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OF NEW TECHNOLOGIES

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Chien–fu Chou and Oz Shy

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Chien-fu Chou* and Oz Shy** †

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Abstract

We formalize an explanation for technology revolutions and growth cycles in a model where consumers and firms benefit from periodic changes in technology which result in the development and marketing of new generations of products. We develop a general equilibrium dynamic differentiated products model in order to explain technological progress via cyclical changes in investment, output, and interest rates as well as the introduction of new products. We characterize the equilibrium and analyze the effects of changing the rate of technology growth, resource endowment, and R&D and production costs on the duration of generations of products and the frequency of technology revolutions, and hence the growth cycles.

Keywords: Technology Revolutions, Gestation of New Technologies

JEL Classification Number: O3, O4

*State University of New York at Albany, Albany, NY 12222, U.S.A.
**Tel-Aviv University, 69978 Tel-Aviv, ISRAEL.
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1. Introduction

We propose a general equilibrium cyclical growth model for analyzing the economic factors which affect the frequency of introductions of new generations of products as well as the time gap between the introduction and marketing dates of new generations of products. It is observed that periodically new generations of products are introduced, and many new generation products are incompatible with older generation products. In addition, we observe that there is a time lag between the introduction date of a new generation and the period in which the new products are marketed (and replace old generation products).\(^1\) Over four decades ago Schumpeter suggested that economic growth is not governed by a continuous capital accumulation but occurs through a sequence of discrete technology revolutions. In his classic book, Schumpeter (1975, p. 83) asserted that:

Those revolutions are not strictly incessant; they occur in discrete rushes which are separated from each other by spans of comparative quiet. The process as a whole works incessantly however, in the sense that there always is either revolution or absorption of the results of revolution, both together forming what are known as business cycles. ... the same process of industrial mutation ... incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one. This process of Creative Destruction is the essential fact about capitalism.

In this paper we develop a framework for modeling endogenous technology revolutions and their implications for cyclical fluctuations of output, investment and growth.

\(^1\)Take for example the microcomputer industry. Although new chips are introduced very frequently, computer firms are reluctant to adopt a chip which is incompatible with older models. Thus, compatibility seems to be a major consideration of whether or not and when to introduce a new machine. Moreover, once a new incompatible machine is introduced, most consumers do not purchase it until it is supported by a large variety of compatible products.
We provide a Schumpeterian model of development in which the developers of new technologies play the key role in moving the economy towards the production of more valuable goods. When innovators believe that a totally new set of technologies will be introduced in the future, all new product developers stop developing products based on old technologies and start developing new generation of products to be marketed in the future. Thus, our contribution here is that we are able to model development not only as a continuous increase in the set of available goods but, following Schumpeter, as a process of creating new technologies for producing new products through the elimination of old products.\textsuperscript{2}

In this paper, we emphasize the distinction between the exogenous accumulation of experience and the \textit{actual} adoption of new technologies for producing new generation products. Assuming the incompatibility of new generation products with old generation products, we are able to obtain that adoptions of new technologies occur only in discrete points in time rather than continuously. Our model endogenously determines the number of new products (new firms) in the economy at each point in time. In addition, we are able to determine the periods in which innovators start developing a new generation of products. In order for a technology revolution to succeed, all innovators developing technologies for a new generation of products must be convinced that consumers will adopt the new generation products at a certain date in the future. In our framework, a single or a small group of innovators alone cannot bring a technological change into the economy if there are no followers.\textsuperscript{3}

We develop a general equilibrium dynamic monopolistic competition model where firms are constantly engaged in product innovation. Once the total number of products reaches a certain level, firms find it profitable to switch to a new generation of products

\textsuperscript{2}For discussions of technology change see Dasgupta (1986), Kamien and Schwartz (1982), and Tirole (1988, Ch. 10).

\textsuperscript{3}A particular feature of the present framework is that if innovators expect a technology revolution to come, then it will come. Hence, our approach is somewhat similar to the sunspots models of Azariadis (1981) and Azariadis and Guesnerie (1986).
which is incompatible with the old generation of products. From that point in time and on, firms develop only new generation products until they find it profitable to start developing products for a newer generation of products, and so on. At each point in time, new consumers enter the market, make purchase of each of the existing goods and exit the market instantaneously. Over time, the variety of existing products gets to be so large that each existing firm makes a small amount of profit. In addition, firms expect consumers to switch to a new (incompatible) generation of products in a given date in the future. Altogether, the value of an old generation firm declines until innovators find it profitable to develop and construct only firms producing new generation products. We call this stage a technology revolution. Since the variety of the new generation of products is very low, consumers purchase old generation products until the variety of the new generation of products reaches a certain level in which new generation products become more attractive to consumers.

In the literature, some of the earlier models of vintage capital are surveyed in Allen (1967). Recently, Aghion and Howitt (1989) provide an alternative approach in which a random successful outcome of innovation is translated into cost reduction and a complete replacement of the intermediate goods. Chari and Hopenhayn (1990) develop an overlapping generations model in which each technology requires vintage specific skills. Their model predicts a lag between the time when a technology appears and the peak of its usage. Jovanovic and Rob (1990) endogenize the frequency of major inventions and their refinements (imitations). Stokey (1988) proposes a growth learning by doing model where new goods replace old goods. The aim of the present paper is to provide a different meaning to technological changes by formalizing the concept of generations of products. In this paper the number of products belonging to a specific generation is increasing over time until producers start developing a new generation of products and stop developing old generation products. At this date, (called the introduction date) the number of products belonging to the old generation of products reaches its
maximal level. Following the introduction date, there is a gestation period in which consumers purchase a fixed number of old generation products and firms develop only new generation products which are still not sold to consumers. The gestation period ends at a date (called the marketing date) when consumers start buying only new generation products.

The paper is organized as follows. In section 2 we develop a technology revolution model. Section 3 defines and classifies two types of dynamic revolution equilibria: an *endless* revolution equilibrium (in which the economy grows indefinitely through discrete technology revolutions), and a *terminated* revolution equilibrium (in which technology revolutions stop at a certain point in time, and the economy stagnates in a growing number of old technology products.) Section 4 characterizes revolution equilibria. Section 5 analyzes the factors which influence the duration and gestation periods of new generations of products as well as interest rate behavior during these periods. Section 6 concludes.

2. The model

We consider an infinitely lived economy producing differentiated products which are indexed on the real line. We say that technology revolution occurs in period $t^*$, if all products developed in periods $t \geq t^*$ are incompatible with products developed before period $t^*$. Thus, we divide time into sub-intervals called generations. A generation is defined as the time interval between two successive revolutions. We index generations by $g$, where $g$ is an integer. By $D^g$ we denote the time when a revolution occurs and generation $g$ firms emerge (D-day). Even when a new generation of products is introduced, consumers may still prefer to purchase only old generation products because of the low variety of the new products. By $M^g$, ($M^g > D^g$), we denote the first date in which generation $g$ products are sold to consumers (M-day). We call the time interval between the introduction date $D^g$ to the marketing date $M^g$ the
gestation period of generation $g$. The length of the gestation period is denoted by $G^g \equiv M^g - D^g$. During generation $g$ gestation period, generation $g$ products are developed but not consumed. We call the length of time between the introduction dates of generations $g$ and $g + 1$ the duration of generation $g$ and denote it by $\Delta^g$, $\Delta^g \equiv D^{g+1} - D^g$.

Figure 1 illustrates the innovation and the consumption patterns over time. Between $t = D^g$ and $t = D^{g+1}$, only generation $g$ products are developed. However, during generation $g$ gestation period (between $t = D^g$ and $t = M^g$) only generation $g - 1$ products are consumed (but old generation $g - 1$ products are no longer being developed). Starting from $t = M^g$ consumers purchase only generation $g$ products.

**INSERT FIGURE 1**

### 2.1 Production and product development

Each product is indexed by $x$, where $x$ is a real number. The set of all available generation $g$ products in period $t$ is denoted by $X^g_t$ which is Lebesgue measurable in $(-\infty, \infty)$. The time $t$ number of actually produced generation $g$ goods is the Lebesgue measure of $X^g_t$ and is denoted by $\mu^g_t = \mu(X^g_t)$. We associate each product with a single firm. Each existing product is produced with a constant marginal cost of $m$ units of labor per unit of output. To develop a new generation $g$ product in period $t$, the innovator has to spend a sunk cost of $F^g_t$ units of labor. We assume that $F^g_t$ declines with the number of existing generation $g$ products. Formally, let

$$F^g_t = F(\mu^g_t) = \left(c^2 + \theta^2 \mu^g_t\right)^{-1/2}. \quad (1)$$

This specification captures the fact that the development cost is high at the beginning of a new generation and is decreasing as more products (within the same generation)
are introduced. In other words, the cost of developing one additional generation $g$ product declines with an increase in the variety of already developed generation $g$ products. Here, $c^{-1}$ measures the development cost of the first product of a generation, and $\theta$ is the cost reduction coefficient. We assume a dynamic monopolistic competition in the product market. Once a fixed development cost is invested, the firm becomes a monopoly. All firms have perfect foresight regarding future demand and interest rates.

We assume a competitive banking system which finances the development cost of new firms. Then, in each period (moment) the monopoly firm reimburses the bank with all the profit made until all the debt is paid. Each bank maximizes intertemporal profits. We also assume that banks are not subject to minimum reserve requirements and can issue unlimited credit as long as they find it profitable.

2.2 Consumers

At each point in time, a new consumer endowed with $L$ units of labor enters the market, sells its labor endowment, makes purchase of each of the existing products and exits the market instantaneously. We denote by $c_t(x)$, $x \in \cup_g X_g^t$, the period $t$ consumption level of good $x$. Consumers derive utility from all (existing) products. Formally, period $t$ consumer's utility function is given by

$$U(\{c_t(x)\}) = \sum_g e^{hD_g} \left\{ \int_{X_g^t} [c_t(x)]^\alpha dx \right\}^{1/\alpha}, \quad 0 < \alpha < 1, \quad h > 0,$$

(2)

where the summation is over existing (previously developed) generations of products, and $D_g$ is the introduction date of the first product of generation $g$. The utility function (2) implies that different generation products are perfect substitutes, and therefore each consumer buys all the existing variety of products which belong only

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4All the results hold for a more general class of $F$ functions satisfying $F' < 0$ and $FF''/3(F)^2 \geq 1$. The present specification enables us to solve for a closed form solution.

5Rob (1991) analyzes sequential entry under demand uncertainty.
to one generation. By the factor $\exp(hD^g)$, we capture the effect of a technological progress on welfare. We can think of a continuous accumulation of knowledge (new technologies) and experience over time, where the rate of technology growth is given by the parameter $h$. The main feature of the paper is that without technology revolutions the new technologies are not being adopted. Once a revolution occurs, the technology embodied in the new generation of products is marked by its introduction date ($D^g$) and its efficiency or utility enhancement is measured by $\exp(hD^g)$. The use of the exponential function allows the possibility of stationary equilibria.

3. Dynamic Equilibrium

We are interested in an equilibrium in which from time to time consumers switch to a new generation of products. We call this a revolution equilibrium. Consider the following economic mechanism. At each moment $t$, banks make loans to new investors. The investors use this credit to purchase new firms from innovators. The innovators pay for the labor needed to develop the new products (new firms). Workers use their labor income to purchase products, and firms use their profit to pay back their loans to the bank. In what follows we characterize the intertemporal equilibrium of all these transactions. At the end of this section, we define and classify revolution equilibria.

3.1 Product market equilibrium

Let $w_t$ denote the period $t$ wage rate and $p_t(x)$ the period $t$ price of product $x$. With no loss of generality, we normalize $w_0 = 1$. Thus, $w_t$ measures the value of period $t$ labor in terms of period 0 labor. Since the utility function (2) is linear with respect to

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6At the introduction date of a new generation, the innovators adopt the most advanced technology available at that time. However, all the products of the same generation use the same technology as the product developed at the introduction date of the generation. Thus, during the life time of a generation the technology embodied into products does not improve. For example, the first generation of computers used a vacuum tube even after the integrated circuit became available. Once consumers switched to the second generation (IC) computers, firms stopped using the vacuum tube.
generations of products, the consumer buys products of the same (latest) generation. Thus, time \( t \) consumer chooses \( c_t(x), x \in X_t^{g} \) to

\[
\max \left\{ \int_{X_t^{g}} [c_t(x)]^\alpha dx \right\}^{1/\alpha} \text{ s.t. } \int_{X_t^{g}} [p_t(x)c_t(x)]dx = w_t L
\]

(3)

In a symmetric CES monopolistic competition market structure, the price of each product is given by \( p_t(x) = w_t m/\alpha \). The consumption level of each brand is found by dividing the income \((w_t L)\) by the price and the variety of existing generation \( g \) products and is given by \( c_t(x) = \alpha L/m \mu_t^g \), see Dixit and Stiglitz (1977). Therefore, the utility level of a period \( t \) consumer purchasing all the available generation \( g \) products is given by

\[
U_t^g = e^{hD^g} \left( \frac{\alpha L}{m} \right) (\mu_t^g)^{1-\alpha}.
\]

(4)

Thus, the time \( t \) equilibrium utility level is increasing with the labor endowment and the variety of generation \( g \) products which is available for sale in time \( t \). The period \( t \) profit of a firm producing a generation \( g \) product is given by

\[
\pi_t(x) = [p_t(x) - w_t m]c_t(x) = \frac{(1 - \alpha) w_t L}{\mu_t^g}, \quad t \in [M^g, M^{g+1}).
\]

(5)

3.2 Labor market equilibrium

Note that \( \dot{\mu}_t^g \equiv d\mu_t^g/dt \) measures the number of \( g \) generation firms constructed in period \( t \). Labor market equilibrium means that the period \( t \) labor demanded for innovation equals period \( t \) aggregate profits in terms of period \( t \) labor. Formally,

\[
\dot{\mu}_t^g F_t^g = \frac{1}{w_t} \int_{X_t^g} \pi_t^g(x)dx = (1 - \alpha) L
\]

(6)

Substituting (1) into (6) yields

\[
\dot{\mu}_t^g \left( \varphi^2 + \theta^2 \mu_t^g \right)^{-1/2} = (1 - \alpha) L, \quad \text{for } D^g \leq t < D^{g+1}.
\]

(7)

Observe that (7) yields identical product expansion paths and identical sunk development cost paths for all generations. This is due to the fact that a constant portion of
labor is used for development. Thus, for any generation \( g \geq 1 \), and for \( t \in [D^g, D^{g+1}] \), the variety of products and sunk cost paths can be found by solving the differential equation (7) and are given by

\[
\mu^g_t = \mu_{t-D^g} \quad \text{and} \quad F^g_t = F(\mu_{t-D^g}), \quad \text{where} \quad \mu_s = \left[ \frac{\theta(1 - \alpha)Ls}{2} + \frac{c}{\theta} \right]^2 - \left( \frac{c}{\theta} \right)^2, \quad s \in [0, \Delta^g],
\]

and for the initial generation \( g = 0 \),

\[
\mu^0_t = \left\{ \frac{\theta(1 - \alpha)Lt}{2} + \left[ \left( \frac{c}{\theta} \right)^2 + \mu^0_{0} \right]^{\frac{1}{2}} \right\}^2 - \left( \frac{c}{\theta} \right)^2, \quad t \in [0, D^1]
\]

3.3 Capital market equilibrium and the banking industry

In equilibrium, each firm’s present value of the stream of profits cannot exceed the sunk product development cost. Hence, if at time \( t \) there is a positive investment in product development, each newly constructed firm breaks even. That is\(^7\),\(^8\)

\[
w_t F^g_t = PV_t^g = \int_t^{M^{g+1}} \pi^g_x d\tau \quad \text{for} \quad D^g \leq t \leq D^{g+1}, \quad \text{where} \quad \pi^g_t(x) \equiv \pi^g_t(x) \text{ for all } x \in X_t^g.
\]

Let \( r_t \) denote the time \( t \) instantaneous interest rate (in terms of labor).\(^9\) Since banks are not subject to reserve requirements, the profit equals zero in a competitive

---

\(^7\)Notice that each period \( \tau \) instantaneous profit is measured in a common unit (period 0 labor). Therefore, further discounting is not needed in measuring the value of firms.

\(^8\)In formulating equation (10) we do not distinguish between producing firms and innovators. We can think of independent competitive innovators maximizing their profits in each period \( t \), which is given by \( \sum_p (p^g_f - F^g)\hat{\mu}^g \), where \( p^g_f \) is the value or price of a new \( g \)-generation firm, \( F^g \) is its construction cost, and \( \hat{\mu}^g \) is the number of newly constructed firms decided by the innovator in period \( t \). In equilibrium, \( p^g_f - F^g \leq 0 \) \(( = 0 \text{ if } \hat{\mu}^g > 0)\). Consider an investor who wishes to own firms. If \( PV_t^g > p^g_f \), then there is an infinite demand for newly constructed firms. Hence, in equilibrium \( PV_t^g \leq p^g_f \), and if \( \hat{\mu}^g > 0 \), then \( PV_t^g = p^g_f \). Thus, firms’ owners pay the innovators the value of the firm which by the previous discussion equals \( F^g \).

\(^9\)The gross rate of return (in terms of period \( \tau \) labor, \( \tau > t \)) on each unit of labor the bank lends in period \( t \) is equal to \( \exp(\int_t^\tau r_s ds) \). Therefore, the bank makes a profit of \( w_t \exp(\int_t^\tau r_s ds) - w_t \) on lending one unit of labor between periods \( t \) and \( \tau \).
banking industry. Hence, in equilibrium \( r_t = -\dot{w}_t/w_t \) and equation (10) becomes

\[
F^g_t = PV^g_t = \int_{\max\{t, M^g\}}^{M^{g+1}} \frac{(1 - \alpha)L}{\mu_t^g} \exp \left(-\int_t^\tau r_s ds\right) d\tau \quad \text{for} \quad D^g \leq t \leq D^{g+1}. \tag{11}
\]

Finally, if the labor market is in equilibrium, then the total amount of outstanding loans is constant since all new loans made to investors are equal to the total repay of loans at each moment. Hence, in each moment there is no net creation of new purchasing power, and consumers’ income is not affected.

### 3.4 Revolution equilibria

Having defined the equilibria in each market, we now define a revolution equilibrium as a sequence of introduction dates and marketing dates. Suppose at time \( t = 0 \) the economy produces \( \mu_0 \) products of generation \( g = 0 \), introduced at \( t = D^0 < 0 \). We consider two types of equilibria. In one, every revolution is followed by a succeeding revolution. In the second type, revolutions stop after a certain point in time.

**DEFINITION 1** A technology revolution equilibrium is a \( \bar{g}, 1 \leq \bar{g} \leq \infty, \mu_{\bar{g}}^g \), and the sequences \( \{D^g, M^g, \mu_t^g\}, 1 \leq g < \bar{g} \) such that \( D^g < M^g < D^{g+1} \), and

(a) for \( t < D^g, \quad \frac{PV^g_t}{F^g_t} < \frac{PV^{g-1}_t}{F^{g-1}_t} \) and

(b) for \( D^{g-1} < t < M^g, \quad \frac{U^g_t}{U^{g-1}_t} < \frac{U^{g-1}_t}{U_t} \).

If \( \bar{g} = \infty \), we say that the revolution equilibrium is **endless**. Otherwise, it is called a **terminated** revolution equilibrium.

Condition (a) states that at the introduction date of generation \( g \), the present value of a dollar invested in developing a product (firm) belonging to the new generation \( g \) (defined in (11)) overtakes that of a dollar invested in developing a product belonging to the old generation \( g - 1 \). Condition (b) states that at the marketing date of generation \( g \), the utility of consuming generation \( g \) goods (given in (4)) overtakes that of generation \( g - 1 \) goods.
**DEFINITION 2** The revolution equilibrium is said to be stationary if it is endless and if there exist $\Delta$, $G$, and $\mu_t$ such that for every generation $g \geq 1$,

$$0 < G < \Delta, \quad D^{g+1} - D^g = \Delta, \quad M^g - D^g = G, \quad \text{and} \quad \mu^g_t = \mu_{t-D^g}. \quad (12)$$

Thus, in a stationary equilibrium revolutions always occur. ($D^g \rightarrow \infty$ as $g \rightarrow \infty$), and all generations have a constant duration and gestation periods.

4. **Characterization of Revolution Equilibria**

4.1 **The endless revolution equilibrium**

In this subsection we show that as long as revolutions occur, all generations of products have identical durations and that every endless revolution equilibrium must be stationary. Then, we demonstrate the existence and uniqueness of the stationary equilibrium. From (7), (11), and condition (a) in definition 1 (evaluated at $t = D^g$), we have that

$$\mu^{g-1}_{D^g} = \frac{F(0)\mu^g_{M^g}}{F(\mu^{g-1}_{D^g})} \quad (\geq \text{ for } g = 1) \quad (> \text{ only if } D^1 = 0). \quad (13)$$

From (4), (8), and condition (b) (evaluated at $t = M^g$), we have that

$$\mu^{g-1}_{D^g} = \mu^{g-1}_{M^g} = \exp \left( \frac{\alpha h}{1-\alpha} (D^g - D^{g-1}) \right) \mu^g_{M^g}. \quad (14)$$

Let $A \equiv \frac{\alpha h}{1-\alpha}$. Using equation (8), (13) and (14) can be written as

$$\mu_{\Delta^{g-1}} = \frac{F(0)\mu_{G^g}}{F(\mu_{\Delta^{g-1}})} \quad \text{and} \quad \mu_{\Delta^{g-1}} = \exp \left( A\Delta^{g-1} \right) \mu_{G^g}. \quad (15)$$

Conditions (15) determine the equilibrium values of $\Delta^{g-1}$ and $G^g$. Eliminating $G^g$, we have that

$$\exp(A\Delta^{g-1}) - \frac{F(0)}{F(\mu_{\Delta^{g-1}})} = 0. \quad (16)$$

\[^{10}\text{A step by step derivation is given in the appendix.}\]
Proposition 1 and the corollary below show that as long as there are revolutions, the duration and the gestation periods of each generation are constant.

**Proposition 1** \(\Delta^g - 1\) and \(G^g\) are independent of \(g\) for \(0 < g < \bar{g}\).

*In particular, the duration and gestation periods are constant and equal for every generation \(0 < g < \bar{g} - 1\).*

**Proof:** In view of (16), define the function \(\phi(\Delta) \equiv \exp(A\Delta) - \frac{F(0)}{F(\mu\Delta)}\). By direct computation we find that \(\phi'' = A^2 e^{A\Delta} + \frac{F(0)[FF'' - 3F']R^2}{F^3} > 0\). Also, observe that \(\phi(0) = 0\). This shows that there can be at most one strictly positive \(\Delta^*\) satisfying (16). Hence, \(\Delta^g\) is independent of \(g\). By (15) \(G^g\) is also independent of \(g\). *Q. E. D.*

**Corollary 1** *Every endless revolution equilibrium is a stationary equilibrium.*

**Proposition 2** Given that \(\theta^2(1 - \alpha)^2 L > 2c_{oh}\), there exists an endless revolution equilibrium.

**Proof:** The condition \(\theta^2(1 - \alpha)^2 L > 2c_{oh}\) implies that \(\phi' < 0\). Also, \(\phi'(\Delta) \to \infty\) as \(\Delta \to \infty\). Since \(\phi'' > 0\), there exists exactly one positive \(\Delta\) solving (16), say, \(\Delta^*\). The existence of the first revolution is shown in the next subsection. *Q. E. D.*

Although an endless revolution equilibrium always exists, note that terminated revolution equilibria also always exist. In a terminated equilibrium, the generation of the last revolution \((g = \bar{g} - 1)\) lasts forever and its product expansion path grows indefinitely, whereas the technological efficiency remains constant at the level of \(\exp(hD^{g-1})\). Thus, similar to Rostow (1961), an economic take-off may not be possible if and the initial variety of old generation products grows indefinitely.

In contrast, the endless revolution equilibrium is characterized by a bounded product variety expansion path and an ever increasing technological efficiency with technological progress rate (approximately equal to \(h\)). Using the overtaking criterion, under the same conditions as in proposition 2, it can be shown that the endless revolution equilibrium is superior to any terminated revolution equilibrium.
4.2 The determination of the first revolution

We now ask how the date of the first revolution \((D^1)\) is being determined. That is, "At what date generation 1 products will be introduced and firms will start developing only generation 1 products?" In this model, two initial conditions are needed to determine the date of the first revolution: the date 0 variety of generation 0 products \((\mu_0^0)\), and the utility enhancement factor of generation 0 products, which is determined by the introduction date of generation 0 products \((D^0)\).

In order to determine \(D^1\), substitute \(g = 1\) into (13) and (14). Then, eliminating \(\mu_{M1}^0\), we have that

\[
\exp(A(D^1 - D^0)) \geq F(0)/F(\mu_{D1}^0), \quad (> 0 \text{ only if } D^1 = 0)
\]  

(17)

Using (1) for \(F(\cdot)\) and the particular solution with the initial condition given in (9) for \(\mu_{D1}^0\) (i.e. setting \(t = D^0\) for the solution), (17) becomes

\[
\exp(-AD^0)\exp(AD^1) \geq \frac{\theta^2(1-\alpha)LD^1}{2c} + \left(1 + \frac{\theta^2}{c^2} \mu_0^0\right)^\frac{1}{2}, \quad (> 0 \text{ only if } D^1 = 0).
\]  

(18)

Equation (18) implicitly determines the introduction date of generation 1 products \((D^1)\) as a function of the two initial conditions \(D^0\) and \(\mu_0^0\). Noting that generation 0 was introduced before period 0 \((D^0 < 0)\), we denote by \(\mu_{|D^0|}\) the stationary product development path given in (8). There are three types of solutions to (18): First, when \(\mu_0^0 > \mu_{|D^0|}\), meaning that the period 0 variety of generation 0 products exceeds its stationary level. Second, when \(\mu_0^0 = \mu_{|D^0|}\), meaning that the variety of generation 0 products is on the stationary path. Third, \(\mu_0^0 < \mu_{|D^0|}\), meaning that the initial variety of generation 0 is low relative to its stationary level.

Once the initial variety is given, the determination of the first revolution \((D^1)\) depends on the generation 0 introduction date \((D^0)\). Figure 2 illustrates the determination of \(D^1\) for the case where \(\mu_0^0 > \mu_{|D^0|}\). Note that the case where \(\mu_0^0 = \mu_{|D^0|}\) means that the two curves cross the vertical axis at the same point (same intercept)
while the case where $\mu_0^2 > \mu_{|D^0|}$ implies that the RHS(18) intercepts the vertical axis at the point higher than the LHS(18), implying that a multiple determination of $D^1$ is possible.

**INSERT FIGURE 2 ABOUT HERE**

Figure 2a shows that if $D^0$ is close to 0, then the first revolution will occur later than period 0. That is, $D^1 > 0$. However, if $D^0$ is far from period 0 (that is, for a given variety, generation 0 was introduced a 'long' time before $t = 0$, and therefore has a low utility enhancement factor) then figure 2b shows that the revolution occurs in period 0. This is the case of *instantaneous revolution equilibrium*.

5. **The Frequency of Technology Revolutions and Interest Rate Behavior**

In this section we analyze the effects of changing the technology growth rate, degree of product substitution, labor endowment, and cost of production on the length and frequency of the revolution cycles in an endless revolution equilibrium. Substituting (8) into (1), yields

$$F(\mu_r^2) = \left[ \frac{\theta^2(1 - \alpha)L(t - D^0)}{2} + c \right]^{-1}. \quad (19)$$

Substituting (19) into (16), we have that

$$\exp \left( \frac{\alpha h}{1 - \alpha} \Delta \right) = 1 + \frac{\theta^2(1 - \alpha)L\Delta}{2c}. \quad (20)$$

The LHS(20) and RHS(20) are drawn in figure 3 as a function of $\Delta$.

**INSERT FIGURE 3**

Perhaps the unique feature of this model is that it enables us to model the factors which influence the frequency of revolutions and the duration of generations of products. We can state the following.
Proposition 3  In an endless revolution equilibrium, the duration of each generation \((\Delta^*)\) increases with a decrease in the technology enhancement rate \((h)\), the resource endowment \((L)\), the development cost parameters \((\theta \text{ and } c^{-1})\), and the degree of intra-generation product substitution \((\alpha)\).

Proof: In figure 3, the RHS(20) increases with \(\theta, L, c^{-1}\), thereby reducing the equilibrium value of \(\Delta\). In terms of figure 3, an increase in \(\alpha\) or \(h\) implies an upward shift of LHS(20).\(^{11}\)

Q. E. D.

It is interesting to analyze the behavior of the interest rate during the life time of a generation. Differentiating equation (11) with respect to \(t\) yields

\[
\frac{dF(\mu_t^2)}{dt} = \begin{cases} 
  r_t F(\mu_t^2) & D^g < t < M^g \\
  -\frac{\pi_t}{w_t} + r_t F(\mu_t^2) & M^g < t < D^{g+1}
\end{cases}
\]

Substituting (5) for \(\pi_t\) and using (19) yields

\[
r_t = \begin{cases} 
  -\frac{\theta^2 (c^2 + \theta^2 \mu_t^2)^{-1/2}(1 - \alpha)L/2}{(c^2 + \theta^2 \mu_t^2)^{-1}c^2(\mu_t^2)^{-1} + \frac{\theta^2}{2} - \frac{\theta^2}{2}} & D^g < t < M^g \\
  (c^2 + \theta^2 \mu_t^2)^{-1}c^2(\mu_t^2)^{-1} + \frac{\theta^2}{2} & M^g < t < D^{g+1}
\end{cases}
\]

Equation (22) shows that during the gestation period, in order to attract innovators (pioneers) to develop products at the beginning of the generation, the equilibrium real interest rate is negative. Note that prior to the marketing date, the new generation firms do not make any profit. Starting from marketing date \(M^g\), the interest rate becomes positive. During the marketing period interest rates decline with time since over time, when variety increases, firms make lower profits.

In this model, because of the monopolistic competition market structure and the CES utility function, prices of products are constant in terms of labor. However, if we interpret the products as intermediate products and the utility function as a final product production function as in Ethier (1982), then the labor productivity (utility level in this paper), which is the inverse of the final product's price, follows a

\(^{11}\)An algebraic proof is given in the appendix.
cyclic growth pattern. The growth rate is highest when new generation products are marketed and declines when the number of marketed products increases. During the gestation period of the subsequent generation, the product expansion stops, and the growth rate becomes zero.

6. Concluding remarks

We develop a dynamic general equilibrium model in order to explain the evolution of technological change and cyclical growth. The main feature of this environment which distinguishes it from previous literature is that a continuous technological progress, measured by \((ht)\) in the utility function, results in a discrete revolution process and persistent growth cycles. We are able to sort out the parameters which affect the duration of each generation of products as well as the frequency of technology revolutions and hence the growth cycles. During the marketing period the initial profit of each firm is high. When the variety of products expands, there is more competition and the profit of existing firms decline. When the new generation of products is marketed, the profit of old generation firms becomes zero, and the profit of new generation firms is at the highest level and so on. At each moment the real interest rate adjusts to equate the present value of all future profit of a newly constructed firm with its development cost.

Finally, the term 'generation of products' used in this paper can also be given a different interpretation. It is possible to view the variety of products belonging to a particular generation as the variety of services supporting a particular technology. With this interpretation, the reason why consumers in the economy do not switch to a new technology immediately after it has been introduced is because the new technology is not supported by a sufficient amount of supporting services.\(^{12}\)

\(^{12}\)In a different context, Chou and Shy (1990) formalize the notion of supporting services by introducing consumers that choose, say, among different brands of computers by taking into consideration
Appendix:

Derivation of equation (13)

Since \( \pi_t^g = 0 \) for \( D^g \leq t \leq M^g \) (gestation period), by (11) we have that

\[
F_t^g = PV_t^g = PV_{D^g} \exp \left( \int_{D^g}^t r_s ds \right) = F(0) \exp \left( \int_{D^g}^t r_s ds \right). \tag{23}
\]

Therefore,

\[
\frac{F(0)}{F(\mu_t^{g-1})} = \exp \left( - \int_{D^g}^t r_s ds \right) \quad \text{for} \quad D^g \leq t \leq M^g. \tag{24}
\]

Substituting (24) into (11) and evaluating (11) for generation \( g - 1 \) yields

\[
F(\mu_{D^g}^{g-1}) = PV_{D^g}^{g-1} = \int_{D^g}^{M^g} \frac{F(0)(1 - \alpha)L}{F(\mu_t^g)\mu_t^{g-1}} d\tau
= \frac{(1 - \alpha)L F(0)}{\mu_{D^g}^{g-1}} \int_{D^g}^{M^g} \frac{1}{F(\mu_t^g)\mu_t^{g-1}} d\tau \quad \text{[since} \quad \mu_{D^g}^{g-1} = \mu_{D^g}^{g-1} \text{ for} \tau \geq D^g]\]
\[
= \frac{(1 - \alpha)L F(0)}{\mu_{D^g}^{g-1}} \int_{D^g}^{M^g} \frac{1}{\mu_t^g} d\mu_t^g = \frac{F(0)}{\mu_{D^g}^{g-1}} \int_{D^g}^{M^g} \mu_t^g d\mu_t^g \quad \text{[by (7)]}
\]
\[
= \frac{F(0)\mu_{M^g}^g}{\mu_{D^g}^{g-1}}. \quad \text{[first theorem of calculus]}
\]

An Algebraic Proof of Proposition 3:

Let \( A \equiv \theta^2 L/(2c) \) and \( B \equiv 1/(1 - \alpha) \). Then, (20) becomes \( \exp((B - 1)h\Delta) = 1 + AB^{-1}\Delta \). By implicit function rule, we have

\[
\frac{\partial \Delta}{\partial A} = - \frac{B^{-1}\Delta}{E} < 0, \quad \frac{\partial \Delta}{\partial h} = - \frac{(B - 1)\Delta e^{(B-1)h\Delta}}{E} < 0, \quad \frac{\partial \Delta}{\partial B} = - \frac{h\Delta e^{(B-1)h\Delta} + AB^{-2}\Delta}{E} < 0,
\]

where \( E = (B - 1)he^{(B-1)h\Delta} - AB^{-1} > 0 \) since the slope of the LHS(20) is greater than that of the RHS(20) at the equilibrium point.

the variety of services supporting each technology (supporting software for each computer brand). In fact, a referee has pointed out to us that our framework is close to the economics of networking, and that in general, a Schumpeterian accumulation of knowledge depend on past generation products and not only on the present.
References


Duration of generation $g$
Only generation $g$ products are developed ($\mu^g_t$ is increasing)

$\Delta^g$

Gestation period of gen. $g + 1$

Gestation period of generation $g$.
Consumers buy only gen. $g - 1$ products.

$G^g$

Consumers buy only generation $g$ products

Marketing period for generation $g$ products

t = $D^g$
introduction
date of generation $g$
t = $M^g$
marketing
date of generation $g$
t = $D^{g+1}$
introduction
date of generation $g + 1$

time

FIGURE 1: Duration and gestation periods of a generation of products
**FIGURE 2: Determination of the first revolution date**
FIGURE 3: Comparative dynamics