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MANAGERIAL EFFORT INCENTIVES,
X-INEFFICIENCY AND INTERNATIONAL TRADE

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ABSTRACT

The paper investigates formally the old idea that competition brought about by international trade yields welfare gains by reducing internal slack — "X–inefficiency" — in firms. The paper employs a contract–theoretic based, general equilibrium, trade model to demonstrate how international trade affects the contractual relationship in firms so as to induce more managerial effort supply. This increased effort supply is shown to be beneficial from a welfare perspective, partly because firms are X–inefficient. But, while these results partly confirm the economic "folklore", the folklore can be questioned on other grounds.

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The idea that international trade yields welfare gains by reducing internal slack in firms, is deeply rooted in the "folklore" of economic thought. Examples abound both in the policy debate and in scientific work. A noticeable recent example is provided by the European Commission's (1988, p. 126) assessment of the consequences of the internal market program, which argues that "...the new competitive pressures brought about by the completion of the internal market can be expected to...produce appreciable gains in internal efficiency...[which will] constitute much of what can be called the dynamic effects of the internal market...".

These arguments have largely been neglected by trade theory. Internal inefficiencies in firms have been hard to reconcile with profit-maximization, at the same time as the arguments about the disciplining effect of trade are usually quite vague. There is a small literature on trade and "X-inefficiency" going back at least to Scitovsky (1958), which includes writings by Balassa (1975), Bergsman (1974), Corden (1970, 1974), and Martin (1978). Typically, it portrays situations in which owners—cum—managers of firms choose the amount of effort to supply, facing a trade-off between profit income and leisure implied by their choice of effort. Occasionally, efficiency is associated directly with the level of effort, even though this usage of the term is criticized by some writers on obvious grounds (e.g. Johnson (1970)). In general, the aim of the literature is to identify circumstances under which the social opportunity cost of leisure exceeds the corresponding private cost, for example due to under-taxation of leisure, and where, as a result, effort is undersupplied from a social point of view. Apart from this small literature on X-inefficiency, arguments about gains from trade due to internal reallocation of resources in firms have not been formally scrutinized, to the best of our knowledge.\footnote{The only exception we are aware of is Horn et al. (1990).} However, the weight attached in the
policy discussion and elsewhere to these types of arguments suggests that they deserve a more careful examination. The aim of this paper is to take a small step in this direction.

The paper examines whether international trade, by increasing the degree of competition in product markets, may bring welfare gains that are related to the existence of a particular, but well-defined, form of internal inefficiency in firms. Specifically, we assume that the owner of a firm must separate himself from the control of the production process, by hiring a manager. The task of the manager is to organize production workers so that they produce a certain output. The number of workers that is required for this task depends partly on firm-specific circumstances that are beyond the control of the owner and the manager, and partly on the amount of "effort" the manager exerts in organizing production. Due to the separation of ownership from control, the owner can observe neither the exogenous circumstances that affect productivity, nor the efforts expended by the manager. The owner can observe resulting costs, but cannot infer whether a high cost is due to unfavorable exogenous circumstances, or to poor organization.

The separation of ownership and control does not cause any deviation from the full information, first-best allocation if managers are risk neutral, sufficiently severe punishments for poor performance can be contracted, and there is symmetric information at the contracting date. However, there are in actuality lower limits to the ex post utility that a contract between an owner and a manager can meaningfully stipulate. The optimal contract may then imply that the benefit to the owner of a marginal unit of effort exceeds the cost of this unit to the manager. The potential Pareto improvement does not come about, since the cost to the owner of inducing the manager to exert the effort is too high due to the limits imposed by the contract technology. The effort level induced by the

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"Effort" is not primarily intended to capture working hours. For instance, it requires "effort" to make uncomfortable decisions, like firing people, or punishing poor personal performance. An alternative interpretation of effort is that of (lack of) "perks" that are not directly observable to firm owners. The larger these perks, the smaller the resources available for production purposes, and the lower the productivity of production workers.
optimal contract is thus inefficiently low in this respect, and variable production costs are consequently inefficiently high. These features of the firm's internal allocation seem to correspond closely to central themes in popular thinking about X-inefficiency. For this reason we denote a firm with the above mentioned contract problem as "X-inefficient", even though our depiction of this form of inefficiency is narrow in comparison to some of the informal writings on the concept. Our approach also differs from that in the earlier formal literature on X-inefficiency and trade, in that the inefficiency considered here is truly internal, in the sense that there are alternative allocations of resources within firms that would increase the total surplus, but that do not come about due to relation-specific problems within firms.

Our main issue is whether international trade, by increasing the degree of competition, has welfare effects that are related to this X-inefficiency. It is often suggested that lack of managerial effort could be detrimental to welfare, since managers influence the marginal productivity of other factors. In a general equilibrium context, with endogenous factor rewards, lack of managerial effort would not only affect owners' profits, but also incomes of other factor owners. To deal with this question, a general equilibrium trade model is developed in which firms are X-inefficient in the above described sense. The model abstracts as far as possible from sources of gains from trade that are already well established. Countries are assumed to have identical preferences and possibly also identical production technologies, and there is one variable factor of production — labor — in fixed supply, that is internationally immobile. There is a fixed number of products, so trade will not affect product variety. Each economy has a given number of firms. With symmetric demand and cost conditions, and with the same number of firms in each industry, trade will not yield any inter-sectoral reallocation of labor. Hence, gains from specialization according to comparative advantage are ruled out, and there are no gains from trade due to exploitation of returns to scale of the conventional type.

There is a large number of homogeneous goods. Each good is produced in each
country by a given and limited number of X–inefficient firms that compete in Cournot fashion. Due to the symmetry of the model, trade need not actually take place in equilibrium. But the mere possibility of trade will nevertheless have real consequences, since it increases the degree of competition, in the sense of increasing the number of firms producing the same output. This will affect firms' decisions with regard to effort in two ways. First, with larger perceived individual demand elasticities, firms will have an incentive to increase output. With increasing output, it becomes profitable to induce managers to exert more effort, and thus to reduce the utilization of labor per unit of output. Second, when all firms want to expand production, the demand for labor will increase, which will increase real wages of production workers. This general equilibrium ramification reinforces the tendency toward larger supply of managerial effort, and lower unit input requirements of production workers. Hence, trade does not affect the source of the internal inefficiency in firms, but changes the circumstances under which it affects the economy.

The main contribution of the paper is to highlight a source of gains from international trade that has not been formally demonstrated (as far as we know): gains from increased managerial effort supply that are attributable to the existence of internal inefficiencies in firms. The increase in the supply of managerial effort induced by trade, increases the joint surplus of the owner and the manager, even though it lowers the owner's surplus. In standard trade theory the gains from trade come from reallocation of resources between firms and sectors. The present model, on the other hand, is constructed so that no reallocation of resources would be observed as a result of the opening up of trade; nevertheless, trade induces each firm to produce a larger output volume.

The content of the paper is as follows. The next section introduces the model, and considers in particular the decision problem facing the owner of an X–inefficient firm. Section II compares the X–inefficient firm to an internally efficient firm, and explains the sense in which firms are "X–inefficient". Section III investigates the allocational
consequences of international trade. Section IV considers welfare aspects of X–inefficiency and trade. Some concluding remarks are made in section V. Part of the formal analysis is relegated to an appendix.

I. THE MODEL OF THE FIRM

Consider a world economy in which goods are produced in \( m \) sectors. In a typical sector \( j \) firm \( i \) produces \( z^i_j \) units of a homogeneous output. The international product market is fully integrated, so there is a uniform international product price \( p^j \) in each sector \( j \) in equilibrium. All individuals have identical preferences. Each individual’s utility depends on his consumption of the \( m \) goods, and on the amount of effort the individual exerts at work. Individuals are risk–neutral in income, and have the indirect utility function

\[
u^s = \frac{I^s}{G(p)} - Z(e^s) \tag{1}\]

where \( I^s \) and \( e^s \) are, respectively, the income and effort supply of individual \( s \), \( p = (p^1, ..., p^m) \) is the vector of goods prices, \( G(p) \) is a price index corresponding to the sub–utility function over goods, and where the function \( Z(e) \) is increasing and strictly convex. The corresponding inverse demand function for sector \( j \)‘s output is \( P^j(X, I) \), where \( X^j \) is world output in sector \( j \), \( X = (X^1, ..., X^m) \), and \( I \) is world income.

A typical firm \( i \) in sector \( j \) has an owner who maximizes the real profit of the firm. This owner employs production workers to perform the actual production, and a manager to organize production. The more effort the manager exerts, the better organized is production, and the smaller is the number of workers that is required to produce a given output volume, all else given. But, the unit input requirement also depend on firm–specific circumstances that affect the productivity of the firm and that are beyond the
control of the manager. Let the managerial effort supply that is required to obtain a unit input requirement $k^{ij}$ be given by the additively separable function (subscripts denote partial derivatives throughout the paper)

$$
e^{ij} = E(k^{ij}, \theta^{ij}); \ E_k < 0, \ E_\theta > 0, \ E_{k\theta} = 0$$

(2)

where the parameter $\theta^{ij}$ captures exogenous firm–specific circumstances that affect the productivity in the firm.

The value of the parameter $\theta^{ij}$ is unknown at the contracting date to both the owner and the prospective manager. The stochastic variables $\theta^{ij}$ are i.i.d. with distribution function $F$ and corresponding density function $f$, supported on the interval $[\underline{\theta}, \overline{\theta}]$. Once the contract is accepted, the manager learns about the state–of–affairs in the firm — i.e. observes a realization of $\theta^{ij}$. Knowing $\theta^{ij}$ the manager decides on how much effort to exert, thus determining the cost level in the firm. For simplicity, it is assumed that the owner of the firm decides on the output volume at the same stage as a manager is contracted.\(^3\)

The owner cannot observe the realization of $\theta^{ij}$, nor the effort expended by the manager. There is therefore a moral hazard problem in the firm, which has to be taken into account when designing the contract. As usual, this type of contract problem can be simplified if it is viewed as a revelation game, in which the manager reports his observation of $\theta^{ij}$. By the revelation principle, the analysis can be confined to incentive compatible (IC) contracts, i.e., contracts that induce the manager to report his private information

\(^3\) An alternative assumption would be to determine output ex post the realization of the firm’s cost. Firms’ outputs would then depend on the realized values of $\theta^{ij}$. (Horn et al. (1991) analyze a partial equilibrium model of this type.) While such an assumption might have some intuitive appeal, it would also complicate the welfare analysis considerably if outputs depend on realized values of $\theta$, since, with a finite number of firms, consumers would be exposed to risk, and a consequence of trade would be to affect consumers’ exposure to such risk. This aspect of trade may be important in special cases, but does not appear to be at the core of the issue at hand.
truthfully. Let $\theta$ be the true realized value of $\theta^i$, let $v(\theta')$ be the wage received by the manager if he reports the value $\theta'$, and let $k'_{1,1}$ be the unit input requirement that the manager is stipulated by the contract to deliver in this case (firm and sector indices are suppressed temporarily). Also, let the manager’s utility from truth-telling be

$$U(\theta) = \frac{v(\theta)}{G(p)} - Z(E(k(\theta), \theta))$$

(3)

As shown in the appendix, truthful reporting maximizes the manager’s *ex post* utility only if

$$U_\theta(\theta) + Z_\theta(E(k(\theta), \theta) E_\theta(k(\theta), \theta) = 0, \ \forall \theta \epsilon (\underline{\theta}, \bar{\theta})$$

(IC)

When designing the contract, there are two additional constraints that the owner has to take into account. First, there is a lower limit to the *ex post* utility level that the manager will accept — a "limited punishment (liability)" (LP) constraint — which is taken to be $\underline{u}$: $U(\theta) \geq \underline{u}, \ \forall \theta \epsilon (\underline{\theta}, \bar{\theta})$. But, (IC) implies that $U_\theta(\theta) < 0$, and thus that the LP constraint can only bind for $\theta = \bar{\theta}$, if it binds at all. The limited punishment constraint is therefore

$$U(\bar{\theta}) \geq \underline{u}$$

(LP)

Secondly, the contract must fulfill the individual rationality (IR) constraint that it yields an expected utility that is no less than what the prospective manager would get in his best alternative occupation, $u^w$.
\[ \int_0^\theta f(\theta) U(\theta) d\theta \geq w^w \] (IR)

The contract problem is similar to that of Laffont and Tirole (1986) who analyzed the optimal regulatory policy for a regulator controlling a private monopoly with superior cost information, which can influence its cost performance in a manner not observable to the regulator. Here, the owner's problem is to design a contract that conveys effort incentives to the manager, while at the same time it restricts as much as possible the rent that the manager acquires due to his superior information. Moreover, when the owner designs the contract he must take into account the interrelatedness between the contract and the optimal output volume.

The present formulation also differs from that of Laffont and Tirole (1986) in that information is here assumed to be symmetric at the contracting date. This assumption is inessential to the main points of the paper, since it will be assumed for most part of the analysis that the IR constraint does not bind in equilibrium. There are a couple of reasons for the inclusion of this constraint. The first is that it makes the model more naturally interpreted as capturing "longer-run" aspects of trade and X-inefficiency, with all employees being identical, at least in an \textit{ex ante} sense. Trade theory is typically concerned with long-run issues, and we want to stay as close as possible to this tradition. Secondly, as will be argued below, it highlights the fact that if trade makes the alternative employments prospects of managers good enough, then the solution to the contracting problem approaches the first best, and hence the X-inefficiency to be defined below disappears. It should also be noted that when the IR restriction is not binding, the LP restriction can be seen as an \textit{ex post} IR constraint. An interpretation is then that the manager, who has a choice between working in the firm, or leaving the firm and obtaining a utility \( \bar{u} \), knows at the contracting date the state of the affairs, i.e. \( \theta \), in his firm.

Let us now consider the owner's decision problem. Let \( w \) be the wage obtained by
production workers, which the firm takes as given. Production workers perform an exogenously given effort level $e^w$. Firms have Cournot conjectures, and the individual firm neglects its impact on the economy’s price index, but takes into account its impact on the price of the sector’s good (i.e., there is a large number of sectors with relatively few firms in each). Formally, the problem facing the owner of firm $i$ in sector $j$ is to choose $x^{ij}$, $k^{ij}(\theta)$, and $v^{ij}(\theta)$, taking into account the constraints (IR), (LP), and (IC), so as to

$$
\max \frac{1}{G(p)} \int_0^\theta [P^j(X,I)x^{ij} - k^{ij}(\theta)w^jz^{ij} - v^{ij}(\theta)]f(\theta)d\theta
$$

(4)

As shown in the appendix, the first—order conditions can then be expressed as

$$
\frac{p^j}{G(p)} \left(1 - \frac{x^{ij}}{X^j} \frac{1}{\gamma^j(p)} \right) = \frac{k^{ij}}{G(p)} \frac{w}{G(p)}
$$

(5)

$$
x^{ij} \frac{w}{G(p)} = -Z_e(E(k(\theta),\theta)) E_k(k(\theta),\theta) + T(\theta)
$$

(6)

where $\gamma^j(p)$ is the market elasticity of demand for sector $j$, and where $k^{ij}$ is the expected number of production workers that is required in order to produce one unit of output, i.e. the expected unit input requirement:

$$
\bar{k}^{ij} \equiv \int_0^\theta k^{ij}(\theta)f(\theta)d\theta
$$

(7)

Equation (5) is just a standard first—order condition with respect to output. The left—hand side of (6) is the gain to the owner of a marginal reduction in the unit input requirement in
state $\theta$. The first term on the right-hand side is the disutility this reduction causes the manager. This is the real resource sacrifice of the lower unit requirement. The term $T(\theta) > 0$ is the part of the marginal cost of reducing the unit input requirement, that stems from the incentive scheme; the formal definition of $T(\theta)$ is given in the appendix.

II. X–INEFFICIENCY

In order to assess the impact of the informational problem in the firm, we first consider the benchmark case where the owner and the manager can observe the realized $\theta^{ij}$, and hence make the contract contingent on this observation. In this case the condition for an optimal choice of output is the same as above, i.e., that marginal revenue equals expected marginal cost. (The fact that output is state–independent introduces a certain inefficiency, which is not of any particular interest in the present context.) The condition for an optimal choice of the unit input requirement, and hence of effort, is given by (6) with $T(\theta) \equiv 0$. The marginal value to the owner of reducing the unit input requirement then equals the marginal disutility this causes the manager, appropriately measured. This implies that there are no gains from trade between the owner and the manager left unexploited – the internal organization of the firm is efficient.

Now return to the informationally constrained firm. Whether in equilibrium the LP constraint, the IR constraint, or both, effectively restrict the firm, depends partly on the discrepancy between $u^w$ and $\bar{u}$. Consider first situations where only the LP constraint is binding, i.e., where $u^w$ is sufficiently low relative to $\bar{u}$. In this case the firm will induce the manager to choose an effort level that is efficient in an *ex post* sense in the state with lowest $\theta$, i.e., when exogenous circumstances that affect the productivity of the firm are as favorable as possible. For all other $\theta$ the effort level is lower than what is *ex post* efficient, given the output volume and the production worker wage rate. Consequently, variable production costs are higher than in the efficient firm, for any given output volume and
production worker wage.\textsuperscript{4} Note that the firm could produce more output with the same work-force if the informational problem caused by the separation of ownership and control, were somehow overcome. Such a reallocation of internal resources would increase the total surplus in the firm. Due to these properties we define a firm in this model to be $X$-inefficient if the LP constraint binds, i.e., if $T(\theta) > 0$, for some $\theta$. This $X$-inefficiency is obviously detrimental to the interests of the owner. But, the information asymmetry benefits the manager, who gets an "information rent", i.e., an expected utility in excess of $u^\psi$.

It is also possible that the IR and the LP constraint bind simultaneously. Then the firm is still $X$-inefficient, but in contrast to the situation where only LP binds, the informational problem does not bring the manager a rent.

Finally, if the required expected utility is sufficiently high relative to the minimum \textit{ex post} utility level, the latter will not be utilized in the contract, and only the IR constraint binds. As shown in the appendix, the $X$-inefficiency vanishes in this case. In this situation the gains from trade between the owner and the manager are exhausted, and the firm chooses the same allocation and expected salary to the manager, as would an informationally unconstrained firm do under the same exogenous circumstances.

This completes the presentation of the firm, and we now turn to the description of the economy in which it interacts with other firms in product and factor markets.

\section*{III. EFFORT SUPPLY AND TRADE}

\textsuperscript{4} Firms are internally inefficient relative to the full information situation. Obviously, this definition of inefficiency could be questioned on the ground that the informational problem in the firm is really part of the technology. According to this latter view, it is no more interesting to compare the actual informationally constrained contract with an informationally unconstrained one, than is it to consider the implications of some other change in the production technology of the firm; see e.g. Stigler's (1976) critical discussion of Leibenstein's (1966) concept "$X$-inefficiency". Our usage of the term "inefficiency" corresponds to that of Holmström and Myerson (1983), who distinguish between incentive efficiency and \textit{ex post} classical efficiency. The present contract is inefficient in the latter, but not the former, sense.
Assume the world economy consists of two countries, Home and Foreign. In the Home (Foreign) economy there are \( L^h \) (\( L^f \)) \( \text{ex ante} \) identical individuals who are not owners of firms. Each economy is taken to be sufficiently large so that \( w \) can be treated as a deterministic variable. Individuals can choose not to enter the labor market, in which case they can perform some non-market activity. For simplicity, let their utility level be \( \bar{u} \) in this case, i.e., the lowest \( \text{ex post} \) utility level that the manager can be exposed to through the contract; this saves on notation but is not essential to the results below. When employment yields (expected) utility levels higher than \( \bar{u} \) all individuals prefer employment. An individual can reject an offer to become a manager, and instead work as production workers. The level \( \text{ex ante} \) reservation utility level \( u^w \) is therefore given by the utility level achieved by production workers:

\[
u^w = \frac{w}{G(p)} - Z_e \tag{8}
\]

If the contract yields an expected utility in excess of \( u^w \), all individuals prefer to become managers instead of production workers. However, it is not possible for individuals to underbid the expected utility in the contract, since there is no other contract that yields appropriate incentives to exert effort, at the same time as it provides lower expected utility.

In each sector there are \( n^h \) domestic firms and \( n^f \) foreign competitors. There is no exit or entry.\(^5\) This assumption removes gains from trade due to exploitation of returns to scale, and allows us to concentrate on the role of firms' internal organization.

For the sake of brevity, we limit attention to situations where the IR constraint

\(^5\) It is not entirely clear how free entry and exit should be modelled. A constraint that requires expected profit to be zero is problematic, since there may then be states in which costs exceed revenue. One possibility might be to assume entry occurs until profit in the least profitable state is zero.
does not bind. We will also impose more structure on the model, in order to keep the analysis simple, and to avoid problems with multiple equilibria. First, the relationship between effort and the unit input requirement is taken to be linear: \( E(\theta, k) = \theta - k \). Secondly, the function capturing the disutility of effort is assumed to be quadratic: \( Z(e) = e^2 / 2 \). These assumptions imply, from the first-order condition (6), that

\[
\dot{y}(\theta) = \theta + \frac{F(\theta)}{f(\theta)} - \frac{z^w}{G(p)} \tag{9}
\]

where the term \( F(\theta)/f(\theta) \) corresponds to the term \( T(\theta) \) in (6). Thirdly, the sub-utility over goods is assumed to be of a CES type. Then, with a large number of sectors, the elasticity of demand \( \gamma^j(p) \) is approximately constant. Finally, it is assumed that demand is symmetric across sectors, and that all firms within a particular economy face identical decision problems, and hence make identical choices. In a symmetric equilibrium \( p^j = p, \gamma^j = \gamma, \) and \( X^j = X. \) Furthermore, for all firms in the same country, \( z^i = z, k^{ij}(\cdot) = k(\cdot), \) and \( u^{ij}(\cdot) = u(\cdot). \)

It should be stressed that the symmetry assumptions, in combination with the assumption of a constant number of firms, remove any possibility of reallocation of labor between sectors and firms in equilibrium. Any impact of trade must hence be due to changes in the allocation of resources within firms.

We can now specify the conditions for general equilibrium in the Home country, for an arbitrary level of Foreign production. To this end, let \( p \) be the numeraire, such that \( p/G(p) = 1; \) the function \( G \) henceforth corresponds to the CES price index. The first-order conditions (5) and (9), after taking the expectations with respect to \( \theta \) in (9), are now

\[
1 - \frac{z^h}{X^\gamma} = k^h \omega^h \tag{10}
\]
\[ \bar{k}^h = \bar{\omega} - z^h \omega^h \quad (11) \]

where \( \omega \equiv w/G(p) \) is the real wage of production workers. Finally, the full employment condition for the domestic labor market requires the demand for production workers plus the demand for managers, to equal the number of individuals in the domestic economy:

\[ mn^h(z^h \bar{k}^h + 1) = L^h \quad \text{(LME)} \]

Equations (10), (11), and (LME) determine the Home economy's general equilibrium values of \( x^h \), \( \bar{k}^h \), and \( \omega^h \), for any output volumes of Foreign firms. Using (10) to solve for \( \omega^h \), and substituting this into (11):

\[ x^h (1 - \frac{z^h \bar{k}^h}{X \gamma}) - (\bar{\omega} - \bar{k}^h)\bar{k}^h = 0 \quad \text{(FOC)} \]

This expression, together with (LME), determines \( \bar{k}^h \) and \( x^h \). The solid lines in Fig. 1 depict the two equations. Clearly, there are always two solutions to the system. However, as shown in the appendix, the solution to the "north–west" is ruled out by second–order conditions for profit maximization, together with an assumption implying that the number of firms \( n \) and the demand elasticity \( \gamma \) are not too small. For instance, with Cobb–Douglas preferences over goods, it suffices that the market share of a Home economy firm is less than 1/4. We will therefore henceforth disregard this solution.

X–Inefficiency and the Autarky Equilibrium

As a preliminary step, let us consider the impact of the X–inefficiency on the Home economy's autarky equilibrium. It was shown above that a manager's incentives to provide effort is weaker in an X–inefficient firm than in an internally efficient firm, \textit{cet. par.}. But,
when the $X$-inefficiency is wide-spread among firms, aggregate variables will be affected, and things are not equal. For instance, the informational problem will affect firms' demand for labor, which in turn will influence real wages. Furthermore, the internal inefficiency implies higher marginal costs, which affects firms' optimal output choices. It is therefore not a priori clear whether in equilibrium managerial effort levels are too low.

In autarky, the labor market equilibrium condition is the same as with trade. But, the absence of international competition affects firms' decision problems; algebraically, $X = n^h z^h$ in the case of autarky, while $X = n^h z^h + n^f z^f$ with trade. The autarky equilibrium with $X$-inefficient firms can thus be derived as the intersection of the solid schedules in Fig. 1, denoted $A^*$.

For an internally efficient firm,

$$\bar{k} = \bar{\vartheta} - zw$$  \hspace{1cm} (12)

where $\bar{\vartheta}$ is the expected value of $\vartheta$. The expression corresponding to (FOC) for the case of $X$-efficient firms, is thus

$$x^h(1 - \frac{x^h I}{X^\gamma}) - (\bar{\vartheta} - \frac{x^h h}{X^\gamma})x^h = 0$$  \hspace{1cm} (FOC*)

This expression is illustrated by the broken schedule in Fig. 1 (note that $\bar{\vartheta} < \vartheta$), and the equilibrium is $A^0$. Hence:

**Proposition 1:**

*In general equilibrium, the managerial effort supply is lower in the $X$-inefficient economy than in the economy with internally efficient firms.*
In each firm the X–inefficiency yields a tendency toward a lower level of managerial effort supply, compared to the case with internal efficiency. At a given real wage the marginal cost will be higher in the X–inefficient economy, and there will thus be a tendency toward a lower demand for labor. In order for the labor market to clear in the X–inefficient economy, it must have a lower product (and real) wage. Furthermore, the X–inefficient economy features smaller output per firm. Both these circumstances reinforces the tendency toward lower effort levels in the X–inefficient economy. Hence, the X–inefficiency exacerbates the product market distortion that is due to imperfect competition.

\textit{X–Inefficiency and International Competition}

It is assumed that whenever trade is permitted, it is unimpeded by transport costs, trade policy interventions, etc. The equilibrium of the Foreign economy is then described by equations analogous to (10), (11), and (LME). Due to the assumed symmetry trade need not actually take place in equilibrium. But, as usual, the mere possibility of trade has real consequences, since it changes the degree of competition in domestic product markets.

The impact of trade is assessed in standard fashion by comparing the above described internationally integrated equilibrium with the Home economy's autarky equilibrium. The qualitative impact of the opening of trade can be seen from a Fig. 2. The LME schedule is unaffected by whether or not the Home economy trades. But, the FOC schedule is drawn conditional on a given value of $\pi^{F}_{x^f}$. Fig. 2 depicts the autarky FOC schedule with a broken line, and the corresponding trade schedule with a solid line. (The latter schedule is drawn conditional on the equilibrium value of $\pi^{F}_{x^f}$ under trade. This value is endogenous to $z^h$ and $\bar{k}$, of course, but it is definitely positive.) In the figure the autarky and trade equilibria are denoted $A^*$ and $f^*$, respectively. Hence, we can infer the following, which applies both to the case when Home economy firms are internally efficient, as well as when they are X–inefficient:
Proposition 2:

*International trade induces managers to exert more effort, and thus to reduce the unit input requirement of labor.*

Intuitively, international integration affects domestic firms through several channels. First, the opening of trade faces a typical Home firm with new competitors in its domestic market. Since outputs are strategic substitutes, the firm has an incentive to reduce its output volume, and consequently to demand less effort from the manager, as can be seen from (11). This *home market* effect of international integration yields a tendency toward higher variable production costs in firms, and is exactly analogous to what would result if competition were increased through the entry of domestic firms. Hence, if firms reduce their usage of labor per unit of output as a result of exposure to international trade, this is not due to the higher competitive pressure *per se*, since this competitive effect weakens incentives to exert effort. Instead, it is due to the particular manner in which competition is brought about by international integration.

But international integration also yields access to Foreign markets. This *market enlargement* effect has consequences opposite to those of the home market effect, and will dominate the latter in, for instance, a symmetric setting, where countries are scaled replicas of each other. International integration leads in this case to increased managerial effort supply at given factor prices and consumer incomes.

An expansion of output is partly achieved with a given work-force, by reduced unit input requirements. However, firms will also increase their demand for labor. In general, this labor could be brought in from other sectors of the economy. But, if the part of economy that produces tradables is large relative to the economy as a whole, as is the case here to an extreme degree, the adjustment must mainly come through changes in product wages. This will in turn affect firm's incentives to save on labor costs, and thus to induce managerial effort supply. This is the *factor market* effect of international integration on
effort incentives — changes in the incentive for firms to induce managerial effort supply, caused by changes in the rewards of other factors.

A very common line of reasoning in the informal literature on X–inefficiency and trade, is that before the opening of trade firms are not on their "production frontiers", whereas exposure to international competition forces firms to move closer to these frontiers. An observer of the economy studied here might mistakenly argue along the same lines. The opening of trade will not lead to any flow of production workers between firms. As a matter of fact, since effort is unobservable, there wouldn't be any observable reallocation of resources at all. Nevertheless, all firms increase their production volumes! It would thus appear as if something has been gained from nothing, and that firms were initially not on their production frontiers.

We have thus seen how international competition can induce increased supply of managerial effort, as the popular arguments referred to in the introduction suggest. However, it is also claimed that this is desirable from a welfare point of view. However, the increased supply of effort is associated with costs, and a welfare evaluation of the consequences of trade must take these costs into account. This is the issue to which we turn next.

IV. TRADE AND WELFARE WHEN FIRMS ARE X–INEFFICIENT

Production workers certainly benefit from trade, since their real wage increases. But there are two other groups of individuals whose welfare must be taken into account: managers and owners of firms. It can be shown that real profits fall as a result of the opening of trade. In order to determine the consequence of trade for managers, note that we can express the expected utility of managers as (using integration by parts)

\[ \int_{\theta} \frac{\partial}{\partial \theta} f(\theta) U(\theta) d\theta = [F(\theta) U(\theta)]_{\theta} - \int_{\theta} \frac{\partial}{\partial \theta} F(\theta) U(\theta) d\theta \]
\[
= \bar{u} + \int_{\theta} F(\theta)U(\theta - k(\theta))d\theta
\]  \hspace{1cm} (13)

Since trade decreases not only average unit input requirement \( \bar{k} \), but also the input requirement in each state \( k(\theta) \), it is clear from (13) that increased international competition benefits managers: in order to induce managers to exert more effort, firm owners have to pay more than the direct disutility of effort.

The question is then whether the gains to employees are smaller or larger than the losses for firm owners, from a social point of view. In order to assess the overall effect, we define a welfare measure \( S \) as the sum of individual expected utilities and expected real profits. (Since utility is linear in income, such an addition is meaningful.) Since payments from firms to production workers and managers merely constitute income transfers and thus net out, and the welfare measure becomes (the number of firms is normalized to unity)

\[
S = z - \int_{\theta} f(\theta)Z(\theta-k(\theta))d\theta
\]  \hspace{1cm} (14)

Consider first the case where firms are internally efficient both before and after the opening of trade. Using the equilibrium conditions, the opening of trade then has the following first-order welfare effect:

\[
dS = -(1 - \bar{k}\omega) \frac{z}{k} \frac{d\bar{k}}{k}
\]  \hspace{1cm} (15)

That is, there are gains from trade since trade leads to higher average effort supply (lower \( \bar{k} \)). Note, however, that in the present context this gain would stem from the fact that
output per firm is too low due to the imperfections in the product markets. With all sectors identical, and the number of firms constant, the desirable expansion in the output per firm cannot come about unless the (average) input requirement per unit of output is reduced. Therefore, if trade induces managers to exert more effort, this will be beneficial from the point of view of society as a whole. But, an increase in the supply of effort is not desirable per se from a private point of view, since firms are internally efficient, i.e. have exploited all potential gains from trade between the owner of the firm and the manager. If firm owners were to choose the socially desirable output volumes, the expected unit input requirement would be socially correct, as would effort levels.

Now turn to the question of whether there are any special welfare consequences of trade in the presence of X–inefficiency, as is so often maintained. Consider the case where firms are X–inefficient both before and after the opening of trade, and where the IR constraint does not bind in either situations. Differentiating (14) and using the conditions for general equilibrium,

\[ dS = - (1 - kw) \frac{\bar{\varphi}}{\bar{k}} d\bar{k} - \int \bar{\varphi} f(\theta) T(\theta) \delta k(\theta) d\theta \]  

(16)

where \( \delta k(\theta) \) is the variation in \( k(\theta) \). Just as in the case of internally efficient firms, an increase in effort enhances welfare because of the monopoly distortion in the product markets. But, when firms are X–inefficient, there is an additional source of gains from trade.

To highlight the source of this additional gain, let us for a moment consider a firm in a partial equilibrium context. Let \( S^i \) be the sum of the expected utilities of the owner and the manager, in a firm \( i \):
\[ S^i = R(x^i, \cdot) - w^kz - \int_{\theta}^{\bar{\theta}} f(\theta)Z(\theta - k^i(\theta)) d\theta \]  \hspace{1cm} (17)

where \( R \) is the firm's revenue. Differentiating \( S^i \), taking into account first-order conditions for profit maximization, and holding other firms' outputs and the production worker wage fixed, yields

\[ dS^i = -\int_{\theta}^{\bar{\theta}} f(\theta) T(\theta) \delta k^i(\theta) d\theta \]  \hspace{1cm} (18)

(18) implies that a reduction in the unit input requirement in each state \( \theta \) increases the owner's and the firm's joint surplus, if \( T(\theta) > 0 \), i.e., if the firm is \( X \)-inefficient. It is then clear from (18) that the second term in (16) captures welfare effects due to the internal reallocation of resources, i.e., increased managerial effort supply, within the \( X \)-inefficient firms:

**Proposition 3:**

The aggregate gains from international trade are partly attributable to the existence of \( X \)-inefficiencies in firms.

In the autarky equilibrium, an increased supply of managerial effort would be socially desirable, due to the difference between the marginal social value of effort, and its marginal cost in terms of disutility for the manager. But the additional effort is not supplied, since the owner cannot command the manager to exert this effort. Instead, the owner has to rely on incentives, and these are too costly to the owner, in comparison with the savings the increased effort would bring about. But these savings depend partly on how much the firm produces, and partly on the wage per production worker. Trade
increases both output per firm and workers' wages, and therefore on both accounts makes it desirable for the owner to induce the manager to supply more effort. The owner is worse off as a result, but the increase in wage payments merely constitute income transfers, and therefore don't affect welfare. It should be emphasized again that this source of gains from trade is unrelated to traditional sources such as exploitation of comparative advantage or increasing returns to scale. Instead, it results from the way in which international trade affects the contractual relationship within firms and, in particular, by increasing the benefits of marginally lower unit input requirements.

The opening of trade implies that managers and workers are better off, owners of firms are worse off, and that the sum of all utilities increases. Whether or not the opening of trade can yield a Pareto improvement, depends on the instruments for redistribution available. Note that workers' gains do not suffice to compensate owners. To make the opening of trade Pareto sanctioned, income has to be redistributed from managers to firm owners. The inherent difficulty with this is that it may distort managers' incentives. But, if redistribution can be achieved by a non-distorting tax, trade can be made Pareto sanctioned. This requires taxation of the ex post utility level $\bar{u}$ (which will hardly ever be experienced by the manager), as well as the realized income.

As was pointed out above, the two utility levels $u^w$ and $\bar{u}$ are crucial for whether or not firms are $X$-inefficient. With increasing utility levels of production workers as a result of the opening of trade, managers' reservation utilities increase. The effect of trade on the limited punishment restriction $\bar{u}$ is less clear. This level is probably affected by the general development level of the economy, but whether the opening of trade is likely to increase $\bar{u}$ more or less than $u^w$ is difficult to say. Let us therefore just note the possibility that trade may have the effect of entirely removing the $X$-inefficiency in firms, by increasing the discrepancy between $u^w$ and $\bar{u}$ sufficiently so that with trade only the IR constraint binds.

Proposition 3 does not hinge on whether or not the Home economy's trading partner
is $X$–inefficient. But, intuitively one would expect that the gains to the Home country from international trade, somehow depend on whether firms in the Foreign economy are $X$–inefficient or not. If firms were internally efficient in the Foreign economy, they should provide a "colder shower" than if they were $X$–inefficient, thus inducing more effort supply in the Home economy, and increasing consumer surplus. On the other hand, firms compete for pure profits, and high marginal costs relative competitors is a drawback.

To assess whether a trading partner with internally efficient firms is preferable to one with $X$–inefficient firms, two international equilibria have to be compared. To this end, recall that (LME) and (FOC) implied that the $z^h$ that is consistent with general equilibrium in the Home economy, is larger the larger is $z^f$. These equations thus define a "general equilibrium reaction function" $z^h = \chi^h(z^f)$, which increases in the Foreign country's production volume, and for which $\chi^h(0)$ yields the Home economy autarky output level per firm. Let the Foreign economy's reaction function $z^f = \chi^f(z^h)$ be defined analogously. The Home economy's output under trade is then given by $z^h = \chi^h(\chi^f(z^h))$, as illustrated in Fig. 3, where the broken line correspond the case of efficient Foreign firms. It is clear that the Home economy will in equilibrium produce less per firm when its trading partner has $X$–inefficient firms, and consequently it will have a higher average unit input requirement, and its managers will exert less effort:

**Proposition 4:**

_The Home economy is better off with a trading partner with internally efficient firms than with a partner with $X$–inefficient firms, regardless of whether firms in the Home own economy are $X$–inefficient or not._

_Reduced $X$–Inefficiency as a Source of Gains from Trade_

In the analysis above we made a precise definition of $X$–inefficiency, but we deliberately abstained from defining a measure of the degree to which the firm is inefficient.
Therefore we cannot address the common claim that trade reduces X-inefficiency. The problem with any measure of X-inefficiency is that it in general seems irrelevant for making inferences about welfare effects of some experiment, such as the opening of trade. Any measure of X-inefficiency involves a comparison of the situation under study with a fictitious, and efficient, situation. The measure indicates a gain that could be made by removing the source of the inefficiency. However, except in special circumstances, trade will not remove the source of the inefficiency, but change the circumstances under which it affects the economy. That is, the opening of trade will in general imply a move from one X-inefficient equilibrium to another. Whether this move constitutes a welfare gain or not must be determined by comparing the initial inefficient situation with the new, inefficient, allocation. Depending on the exact construction of the X-inefficiency measure, the move from one equilibrium to another may imply that the distance to the fictitious efficient economy has increased or decreased. But, this information cannot generally be used as an indicator of the welfare change, since both the "actual" X-inefficient economy, as well as the X-efficient economy, are affected by the opening of trade.

Consider the following simple application of the above reasoning. A standard monopoly firm produces with constant marginal cost, and faces a linear demand. The monopoly pricing gives rise to an inefficiency, the degree of which would typically be measured by the dead-weight loss. Assume that the firm suddenly gets access to another technology with lower marginal costs. Does this constitute a welfare gain? If the dead-weight loss is employed as the welfare criterion, the answer is "no": with linear demand and constant marginal cost, a lower marginal cost implies a higher excess burden of monopoly. But, welfare is usually measured by what is obtained rather than foregone, and the lower marginal cost implies that the sum of consumer and producer surplus has increased. The increase in dead-weight loss simply says that the value of getting rid of the monopoly distortion has increased. But, it does not say much about whether the lower marginal cost is beneficial. Then, why has the value of getting rid of the monopoly
distortion increased? Because the benchmark, the efficient allocation, is different with the lower marginal cost.

Similarly, measures of X-inefficiency capture the value to the firm, or to society, for a given trade regime, of removing the internal inefficiency in firms. Whether this gain increases or decreases as a result of the change in trade regime is in general irrelevant to a welfare assessment of the opening of trade. As long as international trade does not remove the source of the X-inefficiency, there is no general presumption that measures of welfare and measures of the X-inefficiency should be related in any particular way. Thus, statements such as "gains from trade due to reduced X-inefficiency" do not seem meaningful.

V. CONCLUDING REMARKS

A common theme in the policy discussion of the pros and cons of international integration is the idea that international integration, by increasing competition in product markets, changes incentives in firms in such a way as to induce increased effort supply. Furthermore, it is often taken for granted that this yields welfare gains. This paper is an attempt to examine these popular arguments within a simple formal model.

The paper confirms the popular notion in a certain sense: in the model, trade leads to increased managerial effort supply, and firms become more productive in the sense of producing larger output volumes with the same number of employees. Furthermore, the increased supply of effort is beneficial from a welfare point of view, partly because firms are X-inefficient. This source of gains from international trade has not previously been formally captured, as far as we know. But, the paper also argues that it is the existence of the inefficiency, rather than reductions in the amount of inefficiency, which gives increased effort supply its welfare enhancing role.

The internal inefficiency in firms stems from the assumed asymmetry of information. International trade may be welfare improving, not because it directly affects
this asymmetry, but because it changes the circumstances under which the information asymmetry affects the economy. One could also conceive of ways in which trade changes the available information. For instance, if firms have correlated costs \( \phi ^{ij} \), contracts could be made contingent on e.g., costs realizations in other firms. Then, as the number of firms changes due to the opening of trade, the scope for yardstick competition changes, which in turn affects the structure of contracts.

Finally, there are almost as many interpretations of internal, or "X", inefficiencies, as there are observers. We would certainly not ascribe any particular merit to the interpretation of X-inefficiency given here. But a systematic analysis of these popular claims must start somewhere, and the contract-theoretic approach employed here seems to us to be a natural starting point.

APPENDIX

First-Order Conditions for the X-Inefficient Firm

Let \( \theta \) be the realized value of \( \phi ^{ij} \), and let a reported value \( \theta ' \) be associated with a wage \( v^{ij}(\theta ') \) and an unit input requirement \( k^{ij}(\theta ') \). For the contract to be incentive compatible it is then necessary that (omitting firm and sector indices)

\[
\theta = \arg \max_{\theta ', G(p)} \frac{v(\theta')}{G(p)} - Z(E(k(\theta'), \theta))
\]

(A1)

The first-order condition for this maximization problem is

\[
\frac{v(\theta)}{G(p)} - Z_e(E(k(\theta), \theta)) E_{k}(k(\theta), \theta) k_{\theta}(\theta) = 0
\]

(A2)

The restriction (IC) in the main text is obtained from (A2) and the following fact:
\[ U_{\theta}(\theta) = \frac{v_{\theta}(\theta)}{G(p)} - Z_{e}(E(k(\theta),\theta)) \left[ E_{k}(k(\theta),\theta) k_{\theta}(\theta) + E_{\theta}(k(\theta),\theta) \right] \]  

(A3)

The maximization problem stated in (4) can be expressed in a more convenient fashion, by utilizing the definition of \( U(\theta) \). The problem can thus be restated as choosing the \( z, k(\theta) \), and \( U(\theta) \) that solves

\[
\max_{\theta} \int_{\theta}^{\bar{\theta}} \frac{P(X,I)z - k(\theta)z - u}{G(p)} - U(\theta) - Z(E(k(\theta),\theta))f(\theta) d\theta
\]

(A4)

subject to (IC), (LP), and (IR). The Lagrangian \( \Lambda(z, k(\theta), U(\theta), U_{\theta}(\theta), \lambda, \mu(\theta)) \) is

\[
\Lambda = f(\theta) \left[ \frac{P(X,I)z - \omega k(\theta)z - U(\theta) - Z(E(k(\theta),\theta))}{G(p)} \right.
\]
\[
\left. - \lambda(U(\theta) - u) - \mu(\theta)[U_{\theta}(\theta) + Z_{e}(E(k(\theta),\theta))E_{\theta}(k(\theta),\theta)] \right]
\]

(A5)

where \( \lambda < 0 \), and \( \mu(\theta) \) are Lagrange multipliers, and where \( \omega \equiv u/G(p) \).

Optimal choices of \( z, k(\theta) \), and \( U(\theta) \), require (5) and the following two Euler equations with respect to \( k(\theta) \) and \( U(\theta) \):

\[ z\omega = -[Z_{e}(E(k(\theta),\theta)) + \mu(\theta) Z_{ee}(E(k(\theta),\theta))] E_{k}(k(\theta),\theta) \]

(A6)

\[
\frac{d}{d\theta}(\mu(\theta)f(\theta)) = (1 + \lambda)f(\theta)
\]

(A7)

(A7) implies that

\[
\int_{\theta}^{\bar{\theta}} \frac{d}{d\theta}(\mu(s)f(s)) ds = \int_{\theta}^{\bar{\theta}} (1 + \lambda)f(t) dt
\]

(A8)
Let $U^*(\theta)$ denote the optimal value of $U$. The transversality condition

$$
\Lambda U_\theta \left[ U^*(\theta) - U(\theta) \right] \geq 0 \quad \text{at } \theta = \emptyset \\
\leq 0 \quad \text{at } \theta = \emptyset 
$$

(A9)

for all feasible $U(\theta)$ implies that $\mu(\emptyset) = 0$; $\emptyset$ is a free boundary since (LP) does not bind for $\emptyset$. Hence, by (A8),

$$
\mu(\theta) = (1+\lambda) \frac{F(\theta)}{f(\theta)} 
$$

(A10)

Inserting (A10) into (A6), one arrives at (6), with

$$
T(\theta) \equiv (1+\lambda) \frac{F(\theta)}{f(\theta)} Z_{ee}(E(k(\theta),\theta)) E_k(k(\theta),\theta) 
$$

(A11)

If (LP) does not bind for $\emptyset$, $\emptyset$ is a free boundary, and $\mu(\emptyset) = 0$ by the transversality condition (A9). It then follows from (A10) that $\lambda = -1$ in this case. It can also be seen that if (IR) does not bind, (LP) will: $\lambda = 0 \Rightarrow \mu(\emptyset) > 0$ for $\emptyset > \emptyset$.

The Uniqueness of the General Equilibrium

There are more than one potential equilibrium for the general equilibrium system, as is evident from e.g., Figures 1 and 2. The purpose of this part of the appendix is to derive sufficient conditions that exclude intersections between (FOC) and (LME) to the "north—west" as equilibria. To save on space, we introduce some notation. Let $h \equiv x^j w / G(p)$, and let (6) define $k = K(h,\theta)$. Also, let $R(x^j, \cdot)$ be the revenue of the firm, at given output volumes of other firms. Differentiating (7) with respect to $k$ yields
\[ 1 = \frac{\partial}{\partial k} \left[ Z e(\theta-k) + T(\theta) \right] \frac{\partial k}{\partial h} \]  

(A12)

To ensure that the first-order conditions for the contract problem actually maximizes the firm's profit, \( \Lambda \) is assumed to be concave in \((x, k, U, U_\theta)\), which implies

\[ \frac{\partial}{\partial k} \left[ Z e(\theta-k) + T(\theta) \right] \frac{R_{zz}}{G(p)} - \omega^2 > 0 \]  

(A13)

(A12) and (A13) thus imply that

\[ \frac{R_{zz}}{G(p)} < \omega^2 \frac{\partial k}{\partial h} \]  

(A14)

Taking expectations with respect to \( \theta \) for both sides, and utilizing (5), one arrives after some tedious computation at the following expression, where \( s \) is the firm's market share:

\[ \eta \equiv -\frac{\partial h}{\partial k} \frac{2 - s}{\gamma} \left( 1 - \frac{s}{\gamma} \right) \]  

(A15)

We henceforth assume that \( \eta \leq 1/2 \) in order to ensure the uniqueness of the symmetric general equilibrium (the assumption is commented upon below).

Now turn to the (FOC) schedule. The slope of (FOC) is, for given foreign production

\[ \frac{z}{k} \frac{\partial k}{\partial x} \mid_{FOC} = -\frac{\eta}{1 - \eta} \left[ 1 - \frac{s}{1 - ns} \right] \]  

(A16)
The factor in brackets is at most equal to unity, since $ns \leq 1$, and $\gamma > s$, and by minimizing the expression with respect to $s$, under the constraints $ns \leq 1$, and $\gamma > s$, it can be shown to be non-negative. This implies that the value of the RHS of (A16) lies in the interval $(-1, 0)$, since by assumption $\eta \leq 1/2$.

There are three potential types of equilibria, as illustrated in Fig. 4. Consider first the candidate denoted C. At C the (FOC) schedule has a positive slope, which requires that $\eta > 1$. But, this violates the assumption that $\eta \leq 1/2$, and is therefore ruled out. Consider next the equilibrium candidate B. At B we have that

$$\frac{z \, d\bar{k}}{\bar{k} \, dz} \bigg|_{\text{FOC}} > \frac{z \, d\bar{k}}{\bar{k} \, dz} \bigg|_{\text{LME}} = -1$$

which is not possible. The only possible equilibrium is hence of type A.

Finally, a couple of words about the assumption $\eta \leq 1/2$. If this were a standard autarky Cournot model, one would have to assume that $\gamma n > 1$ for an equilibrium to exist (otherwise marginal revenue would be negative). Similarly, in the present case one has to ensure that $n$ is sufficiently large relative to $\gamma$, but here the condition is a bit more stringent. The set of values of the parameters $n$ and $\gamma$ that ensures $\eta \leq 1/2$ can be obtained directly from (A15), even though it is difficult to get an interpretable condition this way. Note instead that what is required is that neither of $n$ and $\gamma$ is too small relative to the other parameter. For instance, in the case of Cobb-Douglas (e.g. $\gamma = 1$) preferences, the assumption that $\eta \leq 1/2$ corresponds to the assumption that the market share of a single firm be no larger than $1/4$.

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Figure 1

Figure 2
Figure 3

Figure 4