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INTERNAL INEFFICIENCIES IN FIRMS

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ABSTRACT

Internal reward structures in firms are often integral parts of their "culture", and are changed infrequently in comparison to decisions about e.g., output prices. This paper investigates how this feature of firm organization provides a mechanism through which product market competition affects firms' internal efficiency. The design of firms' internal organization is modeled as a choice of an incentive structure between a principal and an agent, with strategic implications for firm's competitive positions on the product market. It is shown that - contrary to popular beliefs - there is a negative relation between the competitiveness of the product market and effort incentives.

Keywords: firm organization, competition, incentive contracts

JEL classification #: D20, D82, L22

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Economists often argue that competition promotes efficiency. Indeed, there is ample formal support for the notion that competition, as a rule, promotes *external* (or, *allocative*) *efficiency*, i.e., that it reduces the divergence between equilibrium prices and marginal production costs. The argument is not confined to external efficiency, only. It is frequently alleged that competition reduces *internal inefficiencies*, or organizational slack, in firms. For instance, a common argument is that firms in monopolized industries can survive in spite of inefficient production techniques, or that employees in such firms are able to capture rents due to the absence of competition. A recent prominent example of the asserted link between product market competition and internal inefficiency, is the European Commission's (1988, p. 126) claim that improved internal efficiencies of firms will "...constitute much of what can be called the dynamic effects of the internal market...".

Despite the importance attached to this reasoning in the policy debate as well as in the folklore of microeconomic theory, the suggested relationship has hardly at all been subject to systematic inquiry. Such an analysis must precisely define and explain the existence of an internal inefficiency, as well as introduce mechanisms through which product market competition might affect the organization of the firm. There exists a small and fairly recent literature, exemplified by Hart (1983), Hermalin (1990), Horn *et al.* (1990, 1991), Scharfstein (1988), and Tirole (1988, ch. 1), which employs contract theory to define and explain the existence of internal inefficiencies. Firms are characterized by a contractual relationship between owners and employees. With imperfect information such contracts provide less than perfect incentives compared to a situation with full information. Even though the incentive contract may be the best available given the informational constraints, it does in general lead to an inferior outcome compared to the full information case. In the terminology of Holmström and Myerson (1983) the optimal incentive contract is *incentive efficient* but not, in general, *ex post classically efficient*.

In this literature product market competition is shown to affect firms' contracts, or internal organization, in several different ways. For instance, competition can affect a firm's internal efficiency by altering the information on which contracts can be conditioned; this mechanism has been investigated by Hart (1983) and Scharfstein (1988). Another mechanism, investigated by Hermalin (1990), is that product market competition affects incentives through income effects. Horn et al. (1990, 1991) consider consequences of intensified competition through the exposure to international trade, and through the induced general equilibrium ramifications of trade for factor rewards. In Horn et al. (1990) increased product market competition changes the value of the outside option for one of the parties in the contractual relation within a firm. In Horn et al. (1991) competition tends to drive up rewards to another factor of production, and thus affects the conditions under which the contract between a manager and an owner of a firm is agreed upon.

The starting point of this paper is the observation that decisions about the design of a firm's internal organization are generally of a more long run nature than decisions about production volumes or prices. The latter may be revised from period to period in response to changes in the firm's environment. Changes in a firm's internal organization seem to take place considerably less frequently, and perhaps mainly as a result of a major reshaping of the firm's environment. Organizational design is thus a *strategic* decision which determines the general conditions under which tactical decisions, such as production volumes or prices, are to be taken. In the paper, we model the choice of "organization" as the choice of an incentive structure, or contract, which affects the firm's competitive position on the product market. The particular situation we have in mind is that employees are better informed than the owners about certain aspects of their firm and that they may seek to exploit this to their advantage. Specifically, we model a case where owners are unable to judge whether a high marginal cost is the result of poor performance by their employees or the result of unfavorable external conditions. To remedy the incentive problem, owners may devise more or less elaborated incentive structures. The

purpose of the paper is to investigate how product market competition have an impact on the internal inefficiency of firms by affecting the strategic value of their incentive contracts.

The strategic value of contracts depends on the specific type of competitive interaction on the product market. We therefore consider three different types of product market interaction on a differentiated goods market: Bertrand competition, Cournot competition and a joint profit maximizing cartel. One may view these three types of interactions as successively less competitive. One of the main findings of the paper is a *negative* relation between the competitiveness of the product market interaction and the effort incentives provided by the optimal contract between owners and their employees. Effort incentives are strongest, in a sense made precise below, in the cartel case and weakest with Bertrand competition. This result suggests that there may in fact be a negative relationship between internal efficiency and the degree of competition.

I. THE MODEL OF THE FIRM

Consider a differentiated goods market where two goods are produced by two firms. Each firm is characterized by a contractual relationship between an employer and an employee. To capture the notion that incentive contracts is of a long run nature, a two-stage approach is used, in which contracts are agreed upon prior to the product market interaction. At the first stage, contracting occurs. The employee has superior information about the firm's technology and can influence the cost level by exerting effort, which is unobservable to the employer. The employer offers the employee a contract on a take-it-or-leave-it basis. If accepted, the employee exerts an effort, which results in a production cost level. This occurs simultaneously in both firms. The realized cost is observable by the employer and, as in standard oligopoly models, we assume that he also

can observe the realized cost in the other firm. At the second stage the cost functions have been determined, and the firms meet to interact on the product market.

Asymmetric information about the technology permits the employee to choose an action that cannot be unveiled from the realized cost structure. The structure of the contract problem is similar to that studied by Laffont and Tirole (1986) in a somewhat different context. They were concerned with the design of an optimal regulatory policy for a regulator controlling a private firm, where the firm has superior cost information, and also the possibility to influence its cost performance in a manner not observable to the regulator. In our case, the employer's problem is to design a contract which conveys effort incentives to the employee, while it at the same time restricts as much as possible the rent which the employee acquires due to his superior information.

To capture the effect of product market competition, we study the solution to this contract problem under two different ownership structures. The first is that the two firms have a single owner, and there is simply a two-product monopoly. The contract problem is then to devise incentive contracts for the employees of the two subsidiaries, taking account of product market interdependencies. The common ownership case gives us the solution to the contract problem in the absence of product market competition.

The second case is that the two firms have separate owners, making the product market interaction a case of duopoly competition. When the owner of each firm designs the contract he must take into account that it affects the firm's cost, and hence its competitive position on the product market. The expected profit in each firm is therefore a function, not only of the contract in this firm, but also of the contracts implemented in other firms. Consequently, the design of the optimal incentive contracts is a game between the two owners. The role of product market competition for contract design, and hence the internal organization of firms, can then be elucidated by comparing the solution to this contract design game with the optimal contracts in the case of a common ownership of the two firms.

In the remainder of this section we present the firm's basic contract problem and its solution when firms have common ownership, i.e., the monopoly case. The sense in which the monopoly firm may be internally inefficient, is discussed in section II. Section III describes the contract problems and their solutions when firms have separate owners, and there is product market competition. Section IV contains a discussion of the effects of product market competition on contract design, and hence on firms internal organization and efficiency. Finally, section V provides some concluding comments.

The monopolist's optimal incentive contract

Let x_j denote the output of firm j and let c_j denote its constant unit production cost. Once the employees have exerted their effort, the resulting cost functions $C_j = c_j x_j$ $j=1,2$, are common knowledge among employer and employees.¹ The revenue function of firm j is $R^j(x_j, x_i)$ and its total cost is $C_j + W_j$ where W_j is the payment to the employee. For any realized cost levels c_j , the monopolist's profit maximizing outputs are given by the output functions $x_j(c_j, c_i, P)$ (the index P is used to distinguish these output functions from other ones that occur in the later duopoly analysis).

In each firm the marginal cost c_j is determined partly by a technological productivity parameter θ_j and partly by the effort e_j exerted by the manager: $c_j = \theta_j - e_j$. The employee knows the productivity parameter θ_j while the employer only knows its distribution on the interval $[\underline{\theta}, \bar{\theta}]$. The effort chosen by the employee is also unobservable to the owner. The employee derives utility from his payment W_j and disutility $Z(e_j)$ from the effort he exerts. His net utility is $W_j - Z(e_j)$, where Z is an increasing and strictly convex function.

In order to provide effort incentives the employer can make the payment W_j contingent on the observable variables c_j and c_i . Equivalently, he can design the contract

¹ Henceforth, subscript j indicates both firms 1 and 2. Whenever subscripts i and j occur in the same expression, i indicates "not j ".

as a revelation mechanism, i.e., a mechanism in which the employees have to report something from their domain of private information – the productivity parameters θ_i and θ_j . The employee then receives a payment $W_j(\theta_j, \theta_i)$, which depends on his report and possibly on the report of the employee of the other subsidiary, if he achieves some prescribed cost level $c_j(\theta_j, \theta_i)$. By the *revelation principle* it suffices to study *direct* revelation games in order to characterize the outcome of revelation games. In a direct revelation game, it is an equilibrium strategy to report private information truthfully.

The contract offered to each employee is thus a pair $(W_j(\theta_j, \theta_i), c_j(\theta_j, \theta_i))$. In order to be feasible, the contract must be incentive compatible and individually rational. Let $U_j(\theta_j, \hat{\theta}_j, \theta_i) \equiv W_j(\hat{\theta}_j, \theta_i) - Z(\theta_j - c(\hat{\theta}_j, \theta_i))$, i.e., $U_j(\theta_j, \hat{\theta}_j, \theta_i)$ is the employee's utility when the productivity parameter is θ_j and he reports $\hat{\theta}_j$. The contract $(W_j(\theta_j, \theta_i), c_j(\theta_j, \theta_i))$ is incentive compatible, i.e., truthful reporting is optimal, if and only if

$$E_{\theta_i}[U_j(\theta_j, \theta_j, \theta_i)] \geq E_{\theta_i}[U_j(\theta_j, \hat{\theta}_j, \theta_i)], \quad \forall (\theta_j, \hat{\theta}_j) \in [\underline{\theta}, \bar{\theta}] \quad (1)$$

where E is the expectations operator. The contract is individually rational if and only if

$$E_{\theta_i}[U_j(\theta_j, \theta_j, \theta_i)] \geq u, \quad \forall \theta_j \in [\underline{\theta}, \bar{\theta}] \quad (2)$$

where u is the employee's reservation utility. Incentive compatibility and the definition of

$U_j(\theta_j, \hat{\theta}_j, \theta_i)$ implies that

$$E_{\theta_i}[U_j(\theta_j, \theta_j, \theta_i)] \geq E_{\theta_i}[U_j(\theta_j, \bar{\theta}_j, \theta_i)] > E_{\theta_i}[U_j(\bar{\theta}_j, \bar{\theta}_j, \theta_i)], \quad \forall \theta_j < \bar{\theta}$$

Thus, the individual rationality constraint only binds when $\theta_j = \bar{\theta}$.

The monopolist's contract problem is to devise incentive contracts $(W_j(\theta_j, \theta_i), c_j(\theta_j, \theta_i))$ for his two subsidiaries, that maximizes the expected profit

$$E[R^1(x_1, x_2) + R^2(x_1, x_2) - c_1 x_1 - c_2 x_2 - W_1 - W_2]$$

subject to individual rationality and incentive compatibility constraints. The expectation is taken w r t the distribution of θ_1 and θ_2 .² We assume that they are identically and independently distributed with distribution function $F(\theta_j)$ and density function $f(\theta_j)$.

Let, with a slight abuse of notation, $U_j(\theta_j, \theta_i) \equiv U_j(\theta_j, \theta_j, \theta_i) \equiv W_j(\theta_j, \theta_i) - Z(\theta_j - c(\theta_j, \theta_i))$, i.e., the utility the employee obtains when reporting truthfully. The optimal incentive contracts for the monopolist are then given by functions $c_1(\theta_1, \theta_2)$, $c_2(\theta_1, \theta_2)$, $U_1(\theta_1, \theta_2)$, and $U_2(\theta_1, \theta_2)$ that solves

$$\begin{aligned} \max \quad & E[R^1(x_1, x_2) + R^2(x_1, x_2) - c_1(\theta_1, \theta_2)x_1 - c_2(\theta_1, \theta_2)x_2 \\ & - U_1(\theta_1, \theta_2) - U_2(\theta_1, \theta_2) - Z(\theta_1 - c_1(\theta_1, \theta_2)) - Z(\theta_2 - c_2(\theta_1, \theta_2))] \end{aligned} \quad (\text{MP})$$

$$\begin{aligned} \text{s.t.} \quad & E_{\theta_i}[U_j(\theta_j, \theta_i)] \geq E_{\theta_i}[U_j(\theta_j, \hat{\theta}_j, \theta_i)], \quad \forall (\theta_j, \hat{\theta}_j) \in [\underline{\theta}, \bar{\theta}], \quad j = 1, 2 \\ & E_{\theta_i}[U_j(\theta_j, \theta_i)] \geq u, \quad \forall \theta_j \in [\underline{\theta}, \bar{\theta}], \quad j = 1, 2 \end{aligned}$$

and where $x_j = x_j(c_j, c_i, P)$.

The monopoly problem is analyzed in the appendix. The main features are summarized by the following observations:

² One could view θ_j as a stochastic variable that is realized prior to the determination of the terms of employment. Or, perhaps better, θ_j is deterministic, but unknown to the employer. The distribution of θ_j is then subjective, and captures the employer's lack of information.

OBSERVATION 1:

The optimal required cost functions $c_j(\theta_j, \theta_i)$ satisfy

$$x_j = Z'(\theta_j - c_j) + \frac{F(\theta_j)}{f(\theta_j)} Z''(\theta_j - c_j) \quad (3)$$

The left hand side of (3) is the marginal benefit of an increase in effort. The resulting lower marginal cost increases profits proportionally to output. The right hand side of (3) is the marginal cost of effort. In order to induce effort, it is necessary to pay the employee a marginal compensation in excess of his marginal disutility of effort.

OBSERVATION 2:

The utility received by the agent is independent of the performance of the competitor. The conditional expected payment $E[W_j(\theta_j, \theta_i) | \theta_j]$ consists of a compensation equal to the reservation utility plus the conditional expected disutility of effort, and in addition an information rent.

The fact that the utility received by the agent is independent of the performance of his colleague in the other subsidiary follows from the assumption that the productivity parameters are independently distributed. This replicates a well-known result from the literature on tournament contracts, see e.g., Nalebuff and Stiglitz (1983): nothing can be gained by conditioning the agent's reward on information that is uncorrelated with the agent's performance.

Let $B(\theta_j) \equiv E_{\theta_i}[W_j(\theta_j, \theta_i) - Z(\theta_j - c_j(\theta_j, \theta_i))] - u$, so that $B(\theta_j)$ is the *conditional expected information rent*, i.e., the expected amount in excess of the reservation utility the employee receives as a result of the information asymmetry.

OBSERVATION 3:

$B(\theta_j) = \int_{\theta_j}^{\bar{\theta}} E_{\theta_i} [Z'(t - c(t, \theta_i))] dt$. In particular, $B(\theta_j)$ is decreasing in θ_j , $B(\bar{\theta}) = 0$, and a contract that induces a higher effort level at all θ_j also pays a higher information rent for all $\theta_j < \bar{\theta}$.

If the employer had the same information about the productivity parameters as the employees, and could observe the effort levels, his payment to each agent would be exactly equal to the sum of the reservation utility and the disutility of the effort level he chooses to induce. However, the agents' superior information about the productivity parameters allow them to capture part of the surplus in the firms - the "information rents". In order to give an agent incentives to report truthfully, he must be given a compensation for reporting realizations of the productivity parameter other than the worst outcome $\bar{\theta}$, and by proposition 3, this compensation is larger the lower the reported value is. Without such a compensation the agent would have an incentive to report a high value of the productivity parameter, which would allow him the possibility to slack, particularly if the actual productivity parameter has a low realization.

This completes the description of the contract problem and its solution for the monopolist. We now turn to a discussion of how the optimal incentive contracts may result in internal inefficiencies.

II. INTERNAL INEFFICIENCY

If fully informed, maximizing agents can costlessly sign any type of contract, a firm cannot be internally inefficient, in the sense that there exists unrealized gains from trade between the owners of the firm and its employees. The parties involved in the firm would

be aware of any inefficiency and realize that its removal would bring about a gain, which could be divided in a mutually beneficial way among the agents through an appropriate contract. Maximizing agents would sign such a contract. Thus, in order to reach a situation where internal inefficiencies occur, it is necessary to dispense with at least one of the assumptions about maximizing behavior, full information and costless contracting. In our model there is imperfect information, since the productivity parameter and the effort level cannot be observed by the employer. The optimal incentive contract results in an internal allocation, i.e., an effort level, that in general differs from the one implemented had there been full information. Thus, compared to a benchmark case of full information, firms are internally inefficient.

In order to make the notion of internal efficiency precise, define the *total production cost*, for a given output x_j as $(\theta_j - e_j)x_j + Z(e_j)$, i.e., as the sum of direct production costs and the cost of effort. A firm is defined to be *internally efficient* if the total production cost is minimized, *given the output volume*. The total production cost is strictly convex in the effort level. An internally efficient effort level is thus given by a decision rule $e = E^0(x)$, defined by $x = Z'(e)$. Any deviation from this relationship will be said to imply an *internal inefficiency*.³

A firm may be internally inefficient either because it induces too little or too much effort. Therefore, an increase of effort, at constant output, need not necessarily improve the internal efficiency, either because effort was initially too high, or, if it was initially too low, because the increase of effort overcompensates the initial slack.

³ The effort rule $E^0(x)$ obviously also maximizes social welfare, given the output level, with welfare defined as gross consumer surplus minus total production cost.

Internal inefficiency in a monopoly

For most of the analysis of the paper we employ a linear-quadratic parameterization, i.e., linear product market demands and quadratic disutility of effort, which allow a convenient exposition of our results, in particular for the outcome of the contract design game in the later duopoly analysis. We also assume that the product market demand functions are symmetric in order to focus on a symmetric equilibrium. With a symmetric, linear-quadratic parameterization the monopolist's output functions are linear and symmetric and so are the optimal required cost functions $c_j(\theta_j, \theta_j)$. Taking expectations with respect to θ_j and θ_i on both sides of (3), and integrating by parts, gives

$$x_j^e = \bar{\theta} - c_j^e = \bar{\theta} - \theta^e + e_j \quad (4)$$

where $x_j^e = E[x_j]$, $\theta^e = E[\theta_j]$ and $c_j^e = E[c_j]$.

The symmetric solution to equation (4) is illustrated in Figure 1. Consider first combinations of expected output x^e and expected effort $e = \theta^e - c^e$ which satisfy (4). These points are given by the EE-schedule, which obviously has a positive slope equal to unity. The XX-schedule illustrates combinations of expected output and expected effort which satisfy the monopolist's output function in a symmetric equilibrium. The slope of the XX-schedule can be shown (see appendix) to be larger than unity.

FIG 1 HERE

With the linear-quadratic parameterization the efficient effort schedule is $e = E^0(x) = x$. As depicted in Figure 1, it lies everywhere above the EE-schedule since $\bar{\theta} - \theta^e > 0$.

Thus, in terms of expected values the monopolist is internally inefficient since expected effort is too low for any expected output volume. Comparison of (3) and the efficient rule $e = E^0(x)$ reveals that this internal inefficiency exists for any realization of

(θ_j, θ_i) (except at the lower boundary $\theta_j = \underline{\theta}$ where $F(\underline{\theta}) = 0$). Effort is too low because the employer's trade-off between the cost reduction gain from higher effort, on the one hand, and the cost of inducing that effort, on the other, is distorted by asymmetric information. The marginal cost of inducing effort is higher than the employee's marginal disutility of effort, because the employee's superior information allows him to capture an information rent, which, by proposition 3, is increasing in effort. Thus, the information asymmetry not only affects the division of the surplus in the firm, but also results in a distortion in the allocation of effort in the firm.

The internal inefficiency also has ramifications for consumers, as can easily be seen in Figure 1. With contractable effort the equilibrium would be (e^0, x^0) instead of (e^*, x^*) , where $e^* < e^0$, i.e., the expected marginal cost in the informationally constrained monopoly is higher than in the full information case. Thus, in expected terms, *the conventional monopoly distortion of a too low output is exacerbated by the internal inefficiency*, and the internal inefficiency adversely affect consumers.

III. DUOPOLY COMPETITION

In order to analyze duopoly competition, we shall consider a quantity-setting, conjectural variations duopoly. The conjectural variations should not be interpreted literally. They are only a convenient way of treating different types of oligopolistic interaction in a common formal framework. We start the duopoly analysis by considering the final stage: the outcome of product market competition for some realized cost levels. Then we describe the contract design game.

The duopoly product market

At this stage the employees' effort levels are sunk and the resulting cost functions $C_j = c_j x_j$ are common knowledge among all employers and employees. Thus, the product market competition is a standard duopoly game with constant marginal costs. Let k be an index for the mode of oligopolistic interaction, e.g., Cournot or Bertrand competition. The equilibrium output functions $x_j(c_j, c_i, k)$ are determined by the first order conditions

$$R_j^j(x_j, x_i) + R_i^j(x_j, x_i) v_i(x_j, x_i, k) = c_j \quad (5)$$

where $v_i(x_j, x_i, k)$ is the conjectural variation, and where subscripts on the revenue function denote partial derivatives. Let

$$\begin{aligned} a_j &= R_{jj}^j + R_{ij}^j v_i + R_i^j \frac{\partial v_i}{\partial x_j} \\ b_j &= R_{ji}^j + R_{ii}^j v_i + R_i^j \frac{\partial v_i}{\partial x_i} \\ \Delta &= a_1 a_2 - b_1 b_2 \end{aligned}$$

Standard arguments yield

$$\frac{\partial x_j}{\partial c_j} = \frac{a_i}{\Delta} \quad \text{and} \quad \frac{\partial x_i}{\partial c_j} = -\frac{b_i}{\Delta} \quad (6)$$

Second-order conditions for profit maximization imply $a_j + b_j v_i < 0$. We assume stability of a myopic adjustment process as in Dixit (1983; see eq. (16)), i.e., $a_j < 0$ and $\Delta > 0$. The equilibrium output of a firm is thus decreasing in the own marginal cost. It is increasing in the marginal cost of the other firm if and only if the reaction function, implicitly defined by (5), is downward-sloping, i.e., if and only if $b_i < 0$.

In the subsequent analysis the term $\partial x_i / \partial c_j - v_i \partial x_j / \partial c_j$ will be decisive. From (6) it follows that

$$\frac{\partial x_i}{\partial c_j} - v_i \frac{\partial x_j}{\partial c_j} = \frac{\partial x_j}{\partial c_j} (r_i - v_i) \quad (7)$$

where $r_i = -b_i/a_i$ is the slope of firm i 's reaction function.

Much of the analysis is confined to symmetric and linear demand functions. With proper choice of units the demand function of firm j may then be written as $p^j(x_j, x_i) = \alpha - x_j - \gamma x_i$, where $0 < \gamma < 1$. The closer γ is to unity, the closer to perfect substitutes are the two products.

Three types of duopolistic interaction are examined: Cournot (i.e., $v_j = 0$, $j = 1, 2$), Bertrand (i.e., $v_j = -\gamma$, $j = 1, 2$), and joint profit maximization with respect to output (i.e., $v_j = x_j/x_i$, $j = 1, 2$). Of these, only joint profit maximization warrants a few comments. The idea is that the firms, once their cost levels are determined, can enter into a cartel agreement to produce such quantities that their joint profit is maximized. Each firm then receives its contribution to the joint profit, i.e., $R^j(x_j, x_i) - c_j x_j$. The maximized joint profit is of course always larger than the sum of profits in Cournot- or Bertrand competition, but if the realized cost levels are sufficiently different, one of the firms may prefer to compete rather than to participate in a cartel. We shall however disregard this possibility and simply assume that cost realizations are such that it is in the interest of both firms to participate. The question of interest is whether the knowledge that there will be a product market cartel leads the employers to induce more effort. On the one hand a cartel implies that there is more surplus to divide between the firms, which should provide incentives towards higher effort levels. On the other hand, with less aggressive product market interaction there seem to be less reason to invest in effort to obtain lower marginal costs.

Let $k = C$ denote Cournot competition, $k = B$ Bertrand competition, and $k = P$ joint profit maximization with respect to output. Linear and symmetric demands and constant marginal costs imply linear equilibrium output functions for each of the three types of duopolistic interaction, i.e.,

$$x_j = \beta^k + \frac{\partial x_j^k}{\partial c_j} c_j + \frac{\partial x_j^k}{\partial c_i} c_i, \quad k = C, B, P \quad (8)$$

where the index k indicates that the parameters of the linear equilibrium output functions depend on the type of duopolistic interaction. The expressions for the coefficients of (8) are given in the appendix, but observe that the derivatives $\partial x_j^k / \partial c_j$ and $\partial x_j^k / \partial c_i$ are constants and symmetry of course implies $\partial x_j^k / \partial c_j = \partial x_i^k / \partial c_i$ and $\partial x_j^k / \partial c_i = \partial x_i^k / \partial c_j$. Note also, for future reference, that the slope of the reaction functions, r , is equal to $-\gamma/2$ with Cournot competition, $-\gamma/(2-\gamma^2)$ with Bertrand competition, and $-\gamma$ with joint profit maximization production.

The duopolist's optimal incentive contract

In the duopoly case each firm's employer only receives a report from his own employee. Consequently, the required cost function of firm j can only depend on the report θ_j . The payment W_j can depend on θ_j and possibly on the observable cost performance c_i of the competitor. Thus, the contract offered to the employee is a pair $(W_j(\theta_j, c_i), c_j(\theta_j))$.

The employer's problem is to choose a contract that maximizes the firm's expected profit $E[R^j(x_j, x_i) - c_j x_j - W_j]$, subject to incentive compatibility and individual rationality restrictions. The employer must also take account of how his choice of contract affects his firm's strategic position on the product market, i.e., how his contract will influence the output volumes of the other firm in the subsequent product market interaction.

Using the definition $U_j(\theta_j, c_i) \equiv W_j(\theta_j, c_i) - Z(\theta_j - c_j(\theta_j))$, the optimal incentive contract for firm j is a pair of functions $c_j(\theta_j)$ and $U_j(\theta_j, c_i)$, that solves

$$\begin{aligned} \max \quad & E[R^j(x_j, x_i) - c_j(\theta_j)x_j - U_j(\theta_j, c_i(\theta_i)) - Z(\theta_j - c_j(\theta_j))] \\ \text{s.t.} \quad & E_{\theta_i}[U_j(\theta_j, c_i(\theta_i))] \geq E_{\theta_i}[U_j(\theta_j, \hat{\theta}_j, c_i(\theta_i))], \quad \forall (\theta_j, \hat{\theta}_j) \in [\theta, \bar{\theta}] \\ & E_{\theta_i}[U_j(\theta_j, c_i(\theta_i))] \geq u \end{aligned} \tag{DP}$$

where $x_j = x_j(c_j(\theta_j), c_i(\theta_i), k)$ and $x_i = x_i(c_i(\theta_i), c_j(\theta_j), k)$.

This problem is analyzed in the appendix. The main features are summarized in the following observations:

OBSERVATION 4:

The optimal required cost functions $c_j(\theta_j)$ satisfy

$$\begin{aligned} E_{\theta_i}[x_j] - E_{\theta_i}[R^j_i(x_j, x_i) \frac{\partial x_j}{\partial c_j}(r_i(x_j, x_i) - v_i(x_j, x_i, k))] \\ = Z'(\theta_j - c_j) + \frac{F(\theta_j)}{f(\theta_j)} Z'(\theta_j - c_j) \end{aligned} \tag{9}$$

Conditions (9) shall be compared to the corresponding conditions (3) for the monopolist. As in the monopoly case the right hand side of (9) is the marginal cost of effort. For a given effort level and realization of the productivity parameter the marginal cost of effort is the same for the two types of ownership structures. The left hand side of (9) is the expected marginal benefit, conditional on θ_j of an increase in effort. As in the monopoly case the lower marginal cost increases expected profit proportionally to expected output; this effect is given by the first term on the left hand side of (9). But in contrast to

the monopoly case, a lower marginal cost also induces a different equilibrium output from the other firm which affects expected profits according to the second term on the left hand side. This is the *strategic effect* of the contract.

OBSERVATION 5:

The payment function W is independent of the performance of the competitor and consists of a compensation equal to the reservation utility plus the disutility of effort, and in addition an information rent.

Let $B(\theta_j) \equiv W(\theta_j) - Z(\theta_j - c_j(\theta_j)) - u$, so that $B(\theta_j)$ is the *information rent*, i.e., the amount in excess of the reservation utility the employee receives as a result of the information asymmetry. We can infer the following concerning the information rent:

OBSERVATION 6:

$B(\theta_j) = \int_{\theta_j}^{\bar{\theta}} Z'(t - c_j(t)) dt$. In particular, $B(\theta_j)$ is decreasing in θ_j , $B(\bar{\theta}) = 0$, and a contract that induces a higher effort level at all θ also pays a higher information rent for all $\theta_j < \bar{\theta}$.

This completes the presentation of the contract design stage in the case of a product market duopoly. To investigate the effects of product market competition on optimal contracts we next compare the optimal contracts for the monopolist with the equilibrium contracts in the duopoly case.

IV. EFFORT INCENTIVES AND COMPETITION

Competition, i.e., separate ownership of the two firms, alters the situation depicted in Figure 1 in two ways. First, it attributes a strategic value to the incentive contract, which changes the expected effort induced for any expected output, i.e., the EE-schedule shifts. This *strategic effect* is a direct effect of competition on internal inefficiency in the sense that it affects the equilibrium contract for any given level of output. Secondly, there is an *output effect*, i.e., competition changes the expected output for any expected equilibrium cost. The output effect shifts the XX-schedule. The final effect of competition on expected output and expected effort depends on both these effects.

The strategic effect is given by the second term of the left hand side of conditions (9). By our assumptions $R_i^j(x_j, x_i) \frac{\partial x_j}{\partial c_j} = -\gamma x_j \frac{\partial x_j}{\partial c_j} > 0$, so the sign of the strategic effect depends on the sign of $r_i - v_i$, i.e., on the slope of the reaction curve and the conjectural variation.⁴ Thus, the strategic effect depends on the specific type of competitive interaction on the product market.

Consider first Cournot and Bertrand interaction, where the slope of the reaction functions and the conjectural variations are constants. In these cases the strategic effect can be expressed as $S(k)E_{\theta_i}[x_j]$, where $S(k) = -\gamma \frac{\partial x_j^k}{\partial c_j}(r^k - v^k)$, $k = C, B$. Cournot competition yields $r - v = r = -\gamma/2 < 0$, while Bertrand competition results in $r - v = -\gamma/(2-\gamma^2) + \gamma = \gamma(1-\gamma^2)/(2-\gamma^2) > 0$. As usual these two types of duopolistic interaction result in opposite signs for the strategic effect. With Cournot competition the strategic effect *increases* the marginal benefit of effort, while with Bertrand competition the strategic effect *reduces* the marginal benefit of effort. In these two cases (9) becomes

⁴ The crucial role of the term $r_i - v_i$ is typical of two-stage games. See Eaton and Grossman (1986).

$$(1 - S(k))E_{\theta_i}[x_j] = Z'(\theta_j - c_j) + \frac{F(\theta_j)}{f(\theta_j)}Z''(\theta_j - c_j), \quad k = C, B \quad (10)$$

The linear-quadratic parameterization allows us to explicitly solve (10). In particular, taking expectations with respect to θ_j on both sides of (10), and integrating by parts, gives

$$(1 - S(k))x_j^e = \bar{\theta} - c_j^e = \bar{\theta} - \theta^e + e_j, \quad k = C, B \quad (11)$$

Consider next the cartel case with joint profit maximization with respect to output. The strategic effect is

$$\begin{aligned} E_{\theta_i}[-\gamma x_j \frac{\partial x_j^P}{\partial c_j} (r - v)] &= E_{\theta_i}[-\gamma x_j \frac{\partial x_j^P}{\partial c_j} (-\gamma - \frac{x_i^P}{x_j})] \\ &= \frac{\partial x_j^P}{\partial c_j} (\gamma^2 E_{\theta_i}[x_j] + \gamma E_{\theta_i}[x_i]) < 0 \end{aligned}$$

As with Cournot interaction, but in contrast to Bertrand interaction, the strategic effect in the case of joint profit maximization *increases* the marginal benefit of effort. Again, the linear-quadratic parameterization may be explicitly solved. In particular, we have the following relation between expected output and expected marginal cost

$$(1 - \frac{\partial x_j^P}{\partial c_j} \gamma^2) x_j^e - \gamma \frac{\partial x_j^P}{\partial c_j} x_i^e = \bar{\theta} - c_j^e$$

so in a symmetric equilibrium we have

$$(1-S(P))x^e = \bar{\theta} - c_j^e = \bar{\theta} - \theta^e + e_j \quad (12)$$

where $S(P) = (\gamma^2 + \gamma) \frac{\partial x_j^P}{\partial c_j}$.

Figures 2 and 3 show the effect of competition on effort incentives by comparing equations (11) (the Cournot and Bertrand cases) and (12) (joint profit maximization) with the corresponding equation (4) for the monopolist. The $X^M X^M$ and $E^M E^M$ curves depict the monopolist's output and effort schedules from Figure 1. In Figure 2 the monopoly outcome is compared to that of a Cournot duopoly and that of a joint profit maximizing cartel, respectively. It is clear from (11) and (12) that for any given expected output level, the expected effort is higher compared to the monopoly case due to the strategic effects. In other words, since $1 - S(C) > 1$ and $1 - S(P) > 1$, the effort schedule, $E^C E^C$, defined by (11) for Cournot interaction and the corresponding schedule, $E^P E^P$, defined by (12) for the joint profit maximizing cartel, both lie above $E^M E^M$. Furthermore, since

$$\begin{aligned} 1 - S(C) &= 1 - \gamma^2 \frac{\partial x_j^C}{\partial c_j} = 1 + \gamma^2 \frac{1}{4 - \gamma^2} \\ &< 1 + (\gamma^2 + \gamma) \frac{1}{2(1 - \gamma^2)} = 1 - (\gamma^2 + \gamma) \frac{\partial x_j^C}{\partial c_j} = 1 - S(P) \end{aligned} \quad (13)$$

the effort schedule of the cartel lies above that of the Cournot duopolist.

FIG 2 HERE

With Cournot interaction, or joint profit maximization, the strategic effect unambiguously increases effort incentives for any given level of output. The effect is, somewhat surprisingly, strongest for a joint profit maximization cartel. In both cases, at

the contract design stage of the game, the employer has an incentive to induce more effort, and thus to lower marginal cost, because by doing so he can "slide along the rival's reaction curve" in the product market game, i.e., increase his own output volume and reduce that of the rival. With Cournot or cartel interaction such a movement always increases his profit. The strategic incentive is larger for a participant in a profit maximizing cartel than a Cournot duopolist, because in the former case the rival's reaction to an increase in the other firm's output volume is to decrease its own production by $-\gamma$, while the corresponding value is only $-\gamma/2$ in the Cournot case. This reflects the fact that joint profit maximization internalizes the cross price effects of output changes.

In the cartel case there is no output effect. The output function of the cartel is the same as for the monopolist, so the cartel equilibrium is given by point P in Figure 2. Equilibrium output and effort are unambiguously higher compared to the monopoly outcome M.

With Cournot interaction there is an output effect, and thus a shift in the output schedule. As shown in the appendix, the expected output of the Cournot duopolist in a symmetric equilibrium is always higher than that of the monopolist. Thus, the Cournot output schedule $X^C X^C$ lies to the right of $X^M X^M$. The Cournot outcome is given by point C in Figure 2. Expected output and effort are unambiguously higher compared to the monopolist. Note that the equilibrium expected effort of the Cournot duopolist may be larger than that of the cartel if the output effect is sufficiently strong to neutralize the stronger strategic effect on the cartel. To sum up:

PROPOSITION 1: *With Cournot or cartel interaction on the product market, competition unambiguously increases expected output and effort. Such interaction always promotes effort incentives.*

The Bertrand case is illustrated in Figure 3. Here $1 - S(B) < 1$, so the effort

schedule of the Bertrand duopolist, $E^B E^B$, lies below the monopolist's. Thus, here the strategic effect *reduces* effort incentives. Again, by changing the induced effort, the firm can move along the rival's reaction curve. With Bertrand interaction the firm increases its profit by reducing its output volume, in spite that this induces an increase in the rival's output volume. Thus, the firm has an incentive to lower effort incentives. As in the Cournot case, however, the output schedule of the Bertrand duopolist lies to the right of the monopolist (in fact, for a given expected marginal cost the Bertrand duopolist produces a higher expected output in the symmetric equilibrium than the Cournot duopolist). As a result, the effects on expected output and on expected effort depends on the relative strength of the output effect compared to the strategic effect. In summary:

PROPOSITION 2: *With Bertrand interaction on the product market, competition always reduces effort incentives. The effects on expected output and effort are ambiguous. The strategic effect tends to reduce effort and thus output, while the output effect tends to increase output and thus effort.*

FIG 3 HERE

We have seen that the strategic effect increases effort incentives in the Cournot and cartel cases, while it reduces effort incentives in the Bertrand case. What about the effect of competition on firms' internal efficiency? One can say that competition reduces a firm's internal inefficiency if it shifts the EE-schedule closer to the E^0 -schedule at any given output level, since such a shift lowers total production cost for any given output level. Correspondingly, competition increases internal inefficiency if it shifts the effort-schedule further away from the E^0 -schedule for any given output level.

In the Bertrand case, competition unambiguously *increases* internal inefficiency: at any given output level total production cost is higher than in the monopoly case. This can

be seen in Figure 3 where the Bertrand effort schedule E^B_E everywhere lies below the monopoly effort schedule E^M_E .

The Cournot and cartel cases, however, are ambiguous. In both cases the strategic effect promotes effort incentives in the sense that it shifts the effort schedule upwards compared to the monopoly case. But note that the strategic effect is proportional to expected output, so the shift of the effort schedule is larger at higher expected output levels. Since the monopolist's effort schedule lies below the efficient one, an upward shift improves internal efficiency only as long as it is not too large, i.e., as long as expected output is not too large. Thus, for sufficiently high expected output the strategic effect can in fact reinforce effort incentives to the extent that the internal inefficiency deteriorates, due to too much effort. For any given level of expected output it is of course always possible to compare two effort schedules and conclude which one gives the lowest total production cost, i.e., is more efficient, for that output. Thus, for the case depicted in Figure 2, both the Cournot and the cartel effort schedules are more efficient than the monopoly one for the three relevant output levels.

Finally, it should be noted that in those cases competition reduces internal inefficiency, competition does not affect the *source* of the inefficiency: the asymmetry of information. Instead it introduces a *new* bias on effort, which works in the opposite direction to that of the information asymmetry.

V. CONCLUDING REMARKS

The idea that competition reduces internal inefficiencies, or organizational slack, in firms is almost a folk theorem in economics. Despite its intuitive appeal, this argument has hardly at all been subject to a systematic inquiry. In this paper we have used contract theory to define and explain the existence of internal inefficiencies. Decisions about the

design of a firm's internal organization, interpreted in our case as the design of incentive contracts, are generally of a more long run nature than decisions about production volumes or prices. Consequently, organizational design is a strategic decision which affects the outcome of product market interaction. In such a setting competition among firms gives a strategic value to incentive contracts and this has an impact on their internal inefficiency. But the direction of this impact may reinforce or counteract the inefficiency depending on the exact nature of product market interaction. At a general level this is, of course, a phenomenon which is well-known from industrial organization theory. Nevertheless, it is important to note its implication for the relation between product market competition and internal efficiency: the idea that competition reduces slack is too simplified. Effort incentives are lowest with Bertrand interaction, which is the most competitive of the interactions we consider, in the sense that it leads to lower prices and profits than Cournot or cartel interaction. In the least competitive case, the output cartel, effort incentives are strongest, while Cournot interaction is an intermediate case. Hence, when internal incentive structures are determined prior to product market interaction, there appears to be a negative relation between effort incentives and the competitiveness of product market interaction.

APPENDIX

1. The output functions

The coefficients of the output functions $x_j(c_j, c_i, k)$ are used to construct figures 1 - 3. With the linear-quadratic parameterization it is straightforward to establish that

$$\begin{aligned} \beta^C &= \frac{\alpha(2-\gamma)}{4-\gamma^2} & \frac{\partial x_j^C}{\partial c_j} &= -\frac{2}{4-\gamma^2} & \frac{\partial x_j^C}{\partial c_i} &= \frac{\gamma}{4-\gamma^2} \\ \beta^B &= \frac{\alpha(2-\gamma^2-\gamma)}{(2-\gamma^2)^2-\gamma^2} & \frac{\partial x_j^B}{\partial c_j} &= \frac{-(2-\gamma^2)}{(2-\gamma^2)^2-\gamma^2} & \frac{\partial x_j^B}{\partial c_i} &= \frac{\gamma}{(2-\gamma^2)^2-\gamma^2} \\ \beta^P &= \frac{\alpha(1-\gamma)}{2(1-\gamma^2)} & \frac{\partial x_j^P}{\partial c_j} &= -\frac{1}{2(1-\gamma^2)} & \frac{\partial x_j^P}{\partial c_i} &= \frac{\gamma}{2(1-\gamma^2)} \end{aligned}$$

In the symmetric case where marginal cost is the same for the two products, it is easy to verify that, for any given marginal cost, $x^B > x^C > x^P$. The slope of the XX-schedule in Figures 1 - 3 is $-\left[\frac{\partial x_j^k}{\partial c_j} + \frac{\partial x_j^k}{\partial c_i}\right]^{-1}$, $k = C, B, P$, which in all three cases is larger than unity. The slope of the EE-schedule is unity for the monopoly and $1-S(k)$, $k = C, B, P$ for the duopoly cases. It is easy to verify that slope of the XX-schedule is steeper than that of the EE-schedule in all cases.

2. The monopolist's contract problem

Consider first the solution to the monopolist's contract problem (MP), summarized in Observations 1 - 3. The attention is limited to piecewise differentiable contracts

$(W_j c_j)$. Consequently, U_j is also piecewise differentiable. The analysis of the contract problem becomes simpler if the incentive compatibility constraints are replaced by the corresponding first order conditions on U_j . This so called first order approach is valid provided that the first order conditions are equivalent to the incentive compatibility constraints, i.e., if the first order conditions are sufficient as well as necessary conditions for truthful reporting by the agent. We first establish the validity of the first order approach by the following three lemmas, which are based on results by Laffont and Guesnerie (1984) and Laffont and Tirole (1986).

LEMMA 1: $c_j(\theta_j, \theta_i)$ is nondecreasing in θ_j .

PROOF: Incentive compatibility yields

$$\begin{aligned} E_{\theta_i}[W_j(\theta_j, \theta_i) - Z(\theta_j - c_j(\theta_j, \theta_i))] &\geq E_{\theta_i}[W_j(\hat{\theta}_j, \theta_i) - Z(\theta_j - c_j(\hat{\theta}_j, \theta_i))] \\ E_{\theta_i}[W_j(\hat{\theta}_j, \theta_i) - Z(\hat{\theta}_j - c_j(\hat{\theta}_j, \theta_i))] &\geq E_{\theta_i}[W_j(\theta_j, \theta_i) - Z(\hat{\theta}_j - c_j(\theta_j, \theta_i))] \end{aligned}$$

Adding these inequalities results in

$$E_{\theta_i}[Z(\theta_j - c_j(\hat{\theta}_j, \theta_i)) - Z(\hat{\theta}_j - c_j(\hat{\theta}_j, \theta_i))] \geq E_{\theta_i}[Z(\theta_j - c_j(\theta_j, \theta_i)) - Z(\hat{\theta}_j - c_j(\theta_j, \theta_i))]$$

In particular, there exists some $\tilde{\theta}_i$ such that

$$Z(\theta_j - c_j(\hat{\theta}_j, \tilde{\theta}_i)) - Z(\hat{\theta}_j - c_j(\hat{\theta}_j, \tilde{\theta}_i)) \geq Z(\theta_j - c_j(\theta_j, \tilde{\theta}_i)) - Z(\hat{\theta}_j - c_j(\theta_j, \tilde{\theta}_i)) \quad (\text{A1})$$

Suppose $\theta_j > \hat{\theta}_j$ and suppose, contrary to the assertion of the lemma, that $c_j(\theta_j, \tilde{\theta}_i) < c_j(\hat{\theta}_j, \tilde{\theta}_i)$. Then

$$\theta_j - c_j(\hat{\theta}_j, \tilde{\theta}_i) - (\hat{\theta}_j - c_j(\hat{\theta}_j, \tilde{\theta}_i)) = \theta_j - \hat{\theta}_j = \theta_j - c_j(\theta_j, \tilde{\theta}_i) - (\hat{\theta}_j - c_j(\theta_j, \tilde{\theta}_i)) > 0$$

and

$$\theta_j - c_j(\hat{\theta}_j, \tilde{\theta}_i) < \theta_j - c_j(\theta_j, \tilde{\theta}_i)$$

which together with strict convexity of Z contradicts (A1), Q.E.D.

LEMMA 2: *If $c_j(\theta_j, \theta_i)$ is nondecreasing in θ_j then the first order condition for incentive compatibility is also locally sufficient.*

PROOF: Let $\varphi(\theta_j, \hat{\theta}_j) \equiv E_{\theta_i}[U_j(\theta_j, \hat{\theta}_j, \theta_i)]$, i.e., φ_j denotes the expected utility of agent j when he reports $\hat{\theta}_j$ and his true value is θ_j . If the contract is incentive compatible, i.e., truth-telling is an optimal response for the agent, then the following first and second order conditions are necessarily satisfied

$$\frac{\partial \varphi_j(\theta_j, \theta_j)}{\partial \hat{\theta}_j} = 0 \text{ and } \frac{\partial^2 \varphi_j(\theta_j, \theta_j)}{\partial \hat{\theta}_j^2} \leq 0$$

Since the first order condition must hold for (almost) all θ_j it follows that

$$\frac{\partial^2 \varphi_j(\theta_j, \theta_j)}{\partial \hat{\theta}_j^2} = - \frac{\partial^2 \varphi_j(\theta_j, \theta_j)}{\partial \theta_j \partial \hat{\theta}_j}$$

so the second order condition is equivalent to

$$\frac{\partial^2 \varphi_j(\theta_j, \theta_j)}{\partial \theta_j \partial \hat{\theta}_j} = E_{\theta_i} [Z' \cdot (\theta_j - c_j(\theta_j, \theta_i)) \frac{dc_j(\theta_j, \theta_i)}{d\theta_j}] \geq 0$$

Convexity of Z establishes the lemma, Q.E.D.

LEMMA 3: If $c_j(\theta_j, \theta_i)$ is strictly increasing in θ_j local sufficiency implies global sufficiency.

PROOF: The first order condition is $\partial \varphi_j(\theta_j, \theta_j) / \partial \hat{\theta}_j = 0$ and by lemma 2 this is a local maximum for the agent. Suppose it is not a global one. Then there exists some $\tilde{\theta}_j \neq \theta_j$ such that

$$\frac{\partial \varphi_j(\theta_j, \tilde{\theta}_j)}{\partial \hat{\theta}_j} = 0 = \frac{\partial \varphi_j(\tilde{\theta}_j, \tilde{\theta}_j)}{\partial \hat{\theta}_j}$$

But this is inconsistent, since

$$\frac{\partial^2 \varphi_j(\theta_j, \theta_j)}{\partial \theta_j \partial \hat{\theta}_j} = E_{\theta_i} [Z' \cdot (\theta_j - c_j(\theta_j, \theta_i)) \frac{dc_j(\theta_j, \theta_i)}{d\theta_j}]$$

which is strictly positive by assumption, Q.E.D.

Lemma 1, 2 and 3 provide conditions for the validity of the first order approach. By lemma 1, feasible cost functions for firm j must be nondecreasing in θ_j and by lemma 2 this means that the first order condition implies that the second order condition is locally satisfied. If the required cost function of firm j is strictly increasing in θ_j then the first

order condition is globally sufficient by lemma 3. Thus, one can conclude that the first order condition is equivalent to incentive compatibility if the required cost function is increasing in the own productivity parameter.

OBSERVATION 1:

The optimal required cost functions $c_j(\theta_j, \theta_i)$ satisfy

$$x_j = Z'(\theta_j - c_j) + \frac{F(\theta_j)}{f(\theta_j)} Z''(\theta_j - c_j) \quad (3)$$

PROOF: Replacing the incentive compatibility constraints with the corresponding first order conditions $E_{\theta_i} [U'_j(\theta_j, \theta_i) + Z'(\theta_j - c_j(\theta_j, \theta_i))] = 0, \forall \theta_j$, the Lagrangean for the monopolist's contract problem is

$$\Lambda = \left[\sum_{j=1}^2 \{ R^j(x_1, x_2) - c_j(\theta_1, \theta_2) x_j - U_j(\theta_1, \theta_2) - Z(\theta_j - c_j(\theta_1, \theta_2)) \right. \\ \left. + \lambda_j(\theta_j) [U'_j(\theta_1, \theta_2) + Z'(\theta_j - c_j(\theta_1, \theta_2))] \} \right] f(\theta_j) f(\theta_i)$$

where $x_j = x_j(c_j(\theta_j, \theta_i), c_i(\theta_i, \theta_j), P)$, $j = 1, 2$. The Euler equations w r t U_j , U_i and c_j , c_i are

$$-f(\theta_j) = \frac{\partial}{\partial \theta_j} (\lambda_j(\theta_j) f(\theta_j)), \quad j = 1, 2 \quad (A2)$$

$$\begin{aligned} & [(R^j_j(x_j, x_i) + R^i_j(x_j, x_i) - c_j) \frac{\partial x_j}{\partial c_j} + (R^i_i(x_j, x_i) + R^j_i(x_j, x_i) - c_i) \frac{\partial x_i}{\partial c_j} \\ & - x_j + Z'(\theta_j - c_j) - \lambda_j(\theta_j) Z''(\theta_j - c_j)] f(\theta_i) = \\ & = -x_j + Z'(\theta_j - c_j) - \lambda_j(\theta_j) Z''(\theta_j - c_j)] f(\theta_i) = 0, \quad j = 1, 2 \quad (A3) \end{aligned}$$

Since $\theta_j = \underline{\theta}$ is a free boundary for $j = 1, 2$, $\lambda_j(\underline{\theta}) = 0$, $j = 1, 2$. Equation (A2) then implies $\lambda_j(\theta_j) = -F(\theta_j)/f(\theta_j) < 0$, $j = 1, 2$. Using $\lambda_j(\theta_j) = -F(\theta_j)/f(\theta_j)$ in (A3) results in (3), Q.E.D.

OBSERVATION 2:

The utility received by the agent is independent of the performance of the competitor. The conditional expected payment $E[W(\theta_j, \theta_i) | \theta_j]$ consists of a compensation equal to the reservation utility plus the conditional expected disutility of effort, and in addition an information rent.

PROOF: Equations (3) determine the equilibrium contract cost functions $c_j(\theta_j, \theta_i)$. The remaining part of the contract, the utility functions $U_j(\theta_j, \theta_i)$, can then be recursively solved from the incentive compatibility constraints

$$E_{\theta_i} [U_j'(\theta_j, \theta_i) + Z'(\theta_j - c_j(\theta_j, \theta_i)) \partial c_j(\theta_j, \theta_i) / \partial \theta_i] = 0$$

together with the boundary condition $E_{\theta_i} [U_j(\bar{\theta}_j, \theta_i)] = u$ imposed by the individual rationality constraint. Let $U_j(\theta_j) = E_{\theta_i} [U_j(\theta_j, \theta_i)]$. The assumption about independent distributions of θ_1 and θ_2 ensures that $U_j(\theta_j)$ is incentive compatible. Using $U_j(\theta_j)$ instead of $U_j(\theta_j, \theta_i)$ does not affect the value of the objective function so $U_j(\theta_j)$ must be a solution, which proves the first part of the lemma. The individual rationality constraint directly implies that $E_{\theta_i} [W_j(\theta_j, \theta_i)] \geq u + E_{\theta_i} [Z(\theta_j - c_j(\theta_j, \theta_i))]$ and the incentive compatibility constraint implies that the inequality is strict for all $\theta_j \neq \bar{\theta}$, Q.E.D.

Let $B(\theta_j) \equiv E_{\theta_i} [W_j(\theta_j, \theta_i) - Z(\theta_j - c_j(\theta_j, \theta_i))] - u$, so that $B(\theta_j)$ is the *conditional expected information rent*, i.e., the expected amount in excess of the reservation utility the

employee receives as a result of the information asymmetry.

OBSERVATION 3:

$B(\theta_j) = \int_{\theta_j}^{\bar{\theta}} E_{\theta_i} [Z'(t - c(t, \theta_i))] dt$. In particular, $B(\theta_j)$ is decreasing in θ_j , $B(\bar{\theta}) = 0$, and a contract that induces a higher effort level at all θ_j also pays a higher information rent for all $\theta_j < \bar{\theta}$.

PROOF: $B'(\theta_j) = E_{\theta_i} [\partial W_j(\theta_j, \theta_i) / \partial \theta_j - Z'(\theta_j - c_j(\theta_j, \theta_i))(1 - \partial c_j(\theta_j, \theta_i) / \partial \theta_j)] =$
 $- E_{\theta_i} [Z'(\theta_j - c_j(\theta_j, \theta_i))]$ by incentive compatibility. Employing the boundary condition $E_{\theta_i} [W_j(\bar{\theta}, \theta_i) - Z(\bar{\theta} - c_j(\bar{\theta}, \theta_i))] = u$ gives $B(\bar{\theta}) = 0$, and the integral representation of $B(\theta_j)$ follows. The remaining part of the proposition follows from the fact that $Z'(e)$ is positive and increasing, Q.E.D.

Sufficient conditions

We conclude with a discussion of sufficient conditions. The Euler equations are sufficient if the Lagrangean is concave in $(U_1, U_2, U'_1, U'_2, c_1, c_2)$. Since the Lagrangean is linear in (U_1, U_2, U'_1, U'_2) it suffices if it is concave in c_1 and c_2 . We have

$$\frac{\partial L}{\partial c_j} = -x_j + Z' - \lambda_j Z'' \quad j=1,2$$

so the Lagrangean is concave in c_1 and c_2 if $\frac{\partial^2 L}{\partial c_j^2} < 0$, $j=1,2$ and

$$\begin{aligned} \frac{\partial^2 L}{\partial c_1^2} \frac{\partial^2 L}{\partial c_2^2} - \left(\frac{\partial^2 L}{\partial c_1 \partial c_2} \right)^2 \\ = \left(-\frac{\partial x_1}{\partial c_1} - Z'' + \lambda_1 Z''' \right) \left(-\frac{\partial x_2}{\partial c_2} - Z'' + \lambda_2 Z''' \right) - \frac{\partial x_1}{\partial c_2} \frac{\partial x_2}{\partial c_1} > 0 \end{aligned}$$

The linear-quadratic parameterization implies

$$\frac{\partial^2 L}{\partial c_j^2} = \frac{1}{2(1-\gamma^2)} - 1$$

which is negative since $0 < \gamma < 1$, and

$$\frac{\partial^2 L}{\partial c_1^2} \frac{\partial^2 L}{\partial c_2^2} - \left(\frac{\partial^2 L}{\partial c_1 \partial c_2} \right)^2 = \left(\frac{1}{2(1-\gamma^2)} - 1 \right)^2 - \left(\frac{\gamma}{2(1-\gamma^2)} \right)^2$$

which is positive if $\gamma < 1/2$.

Finally, the validity of the first order approach requires that the solution $c_j(\theta_j, \theta_i)$ is strictly increasing in θ_j . Given that the Lagrangean is concave in c_j , $c_j(\theta_j, \theta_i)$ is strictly increasing in θ_j if $Z'' \cdot (2 - F(\theta_j)f'(\theta_j)/(f(\theta_j))^2) + Z''' \cdot F(\theta_j)/f(\theta_j) > 0$. With our quadratic parameterization of Z the last term vanishes and this condition reduces to $f'(\theta_j)/f(\theta_j) < 2f(\theta_j)/F(\theta_j)$.

3. The duopolist's contract problem

Consider now the duopolist's maximization problem (DP). The methodology is very similar to that applied to the monopolist's problem above. Again, we limit our attention to piecewise differentiable contracts, and begin by establishing the validity of the first order approach.

LEMMA 4: $c_j(\theta_j)$ is nondecreasing.

PROOF: By a similar argument as in the proof of lemma 1, incentive compatibility yields

$$Z(\theta_j - c_j(\hat{\theta}_j)) - Z(\hat{\theta}_j - c_j(\hat{\theta}_j)) \geq Z(\theta_j - c_j(\theta_j)) - Z(\hat{\theta}_j - c_j(\theta_j)) \quad (\text{A4})$$

Suppose $\theta_j > \hat{\theta}_j$. Then

$$\theta_j - c_j(\hat{\theta}_j) - (\hat{\theta}_j - c_j(\hat{\theta}_j)) = \theta_j - \hat{\theta}_j = \theta_j - c_j(\theta_j) - (\hat{\theta}_j - c_j(\theta_j)) > 0$$

and (A4), together with the strict convexity of Z , imply

$$\theta_j - c_j(\hat{\theta}_j) \geq \theta_j - c_j(\theta_j) \quad \text{or} \quad c_j(\theta_j) \geq c_j(\hat{\theta}_j), \quad \text{Q.E.D.}$$

LEMMA 5: If $c_j(\theta_j)$ is nondecreasing then the first order condition for incentive compatibility is also locally sufficient.

PROOF: Let $\varphi(\theta_j, \hat{\theta}_j) \equiv E_{\theta_i} [U_j(\theta_j, \hat{\theta}_j, c_i(\theta_i))]$, i.e., φ_j denotes the expected utility of agent j when he reports $\hat{\theta}_j$ and his true value is θ_j . By a similar argument as in the proof of lemma 2, the second order conditions are

$$\frac{\partial^2 \varphi_j(\theta_j, \hat{\theta}_j)}{\partial \theta_j \partial \hat{\theta}_j} = Z'(\theta_j - c_j(\theta_j)) \frac{dc_j(\theta_j)}{d\theta_j} \geq 0$$

Convexity of Z establishes the lemma, Q.E.D.

LEMMA 3: If $c_j(\theta_j)$ is strictly increasing local sufficiency implies global sufficiency.

PROOF: Same as for lemma 3.

Lemma 4, 5 and 6 provide conditions for the validity of the first order approach for the duopoly case. If the required cost function $c_j(\theta_j)$ is strictly increasing in θ_j , then the first order conditions are equivalent to incentive compatibility.

Next, we characterize the solution of the duopolist's contract problem.

OBSERVATION 4:

The optimal required cost functions $c_j(\theta_j)$, satisfy

$$\begin{aligned} E_{\theta_i}[x_j] - E_{\theta_i}[R_i^j(x_j, x_i) \frac{\partial x_j}{\partial c_j}(r_i(x_j, x_i) - v_i(x_j, x_i))] = \\ = Z'(\theta_j - c_j) + \frac{F(\theta_j)}{f(\theta_j)} Z'(\theta_j - c_j) \end{aligned} \quad (9)$$

PROOF: Replacing the incentive compatibility constraints with the corresponding first order condition $E_{\theta_i}[U'_j(\theta_j, c_i(\theta_i)) + Z'(\theta_j - c_j(\theta_j))] = 0$, $\forall \theta_j$, the Lagrangean for the contract problem is

$$\begin{aligned} L = \{R^j(x_j, x_i) - c_j(\theta_j)x_j - U_j(\theta_j, c_i(\theta_i)) - Z(\theta_j - c_j(\theta_j)) + \mu(\theta_j, \theta_i) \frac{\partial c_j}{\partial \theta_i} \\ + \lambda_j(\theta_j)[U'_j(\theta_j, c_i(\theta_i)) + Z'(\theta_j - c_j(\theta_j))]\} f(\theta_j) f(\theta_i) \end{aligned}$$

where again $x_j = x_j(c_j(\theta_j), c_i(\theta_i), k)$, $x_i = x_i(c_i(\theta_i), c_j(\theta_j), k)$. Here we have written unit cost $c_j(\theta_j, \theta_i)$ as a function of θ_j and θ_i and imposed the restriction $\partial c_j / \partial \theta_i = 0$ to ensure that c_j only depends on θ_j . This puts the maximization problem into a standard framework. The

Euler equations w r t U_j and c_j are

$$-f(\theta_j) = \frac{\partial}{\partial \theta_j} (\lambda_j(\theta_j) f(\theta_j)) \quad (\text{A5})$$

$$\begin{aligned} [(R_j^j(x_j x_i) - c_j) \frac{\partial x_j}{\partial c_j} + R_i^j(x_j x_i) \frac{\partial x_i}{\partial c_j} - x_j + Z'(\theta_j - c_j) - \lambda_j(\theta_j) Z'(\theta_j - c_j)] f(\theta_j) = \\ = \frac{\partial}{\partial \theta_i} (\mu(\theta_j, \theta_i) f_i(\theta_i)) \end{aligned} \quad (\text{A6})$$

Since $\theta_j = \underline{\theta}$ is a free boundary, $\lambda_j(\underline{\theta}) = 0$. Equation (A5) then implies $\lambda_j(\theta_j) = -F(\theta_j)/f(\theta_j) < 0$. Transversality also yields $\mu(\theta_j, \underline{\theta}_i) f_i(\underline{\theta}_i) = \mu(\theta_j, \bar{\theta}_i) f_i(\bar{\theta}_i) = 0$. Integrating (A6) w r t θ_i using (5), (7) and $\lambda_j(\theta_j) = -F(\theta_j)/f(\theta_j)$ results in (9), Q.E.D.

OBSERVATION 5:

The payment function W is independent of the performance of the competitor and it consists of a compensation equal to the reservation utility plus the disutility of effort, and in addition an information rent.

PROOF: Equation (9) and the corresponding condition for the other firm determine the equilibrium contract cost functions $c_j(\theta_j)$. The remaining part of the contract, the payment functions $W_j(\theta_j, c_i)$, can then be recursively solved from the incentive compatibility constraint

$$E_{\theta_i} [W_j'(\theta_j, c_i(\theta_i))] + Z'(\theta_j - c_j(\theta_j)) c_j'(\theta_j) = 0$$

together with the boundary condition $E_{\theta_i} [W_j(\bar{\theta}_j, c_i(\theta_i))] - Z(\bar{\theta} - c_j(\bar{\theta})) = u$ imposed by the individual rationality constraint. It is immediately obvious from these conditions and the

assumption about independent distributions of θ_1 and θ_2 that anything that can be achieved by a payment function $W_j(\theta_j, c_i)$ can also be achieved, with the same expected cost, by the payment $W_j(\theta_j) = E_{\theta_i}[W_j(\theta_j, c_i(\theta_i))]$, i.e., by a payment function which is independent of the performance of the competitor. The individual rationality constraint directly implies that $W_j(\theta_j) \geq u + Z(\theta_j - c_j(\theta_j))$ and the incentive compatibility constraint implies that the inequality is strict for all $\theta_j \neq \bar{\theta}$, Q.E.D.

Let $B(\theta_j) \equiv W(\theta_j) - Z(\theta_j - c(\theta_j)) - u$, so that $B(\theta_j)$ is the *information rent*, i.e., the amount in excess of the reservation utility the employee receives as a result of the information asymmetry.

OBSERVATION 6:

$B(\theta_j) = \int_{\theta_j}^{\bar{\theta}} Z'(t - c(t)) dt$. In particular, $B(\theta_j)$ is decreasing in θ_j , $B(\bar{\theta}) = 0$, and a contract that induces a higher effort level at all θ also pays a higher information rent for all $\theta_j < \bar{\theta}$.

PROOF: $B'(\theta_j) = W'(\theta_j) - Z'(\theta_j - c(\theta_j))(1 - c'(\theta_j)) = -Z'(\theta_j - c(\theta_j))$ by incentive compatibility. Employing the boundary condition $W(\bar{\theta}) - Z(\bar{\theta} - c(\bar{\theta})) = u$ gives $B(\bar{\theta}) = 0$, and the integral representation of $B(\theta_j)$ follows. The remaining part of the proposition follows from the fact that $Z'(e)$ is positive and increasing, Q.E.D.

Sufficient conditions

We conclude our characterization of the solution with a discussion of sufficient conditions. The Euler equations are sufficient if the Lagrangean is concave in $(U_j, U'_j, c_j, \partial c_j / \partial \theta_j)$. Since the Lagrangean is linear in $(U_j, U'_j, \partial c_j / \partial \theta_j)$ it suffices if it is concave in c_j . Using (5) and (7) we have

$$\frac{\partial L}{\partial c_j} = R_i^j \frac{\partial x_j}{\partial c_j} (r_i - v_i) - x_j + Z' - \lambda_j Z''$$

so the Lagrangean is concave in c_j if

$$\frac{\partial^2 L}{\partial c_j^2} = \partial(R_i^j \frac{\partial x_j}{\partial c_j} (r_i - v_i)) / \partial c_j - \frac{\partial x_j}{\partial c_j} - Z'' + \lambda_j Z''' < 0$$

With our linear-quadratic parameterization and Bertrand or Cournot competition this condition reduces to

$$\frac{\partial x_j}{\partial c_j} + Z'' = \frac{\partial x_j}{\partial c_j} + 1 > 0$$

which, from inspection of the coefficients of the output functions, can be seen to be satisfied. With joint profit maximization the concavity condition becomes

$$R_i^j \frac{\partial x_j}{\partial c_j} (v_i \frac{\partial x_j}{\partial c_j} - \frac{\partial x_i}{\partial c_j}) / x_j - \frac{\partial x_j}{\partial c_j} - 1 < 0$$

The first term is negative so this condition is satisfied since $\frac{\partial x_j}{\partial c_j} + 1 > 0$.

Finally, the validity of the first order approach hinges on that the solution $c_j(\theta_j)$ is strictly increasing. Given that the Lagrangean is concave in c_j , $c_j(\theta_j)$ is strictly increasing if $Z'' \cdot (2 - F(\theta_j)f'(\theta_j)/(f(\theta_j))^2) + Z''' \cdot F(\theta_j)/f(\theta_j) > 0$. With our quadratic parameterization of Z the last term vanishes and this condition reduces to $f'(\theta_j)/f(\theta_j) < 2f(\theta_j)/F(\theta_j)$.

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Figure 1

Figure 2

Figure 3