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THEORY OF STAGNATION AND GROWTH

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Vested Interests

In a Positive Theory of Stagnation and Growth

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We study a positive theory of stagnation and growth aimed at understanding the large variations in growth outcomes across actual economies. The theory points to the fundamental role played by *vested interests* in determining policies which are key to the growth process: some agents seek to prevent the adoption of new technologies. We develop a model of technology adoption, and show how technological innovation may sow the seeds of its own destruction. In particular, we find that the equilibrium is characterized by a long cycle of stagnation and growth. Over this cycle, incumbent innovators have sufficient political influence that new technologies are prohibited, and only as these incumbents are phased out of the economy will new innovation occur. In formalizing our theory we make a methodological contribution by characterizing dynamic voting equilibria when voters must forecast the effects of different current policies on future prices and policy outcomes.

“The political economy of technological change is only dimly understood. To some observers, technological change seems to resemble the human life cycle: the vigor of youth is followed by the caution of maturity and finally the feebleness of old age and the ‘climacteric’. ...If we are to understand why the fires of innovation die down, we must propose a model in which technological progress creates the conditions for its own demise.” [Mokyr (1990a); p. 261]

1 Introduction

Motivated by the large variations in medium-and long-run economic performance across countries and within countries over time, we study what we believe is crucial for understanding these observations: the process of technological change. Our approach differs from much of the recent literature on economic growth in that we focus on the socio-political process determining growth-oriented economic policies, rather than on what policies are good per se. The pervasiveness of growth-preventing policies—policies that reduce the incentives for investment and technological innovation—suggests that it is especially important to study policy determination in the context of economic growth. Our way of understanding the existence of such policies rests on a notion of *vested interests*: owners of state-of-the-art technology try to block the emergence of superior technologies by using the political process.

Our point of view leads to a process of technological change which is fundamentally fragile. The first contribution of this paper is to formalize the idea of vested interests in the context of a model of technological change, and, in particular, to give formal meaning to what was hinted at in our opening quote: the possibility that technological progress itself may sow the seeds of its own destruction, leading to a slower pace of technological change and growth than what is technologically efficient. More precisely, we find that our theory can account for policies which imply very poor growth outcomes, and in particular, for cycles with long periods of stagnation and short periods of innovation and growth. These cycles are driven by the dynamic forces created by, on the one hand, the returns to innovation, and on the other, the counteracting vested interests among those who control the old technologies. The cycle of stagnation and growth is an example of how innovation itself *creates* vested interests and, as a consequence, a long period of technological and economic stagnation.

Our second contribution is methodological. We show how to extend existing politico-economic equilibrium theory (e.g., see Persson & Tabellini (1991), Alesina & Rodrik (1991), Perotti (1990), Saint-Paul & Verdier (1992), and Bertola (1992)) to a case where the voting decision is fully forward-looking and involves non-trivial dynamics. In our case, and in general, in order to derive a voter’s policy preference, it is necessary to predict the equilibrium

effects of a change in the current policy on the future path of the economic state variable, and on the future path of policy outcomes. We provide a definition of dynamic politico-economic equilibrium and show how it can be applied in our study of vested interests and technological change.

We build on the vintage human capital model of Chari & Hopenhayn (1991). This setup allows us to study both the process of adoption of new technologies and how it interacts with the distribution of agents across age, skills, and political views. The key economic decision for a consumer in this model is the career choice: he can either choose to work as unskilled throughout his career or to accumulate human capital and become a manager toward the end of his lifetime. The managerial human capital is specific to a technology vintage, and the agent's choice also specifies whether to learn to manage an already existing technology, or to innovate by learning to manage a new and better technology. Innovation requires the agent to be unproductive while learning, whereas learning in a technology where there are already skilled agents gives positive productivity.

The career decisions also affect the consumption streams, so the other aspect of innovation is to shift equilibrium consumption away from current toward future, cheaper products. It is in this context natural, then, that managers of current technologies are against innovation, since it shifts demand away from their products and lowers their profits. On the other hand, current workers will favor innovation, since it shifts equilibrium consumption away from the present, hence lowering the prices of current products and therefore raising the purchasing power of the unskilled. We consider a regulatory policy of a simple kind: either innovation is allowed, or it is not. Since the career and technology adoption choices may induce heterogeneity in skills, they will also determine the views in the population on the desirability of the regulatory policy. We describe the political process determining the policy by letting agents vote on the current policy, with a majority rule aggregating individual views into a chosen policy.

Our equilibrium definition specifies a recursive law of motion for the policy parameter and a law of motion for the state variable—the distribution of agents over vintages—that are both generated by the maximizing behavior of the agents in their two roles as consumers and political participants. The solution of the agent's political decision problem involves evaluating his utility obtained with and without prohibition of technological innovation. The effects of a policy change can be analyzed by finding the future law of motion of the distribution of skills implied by the responses of agents to the new policy—an altogether non-trivial problem since full equilibrium dynamics need to be calculated, whether the economy currently is on a stationary path or not. In particular, it is an essential element of the political equilibrium that the agents need to consider the economic outcomes of *all* policies,

including those which will never be realized. Our equilibrium definition respects rationality also on the part of the voters: we do not limit the voter's ability to forecast or calculate the dynamic effect on future skill distributions and future votes of the different current policies. Despite nontrivial agent heterogeneity, we are able to fully characterize such equilibria for simple example economies.

We find that the properties of the equilibrium of our model depend critically on the characteristics of the initial distribution of economic agents. There are two possible, and very different, long-run economic outcomes: there can be no regulation, and hence sustained technological innovation and growth, or there may be stagnation. The stagnation outcome typically takes the form of a long cycle in which there is a relatively short period of technological innovation, and subsequently a longer period of stagnation, where the previous innovators form the basis of a political majority opposing further innovation. The period of stagnation is interrupted only when the economy reaches its most "backward" state; then, innovation again becomes politically feasible, and the cycle restarts.

The role of conflicting interests for policy formation has long been recognized; a recent study can be found in Olson (1982). Its particular importance for issues related to technological change is perhaps less explored. Mokyr (1990a) argues, as did others (see, e.g. Maine (1890) and Landes (1983)), that vested interests among agents specialized in the old technologies may have played an important role in the way the Industrial Revolution developed in Britain and elsewhere in Europe.¹ According to the hypothesis, the British leaders at the time were much more reluctant than leaders elsewhere in Europe to give in to the political demands by all those opposed to the new technologies and to the general changes that were occurring. The argument potentially accounts for Britain's relative success early on as well as for its industrial problems experienced later—the political success of special interest groups once in favor of technological change now prevented further change. Olson (1982) gives further historical accounts of how coalitions of agents have been successful in blocking technological and economic development. Moreover, many modern development economists emphasize the interaction between political and economic forces as key to understanding the development process (see for example Krueger (1990)). Indeed, the list of circumstances in which vested interests have been detrimental to economic growth can be made extensive, since it can arguably include all those cases in which taxes or regulation have precluded technological change from occurring. We would suggest, for example, that trade barriers,

¹See also Mokyr (1990b) and Mokyr (1992).

high capital taxes, etc. in many cases played this role. One striking and very concrete example of how the incentives for technological innovation have been substantially reduced is Perú; there, opening a new business is associated with next to prohibitively expensive administrative costs, as described in de Soto (1989). Our model attempts to capture some of the forces that lead to vested interests and, in particular, how these forces can arise naturally as a result of the process of technological innovation. In a context of economic growth, we believe that it may be crucial for understanding the political and economic outcomes that the political heterogeneity is not exogenous, but induced by the growth process itself.

Cycles in the adoption of techniques can also occur as a result of special properties of the production technologies, as in Parente (1990) model of technology adoption. Breziz, Krugman, & Tsiddon (1991) study a similar setting with two countries, and they find that different countries adopt new techniques, and become technological “leaders”, at different times in such a way that world-wide production is efficient. The cycles of stagnation and growth that result in our framework do not rely on special aspects of the production technology other than heterogeneity of techniques. The kind of cycles we obtain could naturally also occur in multi-country settings, although we have yet to explore all the implications of trade between countries. Persson & Tabellini (1991), Alesina & Rodrik (1991), Perotti (1990), Saint-Paul & Verdier (1992), and Bertola (1992))

The studies by Persson & Tabellini (1991), Alesina & Rodrik (1991), Saint-Paul & Verdier (1992), and Bertola (1992) are similarly concerned with mechanisms through which agent heterogeneity affects the growth path. Whereas our focus is on the process of technological change and regulation, there the link occurs via voting on the taxation of physical capital. In another set of studies, Perotti (1990), Glomm & Ravikumar (1992), Saint-Paul & Verdier (1991), and Boldrin (1993) analyze how human capital formation is affected by redistribution measures or by the federal funding of expenditures on education. In these papers on capital taxation and on education, the main emphasis has been on how the distribution affects growth. In this sense, our analysis is perhaps a little different since it stresses that the interaction between distribution and growth is two-sided and, indeed, in our particular framework this two-sidedness turns out to be key to the growth process.

In section 2 we describe a version of the model, and we proceed in the following section to describe stationary equilibria for constant, exogenous policies: one stationary equilibrium with technological innovation and growth, and one without. Section 4 then motivates and describes the definition of a political equilibrium. We characterize equilibria for a few simple example economies in section 5. In that context, we show how stagnation, as part of a stagnation/growth cycle or as a permanent phenomenon, does occur in democratic equilibria with fully rational agents. Section 6 concludes with some remarks.

2 The Model

The model consists of overlapping generations of three periods of agents with the measure of agents born each period normalized to be one. These agents are endowed with one unit of time per period.

Preferences: The utility function has the form

$$c_1 + \beta c_2 + \beta^2 c_3,$$

where c_i is the consumption of an agent at age i . Agents have one unit of labor each period.

Production of consumption goods: The consumption good can be produced from technologies of different vintages that are indexed by κ . The production function of each of them is:

$$f_\kappa(\varphi_m, \varphi_n) = \gamma^\kappa \varphi_m^\alpha \varphi_n^{1-\alpha}, \quad 0 < \alpha < 1, \gamma > 1,$$

where φ_m is the input of managers and φ_n is the input of workers. Each new technology hence improves by a factor γ on the best existing technology. Technologies have to be adopted in order: if in a given period the best technology in use is of type κ , then the next technology adopted is by necessity $\kappa + 1$, independently of when it is adopted.

In what follows, we will refer to each technology vintage with its *relative* efficiency: the technologies are relabeled every period and are indexed by τ . The most modern technology that can be used in production in the current period is labeled with $\tau = 1$. A technology that was started in the previous period and is under development by the middle-aged is labeled with $\tau = 0$. If there is innovation and a technology is developed by the current young, we refer to this technology with $\tau = -1$.

Skill accumulation: Any agent can be a worker, but managers need certain skills. There are different ways of achieving managerial skills for a given technology, all of which require two periods of experience:

- An agent can spend two consecutive periods (as young and middle-aged) in a given technology. This can take three forms: an agent can
 1. work in—or “develop”—a new technology. This means spending two periods without the presence of a manager and hence with zero productivity. We label this activity d .
 2. work in any producing technology—an activity labeled n , regardless of the career path of the individuals involved—with positive productivity in both periods.

3. work as young in a technology under development by an agent in the previous generation, with zero productivity, and then work as middle-aged in the same technology with a manager present (with positive productivity). This choice means that the activity is d in the first period and n in the second.
- An agent can spend the second period of life in “retraining”—labeled r —and sit out of production but at the same time learn how to manage a technology currently in production.

Agent types: As young, and before undertaking any activity, agents are identical. The activity choices can, however, imply that agents of the same cohort become different. We let x denote the type of an agent at the beginning of a given period. The variable x is a member of X , which contains all combinations of age and work experience. We use $x = 1$ to refer to a young agent, $x = 2\tau$ to refer to a middle-aged agent with experience in vintage τ , $x = 3\tau$ to refer to an old agent capable of managing technology τ , and $x = 3$ to refer to an old agent without managerial capability. The distribution of agents can hence be described by the distribution $\mu : \sigma(X) \rightarrow \mathbb{R}_+$, where $\sigma(X)$ is a suitable collection of subsets of X . We use μ_x to refer to the measure of agents of type x . Note that our assumptions imply that

$$\mu_1 = 1, \quad \sum_{\tau} \mu_{2\tau} = 1, \quad \text{and} \quad \mu_3 + \sum_{\tau} \mu_{3\tau} = 1.$$

Government Policies: We assume that the assumption that the government has two available regulatory options: either it regulates, or it does not. This assumption allows us to make our main points in the simplest way. Regulation means that young agents are not allowed to develop the innovation technology, or, equivalently, that if they do, they are not allowed to operate this technology two periods hence. Retroactive laws forbidding new technologies are not considered, so that any technology that has already been developed can be operated in the present. We use π to denote policies, with $\pi = 0$ meaning that new technologies can be adopted, and $\pi = 1$ meaning they cannot.

Agent choices: Formally, we denote an agent’s choice by $y = (y_a, y_\tau)$, where $y_a \in \{m, d, n, r\}$ refers to the activity undertaken in the current period, and $y_\tau \in \{-1, 0, 1, 2, \dots\}$ indicates the technology chosen. Of course, y is obviously constrained by the agent’s state, x , and by the policy, π , in effect. We use

$$y \in \Gamma(x, \pi)$$

to summarize these constraints. For example, if $x \neq 3\tau$, i.e. if the agent has no managerial ability, then $y_a \neq m$. Similarly, if $\pi = 1$, then $y_\tau \neq -1$ (recall that $\tau = -1$ is the label

for the technology affected by the regulations). When a middle-aged agent chooses to work in a different technology than the one in which he spent the first period we say that the agent is a “switcher”. Since a switcher cannot ever become a manager, we sometimes need to distinguish this group and we use n_s for this purpose.

Aggregate Activity Measures: It is also useful to define aggregate measures of agents currently undertaking different activities. We use φ to denote measures of agents over all specific activities. The set of possible values for φ is denoted Φ , i.e. $\varphi \in \Phi$. We refer to specific groups of agents by subindexing φ . In this fashion $\varphi_{m\tau}$ is the measure of agents that manage technology τ in the current period, while $\varphi_{n\tau}$, $\varphi_{d\tau}$, and $\varphi_{r\tau}$ are the total number of workers, developers, and retrainees, respectively, in that technology. We use another subindex to refer to the age group of the agents; so, for example, $\varphi_{id\tau}$ is the measure of age i agents developing technology τ . To be perfectly clear, we also keep track of switchers and retrainees by the use of $\varphi_{2n_s\hat{\tau}\tau}$ and $\varphi_{2r\hat{\tau}\tau}$, where $\hat{\tau}\tau$ refers to a switch from $\hat{\tau}$ to τ . Having defined these variables, we need to require $\varphi_{n\tau} = \varphi_{1n\tau} + \varphi_{2n\tau} + \sum_{\hat{\tau}} \varphi_{2n_s\hat{\tau}\tau} + \varphi_{3n\tau}$ for all τ , and similarly for the other activities.

Note that the individual constraints on choices implies that the aggregate choices have to satisfy resource constraints given by the skill distribution. In this vein, the following restrictions are implied:

$$\begin{aligned} \sum_{\tau} \varphi_{1d\tau} + \varphi_{1n\tau} &= \mu_1 (= 1) \\ \varphi_{2d\tau} + \sum_{\hat{\tau}} (\varphi_{2n_s\tau\hat{\tau}} + \varphi_{2r\tau\hat{\tau}}) + \varphi_{2n\tau} &= \mu_{2\tau} \\ \sum_{\tau} (\varphi_{3n\tau} + \varphi_{3m\tau}) &= \mu_3 + \sum_{\tau} \mu_{3\tau} (= 1) \quad \text{and} \quad \varphi_{3m\tau} \leq \mu_{3\tau} \end{aligned}$$

Laws of Motion of States: An agent’s state x and current choice y determine the agent’s future state x' . We write

$$x' = \xi(x, y)$$

to describe this updating: for example, if $x = 1$ and $y = (n, \tau)$, then $x' = 2\tau'$, with $\tau' = \tau + 1$ if there has been growth (so that the best technology in production tomorrow is better than today’s), and $\tau' = \tau$ if there has not.²

Likewise, the current state of the economy and agents’ choices together determine the

²Note that this renormalization of technologies formally requires ξ also to be a function of φ ; we leave this argument out, however, for reasons of notational simplicity.

future distribution μ' . The aggregate law of motion of the skill distribution μ again satisfies a number of constraints that are purely based on accounting for agents' activity choices:

$$\begin{aligned}\mu'_{2,\tau+1} &= \varphi_{1d\tau} + \varphi_{1n\tau} \\ \mu'_3 &= \sum_{\tau, \hat{\tau}} \varphi_{2n_s\tau\hat{\tau}} \quad \text{and} \quad \mu'_{3,\tau+1} = \varphi_{2d\tau} + \varphi_{2n\tau} + \sum_{\hat{\tau}} \varphi_{2\tau\hat{\tau}\tau}\end{aligned}$$

If there is growth, the same formulas apply but with the $\tau + 1$ indices replaced by τ 's. With this the description of the environment is complete.

3 Equilibria for a Given Policy Choice

We now turn to describing equilibria for this environment. We adopt a recursive, first-order Markov equilibrium definition: equilibrium prices and quantities are functions of the current state of the economy.³ In this section we treat the regulatory policy as being given exogenously as a function of the state of the economy. After defining equilibria we characterize the set of steady states associated with constant policies over time. These steady states have a stationary skill distribution.

The decentralization we employ is the following: managers hire workers and become residual claimants to output. Workers are hired at wage rates specific to each technology. Agents choose their activities based on the available earnings prospects. We start by noting that the aggregate state of the economy is given by μ , the distribution of agents across types. The individual state also includes x , the agent's own type. We now describe the problem of a generic young or middle-aged agent:

The Young or Middle-Aged Consumer's Problem

$$\begin{aligned}v(x, \mu) &= \max_y \{W(y, \varphi)/P(\varphi) + \beta v(x', \mu')\} \quad \text{s.t.} \\ y &\in \Gamma(x, \pi) \\ x' &= \xi(x, y) \\ \mu' &= h_\mu(\mu) \\ \varphi &= h_\varphi(\mu) \\ \pi &= \Psi(\mu).\end{aligned}\tag{1}$$

³The focus on first-order equilibria is made for analytical and computational convenience, but we are not aware of the existence of other equilibria.

Here, v is the value function in this dynamic programming problem, and y is the agent's current choice given a value of his state, x , and the aggregate state μ . The maximizing choice for y is labeled $g(x, \mu)$.

The function W is the wage of an agent that takes action y when the aggregate choices are φ , while P is the price of the good. The normalization we use is to set the value of unskilled labor of vintage $\tau = 1$ equal to 1, so that $W((n, 1), \varphi) = 1$ for all $\varphi \in \Phi$. Note that $W((d, \tau), \varphi) = W((r, \tau), \varphi) = 0$, for all τ , and that the assumption of perfect competition completely determines all other prices: $W(n, \tau, \varphi) = \frac{\varphi_{m\tau}^\alpha \varphi_{n\tau}^{1-\alpha} \gamma^{1-\tau}}{\varphi_{m1}^\alpha \varphi_{n1}^{1-\alpha}}$, and $P(\varphi) = (\frac{\varphi_{n1}}{\varphi_{m1}})^\alpha (1 - \alpha)^{-1}$.

The agent is fully aware of the way the future skill distributions, aggregate choices, and policy outcomes depend on the current distribution. This is captured by the functions $h \equiv (h_\mu, h_\varphi)$, and Ψ : $h_\mu(\mu)$ gives tomorrow's skill distribution as a function of today's, $h_\varphi(\mu)$ gives today's aggregate choices as a function of the current state, and $\Psi(\mu)$ tells us the what the current policy outcome is as a function of the state.

Next, we have to characterize the problem of the old agent. His options depend on his type. If his type is 3, i.e. he is unskilled, the problem is just:

$$v(3, \mu) \equiv \max_{\tau} W((n, \tau), \varphi) / P(\varphi)$$

If the old agent has managerial skill he can choose to become a manager in the technology he has expertise in or to work as unskilled. The utility derived from becoming a manager is given by the profits, in units of unskilled labor, associated to this activity. The profits are given by:

$$R(\tau, \varphi) \equiv \max_l \{P(\varphi) \gamma^{1-\tau} l^{1-\alpha} - W((n, \tau), \varphi) l\}.$$

The utility of an old agent of type 3τ is then the maximum between the one obtained as a manager and the highest one obtained as unskilled:

$$v(3\tau, \mu) = \max \{R(\tau, \varphi) / P(\varphi), v(3, \mu)\}.$$

We also use the function $g(x, \mu)$ to denote the activity and the technology that solves the old agent's problem. We will furthermore use $l(\tau, \mu)$ to denote the aggregate demand for unskilled labor in technology τ . This completely determines the problem of all agents now we are ready to define an equilibrium for a given policy Ψ .

Definition 1 *An equilibrium given policy choice Ψ is a set of functions v, g, h, W, P , and l with the following properties*

- (i) (v, g) solve the agents' problems.
- (ii) The consumers' choices g generate h_φ .
- (iii) The aggregate choices h_φ generate h_μ .
- (iv) The price functions, W and P , are competitively determined.
- (v) The market for unskilled labor clears: $l(\tau, \mu) = h_{\varphi_{n\tau}}(\mu) = \varphi_{n\tau}$.

Condition (i) requires agent maximization. The standard recursive equilibrium condition requiring individual choices to be consistent with the given aggregate laws of motions is spelled out in conditions (ii) and (iii). Condition (iv) requires competitive pricing, and condition (v) is a standard market-clearing requirement.

3.1 The Stagnation Steady State

We now study our economy in the case where the policy is exogenous and constant over time. If there is regulation this means that there will be no technological innovation at all. We label the resulting state the “stagnation” steady state. If innovation is allowed, the economy will under certain conditions grow indefinitely, experiencing innovation each period: the growth steady state. In the present and in the next section we will characterize each of these two cases.

We first treat the case of $\pi = 1$, i.e. $\Phi(\mu) = 1$ for all μ ; the case when the adoption of new technologies is not allowed in any state. This situation of perpetual stagnation could also be referred to as a “medieval” equilibrium, since it involves no innovation, and hence no growth, and the existence of “apprentices” and “masters”. We conjecture the precise form of a stationary equilibrium and proceed to verify the conjecture and to show that the conjectured stationary equilibrium is unique.

In particular, we conjecture that there is only one technology in operation and all agents hence make the same choices in the first two periods. With a production technology such that $\alpha \geq \frac{1}{3}$, all agents become masters ($\varphi_m = 1$), while if $\alpha < \frac{1}{3}$, only a fraction $\varphi_m < 1$ become masters and the rest workers.

The price for the good associated with the conjectured equilibrium is

$$P = \left(\frac{3 - \varphi_m}{\varphi_m}\right)^\alpha \frac{1}{1 - \alpha}$$

while the earnings for a manager, R , are

$$R = \frac{\alpha}{1 - \alpha} \frac{3 - \varphi_m}{\varphi_m}$$

so that $\varphi_m \leq 1$ implies that the manager's earnings are higher than those of a worker if and only if $\alpha \leq \frac{1}{3}$. We hence find that the fraction φ_m has to equal 1 if $\alpha \geq \frac{1}{3}$ and that if $\alpha < \frac{1}{3}$, then $\varphi_m = 3\alpha$ and the master's earnings are the same as those of the worker/apprentice.

Are these prices consistent with the conjectured outcomes? The dynamic programming problem can be solved backward in this case, since the agents have a finite time horizon. Since the option of opening up new technologies is not available, there is only one technology to go to. The managerial decision is trivial in the third period and our prices guarantee that the proposed φ_m is consistent with optimization. We have

Proposition 1 *When $\Psi(\mu) = 1$ for all μ , there is a unique stationary equilibrium with one technology in operation. In this equilibrium all agents become managers if $\alpha > \frac{1}{3}$, and a fraction $\varphi_m = 3\alpha$ otherwise.*

PROOF: Existence is guaranteed by the above arguments. The argument for uniqueness relies on stationarity of the skill distribution and on the fact that the career paths in the two technologies could not give the same life-time earnings if their total-factor productivities differ. In order to use this argument formally, we first need to show that there cannot be switching between two technologies. To see this, note that agents who switch always pick the highest spot wage rate, so if there is switching between two technologies, these technologies have to have the same wage rate. But if they do, then masters' rents also have to be equal: if not, then a master career in one of the technologies would give higher total payoff than in the other. But with both wage rates equal in the two technologies, the input ratios also have to be equal, which is impossible since total-factor productivities differ across the technologies.

Now given that there is no switching, stationarity implies that with more than one technology in operation it would have to be true that input ratios are the same across these technologies. This, however, would imply that wages and masters' rents are different in different technologies, since total-factor productivities differ across technologies. In particular, the highest productivity technology would have uniformly higher earnings, violating the assumption that not all agents are in one technology. Hence we conclude that only one technology can be operated in a stationary equilibrium without growth. $\square$

3.2 The Growth Steady State

We now turn to the analysis of the case when innovation is assumed to be allowed in every period: $\pi = 0$. We will focus on the simplest kind of stationary equilibrium with growth, namely one where there is only one technology currently in operation, and two under development. We describe our proposed equilibrium and show the conditions on primitives under which all equilibrium conditions are met. We have not characterized or proved exis-

tence of stationary equilibria for other parameter values. Such values may imply stationary distributions with more than one technology in operation.

The stationary equilibrium we construct has a fraction $\bar{y}$ of the young choosing to develop new technologies. The remainder of the agents work in the only technology under operation and hence switch twice during their life-time. No other actions are taken. In this equilibrium, earnings profiles have to be such that young agents are indifferent between managerial and working careers, so that $1 + \beta\gamma + (\beta\gamma)^2 = (\beta\gamma)^2 R$. Similarly to the steady state without growth, $R = \frac{\alpha}{1-\alpha} \frac{3(1-\bar{y})}{\bar{y}}$, which allows us to solve for $\bar{y}$. We can thus formulate:

Proposition 2 *When $\Psi(\mu) = 0$ for all μ , there is a set of parameter values (β, γ, α) of nonzero measure for which there exists a stationary equilibrium with one technology in operation and two under development. In this equilibrium, $\varphi_{31} = \varphi_{2d} = \varphi_{1d} = \bar{y}$, and $\varphi_{n1} = 3(1 - \bar{y})$, where $\bar{y} = \frac{1}{1 + \frac{1 + \beta\gamma + (\beta\gamma)^2}{\beta\gamma} \frac{1-\alpha}{3\alpha}}$.*

PROOF: We proceed by verifying that all agents maximize. We start with old workers. It is obvious that the switchers cannot do better, as any other alternative involves zero consumption. The managers could choose to become workers; however, the condition on indifference between careers implies $R > 1$ (so long as $\beta\gamma > 0$), so they choose to manage. The same argument applies to middle-aged workers who have been developing the newer technologies. The middle-aged who worked as young could decide not to switch, and instead study their technology for one period on their own and then attract workers and manage this technology the next period. In fact, better still, these agents can retrain in the currently best technology in production, and later become managers for this technology. This would give a life-time income which is dominated by switching if

$$1 + \beta^2 \max_l \{ \gamma l^{1-\alpha} - \gamma^2 l \} \leq (1 + \beta\gamma + (\beta\gamma)^2).$$

It only remains to show that no other activity than to develop technology -1 or to work are chosen by the young. The inequality derived above ensures that it does not pay to go into technology 1, and technologies 2, 3, etc. are clearly worse still. What about technology 0, which is already under development by the current middle-aged? Using it would pay $\beta\gamma$ in the second period, because then the agent would be working. It would, however, give a lower return in the last period. To rule out that the alternative strategy gives higher life-time utility, it has to be that

$$\beta\gamma + \beta^2 \max_l \{ \gamma l^{1-\alpha} - \gamma^2 l \} \leq (1 + \beta\gamma + (\beta\gamma)^2).$$

The inequalities above can be expressed purely in terms of parameters of the economy. If they are met, we are assured that the proposed behavior is indeed optimal. These inequalities are non-trivial restrictions on the parameter space. However, they define a set of parameter values which is of nonzero measure. $\square$

4 Political Equilibrium

In this section we ask how Ψ will be determined if agents are allowed to vote each period over the current regulatory policy. We first define a political equilibrium formally. In the voting decision it will be crucial for the agent to forecast the effects of the current π on current and future prices, which involves predicting future skill distributions as well as future votes. Thereafter we discuss informally the possibility that the stationary equilibria described in the previous section can be political equilibria, i.e. that they can be supported by voting each period over the current π . This discussion will show that it is not possible to answer this question without formally characterizing the dynamic political equilibrium. In the next section we then provide an example which illustrates these issues and which has the properties described in the introduction.

4.1 Definition of Political Equilibrium

We utilize a recursive definition of the dynamic equilibrium. Agents are modeled in their two roles: as consumers and as participants in the political process. The consumer behaves as in the competitive equilibrium with exogenous policy Ψ . We will refer to Ψ as the policy outcome function. We derive political preferences over the current policy alternatives by having the agent evaluate his utility under the equilibrium paths implied by each possible current policy. To calculate an equilibrium path for a given policy today, the agent takes next period's actions and political outcome to be given by those implied by the equilibrium functions. The agent then compares the utility resulting from the two alternative choices, and makes his political decision to maximize utility.⁴

The key mechanism that has to be added to our notion of a political equilibrium is a condition that delivers the Ψ function from first principles. In order to do this, we have to construct a procedure that allows the agent to evaluate his utility under different policies.

First, if the current policy is $\pi = \Psi(\mu)$, the agent will know how to evaluate his utility: it is then simply given by $v(x, \mu)$ in Definition 1 above. Second, for a different policy, $\pi \neq \Psi(\mu)$, the calculation is more difficult: it is necessary to find the equilibrium response to a changed tax rate. This can be thought as a one-period deviation from the policy rule Ψ . Therefore, let

⁴We describe decision rules by functions. This should be interpreted as a selection from the set of maximizing choices, if this set is not a singleton. Our economy of the previous sections does not produce unique choices generally (young agents may be indifferent between careers or policies), and we use functions only for notational convenience.

the function $H(\mu, \pi) \equiv (H_\mu(\mu, \pi), H_\varphi(\mu, \pi))$ be the law of motion of the aggregate state and the aggregate choices for a situation where today's policy is an arbitrary π —which does not have to equal $\Psi(\mu)$ —and tomorrow's aggregate choices and policy are given by the original functions h and Ψ . The problem of the agents can be written compactly in this case if we interpret the function W to also include the returns to managerial activities as:

$$\begin{aligned} V(x, \mu, \pi) &= \max_y \{W(y, \varphi)/P(\varphi) + \beta v(x', \mu')\} \text{ s.t.} \\ y &\in \Gamma(x, \pi) \\ x' &= \xi(x, y) \\ \mu' &= H_\mu(\mu, \pi) \\ \varphi &= H_\varphi(\mu, \pi) \end{aligned} \tag{2}$$

The policy rule that solves this problem is denoted $G(x, \mu, \pi)$. The utility attained for the alternative policy is labeled $V(x, \mu, \pi)$. It is key to note in this maximization problem that tomorrow's value is given by v : this problem evaluates the utility associated to having a one-period departure from the policy specified by Ψ . We denote by $\psi(x, \mu)$ the most preferred political outcome of an agent with individual state x when the aggregate state is μ :

$$\psi(x, \mu) \equiv \arg \max_{\pi} V(x, \mu, \pi) \tag{3}$$

Here as well, we assume for expositional simplicity that there are no ties.

Individual preferences can be aggregated into a political outcome. We use majority rule to aggregate those preferences:

$$\Psi(\mu) = \arg \max_{\pi \in \{0,1\}} \int_{x:\psi(x,\mu)=\pi} \mu(dx). \tag{4}$$

The stage is now set with all the required ingredients for a definition of equilibrium. We have well-defined behavior for given policies, and, given behavioral rules, we have well-defined preferences over such policies. Furthermore, we have a political aggregator or political process that uses these preferences to select an outcome. At this point, it should be obvious that our notion of equilibrium includes a policy outcome function that, when taken as given, generates preferences over policies that, when aggregated, in turn reproduces the given policy outcome function. The political equilibrium can therefore be formally defined as follows:

Definition 2 *A political equilibrium is a set of functions $v, V, g, G, h, H, W, P, l, \psi$, and Ψ with the following properties*

- (i) (v, g) and (V, G) solve (1) and (2), respectively.
- (ii) The consumers' choices g and G generate h_φ and H_φ , respectively.
- (iii) The aggregate choices h_φ and H_φ generate h_μ and H_μ , respectively.
- (iv) The price functions, W and P , are competitively determined and the labor market clears.
- (v) ψ and Ψ are given by (3) and (4).
- (vi) $g(x, \mu) = G(x, \mu, \Psi(\mu))$, $v(x, \mu) = V(x, \mu, \Psi(\mu))$, and $h(\mu) = H(\mu, \Psi(\mu))$.

Conditions (i)-(iv) are parallel to (i)-(v) of Definition 1, extended to include the behavior under the larger set of paths. Condition (v) describes both the political preference forming, and its aggregation through majority rule. Condition (vi) states that a one-period alternative choice of policy π generates the laws of motion under no deviation if $\pi = \Psi(\mu)$. This last condition is redundant as it is implied by the other conditions, and it emphasizes the one-period deviation nature of the G , V , and H functions.

Note that our notion of equilibria applies for a wider class of models than the one described in the previous section. In particular, infinitely-lived agent economies can be thought of as a special case. Our definition leads to a natural procedure for characterizing equilibria numerically, and we apply it to our model in the next section. Also note the generality that lies in the formulation: the variable x here can capture any aspect of the individual's state, which for example could include current stocks of other types of capital. Further, in models with finitely-lived agents like the one in this paper, the age component of x restricts the value functions to be zero for ages outside the life span of agents, while in infinitely-lived agent models, it simply loses the significance it has in our context; the problem is otherwise unchanged.

The political mechanism used in the definition was majority rule, but other political mechanisms can be used as well, e.g. income- or age/type-weighted voting. In particular, we think of a political aggregator function as a function A such that $\Psi(\mu) = A(\psi, \mu)$. It is easy to see that majority rule is a specific function A .

4.2 Could the Stationary Equilibria Be Supported as a Political Outcome?

We turn now to addressing, more on an informal level, the issue of what agents prefer within the context of the two steady states characterized above. To highlight the role of vested interests in our model, note that in the case where there is only one technology in operation the equilibrium prices will satisfy

$$R(\varphi) = \frac{\varphi_n}{\varphi_m} \frac{\alpha}{1 - \alpha}$$

Let $\Psi : \mathcal{M} \rightarrow \{0, 1\}$ be a policy outcome function.

Let π be any policy.

Let $h : \mathcal{M} \rightarrow \mathcal{M} \times \Phi$ jointly denote a specification of the aggregate law of motion of the economy h_μ , and of aggregate actions of all agents h_φ under outcome function Ψ every period. Note that the law of motion of the economy, h_μ , is completely determined by the projection of h_φ over the agents that will be alive the next period.

Let $H : \mathcal{M} \times \{0, 1\} \rightarrow \mathcal{M} \times \Phi$ denote a specification of the aggregate law of motion of the economy, H_μ , together with a specification of aggregate actions of all agents H_φ under policy outcome π today, and Ψ from tomorrow on. As was the case for h_μ , H_μ is completely determined by those components of H_φ that refer to agents who are alive next period.

Let $v(x, \mu; h, \Psi)$ be the value function of an agent of type x when the distribution of types (the aggregate state of the economy) is μ , when the economy's law of motion is given by h , and its policy outcome function is Ψ .

Let $g(x, \mu; h, \Psi)$ be the optimal decision rule of type x agent. This agent is restricted in his choices by his type, and by the policy implemented. Specifically, agents do not choose their age, they can only manage a technology if they have the skill to do so, and they can only start the project that leads to an innovation if it is legal to do so.

Let $V(x, \mu, \pi; H, h, \Psi)$ be the value function of an agent of type x when the distribution of types (the aggregate state of the economy) is μ , when the economy's law of motion is given by H today, and h from tomorrow on, and its policy outcomes are π today and Ψ from tomorrow on.

Let $G(x, \mu, \pi; H, h, \Psi)$ the optimal decision rule associated to value function V .

Let $\psi(x, \mu; H, h, \Psi)$ denote the preferred policy option, when the equilibrium laws of motion and policy outcome function are H, h , and Ψ , of a type x agent when the state of the economy is given by μ .

Finally, we describe the algorithm used in the computation of equilibria.¹²

Step 1: Initialize Ψ .

Step 2: Initialize h .

Step 3: Compute $v(x, \mu; h, \Psi)$ and $g(x, \mu; h, \Psi)$. This is solved by backward induction, noting that old agents have no future to worry about.

Step 4: Compute the law of motion of the economy generated by decision rules $g(x, \mu; h, \Psi)$.

¹²We omit the specification of particular guesses restricting the search, and their verification in the final step of the procedure.

power increases if there is innovation, since innovation shifts equilibrium consumption toward future products with an accompanying decrease in the price of current and next period's products. The relative sizes of the five groups are the key to the policy outcome. In particular, the measure of the group in favor of growth will be $1 + (1 - \varphi_n) + (1 - \varphi_n) = 3 - 2\varphi_n$, and that of the group against is $\varphi_n + \varphi_n = 2\varphi_n$. If the primitives are such that $\varphi_n < \frac{3}{4}$, then growth seems possible as a self-generating outcome of our dynamic political process.

Turning to the no-growth steady state, there are really only three distinct groups: young, middle-aged, and old (assuming, for simplicity, that all old agents become managers). As before, young prefer growth. Old agents prefer not to have growth, for the same reason as in the steady state with growth: innovation comes hand-in-hand with lower equilibrium consumption of the present product, and hence lower equilibrium profits. Finally, middle-aged agents in this case have a harder political decision since their career involves both working at a salary as unskilled and earning a manager's rent in their third period of life. If some young become innovators, and hence lower the current demand for products, the middle-aged agents can enjoy lower product prices in the current period. At the same time, these same agents face lower future profits as managers, since current innovation also competes with the product they will sell in the next period. What, then, do the middle-aged prefer?

The decision making process of the middle-aged in the no-growth situation illustrates the difficult aspect of analyzing dynamic voting models: it requires the ability to predict the future path of the economy under both policies today, and this in turn requires the ability to predict associated future policies, so that the relevant prices (P, P' and R') can be calculated. These prices depend on current and future behavior, as well as on the vote in future periods. With our equilibrium definition above, we choose to tackle this problem upfront. In the next section we show how equilibria can be characterized for a simple example economy. The example illustrates the need for the full-blown definition of equilibrium.

5 Stagnation and Growth in Some Example Economies

In this section we study two simple example economies. In each economy there are three agents per cohort. This facilitates the characterization of equilibrium since it makes the number of possible values of the state vector, μ , finite.⁶ The two economies we study differ

⁶The finite number of agents and the fact that the career choice is discrete make it necessary to use a lottery implementation. In particular, we consider a "planner" for each generation who selects the number

in their parameter values. We use both parameter sets to illustrate, first, that our focal stagnation-and-growth equilibrium exists for a wide range of parameter values, and second, that perpetual stagnation is only possible for some parameter configurations, and then as an additional equilibrium. In other words, both our economies lead to a stagnation-growth equilibrium, and the second economy also has another equilibrium.

Since some aspects of the description of equilibrium are somewhat complex, we first state the main result and describe the key equilibrium components. We postpone a full description of the equilibrium to the end of the section, where we also discuss the second economy and the possibility of multiplicity of equilibria.

5.1 The Main Result

The parameter values in economy 1 are $\alpha = .8$, $\beta = 1.2$, and $\gamma = 2.2$; in economy 2 they are $\alpha = .8$, $\beta = 0.4$, and $\gamma = 3.6$. The parameter values are chosen so that a model period corresponds to 25 years, implying yearly discount rates β of 1.01 and .96, respectively.⁷ The corresponding values for the yearly growth potential, γ , are 1.03, and 1.05, which also seem reasonable given the data.⁸

Since our example has a finite number of values of the state, the equilibrium elements, i.e. the set of functions $v, V, g, G, h, H, W, P, l, \psi$, and Ψ , are functions on a discrete, finite domain. The proof of our main result then involves verifying a large, but finite, number of conditions for individual maximization. For brevity, we only include the central parts of the proof in the presentation. We first state our main result, provide a preliminary discussion of the key features of the equilibrium, and then separately discuss in detail the stagnation-growth cycle as well as the possibility of perpetual, uninterrupted growth. Our main result is:

Proposition 3 *For economies 1 and 2, there exists an equilibrium with the following properties:*

- (1) *There is a set of 28 possible states which the economy will never leave and each of which has only one technology in production.*
- (2) *Agents who develop new technologies never work as unskilled.*

of young agents to innovate, and individual activities are then assigned randomly.

⁷These values are representative for the values used in the empirical overlapping generations literature (see Ríos-Rull (1992a) for a discussion).

⁸If a period were taken to mean 15 years, we would get $\beta \in \{1.01, 0.94\}$, and $\gamma \in \{1.05, 1.09\}$.

- (3) With the exception of young agents, agents with the same age and skill always choose the same activity.
- (4) There are 2 ergodic sets: one with a unique element and perpetual growth ($\pi = 0$), and one 3-period cycle with two periods of stagnation ($\pi = 1$) and one period of growth.
- (5) The other 24 states are transient: 11 lead to the state of perpetual growth, and the remaining 13 lead to the stagnation-growth cycle.

Figure 1 summarizes the mechanics of our recursive political equilibrium. The key parts of the equilibrium can be completely described with 6 of the 28 states. The states in the figure are labeled according to a systematic listing of the 28 states, which is described in subsection 5.2 below. States in which innovation is not allowed are crossed. The labeling of states is made according to a complete listing in section 5.4. State 28 is the state with perpetual growth; the cycle consists of states 1, 4, and 17. The political aspects of the equilibrium are indicated by the dotted arrows: state 1 allows innovation and leads to state 4, but had the vote been against innovation the economy would have stayed in state 1, as indicated by the dotted arrow. Similarly, state 28 would have led to state 25 with a no-vote, and thereafter to the cycle, but the equilibrium outcome given state 28 is to remain in that state. States 20 and 25 are transitory, but are needed in a full description of the equilibrium: the political consideration requires that we look at where the economy is heading for a policy that disagrees with the majority choice. For example, showing that 28 is a steady state involves much more than what was done in section 3.2—it involves verifying all the aspects of Figure 1.

In the following two subsections we will make more precise the nature of the equilibrium. We do this by stating more precise characterizations underlying the above proposition, accompanied by illustrations and proof outlines. The proof outlines focus on the political aspects of the agents' decisions.

5.2 Perpetual growth

We now discuss and verify our steady state with perpetual growth of section 3.2.

Characterization 1 *The equilibrium has is an ergodic set with a unique element. We label this state a steady state, i.e. a state μ^* s.t. $\mu^* = h(\mu^*)$. This steady state has innovation, and its gross long-run growth rate of consumption is equal to γ . There are two innovators and one worker per cohort; all innovators and managers are against growth, and all young agents and workers are in favor of growth. The vote in favor of growth is 5-4.*

OUTLINE OF PROOF: The state labeled 28 in Figure 1 corresponds to the steady state with growth as studied in the section with exogenous policy. Figure 2 describes this steady state in more detail. In it, state 28 is described as well as the actions taken under the winning policy in a box with continuous lines, and under the losing policy, in a box with discontinuous lines. In the figure, the members of the winning coalition are marked with small arrows. The large solid arrow points to the state next period under the winning policy, while the large dashed arrow points to the state to which the economy would have gone under the losing policy. The equilibrium vote is as conjectured in section 3.2. To see this, note first that in the case of $\pi = 0$, two agents innovate, and there is a total of 3 (1 young + 1 middle-aged + 1 old) workers the current period and the same in the next (all consecutive) periods. With the alternative policy, $\pi = 1$, the current number of young workers would be 3 and the total $3 + 1 + 1 = 5$. The next period the state would then be state 25, which differs from state 28 only in the distribution of its middle-aged, since these middle-aged were not able to innovate when young (and all other agents would make the same choices as for $\pi = 0$). In state 25 the the majority vote is not to allow growth, and hence all young agents work. The middle-aged choose to retrain in this state over switching and giving up being able to manage in the consecutive period. Hence, the the total number of workers in state 25 is then $3 + 0 + 1 = 4$. Going back to the voting decision in state 28, the old managers prefer to regulate, since it implies a higher number of current workers, and hence higher demand (i.e. total consumption) and higher R/P (see our price characterization in section 4.2). Old workers have the opposite opinion—they prefer low current demand (i.e. low total consumption) and a low $1/P$, and hence favor innovation. The middle-aged on a working career face a lower present product price for $\pi = 0$ than for $\pi = 1$, but these agents also care about the product price in the following period. This price does depend on the current value of π , since it affects both tomorrow's skill distribution, tomorrow's vote and hence price. As seen in the figure, there will be lower product prices and higher profits in the following period, since the total number of workers in that period is lower if the state is 28 (3) than if the state is 25 (4). Therefore, the middle-aged workers prefer innovation in the current period, giving them higher purchasing power in both periods. Similarly, the middle-aged agents on a managerial career are against growth since $\pi = 1$ implies higher future profits (which is all they care about). All in all, there are 5 agents in favor of innovation, and 4 against. $\square$

This characterization is in line with the conjectures in section 4.2, but it involves more detail: it also checks the effect of the current policy on the prices in the following period, which the middle-aged agents care about. The majority coalition consists of 3 young, 1 middle-aged, and 1 old agents. The key behind supporting the vote in favor of innovation in state 28 is the size of groups—the number of workers per generation is high enough here to make up a total majority together with the young agents. Note, finally, that completeness of the proof requires further exploration into the properties of state 20 and 25. Figures 3 and 4 provide the required information.

5.3 The cycle

Our discussion of the cycle will involve studying in detail how the economy goes from state 1 to state 4 (Figure 5), from state 4 to state 17 (Figure 6), and from state 17 back to state 1 (Figure 7). A description of the main features of this process is

Characterization 2 *There is a three-period cycle, i.e. a state μ^{**} such that $h(h(h(\mu^{**}))) = \mu^{**}$, with the property that $\Psi(\mu^{**}) = 0$ and $\Psi(h(\mu^{**})) = \Psi(h(h(\mu^{**}))) = 1$. Only in state μ^{**} will there be innovation; in that state, 2 among the cohort of young agents become innovators. These innovators then form part of the coalition that decides the vote in the two following periods. More precisely, the coalitions deciding the vote are: in state μ^{**} , the young and the middle-aged ($\pi = 0$); in state $h(\mu^{**})$, the middle-aged innovators and the old managers ($\pi = 1$); and in state $h(h(\mu^{**}))$, the middle-aged and the old innovators from two periods ago. The average long-run growth rate in the stagnation-growth cycle is $\gamma^{\frac{1}{3}}$.*

OUTLINE OF PROOF: We label the three states μ^{**} , $h(\mu^{**})$, and $h(h(\mu^{**}))$ with 1, 4, and 17, respectively. Figures 5 to 7 describe the mechanics of the cycle in detail. First, Figure 5 shows that state 1 is the most backward state—it is the “medieval” state—since all its old agents have the same ability to manage the state-of-the-art technology (they can be “masters”), and the middle-aged have one period of experience in that same technology. The key political element in state 1 is that the middle-aged turn out in favor of breaking out of stagnation. They see, on the one hand, that their current purchasing power increases with innovation, but that on the other, it also takes away demand for the product they will sell the next period as managers. It turns out, however, that in this case the total sales next period, and hence their profits, are the same (with $\varphi_n = 4$) whether there is innovation today or not. This is because saying no to growth today actually also changes the vote the next period: it simply postpones adoption of the new technology one period; if growth is allowed there will be 4 (1 middle-aged and 3 young) workers tomorrow (state 4); if it is postponed one period, there will also be 4 (3 middle-aged and 1 young) workers (state 1). Hence the preference of the middle-aged in state 1 is to allow adoption of new technologies, since this at least raises the current real wage. The winning coalition in state 1 is therefore the young and the middle-aged, who both agree to allow innovation.

Second, Figure 6 shows that the next state, 4, has two groups of middle-aged as a result of the innovation in state 1: there are 2 innovators and 1 worker. This time the winning coalition consists of no-sayers. These comprise the current old and the 2 developers, whose vested interests speak against development: they only care about the number of workers they can attract as buyers the following period, and a no to growth maximizes the work force. First, saying no to growth tips tomorrow’s policy outcome against growth, and hence implies that there will be 3 young workers in that state (state 17) as opposed to 1 if the current vote were to allow innovation and the economy were to go to state 20. Counteracting this is the fact that in state 17, all the middle-aged decide

to retrain, and hence provide no demand—they will consume the period after that, as managers. In the alternative state, state 20, there would be some demand from the middle-aged, namely from the middle-aged agent who did not innovate. However, the net of these two effects is that the middle-aged innovators in state 4 prefer to vote for $\pi = 1$ —the total consumption of the good they produce is higher in state 17 than in state 20 ($4 > 3$). They therefore side with the 3 old managers and make up a majority voting out growth 5-4.

Finally, in state 17 (see Figure 7), there is again a coalition making up a majority against growth. This coalition consists of 2 old innovator-managers and 3 middle-aged workers. The latter agents choose to sit out and retrain in the currently producing technology so as to be able to become managers for this technology in the following period (state 1)—this activity dominates switching, which would have left the economy entirely without managers in the following period. These middle-aged agents form part of the winning political coalition even though they are actually indifferent with respect to the current policy: the current wage rate is irrelevant to them, and both policies imply the same number of workers the following period: state 1 as well as state 4 have four workers (although with different age compositions).⁹

One period later the economy is back in state 1, after the two periods of stagnation. But in state 1, as described, growth is more attractive again, and the cycle begins anew. The cycle of growth and stagnation is driven by vested interests: only when there are no technology developers around will new skills be tolerated; and once the new skills have been accumulated, further innovation is restricted until the innovators disappear completely from the economy. $\square$

To complete the discussion of the cycle, it would also be necessary to support the asserted claims about what happens in states 20, which, in turn, involves understanding that state 20 leads to state 28—the state with perpetual growth. It is straightforward to provide these arguments, and Figures 2 to 4 summarize the necessary information. Note, therefore, that in order to verify that the cycle is an equilibrium (and also in order to verify that state 28 is a steady state) it is necessary to discuss all the states 1, 4, 17, 20, 25, and 28.

5.4 A complete characterization

We now provide a complete description of the equilibrium elements—for all 28 states—in Tables 1-2. The first of these tables describes all the aspects of the economics, and the second

⁹Although it is true that the winning majority may seem weak since a part of it is indifferent, there are minor alterations of the model which would break the indifference and produce the same outcome. One such alteration is that middle-aged workers, while still being perfect substitutes with young workers, have slightly higher productivity than the young workers.

describes the induced policy preferences. Table 1 lists the 28 values of the state variable in the first 6 columns. The index s represents the state. The first three columns describe the number of middle-aged agents of different types $(\mu_{20}, \mu_{21}, \mu_{22})$, and the next three columns list the number of old agents of different types $(\mu_{31}, \mu_{32}, \mu_{33})$. As regards the properties of the different states, note that state 1 is the medieval state, and that states 23 and 28 are the potential steady states with growth in every period.

Table 1 moreover describes the laws of motion of the state variable, μ . The economic actions are not described explicitly. However, if the current state is μ and next period's state is μ' , then it is clear what today's choices of the currently middle-aged and young must be. The actions of the currently old agents cannot be read off tomorrow's state, but it is true throughout that old behave according to our conjecture: if they have skills in the newest technology, they manage this technology; if they have no skills, they work in the newest technology; and if they have skills in technology 2, prices are such that they choose not to operate their technology, but instead work as unskilled in technology 1.

The right-hand side of the table describes the functions h_μ, H_μ , and the political outcome function Ψ . This information enables us to completely describe the dynamics of the skill distribution and the policy outcome for any initial skill distribution in the set of 28. It also shows how alternative policies would affect the system over time: this is described by $H_\mu(\mu, \Psi^c(\mu))$ (here c denotes the alternative policy). This information is crucial to the agents when they form their preferences over the different policies.

Table 2 then describes the policy preferences $\psi(x, \mu)$ for each type of agent, i.e. agents of type $x \in \{1, 20, 21, 22, 31, 32, 33\}$. It also depicts the aggregate voting outcome.

It is clear from inspection of Table 1 that 16 out of the 28 states lead to the cycle, and the remaining 12 to perpetual growth. Along the paths toward stagnation, it is possible that innovation is initially allowed, but eventually there is a large enough vested interest in the old technologies that the economy ends up in the cycle of stagnation and growth. The longest transition to the stagnation cycle in our example spans three periods, for example from state 14, via 12 and 25, to state 1.

The transition to prosperity is at most 2 periods long. Along all these paths growth is allowed. Hence, in our example, a no-to-growth once is a sufficient, but not necessary, signal that the economy will end up in stagnation.

It is clear for our economy that it is crucial that the agents take into account how a change in the current policy affects the future path of skill distributions and policy outcomes. Indeed, in all states we find that a change in the current value of π also implies a change in the policy next period, and these effects are key to the equilibrium characteristics.

What properties of the skill distribution are indicative of where the economy is heading?

We note that the composition of the middle-aged is important in this respect. If either all or none among the middle-aged are innovators, then the economy will end up in the stagnation cycle. When the number of middle-aged innovators, who all are against growth, together with the number of old managers is large enough to form a majority (≥ 5), then growth will be voted down, and the stagnation cycle result. When there is only one manager but three middle-aged innovators (states 14 and 22), this coalition does not have a majority, and growth is allowed. Even here, however, the stagnation cycle will result: allowing growth for our parameter values always means that two young agents will innovate and one young agent work, and the two young innovators will form a majority coalition against innovation in the next period with the three currently middle-aged innovators.

Also in the case where there are no middle-aged innovators, the political equilibrium leads to low average growth (i.e. to the cycle). In states 9, 13, 17, 21, and 25, the middle-aged agents retrain and in fact are indifferent with respect to the policy outcome: whether or not there will be regulation, next period's profits will be the same.¹⁰ In some of these states there will be innovation, and in others there will not.

When there are 1 or 2 innovators among the currently middle-aged, growth may or may not result; the middle-aged workers are now sure that they will not be managers next period (since there are better managers around), and as workers they prefer growth, because growth keeps the labor force smaller and they get higher wages that way. In these cases, the political decisions of the currently old agents are pivotal, and depending on the exact distribution, the economy goes to the cycle or to state 28.

5.5 Perpetual stagnation

For the second economy, i.e. for the second set of parameter values, there is also a different equilibrium: an equilibrium without cycles, but instead with perpetual stagnation, as described in the section above on steady states for given policies.

Proposition 4 *For the second economy there exists an equilibrium satisfying the same conditions as (1)-(3) of Proposition 3, with*
(4') There are 2 ergodic sets with a unique element each: one with perpetual growth ($\pi = 0$), and one with perpetual stagnation ($\pi = 1$).

¹⁰Had these agents abstained, there would have been a reverse outcome in states 9, 17, and 25. However, footnote 9 in the previous subsection applies: there is a minor alteration of the the model which would give the same result without this indifference.

(5') *The other 26 states are transient: 11 lead to perpetual growth, and the remaining 15 lead to perpetual stagnation.*

This second type of equilibrium is described in Tables 3-4. In the medieval state 1, the one that is associated with a long spell of previous stagnation, the policy is not to allow growth; in particular, the middle-aged do not find it optimal to support growth (see Table 4). In this equilibrium, the middle-aged are faced with the nontrivial choice indicated in Section 4: these agents realize that whereas the number of young workers the next period is unaffected by the current policy—tomorrow's state, whether 1 or 4, is one where innovation is not allowed—the current policy matters for the number of middle-aged workers they will have the next period. Hence if innovation is prohibited in the current period, there will be a larger number of workers next period, but there will also be a larger number of competing workers in the current period, than if innovation is allowed. The political preference of these agents therefore involves an intertemporal consideration, and this equilibrium will only exist for some parameter values. In terms of our two sets of parameter values, the discount factor is relatively lower in the second economy, and in this case the payoff from allowing the young to innovate—the higher current purchasing power—is greater than the discounted cost—the lower future profits—so the middle-aged will vote yes to growth in this case. This explains why for lower discount factors this type of equilibrium does not exist.

6 Concluding Remarks

We constructed a model where vested interests were responsible for the prohibition of technological innovation. In the economy studied, there is a permanent tension in the political process between agents with different types of human capital. Growth is always a possibility, and whether it occurs or not depends on properties of the current distribution of agents over capital specialization. The particular features of this distribution that matter to the outcome can be explained from economic and political fundamentals. We have three final remarks.

First, to summarize the fundamentals that matter to the long run growth outcome proved surprisingly hard for some initial distributions even in this simple economy—it is clear what policy *outcomes* lead to growth, but to understand their *determination* is a much less trivial task. We saw that the middle-aged agents were important to the final outcome, and that the distribution of old agents mattered as well. Moreover, neither were the particular distributions of skills within these groups that lead to growth trivial to characterize. Hence, it was necessary to study subtle aspects of the interplay between economics and politics to fully understand the long-run growth outcome. Our results indicate that it may be difficult in many cases to identify reasons for growth based on analyzing crude macroeconomic aggregates—it

is not obvious that the key characteristics of the distribution of skills are either observable or easily summarizable with simple statistics.

Second, Mokyr (1992) argues that something he labels “Cardwell’s Law” is likely to hold in “closed” economies. This law says that all closed economies will, due to the kind of vested interests discussed here, eventually and permanently stagnate. What, then, makes our model predict that some economies will not stagnate in the long run? The key element is that our economies have entirely democratic political institutions; Mokyr assumes that the power of incumbents can be increased over time in a supposedly endogenous fashion. Our overlapping generations setup naturally gives rise to a kind of mixing in the distribution that, at least for some economies, is sufficient to make stagnation avoidable.

Third, our positive analysis is far away from understanding what countries or regions might want to *do* to avoid stagnation outcomes. A general problem is that the set of policies considered, as in most political equilibrium theory, is severely restricted. Alternative policy packages in our framework, e.g. those including yes to growth but compensation to the agents losing from the introduction of new techniques, might well attract a majority of voters. We rule out these packages with a vague reference to real world economies.¹¹ But why aren’t these kinds of packages used more often in real economies? This is the key question that needs to be addressed in order to understand the role of different political institutions.

¹¹Note that in fact not all compensation schemes would work in our model as some transfers from the young to the old will imply negative consumption.

APPENDIX

Computing dynamic political equilibria for economies with an infinite number of agents is a formidable task when heterogeneity is allowed among these agents. The problem is that our equilibrium requires a fully dynamic analysis, and it is well known that the computational complexity for dynamic models increases very rapidly in the number of agents in the economy. In our case, it would be necessary to find a specific mapping (a fixed point) from the set of possible distributions of types into itself. These sets are large; in particular, in our example, they are the product set of a two-dimensional unit simplex and a three dimensional unit simplex. Moreover, the political features imply that for every iteration in the policy outcome function we have to solve a fixed point problem of the type described above. For these reasons, we have chosen to compute equilibria for a model with a finite number of agents in each generation.

The example we are considering has three agents born each period. This is the smallest number that makes ties impossible, as the total number of people alive at any point in time, nine, is an odd number. The rest of the features of the example are the same as in the model above. Now the problem remains at a manageable dimension, as there is only a finite number of possible distributions of agents. However, the introduction of this small number of agents does not come for free: there are a number of small issues that arise and that have to be taken care of. In the next few paragraphs, we describe these issues.

With a small number of agents, the behavior of each agent has aggregate implications. In our present context, we are not interested in strategic behavior, and we consequently try to abstract as much as possible from issues of market power. This means that we consider price taking behavior: consumers and firms optimize treating prices as beyond their control.

In our model, the only choice of an agent is which technology to join. This is a nonconvex choice. If identical agents take different actions in equilibrium, then it has to be because these actions lead to the same level of utility. In environments with a continuum of agents this can be achieved with lotteries, (see Rogerson (1988) for a general implementation of these market institutions), or by using the fact that the utility of the different choices is a continuous function of the fraction of agents that choose each activity, with the equilibrium having a partition of the population that delivers equal utility to all types (for an example see Ríos-Rull (1992b)). In environments with a finite number of agents, the fractions route cannot be followed, but lotteries can be implemented. In our context, then, we use lotteries as a decentralizing device.

The lottery equilibrium has the decisions made by the young agents taken collectively. A planner representing the young agents chooses the alternative that provides the maxi-

imum total utility to the generation as a whole. The solution is implemented by solving a generation-wide planning problem where agents act collectively **only** when young; in later periods agents act on their own (we consider commitment not to be possible, both with respect to future choices of technology and with respect to future policy outcomes). We assume that the planner representing the young agents solves a maximization problem in which full account is taken of the impact of the choices of the young on the future distribution of agents. Lotteries are then used to decide on whose lot it is to work, and on whose lot it is to study.

The computational procedure that we use is straightforward, and it is described in detail below. One important, and normally unfortunate, feature of our methods is that the procedures themselves do not guarantee convergence to an equilibrium. This makes intelligent guesses about equilibrium properties crucial as inputs in the computational algorithm. We therefore make certain guesses initially, impose the implications of these guesses on the state space and behavior of agents, run the algorithm, and verify the guesses at the end.

The guesses are: (i) only one technology—the most recent one for which there are skilled old agents—is operated at each point in time; (ii) only skills in the two most recent technologies are possible; (iii) innovators always choose to continue their career; and (iv) middle-aged and old agents with the same skill behave the same way (no lotteries are needed for these agents).

Working through the implications of these guesses, one arrives to the conclusion that there are 28 possible distributions μ , listed in Table 1 in Section 5. A first look at the problem at hand shows that the set of possible laws of motions is the set of all mappings from 28-element-sets into itself. Many of these 28^{28} laws of motion are not feasible, but the remaining number of possibilities is still very large. Associated to this set of 28 states is the set of possible policy outcome functions: this set is exactly the set of all possible ordered combinations of 0's and 1's of size 28: a set with 2^{28} elements. The size of these numbers gives an idea of the size of the computations involved in a general procedure for finding the equilibrium fixed point, and hence of the importance of good guesses.

Numerical algorithm:

We now describe the iterative procedure used to compute the equilibria. For this, we recall some notation.

Let X be the set of all possible types, specifying properties like age, managerial abilities, wealth or anything additional that makes people different.

Let $\mathcal{M}$ be the set of all possible distributions. $\mu \in \mathcal{M}$ is a measure on X .

Let Φ be the set of measures of agents actions. $\varphi \in \Phi$ is one such measure.

$$P(\varphi) = \left(\frac{\varphi_n}{\varphi_m} \right)^\alpha \frac{1}{1 - \alpha}$$

which are both given in terms of the numéraire (units of unskilled labor). We therefore have

$$\text{Current payoff for a manager} = R/P = \left(\frac{\varphi_n}{\varphi_m} \right)^{1-\alpha} \alpha$$

$$\text{Current payoff for a worker} = 1/P = \left(\frac{\varphi_n}{\varphi_m} \right)^{-\alpha} (1 - \alpha).$$

Observe that as the number of young workers increases, the equilibrium consumption level for the current product increases. When the young innovate, they effectively also choose future over present products. So as the number of young innovators rises, the demand for current products goes down, as represented, then, by a decrease in φ_n . Note furthermore that R/P is increasing in φ_n and that $1/P$ is decreasing in φ_n , so with innovation, the payoff of managers goes down and the payoff of workers goes up. In short, managers are worse off with innovation, since it shifts demand toward future products, resulting in lower equilibrium profits; and workers are better off with innovation, since the worker's purchasing power goes up—the innovation of future products makes the demand, and hence the price, for the current product go down in equilibrium.⁵

We now use the simple characterization above in order to discuss political aspects of the two steady states. First, when there is perpetual growth, there are five groups in distinct situations: young agents, and middle-aged and old with managerial or working careers. First, young agents will always prefer growth, since it makes higher-productivity technologies—cheaper products—available. Second, agents on a managerial career path either develop a new technology (the middle-aged) or are ready to manage a firm (the old). Since the payoff of these agents is the current and future managerial profits and the above arguments imply that innovation today will shift demand (equilibrium consumption) toward the products produced two periods hence and consequently lower this payoff, these agents will be against innovation. Third, working agents, both middle-aged and old, prefer growth: these agents' purchasing

⁵There is an alternative way, using labor market interactions, to describe why there are vested interests in this framework. The argument rests on noting that managers and unskilled workers are complements in production, whereas unskilled workers are perfect substitutes for each other. Since innovation decreases the number of unskilled workers, managers will therefore favor $\pi = 1$: it increases their marginal product. Workers, on the other hand, favor $\pi = 0$ because innovation increases their marginal productivity by decreasing the size of the labor force.

Check whether this law of motion coincides with the assumed law of motion h . If it does, proceed to step 5, if it does not, update the law of motion h taking into account decisions g and go back to Step 3.

Step 5: Initialize H .

Step 6: Compute $V(x, \mu, \pi; H, h, \Psi)$, and $G(x, \mu, \pi; H, h, \Psi)$. These objects are computed in one step. They arise in a one step iteration where agents evaluate the future using function v , not V , but take into account that tomorrow's distribution μ' is given by H as opposed to h , and that today's policy outcome is given by Ψ^c , and not by Ψ . (If $\pi = \Psi$, then $V = v, G = g$, and $H = h$.)

Step 7: Compute the law of motion of the economy generated by decision rules G . Check whether this law of motion coincides with the assumed law of motion H . If it does, proceed to step 8. If it does not, update the law of motion H taking into account decisions G and go back to Step 6.

Step 8: Compute $\psi(x, \mu; H, h, \Psi)$ by letting agents choose what policy they prefer. This is done by strict comparison of the values of v and $V(\neq v)$. If the first is higher then $\psi(x, \mu; H, h, \Psi) = \Psi(\mu)$; otherwise $\psi(x, \mu; H, h, \Psi) = \pi$.

Step 9: Compute the winners choice. This is done by computing $\int_X \psi(x, \mu) \mu(dx)$, for every μ . We use democracy (one agent one vote), so if this expression has a value of more than half the amount of agents, then the winner is to choose the policy equal to 1. Otherwise the policy chosen is 0. Next, compare the election winner with policy Ψ . If they differ, update policy Ψ using the winner's information; if they coincide, we are done.

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μ_{20}	μ_{21}	μ_{22}	μ_{31}	μ_{32}	μ_3	s	$h_\mu(\mu)$	$H_\mu(\mu, \Psi^c(\mu))$	$\Psi(\mu)$
0	3	0	3	0	0	1	4	1	0
3	0	0	3	0	0	2	9	12	1
1	2	0	3	0	0	3	16	13	0
2	1	0	3	0	0	4	17	20	1
0	3	0	0	0	3	5	4	1	0
3	0	0	0	0	3	6	12	9	0
1	2	0	0	0	3	7	16	13	0
2	1	0	0	0	3	8	20	20	0
0	0	3	3	0	0	9	1	4	1
3	0	0	3	0	0	10	9	12	1
1	0	2	3	0	0	11	24	21	0
2	0	1	3	0	0	12	25	28	1
0	0	3	1	2	0	13	4	1	0
3	0	0	1	2	0	14	12	9	0
1	0	2	1	2	0	15	24	21	0
2	0	1	1	2	0	16	28	25	0
0	0	3	2	1	0	17	1	4	1
3	0	0	2	1	0	18	9	12	1
1	0	2	2	1	0	19	24	21	0
2	0	1	2	1	0	20	28	25	0
0	0	3	1	0	2	21	4	1	0
3	0	0	1	0	2	22	12	9	0
1	0	2	1	0	2	23	24	21	0
2	0	1	1	0	2	24	28	25	0
0	0	3	2	0	1	25	1	4	1
3	0	0	2	0	1	26	9	12	1
1	0	2	2	0	1	27	24	21	0
2	0	1	2	0	1	28	28	25	0

Table 1: Equilibrium Distributions, Laws of Motion, and Voting Outcomes for Economies 1 and 2

s	ψ_1	ψ_{20}	ψ_{21}	ψ_{22}	ψ_{31}	ψ_{32}	ψ_3	Totals (0,1)
1	0	-	0	-	1	-	-	6-3
2	0	1	-	-	1	-	-	3-6
3	0	1	0	-	1	-	-	5-4
4	0	1	0	-	1	-	-	4-5
5	0	-	1	-	-	-	0	6-3
6	0	1	-	-	-	-	0	6-3
7	0	1	0	-	-	-	0	8-1
8	0	1	0	-	-	-	0	7-2
9	0	-	-	1	1	-	-	3-6
10	0	1	-	-	1	-	-	3-6
11	0	1	-	0	1	-	-	5-4
12	0	1	-	0	1	-	-	4-5
13	0	-	-	0	1	0	-	8-1
14	0	1	-	-	1	0	-	5-4
15	0	1	-	0	1	0	-	7-2
16	0	1	-	0	1	0	-	5-4
17	0	-	-	1	1	0	-	4-5
18	0	1	-	-	1	0	-	4-5
19	0	1	-	0	1	0	-	6-3
20	0	1	-	0	1	0	-	5-4
21	0	-	-	0	1	-	0	8-1
22	0	1	-	-	1	-	0	5-4
23	0	1	-	0	1	-	0	7-2
24	0	1	-	0	1	-	0	6-3
25	0	-	-	1	1	-	0	4-5
26	0	1	-	-	1	-	0	4-5
27	0	1	-	0	1	-	0	6-3
28	0	1	-	0	1	-	0	5-4

Table 2: Individual Voting Choices For Economies 1 and 2

μ_{20}	μ_{21}	μ_{22}	μ_{31}	μ_{32}	μ_3	s	$h_\mu(\mu)$	$H_\mu(\mu, \Psi^c(\mu))$	$\Psi(\mu)$
0	3	0	3	0	0	1	1	4	1
3	0	0	3	0	0	2	9	12	1
1	2	0	3	0	0	3	16	13	0
2	1	0	3	0	0	4	17	20	1
0	3	0	0	0	3	5	4	1	0
3	0	0	0	0	3	6	12	9	0
1	2	0	0	0	3	7	16	13	0
2	1	0	0	0	3	8	20	20	0
0	0	3	3	0	0	9	1	4	1
3	0	0	3	0	0	10	9	12	1
1	0	2	3	0	0	11	24	21	0
2	0	1	3	0	0	12	25	28	1
0	0	3	1	2	0	13	4	1	0
3	0	0	1	2	0	14	12	9	0
1	0	2	1	2	0	15	24	21	0
2	0	1	1	2	0	16	28	25	0
0	0	3	2	1	0	17	1	4	1
3	0	0	2	1	0	18	9	12	1
1	0	2	2	1	0	19	24	21	0
2	0	1	2	1	0	20	28	25	0
0	0	3	1	0	2	21	4	1	0
3	0	0	1	0	2	22	12	9	0
1	0	2	1	0	2	23	24	21	0
2	0	1	1	0	2	24	28	25	0
0	0	3	2	0	1	25	1	4	1
3	0	0	2	0	1	26	9	12	1
1	0	2	2	0	1	27	24	21	0
2	0	1	2	0	1	28	28	25	0

Table 3: Equilibrium Distributions, Laws of Motion, and Voting Outcomes for Economy 1

s	ψ_1	ψ_{20}	ψ_{21}	ψ_{22}	ψ_{31}	ψ_{32}	ψ_3	Totals (0,1)
1	0	-	1	-	1	-	-	3-6
2	0	1	-	-	1	-	-	3-6
3	0	1	0	-	1	-	-	5-4
4	0	1	0	-	1	-	-	4-5
5	0	-	1	-	-	-	0	6-3
6	0	1	-	-	-	-	0	6-3
7	0	1	0	-	-	-	0	8-1
8	0	1	0	-	-	-	0	7-2
9	0	-	-	1	1	-	-	3-6
10	0	1	-	-	1	-	-	3-6
11	0	1	-	0	1	-	-	5-4
12	0	1	-	0	1	-	-	4-5
13	0	-	-	0	1	0	-	8-1
14	0	1	-	-	1	0	-	5-4
15	0	1	-	0	1	0	-	7-2
16	0	1	-	0	1	0	-	5-4
17	0	-	-	1	1	0	-	4-5
18	0	1	-	-	1	0	-	4-5
19	0	1	-	0	1	0	-	6-3
20	0	1	-	0	1	0	-	5-4
21	0	-	-	0	1	-	0	8-1
22	0	1	-	-	1	-	0	5-4
23	0	1	-	0	1	-	0	7-2
24	0	1	-	0	1	-	0	6-3
25	0	-	-	1	1	-	0	4-5
26	0	1	-	-	1	-	0	4-5
27	0	1	-	0	1	-	0	6-3
28	0	1	-	0	1	-	0	5-4

Table 4: Individual Voting Choices For Economy 1

Figure 1

**EQUILIBRIUM
MECHANICS**

TO CYCLE

PERPETUAL GROWTH

TO GROWTH

Figure 2

STATE #28	μ	actions $\pi=0$
# young	3	d:2, n:1
# m-a, $\tau=0$	2	d
$\tau=1$	0	-
$\tau=2$	1	n
# old, $\tau=1$	2	m
$\tau=2$	0	-
# old, no skill	1	n
CHOSEN POLICY		0
AGGR. # MGRS		2
AGGR. # WKRS		3 ₁₊₁₊₁

STATE #20	μ	actions $\pi=0$	actions $\pi=1$
# young	$\rightarrow 3$	d:2, n:1	n:3
# m-a, $\tau=0$	2	d	d
$\tau=1$	0	-	-
$\tau=2$	$\rightarrow 1$	n	n
# old, $\tau=1$	2	m	m
$\tau=2$	$\rightarrow 1$	n	n
# old, no skill	0	-	-
CHOSEN POLICY (π)		0	1
AGGR. # MGRS (φ_m)		2	2
AGGR. # WKRS (φ_n)		3 ₁₊₁₊₁	5 ₃₊₁₊₁

STATE #25	μ	actions $\pi=1$
# young	3	n:3
# m-a, $\tau=0$	0	-
$\tau=1$	0	-
$\tau=2$	3	r
# old, $\tau=1$	2	m
$\tau=2$	0	-
# old, no skill	1	n
CHOSEN POLICY		1
AGGR. # MGRS		2
AGGR. # WKRS		4 ₃₊₀₊₁

Figure 3

Figure 4

STATE #4	μ	actions $\pi=1$
# young	3	n:3
# m-a, $\tau=0$	2	d
$\tau=1$	1	n
$\tau=2$	0	-
# old, $\tau=1$	3	m
$\tau=2$	0	-
# old, no skill	0	-
CHOSEN POLICY		1
AGGR. # MGRS		3
AGGR. # WKRS		4 3+1+0

STATE #1	μ	actions $\pi=0$
# young	3	d:2, n:1
# m-a, $\tau=0$	0	-
$\tau=1$	3	n
$\tau=2$	0	-
# old, $\tau=1$	3	m
$\tau=2$	0	-
# old, no skill	0	-
CHOSEN POLICY		0
AGGR. # MGRS		3
AGGR. # WKRS		4 1+3+0

STATE #25	μ	actions $\pi=0$	actions $\pi=1$
# young	3	n:3	d:2, n:1
# m-a, $\tau=0$	0	-	-
$\tau=1$	0	-	-
$\tau=2$	$\rightarrow 3$	r	r
# old, $\tau=1$	$\rightarrow 2$	m	m
$\tau=2$	0	-	-
# old, no skill	1	n	n
CHOSEN POLICY (π)		1	0
AGGR. # MGRS (φ_m)		2	2
AGGR. # WKRS (φ_n)		4 3+0+1	2 1+0+1

Figure 5

Figure 6

STATE #20	μ	actions $\pi=0$
# young	3	d:2, n:1
# m-a, $\tau=0$	2	d
$\tau=1$	0	-
$\tau=2$	1	n
# old, $\tau=1$	2	m
$\tau=2$	0	-
# old, no skill	1	n
CHOSEN POLICY		0
AGGR. # MGRS		2
AGGR. # WKRS		3 1+1+1

STATE #17	μ	actions $\pi=1$
# young	3	n:3
# m-a, $\tau=0$	0	-
$\tau=1$	0	-
$\tau=2$	3	r
# old, $\tau=1$	2	m
$\tau=2$	0	-
# old, no skill	1	n
CHOSEN POLICY		1
AGGR. # MGRS		2
AGGR. # WKRS		4 3+0+1

STATE #4	μ	actions $\pi=0$	actions $\pi=1$
# young	3	n:3	d:2, n:1
# m-a, $\tau=0$	$\rightarrow 2$	d	d
$\tau=1$	1	n	n
$\tau=2$	0	-	-
# old, $\tau=1$	$\rightarrow 3$	m	m
$\tau=2$	0	-	-
# old, no skill	0	-	-
CHOSEN POLICY (π)		1	0
AGGR. # MGRS (φ_m)		3	3
AGGR. # WKRS (φ_n)		4 3+1+0	2 1+1+0

Figure 7

STATE #4	μ	actions $\pi=0$
	3	n:3
	2	d
	1	n
	0	-
	3	m
	0	-
	0	-
CHOSEN POLICY		1
AGGR. # MGRS		3
AGGR. # WKRS		4 3+1+0

STATE #1	μ	actions $\pi=0$
# young	3	d:2, n:1
# m-a, $\tau=0$	0	-
$\tau=1$	3	n
$\tau=2$	0	-
# old, $\tau=1$	3	m
$\tau=2$	0	-
# old, no skill	0	-
CHOSEN POLICY		0
AGGR. # MGRS		3
AGGR. # WKRS		4 1+3+0

STATE #17	μ	actions $\pi=0$	actions $\pi=1$
# young	3	n:3	d:2, n:1
# m-a, $\tau=0$	0	-	-
$\tau=1$	0	-	-
$\tau=2$	$\rightarrow 3$	r	r
# old, $\tau=1$	$\rightarrow 2$	m	m
$\tau=2$	0	-	-
# old, no skill	1	n	n
CHOSEN POLICY (π)		1	0
AGGR. # MGRS (φ_m)		2	2
AGGR. # WKRS (φ_n)		4 3+0+1	2 1+0+1