Seminar Paper No. 615

INFLATION FORECAST TARGETING: IMPLEMENTING AND MONITORING INFLATION TARGETS

by

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Abstract

Inflation targeting is shown to imply inflation forecast targeting: the central bank’s inflation forecast becomes an intermediate target. Inflation forecast targeting simplifies both implementing and monitoring of monetary policy. The inflation forecast is actually an ideal intermediate target: it is most correlated with the goal, easier to control than the goal, more observable than the goal, and very transparent. Money growth targeting generally leads to higher inflation variability than inflation targeting. In the rare special cases when either money growth or the exchange rate is the best intermediate target, inflation forecast targeting automatically implies the relevant intermediate target.

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1 Introduction

In recent years a number of countries (New Zealand, Canada, U.K., Sweden, Finland, Australia and Spain) have instituted explicit inflation targeting. An inflation targeting regime has several characteristics. The crucial one is a quantitative inflation target, typically 2 percent per year. In most cases there is also an explicit tolerance interval around the inflation target, typically ±1 percentage point. Finally, there is no explicit intermediate target, such as a money growth target or an exchange rate target (except for Spain which, as a participant of ERM, also has an exchange rate target). As argued in Leiderman and Svensson (1995, Introduction) the last characteristic is not crucial; (temporary) intermediate targets are not inconsistent with an inflation target, as long as the inflation target has priority if a conflict arises.\footnote{See the papers in Leiderman and Svensson (1995) and Haldane (1995), as well as Ammer and Freeman (1995) and McCallum (1995), for discussion of and details on inflation targeting.}

The purpose of this paper is to examine inflation targeting with regard to potential problems with its implementation by the monetary authority and its monitoring by the public and market agents. Inflation targeting has some obvious general advantages, and some potentially serious problems. The general advantages includes that it focuses monetary policy directly on achieving the goal of low and stable inflation. With a specified quantitative target, it provides an ex post measurement of monetary policy performance, namely realized inflation relative to the inflation target. It also provides measurement of the credibility of monetary policy, in the form of measures of inflation expectations relative to the inflation target. Both these measurements simplify the evaluation of monetary policy and thereby the accountability of monetary policy is increased. By increasing accountability, inflation targeting may serve as a potential commitment mechanism, increasing the likelihood that it will succeed in maintaining low and stable inflation, as well as anchoring and stabilizing inflation expectations.

More specifically, as demonstrated in Svensson (1996a), in a framework where discretionary monetary policy leads to an inflation bias (for instance due to an implicit employment target that exceeds the natural rate of employment), a low inflation target may also reduce or even remove the inflation bias. In some cases it may lead to the same equilibria as the linear inflation contracts proposed by Walsh (1995) and extended by Persson and Tabellini (1993), but be relatively easier to implement. Since a low inflation target need not distort the relative output/inflation variability, ‘inflation-target conservative’ objectives (that is, with a lower inflation target) for the central bank may lead to better equilibria than Rogoff’s (1985) ‘weight conservative’ objectives.
for the central bank (that is, with a higher weight on inflation stabilization).

However, inflation targeting faces some potentially serious problems with regard to both its implementing and its monitoring. First, inflation targeting may be difficult to implement, for the simple reason that central banks have imperfect control over inflation. Current inflation is essentially predetermined by previous decisions and contracts, which means that central banks can only affect future inflation. ‘Long and variable lags’, and variable strength, in the effect of monetary policy on future inflation make decisions on current instrument setting inherently difficult. Inflation is also affected by other factors than monetary policy, in particular disturbances that occur within the ‘control lag’ between the instrument change and the resulting effect on inflation. Second, the imperfect control over inflation makes monitoring and evaluation of monetary policy by the public inherently difficult. For instance, with a control lag of 1.5-2 years, it appears that current monetary policy cannot be evaluated until realized inflation has been observed 1.5-2 years later. However, that realized inflation is the result of several other factors than monetary policy, in particular disturbances that monetary policy cannot respond to due to the control lags. Thus, measuring monetary policy performance is not straightforward. A central bank may argue that a particular deviation of realized inflation from the inflation target is due to factors outside its control, and that it should hence not be held accountable for the deviation.

With implementation, monitoring and evaluation made more difficult, accountability is less improved, and the potential commitment mechanism is correspondingly weakened. Sceptics and critics may argue that the merits of inflation targeting are highly dubious, and that less sophisticated money growth targeting or exchange rate targeting is a safer way to achieve low inflation.2

This paper argues that the potentially serious problems with implementing and monitoring inflation targeting have a simple but powerful solution. Inflation targeting implies inflation forecast targeting: The central bank’s inflation forecast becomes an intermediate target.3 Making

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1 Cm. von Hagen (1996).
2 The idea that long lags imply that forecasts should be targeted rather than current values goes back at least to Gordon (1985) and Hall (1985), and is further discussed with regard to nominal GDP targeting in Hall and Mankiw (1990). With regard to explicit inflation forecast targeting, see King (1994, p. 118): “The use of an inflation target does not mean that there is no intermediate target. Rather, the intermediate target is the expected level of inflation at some future date chosen to allow for the lag between changes in interest rates and the resulting changes in inflation. In practice, we use a forecasting horizon of two years.” See also Bowen (1995, p. 57): “The most appropriate guide to monetary policy [under inflation targeting] is the best obtainable forecast of the probability distribution for inflation, over a time horizon defined by how long it takes for a change in monetary policy to affect inflation. Such a forecast must use information from a wide variety of sources. It can be thought of as an intermediate target: monetary policy is to be adjusted to maximize the probability forecast at the time of the policy adjustment—of inflation falling within the target range by the time the adjustment has taken effect.”
this explicit simplifies both implementation and monitoring of monetary policy. The central bank's inflation forecast is indeed an ideal intermediate target: it is by definition the current variable that is most correlated with the goal, it is more controllable than the goal, and it can be made more observable than the goal. It can also be made very transparent, and may therefore facilitate the central bank's communication with the public, and the public's understanding of monetary policy. In the (rare) special cases when either money growth targeting or exchange rate targeting is the optimal arrangement, inflation targeting will automatically imply that arrangement. Very sophisticated money growth targeting can be made equivalent to inflation targeting, but it is much less transparent, whereas simple money growth targeting is inefficient compared to inflation targeting.

By the central bank's inflation forecast I mean the central bank's own structural forecast, based on its view and model of the fundamental determinants of inflation and the transmission mechanism of monetary policy. Within a discussion of nominal GDP targeting, Hall and Mankiw (1994) have argued that the central bank should target outside forecasters' consensus forecast (of nominal GDP) rather than its own structural forecast. Woodford (1994) has shown, however, that targeting other forecasters' forecasts is problematic, if these forecasters incorporate in their forecasting procedure the central bank's feedback rule from their forecasts. Instability, multiple equilibria, or even non-existence of equilibria may result. These problems are avoided if the central bank targets its own structural forecast.

The paper's outline is as follows. Section 2 discusses the implementation of inflation targeting and demonstrates, with the help of a very simple model, that inflation targeting implies inflation forecast targeting. Section 3 discusses public monitoring and evaluation of inflation targeting. Section 3 shows that the inflation forecast is indeed an ideal intermediate target. Section 5 discusses the relation of inflation targeting to money targeting. Section 6 examines some extensions and complications. Section 7 concludes. Appendices A and B contain some technical points.

2 Implementing inflation targeting: Inflation forecast targeting

This section argues that the solution to the potential problem in implementing inflation targeting consists of making the central bank's inflation forecast an explicit intermediate target. Although this is a very straightforward result that hardly requires a model, I believe that it is best demonstrated with the help of a very simple model. Although the result can be demonstrated
in a much more elaborate model with an explicit role for agents’ expectations, it is sufficient to use a much simpler one in this case. The model nevertheless has some structural similarity to more elaborate models used by some central banks.\footnote{I believe these issues on implementation and monitoring inflation targeting can be discussed without necessarily assuming the systematic discretionary inflation bias (due to ‘time-consistency’ problems) emphasized in the modern ‘principal-agent’ approach to central banking (for instance in the work by Barro and Gordon (1983), Rogoff (1985), Cukierman (1992), Walsh (1995b), Persson and Tabellini (1993) and Svensson (1996a), and disputed in the ‘traditional’ approach (for instance in McCallum (1995) and Romer and Romer (1999)); see Tabellini (1995) for discussion of these approaches. Therefore the model here does not include any source of discretionary inflation bias, although this can easily be added without affecting the results.}

Consider therefore the model

\begin{alignat}{2}
\pi_{t+1} &= \pi_t + \alpha_1 y_t + \alpha_2 x_t + \epsilon_{t+1} \quad (2.1) \\
y_{t+1} &= \beta_1 y_t - \beta_2 (i_t - \pi_t) + \beta_3 x_t + \eta_{t+1} \quad (2.2) \\
x_{t+1} &= \gamma x_t + \theta_{t+1}, \quad (2.3)
\end{alignat}

where $\pi_t = p_t - p_{t-1}$ is the inflation (rate) in year $t$, $p_t$ is the (log) price level, $y_t$ is an endogenous variable (log) output (relative to potential output), say), $x_t$ is an exogenous variable, $i_t$ is the monetary policy instrument (the repo rate, say), and $\epsilon_t$, $\eta_t$ and $\theta_t$ are i.i.d. shocks in year $t$ that are not known in year $t - 1$. The coefficients $\alpha_1$ and $\beta_2$ are assumed to be positive; the other coefficients are assumed to be nonnegative; $\beta_1$ and $\gamma$ in addition fulfill $\beta_1 < 1$, $\gamma < 1$. The change in inflation is increasing in lagged output and the lagged exogenous variable. Output is serially correlated, decreasing in the lagged ‘real’ repo rate, $i_t - \pi_t$,\footnote{With this somewhat imprecise terminology, $i_t$ is interpreted as a repo rate with a short maturity (2 weeks, say) and the current 1-year inflation rate is considered a proxy for the expected 2-week inflation rate.} and increasing in the lagged exogenous variable. The long-run natural output level is normalized to equal zero. The repo rate affects output with a one-year lag, and hence inflation with a two-year lag, the control lag in the model. That the instrument affects inflation with a longer lag than it affects output is the crucial property of the model. It is consistent with results from a number of VAR-studies.\footnotemark[6]
Suppose monetary policy is conducted by a central bank with an inflation target $\pi^*$ (say 2 percent per year). Interpret inflation targeting as implying that the central bank’s objective in period $t$ is to choose a sequence of current and future repo rates $\{i_{r}\}_{\tau=t}^{\infty}$ so as to minimize

$$E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L(\pi_{\tau}),$$

(2.4)

where $E_t$ denotes expectations conditional upon (the central bank’s) information available in year $t$, the discount factor $\delta$ fulfills $0 < \delta < 1$, and the period loss function $L(\pi_{\tau})$ is

$$L(\pi_{\tau}) = \frac{1}{2} (\pi_{\tau} - \pi^*)^2.$$

(2.5)

That is, the central bank wishes to minimize the expected sum of discounted squared future deviations of inflation from the target.\(^7\)

It is crucial here that inflation targeting is interpreted as implying a single objective; that the inflation rate is the only variable in the period loss function (2.5). Svensson (1996a) has argued that inflation targeting may in practice be interpreted by central banks as involving additional objectives for output or employment. The consequences of that interpretation are discussed in section 6.

Since the repo rate affects inflation with a two-year lag, it is practical to express $\pi_{t+2}$ in terms of year $t$ variables and $t+1$ and $t+2$ disturbances:

$$\pi_{t+2} = (\pi_t + \alpha_1 y_t + \alpha_2 x_t + \epsilon_{t+1}) + \alpha_1 \left[\beta_1 y_t - \beta_2 i_t + \beta_3 \pi_t + \gamma_{t+1}\right] + \alpha_2 (\gamma x_t + \theta_{t+1}) + \epsilon_{t+2}$$

$$= a_1 \pi_t + a_2 y_t + a_3 x_t - a_4 i_t + \epsilon_{t+1} + \alpha_1 \eta_{t+1} + \alpha_2 \theta_{t+1} + \epsilon_{t+2},$$

(2.6)

where

$$a_1 = 1 + \alpha_1 \beta_2, \quad a_2 = \alpha_1 (1 + \beta_1), \quad a_3 = \alpha_1 \beta_3 + \alpha_2 (1 + \gamma) \text{ and } a_4 = \alpha_1 \beta_2.$$

(2.7)

Since in this simple case the repo rate in year $t$ will not affect the inflation rate in year $t$ and $t+1$, but only in year $t+2$, $t+3$, etc., and the repo rate in year $t+1$ will only affect the inflation rate in year $t+3$, $t+4$, etc., we realize that we can find the solution to the optimization problem by assigning the repo rate in year $t$ to the inflation target for year $t+2$, the repo rate in year

\(^7\) Such policies will never be optimal with the loss functions to be used in this paper. If such policies were attempted, the presumption is that the model would break down.

\(7\) Since the central bank does not have perfect control over inflation it is not meaningful to minimize the realized squared deviations, only the expected squared deviations (conditional upon the information available when the repo rate is set).
$t + 1$ to the inflation target for year $t + 3$, etc. Thus, the central bank can find the optimal repo rate in year $t$ as the solution to the simple period-by-period problem

$$\min_{i_t} E_t \delta^2 L(\pi_{t+2})$$

(2.8)

(see appendix A for details).

The first-order condition for minimizing (2.8) with respect to $i_t$ is

$$\frac{\partial E_t \delta^2 L(\pi_{t+2})}{\partial i_t} = E_t \left[ \delta^2 (\pi_{t+2} - \pi^*) \frac{\partial \pi_{t+2}}{\partial i_t} \right] = -\delta^2 a_4 \left( \pi_{t+2|t} - \pi^* \right) = 0,$$

where $\pi_{t+2|t}$ denotes $E_t \pi_{t+2}$, and where I have used that by (2.6) $\frac{\partial \pi_{t+2}}{\partial i_t} = -a_4$. It follows that the first-order condition can be written

$$\pi_{t+2|t} = \pi^*.$$  

(2.9)

That is, the repo rate in year $t$ should be set so that the forecast of the one-year ‘forward’ inflation rate from year $t + 1$ to year $t + 2$, conditional upon information available in year $t$, equals the inflation target. Although a more precise terminology for this forecast would be the one-to-two-year forecast, I shall for simplicitly call it the two-year forecast. (It should not be confused with the forecast of the average inflation rate between year $t$ and year $t + 2$.) Thus, the two-year inflation forecast can be considered an explicit intermediate target.\(^8\)

It follows that the inflation targeting loss function (2.5) can be replaced by an intermediate loss function $L^i(\pi_{t+2|t})$, the inflation forecast targeting loss function

$$L^i(\pi_{t+2|t}) = \frac{1}{2} \left( \pi_{t+2|t} - \pi^* \right)^2.$$  

(2.10)

Instead of minimizing the expected squared deviations of the future two-year inflation rate $\pi_{t+2}$ from the inflation target as in (2.8), the central bank can minimize the squared deviation of the current two-year inflation forecast $\pi_{t+2|t}$ from the inflation target,

$$\min_{i_t} L^i(\pi_{t+2|t}).$$  

(2.11)

Since the first-order condition is the same, (2.9), the same optimal repo rate results.

The two-year inflation forecast is by (2.6)

$$\pi_{t+2|t} = a_1 \pi_t + a_2 y_t + a_3 x_t - a_4 i_t.$$  

(2.12)

\(^8\) An alternative objective function for an inflation targeting regime is to maximize the probability that future inflation falls within a symmetric band around the inflation target. With a symmetric probability distribution for future inflation, which is the case in the model used here, this results in the same intermediate target (2.9).
Setting this equal to the inflation target, (2.9), leads to the central bank’s optimal reaction function,

\[ i_t = \frac{1}{a_4} (-\pi^* + a_1\pi_t + a_2y_t + a_3x_t) \]
\[ = \pi_t + b_1(\pi_t - \pi^*) + b_2y_t + b_3x_t, \]  
\[ (2.13) \]

where I have used (2.7) and

\[ b_1 = \frac{1}{\alpha_1\beta_2}, \quad b_2 = \frac{1 + \beta_1}{\beta_2} \quad \text{and} \quad b_3 = \frac{\alpha_1\beta_3 + \alpha_2(1 + \gamma)}{\alpha_1\beta_2}. \]
\[ (2.14) \]

This reaction function is of the same form as the Taylor rule (1993), except that it also depends on the exogenous variable. The real repo rate \( i_t - \pi_t \) is increasing in the excess of current inflation over the inflation target, in current output, and in the current exogenous variable. The instrument depends on current inflation, not because current inflation is targeted (current inflation is predetermined) but because current inflation together with output and the exogenous variable predict future inflation.

With this reaction function the two-year inflation forecast will equal the inflation target, for all values of \( \pi_t, y_t \) and \( x_t \). If the inflation forecast exceeds (falls short of) the inflation target, the repo rate should be increased (decreased) until the inflation forecast equals the target. If the current inflation rate increases, output increases, or the exogenous variable increases, the repo rate should be increased, in order to keep the inflation forecast equal to the inflation target.

Actual inflation in year \( t + 2 \) will in equilibrium be

\[ \pi_{t+2} = \pi_{t+2|t} + \epsilon_{t+1} + \alpha_1\eta_{t+1} + \alpha_2\theta_{t+1} + \epsilon_{t+2} \]
\[ = \pi^* + \epsilon_{t+1} + \alpha_1\eta_{t+1} + \alpha_2\theta_{t+1} + \epsilon_{t+2}. \]  
\[ (2.15) \]

It will deviate from the inflation target and the two-year inflation forecast by the forecast error,

\[ \pi_{t+2} - \pi_{t+2|t} = \epsilon_{t+1} + \alpha_1\eta_{t+1} + \alpha_2\theta_{t+1} + \epsilon_{t+2}, \]
\[ (2.16) \]
due to the disturbances that occur within the control lag, after the central bank has set the instrument.

Clearly the central bank cannot prevent the deviations from the inflation target that are caused by disturbances occurring within the control lag. At best it can only control the deviations of the two-year forecast from the target. It can therefore be argued that the central bank should be held accountable for the forecast deviations from the target rather than the realized inflation deviations, if the forecast deviations can be observed.
Equilibrium output will by (2.1) and (2.15) be given by

$$
y_{t+1} = \frac{\pi_{t+2} - \pi_{t+1} - \alpha_2 x_{t+1} - \varepsilon_{t+2}}{\alpha_1}
= \frac{\epsilon_{t+1} + \alpha_1 \eta_{t+1} + \alpha_2 \theta_{t+1} + \varepsilon_{t+2}}{\alpha_1}
= \frac{\epsilon_t + \alpha_1 \eta_t + \alpha_2 \theta_t + \alpha_2 (\gamma x_t + \theta_{t+1}) + \varepsilon_{t+1} + \varepsilon_{t+2}}{\alpha_1}
= -\frac{\alpha_2}{\alpha_1} \gamma x_t - \frac{1}{\alpha_1} \epsilon_t - \eta_t - \frac{\alpha_2}{\alpha_1} \theta_t + \eta_{t+1}.
$$

(2.17)

To generalize from this example, inflation targeting implies a simple rule for its implementation. The central bank’s inflation forecast for the horizon corresponding to the control lag (2 years in the example) becomes an intermediate target, and the instrument should hence be set so as to make the inflation forecast equal to the inflation target. Thus, if the inflation forecast is above (below) the target, the repo rate should be increased (decreased). This simple rule results in the optimal reaction function for the central bank. Since the inflation forecast depends on all relevant information, the instrument will be a function of all relevant information.

Adjusting the instrument so the inflation forecast equals the target is the best the central bank can do. Ex post inflation will differ from the target, because of forecast and control errors, for instance due to disturbances that occur within the control lag. If the central bank is competent, the mean forecast errors will be zero, and the variance of the forecast errors minimized. The central bank should be accountable for deviations of the inflation forecast from the target, but not for the unavoidable deviations of realized inflation from the target.

Note that the inflation forecast is a \textit{structural} forecast, based on a structural model of the fundamental determinants of inflation and of the monetary policy transmission mechanism. As mentioned in the introduction, the problem Woodford (1994) has raised regarding central bank targeting of outside forecasters’ consensus forecast is then avoided.

The central bank’s inflation forecast will in practice have to combine both formal and informal components, for instance with judgemental adjustments of more formal structural forecasts. Forecasts will hardly ever be purely mechanical. This view is strengthened by the results of Cecchetti (1995), who has examined mechanical reduced-form inflation forecasts for the United States, with rather negative results. Forecast errors are sizeable, and there are frequent structural shifts in the forecast equations. However, forecast errors for one-year inflation rates, for instance for the one-to-two-year inflation rate emphasized in the model used here, are smaller than for one-quarter inflation rates. As emphasized by Kohn (1995), more structural modeling and use
of extramodal information and judgment by forecasters are likely to produce forecasts with acceptable precision. In addition, forecasting inflation is likely to be easier in a situation when the central bank actively pursues inflation targeting and, importantly, the public expects the central bank to pursue inflation targeting so that inflation expectations are stabilized.

3 Monitoring inflation targeting

In the model used above, there is no specific need to monitor monetary policy in order to ensure that the central bank implements inflation targeting. If the central bank has the preferences described by (2.4) and (2.5), it will behave according to the optimal reaction function (2.13) with or without any monitoring by outsiders. Let me now consider a simple modification of the setup which results in a need for outside monitoring.

Consider the inflation target $\pi^*$ in (2.5) as the 'official' explicit inflation target, assigned to the central bank by society. Suppose, however, that the central bank has its own implicit inflation target that may deviate from that assigned by society. More specifically, assume that the central bank has an intertemporal loss function of the form (2.4) with the same discount factor $\delta$ but a time-varying period loss function $L_t^b(\pi)$ given by

$$L_t^b(\pi_t) = \frac{1}{2} (\pi_t - \pi_t^b)^2$$

$$\pi_t^b = \pi^* + z_t$$

$$z_{t+1} = (1 - \rho)z + \rho z_t + \xi_{t+1},$$

where the central bank's implicit inflation target $\pi_t^b$ deviates from the explicit one, the deviation $z_t$ follows an AR(1) process, the unconditional mean $\bar{z}$ is constant, $|\rho| < 1$ and $\xi_t$ is i.i.d.

The central bank's decision problem then becomes

$$\min_{\pi_t} E_t^b \delta^2 L_{t+2}^b(\pi_{t+2}).$$

(3.4)

The first-order condition is

$$\frac{\partial E_t^b \delta^2 L_{t+2}^b(\pi_{t+2})}{\partial \pi_t} = -\delta^2 a_4 (\pi_{t+2|t} - \pi^* - z_{t+2|t}) = -\delta^2 a_4 \left[ \pi_{t+2|t} - \pi^* - (1 - \rho^2)\bar{z} - \rho^2 z_t \right] = 0.$$

Thus, the first-order condition can be written

$$\pi_{t+2|t} = \pi^* + (1 - \rho^2)\bar{z} + \rho^2 z_t.$$

(3.5)
The corresponding reaction function will be

\[ i_t = \pi_t + b_1 \left[ \pi_t - \pi^* - (1 - \rho^2) \bar{z} - \rho^2 z_t \right] + b_2 y_t + b_3 x_t. \]  

(3.6)

where the \( b \) coefficients are given by (2.14).

Equilibrium inflation in year \( t + 2 \) will be

\[ \pi_{t+2} = \pi^* + (1 - \rho^2) \bar{z} + \rho^2 z_t + \epsilon_{t+1} + \alpha_1 \eta_{t+1} + \alpha_2 \theta_{t+1} + \epsilon_{t+2}, \]  

(3.7)

and equilibrium output will fulfill

\[ y_{t+1} = \frac{\pi_{t+2} - \pi_{t+1} - \alpha_2 x_{t+1} - \epsilon_{t+2}}{\alpha_1} \]

\[ = \rho^2 (z_t - z_{t-1}) - \frac{\alpha_2}{\alpha_1} \gamma x_t - \frac{1}{\alpha_1} \epsilon_t - \eta_t - \frac{\alpha_2}{\alpha_1} \theta_t + \eta_{t+1}. \]  

(3.8)

Thus, if the central bank’s implicit inflation target deviates from the explicit one by \( z_t \) in year \( t \), the central bank will choose the repo rate so as to set its two-year inflation forecast above the explicit inflation target by \((1 - \rho^2) \bar{z} + \rho^2 z_t\), the expected \( t + 2 \) inflation target deviation. Compared to the situation when the central bank shares society’s inflation target, the equilibrium inflation in year \( t + 2 \) will deviate by that same amount, and for given \( \pi_t, y_t \) and \( x_t \), the repo rate in year \( t \) will be lower by \( b_1 \left[(1 - \rho^2) \bar{z} + \rho^2 z_t \right] \).

Can public monitoring of the central bank prevent these deviations? Suppose the public cannot directly observe the central bank’s implicit inflation target \( \pi^b_t \), so that the latter is private information to the central bank. Assume in the simplest case that the public has the same information about the model (2.1)-(2.3) as the central bank, and that the public observes \( \pi_t, y_t, x_t \) and \( i_t \) in year \( t \) (and hence can extract the disturbances \( \epsilon_t, \eta_t \) and \( \theta_t \)). Even though the public does not directly observe the central bank’s inflation target \( \pi^b_t \), it can infer the relevant deviation \((1 - \rho^2) \bar{z} + \rho^2 z_t\), either from comparing the current instrument with that corresponding to the optimal reaction function (2.13), or by using (2.12) to form an inflation forecast \( \pi_{t+2|t} \) and observe its deviation from the explicit inflation target. (Note that the public need not know the stochastic process (3.3) for the central bank’s deviations from the inflation target.)\(^9\)

Thus, the public can spot deviations of the inflation forecast from the explicit inflation target, and by criticizing the central bank for such deviations reduce or even eliminate such deviations. More specifically, consider such public criticism as equivalent to giving the central

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\(^9\) Cf. Cukierman and Meltzer (1986) and Faust and Svensson (1996) for analysis of situations when the central bank preferences cannot be perfectly inferred but instead are estimated by the public with a Kalman filter.
bank an additional loss in year $t$ equal to

$$\varphi L_t^i(\pi_{t+2|t}),$$

where $L_t^i(\pi_{t+2|t})$ is given by (2.10) and the parameter $\varphi > 0$ measures the intensity of the criticism. Consider further the central bank's behavior in the face of such monitoring as minimizing in year $t$ the total loss

$$E_t \delta^2 L_{t+2}^b(\pi_{t+2}) + \varphi L_t^i(\pi_{t+2|t}).$$

The first-order condition with respect to $i_t$ will be

$$\delta^2 \left[ \pi_{t+2|t} - \pi^* - (1 - \rho^2) \bar{z} - \rho^2 z_t \right] + \varphi \left( \pi_{t+2|t} - \pi^* \right) = 0,$$

hence

$$\pi_{t+2|t} = \pi^* + \frac{(1 - \rho^2) \bar{z} + \rho^2 z_t}{1 + \varphi/\delta^2}.$$

By intensive criticism, that is, a large $\varphi$, the public can enforce that the central bank's inflation forecast is close to the explicit inflation target.

In the real world, how can the public monitor and evaluate monetary policy with an inflation target? How can the central bank's inflation forecast become observable by the public, so the public can detect deviations from the explicit inflation target? The best way to make the central bank's inflation forecast observable by the public and to allow the most thorough monitoring of monetary policy, I believe, is for the central bank to reveal the details of its forecast to the public. This involves revealing the central bank's model, information and assumptions, in order to allow public scrutiny and discussion of these, including comparison with outsiders' forecasts and analysis. In terms of the model used above, this involves revealing the model (2.1)-(2.3) and its coefficients to the public, as well as the central bank's information about the current state of the economy. Full revelation and public scrutiny is likely to provide the best incentive for high-quality analysis and forecasting by the central bank and to minimize the risk of self-serving bias in the central bank's forecast. An example of this is the increasing occurrence, and increasing quality, of Inflation Reports by inflation targeting central banks, although a fair amount of detail in analysis and assumptions is still kept secret.\(^{10}\)

\(^{10}\)To the extent that the central bank has objectives that deviate from the official ones, it may have an incentive to misrepresent its model and information (in addition to its objectives). The central bank's incentives to misrepresent the truth and mechanism design to ensure truth-telling is an increasingly relevant subject for future research. See Persson and Tabellini (1993) and Walsh (1995a) for examples of incentive schemes that induce the central bank to reveal the truth.
Central banks have a strong tradition of secrecy (mostly for no good reasons, I believe). If an inflation targeting central bank keeps essential components of its inflation forecast secret and thus prevents public observation and scrutiny of it, there is still ample opportunities to monitor the inflation targeting. Sophisticated observers of monetary policy can, and certainly will, publish their own inflation forecasts and scrutinize monetary policy with the help of these. Less sophisticated observers can always obtain publicly available inflation forecasts by reputable forecasters, for instance in the convenient form of “Consensus Forecasts” already made available by specialized publishers. Such forecasts are frequently published and updated with new information, allowing continuous observation of outsiders’ inflation forecasts, even if the central bank is secretive about its forecast.

Central banks can be legally obliged to provide information to the public. It is also possible for governments to create an independent body, separate from the central bank, that monitors monetary policy. This may be a possibility that has received insufficient attention in discussions of central bank reform.11

Thus, outsiders have ample opportunities to monitor and evaluate the central bank’s policy, either with the central bank’s own analysis and forecast available, or with that of outside forecasters and analysts. In the simplest form, monitoring inflation targeting then consists of observing whether available inflation forecasts are on target or whether they systematically exceed or fall short of the target, in which case the direction (although not the magnitude) of the warranted correction of monetary policy is obvious, since the principles of inflation targeting monetary policy are so simple and transparent.

Normally the central bank and sophisticated outside observers should have approximately the same information about the state of the economy and approximately the same model. There is no reason for systematic biases in information or models between the central bank and these sophisticated observers.12 From this point of view, the example above may be rather realistic. The central bank has a distinct information advantage, though, with regard to the planned future path for the instrument, especially if this is related to implicit monetary policy objectives that deviate from the official ones. The current instrument setting is observable to the public, but the central bank’s plan for future instrument levels is not. Inflation forecasts for longer horizons than

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11 Bank of England’s Inflation Report is arguably the best current monetary policy report by any central bank. It has sometimes been suggested that this may partly be related to the fact that the Government rather than the Bank controls monetary policy in the U.K., and that there is a risk that the Report might be more self-serving if the Bank itself would control monetary policy.

12 For support of this view from inside Bank of England, see Briault, Haldane and King (1995).
the control lag (that is, horizons longer than 2 years in the example above) will be contingent on expected future instrument settings. This means that there could be systematic differences between the central bank’s and outsiders inflation forecasts for longer horizons, depending upon differences between the instrument plan of the central bank and the instrument expectations of the outsiders. For instance, if the central bank longer term inflation forecast is below outsiders’ forecasts, this should correspond to a situation when the central bank plans a less expansionary monetary policy than expected by the outsiders, which in turn should correspond to the central bank having a lower implicit inflation target than the public believes.

The public’s consensus expectations about the future repo rate can be inferred from the implicit forward interest rate curve that can be estimated from money market yield curves or directly observed on the futures interest rate market. Thus, the central bank can easily compare its repo rate plan to the forward rate curve for systematic discrepancies. Such discrepancies, along with corresponding discrepancies in inflation forecast, is a symptom of credibility problems, in the sense that the implicit objectives of the central bank deviates from what the public’s estimate of these objectives. One possible remedy to such credibility problems is more revelation of central bank plans and analysis. If the public’s expectations about the future repo rate coincide with the central bank’s plan for the instrument, but the public’s inflation forecasts differ from the central bank’s, this is an indication of differences in models or information between the public and the bank. More revelation by the central bank about its models and information may also be a remedy to that situation.

Ideally, the central bank’s implicit objectives coincide with the explicit inflation target, and the public understands the central bank’s implicit reaction function and has similar models and information as the bank has. Then the bank’s instrument plan would be consistent with the forward rate curve and the public’s and the bank’s inflation forecasts should be similar and equal to the explicit inflation target, both for the horizon corresponding to the control lag and for longer horizons.

More sophisticated evaluation of monetary policy would examine and compare the ex post forecast errors of the central bank and outside forecasts with respect to bias and variance. This requires more than the current few years of data from the inflation targeting regimes, though.

The transparency of inflation forecast targeting might help improving the sometimes deficient state of current monetary policy debate in the media in inflation targeting countries (the debate

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for instance frequently includes requests for lower interest rates without reference to inflation forecasts, sometimes when inflation forecasts clearly exceed targets, cf. the discussion in King (1996)). Perhaps it would then be more natural for debaters to specify whether they share or have different targets, forecasts, estimates of instrument effects and control lags, etc.

4 An ideal intermediate target

A good intermediate target for monetary policy is highly correlated with the goal, easier to control by the central bank than the goal, easier to observe by both the central bank and the public than the goal, and transparent so that central bank communication with the public and public understanding and prediction of monetary policy is facilitated (cf. Brunner and Meltzer (1967), Friedman (1990) and McCallum (1990)). From this perspective, the central bank’s inflation forecast appears to be an ideal intermediate target.

First, the inflation forecast $\pi_{t+2|t}$ is by definition the year $t$ variable that has the highest correlation with $t + 2$ inflation, since it minimizes the variance of forecast errors and by (2.12) uses all the relevant information in $\pi_t$, $y_t$ and $x_t$, rather than an arbitrary subset of the available information.

Second, by definition the inflation forecast $\pi_{t+2|t}$ is more controllable than inflation $\pi_{t+2}$ itself. The effect of the instrument on the inflation forecast is the same as the effect on mean inflation, and the variance of the inflation forecast is less than that of inflation, since the forecast errors (2.16) are subtracted.

Third, the inflation forecast is easier to observe by the central bank than inflation. The forecast $\pi_{t+1|t}$ is (continuously) observable by the central bank in year $t$, since it depends on year $t$ information; it is not necessary to wait until year $t + 2$ to observe realized inflation. Also, realized inflation is affected by additional disturbances. As argued in section 3, the inflation forecast can also be made observable by the public, either because the central bank reveals its forecast to the public, or because outside forecasters’ inflation forecasts are easily accessible. This facilitates outside monitoring of the central bank.

Fourth, inflation forecast targeting is very transparent. Although the construction of the forecast is difficult and resource-demanding, the monetary policy conclusions from a given inflation forecast are straightforward: If the forecast is above (below) the target, monetary policy should be adjusted in a contractionary (expansionary) direction. If the forecast is on target, monetary policy is appropriate. I cannot imagine simpler principles, and I cannot imagine any-
thing easier to explain to the public, or anything more conducive to public understanding of monetary policy.

Inflation forecast targeting also makes monetary policy easier to predict. Predicting monetary policy becomes equivalent to predicting future inflation, which implies that the information relevant for predicting monetary policy is precisely the information relevant to predicting inflation.

The transparency of inflation forecast targeting also focuses, simplifies and motivates the work inside the central bank. It provides strong incentives to improve the central bank’s understanding and structural models of the economy, to hire competent staff, and to provide high-quality analysis, especially if the central bank chooses or is required to make its model, analysis and forecast public. It helps to clarify for what the central bank can, and cannot, be accountable.

5 Money growth targeting

Inflation forecast targeting generally uses all relevant information for predicting future inflation. This information may include some measure of the money stock, but normally also other macro variables. In the (rare) special case when future inflation is best predicted by just the growth rate of some money aggregate, that is, money growth is a sufficient statistic for future inflation, inflation forecast targeting will be equivalent to money growth targeting. Similarly, if future inflation for a small open economy is best predicted only by the rate of exchange rate depreciation, inflation targeting will be equivalent to exchange rate (depreciation) targeting. But normally money growth or exchange rate depreciation are not sufficient statistics for future inflation; that is, other information has additional predictive value. Then money growth targeting or exchange rate targeting is inefficient and leads to a worse outcome than inflation forecast targeting.\(^\text{14}\)

To illustrate this within the above model, add the following money demand function:

\[ m_{t+1} - p_{t+1} = y_{t+1} - \kappa i_t + \nu_{t+1}, \]

where \( m_t \) is (the log of) some monetary aggregate (M3, say), the income velocity for simplicity is unity, the coefficient \( \kappa \) is positive, the repo rate affects money demand with a lag, and \( \nu_{t+1} \) is an i.i.d. disturbance.

\(^{14}\) See Friedman (1990, 1995) for a more general discussion of money growth targeting.
This formulation takes into the account the fact that the monetary aggregate cannot be an instrument of the central bank, in the sense that the central bank does not have perfect control of it. The broader the aggregate, the less control has the central bank. It can even be disputed that such a narrow an aggregate as the monetary base is under complete control of the central bank, cf. Goodhart (1994). In (5.1) above, the central bank can affect the monetary aggregate by affecting the money demand, via the direct lagged effect on money demand of the instrument, the repo rate, and via the indirect effect of the instrument on aggregate demand for output. The supply of money then adjusts to money demand by endogenous adjustment of the monetary base. (The price level in (5.1) is predetermined by (2.1).)

First-difference (5.1), which gives

\[ \mu_{t+1} = \pi_{t+1} + y_{t+1} - y_t - \kappa i_t + \kappa i_{t-1} + \nu_{t+1} - \nu_t, \]  

(5.2)

where \( \mu_{t+1} = m_{t+1} - m_t \) denotes (the) money growth (rate).

Since the repo rate affects money growth with a one-year lag, rewrite \( \mu_{t+1} \) in terms of year \( t \) variables and \( t + 1 \) disturbances:

\[
\begin{align*}
\mu_{t+1} &= \pi_{t+1} + y_{t+1} - y_t - \kappa i_t + \kappa i_{t-1} + \nu_{t+1} - \nu_t \\
&= (\pi_t + \alpha_1 y_t + \alpha_2 x_t + \epsilon_{t+1}) + \\
&\quad (\beta_1 y_t - \beta_2 \pi_t + \beta_3 x_t + \eta_{t+1}) - y_t - \kappa i_t + \kappa i_{t-1} + \nu_{t+1} - \nu_t \\
&= d_1 \pi_t + d_2 y_t + d_3 x_t - d_4 i_t + \kappa i_{t-1} - \nu_t + (\epsilon_{t+1} + \eta_{t+1} + \nu_{t+1}),
\end{align*}
\]  

(5.3)

where

\[ d_1 = 1 + \beta_2, \quad d_2 = \alpha_1 + \beta_1 - 1, \quad d_3 = \alpha_2 + \beta_3, \quad \text{and} \quad d_4 = \beta_2 + \kappa. \]  

(5.4)

The one-year money growth forecast is hence

\[ \mu_{t+1|t} = d_1 \pi_t + d_2 y_t + d_3 x_t - d_4 i_t + \kappa i_{t-1} - \nu_t. \]  

(5.5)

Eliminate \( i_t \) between (5.5) and (2.12), and express the two-year inflation forecast in terms of year \( t \) variables (aside from the repo rate) and the one-year money growth forecast,

\[
\begin{align*}
\pi_{t+2|t} &= a_1 \pi_t + a_2 y_t + a_3 x_t - \frac{a_4}{d_4} \left( -\mu_{t+1|t} + d_1 \pi_t + d_2 y_t + d_3 x_t + \kappa i_{t-1} - \nu_t \right) \\
&= f_1 \pi_t + f_2 y_t + f_3 x_t - f_4 i_t - f_5 \nu_t + f_6 \mu_{t+1|t},
\end{align*}
\]  

(5.6)

where

\[ f_1 = a_1 - \frac{a_4 d_1}{d_4}, \quad f_2 = a_2 - \frac{a_4 d_2}{d_4}, \quad f_3 = a_3 - \frac{a_4 d_3}{d_4}, \quad f_4 = \frac{a_4 \kappa}{d_4} \quad \text{and} \quad f_5 = \frac{a_4}{d_4}. \]  

(5.7)
Let $\mu_{t+1|t}^*$ denote the one-year money growth forecast that makes the two-year inflation forecast equal to the inflation target and hence fulfills,

$$\pi^* = f_1\pi_t + f_2 y_t + f_3 x_t - f_4 i_{t-1} + f_5 \nu_t + f_5 \mu_{t+1|t}^*.$$

This results in

$$\mu_{t+1|t}^* = \frac{1}{f_5} \left( \pi^* - f_1 \pi_t - f_2 y_t - f_3 x_t + f_4 i_{t-1} - f_5 \nu_t \right) = \pi^* - g_1 (\pi_t - \pi^*) - g_2 y_t - g_3 x_t + \kappa (i_{t-1} - \pi^*) - \nu_t,$$

where

$$g_1 = \frac{f_1}{f_5}, \quad g_2 = \frac{f_2}{f_5} \quad \text{and} \quad g_3 = \frac{f_3}{f_5}. \quad (5.9)$$

It follows that we can interpret $\mu_{t+1|t}^*$ as a conditional one-year money growth target that depends on the information available in year $t$, in this case on $\pi_t, y_t, x_t, i_{t-1}$ and $\nu_t$. The repo rate $i_t$ should then be chosen so as to minimize

$$E_t \frac{1}{2} \left( \mu_{t+1} - \mu_{t+1|t}^* \right)^2$$

(subject to (5.3)), or, equivalently, chosen so as to fulfill the first-order condition that the one-year money growth forecast equals the money growth target,

$$\mu_{t+1|t} = \mu_{t+1|t}^*. \quad (5.10)$$

Fulfilling (5.10) will imply the reaction function (2.13) and is equivalent to fulfilling (2.9).

Note that money growth targeting implies money growth forecast targeting, for the simple reason that money growth reacts with a lag to the instrument and is imperfectly controlled.

We can also consider an unconditional money growth target, $\mu^*$, that is constant over time. We realize from (5.2) and (5.8) that the unconditional money growth target must equal the inflation target,

$$\mu^* = \pi^*, \quad (5.11)$$

in order to cause average inflation to be equal to the target.

Suppose the repo rate is set so as to fulfill the unconditional money growth target,

$$\pi^* = \mu_{t+1|t} = d_1 \pi_t + d_2 y_t + d_3 x_t - d_4 i_t + \kappa i_{t-1} - \nu_t.$$ 

This results in the reaction function

$$i_t = \frac{1}{d_4} \left[ -\pi^* + d_1 \pi_t + d_2 y_t + d_3 x_t + \kappa i_{t-1} - \nu_t \right]$$

$$= \pi_t + h_1 (\pi_t - \pi^*) + h_2 y_t + h_3 x_t + h_4 (i_{t-1} - \pi_t) - h_1 \nu_t, \quad (5.12)$$
where
\[ h_1 = \frac{1}{\beta_2 + \kappa}, \quad h_2 = \frac{\alpha_1 + \beta_1 - 1}{\beta_2 + \kappa}, \quad h_3 = \frac{\alpha_2 + \beta_2}{\beta_2 + \kappa} \quad \text{and} \quad h_4 = \frac{\kappa}{\beta_2 + \kappa}. \] (5.13)

This reaction function should be compared with the optimal reaction function (2.13). It will result in a different equilibrium, with average inflation equal to \( \pi^* \), but with more variability of inflation. Due to the persistence of both output and the exogenous variable, there will be persistent deviations of both realized inflation and conditional inflation expectations from the inflation target. The equilibrium will be inefficient, since the intertemporal loss (2.5) will be higher.

Thus, although the sophisticated conditional money growth targeting (5.10) can achieve the same equilibrium as the optimal reaction function (2.13), it is less direct and less transparent. Its role is only to induce the correct reaction function (2.13). Unconditional money growth targeting (5.11) is perhaps more transparent than the conditional one. It will result in long-run average inflation equal to the target, but inflation and inflation expectations will be more variable and show persistent deviations from the target, and unconditional money growth targeting will hence be inefficient.

Bundesbank’s money growth targets are formulated from a simple quantity equation relationship, such that the money growth target equals the sum of the implicit inflation target of 2 percent, previously called ‘unavoidable’ inflation and now called ‘normative’ inflation, and the capacity growth forecast, less the forecast of the velocity trend (von Hagen (1995)). In terms of the model used here, both the capacity growth forecast and the velocity trend forecast are zero. Hence, Bundesbank’s money growth target in this model corresponds to the unconditional \( \mu^* \). Thus, adherence to this money growth target would be inefficient. However, as emphasized for instance in von Hagen (1995, 1996), Bundesbank has a most flexible approach to its money growth target, frequently deviating from the money growth target when the inflation forecast is consistent with the inflation target. This might be interpreted as a somewhat nontransparent attempt to adhere to the conditional money growth target (5.8).

Can unconditional money growth targeting ever be optimal? Consider the expression for the two-year inflation target as a function of the one-year money growth forecast and year \( t \) variables other than the repo rate, (5.6). Consider the special case when
\[ f_1 = f_2 = f_3 = f_4 = 0, \quad f_5 = 1 \quad \text{and} \quad \nu_t = 0, \] (5.14)
that is when money growth is a sufficient statistic for future inflation, and when there are no
disturbances to money demand. Then unconditional money growth targeting would be optimal. The conditions (5.14) on the \( f \)-coefficients cannot be fulfilled in the model used here. Nevertheless, if they could be fulfilled (which requires some other transmission mechanism for monetary policy than assumed in the above model), inflation targeting would imply unconditional money growth targeting.

Generally, inflation targeting will imply some simple money growth targeting if and only if such money growth targeting is appropriate.

The previous discussion can be adapted to exchange rate targeting, with similar conclusions. Inflation targeting will automatically imply exchange rate targeting if, and only if, exchange rate targeting is optimal.\(^{15}\)

6 Extensions

6.1 Output stabilization

The above results are very straightforward when inflation targeting is interpreted as implying one objective only, in the sense that inflation is the only argument of the period loss function (2.5). If inflation targeting is interpreted as involving additional objectives, as in Svensson (1996a), the situation will be somewhat more complicated. Generally, additional objectives motivate temporary deviations of the inflation forecast from the inflation target.

Let me consider the case when there are additional stabilization objectives with regard to real variables, like output or employment. For instance, in the model used above, monetary policy has temporary effects on output. Although the average output level, the long-run natural level of output, cannot be affected by monetary policy, monetary policy can, potentially, stabilize output around its average level to some extent.\(^{16}\)

Simplify the model by eliminating the effect of the exogenous variable, that is, set

\[ \alpha_2 = \beta_3 = 0. \tag{6.1} \]

Then the model is

\[ \pi_{t+1} = \pi_t + \alpha_1 y_t + \epsilon_{t+1} \tag{6.2} \]

\(^{15}\) See Persson and Tabellini (1996) for a comparison of inflation targeting, money growth targeting and exchange rate targeting for 'ins' and 'outs' of the EMU.

\(^{16}\) The explicit formulation of (2.1) implies that a monetary policy that results in a steady increase in inflation has a permanent effect on output. A reasonable microeconomic derivation of (2.1) would certainly show that (2.1) will break down if such a policy was implemented. Such policies will never be optimal for the constant inflation target used here.
\[ y_{t+1} = \beta_1 y_t - \beta_2 (i_t - \pi_t) + \eta_{t+1}. \] (6.3)

Consider the intertemporal loss function
\[ E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L(\pi_{\tau}, y_{\tau}), \] (6.4)
when the period loss function is
\[ L(\pi_{\tau}, y_{\tau}) = \frac{1}{2} \left( (\pi_{\tau} - \pi^*)^2 + \lambda y_{\tau}^2 \right), \] (6.5)
where \( \lambda > 0 \) is the relative weight on output stabilization. The case examined in previous sections corresponds to \( \lambda = 0 \).

In appendix B, it is shown that the first-order condition for minimizing (6.4) over the repo rate can be written
\[ \pi_{t+2|t} - \pi^* = -\frac{\lambda}{\delta \alpha_1 k} y_{t+1|t}, \] (6.6)
where the coefficient \( k \geq 1 \) is given by
\[ k = \frac{1}{2} \left( 1 - \frac{\lambda (1 - \delta)}{\delta \alpha_1^2} + \sqrt{\left( 1 + \frac{\lambda (1 - \delta)}{\delta \alpha_1^2} \right)^2 + \frac{4 \lambda}{\alpha_1^2}} \right). \] (6.7)

That is, the two-year inflation forecast should equal the inflation target only if the one-year output forecast equals the natural output rate. Else it should exceed the inflation target in proportion to how much the one-year output forecast falls short of the natural output level. The proportionality coefficient, \( \frac{\lambda}{\delta \alpha_1 k} \), is increasing in the relative weight on output stabilization, \( \lambda \), and decreasing in the (short-run) inflation/output trade-off, \( \alpha_1 \) (see appendix B).

Since the one-year output forecast and the two-year inflation forecast fulfill
\[ y_{t+1|t} = \beta_1 y_t - \beta_2 (i_t - \pi_t) \]
\[ \pi_{t+2|t} = \pi_t + \alpha_1 (1 + \beta_1) y_t - \alpha_1 \beta_2 (i_t - \pi_t), \]
it follows from (6.6) (see appendix B) that the reaction function can be written
\[ i_t = \pi_t + \frac{\delta \alpha_1 k}{\lambda \beta_2} (\pi_{t+2|t} - \pi^*) + \frac{\beta_1}{\beta_2} y_t \] (6.8)
\[ = \pi_t + \tilde{b}_1 (\pi_t - \pi^*) + \tilde{b}_2 y_t, \] (6.9)
where
\[ \tilde{b}_1 = \frac{\delta \alpha_1 k}{\beta_2 (\lambda + \delta \alpha_1^2 k)} \text{ and } \tilde{b}_2 = \frac{1}{\beta_2} \left( \frac{\delta \alpha_1^2 k}{\lambda + \delta \alpha_1^2 k} + \beta_1 \right). \] (6.10)
The real repo rate is increasing in the excess of the two-year inflation forecast over the inflation target, (6.8), or in the excess of current inflation over the inflation target, (6.9), in addition to being increasing in output. We see that $\lambda = 0$ results in $\tilde{b}_1 = b_1$ and $\tilde{b}_2 = b_2$, the single-objective reaction function (2.13).

Output and inflation will in equilibrium, by (6.2), (6.3) and (6.9), obey

$$y_{t+1} = \frac{\delta \alpha_1 k}{\lambda + \delta \alpha_2 k} (\pi_t - \pi^*) - \frac{\delta \alpha_2 k}{\lambda + \delta \alpha_2 k} \eta_{t+1},$$

$$\pi_{t+1} - \pi_t = \alpha_1 y_t + \epsilon_{t+1}$$

$$= \frac{\delta \alpha_2 k}{\lambda + \delta \alpha_2 k} (\pi_{t-1} - \pi^*) - \frac{\delta \alpha_2 k}{\lambda + \delta \alpha_2 k} \eta_{t-1} + \alpha_1 \eta_t + \epsilon_{t+1}.$$

We see that both output and inflation are mean-reverting, output towards the natural output level and inflation towards the inflation target.

In summary, additional objectives motivate deviations of the inflation forecast from the inflation target. In this example, a one-year output forecast that falls short of the natural rate is a reason for allowing the two-year inflation forecast to exceed the inflation target. How much the inflation forecast may exceed the inflation target for a given shortfall of the output forecast relative to the natural output level depends positively on the relative weight on output stabilization and negatively on the (short-run) inflation/output trade-off.\(^{17}\)

This case can be interpreted as involving a constant long-run inflation target, $\pi^*$, and a variable short-run inflation target, $\pi^* - \frac{\lambda}{\delta \alpha_2 k} y_{t+1}|t|$, the latter deviating from the long-run inflation target depending upon the output forecast $y_{t+1}|t|$.

Thus, additional objectives make inflation targeting more complicated, but not overly so. The central bank needs to explain deviations of the inflation forecast from the inflation target with reference to its additional objectives. The outside monitoring of the central bank needs to be somewhat more sophisticated.

It seems that, while inflation targeting is still relatively new, a realistic strategy for a central bank learning how to implement inflation targeting is to disregard additional objectives and focus on the simple case of a single inflation target ($\lambda = 0$ in the model above). Implementing and monitoring this single target may be a sufficient challenge for the central bank and outside observers for a while. With increased experience, improved performance and increased credibility of inflation targeting, it may be appropriate to increase ambitions and cautiously incorporate

\(^{17}\) Note that an instrument-smoothing objective would similarly make the inflation forecast temporarily deviate from the inflation target in order to reduce the necessary instrument change, cf. Goodhart (1996) for a recent discussion.

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additional stabilization objectives \((\lambda > 0)\).

6.2 Supply and demand shocks

The previous discussion has not been explicit on (aggregate) supply and demand shocks. Demand shocks generally increase inflation and output. Supply shocks reduce inflation and/or increase output. Therefore demand shocks do not create any conflict between stabilizing inflation and stabilizing output. Supply shocks create such a conflict, though.

In the model (2.1)-(2.3), \(\beta_3 > 0\) implies that the exogenous variable can be interpreted as a persistent demand shock. Inflation targeting implies that demand shocks are neutralized, if possible. This occurs in the equilibrium described in section 2, since equilibrium inflation and output in (2.15) and (2.17) are independent of the coefficient \(\beta_3\). In the model, \(\alpha_2 > 0\) implies that the exogenous variable can also be interpreted as a negative supply shock, since even if \(\beta_3 = 0\) it increases inflation. In the equilibrium in section 2, the effect of supply shocks on inflation is neutralized, if possible, but not the effect on output. In equilibrium, inflation \(\pi_{t+1}\) depends on the coefficient \(\alpha_2\) only in the term that contains \(\theta_{t+1}\) in (2.15), since monetary policy cannot, due to the control lag, neutralize that disturbance. Inflation does not depend on the level of the exogenous variable. In contrast, output \(y_{t+1}\) will in equilibrium depend on both the disturbance \(\theta_t\) and the exogenous variable \(x_t\), via the term \(-\alpha_2 \frac{\alpha_2}{\alpha_1} (\gamma x_t + \theta_t)\) in (2.17). Thus, fluctuations in output are used to neutralize the effect of supply shocks on inflation in the model.

With an additional output stabilization objective, the effect of supply shocks will be optimally distributed between inflation and output movements.

6.3 The role of bands

Several inflation targeting regimes have an explicit band for inflation, either in the form of a target band without an explicit inflation point target, or in the form of a band around an explicit inflation (point) target. These bands can potentially be interpreted in several ways. First, one may ask whether inflation is supposed to remain within the band for all of the time, or for most of the time. The announced bands are typically 2 percentage points wide, which together with the imperfect control over inflation makes it rather likely that inflation will sometimes move outside the bands. It may then be more transparent to the public if the central bank explicitly acknowledges this, for instance by announcing that it expects to keep inflation within the band \(x\) percent of the time. Apparently, inflation targeting central banks have so far shunned such
precise statements. From an analytical viewpoint it seems natural to interpret the bands as a confidence interval, proportional to the unconditional standard deviation of inflation, the square root of the sum of the variance of the conditional expectation of inflation and the variance of the inflation forecast errors.

In addition to bands for realized inflation, we can also conceive of potential bands for inflation forecasts, for conditional expectations of inflation. Bands for inflation forecasts would then be proportional to the unconditional standard deviation of inflation forecasts.

6.3.1 A single objective

Consider first the situation when inflation targeting is interpreted as a single objective, in the sense that only inflation enters the central bank's loss function. The band for realized inflation is then proportional to the standard deviation of possible deviations of realized inflation from the inflation target, due to uncertainty about, and imperfect control of, future inflation. It indicates an anticipated 'unavoidable' variability of inflation. The bandwidth is proportional to the unconditional standard deviation of the sum of control and forecast errors, but not deviations of the inflation forecast (the intermediate target) from the inflation target, since the inflation forecast should always equal the inflation target with a single objective. In terms of the model, the bandwidth would be proportional to $\sqrt{2\sigma_t^2 + \sigma_n^2 + \sigma_\theta^2}$, cf. (2.15).

With a single objective there is hence room for a very narrow band for the inflation forecast at most; in the stylized model above there is a zero band for the inflation forecast.

6.3.2 Multiple objectives

With multiple objectives, conscious deviations of the inflation forecast from the inflation target occurs. The band for realized inflation then incorporates the unconditional variance of the deviation of inflation forecasts (short-run inflation targets) from the inflation target (the long-run inflation target), in addition to the variance of forecast errors.

There is hence an implicit band for the inflation forecast, in proportion to the unconditional standard deviation of the inflation forecast, which will be increasing in the weight on additional objectives. In the above model, that unconditional standard deviation will be increasing in $\lambda$, the relative weight on output stabilization, but also depend on the other parameters of the problem.

A wide band could then potentially indicate that the central bank has a relatively high $\lambda$ and
hence a significant output stabilization objective. A narrow band could indicate a commitment to a low or even zero $\lambda$, cf. the discussion in Svensson (1996a).

6.3.3 Accountability

The band for realized inflation can be used to increase the accountability of the central bank, and the central bank may be subject to sanctions if realized inflation moves outside the band. The most explicit example is New Zealand, where the Governor may then be relieved from his post. As discussed by Walsh (1993), this is an optimal arrangement even though inflation is also subject to disturbances outside the control of the central bank, if only realized inflation and not the actions of the central bank can be observed and verified. The optimal bandwidth can then be chosen so as to achieve the optimal trade-off between type I and type II errors (the Governor is fired (because of unanticipated and unobservable disturbances) even though he/she has chosen the (ex ante) appropriate policy, or the Governor is retained even though he/she has chosen an (ex ante) inappropriate policy).

If the central bank's inflation forecast can be observed and verified, it is better to make sanctions conditional upon a band for the inflation forecast rather than realized inflation, since then the noise (and injustice) from unobserved disturbances is eliminated. It remains to be seen whether a central bank's inflation forecasts can be made so observable and verifiable as to allow sanctions to be conditional on forecasts rather than outcomes. If the amount of private information of the central bank is substantial, this may require rather sophisticated incentive schemes in order to induce the central bank to reveal the necessary information, cf. Persson and Tabellini (1993) and Walsh (1995a).

7 Conclusions

Although inflation targeting has several general advantages, it faces some potentially very serious problems with regard to both its implementing by the monetary authority and its monitoring by the public. Implementation is difficult due to the imperfect control of inflation. Monitoring is difficult since inflation reacts with long and variable lags to changes in the monetary policy instrument, and since inflation is affected by other factors than monetary policy. This paper argues that there is a straightforward solution to these problems, namely that inflation targeting can be implemented as inflation forecast targeting, in the sense that the monetary authority's inflation forecast is treated as an explicit intermediate target.
The monetary authority can then implement inflation targeting by simply setting the instrument such that its inflation forecast for a horizon corresponding to the control lag equals the inflation target. Ex post realized inflation will deviate from the inflation forecast due to disturbances that occur within the control lag, but these are beyond the control of the monetary authority. The best the monetary authority can do is to get its inflation forecast equal to the inflation target, which will minimize the squared deviations of realized inflation from the inflation target. In order to avoid the problems of instability, multiplicity and even non-existence of equilibria that Woodford (1994) has emphasized, the intermediate target should be the monetary authority's own structural forecast, based on the fundamental determinants of inflation, and not a consensus forecast of outside observers.

The public can monitor and evaluate inflation targeting by observing and scrutinizing the monetary authority's inflation forecast, in the favorable situation when the monetary authority reveals the details of its forecast. The public can also use inflation forecasts of other forecasters' forecasts for this purpose, especially if the monetary authority keeps its forecast secret to some extent. A healthy competition is likely to arise between the monetary authority and outside professional or academic forecasters with regard to the quality of the analysis and forecasts. Although the construction of inflation forecasts is difficult and demanding, the monetary policy conclusions for a given forecast follow simple and intuitive principles: If the forecast is on target, monetary policy is appropriate. If the forecast is above (below) the target, monetary policy should be tightened (eased). This should improve monetary authority communication with the public, the public's understanding of monetary policy, and the predictability of monetary policy.

Setting the instrument to make the inflation forecast equal to the inflation target is an example of a 'target rule', which if applied by the monetary authority, would result in an endogenous optimal reaction function expressing the instrument as a function of the available relevant information. This is different from an 'instrument rule' that directly specifies the reaction function for the instrument. I believe a target rule is a more advantageous arrangement, since it focuses on the essential, that is, to achieve the target, and allows more flexibility in finding the corresponding reaction function. More specifically, with new information about structural relationships, changes in exogenous variables, etc., a target rule implies automatic revisions of the reaction function. An explicit instrument rule either requires more confidence in the structural model and its stability, or requires frequent revision that may be difficult to motivate and less transparent. If the monetary authority has private information, the advantage of a target rule
over an instrument rule is an intuitive principal-agent result. In other words, target rules are inherently more stable than instrument rules, and easier to identify, motivate and verify.\textsuperscript{18}

However, it might be argued that the lack of knowledge, and resulting disagreement, about the appropriate macroeconomic model not only make instrument rules inferior to target rules, but that they may be so substantial as to make both the implementing and monitoring of the target rule proposed here too difficult. Imperfect knowledge about the model certainly poses a general problem for monetary policy. But the inflation targeting framework outlined above provides very strong incentives for the monetary authority to improve its understanding of the economy and its control of inflation, especially if the monetary authority chooses or is obliged to reveal its model and analysis in detail to the public. It then has strong incentives to increase the level of its competence, to hire the best staff, and generally concentrate its resources on understanding and controlling inflation.

The inflation targeting framework is especially straightforward to implement and monitor when it is interpreted as involving a single objective only. With additional objectives, like stabilizing output or employment, the inflation targeting framework becomes somewhat more complex. It can then be interpreted as having a constant long-run inflation target equal to the announced constant inflation target, and a flexible short-run inflation forecast target which takes the additional objectives into account. For instance, with an output stabilizing objective, the inflation forecast may exceed the long-run inflation target in proportion to how much the output forecast falls short of the natural output level. Thus both the implementation and monitoring of inflation targeting becomes a bit more complex, but not overly so. Still, it may be wise for monetary authorities that have recently started inflation targeting to initially interpret it as involving only one objective, and first learn to fulfill the single inflation (forecast) target. With increasing experience and competence of the monetary authority, and increasing understanding of inflation targeting by the public, it may eventually be appropriate to increase the ambition and take additional (reasonable) monetary policy objectives into account. Systematic deviations of the inflation forecast from the inflation target obviously need to be thoroughly motivated in terms of reasonable additional objectives, for instance in order not to give the impression of abandoning the framework.

Generally I find it unlikely that monetary authorities have much private information, relative

\textsuperscript{18} Taylor (1996) argues for instrument rules. McCallum has in several papers, for instance McCallum (1990), argued for an instrument rule in the form of a monetary base rule. Goodhart (1994) questions whether the monetary base is under sufficient control of the central bank to qualify as an instrument.
to sophisticated outside observers, about the state of the economy and the behavior of the economy, but that their private information mostly concern their own implicit objectives and their corresponding plans for the future instrument. This is consistent with an important role for outside observers in detecting and preventing monetary authority deviations from explicit objectives. However, to the extent that monetary authorities do have private information about the state and behavior of the economy, the issue of their incentives to truthfully reveal their information to the public becomes most important. There seem to be room for considerable future research both on the extent of such private information, and on possible incentive schemes to induce monetary authorities to reveal the truth.19

Inflation targeting implicitly or explicitly allows base drift of the price level. The price level then becomes non-stationary and integrated of order one, with price level uncertainty increasing in the forecasting horizon. Once central banks have learned to successfully target inflation, more ambitious price level targeting, which makes the price level stationary and reduces long-term price uncertainty, may be a realistic alternative. In Svensson (1996b) it is demonstrated that price level targeting, counter to conventional wisdom, may also actually reduce rather than increase short-term inflation variability.

References


19See for instance Persson and Tabellini (1993) and Walsh (1995a) for examples of the latter.


A Period-by-period optimization

The problem to choose \( \{i_\tau\}_{\tau=t}^{\infty} \) so as to minimize

\[
E_t \frac{1}{2} \sum_{\tau=t}^{\infty} \delta^{\tau-t} (\pi_\tau - \pi^*)^2,
\]

where each \( i_\tau \) depends on information available in period \( \tau \), can be written as a sequence of one-period problems

\[
\min_{i_t} E_t \frac{1}{2} \delta^2 (\pi_{t+2} - \pi^*)^2 + E_t \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} \min_{i_\tau} E_t \frac{1}{2} (\pi_{\tau+2} - \pi^*)^2,
\]

since \( \pi_{\tau+2} \) according to (2.12) can be controlled by \( i_\tau \) and is not affected by \( i_{\tau+1}, i_{\tau+2}, \ldots \). This can for instance be seen from the first-order condition for the problem

\[
\min_{i_t} E_t \frac{1}{2} \sum_{\tau=t}^{\infty} \delta^{\tau-t} (\pi_\tau - \pi^*)^2,
\]

which is

\[
E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} (\pi_\tau - \pi^*) \frac{\partial \pi_\tau}{\partial i_t} = \sum_{\tau=t+2}^{\infty} \delta^{\tau-t} (\pi_{\tau|t} - \pi^*) \frac{\partial \pi_{\tau|t}}{\partial i_t} = 0,
\]

where I have used that \( \frac{\partial \pi_{\tau|t}}{\partial i_t} \) due to the linearity of the model will be constant. Due to (2.12) \( i_t \) can be chosen such that

\[
\pi_{t+2|t} - \pi^* = 0.
\]

Similarly, \( i_\tau, \tau = t + 1, t + 2, \ldots \), can be chosen such that

\[
\pi_{\tau+2|\tau} - \pi^* = 0.
\]

Due to the law of iterated expectations it follows that

\[
\pi_{\tau|t} - \pi^* = 0, \quad \tau = t + 3, t + 4, \ldots
\]

and the first-order condition (A.3) can be fulfilled with each term in the sum equal to zero. It is clear that this must correspond to a global minimum of the problem. The one-period problem

\[
\min_{i_\tau} E_t \frac{1}{2} (\pi_{\tau+2} - \pi^*)^2
\]

results in (A.4), so a sequence of one-period problems as in (A.2) will result in the global minimum.
B Output stabilization

B.1 One-year control lag

In order to derive the first-order condition (6.6) it is practical to first study the simpler problem

\[
V(\pi_t) = \min_{y_t} \left\{ \frac{1}{2} \left[ (\pi_t - \pi^*)^2 + \lambda y_t^2 \right] + \delta E_t V(\pi_{t+1}) \right\}
\]

subject to

\[
\pi_{t+1} = \pi_t + \alpha_1 y_t + \epsilon_{t+1},
\]

where output \( y_t \) is regarded as a control variable.

The indirect loss function \( V(\pi_t) \) will be quadratic,

\[
V(\pi_t) = k_0 + \frac{1}{2} k (\pi_t - \pi^*)^2,
\]

where the coefficients \( k_0 \) and \( k \) remain to be determined. The first-order condition is

\[
\lambda y_t + \delta E_t V_\pi(\pi_{t+1}) \alpha_1 = \lambda y_t + \delta \alpha_1 k \left( \pi_{t+1|t} - \pi^* \right) = 0,
\]

where I have used (B.3). This can be written

\[
\pi_{t+1|t} - \pi^* = -\frac{\lambda}{\delta \alpha_1 k} y_t.
\]

The decision rule for output fulfills

\[
y_t = -\frac{\delta \alpha_1 k}{\lambda} (\pi_{t+1|t} - \pi^*).
\]

where I have used that by (B.2)

\[
\pi_{t+1|t} = \pi_t + \alpha_1 y_t.
\]

Then the equilibrium inflation forecast fulfills

\[
\pi_{t+1|t} = \pi_t + \alpha_1 y_t = \pi^* + \left( 1 - \frac{\delta \alpha_1^2 k}{\lambda + \delta \alpha_1^2 k} \right) (\pi_t - \pi^*) = \pi^* + \frac{\lambda}{\lambda + \delta \alpha_1^2 k} (\pi_t - \pi^*).
\]

In order to identify \( k \) I exploit the envelope theorem for (B.1) and (B.3) and use (B.5), which gives

\[
V_\pi(\pi_t) = k (\pi_t - \pi^*) + \delta k \left( \pi_{t+1|t} - \pi^* \right) = \left( 1 + \frac{\delta \lambda k}{\lambda + \delta \alpha_1^2 k} \right) (\pi_t - \pi^*).
\]
Identification of the coefficient for $\pi_t - \pi^*$ gives

$$k = 1 + \frac{\delta \lambda k}{\lambda + \delta \alpha_1^2 k}.$$  

The right-hand side is equal to unity for $k = 0$ and increases towards $1 + \frac{\lambda}{\alpha_1^2}$ for $k \to \infty$. We realize that there is a unique positive solution that fulfills $k > 1$. It can be solved analytically:

$$k^2 - \left(1 - \frac{\lambda (1 - \delta)}{\delta \alpha_1^2}\right) k - \frac{\lambda}{\delta \alpha_1^2} = 0,$$

$$k = \frac{1}{2} \left( 1 - \frac{\lambda (1 - \delta)}{\delta \alpha_1^2} + \sqrt{\left(1 - \frac{\lambda (1 - \delta)}{\delta \alpha_1^2}\right)^2 + \frac{4 \lambda}{\delta \alpha_1^2}} \right)$$

$$= \frac{1}{2} \left( 1 - \frac{\lambda (1 - \delta)}{\delta \alpha_1^2} + \sqrt{\left(1 - \frac{\lambda (1 - \delta)}{\delta \alpha_1^2}\right)^2 + \frac{4 \lambda (1 - \delta)}{\delta \alpha_1^2} + \frac{4 \lambda}{\alpha_1^2}} \right)$$

$$= \frac{1}{2} \left( 1 - \frac{\lambda (1 - \delta)}{\delta \alpha_1^2} + \sqrt{\left(1 + \frac{\lambda (1 - \delta)}{\delta \alpha_1^2}\right)^2 + \frac{4 \lambda}{\alpha_1^2}} \right) \geq 1. \quad (B.6)$$

### B.2 Two-year control lag

After these preliminaries, consider the problem

$$\min_{\pi_t} \sum_{\tau=0}^{\infty} \delta^\tau L(\pi_{t+\tau}, y_{t+\tau})$$

subject to

$$L(\pi_t, y_t) = \frac{1}{2} \left[ (\pi_t - \pi^*)^2 + \lambda y_t^2 \right]$$

$$\pi_{t+1} = \pi_t + \alpha_1 y_t + \epsilon_t$$

$$y_{t+1} = \beta_1 y_t - \beta_2 (\pi_t - \pi^*) + \eta_{t+1}.$$

We realize that this can be formulated as

$$V(\pi_{t+1}|t) = \min_{y_{t+1}|t} \left\{ \frac{1}{2} \left[ (\pi_{t+1}|t - \pi^*)^2 + \lambda y_{t+1}|t^2 \right] + \delta E_t V(\pi_{t+2}|t+1) \right\}$$

subject to

$$\pi_{t+2}|t+1 = \pi_{t+1} + \alpha_1 y_{t+1}$$

$$= \pi_{t+1}|t + \alpha_1 y_{t+1}|t + (\epsilon_{t+1} + \alpha_1 \eta_{t+1}).$$
where $y_{t+1|t}$ is regarded as the control, and where the optimal repo rate can be inferred from

$$i_t - \pi_t = -\frac{1}{\beta_2} y_{t+1|t} + \frac{\beta_1}{\beta_2} y_t.$$  

This problem is analogous to the problem (B.1) subject to (B.2). Thus, in analogy with (B.4), the first-order condition can be written

$$\pi_{t+2|t} - \pi^* = -\frac{\lambda}{\delta \alpha_{1k}} y_{t+1|t},$$  

(B.7)

and the reaction function will fulfill

$$i_t - \pi_t = -\frac{1}{\beta_2} y_{t+1|t} + \frac{\beta_1}{\beta_2} y_t$$

$$= \frac{\delta \alpha_{1k}}{\lambda \beta_2} (\pi_{t+2|t} - \pi^*) + \frac{\beta_1}{\beta_2} y_t$$

$$= \frac{\delta \alpha_{1k}}{\lambda \beta_2} [\pi_t - \pi^* + \alpha_1 (1 + \beta_1) y_t - \alpha_1 \beta_2 (i_t - \pi_t)] + \frac{\beta_1}{\beta_2} y_t$$

$$= \frac{\delta \alpha_{1k}}{\beta_2 (\lambda + \delta \alpha_{1k}^2)} (\pi_t - \pi^*) + \frac{1}{\beta_2} \left( \frac{\delta \alpha_{1k}^2}{\lambda + \delta \alpha_{1k}^2} + \beta_1 \right) y_t,$$

where I have used

$$\pi_{t+2|t} = \pi_t + \alpha_1 (1 + \beta_1) y_t - \alpha_1 \beta_2 (i_t - \pi_t),$$

and where $k$ will obey (B.6).

The coefficient $\frac{\lambda}{\delta \alpha_{1k}}$ in (B.7) will be (i) increasing in $\lambda$ and (ii) decreasing in $\alpha_1$. To show (i), consider

$$z = \frac{k}{\lambda} = \frac{1}{2} \left[ \frac{1}{\lambda} - \frac{1 - \delta}{\delta \alpha_{1k}^2} + \sqrt{\left( \frac{1}{\lambda} + \frac{1 - \delta}{\delta \alpha_{1k}^2} \right)^2 + \frac{4 A}{\lambda \alpha_{1k}^2}} \right] = \frac{1}{2} \left[ w - A + \sqrt{(w + A)^2 + 4 A B w} \right],$$

where

$$w = \frac{1}{\lambda}, \quad A = \frac{1 - \delta}{\delta \alpha_{1k}^2} > 0, \quad B = \frac{\delta}{1 - \delta} > 0.$$  

It is straightforward to show that $\frac{\partial z}{\partial w} > 0$, hence that $\frac{\partial (k/\lambda)}{\partial \lambda} < 0$, and $\frac{\partial (k/\lambda)}{\partial \lambda} > 0$. To show (ii), consider

$$v = \alpha_1 k = \frac{1}{2} \left[ \alpha_1 - \frac{C}{\alpha_1} + \sqrt{\left( \alpha_1 + \frac{C}{\alpha_1} \right)^2 + 4 \lambda} \right],$$

where

$$C = \frac{\lambda (1 - \delta)}{\delta} > 0.$$  

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It is sufficient to show that $\frac{\partial v}{\partial \alpha_1} > 0$. Thus,

$$2 \frac{\partial v}{\partial \alpha_1} = 1 + \frac{C}{\alpha_1^2} + \frac{2 \left(1 + \frac{C}{\alpha_1^2}\right) \alpha_1 \left(1 - \frac{C}{\alpha_1^2}\right)}{2 \sqrt{(\alpha_1 + \frac{C}{\alpha_1})^2 + 4\lambda}}$$

$$= \left(1 + \frac{C}{\alpha_1^2}\right) \left(1 + \frac{\alpha_1 - \frac{C}{\alpha_1}}{\sqrt{(\alpha_1 + \frac{C}{\alpha_1})^2 + 4\lambda}}\right)$$

$$= \left(1 + \frac{C}{\alpha_1^2}\right) \frac{\sqrt{(\alpha_1 + \frac{C}{\alpha_1})^2 + 4\lambda} + (\alpha_1 - \frac{C}{\alpha_1})}{\sqrt{(\alpha_1 + \frac{C}{\alpha_1})^2 + 4\lambda}} > 0.$$