prove the existence of symmetric equilibria.

We denote the common primary sector package by  $(w_p, e_p)$ , the wage offered by secondary sector firms by  $w_s$ , and the fraction of vacancies arising in primary sector firms by  $\varphi$ . A symmetric distribution  $H(w, e)$  is thus characterized by four variables,  $w_s$ ,  $w_p$ ,  $e_p$ , and  $\varphi$ . Symmetric equilibria in which some firms incur monitoring costs and some do not, ie,  $0 < \varphi < 1$ , are (symmetric) "dual labor market equilibria." Two degenerate cases can also arise. If  $\varphi = 0$ , we have "pure secondary sector equilibria," in which no firms require effort above the minimum level; if  $\varphi = 1$ , we have "pure primary sector equilibria," in which all firms require effort above this level.

To prove the existence of symmetric equilibria we construct a map that takes any initial symmetric distribution  $H^0(w, e)$  into a new symmetric distribution  $H^1(w, e)$ . To use Brouwer's Theorem to show that this map has a fixed point, we must establish that the map is continuous and defined on a compact set. As we have assumed an upper bound on effort, the set of possible quadruples  $\{w_s, w_p, e_p, \varphi\}$  is closed and bounded. Thus, to prove existence we need to prove continuity.

The map from  $H^0$  to  $H^1$  has three basic components. First, optimal behavior by workers generates the contaminated distribution  $F_U(\theta)$  and the probabilities  $q(w)$  and  $p(w, e)$  that enter firms' decisions. Second, given  $q(w)$  and  $p(w, e)$ , firms compute the optimal secondary sector wage offer,  $w_s$ , and the optimal primary sector package  $(w_p, e_p)$ . Finally, given the updated  $(w_s, w_p, e_p)$ , firms optimally allocate vacancies across sectors, producing an updated value of  $\varphi$ .

The continuity of the map that takes  $H^0$  to  $H^1$  is established by demonstrating the continuity of each step. Thus, we need first to demonstrate the continuity of  $F_U(\theta)$  and of  $q(w)$  and  $p(w, e)$  in the variables comprising  $H^0$ . Second, we need to demonstrate the continuity of the optimal secondary sector and primary sector choices in the variables comprising  $H^0$ .

To do this we show that these optimizing values are unique, so that the Maximum Theorem can be applied to establish continuity. Finally, given optimal behavior in both sectors, we need to show that optimal sectoral choice generates a unique  $\varphi$ . These results are given in the following three propositions.

Proposition 9: The distribution  $F_U(\theta)$  and the probabilities  $q(w)$  and  $p(w,e)$  are continuous in the variables comprising  $H^0$ .

Proof: Given in the Appendix.

Proposition 10: Given suitable restrictions on  $f(\theta)$  and  $g(\xi)$ , there exists a unique solution  $w_S$  to the secondary sector firm maximization problem and a unique solution  $(w_P, e_P)$  to the primary sector firm maximization problem.

Proof: Given in the Appendix.

Proposition 11: Given  $\{w_S, w_P, e_P\}$ , there is a unique  $\varphi$  reflecting optimal sectoral choice by firms.

Proof: Given in the Appendix.

This completes our characterization of the map from the set of quadruples  $(w_S, w_P, e_P, \varphi)$  into itself. This map is a continuous function defined on a compact set, so we can apply Brouwer's Theorem. Thus, we have proven the following proposition.

Proposition 12: Given suitable restrictions on  $f(\theta)$  and  $g(\xi)$ , a symmetric equilibrium exists.

The existence of a symmetric equilibrium does not guarantee per se the existence of a symmetric dual labor market equilibrium, ie, an equilibrium in which  $0 < \varphi < 1$ . However, it is clear that for suitable values of the exogenous parameters of the model such an equilibrium must exist. For example, the equilibrium quadruple  $(w_P, e_P, w_S, \varphi)$  depends on the underlying parameters of the monitoring technology,  $\mu$  and  $m$ , in a continuous manner. Suppose  $\varphi = 0$ , so that all vacancies are in the secondary sector. If  $\mu$  is increased and/or  $m$  is reduced sufficiently, then a dual labor market equilibrium arises. Similarly, if  $\varphi = 1$ , reducing  $\mu$  and/or increasing  $m$  produces a dual labor market equilibrium.

## 6. Characteristics of Dual Labor Market Equilibria

The most interesting equilibria in our model are those in which the secondary and primary sectors coexist. In this section we investigate the characteristics of such equilibria. In particular, we establish two results.



First, the primary sector wage exceeds the secondary sector wage when  $\gamma$  is sufficiently large. Second, given that the primary sector wage is at least as large as the secondary sector wage, the primary sector effort requirement cannot be arbitrarily close to the minimum (secondary sector) effort requirement. In this sense there is a separation between the two sectors. The fact that these two results require a "sufficiently large"  $\gamma$  should not be surprising. The added productivity gained by incurring monitoring costs in the primary sector increases with  $\gamma$  since the difference in productivity between a nonshirker and a shirker is  $\gamma(e-1)$ .

After discussing these two results, we describe the sense in which the model yields rationing, ie, situations in which a worker who has taken a secondary sector job would prefer a corresponding primary sector job.

Proposition 13: Given  $\gamma$  sufficiently large,  $w_p > w_s$  in any dual labor market equilibrium.

Proof: Given in the Appendix.

The intuition for this result is as follows. The benefit of having primary sector employees meet the effort requirement is increasing in  $\gamma$ . That is, a firm's incentive to encourage its workers to meet its effort requirement by paying an attractive wage is increasing in  $\gamma$ . At the same time,  $\gamma$  does not affect workers' decisions directly, so the cost to the firm of using its wage offer to elicit more effort is independent of  $\gamma$ . Thus, as  $\gamma$  increases, the primary sector wage must rise relative to the secondary sector wage.

For low values of  $\gamma$  (those close to 1) the only possible dual labor market outcome would entail  $w_p < w_s$ . In the primary sector monitoring costs as well as wage costs must be covered, and for  $\gamma$  close to 1, revenues do not suffice. Of course, in this case a dual outcome is extremely unlikely, and if it occurs, the primary sector must be very small.<sup>8</sup>

Thus, in "almost all" dual labor market equilibria the primary sector wage exceeds the secondary sector wage, as one would expect. It follows that the primary sector effort requirement is "discretely greater than" the secondary sector minimum of one.

---

<sup>8</sup>If  $w_p < w_s$ , a worker would have to draw an extremely high value of  $\xi$  to make meeting a primary sector job's effort requirement worthwhile. The only way a primary sector job could survive would be by waiting for lucky matches. This outcome can be ruled out in a variety of ways, eg, by placing an upper bound on the support of  $\xi$ .

Proposition 14: If  $w_P \geq w_S$ , the primary sector effort requirement satisfies  $e_P \geq 1 + \frac{m}{\gamma}$ .

Proof: Given in the Appendix.

Proposition 14 implies that any symmetric dual labor market equilibrium (in which  $w_P \geq w_S$ ) must involve separation. The difference between the primary and secondary sector effort requirements can be no less than  $m/\gamma$ . The proof of this proposition does not rely on symmetry of dual labor market equilibria. Even if a continuum of wage/effort requirement packages were offered in the primary sector, the lowest primary sector effort requirement could not be arbitrarily close to the secondary sector requirement, ie, asymmetric dual labor market equilibria must also entail separation.

We have established that the primary sector wage is greater than the secondary sector wage, but, at the same time, primary sector jobs require an effort above the secondary sector minimum. Thus, it is not obvious a priori that workers prefer the primary sector. To be consistent with the dual labor market literature, workers who take secondary sector jobs should prefer to work in the primary sector.

We consider only the case in which  $\gamma$  is sufficiently high to ensure that  $w_P > w_S$ . To determine whether a particular worker prefers a primary sector job to its secondary sector counterpart (ie, a secondary sector job with the same  $\xi$ ) we examine the relevant value functions:

$$\begin{aligned} V_N(w_S, \xi, 1; \theta) &= \frac{w_S^{+\xi-1}}{r+\delta} + \frac{\delta}{r+\delta} U(\theta), \\ V_N(w_P, \xi, e_P; \theta) &= \frac{w_P^{+\xi-e_P}}{r+\delta} + \frac{\delta}{r+\delta} U(\theta), \text{ and} \\ V_S(w_P, \xi; \theta) &= \frac{w_P^{+\xi-1}}{r+\mu} + \frac{\mu}{r+\mu} U(\theta). \end{aligned}$$

A comparison of the first two value functions implies that a worker always prefers not shirking on a primary sector job to the corresponding secondary sector job so long as  $w_P - w_S > e_P - 1$ .

Suppose this inequality does not hold. It is still the case that some workers who have secondary sector jobs would prefer to shirk on the corresponding primary sector job. Consider, eg, a worker of type  $\theta$  with a secondary sector job who is just indifferent between this job and unemployment. For this worker,  $V_N[w_S, \xi_A(w_S, \theta); \theta] = U(\theta)$ , implying  $U(\theta) = \frac{w_S^{+\xi_A-1}}{r}$ . Holding  $\xi$  fixed at  $\xi_A$ , the value of shirking on a  $(w_P, \xi_A, e_P)$  job

exceeds the value of a  $(w_S, \xi_A, 1)$  job so long as  $w_P > w_S$ . Holding  $\theta$  fixed and allowing  $\xi$  to increase (lower values of  $\xi$  lead the worker to reject secondary sector jobs), the value of shirking on a primary sector job exceeds the value of the corresponding secondary sector job if  $w_P - w_S > \frac{(\mu - \delta)(\xi - \xi_A)}{(r + \delta)}$ . Since  $w_P > w_S$ , there is a range of values for  $\xi$  for which a worker holding a secondary sector job will prefer shirking on a primary sector job. This range increases with  $\gamma$  as higher values of  $\gamma$  are associated with a greater spread between  $w_P$  and  $w_S$ . Since  $\xi_A(w_S; \theta)$  increases with  $\theta$ , workers with higher values of  $\theta$  prefer shirking in primary sector jobs to having secondary sector jobs for a wider range of  $\xi$ .

Thus our model can generate an equilibrium that is consistent with several of the main premises of dual labor market analysis, ie, an equilibrium in which the primary sector requires greater effort, but rewards workers sufficiently so that workers on secondary sector jobs prefer primary sector jobs.

## 7. Conclusion

In this paper we presented a model in which dual labor market outcomes arise as Nash equilibria even though firms are identical ex ante. These equilibria not only involve dualism in the sense that there are two sectors, but the characteristics of the sectors accord with those described in the dual labor market literature: primary sector effort requirements and wages are higher than in the secondary sector and workers prefer to work in the primary sector. Our model thus provides a theoretical basis for an essentially empirical construct that has proven useful to labor economists. The model could be used to analyze policy questions within a dual labor market context. For example, the effects of a minimum wage on the primary sector package and on the sizes of the two sectors could be examined.

The fact that firms are identical ex ante makes our model consistent with the argument in the dual labor market literature that technology choice is endogenous. More importantly, we need identical firms in order to demonstrate that dualism can arise as a result of the common monitoring problem that firms face; in particular, there is no need to assume differences in technology across firms to generate a dual labor market outcome.

Our monitoring technology is the key assumption that allows dual labor market equilibria. The technology we chose is extreme in the sense that even small deviations from the required effort level result in shirkers being fired, if detected. This leads to a zero-one choice for employed workers: to shirk or work, and it is this dichotomy that drives our result. A smooth monitoring technology could produce the same result if the expected penalty to shirking were to rise sufficiently rapidly with deviations from the effort requirement.

In proving existence we worked with symmetric equilibria. Whether asymmetric equilibria exist in this model is an open question. However, our focus on symmetric equilibria is not essential to the dual labor market result. Even if we were to allow for the possibility of a range of  $(w, e)$  combinations among "primary sector" jobs, Proposition 14 establishes the required separation. The gap between the lowest primary sector effort requirement and the minimum effort level is ensured by the monitoring cost.

While the model yields equilibria that are consistent with many of the characteristics described in the dual labor market literature, it does not capture all the features of dual labor markets. Most descriptions of primary sector jobs include not only higher wages and effort levels, but also better opportunities for advancement, more on-the-job training, lower turnover rates, etc. A more detailed analysis of the primary sector employment package would be needed to account for advancement opportunities and on-the-job training. As the model now stands the turnover rate in the primary sector is above that in the secondary sector since the primary sector contains shirkers who have a higher separation rate. Had we modeled employed workers as searching, the primary sector would have had the lower turnover rate as workers in the secondary sector would quit to take jobs in the primary sector, but not vice versa. While these refinements are important, we chose to focus our attention on the difference between sectors in wages and effort requirements so as not to obscure the main result.

The model could also be extended by considering more complicated contractual arrangements between firms and workers. One possibility, which is often analyzed in the context of efficiency wage models (eg, Carmichael [1985]), would be to allow firms to require workers to post a bond upon taking a job. An analysis of bonding is beyond the scope of our paper. However, we can offer two conjectures about the effect of bonding in our model. First, because workers are heterogeneous, bonding would ameliorate but not completely eliminate shirking. Second, and more interestingly,

bonding could well increase the size of the secondary sector. Bonding reduces the need for primary sector employers to pay an efficiency wage to induce workers not to shirk. Those workers who are fortunate enough to get a primary sector job are less well off than they would have been in the absence of the bond. In a one-sector efficiency wage model this does not cause workers to reject primary sector jobs since the only option is unemployment. However, in a two-sector model workers have the option of secondary sector employment.

Another interesting possibility would be to allow firms to offer a menu of choices to workers, ie, to allow workers to self-select. For example, one might imagine that a firm, upon being contacted by a job seeker, might offer the applicant a choice of a primary sector versus a secondary sector "work station"; ie, the firm might offer the applicant a (two-point) menu of job types. Similar to bonding, we conjecture that this form of contractual complication would reduce (but not eliminate) shirking in the primary sector by diverting some workers into secondary sector jobs.

Even without extensions, however, the model performs its basic function, namely, it establishes the possibility of an endogenously generated dual labor market outcome.

## APPENDIX

Proof of Proposition 1: Substituting (1) and (2) into (3) gives a mapping of the form  $U(\theta) = T[U(\theta)]$ . It is straightforward to check that  $T(\cdot)$  is a contraction for each  $\theta$ , ensuring the existence of a unique function  $U(\theta)$ . To establish the uniqueness of  $V_N(\cdot)$  and  $V_S(\cdot)$ , substitute the unique  $U(\cdot)$  into (1) and (2). QED.

Proof of Proposition 2:

$$V_N(w, \xi, e; \theta) = \frac{w + \xi - e}{r + \delta} + \frac{\delta}{r + \delta} U(\theta) \geq U(\theta) \text{ iff } w + \xi \geq rU(\theta) + e;$$

$$V_S(w, \xi; \theta) = \frac{w + \xi - 1}{r + \mu} + \frac{\mu}{r + \mu} U(\theta) \geq U(\theta) \text{ iff } w + \xi \geq rU(\theta) + 1.$$

If either of these conditions is satisfied, the worker accepts. Since the condition  $V_S(w, \xi; \theta) \geq U(\theta)$  is more easily satisfied than  $V_N(w, \xi, e; \theta) \geq U(\theta)$ , the acceptance decision is determined only by  $w$  and  $\xi$ . QED.

Proof of Proposition 3: From (5) we have  $\frac{\partial \xi_A(w, \theta)}{\partial w} = -1$  and  $\frac{\partial \xi_A(w, \theta)}{\partial \theta} =$

$rU'(\theta)$ ; thus, we need to show  $rU'(\theta) > 0$ . We can write  $U(\theta)$  as:

$$U(\theta) = \frac{\theta}{r + a} + \frac{a}{r + a} \left\{ \int_{N(\theta)} V_N(w, \xi, e; \theta) dH(w, e) dG(\xi) + \int_{S(\theta)} V_S(w, \xi; \theta) dH(w, e) dG(\xi) + \int_{R(\theta)} U(\theta) dH(w, e) dG(\xi) \right\},$$

where  $N(\theta)$  is the no-shirk region for an individual of type  $\theta$ ; ie,

$$N(\theta) = \{(w, \xi, e) : V_N(w, \xi, e; \theta) \geq V_S(w, \xi; \theta)\}.$$

The shirk region,  $S(\theta)$ , and the reject region,  $R(\theta)$ , are defined analogously. Using (1) and (2) and rearranging gives:

$$rU(\theta) = \theta + a \left\{ \int_{N(\theta)} \left[ \frac{w + \xi - e - rU(\theta)}{r + \delta} \right] dH(w, e) dG(\xi) + \int_{S(\theta)} \left[ \frac{w + \xi - 1 - rU(\theta)}{r + \mu} \right] dH(w, e) dG(\xi) \right\}$$

Differentiating and collecting terms gives:

$$rU'(\theta) = 1 / \left[ 1 + \frac{a}{r + \delta} \int_{N(\theta)} dH(w, e) dG(\xi) + \frac{a}{r + \mu} \int_{S(\theta)} dH(w, e) dG(\xi) \right] > 0. \text{ QED.}$$

Proof of Proposition 4: From (7) we have:

$$\frac{\partial \xi_N(w, e, \theta)}{\partial w} = -1, \quad \frac{\partial \xi_N(w, e, \theta)}{\partial e} = \frac{r + \mu}{\mu - \delta}, \text{ and } \frac{\partial \xi_N(w, e, \theta)}{\partial \theta} = rU'(\theta). \text{ In the}$$

proof of Proposition 3 we showed that  $rU'(\theta) > 0$  for all  $\theta$ . QED.

### Derivation of $u(\theta)$

We use steady-state flow conditions to derive the  $\theta$ -specific unemployment rates. For each  $\theta$ , (i) the rates of flow of nonshirkers into and out of unemployment must be equal and (ii) the corresponding rates for shirkers must also be equal. Let  $n(\theta)$  denote the probability that an individual of type  $\theta$  is employed and not shirking; let  $s(\theta)$  be the probability that he or she is employed and shirking. The rate of flow into unemployment of nonshirkers of type  $\theta$  is  $\delta n(\theta)$ ; the corresponding rate of flow for shirkers of type  $\theta$  is  $\mu s(\theta)$ .

The flows out of unemployment of workers of type  $\theta$  consist of new hires. The flow of offers to unemployed workers of type  $\theta$  is  $a[1-n(\theta)-s(\theta)]$ . To compute the flow rates of new hires this offer arrival rate needs to be multiplied by the relevant acceptance probability. Let  $q^*(\theta)$  denote the probability that an unemployed worker of type  $\theta$  accepts a random offer. This probability is:

$$(A1) \quad q^*(\theta) = \int \{1 - G[\xi_A(w, \theta)]\} dH(w, e).$$

Similarly, let  $p^*(\theta)$  be the probability that a worker accepts a job and chooses to meet its effort requirement. This probability is:

$$(A2) \quad p^*(\theta) = \int \{1 - G[\xi_N(w, e, \theta)]\} dH(w, e).$$

The rate of flow of workers of type  $\theta$  into jobs on which they will not shirk is thus  $a[1-n(\theta)-s(\theta)]p^*(\theta)$ . In steady-state this must equal  $\delta n(\theta)$ . The flow of new hires who shirk is the difference between the total outflow from unemployment and the outflow of nonshirkers; thus, the flow of workers of type  $\theta$  into jobs on which they will shirk is  $a[1-n(\theta)-s(\theta)][q^*(\theta)-p^*(\theta)]$ . This expression must equal  $\mu s(\theta)$  in equilibrium.

Equating flow rates into and out of unemployment for workers of type  $\theta$  gives:

$$\begin{aligned} \delta n(\theta) &= a[1-n(\theta)-s(\theta)]p^*(\theta) \\ \mu s(\theta) &= a[1-n(\theta)-s(\theta)][q^*(\theta)-p^*(\theta)]. \end{aligned}$$

These steady-state conditions, plus the identity  $n(\theta)+s(\theta)+u(\theta) = 1$ , can be solved for  $u(\theta)$ :

$$u(\theta) = \mu\delta / \{\mu\delta + a\mu p^*(\theta) + a\delta[q^*(\theta)-p^*(\theta)]\}.$$

Proof of Proposition 5: The population density  $f(\theta)$  is continuously differentiable by assumption. To show that  $f_U(\theta)$  is continuously

differentiable, we need to show that  $q^*(\theta)$  and  $p^*(\theta)$  have this property. By Propositions 3 and 4,  $\xi_A$  and  $\xi_N$  are continuously differentiable in  $\theta$ ; by

assumption,  $G$  is continuously differentiable in its argument. The continuous differentiability of  $q^*(\theta)$  and  $p^*(\theta)$  then follows directly from equations (A1) and (A2). QED.

Proof of Proposition 8: Substituting the expression for  $B$  into (19) gives an expression of the form  $\Pi = T(\Pi)$ . It is straightforward to check that  $T(\cdot)$  is a contraction, ensuring the existence of a unique value  $\Pi$ . To establish the corresponding properties for  $R(w)$ ,  $N(w, e)$  and  $S(w)$ , substitute the unique  $\Pi$  into (13), (15), and (16). QED.

Proof of Proposition 9: The continuity of  $F_U(\theta)$  and of  $q(w)$  and  $p(w, e)$  in the variables comprising  $H^0$  all depend on the continuity of  $U(\theta)$  in those variables. In the case of a symmetric initial  $H$ ,  $U(\theta)$  is implicitly defined by:

$$(A3) \quad rU(\theta) - \theta - a \left[ \varphi \int_{\xi_N}^{\infty} \left[ \frac{w_P + \xi - e_P - rU(\theta)}{r + \delta} \right] g(\xi) d\xi + \right. \\ \left. \varphi \int_{\xi_P}^{\xi_N} \left[ \frac{w_P + \xi - 1 - rU(\theta)}{r + \mu} \right] g(\xi) d\xi + (1 - \varphi) \int_{\xi_S}^{\infty} \left[ \frac{w_S + \xi - 1 - rU(\theta)}{r + \delta} \right] g(\xi) d\xi \right] = 0,$$

where  $\xi_P \equiv \xi_A(w_P, \theta)$  and  $\xi_S \equiv \xi_A(w_S, \theta)$ . Differentiating with respect to  $w_S$ ,  $w_P$ ,  $e_P$ , and  $\varphi$ , and applying the Implicit Function Theorem gives the desired result. QED.

Proof of Proposition 10: The optimal wage for the firm, should it require only the minimum effort level, maximizes  $q(w)R(w) + [1 - q(w)]\Pi$ , where  $R(w) = \frac{\gamma - w - c}{r + \delta} + \frac{\delta}{r + \delta}\Pi$ . In considering the maximization problem we impose the long-run equilibrium condition  $\Pi = 0$  in advance. Thus, the optimal wage  $w_S$  is unique if  $q(w)R(w)$  is concave.

The first-order condition for this problem can be written:

$$q_w(w) [\gamma - w - c] - q(w) = 0,$$

and the second-order condition is:

$$q_{ww}(w) [\gamma - w - c] - 2q_w(w) < 0.$$

From the first-order condition,  $\gamma - w - c = q(w)/q_w(w)$ ; thus, the second-order condition can be written as:



$$q_{ww}(w)q(w) < 2q_w(w)^2.$$

The acceptance probability is:

$$q(w) = \int \{1 - G[rU(\theta)+1-w]\} f_U(\theta) d\theta;$$

thus:

$$q_w(w) = \int g[rU(\theta)+1-w] f_U(\theta) d\theta$$

$$q_{ww}(w) = - \int g' [rU(\theta)+1-w] f_U(\theta) d\theta.$$

By Proposition 5,  $f_U(\theta)$  is continuously differentiable; so, the condition on  $q(w)$  required for the existence of a unique maximizing value  $w_S$  can be satisfied by suitably restricting  $f(\theta)$  and/or  $g(\xi)$ . For example, the condition is satisfied if  $\xi$  is distributed as a uniform random variable.

Should the firm choose to incur monitoring costs, its optimal wage/effort requirement package maximizes:

$$p(w,e) [N(w,e) - S(w)] + q(w)S(w).$$

Again, we impose the long-run equilibrium condition  $\Pi = 0$  in advance, so  $N(w,e) = \frac{\gamma e - w - c - m}{r + \delta}$  and  $S(w) = \frac{\gamma - w - c - m}{r + \mu}$ . The no-shirk probability is:

$p(w,e) = \int \{1 - G[rU(\theta)+1-w+(\frac{r+\mu}{\mu-\delta})(e-1)]\} f_U(\theta) d\theta$  so that  $p_e(w,e) = -(\frac{r+\mu}{\mu-\delta})p_w(w,e)$ . Sufficient conditions for the existence of a unique optimal wage/effort requirement package include:

$$q_{ww}(w)q(w) < 2q_w(w)^2$$

$$p_{ww}(w,e)p(w,e) < 2p_w(w,e)^2$$

$$p_{ee}(w,e)p(w,e) < 2p_e(w,e)^2,$$

which can be satisfied by suitably restricting  $f(\theta)$  and/or  $g(\xi)$ . There are a few additional sufficient conditions. If  $\xi$  is a uniform random variable, these reduce to:  $4\gamma(r+\delta) > (\mu-\delta)(\gamma-1)^2$ . For suitable values of  $\gamma$ ,  $\mu$ , and  $\delta$ , a unique optimal wage/effort requirement package exists. QED.

Proof of Proposition 11: In a dual labor market equilibrium,  $\varphi$  equates the value of meeting an applicant for a secondary sector firm to the analogous value for a primary sector firm; ie,  $\varphi$  solves:

$$(A4) \quad q(w_S; \varphi)R(w_S) - p(w_P, e_P; \varphi)N(w_P, e_P) - [q(w_P; \varphi) - p(w_P, e_P; \varphi)]S(w_P) = 0.$$

For this equation to yield a unique value of  $\varphi$  it is sufficient that the above equation be continuous and monotonic in  $\varphi$ . We have already shown that

$q(\cdot)$  and  $p(\cdot)$  are continuous in  $\varphi$ ; thus, monotonicity is the property we need to verify.

Monotonicity of (A4) in  $\varphi$  is established if the derivative of the LHS of (A4) with respect to  $\varphi$  is of the same sign for all  $\varphi$ . This derivative is:

$$\begin{aligned} & \frac{\partial q(w_S)}{\partial \varphi} [(r+\mu)(\gamma - w_S - c)] - \left[ \frac{\partial q(w_P)}{\partial \varphi} - \frac{\partial p(w_P, e_P)}{\partial \varphi} \right] [(r+\delta)(\gamma - w_P - c - m)] \\ & - \frac{\partial p(w_P, e_P)}{\partial \varphi} [(r+\mu)(\gamma e_P - w_P - c - m)]. \end{aligned}$$

Since firms take  $F_U(\theta)$  as given, the partials of  $q(\cdot)$  and  $p(\cdot)$  with respect to  $\varphi$  depend on  $\frac{\partial U(\theta; \varphi)}{\partial \varphi}$ . That is, variations in  $\varphi$  affect the acceptance and no-shirk probabilities by affecting the value  $U(\theta)$  among the unemployed. Differentiating (A3) with respect to  $\varphi$ , we find that the sign of this derivative depends on the difference between the expected value of an acceptable primary sector offer and the expected value of an acceptable secondary sector offer, which must be positive. That is,  $\frac{\partial U(\theta; \varphi)}{\partial \varphi} > 0$ . Given  $\frac{\partial U(\theta; \varphi)}{\partial \varphi} > 0$ , the partials of  $q(\cdot)$  and  $p(\cdot)$  with respect to  $\varphi$  depend only on the form of  $g(\xi)$ . With minimal restrictions on the distribution of  $\xi$ , (A4) is monotonic in  $\varphi$ . For example, if  $\xi$  is a uniform random variable, the above derivative is positive for all  $\varphi$ .

If there is no  $\varphi \in [0,1]$  solving (A4), then we have a degenerate solution. If the LHS of (A4) is positive for all  $\varphi$  in this range, we have a pure secondary sector solution, ie,  $\varphi = 0$ . If the LHS of (A4) is negative for all  $\varphi \in [0,1]$ , we have a pure primary sector solution, ie,  $\varphi = 1$ . QED.

Proof of Proposition 13: The optimal secondary sector wage is given by

$$w_S = \gamma - c - \frac{q(w_S)}{q_w(w_S)}.$$

The corresponding primary sector first-order condition with respect to the wage can be written as:

$$\begin{aligned} & p_w(w_P, e_P) \{ (r+\mu)\gamma(e_P-1) + (\mu-\delta)(\gamma - w_P - c - m) \} - (\mu-\delta)p(w_P, e_P) \\ & + q_w(w_P)(r+\delta)(\gamma - w_P - c - m) - q(w_P)(r+\delta) = 0. \end{aligned}$$

The primary sector first-order condition with respect to  $e$  is:

$$p_e(w_P, e_P) \{ (r+\mu)\gamma(e_P-1) + (\mu-\delta)(\gamma - w_P - c - m) \} + \gamma(r+\mu)p(w_P, e_P) = 0.$$

This condition, together with  $p_e(w_P, e_P) = -\left(\frac{r+\mu}{\mu-\delta}\right)p_w(w_P, e_P)$ , implies

$$p_w(w_P, e_P) \{ (r+\mu)\gamma(e_P-1) + (\mu-\delta)(\gamma - w_P - c - m) \} = \gamma(\mu-\delta)p(w_P, e_P).$$

Substituting this into the first-order condition with respect to  $w$  and rearranging gives:

$$w_P = \gamma - c - m - \frac{q(w_P)}{q_w(w_P)} + \frac{(\gamma-1)(\mu-\delta)p(w_P, e_P)}{(r+\delta)q_w(w_P)}.$$

Thus,  $w_P > w_S$  so long as  $\gamma$  is sufficiently large. QED.

Proof of Proposition 14: Suppose that a primary sector firm has an optimal effort requirement of  $e_P$ . The value to the firm of a nonshirker is  $N(w_P, e_P)$

$= \frac{\gamma e_P - w_P - c - m}{r+\delta} + \frac{\delta}{r+\delta} \Pi$ . The value of a worker to a secondary sector firm is

$R(w_S) = \frac{\gamma - w_S - c}{r+\delta} + \frac{\delta}{r+\delta} \Pi$ . A minimum requirement for incurring monitoring costs to be profitable is that  $N(w_P, e_P) > R(w_S)$  since the primary sector firm also has some shirkers. This requires that  $\gamma(e-1) > (w_P - w_S) + m$ . Given  $w_P \geq w_S$ , the minimum requirement to ensure  $N(w_P, e_P) > R(w_S)$  is  $e_P > 1 + m/\gamma$ . QED.

## References

1. J. Albrecht and S. Vroman, "Nonexistence of Single-Wage Equilibria in Search Models with Adverse Selection," Review of Economic Studies, forthcoming.
2. J. Bulow and L. Summers, "A Theory of Dual Labor Markets with Application to Industrial Policy, Discrimination, and Keynesian Unemployment," Journal of Labor Economics, July 1986, part 1, pp 376-414.
3. K. Burdett and D. Mortensen, "Search, Layoffs, and Labor Market Equilibrium," Journal of Political Economy, August 1980, pp 652-72.
4. L. Carmichael, "Can Unemployment Be Voluntary?: Comment," American Economic Review, December 1985, pp 1213-1214.
5. W. Dickens and K. Lang, "A Test of Dual Labor Market Theory," American Economic Review, September 1985, pp 792-805.
6. P. Doeringer and M. Piore, Internal Labor Markets and Manpower Analysis, Lexington, Mass.: Heath, 1971.
7. C. Shapiro and J. Stiglitz, "Involuntary Unemployment as a Worker Discipline Device," American Economic Review, June 1984, pp 433-44.