Seminar Paper No. 592

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April 1995

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April 24, 1995

Abstract

In this paper we show that fluctuations in distortive taxes account for some of the key features of the Swedish post-war business cycle. The empirical fit of a simple stochastic growth model is dramatically improved when it is amended to include imperfectly predictable fluctuations in payroll taxes, consumption taxes and the degree of progressivity of income taxes. Indeed, using the simulated method of moments, SMM, we find that for large sets of conventional moments the tax-model cannot be statistically rejected, whereas a no-tax-model is always strongly rejected. In particular, allowing for stochastic fiscal policy brings the model’s predicted correlation between labor input and the real wage, as well as the standard deviation of labor input and consumption relative to that of output, much more in line with the empirical evidence.

Keywords: Business cycles, stochastic growth model, fiscal policy, simulated method of moments.


*We thank Torsten Persson, Paul Söderlind, Anders Warne and seminar participants at the Stockholm School of Economics and IIES for their helpful comments and suggestions, and Carl Mannerfels fond and Jan Wallanders och Tom Hedelius’ stiftelse för samhällsvetenskaplig forskning for financial support. Special thanks are owed to Bodil Hulgaard of the Ministry of Finance, who advised us on income tax legislation. Correspondence to: Paul Klein, Institute for International Economic Studies, Stockholm University, S-106 91 Stockholm, Sweden. Phone: +46 8 162326. Fax: +46 8 161443. E-mail: kleinp@iies.su.se.
1. Introduction

Is fiscal policy important for the business cycle? In the early general equilibrium business cycle literature, the view was that it is not. For example, Kydland & Prescott (1987 p.22) state that "Our view is that public-finance considerations are not the principal factor driving the cycle and that abstracting from them at this stage is warranted." In this paper, we dissent from that view. Indeed, we argue that fluctuations in distortive taxes and government spending can account for some of the key features of the Swedish post-war business cycle.

Our approach to this issue is to ask whether and to what extent the introduction of imperfectly predictable fluctuations in fiscal policy can improve the empirical fit of a simple stochastic growth model. This question has recently been addressed for the United States by Christiano & Eichenbaum (1992), Braun (1994), McGrattan (1994) and McGrattan, Rogerson & Wright (1993). In these papers it is found that models with stochastic fiscal policy empirically outperform traditional real business cycle models. Hence, the general conclusion is that fluctuations in government consumption, labor taxes and capital taxes strongly affect the U.S. business cycle.

Although there are many other extensions of the standard stochastic growth model one might consider, several reasons suggest that fiscal policy must be included in any satisfactory account of economic fluctuations in Sweden. First, government spending in Sweden is huge. For example, total government expenditure as a percentage of GDP was 43.9 in 1970 and 65.4 in 1985. If fiscal policy matters for the business cycle in the U.S., where government spending is relatively much smaller than in Sweden, it seems likely that it matters in Sweden too.

Second, taxes and spending have changed considerably over time. Between 1947 and 1990, the central government income tax schedule was changed 22 times. And when the schedule was nominally unchanged, inflation and real growth caused an automatic steepening of the effective tax schedule.¹ During the same period, local government taxes,

¹When measuring the parameters that characterize the empirical income tax schedule, we adjust for inflation and real growth so as to take this "bracket creep" phenomenon into account.
consumption taxes, and payroll taxes have also fluctuated; in particular, they have risen.

Third, using Swedish data, Lundvik (1992) shows that models of a two-sector open economy with terms-of-trade shocks and interest rate shocks, in general perform better than a benchmark model. However, these models - which all abstract from fiscal policy - are nevertheless rejected at any reasonable level of significance. Indeed, Lundvik (1992, p.117) himself says the following: "How should we interpret the fact that the models are rejected? ... There are a lot of possible sources of fluctuations that the models abstract from ... especially shocks to public spending and taxes ..." To sum up, the motives for studying the effects of fiscal policy on the Swedish business cycle seem strong.

In addition to these arguments, there exist a number of empirical regularities or "stylized facts" which general equilibrium business cycle models have traditionally failed to reproduce, and which can potentially be explained and understood by considering the effects of stochastic fiscal policy. In particular, we stress the following three puzzles.

First, in most industrialized countries labor input is nearly as volatile as output, and in some countries even more so. The typical way to rationalize this in a stochastic growth model is to postulate (or derive) a high intertemporal substitutability of leisure. This approach has provoked much criticism. Indeed, some authors have even claimed that the general equilibrium approach to the business cycle should be abandoned altogether because of its reliance on this implausible feature. In contrast, our approach to this puzzle does not rely on an implausibly high degree of substitutability of leisure: it relies on the volatility of taxes on labor. Intuitively, if income and payroll taxes fluctuate over time, it is optimal to work hard when taxes are relatively low and to take time off when they are relatively high. Thus, stochastic taxes should in principle increase the model's predicted volatility of labor input, and bring the implications of the model closer to the empirical facts.

Second, for many European countries, including Sweden, consumption tends to be about as volatile as output. Again, one might deal with this by postulating a high degree

\[2\] Danthine & Donaldson (1993) provide a thorough survey and discussion of these issues. See also Backus & Kehoe (1992) for some additional evidence.

\[3\] See, for example, Kydland & Prescott (1982) and Hansen (1985).

\[4\] See Mankiw (1989).
of substitutability over time. A more popular explanation is to claim that households do not have access to a well-functioning capital market, and are hence unable to smooth consumption as much as they would like. In this paper, we offer another explanation: stochastic taxes on consumption. In a static model, a consumption tax is equivalent to an income tax. However, in a dynamic setting, a temporarily high tax rate on consumption provides an incentive to postpone consumption to a later date, when the tax rate is likely to be lower. Consequently, a fluctuating tax on consumption should increase the predicted volatility of consumption, and thus (again) bring the implications of theory closer to the facts of reality.\textsuperscript{5}

Finally, in nearly all countries the correlation (at business-cycle frequencies) between labor input and the real wage is close to zero or weakly positive. In Sweden, the correlation has even tended to be negative, depending on the frequencies and period studied.\textsuperscript{6} However, most general equilibrium business cycle models are driven by shocks to technology only. Intuitively, this means that all fluctuations are along a fixed labor supply curve, with labor demand fluctuating as technology shifts the marginal product of labor. This leads to the prediction of a strong positive correlation between labor input and the real wage. Hence, we have another empirical regularity which traditional business cycle models fail to reproduce. To deal with this puzzle, it is again natural to consider the effects of stochastic fiscal policy. As in Christiano & Eichenbaum (1992), McGrattan (1994) and Braun (1994), we interpret shocks to income taxes and payroll taxes as shocks to labor supply. Hence these shocks should tend to bring down the correlation between labor input and the real wage, and bring the model’s prediction more in line with the empirical evidence.

The approach in this paper can be summarized as follows. In order to study if fiscal variables and economic behavior are related in a way that standard economic theory can explain, we build two stochastic general equilibrium models; one with fiscal policy, and, as a benchmark, one without fiscal policy. We then check whether and to what extent the

\textsuperscript{5}This empirical puzzle is less pronounced for the United States where consumption is considerably less volatile than output. This smoothing behavior is just what the standard stochastic growth model predicts.

\textsuperscript{6}For a thorough treatment of this issue, see Hassler, Lundvik, Persson & Söderlind (1992).
fiscal policy version performs better than its benchmark rival. This question is addressed in a precise way: By using the simulated method of moments (SMM), we are able to perform statistical tests of our models.\footnote{Given that the parameters we estimate by SMM characterize preferences and technology rather than private behavior directly, the estimates are immune to the Lucas critique. Consequently, if our model is not rejected by the evidence, it can be used for welfare and policy analysis.}

The results favor the view that fluctuations in distorting taxes and government consumption are important for the business cycle. The model which includes fiscal variables systematically and strongly outperforms the standard real business cycle model in reproducing the most salient facts of the Swedish business cycle. Actually, for large sets of conventional moments the model with fiscal policy cannot be statistically rejected, whereas the model without taxes is always rejected at the 5-percent significance level. Moreover, the model with fiscal policy performs well with respect to the empirical puzzles described above. This means that the near-zero correlation between labor input and the real wage, as well as the rather high variability of both labor input and consumption relative to that of output, are well reproduced by the model with taxes.

The remainder of the paper is organized as follows. Section 2 describes the general equilibrium model. The dataset is presented in Section 3, and the methodological approach is laid out and discussed in Section 4. Section 5 presents the results, before conclusions are drawn and discussed in Section 6.

2. The general equilibrium model

Our model economy consists of infinitely many identical households, each of which faces a tax system whose structure closely resembles that of post-war Sweden. A representative household chooses leisure and consumption in each period so as to maximize

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \]  

\[
(2.1)\]
where $\tilde{\beta}$ is the subjective discount factor. The period utility function, $U$, is assumed to be of the conventional form

$$U(c_t, h_t) = \lim_{\delta \to \sigma} \frac{c_t^\alpha (1 - h_t)^{1-\alpha})^{1-\delta} - 1}{1 - \delta}$$

(2.2)

where $c_t$ is consumption of the single good and $h_t$ is the share of available time spent in paid employment. The parameter $\alpha$ is the weight attached to consumption relative to leisure, whereas $\sigma$ is the coefficient of relative risk aversion as well as the reciprocal of the intertemporal elasticity of substitution. The flow budget constraint facing the household is

$$(1 + \tau_t^c) c_t + i_t = \hat{y}_t - a_t \hat{y}_t^b + tr_t,$$

(2.3)

where $\tau_t^c$ is the consumption tax (the distinction between a sales tax and a value added tax being irrelevant here), $i_t$ is (private) investment and $\hat{y}_t$ is the income of the household before income tax and lump sum transfers from the government.\(^8\) The variables $a_t$ and $b_t$ describe the income tax schedule. In particular, $b_t$ may be interpreted as the degree of progressivity of the income tax schedule; thus $b_t = 0$ represents a poll tax, $b_t = 1$ a proportional tax, and $b_t > 1$ a progressive tax.\(^9\) Finally, $tr_t$ is a lump sum transfer from the government.

The reason for choosing an iso-elastic specification of the income tax schedule rather than a linear one is (apart from enhanced realism) the following. A linear tax schedule in the model would be characterized by a single variable: its slope. This slope’s empirical counterpart would, presumably, be the average marginal tax rate paid by households. But the issue that we are mainly concerned with is whether households respond in various ways

---

\(^8\)It should be noted that the model contains a deterministic labor-augmenting rate of technical change $\gamma$, and that the model has been stationarized (growth-adjusted) to take care of this. This implies that the effective subjective discount factor actually is of the rather strange form $\tilde{\beta} = \beta (1 + \gamma)^{\sigma(1-\sigma)}$ where $\beta$ has the usual interpretation as the weight the household attaches to next period’s utility. Thus, convergence of the maximand requires that $\tilde{\beta} < 1$, but not that $\tilde{\beta} < 1$. It also means that output, $\hat{y}_t$, (and similarly for the other non-stationary variables) should be interpreted as output per capita at time $t$ divided by $(1 + \gamma)^t$. This method of stationarizing a growth model is described in detail in Cooley & Prescott (1993).

\(^9\)Note that depreciation is not tax-deductible. This can easily be amended if preferred, but it would probably not matter much unless a richer treatment of corporate taxation were introduced. (See also footnote 11.) More importantly, the progressive tax schedule applies to growth-adjusted income. This prevents income taxes from growing faster than income over time.
to exogenous changes in fiscal policy variables. Thus it would be problematic if some of these variables could be controlled to any extent by a single household. But such is exactly the case with the marginal tax rate. In a progressive tax system, the marginal tax rate depends on the quantity of labor supplied by the individual household. In contrast, for our Swedish data, the marginal tax rate divided by the average tax rate (the elasticity of the tax schedule) is nearly constant along the tax schedule. At least this holds for income levels which are neither very low nor very high. Thus, the variables \( a_t \) and \( b_t \) are exogenous in the desired sense.

In addition to that, it actually matters for behavior whether the marginal tax rate is constant or increasing. This is shown for the static case by Persson (1983). Using a specification very similar to ours, he finds that even when preferences are such that a change in a proportional tax rate has no effect on labor supply, the degree of progressivity does have a negative effect on hours worked.\(^\text{10}\)

The representative household’s income is given by

\[
\hat{y}_t = \frac{1}{1 + \tau_t^w} w_t h_t + r_t k_t
\]

(2.4)

where

\[
w_t = z_t (1 - \theta) K_t^\theta H_t^{1-\theta}
\]

(2.5)

and

\[
r_t = z_t \theta K_t^{\theta-1} H_t^{1-\theta}.
\]

(2.6)

In equation (2.4) we encounter the payroll tax \( \tau_t^w \). \( w_t \) is the unit labor cost of an employer.

\(^\text{10}\)The question may arise as to why other taxes are not included, such as a corporation tax. One reason is that income taxes, value added taxes (including various excise fees), and payroll taxes generate the bulk of government revenue. In the fiscal year of 1979/80, state income taxes, payroll taxes and consumption taxes generated 87 per cent of central government revenue. The corresponding figure for 1988/89 was 88 per cent. Similarly, in 1980, 68 per cent of the revenue of local authorities consisted of local income taxes (42 per cent) and central government grants (26 per cent), the remainder being mainly fees charged for services provided. Moreover, given the assumption of perfect competition, the introduction of a corporation tax would raise a host of subtle issues which are beyond the scope of this paper.
and \( r_t \) is the pre-tax return on capital, \( k_t \). Note that the wage as well as the return on capital is exogenously given to the household, and that the return on period \( t \) capital is received in period \( t \). Equations (2.5) and (2.6) are profit maximization conditions of a representative firm in a perfectly competitive industry which produces the economy’s single good using a Cobb-Douglas production function. \( z_t \) is the exogenously given level of technology, and \( \theta \) is the capital share of income. Capital letters denote population averages. The law of motion for the capital stock is given by

\[
(1 + \gamma) k_{t+1} = (1 - \delta) \, k_t + i_t
\]  
(2.7)

where \( \delta \) is the depreciation rate. Note that each unit of growth-adjusted period \( t \) capital corresponds to \( (1 + \gamma)^{-1} \) units of growth-adjusted period \( t+1 \) capital. (\( \gamma \) is a deterministic labor-augmenting rate of technical change, see footnote 8.)

The government’s budget constraint is

\[
tr_t + g_t Y_t = \tau_c^c C_t + \frac{\tau_t^w}{1 + \tau_t^w} w_t H_t + a_t \dot{Y}_t^b
\]  
(2.8)

where \( g_t \) is the ratio of (useless) government consumption to output. Since technology exhibits constant returns to scale, we have \( y_t = w_t h_t + r_t k_t \), where \( y_t \) is output. Note that due to the existence of a payroll tax, the income of the household, \( \dot{g}_t \), is not equal to output, \( y_t \). Since there is no government debt, any surplus or deficit is immediately transferred lump sum to the households. We assume that the lump-sum transfer, \( tr_t \), is determined residually.

In this economy, then, the household’s problem is to solve

\[
Max_{(h_t, r_t+1)_{t=0}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \right]
\]  
(2.9)

subject to (2.3) - (2.8), and where the household also takes into consideration the stochastic process governing the evolution of the fiscal policy variables, as well as the aggregate de-

\[11\] Notice that the model economy we consider is closed. Since there is just one good there is no international trade, and international borrowing is not allowed. See Section 6 for further discussion of this issue.
cision rules $H_t = H(b_t, \tau_t^w, \tau_t^c, g_t, z_t, K_t)$ and $K_{t+1} = K'(b_t, \tau_t^w, \tau_t^c, g_t, z_t, K_t)$. These matter to the household, because they determine the stochastic price processes. The functions $H$ and $K'$ are given to and known by each household. Corresponding to the aggregate decision rules we have the individual household’s decision rules $h_t = h(b_t, \tau_t^w, \tau_t^c, g_t, z_t, K_t, k_t)$ and $k_{t+1} = k'(b_t, \tau_t^w, \tau_t^c, g_t, z_t, K_t, k_t)$.\(^{12}\) We impose, of course, as market equilibrium conditions, that the aggregate decision rules are compatible with the household’s decision rules.\(^{13,14}\) Thus, finding the equilibrium is essentially a matter of solving a fixed point problem: each household’s strategy (decision rule) must be optimal given the (average) strategy of all the other households.

3. Data

The data sample consists of 40 annual observations from 1951 to 1990. Figure 1 plots the per capita values (in natural logs) of GDP, private consumption, private investment, worked hours, and the hourly product wage, all in real terms. GDP and consumption are calculated at factor cost, i.e. excluding indirect taxes and subsidies. The series over consumption includes durables purchases. This partly accounts for the rather high volatility of consumption as we define it. The main reason for not excluding durables purchases is that these goods, of course, are subject to the consumption tax too. And we want to study whether fluctuations in the consumption tax (including various excise fees)

\(^{12}\)The question may arise as to whether such aggregate decision rules exist, i.e. whether it is optimal for the households to use a stationary feedback rule. The answer is yes. To establish this (informally), we begin by noting that the sequence of control vectors is allowed to be stochastic. But each household, when choosing time $t$ decisions, is allowed to use no more information than is available to it at time $t$. This requirement is clearly fulfilled by letting the household choose a feedback rule (Markov strategy) of the form $h_t = h(t, b_t, \tau_t^w, \tau_t^c, g_t, z_t, K_t, k_t)$, $k_{t+1} = k'(t, b_t, \tau_t^w, \tau_t^c, g_t, z_t, K_t, k_t)$. It should also be clear that the household cannot do better by conditioning her choices on the entire history rather than just the current state. Since the exogenous state vector is a Markov process, the household loses nothing by conditioning its actions only on the current state. Given the infinite horizon, the $t$-specificity of the feedback rules is only needed to take care of deterministic growth. To avoid it, the non-stationary variables are divided by $(1 + \gamma)^t$. Thus, the problem looks the same in every period. This is what justifies the imposition of a single time-invariant (stationary) feedback rule. Thus we may impose $h_t = h(b_t, \tau_t^w, \tau_t^c, g_t, z_t, K_t, k_t)$, $k_{t+1} = k'(b_t, \tau_t^w, \tau_t^c, g_t, z_t, K_t, k_t)$ and analogously at the aggregate level.

\(^{13}\)More precisely, we require that $H(b_t, \tau_t^w, \tau_t^c, g_t, z_t, K_t) \equiv h(b_t, \tau_t^w, \tau_t^c, g_t, z_t, K_t, k_t)$ and $K'(b_t, \tau_t^w, \tau_t^c, g_t, z_t, K_t) \equiv k'(b_t, \tau_t^w, \tau_t^c, g_t, z_t, K_t, k_t)$.

\(^{14}\)The household also faces a no-Ponzi-game condition which means that the limit of the net present discounted value of assets must, with probability 1, be non-negative where the discount rate equals the after tax interest rate.
are sufficient to explain the relative volatility of consumption. Moreover, any classification of goods into durables and non-durables is bound to be controversial. It should also be mentioned that inventory investment is not included in the empirical data series on investment.\textsuperscript{15} This is because inventories do not feature in the model, so there is no hope of reproducing its time series properties.

The fiscal policy variables are plotted in Figure 2. A summary of the method of estimating the degree of progressivity of the income tax schedule is provided in the Appendix. The approach basically involves fitting a curve with constant elasticity to a function that relates income earned to income tax paid, using a least squares criterion. From the figure one can notice that the tax system has been progressive throughout the period \((b_t > 1)\).

Furthermore, the consumption tax includes both VAT (\textit{moms}) as well as various other indirect taxes and excise fees, such as taxes on alcohol, motor vehicles, horse race betting etc. This explains why the consumption tax has been, for example, above 30\% since the early eighties, although the rate of VAT has been lower than that during the same period.

Finally, a few comments on the measurement of payroll taxes (mainly social insurance contributions) are in order. To some extent, these compulsory contributions generate claims on future transfer payments in the form of pensions etc. from the government in proportion to the contributions paid. To the extent that this is the case, payroll taxes do not distort behavior in the way that an ordinary tax does. Ideally, one would like to separate, for each year, the insurance premium/savings part of payroll taxes from the distortive tax part. But since this would raise difficult conceptual issues as well as involve an extremely cumbersome data collection exercise, we avoid such a separation and treat the entire payroll tax as a tax in the ordinary distortive sense.\textsuperscript{16}

\textsuperscript{15}This implies that the accounting identity \(y = c + i + g\) does not hold in the empirical series. But since Sweden is an open economy, it would not hold in any case.

\textsuperscript{16}For an account of our data sources, see the Appendix.
4. Method

4.1. Solution Algorithm

In practice, it is impossible to find the equilibrium decision rule; no analytic solution exists, and a computer can at best find an approximation since the exact decision rule is infinite-dimensional. In this paper we have chosen the conventional method of approximating the original problem around its steady state by a linear-quadratic problem. This approximation of the problem can then be solved rather swiftly. We should perhaps stress that the linear-quadratic approximation is around the natural logs of all the variables except for the tax variables $b_t$, $\tau_t^c$, and $\tau_t^w$.\textsuperscript{17}

4.2. Filtering

For data to be amenable to estimation, it must somehow be rendered stationary. A convenient way of doing this in practice is to use the HP-filter introduced into economics by Hodrick & Prescott (1980). This filter decomposes a series into a cyclical component and a trend component.\textsuperscript{18} The HP-filter is flexible in that one may choose different values of a smoothness parameter, $\lambda$, depending on the frequencies that one is mostly concerned with. In order to capture what we usually mean by the business cycle (i.e. fluctuations with a period of 3-8 years), we set this smoothness parameter, $\lambda$, to 100.\textsuperscript{19} In our data set this implies that real GDP exhibits about 8 cycles during the studied 40-years interval. This can be seen in Figure 3 which shows the HP-filtered series from Figure 1. Since the filter is applied to the natural logs of the series, the fluctuations can be interpreted as percentage deviations from their trend.

After this subsection, all non-stationary variables as well as hours worked per head should be interpreted as having been submitted to HP-filtration. Note that all the fiscal

\textsuperscript{17}For the purposes of dynamic programming, $a_t$ is not a state variable since it is an exact function of $b_t$.

\textsuperscript{18}The HP-filter is more thoroughly described in the Appendix.

\textsuperscript{19}We have also performed the simulations with $\lambda$ equal to 20 and 200, respectively. These values imply that the business cycle (for real GDP) is about 4 years and 8 years, respectively, for Swedish post-war data. The results from these simulations are similar to those reported in Section 5 (where $\lambda$ is set to 100). In particular, the model that includes fiscal variables always outperforms the benchmark model.


Rather than estimate the constant, we equated the unconditional mean $\mu = (I - \varphi)^{-1}v$ of the process to the sample mean of the empirical series and estimated the system without a constant. The estimation results are as follows:

$$\hat{\mu} = \begin{bmatrix} -1.30 \\ 1.75 \\ 0.24 \\ 0.20 \end{bmatrix}$$

(4.3)

$$\hat{\phi} = \begin{bmatrix} 0.86 & 0.10 & -0.01 & 0.08 \\ -0.06 & 0.28 & -0.01 & 0.01 \\ 0.13 & -0.72 & 0.93 & -0.10 \\ 0.04 & -0.14 & 0.04 & 0.92 \end{bmatrix}$$

(4.4)

$$\hat{\Sigma} = \begin{bmatrix} 0.0013 & 0.0004 & 0.0002 & 0.0003 \\ 0.0004 & 0.0164 & -0.0001 & 0.0002 \\ 0.0002 & -0.0001 & 0.0002 & 0.0000 \\ 0.0003 & 0.0002 & 0.0000 & 0.0002 \end{bmatrix}$$

(4.5)

Since $e^{-1.3} \approx 0.27$, the time average of the ratio of government consumption to GDP is 27%, that of payroll taxes is 20%, and that of consumption taxes is 24%. Also, the income tax system has on average been progressive during the studied period. Moreover, all fiscal policy variables are quite highly autocorrelated.

For the existence of a steady state (which we must have in order to use the solution algorithm we have chosen), we require that the exogenous state vector be stationary. For this it suffices that all the eigenvalues of $\varphi$ lie strictly inside the unit circle in the complex plane. Fortunately, this is the case for our estimate of $\varphi$. 
4.4. SMM

Heuristically, simulated method of moments estimation consists in choosing those parameter values that produce the best match between empirical and simulated moments.\textsuperscript{23} To test the model, we use the fact that sample moments are functions of time averages, and hence asymptotically normally distributed. Thus, multiplying the vector of differences between empirical and simulated moments around the inverse of a consistent estimate of its asymptotic variance matrix yields a test statistic which is asymptotically distributed as $\chi^2$ with degrees of freedom equal to the number of moments minus the number of parameters being estimated.

When interpreting the test statistics (and the associated p-values) it is worth keeping in mind what it is that is being tested. The null hypothesis is not, as in our reading of Christiano & Eichenbaum (1992) and Braun (1994), that a certain subset of the model's moments are equal to the corresponding empirical sample moments (the latter regarded as fixed numbers). Rather, the hypothesis is that a certain subset of the model's (true, not simulated) moments are equal to the (true, not sample) moments of the stochastic process of which our observed macroeconomic time series are but a single realization. In other words, our population is not the actual history, but, as is the convention in statistics, the set of possible histories. Thus, sampling uncertainty affects not only our parameter estimates, but also our sample standard deviations and correlations which should be seen as estimates of their population counterparts.

4.5. Choice of moments

In all of the estimations, we use only standard deviations, relative standard deviations, contemporaneous correlations, and first-order autocorrelations. The main reason for this restriction is that the set of moments would otherwise be too large, and that the moments we do use are the easiest to interpret.

Also, the relative variability of the real wage is never considered in estimation, simply because there is no hope of the model reproducing it. Presumably for (implicit) insurance

\textsuperscript{23}For details, see the Appendix.
reasons, the real wage is in reality not allowed to fluctuate with the same amplitude as the marginal product of labor. Nevertheless, while an insurance contract might dampen fluctuations, it should not influence comovements (defined as correlations). In particular, the model should be able to explain the correlation between the real wage and hours worked.

We estimate the models with four different sets of moments. The first set (set 1) is chosen so as to highlight the contrast between the model with fiscal policy and the model without fiscal policy. In particular, it contains the three moments we discussed in the introduction: the relative variability of hours worked and of consumption to that of output, and the correlation between hours worked and the real wage. The following 9 moments were included in set 1:

\[
\begin{pmatrix}
\sigma_{\ln y_t}, \sigma_{\ln h_t}/\sigma_{\ln y_t}, \sigma_{\ln c_t}/\sigma_{\ln y_t}, \\
Corr(\ln h_t, \ln h_{t-1}), Corr(\ln y_t, \ln y_{t-1}), Corr(\ln c_t, \ln c_{t-1}), \\
Corr(\ln h_t, \ln y_t), Corr(\ln h_t, \ln c_t), Corr(\ln h_t, \ln w_t).
\end{pmatrix}
\]

To see whether the model with fiscal policy could outperform its rival on a larger set of moments, we included in set 2 all (relative) standard deviations except the one of the real wage, all contemporaneous correlations and all first-order autocorrelations of the endogenous variables.\textsuperscript{24}

Third, the model with fiscal policy is estimated with a set of moments including the exogenous variables. The reason for doing this is to see whether the mechanism we use to explain fluctuations of and comovements between hours, output, consumption, investment and real wages is the right one, i.e. whether private behavior reacts to fiscal shocks in the way that our model predicts. Thus, in addition to the moments included in set 1 and 2, sets 3 and 4 include all contemporaneous correlations between endogenous and exogenous variables, except those involving \(w_t\) and \(g_t\), respectively. Actually, it is not possible to include much more than that; if the number of moments is close to or exceeds the number of observations, the variance matrix of the sample moments will typically

\textsuperscript{24}We also excluded moments including the capital stock, since we do not have data on the Swedish capital stock.
become singular.

5. Results

In this section we report the parameter estimates and the goodness-of-fit measures from the SMM procedure. As already mentioned, the first and second sets of moments include no fiscal policy variables. This makes the comparison of the models with and without taxes straightforward. However, in order to study if the model economy reacts to fiscal shocks in the predicted way, results are also reported (see Section 5.3) where the sets of moments are extended to also include correlations between endogenous variables and exogenous fiscal policy variables.

5.1. Set 1

Focusing on 9 of the most common moments the results are as follows. The parameter estimates for the model without fiscal policy are

\[ (\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{\sigma}, \hat{\delta}, \hat{\rho}, \hat{\sigma}_x^2) = (0.055, 0.685, 0.59, 0.30, 0.15, 0.905, 0.00031), \]  

(5.1)

whereas for the model with fiscal policy the estimates are

\[ (\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{\sigma}, \hat{\delta}, \hat{\rho}, \hat{\sigma}_x^2) = (0.16, 0.995, 0.31, 6.4, 0.15, 0.995, 0.00026). \]  

(5.2)

It can immediately be noticed that the parameter estimates are unreasonable in the no-tax model, indicating that this model is misspecified. In particular, the estimated reciprocal of the intertemporal elasticity of substitution, $\hat{\sigma}$, is implausibly low, implying a very high willingness to substitute leisure and consumption over time. In contrast, the estimated parameters in the model which includes fiscal policy, are very close to those that would have been obtained via a conventional calibration procedure.\textsuperscript{25}

\textsuperscript{25}The exception is $\hat{\delta}$, which is higher than what is usually assumed. However, if $\delta$ is fixed at 0.10 the model can still not be rejected.
The decision rules are found to be the following. For the no-tax model they are

\[
\begin{bmatrix}
\ln H_t - \ln \bar{H} \\
\ln K_{t+1} - \ln \bar{K}
\end{bmatrix} =
\begin{bmatrix}
-0.17 & 0.46 \\
0.76 & 0.61
\end{bmatrix}
\begin{bmatrix}
\ln K_t - \ln \bar{K} \\
\ln z_t
\end{bmatrix}
\] (5.3)

and for the tax model they are

\[
\begin{bmatrix}
\ln H_t - \ln \bar{H} \\
\ln K_{t+1} - \ln \bar{K}
\end{bmatrix} =
\begin{bmatrix}
-0.10 & 0.16 & 0.29 & -0.02 & -0.41 & -0.73 \\
0.84 & 0.24 & -0.04 & -0.01 & 0.04 & -0.09
\end{bmatrix}
\begin{bmatrix}
\ln K_t - \ln \bar{K} \\
\ln z_t \\
\ln g_t - \ln \bar{g} \\
b_t - \bar{b} \\
\tau_t^c - \bar{\tau}_c \\
\tau_t^w - \bar{\tau}_w
\end{bmatrix}
\] (5.4)

where a bar denotes a steady state value. In the model without taxes only technology shocks are present, and the household reacts quite strongly to these shocks. This can be contrasted with the tax-model where the household does not react that much to these shocks, but instead reacts to the shocks introduced by stochastic fiscal policy. Moreover, the decision rules seem to be reasonable. In particular it can be noticed that the coefficient of savings on the consumption tax is positive; if the consumption tax is temporarily above its mean, consumption is postponed. This illustrates clearly the non-equivalence between an income tax and a consumption tax in a dynamic model. One can also note the strong negative effect of an increase in the payroll tax on hours worked. It may seem puzzling that the coefficients on \((b_t - \bar{b})\) are so close to zero; presumably the reason is that any increase in \(b_t\) is always offset by a decrease in \(a_t\).

The results from the SMM estimation of the moments and the tests for the goodness of fit are reported in Table 1. We immediately see that the model with fiscal policy variables strongly outperforms the model without taxes. All 9 moments are closer to their empirical values in the model which includes distortive taxes. In particular, allowing for stochastic fiscal policy decreases the model’s predicted correlation between hours and real wages in

\[26\text{Note that } \ln K_t \text{ is stationarized through multiplication by } (1 + \gamma)^{-t}.\]
a way that makes the prediction quite close to the empirical correlation. Moreover, the estimations of the variability of both worked hours and consumption (relative to that of output) are much closer to their empirical values when fiscal policy variables are included in the model. Together this implies that the model with fiscal policy cannot be rejected at standard significance levels, whereas the model without it is rejected at the 5 percent significance level.

5.2. Set 2

The above conclusions are unchanged when we study how the models perform with respect to 10 additional moments. The parameter estimates for the model without fiscal policy when the original 9 plus the new 10 moments are included are

$$
(\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{\sigma}, \hat{\delta}, \hat{\rho}, \hat{\sigma}_z^2) = (0.18, 0.825, 0.39, 0.60, 0.12, 0.985, 0.00027),
$$

(5.5)

whereas for the model with fiscal policy the estimates are

$$
(\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{\sigma}, \hat{\delta}, \hat{\rho}, \hat{\sigma}_z^2) = (0.12, 0.97, 0.19, 9.0, 0.07, 0.995, 0.00009).
$$

(5.6)

The estimates are again implausible in the no-tax model, indicating that this model is misspecified, whereas they are quite reasonable for the tax-model.

The decision rules are very similar to the ones found when using the first set of moments. Hence, in the tax-model the household responds to a variety of shocks, including fiscal policy shocks. And the reactions to these shocks seem to be what one might anticipate.

The results from the estimation of the 19 moments and the tests for the goodness of fit are reported in Table 2. Again we note that the model with fiscal policy strongly outperforms the model without fiscal policy. The predicted correlation between hours and real wages and the variability of both worked hours and consumption (relative to that of output) are again much more in line with their empirical values when fiscal policy is taken
into account. In addition to these observations, one can note that the positive correlations between the real wage and both output and consumption decrease and become closer to their empirical counterparts when the model is amended to include distortive taxes. A potential explanation for these observations is that fluctuations in the real wage to a certain extent consist of fluctuations in payroll taxes, (recall that the real wage is defined as the product real wage). Hence, an increase in the real wage does not necessarily imply an increase in the household's real disposable income.

The other moments are also in general closer to their empirical values in the tax-model. Actually, the tax-model's predictions are closer to the empirical values than the no-tax model's predictions for 17 of the 19 moments. Together this again implies that the tax model cannot be statistically rejected, whereas the model without taxes is rejected at any reasonable significance level.

5.3. Sets 3 and 4

In order to study if the economy reacts to fiscal shocks in the way predicted by the model, we also report the results from estimations where 12 additional moments - related to fiscal policy variables - are included. The parameter estimates and the decision rules are in these cases very similar to the ones reported for the tax-models in the previous two sections.

The estimated moments and the tests for goodness of fit are reported in Table 3. The conclusions with respect to the endogenous variables' correlations, autocorrelations and relative variability are the same as in the previous two sections. The predicted correlations between the endogenous variables and the tax variables are also, in general, in line with the empirical correlations. In particular one can notice that the negative correlation between hours and the degree of income tax progressivity is well predicted by the model. More interesting, perhaps, is that the positive correlation between the degree of income tax progressivity and consumption is well predicted by the model as well. A possible reason for this result is that a higher degree of income tax progressivity is always associated with lower income taxes on average (an increase in $b_t$ is always offset by a decrease in $a_t$). Moreover, the correlations between the consumption tax and consumption on the one
hand, and investment on the other, have the expected signs and are fairly well predicted by the model. This pattern is consistent with the argument that the household postpones consumption (and saves more) when the consumption tax is temporarily high.

Finally, one should note that although the model quantitatively exaggerates the negative correlation between some tax variables and the endogenous variables, the model can still not be rejected when the 9 first moments are studied together with the 12 tax-moments. Only when all 31 moments are included is the model statistically rejected.

6. Conclusions and Discussion

The main conclusion in this paper is that fiscal shocks do matter for the business cycle. The introduction of stochastic fiscal policy dramatically improves the empirical fit of the simplest stochastic growth model. In the course of making this point, we show that the fluctuations in distorting taxes provide a plausible explanation for several empirical puzzles. In contrast to a simple stochastic growth model, the model that takes fiscal policy shocks into account predicts a close to zero correlation between the product real wage and labor input, as well as a high variability in both worked hours and consumption (relative to that of output). These three predictions are all consistent with Swedish postwar data.

It is perhaps worth stressing that the results in this paper are substantive, and not purely mechanical. The extension of a general equilibrium model does not automatically improve its empirical performance in the way that adding regressors to a regression model automatically increases $R^2$. Indeed, an extended model may easily perform worse than a simple one.

One may wonder if the tests for accepting or rejecting the model have any power, given the small number of observations (40) and the possible instability over time of some sample correlations. One answer is that the tests are evidently powerful enough to reject the model without fiscal policy at any reasonable significance level. Another answer is that in other studies these tests (or similar ones) usually reject the models, see e.g. Lundvik (1992), Christiano & Eichenbaum (1992) and Braun (1994). In particular, Braun, who has less data points than we do, can avoid rejection only by including labor indivisibility
as well as fiscal policy, and by confining his attention to no more than 8 moments at a
time.

However, the question does arise as to whether our extension is the right one. If
the model correctly reproduces only moments involving endogenous variables, one might
be tempted to dismiss the results by claiming that the addition of extra noise of any
kind tends to improve the empirical performance of general equilibrium models. The
evidence from Table 3 is mixed but, on the whole, encouraging. Only when 31 moments
are included can we reject the model. And even then, the fit is qualitatively impressive.
Nearly all correlation coefficients have the right sign, and most even have the right order
of magnitude.

Movements in international variables could be an alternative source of shocks to the
Swedish economy. A natural extension of our model, then, would be to consider an open
economy. However, it should be kept in mind that capital controls were in force during the
entire period we study, except for the last two years. This means that the ability of the
private sector to borrow and invest abroad was, in fact, quite restricted. Moreover, both
Lundvik (1992) and Hassler (1994) show that fluctuations in international variables - such
as foreign GDP, the foreign interest rate, terms of trade etc - seem to have only a small
impact on fluctuations in Swedish macroeconomic variables, except for those variables
which explicitly involve the foreign sector such as exports and imports. Still, in order to
distinguish between the relative importance of various international as well as domestic
shocks for the business cycle, and in order to explain the movements and comovements of
those variables which explicitly involve the foreign sector, it would be fruitful to consider
open economy models with fiscal policy shocks.

Indeed, there are reasons to think that the introduction of a fluctuating tax on con-
sumption into an open economy model may have the potential to explain not just the high
relative variability of consumption in most countries, but also a particularly recalcitrant
stylized fact of the international business cycle, namely that output tends to be more
strongly correlated across countries than consumption.

Finally, our decision to include durables purchases in consumption means that we
do not explain the variability of such purchases relative to that of non-durables pur-
chases. To deal with this, we need a model with household production in the style of Benhabib, Rogerson & Wright (1991). In particular, it would be interesting to see if a model with household production and stochastic taxes on purchases of both durables and non-durables could reproduce the time series properties of these purchases without postulating implausibly large shocks to technology in home production.
A. SMM

The following brief summary is based on, but very slightly extends, the work of Lee & Ingram (1991). The idea behind SMM is very similar in spirit to that behind GMM. Suppose we have a model which is fully specified except for an unknown parameter vector $\beta \in \mathbb{R}^K$, and which can be used to simulate $K$ independent $l$-dimensional sequences $\left\langle y_{t,i}^\beta \right\rangle_{t=1}^N$, $i = 1, 2, ..., K$, and have an observed $l$-dimensional sequence $\left\langle x_t \right\rangle_{t=1}^T$. Suppose $n \triangleq \frac{NK}{T} > 1$.\textsuperscript{27,28} Without loss of generality, let the mean of these series be 0. We now want to do the following: (1) Estimate the model’s parameters. (2) Test the model. We begin by specifying a "raw" moment function $v : \mathbb{R}^l \to \mathbb{R}^{(l+1)/2}$ defined by

$$v(x_t) \triangleq \text{vech} \left( x_t x_t' \right).$$

Thus $v(x_t)$ is just the vector of all possible products between the elements of $x_t$ arranged in suitable order. Now define

$$v_T \triangleq \frac{1}{T} \sum_{t=1}^T v(x_t),$$

$$v_{N,i}^\beta \triangleq \frac{1}{N} \sum_{t=1}^N v \left( y_{t,i}^\beta \right).$$

If we wanted only variances and covariances as moments, the above would suffice. But since it is easier to interpret standard deviations, relative standard deviations and correlation coefficients, we go one step further and define a function $m : \mathbb{R}^{(l+1)/2} \to \mathbb{R}^l$. Since sample standard deviations and correlation coefficients are functions of the raw moments in $v_T$ and $v_{N,i}^\beta$, this is all we need. So finally, define

\textsuperscript{27}The symbol $\triangleq$ denotes an equality which is decreed as a definition.

\textsuperscript{28}The reason for simulating $K$ independent sequences of length $N$ rather than a single sequence of length $NK$ is the gain in efficiency. This gain is large because of the high degree of persistence in our simulated series.
\[ m_T \triangleq m(v_T) \quad (A.4) \]
\[ m_{N,i}^\beta \triangleq m\left(v_{N,i}^\beta\right) \quad (A.5) \]
\[ m_N^\beta \triangleq \frac{1}{K} \sum_{i=1}^{K} m_{N,i}^\beta \quad (A.6) \]

If the model is true, then, for some \( \beta \in \mathbb{R}^k \), we should presumably have

\[ \underset{T \to \infty}{\operatorname{p.lim}} [m_T] = \underset{N,K \to \infty}{\operatorname{p.lim}} [m_N^\beta] = \mathbf{m} \quad (A.7) \]

where \( \mathbf{m} \in \mathbb{R}^j \) is a non-random vector of population moments. This motivates the following estimator of \( \beta \) (the SMM estimator)\(^{29}\):

\[ \hat{\beta} \triangleq \arg \min_{\beta} \left[ m_T - m_N^\beta \right]' \hat{W} \left[ m_T - m_N^\beta \right] \quad (A.8) \]

where \( \hat{W} \) is some, possibly random, positive definite matrix such that \( \underset{T \to \infty}{\operatorname{p.lim}} \hat{W} = W \), where \( W \) is non-random. Then, under various technical assumptions\(^{30}\), \( \hat{\beta} \) is consistent and asymptotically normal. The optimal choice of \( W \) is the inverse of the asymptotic variance matrix\(^{31}\) of \( \left( m_T - m_N^\beta \right) \), i.e.

\[ W = \left[ \frac{\partial m(v_T)}{\partial v_T} \left( 1 + \frac{1}{n} \right) S \frac{\partial m(v_T)}{\partial v_T} \right]^{-1} \quad (A.9) \]

where \( S \) is the asymptotic variance matrix of \( v_T \).

A suitable estimator of \( S \) is given by Newey & West (1987) as

\(^{29}\)Note the abuse of notation: here \( \beta \) is not the subjective discount factor.

\(^{30}\)In particular, our series must be stationary and ergodic.

\(^{31}\)When we say that a sequence of random variables \( X_T \) is asymptotically normal with asymptotic variance matrix \( V \), we mean that

\[ \sqrt{T} [X_T - E(X_T)] \overset{d}{\to} N(0, V) \]

as \( T \to \infty \).
\[ \hat{S} = \hat{R}_0 + \sum_{i=1}^{p} \left( 1 - \frac{i}{p+1} \right) (\hat{R}_i + \hat{R}_i') \]  

(A.10)

where \( p \) is some suitable integer-valued strictly increasing function of \( T \) and

\[ \hat{R}_i = \frac{1}{T} \sum_{t=i+1}^{T} (v(x_t) - v_T)(v(x_{t-i}) - v_T)' . \]  

(A.11)

To test the model, we use the fact that if the weighting matrix is chosen optimally, then, as \( T \to \infty \) (keeping \( n \) fixed), we have

\[ T \left[ m_T - m_N^\beta \right]' \hat{W} \left[ m_T - m_N^\beta \right] \xrightarrow{d} \chi^2 (j - k) \]  

(A.12)

where \( j \) is the number of moments and \( k \) is the number of estimated parameters.\(^{32}\)

In each estimation, the \( K \) random sequences \( \left\{ \varepsilon_{t,i} \right\}_{i=1}^{N} \) and \( \left\{ \xi_{t,i} \right\}_{i=1}^{N} \) are fixed. As a reasonable compromise between speed and efficiency, we set \( N = 300 \) and \( K = 10 \). In practice, minimization of (A.8) is done by a grid search where each parameter takes on three different values. In order to determine where to search for the minimizing parameters, i.e. in order to find the centre and the steps of the grid, the effects of large variations for each parameter at a time was considered before the final grid search was performed.

**B. The Hodrick-Prescott (HP) filter**

The HP-filter works as follows: For a given value of \( \lambda \), the stationary component \( \langle x_t' \rangle \) of the sequence \( \langle x_t \rangle \) is defined as follows.

\[ \langle x_t' \rangle_{i=1}^{T} = \min \left\{ \sum_{t=1}^{T} (x_t')^2 + \lambda \sum_{t=2}^{T-1} \left( x_{t+1} - x_{t+1}' - 2x_t + 2x_t' + x_{t-1} - x_{t-1}' \right)^2 \right\} \]  

(B.1)

\(^{32}\) We are aware of the possibility that the small sample distribution of our test statistic may deviate significantly from its asymptotic distribution. Ideally, this issue should be settled by performing a Monte Carlo study. However, since a single estimation of our model takes several hours, it would be extremely time-consuming to perform such a study.
Convenient features of the HP-filter include the following. First, it leaves a linear trend undistorted. Thus if data are first multiplied by $(1 + \gamma)^t$ and then logged, the stationary component will be the same regardless of $\gamma$. Thus there is no real need to estimate $\gamma$ with any precision (although its value does affect the steady state of the stationarized general equilibrium model, so we must choose some value for it; we decided to fix it at 0.024 to match the average rate of growth during the period). Second, HP-filtering obviates the need for choosing an appropriate unit for measuring output etc. Two series measuring the same thing in different units will yield the same stationary component if they are first logged and then HP-filtered. Given that it is the logarithms of variables that are filtered, the filtered data series can be interpreted as percentage deviations from trend. Finally, the HP-filter is flexible in that one may choose different values of $\lambda$ depending on the frequencies that one is concerned with. A low value of $\lambda$ means that the cyclical component contains mainly the high frequencies of the original series, while the opposite is true for high values of $\lambda$.

C. Data

C.1. Income taxes

See Klein (1995) for details on the collection of data on the income tax schedule. For the purposes of interpreting the results in this paper the following should suffice.

When reconstructing the income tax schedule, we use statistics on how much income tax $T_i$ was paid on average by those who earned an income in an interval $[\hat{y}_i, \hat{y}_{i+1}]$.\textsuperscript{33,34} Approximating the income earned by each individual or household in each interval by $\bar{y}_i \triangleq (\hat{y}_i + \hat{y}_{i+1}) / 2$, we get a finite number of pairs $(\bar{y}_i, T_i)$, each associated with the number of individuals or households (assessments) in interval $i$. We then fit a curve to these observations. In particular, we run a weighted least square regression of $\ln T_i$ on

\textsuperscript{33}Slightly abusing the notation, $\hat{y}$ is here defined as in equation (2.3) except that it is not growth-adjusted.

\textsuperscript{34}Strictly speaking, there exist no statistics relating income earned to income tax paid for the period 1951-1971. For those years we instead consult the relevant legislation in order to determine what the relation should have been between income and taxes.
ln \( \hat{y}_t \), where the weights are given by the number of assessments in each band. We then measure \( b_t \) as the estimated slope from this regression. Meanwhile, in order to adjust for growth, \( \ln a_t \) is given by

\[
\ln a_t = \text{constant}_t + t (\text{slope}_t - 1) \ln (1 + \gamma)
\]  

(C.1)

The consumption tax is estimated for each year by relating the amount of total tax revenues from consumption to the amount of total private consumption, whereas the payroll tax for each year is found by relating the total compensation of employees to the total amount of employers' contributions to social security, private pensions etc.

The data is, of course, available upon request.

C.2. Sources

For legislation pertaining to income taxation, we consulted *Svensk Författningssamling*, various years and issues. For local income tax rates, we used *Årshok för Sveriges kommuner* (Central Bureau of Statistics, Stockholm, various years). For data on the distribution of income for the years 1952-71, we used *Skattetaxeringarna samt fördelningen av inkomst och förmögenhet* (Central Bureau of Statistics, Stockholm, various years). The source of all figures for 1972-91 is *Riksrevisionsverkets tazoringsstatistiska undersökning* (Central Bureau of Statistics, Stockholm, various years).

For consumption tax revenues we used *Statens Finanser* and *Utsaltet av statens finanser* (Riksrevisionsverket, Stockholm, various years). All other variables are taken from the annual Swedish national accounts (Central Bureau of Statistics, Stockholm, various years).
References


policy series are treated as stationary. Consequently they are not filtered.\footnote{The results are qualitatively similar if the fiscal policy series are filtered. The main effect of filtering is that the absolute magnitudes of the correlations which include fiscal policy variables become greater.}

4.3. The VAR model

In the general equilibrium model fiscal policy is treated as an exogenous vector-valued stochastic process. This process is modeled as a \( \text{VAR}(p) \) where the parameters are estimated by least squares. Using the parameter estimates, we then estimate the remaining parameters using SMM. Since \( \ln a_t \) and \( b_t \) empirically are highly collinear, we excluded \( \ln a_t \) from the estimation. (In the general equilibrium model, the two variables are treated as perfectly correlated with a negative affine relationship defined by the OLS regression of \( \ln a_t \) on \( b_t \) and a constant.)\footnote{The result from this OLS regression is \( \ln a = 10.8 - 11.4b \) with \( R^2 = 0.93 \).} The choice of order \( p \) was settled using a likelihood ratio test. The hypothesis \( p = 1 \) could not be rejected against the alternative hypothesis \( p = 2 \) at the 5 percent significance level.\footnote{Had this not been the case, we would have had to extend the state space in order to preserve the Markov property of the fiscal policy process.}

Since we have no data on the capital stock, the Solow residual, \( z_t \), cannot be measured. Hence, \( \rho \) and \( \sigma_z^2 \) given in

\[
\ln z_{t+1} = \rho \ln z_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N \left(0, \sigma_z^2\right)
\]  

are estimated by SMM along with the preference and technology parameters of the general equilibrium model.

The following \( \text{VAR}(1) \) model for the fiscal variables government consumption, degree of income tax progressivity, consumption tax and payroll tax is used.

\[
\begin{bmatrix}
\ln g_{t+1} \\
b_{t+1} \\
\tau_{t+1}^c \\
\tau_{t+1}^{\text{w}}
\end{bmatrix}
= \begin{bmatrix} 1 \\ \ln g_t \\ b_t \\ \tau_t^c \\ \tau_t^{\text{w}} \end{bmatrix} + \begin{bmatrix} v \\ \varphi \end{bmatrix}
\xi_{t+1} \sim N \left(0, \Sigma\right).
\]  

(4.2)

Table 1. Estimation of moments and test for goodness of fit.
Set 1: 9 moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>No-Tax Model</th>
<th>Tax Model</th>
<th>Empirical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\ln y_t}$</td>
<td>0.022</td>
<td>0.018</td>
<td>0.016</td>
</tr>
<tr>
<td>$\sigma_{\ln h_t}/\sigma_{\ln y_t}$</td>
<td>0.36</td>
<td>0.55</td>
<td>0.64</td>
</tr>
<tr>
<td>$\sigma_{\ln c_t}/\sigma_{\ln y_t}$</td>
<td>0.79</td>
<td>1.07</td>
<td>1.07</td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \ln h_{t-1})$</td>
<td>0.40</td>
<td>0.52</td>
<td>0.56</td>
</tr>
<tr>
<td>$\text{Corr} (\ln y_t, \ln y_{t-1})$</td>
<td>0.63</td>
<td>0.51</td>
<td>0.45</td>
</tr>
<tr>
<td>$\text{Corr} (\ln c_t, \ln c_{t-1})$</td>
<td>0.77</td>
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<td>0.48</td>
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<tr>
<td>$\text{Corr} (\ln h_t, \ln y_t)$</td>
<td>0.70</td>
<td>0.61</td>
<td>0.52</td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \ln c_t)$</td>
<td>0.43</td>
<td>0.31</td>
<td>0.26</td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \ln w_t)$</td>
<td>0.44</td>
<td>0.07</td>
<td>0.00</td>
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</table>

$\chi^2(2) = 8.02, \quad p\text{-value} = 0.018$
<table>
<thead>
<tr>
<th>Moment</th>
<th>No-Tax Model</th>
<th>Tax Model</th>
<th>Empirical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\ln y_t}$</td>
<td>0.020</td>
<td>0.014</td>
<td>0.016</td>
</tr>
<tr>
<td>$\sigma_{\ln h_t} / \sigma_{\ln y_t}$</td>
<td>0.33</td>
<td>0.82</td>
<td>0.64</td>
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<tr>
<td>$\sigma_{\ln c_t} / \sigma_{\ln y_t}$</td>
<td>0.74</td>
<td>1.09</td>
<td>1.07</td>
</tr>
<tr>
<td>$\sigma_{\ln i_t} / \sigma_{\ln y_t}$</td>
<td>2.92</td>
<td>2.58</td>
<td>2.40</td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \ln h_{t-1})$</td>
<td>0.36</td>
<td>0.50</td>
<td>0.56</td>
</tr>
<tr>
<td>$\text{Corr} (\ln y_t, \ln y_{t-1})$</td>
<td>0.54</td>
<td>0.50</td>
<td>0.45</td>
</tr>
<tr>
<td>$\text{Corr} (\ln c_t, \ln c_{t-1})$</td>
<td>0.69</td>
<td>0.47</td>
<td>0.48</td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \ln y_t)$</td>
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<td>0.80</td>
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<tr>
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<td>0.51</td>
<td>0.26</td>
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<tr>
<td>$\text{Corr} (\ln h_t, \ln w_t)$</td>
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<td>0.00</td>
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<td>$\text{Corr} (\ln i_t, \ln i_{t-1})$</td>
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</tr>
<tr>
<td>$\text{Corr} (\ln w_t, \ln w_{t-1})$</td>
<td>0.67</td>
<td>0.50</td>
<td>0.65</td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \ln i_t)$</td>
<td>0.98</td>
<td>0.37</td>
<td>0.47</td>
</tr>
<tr>
<td>$\text{Corr} (\ln y_t, \ln c_t)$</td>
<td>0.95</td>
<td>0.67</td>
<td>0.54</td>
</tr>
<tr>
<td>$\text{Corr} (\ln y_t, \ln i_t)$</td>
<td>0.90</td>
<td>0.58</td>
<td>0.63</td>
</tr>
<tr>
<td>$\text{Corr} (\ln y_t, \ln w_t)$</td>
<td>0.97</td>
<td>0.57</td>
<td>0.52</td>
</tr>
<tr>
<td>$\text{Corr} (\ln c_t, \ln i_t)$</td>
<td>0.72</td>
<td>0.69</td>
<td>0.33</td>
</tr>
<tr>
<td>$\text{Corr} (\ln c_t, \ln w_t)$</td>
<td>1.00</td>
<td>0.42</td>
<td>0.44</td>
</tr>
<tr>
<td>$\text{Corr} (\ln i_t, \ln w_t)$</td>
<td>0.76</td>
<td>0.46</td>
<td>0.21</td>
</tr>
</tbody>
</table>

$\chi^2(12) = 100.01$ \hspace{1cm} $p$-value $= 0.000$ \hspace{1cm} $16.32$ \hspace{1cm} 0.177
Table 3. Estimation of moments and test for goodness of fit.  
Set 3 and 4: 21 and 31 moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Tax Model</th>
<th>Tax Model</th>
<th>Empirical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\ln y_t}$</td>
<td>0.018</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td>$\sigma_{\ln h_t} / \sigma_{\ln y_t}$</td>
<td>0.61</td>
<td>0.82</td>
<td>0.64</td>
</tr>
<tr>
<td>$\sigma_{\ln c_t} / \sigma_{\ln y_t}$</td>
<td>1.05</td>
<td>0.93</td>
<td>1.07</td>
</tr>
<tr>
<td>$\sigma_{\ln i_t} / \sigma_{\ln y_t}$</td>
<td>2.31</td>
<td>2.40</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \ln h_{t-1})$</td>
<td>0.52</td>
<td>0.51</td>
<td>0.56</td>
</tr>
<tr>
<td>$\text{Corr} (\ln y_t, \ln y_{t-1})$</td>
<td>0.50</td>
<td>0.50</td>
<td>0.45</td>
</tr>
<tr>
<td>$\text{Corr} (\ln c_t, \ln c_{t-1})$</td>
<td>0.48</td>
<td>0.46</td>
<td>0.48</td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \ln y_t)$</td>
<td>0.66</td>
<td>0.83</td>
<td>0.52</td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \ln c_t)$</td>
<td>0.26</td>
<td>0.44</td>
<td>0.26</td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \ln w_t)$</td>
<td>0.07</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>$\text{Corr} (\ln i_t, \ln i_{t-1})$</td>
<td>0.47</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr} (\ln w_t, \ln w_{t-1})$</td>
<td>0.50</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \ln i_t)$</td>
<td>0.63</td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr} (\ln y_t, \ln c_t)$</td>
<td>0.60</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr} (\ln y_t, \ln i_t)$</td>
<td>0.76</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr} (\ln y_t, \ln w_t)$</td>
<td>0.57</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr} (\ln c_t, \ln i_t)$</td>
<td>0.61</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr} (\ln c_t, \ln w_t)$</td>
<td>0.42</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr} (\ln i_t, \ln w_t)$</td>
<td>0.44</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \ln h_t)$</td>
<td>-0.14</td>
<td>-0.09</td>
<td>-0.09</td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \tau_t)$</td>
<td>0.04</td>
<td>-0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>$\text{Corr} (\ln h_t, \tau_{t-1})$</td>
<td>-0.21</td>
<td>-0.26</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\text{Corr} (\ln y_t, \tau_t)$</td>
<td>-0.02</td>
<td>-0.04</td>
<td>-0.18</td>
</tr>
<tr>
<td>$\text{Corr} (\ln y_t, \tau_{t-1})$</td>
<td>0.01</td>
<td>-0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>$\text{Corr} (\ln y_t, \tau_{t+1})$</td>
<td>-0.08</td>
<td>-0.16</td>
<td>0.02</td>
</tr>
<tr>
<td>$\text{Corr} (\ln c_t, \ln h_t)$</td>
<td>0.08</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>$\text{Corr} (\ln c_t, \tau_t)$</td>
<td>-0.26</td>
<td>-0.31</td>
<td>-0.08</td>
</tr>
<tr>
<td>$\text{Corr} (\ln c_t, \tau_{t-1})$</td>
<td>-0.14</td>
<td>-0.16</td>
<td>0.04</td>
</tr>
<tr>
<td>$\text{Corr} (\ln i_t, \ln h_t)$</td>
<td>-0.20</td>
<td>-0.24</td>
<td>0.01</td>
</tr>
<tr>
<td>$\text{Corr} (\ln i_t, \tau_t)$</td>
<td>0.05</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>$\text{Corr} (\ln i_t, \tau_{t-1})$</td>
<td>-0.14</td>
<td>-0.25</td>
<td>-0.10</td>
</tr>
<tr>
<td>$\chi^2(14 \text{ or } 24)$</td>
<td>14.63</td>
<td>60.72</td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>0.404</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Natural logs of per capita variables
Figure 2: Fiscal policy variables
Figure 3: HP-Filtered series, $\lambda = 100$