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OPTIMAL SAVING, INTEREST RATES
AND ENDOGENOUS GROWTH

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Abstract

The main point of this paper is that the apparent failure of economists thus far to establish a positive empirical link between interest rates and saving does not, by itself, discredit the hypothesis of a direct structural relationship between the two, *ceteris paribus*, because this structural relationship may be shifting about in response to changes in exogenous variables such as tastes and technology in a way that is consistent with *any* type of reduced-form correlation between interest rates and saving in the data. This point is demonstrated within a simple model of optimal saving, interest rates, and economic growth. The different implications of endogenous *versus* exogenous growth are explored in this context.

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by Thorvaldur Gylfason

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University of Stockholm; and CEPR.

1. Introduction

Econometric research is widely viewed by economists as having failed to establish a clear empirical relationship between interest rates and the saving behavior of households. Many studies, it is true, have reported significantly positive effects of interest rates on saving propensities in several countries. Others have concluded that no such evidence can be distilled from the data. Others still have indicated an inverse relationship between saving propensities and interest rates. Table 1 provides a glimpse of the variety of results that have been reported in twenty-four empirical studies over the past quarter of a century. Summarizing the evidence, Professor Alan S. Blinder of Princeton University, a member of President Clinton's Council of Economic Advisers, has recently said that "... there is *zero* evidence that tax incentives that enhance the rate of return on saving actually boost the national saving rate. *None. No* evidence. Economists now accept that as a consensus view." (Interview in *Challenge*, September-October 1992, p. 16).

Table 1. Overview of empirical results

Study	$\delta s/\delta r$	Country	Period
Wright (1967, 1969)	0.5	USA	1897-1959
Taylor (1971)	2.0	USA	1953-69
Heien (1972)	4.4	USA	1948-65
Juster & Wachtel (1972)	0.7	USA	1954-72
Blinder (1975)	0.0	USA	1949-72
Boskin (1978)	0.7	USA	1929-69
Fry (1978)	0.2	7 LDCs	1962-72
Howrey & Hymans (1978)	0.0	USA	1951-74
Blinder (1981)	0.0	USA	1953-77
Gylfason (1981)	0.7	USA	1952-78
Mankiw (1981)	0.0	USA	1948-80
Summers (1981)	4.5	Calibrated	
Carlino (1982)	0.0	USA	1957-78
Evans (1983)	1.0	Calibrated	
Friend & Hasbrouck (1983)	0.0	USA	1932-80
Giovannini (1983)	0.0	7 LDCs	1964-80
Blinder & Deaton (1985)	0.0	USA	1954-84
Mankiw <i>et al.</i> (1985)	0.5	USA	1950-81
Montgomery (1986)	0.0	USA	1953-82
Baum (1988)	0.0	USA	1952-82
Campbell & Mankiw (1989)	0.0	USA	1953-85
Campbell & Mankiw (1991)	0.0	5 MDCs	1957-88
Barro (1992)	0.6	10 MDCs	1957-90

Note: $\delta s/\delta r$ denotes the effect on the saving rate s (i.e., aggregate saving as a proportion of gross national product) of an increase in the interest rate r by one percentage point. The values of $\delta s/\delta r$ shown above were computed from estimates of elasticities by assuming $s = 0.1$ and $r = 0.04$ when representative values of s and r were not presented or could not be deduced from the study in question.

The representative estimates shown in Table 1 can be interpreted in at least three different ways. First, the fact that one-half of the studies, or 12 out of 24, failed to corroborate a significantly positive relationship between interest rates and saving may be considered as an indication that this relationship, if it exists at all, can hardly be strong. This impression may be strengthened by five studies (Houthakker and Taylor 1970; Weber 1970, 1975; and Springer 1975, 1977) that have reported a significantly *negative* relationship between interest rates and saving in the United States--without, however, quantifying the relationship (which is why these five studies are not included in Table 1). On the other hand, the remaining twelve studies included in the table seem to indicate a fairly strong positive link between interest rates and saving: the average estimate of $\delta s / \delta r$ in the 24 studies is 0.7. This estimate is consistent with a long-run elasticity of saving rates with respect to real interest rates of about 0.3. Put differently, if $\delta s / \delta r = 0.7$, then an increase in the interest rate from 4 percent to 7 percent annually will raise the saving rate from 10 percent to 12 percent of national income. The equations estimated in almost all the studies reviewed above were derived explicitly or implicitly from the theory of intertemporal choice. A useful and detailed survey of empirical work on saving behavior is provided by Smith (1991).

The third interpretation that I find most plausible is that saving rates and interest rates are *jointly* determined endogenous variables in macroeconomic analysis. As virtually any other pair of (stationary) endogenous macroeconomic variables, saving rates and interest rates can move in the same direction or in opposite directions depending on the movements of the exogenous variables that affect both of them. Therefore, the observation that saving rates and interest rates can move all over the map in principle and in practice does *not* by itself constitute a legitimate refutation of the hypothesis that the optimal propensity to save is stimulated by an increase in interest rates, *other things being equal*. For example, Hall's (1988, p. 365) conclusion that "... periods of

high expected real interest rates have not been periods of rapid growth of consumption" is not *per se* inconsistent with a positive partial or causal relationship between saving and interest rates.

This, however, has not been the prevalent interpretation of the literature thus far. Rather, the apparent absence of a strong empirical link between interest rates and saving according to a half of the studies summarized above has generally not been considered surprising in view of the neoclassical theory of intertemporal choice developed by Fisher (1907, 1930) and formalized by Ramsey (1928). This view is expressed by Deaton (1992, p. 61) among others. According to this theory, the optimal *rate of growth* of consumption is positively and unambiguously related to the real rate of interest by the Ramsey rule. In one simple formulation, this rule implies that

$$(1.1) \quad \frac{\Delta C_t}{C_t} = (r - \rho)\sigma,$$

where Δ is the first-difference operator in discrete or continuous time; C_t is real consumption at time t ; r is the real rate of interest (and is exogenously determined from the representative consumer's point of view); ρ is the subjective rate of time preference, a constant; and σ is the elasticity of intertemporal substitution, also a constant by assumption. Thus, an increase in interest rates stimulates saving unambiguously as long as $\sigma > 0$ in the sense that it induces the individual to postpone consumption. The extent of the postponement depends solely on the elasticity of substitution. Hansen and Singleton (1983) and others (including several of the studies reviewed in Table 1) have reported positive and in some cases quite high estimates of σ , while Hall's (1988) empirical conclusion is that σ is close to zero. On the other hand, Campbell and Mankiw (1989) and Deaton (1992) consider the simple Ramsey equation (1.1) and extensions thereof too simple to serve as reliable guides to the extent of intertemporal substitution.

In the above simple formulation of the solution to the Fisher-Ramsey problem, the *level* of consumption at a point in time is

$$(1.2) \quad C_t = C_0 e^{(r-\rho)\sigma t}.$$

Initial consumption C_0 is determined from boundary conditions:

$$(1.3) \quad C_0 = [\sigma\rho + (1-\sigma)r]W_0.$$

Here W_0 denotes initial wealth, that is, the present discounted value of current and future income from labor and interest.

Equations (1.2) and (1.3) together imply that consumption (and hence also saving) at any given time bears an ambiguous relation to the interest rate for given initial consumption or wealth, depending on the elasticity of substitution:

$$(1.4) \quad \frac{dC_t}{dr} = [(1-\sigma)(1+\sigma r t) + \sigma^2 \rho t] e^{(r-\rho)\sigma t} W_0.$$

Specifically, an elasticity of substitution above unity ($\sigma > 1$) is necessary (but not enough) to derive a negative relationship between consumption and interest in this case. On the other hand, an elasticity of substitution below unity ($\sigma < 1$) implies a positive relationship between the two for given W_0 . This well-known result has generally been regarded as a dynamic confirmation of the view that static substitution effects and income effects of interest rate changes on consumption and saving may conflict because lenders may react to higher interest rates by spending more and saving less out of given income. In a wide class of models, substitution effects outweigh income effects only when the elasticity of intertemporal substitution exceeds one (see, for example, Hall 1978, Summers 1981, Mankiw, Rotemberg, and Summers 1985, and Deaton 1992).

This interpretation is not without problems, however, for it can be shown that, with certain commonly assumed forms of utility functions, income effects cancel out on aggregation across individuals, leaving only substitution effects of changes in interest rates on aggregate consumption and saving. This is because borrowers, in contrast to lenders, become worse off when interest rates

rise, and may react by spending less and saving more out of a given income. If the reactions of borrowers offset those of lenders, the remaining effect of an increase in interest rates on aggregate saving is unambiguously positive.

Even so, there is a valid reason why interest rates and saving seem to be sometimes positively correlated in macroeconomic data, and sometimes not. To repeat, saving rates and interest rates are jointly endogenous macroeconomic variables that move about in response to a multitude of exogenous forces that impinge on the economic system. As a rule, any such pair of stationary endogenous macroeconomic variables can move in the same direction or in opposite directions, depending on the character and constellation of the exogenous phenomena that cause both of them to change. Therefore, the apparent failure of economists thus far to establish a positive empirical link between interest rates and saving does not, by itself, discredit the hypothesis of a direct structural relationship between the two, *ceteris paribus*, because this structural relationship may be shifting about in a way that is consistent with *any* type of reduced-form correlation between interest rates and saving in the data.

To set the stage for a further exploration of this issue, a simple model of optimal saving, interest rates, and economic growth is presented in Section 2. The model features an unambiguously positive structural relationship between the propensity to save and the interest rate, but it does not preclude the possibility of a negative reduced-form correlation between the two in response to changes in exogenous variables that reflect tastes and technology. The same applies to saving and growth: they can be either positively or negatively correlated when growth is endogenous, as we shall see. The quantitative properties of the model are subsequently considered in the light of numerical examples in Section 3. The simulations presented there confirm that variations in the exogenous parameters of the model within a reasonable range can produce a seemingly negative reduced-form relationship between saving and

interest rates, even though a positive structural link between the saving rate and the interest rate has been built into the model. The results are summarized in Section 4 that concludes with a brief discussion of their relevance for public policy.

2. Analytical framework

In this section, a simple model is developed for the purpose of exploring the interaction of saving behavior, interest rates, and economic growth under different assumptions about the nature of the growth process.

A. Consumption and saving

Consider an economy where infinitely lived households choose a path of consumption C_t and of real and financial assets A_t , including money, so as to maximize their utility over time:

$$(2.1) \quad \int_0^{\infty} \left(\frac{1}{1 - 1/\sigma} \right) \left(C_t^\lambda A_t^{1-\lambda} \right)^{1 - 1/\sigma} e^{-\rho t} dt.$$

Utility depends on current consumption and accumulated saving, i.e., assets, so that an increase in income that increases both current consumption and asset holdings intended for future consumption increases utility through *both* channels as long as $\lambda < 1$. The elasticity of substitution of consumption for assets is set equal to 1 for simplicity by assuming the inner function within the second pair of parentheses to have a Cobb-Douglas form, while σ is the elasticity of intertemporal substitution of the composite Cobb-Douglas bundle of consumption and assets and ρ is the rate of time preference as before. All the parameters (σ , λ , $1-\lambda$, and ρ) are positive and constant by assumption. Labor supply is left out of the utility function to avoid complexity.

The maximization of the integral in (2.1) takes place subject to the constraint that the accumulation of assets (saving, in other words) equals total income less

consumption:

$$(2.2) \quad \Delta A_t = Y_t + rA_t - C_t$$

where Y_t is real labor income and rA_t is real interest income. The steady-state solution to this dynamic maximization problem involves

$$(2.3) \quad \frac{C_t}{A_t} = \left(\frac{\lambda}{1-\lambda} \right) \left[\left(\frac{1}{\sigma} \right) g + \rho - r \right] \equiv f(g, r).$$

Here g is the optimal rate of growth of consumption and asset holdings along the steady-state equilibrium path. Consumption is thus proportional to asset holdings in long-run equilibrium. The ratio of consumption to assets is *inversely* related to the interest rate (i.e., $\delta f / \delta r < 0$) as long as $0 < \lambda < 1$. Moreover, this ratio is *directly* related to the rate of growth (i.e., $\delta f / \delta g > 0$) if $0 < \lambda < 1$ and $\sigma > 0$ as we have assumed. Without assets in the utility function (i.e., with $\lambda = 1$), the equilibrium solution to the above problem simplifies to the familiar Ramsey rule, $g = (r - \rho)\sigma$.

Equations (2.2) and (2.3) enable us to describe the long-run relationships among consumption, saving, asset holdings, the interest rate, and the rate of growth in a particularly simple way:

$$(2.4) \quad c = \frac{C_t}{Y_t} = \frac{f(g, r)}{f(g, r) + g - r},$$

$$(2.5) \quad s = \frac{S_t}{Y_t} = \frac{g}{f(g, r) + g - r},$$

$$(2.6) \quad a = \frac{A_t}{Y_t} = \frac{1}{f(g, r) + g - r}.$$

These equations are obtained as follows. First, because $s = S_t/Y_t = \Delta A_t/Y_t = (\Delta A_t/A_t)(A_t/Y_t)$ we have $s = ga$ in the steady state by definition. Moreover, we also have $s = 1 + ra - c$ by equation (2.2). But $c = C_t/Y_t = (C_t/A_t)(A_t/Y_t)$ which equals $f(g, r)a$ in the steady state by equation (2.3). Thus we can see that $s = ga =$

$1 + ra - f(g,r)a$ which, when we solve for a , gives equation (2.6). It follows directly that $c = f(g,r)a$ gives equation (2.4) and that $s = ga$ gives equation (2.5). These equations--(2.4), (2.5), and (2.6)--could also be derived from a utility function with a nonunitary elasticity of substitution between consumption and assets, that is, one in which the inner Cobb-Douglas function in (2.1) was replaced by a CES function; in that case, the middle term in equation (2.3) would become slightly more complicated, but the qualitative properties of $f(g,r)$ would remain unchanged.

Because $\delta f / \delta r < 0$ for given growth by equation (2.3), both the saving rate (s) and the asset ratio (a) are unambiguously positively related to the interest rate (r) for given growth (g) by equations (2.5) and (2.6). It is noteworthy that these relationships are independent of the intertemporal elasticity of substitution (σ). On the other hand, the direction of the effect of an increase in the interest rate on the propensity to consume (c) is indeterminate by equation (2.4). An increase in the rate of interest can conceivably increase *both* the propensities to consume and to save because $c + s = 1 + ra$ by equation (2.2).

The optimal saving rate can now be expressed in terms of the structural parameters of the intertemporal utility function (λ , σ , and ρ) as well as the of interest rate (r) and the rate of growth (g) by substituting equation (2.3) into equation (2.5). This yields

$$(2.7) \quad s = \frac{(1 - \lambda)g}{\left[1 - \lambda \left(1 - \frac{1}{\sigma}\right)\right]g + \lambda\rho - r}.$$

Here again we have an unambiguously positive nonlinear relationship between the saving rate and the interest rate, regardless of the value of σ . Specifically, if $\sigma = 1$, the expression for the saving rate simplifies to $s = (1 - \lambda)g / (g + \lambda\rho - r)$. This means that $\delta s / \delta r = s / (g + \lambda\rho - r)$. The corresponding interest elasticity of the saving rate is $\varepsilon = (\delta s / \delta r)(r / s) = r / (g + \lambda\rho - r)$; this elasticity is positive as

long as $g > r - \lambda\rho$. Also, we now see that increased flexibility of consumption over time (i.e., an increase in σ) and increased patience (i.e., a decrease in ρ) both lead to increased saving, other things being equal. Moreover, increased growth lifts the saving rate as long as $r > \lambda\rho$ (to see this, divide through equation (2.7) by g). Lastly, if assets are removed from the utility function (i.e., if we set $\lambda = 1$), equation (2.3) boils down to the Ramsey rule, $g = (r-\rho)\sigma$, and the expression for the optimal saving rate simplifies to $s = (r-\rho)\sigma a$ by equations (2.5) and (2.7).

The upward-sloping saving function (the SS schedule) in Figure 1 shows the pairs of saving rates (s) and interest rates (r) that satisfy equation (2.5) or, equivalently, equation (2.7). This schedule describes the representative household's optimal consumption plan over time given the exogenous or endogenous rate of growth (g) and the taste parameters λ , σ , and ρ .

B. Production and investment

In order to close the model, we now need to specify the mechanism by which the rate of growth and the rate of interest are determined. Let us assume a Cobb-Douglas production function for simplicity:

$$(2.8) \quad Q_t = B_t N_t^{1-\beta} A_t^\beta,$$

where $Q_t (= Y_t + rA_t)$ is total output, B_t is a technological shift parameter, N_t is labor, and $0 < \beta \leq 1$. Money and other financial assets are included as factors of production in addition to capital on the grounds that they enable firms to economize on the use of other inputs (Fischer 1974). Firms maximize profits by equating the marginal product of their real and financial capital $\beta(Q_t/A_t) = \beta(Q_t/Y_t)(Y_t/A_t) = \beta[(Y_t + rA_t)/Y_t](Y_t/A_t) = \beta(1 + ra)/a$ to the exogenously given interest rate r . Because $s = ga$ and $a = [\beta/(1-\beta)]/r$, we now obtain

$$(2.9) \quad s = \left(\frac{\beta}{1-\beta} \right) \left(\frac{g}{r} \right).$$

Thus, the rate of saving (and investment) is inversely related to the interest rate

for given productivity (by which is meant the contribution of capital to output as measured by the output elasticity β in the production function (2.8)) and for given growth on the supply side of the economy. An essentially similar negative relationship between investment and interest can also be derived from an explicitly intertemporal model where firms maximize their present discounted value and where new capital is costly to install (see, e.g., Kouri 1982).

This relationship is illustrated by the downward-sloping investment function (the *II* schedule) in Figure 1. Just as the *SS* schedule traces the combinations of interest rates (r) and saving rates (s) that maximize the utility of consumers over time for given tastes (i.e., λ , σ , and ρ) and growth (g), as we have seen, the *II* schedule shows the pairs of r and s that earn firms maximum profit from their investments for given productivity (β) and growth (g). This suffices to close the model: optimal saving and the interest rate can now be determined through the interplay of consumers and producers for given values of the exogenous parameters of the system, with or without endogenous growth.

C. Equilibrium with exogenous growth

Let us begin by assuming constant returns to scale in production and decreasing returns to capital (as in Solow 1956). Then the rate of growth of output and of capital equals the exogenously determined rate of growth of the labor force $n = \Delta N_t / N_t$, adjusted for efficiency. In this case, equations (2.7) and (2.9) form a closed system in which the saving rate and the interest rate are determined in reduced form by the underlying taste and technology parameters of the model:

$$(2.10) \quad s = \frac{1 - \lambda + \left(\frac{\beta}{1 - \beta} \right)}{1 - \lambda \left(1 - \frac{1}{\sigma} - \frac{\rho}{n} \right)},$$

$$(2.11) \quad r = \frac{\lambda \rho + \left[1 - \lambda \left(1 - \frac{1}{\sigma} \right) \right] n}{1 + (1 - \lambda) \left(\frac{1 - \beta}{\beta} \right)}.$$

A quick glance at this pair of reduced-form equations and the underlying structural equations (2.7) and (2.9), and at the corresponding intersection of the two familiar-looking schedules in Figure 1, suggests why interest rates and optimal saving can move all over the map in principle and in practice, and why it must therefore be difficult to identify the structural relationship between saving and interest rates in econometric work. Specifically, supply shocks tend to shift one or both schedules in the figure in such a way that the interest rate and the saving rate move in the same direction. For example, increased growth (i.e., an increase in $n = g$) and increased marginal productivity of capital (i.e., an increase in β) shift the investment function to the right by equation (2.9), whereas the saving function can shift either to the right or left depending on whether r is larger or smaller than $\lambda \rho$ (if n rises) or it stays put (if β rises) by equation (2.7). In any case, equation (2.11) shows that the interest rate rises with increased growth (because $1 - \lambda[1 - (1/\sigma)] > 0$) and productivity, and the saving rate also rises by equation (2.10). This explains the plus signs in the first two columns of Table 2 that summarizes the effects of changes in the exogenous parameters of the model on the interest rate and the saving rate.

On the demand side, we see from equation (2.7) that an increase in the elasticity of intertemporal substitution σ and in patience (i.e., a decrease in the discount rate ρ) shifts the saving function to the right without affecting the

investment function by equation (2.9). Therefore, saving increases and the rate of interest falls. This explains the mixed sign pattern shown in the last two columns of Table 2.

**Table 2. The interest rate, saving, and exogenous growth:
Comparative statics**

	n	β	σ	ρ
r	+	+	-	+
s	+	+	+	-

In sum, then, the table shows why interest rates and saving move sometimes in the same direction and sometimes not, depending on the source of the exogenous shocks to the system. Even so, growth, interest, and saving remain fundamentally positively correlated for given technology (β) and tastes (σ and ρ). Put differently, the rates of growth, interest, and saving cannot move in opposite directions in this model unless at least one of the two key taste parameters (σ or ρ) changes. The model thus preserves, through optimal saving, the fundamental positive link between interest and growth associated with the Golden Rule.

A special case. Before proceeding to the case of endogenous growth, it may be instructive to consider a special version of the exogenous-growth model above in which assets do not enter the intertemporal utility function so that $\lambda = 1$ in expression (2.1). In this case, the dynamic utility maximization by the representative household leads to the Ramsey rule, $g = (r - \rho)\sigma$. This is confirmed by substituting $\lambda = 1$ into equations (2.10) and (2.11), for thus we get

$$(2.12) \quad r = \rho + \frac{n}{\sigma},$$

$$(2.13) \quad s = \frac{\left(\frac{\beta}{1-\beta} \right)}{\left(\frac{\rho}{n} + \frac{1}{\sigma} \right)}.$$

These reduced-form equations have all the same comparative-statics properties as the preceding pair of equations (compare Table 2) except increased productivity now leaves the rate of interest unaffected because the *SS* schedule in Figure 1 becomes horizontal by equation (2.12).

D. Endogenous growth

We now turn to the case where the rate of growth is modeled as an endogenous variable. Suppose technology is embodied in capital so that $B_t = A_t^{1-\beta}$ in equation (2.8) at the macroeconomic level. Aggregate production is then characterized by increasing returns to scale and constant returns to capital, broadly defined (see Romer 1986):

$$(2.14) \quad Q_t = N_t^{1-\beta} A_t.$$

Profit maximization by each individual firm requires equality between the interest rate and the marginal product of capital at the microeconomic level as before, compare equation (2.8). This now implies that

$$(2.15) \quad r = \beta N_t^{1-\beta}$$

by equation (2.14). Thus, the representative firm does not reckon with technological spill-over effects reflected in the macroeconomic production function in equation (2.14) by assumption. A stationary interest rate requires N_t to be held constant because of the scale effects of employment on growth and interest in this version of the model. Output grows at the same endogenously determined rate as consumption and asset holdings, so that the propensities to consume and save and the asset ratio (c , s , and a) are constant in equilibrium as before. Notice also that now the ratio of asset holdings to output is given by

$A_t/Q_t = N_t^{\beta-1}$ by equation (2.14), so that $s = ga = g(A_t/Q_t)(Q_t/Y_t) = gN_t^{\beta-1}(1 + ra) = gN_t^{\beta-1}[1 + r(s/g)] = gN_t^{\beta-1}/[1 - rN_t^{\beta-1}]$. Therefore, $\delta s/\delta r > 0$ as before.

When the labor force is exogenously determined and growth is endogenous, the roles of the interest rate and growth are interchanged in the model. In the *Solow version* of the model, the rate of growth was determined solely by the exogenous rate of growth of the efficiency-adjusted labor force, and the interest rate and the saving rate adjusted as shown in Figure 1. In the *Romer version* of the model, on the other hand, the interest rate depends solely on productivity and employment both of which are exogenously given by assumption. Thus, the interest rate now becomes an exogenous variable instead of growth. In Figure 1, the interest rate is now represented by a horizontal line (not shown) as implied by equation (2.15) and the rate of growth adjusts to ensure that the *SS* and *II* schedules intersect at that exogenously determined rate of interest. Now, for example, an increase in intertemporal substitution (σ) will shift the *SS* schedule to the right as before and thus trigger an increase in growth that will shift the *SS* schedule back to the left and the *II* schedule to the right until the two schedules intersect on the horizontal interest-rate line that has not moved by assumption. The outcome of the experiment is thus that saving and growth go up, but the interest rate remains the same.

An alternative description of this endogenous-growth version of the model is provided in Figure 2 where the rate of growth now appears on the vertical axis instead of the interest rate. The downward-sloping *SS* schedule is based on equation (2.7); it traces the combinations of saving rates and growth rates that keep utility at a maximum over time for a given interest rate, on the assumption that $r > \lambda\rho$. The *SS* schedule in Figure 2 shifts up and to the right (i) when the interest rate (r) increases, (ii) when the intertemporal elasticity of substitution (σ) increases, and (iii) when the discount rate (ρ) decreases, compare equation (2.7). The upward-sloping *II* schedule is derived from

equation (2.9) which implies that $g = [(1-\beta)/\beta]rs$. It shows the pairs of saving rates and growth that bring firms maximum profit for given interest rates and productivity. The II schedule rotates to the right when productivity improves (i.e., when β increases), and to the left when the interest rate (r) increases.

The system can now be summarized by the following pair of reduced-form equations for the rates of optimal saving and endogenous growth:

$$(2.16) \quad s = \frac{1 - \lambda + \left(\frac{\beta}{1 - \beta} \right) \left(1 - \frac{\lambda \rho}{r} \right)}{1 - \lambda \left(1 - \frac{1}{\sigma} \right)},$$

$$(2.17) \quad g = \frac{r \left[1 + (1 - \lambda) \left(\frac{1 - \beta}{\beta} \right) \right] - \lambda \rho}{1 - \lambda \left(1 - \frac{1}{\sigma} \right)}.$$

Equation (2.17) shows how the rate of growth is determined endogenously by tastes and technology in addition to the interest rate. We can now see that an exogenous increase in the marginal product of capital (i.e., an increase in β), by rotating the II schedule down and to the right in Figure 2, decreases the rate of growth and increases the saving rate, provided that $r > \lambda \rho$. Here, then, we have a case where saving and growth move in opposite directions. On the other hand, an increase in the interest rate, by rotating the II schedule up and to the left in Figure 2 and by shifting the SS schedule up and to the right at the same time, increases both the saving rate and the rate of growth. Here we see again how saving, growth, and the rate of interest all move in the same direction for given technology (β) and tastes (σ and ρ). Changes in the taste parameters influence saving and growth *via* the SS schedule without changing the interest rate. For example, increased patience (i.e., a decrease in ρ) and increased intertemporal substitutability (i.e., an increase in σ) shift the SS schedule to the

right, and thus stimulate saving and growth for a given rate of interest, compare equation (2.15). Once again, any conceivable pattern of interest rates and saving--and, now, growth--is possible, especially if two or more exogenous parameters change at the same time (see Table 3). But if r , s , and g do, in fact, move in opposite directions, either tastes or technology must be changing in the background in this model.

**Table 3. The interest rate, saving, and endogenous growth:
Comparative statics**

	r	β	σ	ρ
s	+	+	+	-
g	+	-	+	-

A special case. Again, the case without assets in the utility function (i.e., with $\lambda = 1$) is of interest. In this case, as before, dynamic optimization yields the Ramsey rule and a corresponding saving function:

$$(2.18) \quad g = \sigma(r - \rho),$$

$$(2.19) \quad s = \left(\frac{\beta\sigma}{1-\beta} \right) \left(1 - \frac{\rho}{r} \right).$$

These equations are obtained by substituting $\lambda = 1$ into equations (2.16) and (2.17). Equation (2.19) implies that saving rises unambiguously in response to an increase in the rate of interest given the two basic taste parameters of the representative consumer, that is, the discount rate (ρ) and the intertemporal elasticity of substitution (σ), and also given productivity (β). Specifically, we now have

$$(2.20) \quad \frac{\partial s}{\partial r} = \frac{\beta\sigma\rho}{(1-\beta)r^2} > 0.$$

Therefore, the interest elasticity of the saving rate is

$$(2.21) \quad \varepsilon = \frac{\partial s}{\partial r} \frac{r}{s} = \frac{\rho}{r - \rho}$$

which is positive as long as $r > \rho$, that is, as long as the growth rate (g) and saving rate (s) are positive, compare equations (2.18) and (2.19). Notice also that there is no way for the saving rate to change by equation (2.19) without either a change in the taste parameters (ρ or σ) or in productivity (β) or in the interest rate triggered by a productivity shock on the supply side of the economy by equation (2.15). In other words, the saving rate reacts to external shocks or to changes in economic policy in this model only if the underlying parameters reflecting tastes and technology do so. We shall return to this in Section 4.

A numerical illustration. To convey an idea of the quantities involved in this simple version of the endogenous-growth model without assets in the utility function (i.e., with $\lambda = 1$), a numerical illustration may be useful at this point. Assume the following parameter values for example: $\sigma = 1$, $\rho = 0.025$, $\beta = 0.2$, and $k = Q_t/A_t = 0.25$. Then we have $r = 0.05$ by equations (2.14) and (2.15), $g = 0.025$ by equation (2.18), and $s = 0.125$ by equation (2.19). Moreover, assets are five times larger than labor income: $a = 5$ (because $s = ga$). By implication, interest income is one-fourth of labor income: $ra = 0.25$. This, in turn, means that the propensity to consume out of labor income exceeds one: $c = 1.125$ (because $c = 1 + ra - s$), but the propensities to consume and save out of total income sum to one: $cY_t/Q_t = 0.9$ and $sY_t/Q_t = 0.1$. These values rhyme reasonably well with actual numbers observed in the real world. They do, however, entail a stronger structural link between saving and interest rates than has been found in most empirical studies (compare Table 1). Specifically, the above values imply that $\delta s/\delta r = 2.5$ by equation (2.20) and the corresponding elasticity is 1 by equation (2.21).

3. Numerical examples

Now the time has come to ask: If the interaction of saving behavior, interest

rates, and economic growth is governed by the relationships developed in the preceding section, how do these variables react to changes in the exogenous variables that determine them? Do variations of the underlying exogenous parameters of the model within a reasonable range produce a suggestive pattern of movement of the endogenous variables?

To seek an answer to these questions, we now proceed to simulate the behavior of saving rates, interest rates, and growth under various assumptions about the values of the exogenous parameters of the model and about exogenous *versus* endogenous growth. This will also enable us to investigate the sensitivity of the interaction of the three main variables of the model to variations in individual parameters and assumptions.

A. Structural parameters

Each of the five main parameters of the model is assumed to take one of five values as shown in Table 4.

Table 4. Assumed values of structural parameters

λ	σ	ρ	n	β
0.5	0.25	0.001	0.001	0.05
0.625	0.5	0.025	0.01	0.10
0.75	1.0	0.05	0.02	0.15
0.875	2.0	0.075	0.03	0.20
1.0	4.0	0.1	0.04	0.25

In the first column of the table, the value of λ ranges in four equal steps from 0.5 up to 1, in which case assets disappear from the intertemporal utility function in (2.1). Next, the elasticity of intertemporal substitution σ is centered on 1, which means that the intertemporal utility function in (2.1) is logarithmic, and σ is allowed to rise and fall from its central value by a factor of two and

four. Then, the discount rate ρ ranges in steps of 2.5 percent from approximately zero to 10 percent. The growth rate ranges from zero to 4 percent. Lastly, the output elasticity of capital in a broad sense spans the range from 0.05 to 0.25.

B. Simulations of saving and interest

We begin by computing the saving rate and the interest rate from the reduced-form equations (2.10) and (2.11) using all possible combinations, $3^5 = 243$ in number, of the bold-faced parameter values displayed in Table 4. (If all possible permutations were included, their number would rise to $5^5 = 3125$.) The "observations" so obtained are presented in Figure 3. The calibrated saving rates and interest rates lie within a plausible range: the saving rate is mostly between 3 percent and 30 percent of income and the interest rate is between 1 percent and 14 percent per year. The mean values of s and r in the figure are 12 percent and 5 percent (with standard deviations of 8 and 3 around the means).

Figure 3 reveals what appears to be an *inverse* relationship between saving and interest. A linear regression of the saving rate on the interest rate through the points in the figure seems to confirm this impression:

$$(3.1) \quad s = 0.20 - 1.60r$$

(22.9)(9.9)

where t -statistics are shown within parentheses and $R^2 = 0.29$. (In view of the nonlinear pattern observed in the figure, a regression of s on $\ln(r)$ produces a slightly better fit.) If these were actual observations, the statistical researcher might be tempted to infer a significantly negative relationship between the propensity to save and the interest rate from this sample. Such an inference would be invalid, however. This is because a *positive* structural relationship between the saving rate and the interest rate has been built into the model through equation (2.7).

A conclusion that *can* be drawn from Figure 3 and from the regression

equation (3.1) is that the saving function shifts about more than the investment function in the simulation, compare Figure 1. In other words, demand shocks dominate supply shocks in this experiment. A negative reduced-form correlation between saving and interest is *not* evidence against a positive structural relationship between saving rates and interest rates. Moreover, from the "observations" shown in Figure 3 it should, in principle, be possible to identify the positively sloped saving schedule. In practice, however, that has proved difficult, not only in empirical studies of saving behavior (recall Table 1), but also in other fields of economics. Supply responses to price changes are notoriously hard to quantify with confidence, for example.

C. Simulations of saving and growth

Saving and growth can be approached in the same way, based on the endogenous-growth version of the model presented in Section 2, equations (2.16) and (2.17). We assume the same values of the exogenous structural parameters as in Table 4, except the rate of growth is now replaced by the interest rate that is assumed to vary from 0.001 to 0.1, just as the discount rate ρ in the middle column of the table.

A calculation of the equilibrium rates of saving and endogenous growth using all 243 possible combinations of the bold-faced parameter values in Table 4, in addition to the same three values for r as for ρ , yields the results shown in Figure 4. The mean values of s and g in the figure are 12 and 4 percent (with standard deviations of 28 and 6 around the means). The parameter constellations assumed produce a few extreme values of both variables, including some negative ones (when r is too far below $\lambda\rho$, compare equations (2.16) and (2.17)). Even so, the results indicate a clear positive reduced-form relationship between saving and growth in the sample. This impression is confirmed by a linear regression of the growth rate on the saving rate:

$$(3.2) \quad g = 0.02 + 0.18s$$

$$(7.6)(26.9)$$

where t-statistics appear in parentheses as before and $R^2 = 0.75$. The coefficient on the saving rate, 0.18, indicates that the rate of growth is quite responsive to variations in the underlying structural parameters. This does *not* mean, however, that saving and growth cannot move in opposite directions. Rather, it means that the saving schedule is less stable than the investment schedule in the simulation behind Figure 2. Put differently, the simulation suggests that tastes are more volatile than technology in this example as before. Notice also that the effects of changes in the parameters on growth are permanent, not transitory, by the nature of the endogenous growth process assumed.

D. Sensitivity tests

It now remains to apply the framework developed above to study the sensitivity of individual components of the model to variations in the exogenous structural parameters. It is of particular interest to review the sensitivity of the interplay of saving and interest rates to changes in the elasticity of intertemporal substitution, in view of the widely held notion that the shape of the saving function is closely linked to that parameter. In the present model, however, the direction of the structural demand-side relationship between the saving rate and the interest rate is independent of the elasticity of intertemporal substitution (σ) as we have seen, compare equation (2.7). It now remains to see how the responsiveness of the saving rate to exogenous shocks varies with the elasticity of substitution.

For example, let us assume three of the parameters shown in Table 4 to take the following values: $\lambda = 0.875$, $\rho = 0.05$, and $\beta = 0.2$. We can then compute the response of the saving rate (s) and the interest rate (r) to changes in the exogenously given growth rate from $g = 0.001$ to $g = 0.04$ for each of the five values of the elasticity of substitution shown in the table. The consequences of

doubling σ in four steps from 0.25 to 4 are presented in Figure 5 where each of the five schedules shown corresponds to one of the five assumed values of σ as indicated. The figure shows how the slope of the reduced-form relationship between s and r resulting from increased economic growth decreases as the value of σ increases. In other words, the greater the elasticity of intertemporal substitution, the greater appears the positive response of saving to interest rates, as expected.

Turning to endogenous growth at last, we can also simulate the response of the saving rate (s) and the rate of growth (g) to changes in the now exogenously given interest rate in steps of 2.5 percent from $r = 0.025$ up to $r = 0.125$ for each of the five values of the elasticity of substitution (σ) shown in Table 4 and for $\lambda = 0.875$, $\rho = 0.05$, and $\beta = 0.2$ as before. The simulation shows how the response of saving rates to increased interest rates increases with the value of σ (Figure 6). Saving is less responsive to interest rates when they are high than when they are low. Even so, saving responds significantly to interest rates unless very high real interest rates coincide with very low values of the elasticity of intertemporal substitution (σ) in this experiment.

4. Discussion

The simultaneous endogeneity of saving rates and interest rates in the economic system makes it possible for them to move in the same direction or in opposite directions, over time and across countries, depending on the exogenous forces that affect both of them. In this light, it is not surprising that the structural links between saving and interest derived from the neoclassical theory of intertemporal choice have proven hard to establish in econometric research. Here we have seen examples of how saving rates and interest rates may seem to be *negatively* correlated in economic data (that is, in reduced form) despite a strongly *positive* structural link between the two by hypothesis.

What can monetary and fiscal authorities infer from this about the

formulation of interest rate policies? In particular, does it make sense to maintain high real interest rates through monetary restraint, or to tax interest income lightly, in the hope of stimulating saving--and, perhaps, growth--if observed saving and interest rates are inversely related in practice as, for example, in Figure 3?

The arguments presented in this paper suggest an answer to these questions, in two parts.

In the exogenous-growth version of the model presented here, the long-run relationship between saving and interest rates is influenced by three exogenous variables (compare Figure 1): (i) growth, which is exogenous by assumption; (ii) the discount rate, which is essentially a psychological parameter; and (iii) the elasticity of intertemporal substitution, a quantity that can be influenced by institutional change. For example, a liberalization of borrowing constraints in the banking system (with a commensurate strengthening of bank supervision) would create conditions for increased flexibility in consumption and saving. Policies thus geared toward increased intertemporal substitution would increase saving and investment and lower interest rates at the same time. Moreover, if a distinction between gross and net interest and hence also between the return on saving and the cost of investment were introduced into the story, saving and investment could be stimulated by a favorable tax treatment, for instance. Also, a distinction between private and public saving would introduce a possible link between fiscal policy and total saving in the model.

However, even if the saving rate can be affected by fiscal policy and by institutional change, there is no way for high-interest policies through monetary restraint to promote saving in the long run in the exogenous-growth version of the model without further extension. Essentially, this is because real variables such as the real interest rate cannot be influenced in the long run by nominal variables such as money and credit in this version of the model. Put

differently, the marginal product of capital is beyond the reach of the central bank in Solow's model of growth. In other words, the exogeneity of growth makes it impossible for the monetary authorities to influence saving and investment in the long run.

In the endogenous-growth version of the model, on the other hand, public policies to increase productivity and growth stimulate saving and increase interest rates at the same time (compare Figure 2 and Table 3). In this version, the marginal productivity of capital and the rate of growth of output and other real variables in the long run are modeled in a way that allows them to be influenced by a number of variables: these include employment (compare equation (2.15)), trade policy and research and development (Grossman and Helpman 1991), fiscal policy (Barro 1990, Barro and Sala-i-Martin 1992), redistribution policies (Persson and Tabellini 1991), and monetary policy (Fischer 1991). The endogeneity of growth thus opens a new channel through which the government, including the monetary authorities, may be able to influence saving, investment, and interest rates, and, of course, growth. Figures 1 and 2 are intended to bring this out: in Figure 1, the exogeneity of growth makes real interest rates immune to changes in monetary policy in the long run, whereas in Figure 2 monetary policy can possibly be used to influence real interest rates, saving, investment, and growth permanently.

Even so, the contrast between the two versions of the growth model studied here, and between models of exogenous *versus* endogenous growth in general, should not be exaggerated. In practice, it may prove difficult to distinguish the properties of endogenous growth processes from the medium-term properties of neoclassical growth models where the rate of growth adjusts slowly along the transition path of the economy to a steady state.

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Figure 1. Saving and interest

Figure 2. Saving and growth

Figure 3. Saving
and interest

Figure 4. Saving and growth

Figure 5. Saving,
interest, and
substitution

Figure 6. Saving,
interest, and
endogenous growth