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THE INFORMATION IN SWEDISH SHORT–MATURITY FORWARD RATES

by

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Abstract

In this paper we empirically study the relationship between implicit forward rates and corresponding interest rates in the short-end of the Swedish term structure. The interest rates and forward rates seem to be integrated of order one and cointegrated. We find that the forward rates, for all maturities, contain information about future interest rates. Further, in contrast, to previous empirical studies we cannot reject the joint hypothesis of rational expectations and no term premium. However, the results should be treated with caution since we also find parameter instability.

Keywords: Expectations theory, forward rates, interest rates, term premium, term structure.

JEL classification number: E43

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1. Introduction

One strand of the empirical literature on the term structure of interest rates is concerned with testing forward interest rates as predictors of future spot interest rates. The purpose of this paper is to examine the short-end of the term structure of Swedish Treasury Bills. More specifically, the following questions will be addressed: Does the short-end of the term structure of interest rates contain information about future movements in spot interest rates? If so, how precise is this information, is the forward interest rate an unbiased predictor of future spot rates?

In his pioneering paper Fama (1984) finds evidence that the forward rate-spot rate differential helps to predict changes in the spot rate one month ahead, but the predictions are less accurate as the forecast horizon increases. Fama uses U.S. t-bills for the period 1959-82.¹ Hörgren (1986) has carried out a study in the Swedish money market that is similar to Fama’s. But, instead of studying the forecast power several periods in the future, he studies the forecast power of forward rates with different maturities.² For the period 1980-85 Hörgren finds that the level of forward rates contain information on both the level of future interest rates and the premium for all studied maturities. The hypothesis of the forward rate as an unbiased predictor of future interest rates is thus rejected. The results further show that the premium is more important for longer term maturities, and that the predictive power of forward-spot rate differential on future changes in the spot rate is weak.³

Two drawbacks with Hörgren’s study are that he uses interest rates on bank certificates of deposit with possible default premiums, and that the sample period is relatively short. The database we use in this study should be better than the earlier used, partly because the Swedish capital market in the early eighties was undeveloped and highly regulated, and partly because we now have access to interest rates of Swedish t-bills instead of bank certificates. Further, in this

¹Mishkin (1988) refines and updates Fama’s study by using an econometric technique that obtain a correct variance-covariance matrix. Still, the results are overall consistent with those of Fama.

²This is partly due to the fact that for certain maturities all required quotations for computing forward rates do not exist.

³Ekdahl and Warne (1990) also provide a study of the Swedish term structure of interest rates. By using a VAR-model they study the relationship between monthly interest rates of 1-month t-bills and 5-year t-bonds for the period 1984-89. Campell and Shiller (1991) studies the whole spectrum of the U.S. term structure (from 1 month up to 10 years) while Fama and Bliss (1987) focus on the information in long-maturity forward rates. The general result is that the forecast power deteriorates with the forecast horizon in the shorter end of the term structure, but it improves with the horizon for multi-year predictions. See Shiller (1990) for an overview of theoretical and empirical research on the term structure.
study we make use of the time series properties of the interest rates according to Engle and Granger (1987). On the basis of these properties we can develop the specifications and perform some new tests. Contrary to previous findings in the literature, we now find that the joint hypothesis of no term premium and rational expectations cannot be rejected.

The paper is organized as follows. Section 2 describes the theory and introduce some notation. In section 3 we present the data and conduct some preliminary tests. The results from the regressions are given in section 4. Finally, in section 5, we summarises our major findings and discuss the empirical results.

2. The Expectations Theory of the Term Structure

According to the expectations theory of the term structure the interest rate of a long term bond can be expressed as an average of current and expected future short term rates. Let \( r(t + n, t + m + n) \) denote the interest rate on a pure discount bond purchased at time \( t + n \) that matures at time \( t + m + n \) and let \( f(t, t + n, t + m + n) \) denote the forward rate at trade date \( t \) applying for a loan from the settlement date \( t + n \) to the maturity date \( t + m + n \). We measure the term \( m \) and the horizon \( n \) in months.

Assuming continuously compounded interest rates, forward rates can be defined by

\[
f(t, t + n, t + n + m) \equiv \frac{(m + n)r(t, t + m + n) - nr(t, t + n)}{m} \tag{2.1}
\]

It follows that the spot rate at time \( t \) with \( m \) months to maturity can be expressed as an average of \( m \) one-month forward rates according to the following

\[
r(t, t + m) = [r(t, t + 1) + f(t, t + 1, t + 2) + \ldots + f(t, t + m - 1, t + m)] / m, \tag{2.2}
\]

where \( r(t, t + 1) = f(t, t + 1) \). As a result, forward rates can recursively be computed from (2.2). The parity holds, of course, also when one is considering forward rates with different maturities as long as they sum up to a maturity of \( m \) months.

In a more general form a term premium \( \theta(t, n, m) \) can be defined as the difference between the forward rate \( f(t, t + n, t + m + n) \) and the corresponding expected interest rate \( E_t[r(t + n, t + m + n)] \)

\[
\theta(t, n, m) \equiv f(t, t + n, t + m + n) - E_t[r(t + n, t + m + n)], \tag{2.3}
\]

where \( E_t \) represent the mathematical expectations conditional on the information set available at time \( t \). If one substitutes equation (2.3), when the maturity is equal to one month, into (2.2) one can see that the interest rates on a long term bond are simply averages of current and expected future short term interest rates plus a possible term premium.
The expectations theory states that the term premium might be dependent on \( n \) and \( m \) but is not dependent of time. According to the pure expectations theory the premium is zero. If it is assumed that the expectations theory holds, and the expectations operator is removed from equation (2.3), the difference between the forward rate and the future spot rate can be written as the sum of the term premium and an expectation error, \( \nu(t + n) \). That is,

\[
f(t, t + n, t + m + n) - r(t + n, t + m + n) = \theta(n, m) + \nu(t + n). \tag{2.4}
\]

So, equation (2.4) merely decomposes the difference into the anticipated component, the term premium, and the unanticipated component, the expectation error. The realized difference is called excess return.

The expectations theory with a constant term premium does not postulate the sign of the term premium unless agents risk preferences and investment horizons are defined. If an investor is risk averse and has a long investment horizon he requires a higher return to invest in a sequence of short papers since the interest rates in the coming periods are uncertain. This means that the term premium is negative and the expected return on a long term bill is lower. Thus, the yield curve would on average be downward sloping. On the other hand, the premium can be positive if the investor is uncertain about his investment horizon. It is then possible that he regards the investment in the long term bill as more uncertain than the sequential investment. The expected return would then be higher for the long term bill and the yield curve on average upward sloping.\(^4\)

In the following sections we empirically study the relationship between forward rates and corresponding future spot rates assuming that the term premium is zero and that agents form rational expectations. If both these conditions hold, then \( f(t, t + n, t + m + n) \) is an unbiased predictor of \( r(t + n, t + m + n) \).

3. The Data and Some Preliminary Tests

3.1. The Data

The dataset consists of monthly observations of interest rates for Swedish Treasury Bills (Statsskuldsväxlar) with 1, 2, 3, 6 and 12 months to maturity. The sample covers the period January 1984 to July 1992, providing 103 observations in total.\(^5\)

\(^4\) Other theories of the term structure of interest rates give different presumptions pertaining to the sign of the term premium. The liquidity theory of interest predicts a positive term premium while according to the more general preferred habitat theory the term premium might be positive or negative. In summary, unless we define what is uncertain for the investor and his risk preferences the term premium in our model can be positive as well as negative.

\(^5\) We have received our data from Sveriges Riksbank (the Swedish central bank), and we are grateful to Camilla Falkner for collecting them.
We have chosen to examine this sample period because $t$-bills were first introduced in the Swedish money market in 1982 and the preceding period is characterized by a highly regulated credit market. Daily quotations are only available since 1984.

The $t$-bills are pure discount bonds and are issued each month as 180 or 360 days bills. Every banking day there are quotations on $t$-bills with 1, 2, 3, 6 and 12 months to maturity, but these are not the exact maturities. Since $t$-bills generally are issued in the middle of the month, and we have access to daily observations, quotations on the 15:th of every month were chosen. When an observation was unavailable because of a holiday, an observation on the next adjacent banking day was chosen. By this way of selecting the sample we try to match the observations with the actual yield to maturity.

The interest rates are quoted as annualized simple interest rates. From these we compute continuously compounded interest rates and forward rates. The forward rates correspond to the interest rates with 1, 3 and 6 months to maturity 1, 3 and 6 months ahead, and a 1 month interest rate 2 months ahead.

The term structure of interest rates are plotted against time and yield to maturity in figure 1. Maturities shown are 1, 2, 3, 6 and 12 months. We can see that there have been approximately parallel shifts over time in the term structure. But we can also see that at times the term structure has been upward-sloping and at times downward-sloping.

Summary statistics for compounded interest rates are collected in table 1. As a comparison, statistics for the forward rates are also reported. The means for the interest rates, for the whole period, decrease with increasing maturity, which implies an on average downward-sloping yield curve. We can see that the standard deviations of the interest rates are decreasing with the term. We also note that the interest rates minimum is approximately equal for all maturities but the maximum is decreasing with maturity. That is, the band for the interest rates is decreasing with term to maturity.

3.2. Integration and Cointegration Properties of the Data

Traditional test statistics used for inference are based on the assumption that the underlying time series are stationary processes. When variables used in regressions are non-stationary the distributions may be non-standard and statistical inferences must be treated with caution. So, before any regressions are done we want to study the characteristics of the time series themselves. A test for unit

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6The deregulation of the Swedish credit market in the eighties opened up possibilities for a well functioning secondary market. Except for a temporary turnover tax in 1989, the process has led to an increasing liquidity. See Englund (1990) for a review of the deregulation process, and Viotti and Wissen (1991) for a description of the Swedish credit market as it is today.

7Interest rates are computed according to the convention of 30 days a month and 360 days a year.
roots can for example be thought of as a pre-test to avoid "spurious regressions" situations in the sense of Granger and Newbold (1986).

We perform two univariate tests for unit roots. The first test is the Dickey Fuller test, which is based on the t statistic associated with the $\rho$-coefficient in the following regression estimated by ordinary least square:

$$\Delta x_t = \rho x_{t-1} + \sum_{i=1}^{k} \theta_i \Delta x_{t-i} + \epsilon_t$$  \hspace{1cm} (3.1)

where $\Delta$ is the difference operator. We select the lag order $k$ to be large enough to ensure that $\epsilon_t$ is a serially uncorrelated disturbance.\footnote{Serial correlation is tested, on the five percent significance level, by a Lagrange Multiplier test of first order autocorrelation and the Ljung-Box statistic of correlation at lag twelve.} When $k = 0$ the test is known as the Dickey Fuller (DF) test, while if $k > 0$ it is termed the augmented DF (ADF) test. The null hypothesis of non-stationarity is rejected if $\rho$ is negative and significantly different from zero. Secondly, we also perform the non-parametric unit root test suggested by Perron (1988), denoted $Z_t$. This test is similar to the DF test but imposes less structure on the disturbance term, $\epsilon_t$, in (3.1). Critical values are tabulated in Fuller (1976).

Table 2 gives the test statistics with significance levels under the null hypothesis that the series have a unit root. The tests are applied to the levels and to the first differences of the series. For all interest rates and forward rates, in levels, the null cannot be rejected. However, for the differences we can reject the null hypothesis at the one percent significance level. This indicates that the series are integrated of order one.\footnote{The result that the interest rates for Swedish Treasury Bills are integrated of order one is consistent with Ekdahl (1990).}

We continue by testing if a linear combination of the forward rates and the corresponding interest rates is stationary, i.e. if they are cointegrated. Granger (1986) derives some necessary (but not sufficient) conditions that must hold for an optimal forecast series. One condition is that it must be integrated of the same order as the realized series. Further, the optimal forecast series must be cointegrated with the realized series if the information set includes all earlier observations of the series. Cointegration is also a precondition for the existence of a stable linear relationship between the variables. From this reasoning it follows that the relationship between spot and forward rates ought to be characterized by cointegration as a necessary condition for unbiasedness.

To test the hypothesis of non-cointegration we follow the recommendations of Engle and Granger (1987). This means that we perform the standard tests of the unit root hypothesis applied to the residuals from the cointegration regression of $r(t + n, t + m + n)$ on a constant and $f(t, t + n, t + m + n)$. We use the earlier described tests, the DF, ADF and the $Z_t$ tests, to examine whether the estimated residuals from the cointegration regression are stationary. Phillips and Ouliaris
(1990) tabulate the critical values for these residual based tests.\textsuperscript{10}

The results of the cointegration regressions are reported in table 3 (disregard the point estimates of $\alpha$ and $\beta$ for the moment). The null hypothesis of non-cointegration can be rejected for all maturities, i.e. the forward rates and the associated spot rates are cointegrated. For maturities up to 3 months we reject non-cointegration on the one percent significance level, while for the 6 months maturity rate the ADF-test reject non-cointegration on the five percent significance level and the $Z_t$-test reject on the ten percent level. Thus, given that the interest and forward rates are integrated of order one, the necessary condition of cointegration for unbiasedness seems to be fulfilled. In the following sections sufficient conditions will be tested.

4. Empirical Results

If the joint hypothesis of rational expectations and risk neutrality holds, the forward rate should be equal to the market’s expectations of the corresponding future spot rate. This was written in section 2 as

$$E_t[r(t + n, t + m + n)] = f(t, t + n, t + m + n).$$  

(4.1)

The actual realization of $r(t + n, t + m + n)$ will differ from the expected level by an expectations forecasting error, $\nu(t + n)$. The first idea for testing this joint hypothesis is to run the level regression

$$r(t + n, t + m + n) = \alpha_1 + \beta_1 f(t, t + n, t + m + n) + \nu(t + n).$$  

(4.2)

In an estimated version of this “direct test” the null hypothesis would be $\alpha_1 = 0$ and $\beta_1 = 1$. Then, if the time series $r(t+n, t+m+n)$ and $f(t, t+n, t+m+n)$ are stationary, an ordinary $F$-test could be used to test this joint hypothesis. But, if the variables are non-stationary, as they seem to be according to the tests performed in section 3.2, the distributions of the tests would be non-standard and statistical inference must be treated with caution. In the following sections we try to estimate the predictive power of forward rates on spot rates by using three different models.

It should be noted that we always examine the joint hypothesis of risk-neutrality (no premium) and rationality (rational use of all information). If both these parts hold, then the current forward rate is an unbiased predictor of the future spot rate. Sometimes this is called an “efficiency-test” although a rejection of the joint hypothesis does not necessarily imply “market-inefficiency”. For example, if the slope in the regression-equation is significantly different from unity,

\textsuperscript{10}An alternative way to test for cointegration is by the likelihood ratio test in Johansen (1988). This would be preferable in a multivariate setting, but in a bivariate case (as in ours) the residual based tests suffice.
this could either depend on some expectation bias or arise from a premium which
is in some way correlated with the level of the interest rate. The same reasoning
gives that a non-zero intercept could be due both to a systematic bias in market
expectations or a constant term premium.

4.1. The Restricted Cointegration Regression

When the series \( r(t+n, t+m+n) \) and \( f(t, t+n, t+m+n) \) are non-stationary but
cointegrated the regression equation (4.2) becomes the cointegration regression.

Conventional test statistics are not valid when the variables in the regression
are non-stationary, but we still have consistent estimates if cointegration holds. In
fact, Stock (1987) has shown that the regression parameter estimates are super-
consistent, which means that they converge even faster to their true values than
normal OLS estimates. The results from the cointegration regression are given
in table 3. We see that the point estimates of the constants and slope coefficients
are fairly close to 0 and 1, especially for the shorter maturities. However, these
estimates are biased in finite samples, and since the distribution for the estimated
coefficients are unknown it is not possible to make inference.

As an alternative to a formal \( F \)-test we instead test if the residuals are non-
stationary under the restriction that \( \alpha_1 = 0 \) and \( \beta_1 = 1 \). This is equivalent
with a test for non-stationarity of the negative excess return. The results are
reported in table 4.\(^{11}\) Under the restriction we reject non-stationarity for all
maturities. The tests thus seem to support the null hypothesis with regard to
\( \beta_1 \). The hypothesis of a zero \( \alpha_1 \) cannot be tested with this method, since the
stationarity of a time-series is not affected by adding a constant.

So, although we now have consistent estimates for a “long-run” relationship
we still want to test for unbiasedness and estimate confidence bands both for the
intercepts and the slope-coefficients. In the following two sections we therefore
develop the model and generate more general regression specifications.

4.2. The Simple Difference Test

A common way to overcome the problems of non-stationary series has been to
estimate the following regression for the first difference of the interest rate\(^{12}\)

\[
\begin{align*}
\alpha_2 + \beta_2[f(t, t+n, t+m+n) - r(t, t+m)] + \epsilon(t+n) \\
r(t+n, t+m+n) - r(t, t+m) =
\end{align*}
\]

\(^{11}\)When the cointegrating vector is restricted the test statistics should be evaluated by using
the standard unit root distribution in Fuller (1976).

\(^{12}\)See for example Fama (1984) and Hörmgren (1986). In section 3.2 we concluded that the
interest rate series are integrated of order one. We have also performed the tests for non-
stationarity of the transformed series, \( f(t, t+n, t+m+n) - r(t, t+m) \). They do not appear
to be non-stationary at the five percent level.

7
where the error term $\epsilon(t+n)$ is assumed to satisfy the usual conditions for regression analysis. This equation is equivalent with (4.2) under the null hypothesis of $\alpha_2 = 0$ and $\beta_2 = 1$, and the regression can then be interpreted as how an expected change in the interest rate relates to an actual change in the spot rate.

The results from regressions (4.3) are shown in table 5. Since we use overlapping data, we expect the error terms to be autocorrelated. The hypothesis of non-correlation, examined by using the Ljung-Box statistic (denoted $Q_{LB}$), can not be rejected only for the $r(t+1, t+2) - r(t, t+1)$ regression. For the other three regressions we therefore reestimate the parameter covariance matrix using the generalized method of moments imposing the weighting scheme proposed by Newey and West (1987). The test of the joint hypothesis is in this case chi-square distributed with two degrees of freedom as there is no small-sample adjustment.

All slope-coefficients are positive and significant. This implies that the forward rates contain information about the future spot rates. Actually, the slope coefficients are never significantly different from one. Further, the constant terms are always positive but never significantly different from zero. Together this implies that we cannot reject the null hypothesis of unbiasedness for any maturity or forecast horizon. That is, the joint hypothesis of risk-neutrality and rational expectations cannot be rejected. However, the precision in the estimates decreases with maturity indicated by larger standard errors. This is expected since the problem with overlapping data is more severe for longer maturities. Still, the larger variance is not enough to overturn the conclusions about the expectation theory. The results imply that the forward rate is an unbiased predictor of the future spot rate for all tested maturities.

In the next section we use the results from section 3.2 regarding the time-series properties of the series and model an even more general regression specification.

### 4.3. The Error Correction Model

Engle and Granger (1987) have shown that a model with cointegrated time series has a correspondence in an error-correction mechanism (ECM) representation and vice versa. A common interpretation of this model is that it combines short run dynamics with adjustments to a long run steady-state path. The market is supposed to constantly adjust along this path whenever the system is outside the equilibrium. In some studies this could be a plausible interpretation, but in this paper we do not explicitly model any dynamic relationships nor adjustment paths. Still, the error correction representation follows from the time series properties of

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13When $m$-months predictions are made more than once during one $m$-months period the same information is partly used for successive observations which can generate autocorrelated residuals. For example, a series of multi-months changes of a variable that are observed every month generates residuals that follow a $MA(q)$ process where $q$ is the length of the forecast horizon minus one. Thus, two-months-ahead forecasts that are observed every month should have residuals following a $MA(1)$ process.
the data, and it is an appropriate device for testing the expectation theory. Put differently, by using the ECM we can test the hypothesis of non-cointegration between the forward rates and corresponding spot rates while we simultaneously study short run fluctuations in the series.

Estimation of an ECM follows a straightforward two-step procedure. First, a prior levels regression is performed, like regression (4.2). The lagged residuals from this regression, \( \hat{\nu}(t) \), are then entered into the model where the original variables are differenced. This allows the hypothesis of non-cointegration to be tested. In our case this turns into the following regression equation;

\[
r(t + n, t + m + n) - r(t, t + m) = \\
\beta_3 \hat{\nu}(t) + \beta_4 [f(t, t + n, t + m + n) - f(t - n, t + m)] + \mu(t + n)
\]

where \( \mu(t + n) \) is an i.i.d. residual when \( n \) is equal to one.\textsuperscript{14} The long run equilibrium for the relationship between the forward rates and spot rates are given by equation (4.2). Any deviation from this equilibrium results in a non-zero disturbance term, \( \hat{\nu}(t) \neq 0 \), which is taken into account when predicting future spot rates. The test for the joint hypothesis of risk neutrality and rational expectations in the error correction model is to test the hypothesis of \( -\beta_3 = \beta_4 = 1 \). This hypothesis implies that agents are assumed to adjust completely for any expectation-errors made in the preceding period, and at the same time they use all information given by changes in the forward rates. If there were no corrections at all the series would drift apart over time, and the estimated \( \beta_3 \) value would go towards zero. This in turn, would imply that the series are not cointegrated.

The results from the regressions are shown in table 6. Again we have problems with overlapping observations. The disturbance terms seem to fulfill the required conditions only in the \( r(t + 1, t + 2) - r(t, t + 1) \) regression. For the other three regressions we therefore use the generalized method of moments to obtain correct standard errors.

Although interpretation of the coefficients is slightly different in this test, overall the results are in line with the two earlier tests. Since the ECM specification is more general than the previously used models the results in this section reinforces the earlier conclusions. The overall fit of the ECM specification is even better than in the simple difference test. All coefficients have the right sign and are significantly different from zero. That the estimated \( \beta_3 \)-values are strongly significant imply that we again conclude that cointegration holds for all maturities. Further, all coefficients are close to the null hypothesis values of -1 and 1 respectively. As in the simple difference test the problem with overlapping data is more severe for longer maturities. This is indicated by a larger variance around the point estimates for the longer maturities. Still, the hypothesis of unbiasedness

\textsuperscript{14}One may also include lagged difference terms of both interest rates and forward rates as independent variables. In our case these terms never significant and the results are reported where all such terms are restricted to zero.
cannot be rejected for any regression. Thus, the conclusion that forward rates are unbiased predictors of future spot rates is confirmed.

4.4. Parameter Stability

In this section we make an informal examination of the stability of the regression coefficients in the simple difference test and the error correction model. The procedure we use is a simple rolling regression technique. It consists of running all the regressions for successive 48-months subperiods. The first estimate is based on the first 48 observations, the second on a regression where a new observation is added while the oldest one is dropped, etc. Thus, the first estimation is for the 1984:2-1988:1 subperiod, while the last estimation is for the 1988:8-1992:7 subperiod. The estimates of the coefficients and 95 percent confidence bands are plotted in figure 2a – h and 3a – h.

In both tests we find the coefficients to be fairly stable over all subperiods for the 1 month interest rate one and two months ahead. For the other two regressions the stability declines with months to maturity. Although the magnitude of the variations in the coefficients varies dramatically we always have the same pattern. Under certain subperiods the unbiasedness hypothesis can clearly be rejected for both regressions while for other subperiods the opposite holds.

In the simple difference test we most often have positive estimated intercepts, although we can reject significance most of the times. The slope estimates are higher for the earliest and latest subperiods. For subperiods in the middle of the entire sample the slope estimates fall (sometimes significantly) below one and for some subperiods the estimates are not even significantly different from zero.

In the ECM we again have the same pattern. Unbiasedness cannot be rejected for subperiods in the beginning and the end of the entire sample. For subperiods in the middle of the sample we also note that for longer maturities the absolute value of the coefficients often are less than one, and occasionally not significantly different from zero.

Our interpretation of these results is simply that if we had had access to a data set of a shorter time span (4 years) our conclusions could have been very different. This fact seems to indicate that the length of sample period is quite important for a given frequency of the data.

5. Summary and Discussion

In this paper we empirically study the relationship between implicit forward rates and corresponding future spot interest rates in the short-end of the Swedish term structure. In summary the major findings are as follows: The interest and forward rates seem to be integrated of order one and clearly cointegrated. The unbiasedness hypothesis cannot be rejected for any tested maturity or forecast
horizon. That is, the joint hypothesis of rational expectations and no term premium cannot be rejected. The precision in the estimates are lower for forward rates with longer maturity, but this increase in uncertainty is not enough to overturn our conclusions. However, the results should be treated with caution since we also find some parameter instability.

The results in this paper are stronger than what has been found in similar studies. Shiller (1990, table 13.2) summarizes the results from several studies where the expectation theory has been tested for different countries and different time periods. The general result is that the forward-spot rate differential contains some information about future spot rate changes, but it is not an unbiased predictor. In Hörngren’s study (1986) for the Swedish market the comparable estimates are not significantly different from zero.

Excess return consists of two components; an expectation error and a term premium. From our data it is not possible to test how the two components separately affect the relationship between forward rates and future spot rates.

If the rational expectations postulate holds and a constant term premium exists, we would expect the intercept to be different from zero and the slope coefficient to equal unity in the simple difference test. But if the term premium is correlated with the interest rate we would expect slope coefficients different from one. A slope coefficient greater than one would indicate that the term premium is negative when long term interest rates are higher than short term rates, but positive when the opposite holds. If we alternatively assume the sign of the premium, say negative, we would again expect slope coefficients greater than one when the yield curve is positively sloped, but we would expect the slope coefficients to be less than one when the yield curve is negatively sloped. We would also expect a positive constant when a linear regression is run for the whole period. In our regressions are the estimated intercepts always close to zero while the estimated slope coefficients are not significantly different from one. However, it would still be interesting to run separate regressions for periods when the yield curve is positively sloped and negatively sloped respectively, since there seem to exist some parameter instability.

Is it reasonable to assume that agents form rational expectations and on average make no mistakes in forecasting the future? Froot (1989) uses both implicit forward rates and survey data to measure the market’s expectations of future interest rates. When using forward rates he finds that predictions of changes in future interest rates are biased both for three months horizon and six months horizon. But as he also has survey data available he can directly test the rational expectations postulate. The results now show that there is no significant bias in predicting future interest rates three months ahead but there is still a bias for six months ahead predictions. This implies that the bias in short maturities is primarily due to a risk premium, while for a longer forecast horizon the expectations error becomes more important. To our knowledge, no such study has been performed for the Swedish market. Still, we are inclined to argue that
for longer sample periods rational expectations is not an implausible assumption. But under certain shorter sample periods it is still possible that the market does not have rational expectations. By comparing the level of the interest rates (see figure 1) with the results from the rolling regressions, it seems to be a greater probability of rejecting the unbiasedness hypothesis when the interest rates are low, especially for longer maturities. A possible explanation could simply be that the risk premium is time-varying. But maybe it is more plausible to guess that under some subperiods the rational expectations postulate did not hold. If we had access to reliable survey expectations data for market expectations of future interest rates, we could test the auxiliary assumption of rational expectations and then proceed with a more formal study of the expectations theory.
Table 1. Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Min (date)</th>
<th>Max (date)</th>
</tr>
</thead>
<tbody>
<tr>
<td>r(t, t + 1)</td>
<td>12.15</td>
<td>2.11</td>
<td>8.64 (1987:6)</td>
<td>17.75 (1985:5)</td>
</tr>
<tr>
<td>r(t, t + 2)</td>
<td>12.11</td>
<td>2.06</td>
<td>8.70 (1986:9)</td>
<td>17.56 (1985:5)</td>
</tr>
<tr>
<td>r(t, t + 3)</td>
<td>12.06</td>
<td>2.00</td>
<td>8.72 (1986:10)</td>
<td>17.37 (1985:5)</td>
</tr>
<tr>
<td>r(t, t + 6)</td>
<td>11.96</td>
<td>1.84</td>
<td>8.70 (1986:10)</td>
<td>16.94 (1985:5)</td>
</tr>
<tr>
<td>r(t, t + 12)</td>
<td>11.78</td>
<td>1.67</td>
<td>8.62 (1986:10)</td>
<td>16.10 (1985:5)</td>
</tr>
<tr>
<td>f(t, t + 1, t + 2)</td>
<td>12.07</td>
<td>2.05</td>
<td>8.69 (1986:10)</td>
<td>17.61 (1985:6)</td>
</tr>
<tr>
<td>f(t, t + 2, t + 3)</td>
<td>11.95</td>
<td>1.94</td>
<td>8.19 (1986:12)</td>
<td>17.00 (1985:5)</td>
</tr>
<tr>
<td>f(t, t + 3, t + 6)</td>
<td>11.87</td>
<td>1.72</td>
<td>8.68 (1986:10)</td>
<td>16.51 (1985:5)</td>
</tr>
<tr>
<td>f(t, t + 6, t + 12)</td>
<td>11.60</td>
<td>1.56</td>
<td>8.55 (1986:10)</td>
<td>15.27 (1985:5)</td>
</tr>
</tbody>
</table>

13
Table 2. Tests for Unit Roots

<table>
<thead>
<tr>
<th>Variable</th>
<th>DF</th>
<th>ADF</th>
<th>( Z_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(t, t + 1) )</td>
<td>-0.38</td>
<td>-</td>
<td>-0.13</td>
</tr>
<tr>
<td>( r(t, t + 2) )</td>
<td>-0.30</td>
<td>-</td>
<td>-0.17</td>
</tr>
<tr>
<td>( r(t, t + 3) )</td>
<td>-0.25</td>
<td>-0.26</td>
<td>-0.15</td>
</tr>
<tr>
<td>( r(t, t + 6) )</td>
<td>-0.22</td>
<td>-0.22</td>
<td>-0.14</td>
</tr>
<tr>
<td>( r(t, t + 12) )</td>
<td>-0.17</td>
<td>-0.21</td>
<td>-0.15</td>
</tr>
<tr>
<td>( f(t, t + 1, t + 2) )</td>
<td>-0.31</td>
<td>-</td>
<td>-0.20</td>
</tr>
<tr>
<td>( f(t, t + 2, t + 3) )</td>
<td>-0.23</td>
<td>-</td>
<td>-0.18</td>
</tr>
<tr>
<td>( f(t, t + 3, t + 6) )</td>
<td>-0.18</td>
<td>-0.18</td>
<td>-0.13</td>
</tr>
<tr>
<td>( f(t, t + 6, t + 12) )</td>
<td>-0.16</td>
<td>-</td>
<td>-0.18</td>
</tr>
<tr>
<td>( \Delta r(t, t + 1) )</td>
<td>-11.15**</td>
<td>-</td>
<td>-13.21**</td>
</tr>
<tr>
<td>( \Delta r(t, t + 2) )</td>
<td>-9.91**</td>
<td>-</td>
<td>-10.48**</td>
</tr>
<tr>
<td>( \Delta r(t, t + 3) )</td>
<td>-9.15**</td>
<td>-</td>
<td>-9.50**</td>
</tr>
<tr>
<td>( \Delta r(t, t + 6) )</td>
<td>-8.99**</td>
<td>-</td>
<td>-9.12**</td>
</tr>
<tr>
<td>( \Delta r(t, t + 12) )</td>
<td>-8.63**</td>
<td>-6.23**</td>
<td>-8.59**</td>
</tr>
<tr>
<td>( \Delta f(t, t + 1, t + 2) )</td>
<td>-11.00**</td>
<td>-</td>
<td>-11.51**</td>
</tr>
<tr>
<td>( \Delta f(t, t + 2, t + 3) )</td>
<td>-9.62**</td>
<td>-</td>
<td>-9.74**</td>
</tr>
<tr>
<td>( \Delta f(t, t + 3, t + 6) )</td>
<td>-9.22**</td>
<td>-</td>
<td>-9.30**</td>
</tr>
<tr>
<td>( \Delta f(t, t + 6, t + 12) )</td>
<td>-8.66**</td>
<td>-</td>
<td>-8.63**</td>
</tr>
</tbody>
</table>

Note: * and ** denote rejection of a unit root on the 5-percent and 1-percent significance levels, respectively. The Augmented Dickey Fuller test has one lagged dependent variable included in the estimated equation.
Table 3. The Cointegration Regression

\[ r(t + n, t + m + n) = \alpha_1 + \beta_1 f(t, t + n, t + m + n) + \nu(t + n) \]

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>( \hat{\alpha}_1 )</th>
<th>( \hat{\beta}_1 )</th>
<th>( R^2 )</th>
<th>DF</th>
<th>ADF</th>
<th>( Z_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(t + 1, t + 2) ) (1984:2-19927)</td>
<td>0.010</td>
<td>0.920</td>
<td>0.794</td>
<td>9.54**</td>
<td>-</td>
<td>-10.13**</td>
</tr>
<tr>
<td>( r(t + 2, t + 3) ) (1984:3-19927)</td>
<td>0.016</td>
<td>0.876</td>
<td>0.649</td>
<td>-</td>
<td>-6.78**</td>
<td>-5.03**</td>
</tr>
<tr>
<td>( r(t + 3, t + 6) ) (1984:4-19927)</td>
<td>0.017</td>
<td>0.864</td>
<td>0.551</td>
<td>-</td>
<td>-5.07**</td>
<td>-4.56**</td>
</tr>
<tr>
<td>( r(t + 6, t + 12) ) (1984:7-19927)</td>
<td>0.041</td>
<td>0.654</td>
<td>0.309</td>
<td>-</td>
<td>-3.62*</td>
<td>-3.29</td>
</tr>
</tbody>
</table>

Note: * and ** denote rejection of non-cointegration on the 5-percent and 1-percent significance levels, respectively. The Augmented Dickey Fuller tests have 1, 1 and 4 lagged dependent variables included in the estimated equation, respectively.
<table>
<thead>
<tr>
<th>Variable</th>
<th>DF</th>
<th>ADF</th>
<th>Z_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r(t + 1, t + 2) - f(t, t + 1, t + 2)$ (1984:2-1992:7)</td>
<td>-9.80*</td>
<td>-</td>
<td>-9.85**</td>
</tr>
<tr>
<td>$r(t + 2, t + 3) - f(t, t + 2, t + 3)$ (1984:3-1992:7)</td>
<td>-</td>
<td>-6.94*</td>
<td>-5.72**</td>
</tr>
<tr>
<td>$r(t + 3, t + 6) - f(t, t + 3, t + 6)$ (1984:4-1992:7)</td>
<td>-</td>
<td>-5.28*</td>
<td>-4.67**</td>
</tr>
<tr>
<td>$r(t + 6, t + 12) - f(t, t + 6, t + 12)$ (1984:7-1992:7)</td>
<td>-</td>
<td>-3.42*</td>
<td>-3.50*</td>
</tr>
</tbody>
</table>

Note: * and ** denote rejection of non-cointegration on the 5-percent and 1-percent significance levels, respectively. The Augmented Dickey Fuller tests have 1, 1 and 4 lagged dependent variables included in the estimated equation, respectively.
Table 5. The Simple Difference Test

\[ r(t+n, t+m+n) - r(t, t+m) = \alpha_2 + \beta_2[f(t, t+n, t+m+n) - r(t, t+m)] + \epsilon(t+n) \]

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>$\hat{\alpha}_2$</th>
<th>$\hat{\beta}_2$</th>
<th>$R^2$</th>
<th>$Q_{LB}(12)$</th>
<th>$F$-test</th>
<th>$\chi^2$-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r(t+1, t+2) - r(t, t+1)$ (1984:2-1992:7)</td>
<td>0.001 (0.001)</td>
<td>0.918** (0.156)</td>
<td>0.258</td>
<td>17.433</td>
<td>0.511</td>
<td>-</td>
</tr>
<tr>
<td>$r(t+2, t+3) - r(t, t+1)$ (1984:3-1992:7)</td>
<td>0.002 (0.001)</td>
<td>1.205** (0.202)</td>
<td>0.293</td>
<td>59.845**</td>
<td>-</td>
<td>3.225</td>
</tr>
<tr>
<td>$r(t+3, t+6) - r(t, t+3)$ (1984:4-1992:7)</td>
<td>0.002 (0.002)</td>
<td>1.427** (0.311)</td>
<td>0.249</td>
<td>74.153**</td>
<td>-</td>
<td>2.652</td>
</tr>
<tr>
<td>$r(t+6, t+12) - r(t, t+6)$ (1984:7-1992:7)</td>
<td>0.004 (0.003)</td>
<td>1.347** (0.487)</td>
<td>0.230</td>
<td>136.330**</td>
<td>-</td>
<td>2.271</td>
</tr>
</tbody>
</table>

Note: Newey and West (1987) standard errors for the coefficients are given in parentheses, except in the $r(t+1, t+2) - r(t, t+1)$ regression. We use 1, 2 and 5 lags in the regressions, respectively. * and ** denote significance on the 5-percent and 1-percent levels, respectively.
Table 6. The Error Correction Model

\[ r(t+n,t+m+n) - r(t,t+m) = \beta_3 \nu(t) + \beta_4 [f(t,t+n,t+m+n) - f(t-n,t,t+m)] + \mu(t+n) \]

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>( \hat{\beta}_3 )</th>
<th>( \hat{\beta}_4 )</th>
<th>( R^2 )</th>
<th>( Q_{LB}(12) )</th>
<th>( F)-test</th>
<th>( \chi^2)-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(t+1,t+2) - r(t,t+1) ) (1984:3-1992:7)</td>
<td>-0.921**</td>
<td>0.879**</td>
<td>0.280</td>
<td>14.983</td>
<td>0.321</td>
<td>-</td>
</tr>
<tr>
<td>( r(t+2,t+3) - r(t,t+1) ) (1984:5-1992:7)</td>
<td>-1.151**</td>
<td>1.120**</td>
<td>0.324</td>
<td>44.855**</td>
<td>-</td>
<td>0.864</td>
</tr>
<tr>
<td>( r(t+3,t+6) - r(t,t+3) ) (1984:7-1992:7)</td>
<td>-1.180**</td>
<td>1.100**</td>
<td>0.258</td>
<td>71.259**</td>
<td>-</td>
<td>0.748</td>
</tr>
<tr>
<td>( r(t+6,t+12) - r(t,t+6) ) (1985:1-1992:7)</td>
<td>-1.086**</td>
<td>0.858**</td>
<td>0.315</td>
<td>137.724**</td>
<td>-</td>
<td>3.983</td>
</tr>
</tbody>
</table>

Note: Newey and West (1987) standard errors for the coefficients are given in parentheses, except in the \( r(t+1,t+2) - r(t,t+1) \) regression. We use 1, 2 and 5 lags in the regressions, respectively. * and ** denote significance on the 5-percent and 1-percent levels, respectively.
References


Figure 1. The Term Structure of Interest Rates
Interest Rates Against Time and Time to Maturity
Figure 2a–h. Rolling Regressions — The Simple Difference Test
Point Estimates and 95% Confidence Bands

2a. $\alpha_2$, 1-month one month ahead

2b. $\beta_2$, 1-month one month ahead

2c. $\alpha_2$, 1-month two months ahead

2d. $\beta_2$, 1-month two months ahead

2e. $\alpha_2$, 3-months

2f. $\beta_2$, 3-months

2g. $\alpha_2$, 6-months

2h. $\beta_2$, 6-months
Figure 3a–h. Rolling Regressions – The Error Correction Model
Point Estimates and 95% Confidence Bands

3a. $\beta_3$, 1-month one month ahead

3b. $\beta_4$, 1-month one month ahead

3c. $\beta_3$, 1-month two months ahead

3d. $\beta_4$, 1-month two months ahead

3e. $\beta_3$, 3-months

3f. $\beta_4$, 3-months

3g. $\beta_3$, 6-months

3h. $\beta_4$, 6-months