A COMPARISON OF EXCHANGE RATE REGIMES IN THE PRESENCE OF
IMPERFECT CAPITAL MARKETS

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1. Introduction

The renewed interest in systematic evaluations of exchange rate regimes has brought about new approaches which are based on microeconomic foundations, including microeconomic foundations of monetary theory (see Helpman and Razin (1979), Kareken and Wallace (1979), Helpman (1979), Lucas (1980), Lapan and Enders (1980) and Persson (1980)). The main common feature of all these studies is the use of utility levels (or expected utility levels) -- which are derived from individual maximization -- to evaluate the relative desirability of alternative exchange rate regimes. A common conclusion that has emerged so far is that in the presence of self-fulfilling expectations and in the absence of imperfections, all exchange rate regimes are equally efficient.^{1/} However, efficiency does not always imply identical allocations.

Helpman (1979) distinguishes between two types of fixed exchange rate regimes; a one-sided peg and a cooperative peg. In the one-sided peg regime, which is essentially the fixed exchange rate regime employed in Helpman and Razin (1979), the government stabilizes the exchange rate by borrowing foreign currency whenever it is required to sell foreign currency and by lending foreign

currency whenever it is required to buy foreign currency. In this case exchange rate stabilization operations change not only the distribution of world money but also its aggregate quantity. In particular, it changes the quantity of money of the pegging country without affecting the quantity of money in the passive country. In the cooperative peg (formally specified in Kareken and Wallace (1979)), countries cooperate by providing each other, or a common exchange rate stabilizing authority, with monies required for exchange rate stabilization. Reserve holdings play no essential role in this regime. In this case exchange rate stabilization operations change the composition of world money in terms of volumes of national currencies, but not the aggregate value of world money. This is a central feature of commonly used macroeconomic models of fixed exchange rate economies (e.g., Mundell (1968, ch.18).) In this system real allocations depend on the exchange rate levels and on monetary policies of each country, since by unilaterally increasing the stock of money a country can gain real resources. No such gains can be made in the one-sided peg regime, whose real allocation coincides with the real allocation of a floating exchange rate regime (see Helpman (1979)) ³

The new studies provide a framework and a benchmark for evaluations of exchange rate regimes. The next interesting question, as we see it, is how do exchange rate regimes compare in the presence of departures from this benchmark. Namely, if one departs from the frictionless world in which all regimes are equally efficient, which regime performs better in the presence of frictions? Interesting frictions include price rigidities, uncertainty with incomplete markets, departures from self-fulfilling expectations, etc.

In the present study we take up the problem of uncertainty and incompleteness of financial markets. A detailed verbal description of the main features of our framework is presented in Section 2. We have chosen to compare a floating

exchange rate regime with a one-sided peg, since we believe that the one-sided peg regime is more relevant. Under our specification of the frictions, the one-sided peg and floating regimes do not yield the same allocations (except if the uncertainty degenerates to certainty).

We describe the operation of an economy under each regime in Sections 3-7. This discussion brings out many important features of each regime and points out new elements which stem from the structure of financial markets. A detailed discussion on necessary absorption policies in the one-sided peg regime is also provided. In the last section, Section 8, we present a welfare comparison of the two regimes. We provide a set of sufficient conditions under which the floating exchange rate regime performs better than the fixed exchange rate regime. The question of whether in the present context there are circumstances in which a fixed exchange rate regime may perform better than a floating exchange rate regime has, unfortunately, to remain at this stage an open question.

2. Main Features of the Framework

Our main focus is on a small country that consumes and produces a single good. There are discrete time periods which coincide with payment periods, and wages and dividends are paid out at the end of each period.

We assume that the world economy is a monetary economy. By this we mean that all payments are made in the form of money, so that money is a side in each market transaction. In particular, goods and financial assets are acquired in exchange for money. Domestic goods and financial assets are paid for with domestic money while foreign goods and assets are paid for with foreign money.

At the beginning of each period, trade in financial assets takes place. At this instant of time home money is exchanged for home currency denominated bonds and foreign currency is exchanged for foreign currency denominated bonds. In addition, home currency is exchanged for foreign currency in the foreign exchange market or in the home country's central bank, depending on the exchange rate regime.^{2/} We assume that there is only one type of foreign money and that each bond is a one-period bond with a fixed nominal interest rate. Before the closing of the financial markets each individual receives his previous period's income on account of wages and dividends. Taxes and previous period's debt, including interest, are also paid and government transfers received during this instant of time. All the abovementioned financial transactions are made extremely fast; in our limiting formulation they take no time at all.

After completion of the financial transactions at the beginning of a period, an individual ends up with desired stocks of each type of money. He can use the money to buy goods during the period and transfer whatever money remains at the end of the period to the next period. We assume that no financial markets open during a period, except for the market for currencies. This assumption is meant to capture differential transaction costs in bonds and currency markets. We make this shortcut in order to avoid explicit treatment of transaction costs. An explicit modelling of transaction costs extremely complicates our model, which as simple as it might appear is already difficult to handle in the main analysis. Our currency markets are spot markets. We do not introduce forward markets for currencies because under our assumptions they turn out to be redundant, i.e., they provide trading opportunities which can be obtained by a combination of domestic and foreign bonds coupled with spot currency markets.

In the absence of uncertainty this economy has rather simple features. If the interest rate on home currency denominated bonds is positive, maximizing individuals will not use home money as a store of value, in the sense that they will not use home money to transfer purchasing power from one period to the next. This procedure minimizes interest costs of money holdings. As a result, all money holdings at the beginning of a period will be spent during the period on home goods. Since aggregate home money holdings at the beginning of a period are equal in equilibrium to the supply of money, the home price level during a period has to be equal to the ratio of the stock of money to home output. This is, of course, the strict quantity equation with a unitary velocity of circulation. In a floating exchange rate regime the ratio of domestic

to foreign prices determines the exchange rate (since in this world purchasing power parity has to hold due to maximizing behavior, and we have no nontraded goods), while in a fixed exchange rate regime the domestic supply of money will adjust through the balance of payments so as to keep domestic prices in line with foreign prices multiplied by the fixed exchange rate. It can also be shown that in the absence of uncertainty the equilibrium allocation of consumption that results in a floating exchange rate regime coincides with the equilibrium allocation of consumption that results in a one-sided peg regime (a regime in which the pegging country engages in foreign borrowing and lending to stabilize its exchange rate, without holding reserves that do not bear interest), so that the same welfare level is achieved in both regimes. (For more details about this see Helpman (1979).)

It should also be clear that in the absence of uncertainty there is no need to reopen financial markets after their closing at the beginning of a period. This is so since when the financial transactions are carried out, individuals know already the prices they will have to pay and so they can build up their most desired holdings of monies and bonds. If financial markets open during the period, they will not be active, because due to the absence of new information no-one will regret the financial decisions that were made at the beginning of the period.

In the presence of uncertainty, however, the main conclusions from the certain environment change substantially. Individuals make financial decisions at the beginning of a period, before they know commodity prices. As a result, the question of whether currency markets reopen after the resolution of the temporal uncertainty or not, and similarly with regard to bond markets is highly significant. We assume that after the resolution

of the temporal uncertainty bond markets do not reopen (they reopen only at the beginning of the next period) while currency markets do reopen. The reopening of currency markets assures purchasing power parity which will not hold if currency markets do not reopen.

Now a positive interest rate on home currency denominated bonds does not prevent home money from being used as a store of value. Since at the instant of time at the beginning of a period in which financial decisions are made the individual does not know the commodity prices that will prevail during the period, nor does he know his income during the period (that will be paid out to him at the beginning of the next period), he might desire to build up money holdings which include a precautionary component. If, say, other things being equal, it turns out that commodity prices are relatively low, he will spend all his money on goods. If, however, commodity prices turn out to be relatively high, he might decide to spend only part of his money on goods and keep the rest of it until the next period. Since the bond market does not reopen during the period, the individual cannot invest his idle funds in an interest bearing asset. In this framework all three well-known motives for holding money are present; the transactions, the store of value and the precautionary motives. As a result, the simple quantity equation does not hold any more; i.e., the ratio of the stock of money to domestic output need not be equal to the domestic price level. One major implication of these new features is that the welfare level attained in a floating exchange rate system is not necessarily the same as the welfare level attained in a one-sided peg system. The differences that arise are one of the concerns of the present paper.

3. The Home Country's Private Sector

In order to formalize the ideas that were presented in the previous section, we develop a simple model which contains their main ingredients. In our model there are two periods. All borrowing and lending is done at the beginning of the first period, before the uncertainty is resolved. During the first period all the uncertainty is resolved. This means that once the first period uncertainty is resolved, individuals know also all relevant variables (such as prices) in the second period. We assume that economic agents know the state contingent prices, exchange rates and incomes, but they do not know which state will be realized. This is a form of self-fulfilling expectations. However, the probabilities which agents attach to states remain subjective.

Under our assumptions, the decision problem of a representative individual in the home country can be written as follows:

$$\text{Choose } M_H, M_F, B_H, B_F, [c_1(\alpha), c_2(\alpha), m_H(\alpha), m_F(\alpha)]_{\alpha=1}^S \text{ to maximize}$$

$$Eu[c_1(\alpha)] + \delta Eu[c_2(\alpha)]$$

subject to

$$(1) \quad M_H + e_O M_F = M + B_H + e_O B_F$$

$$(2) \quad e_1(\alpha) p_{F1}(\alpha) c_1(\alpha) = M_H + e_1(\alpha) M_F - m_H(\alpha) - e_1(\alpha) m_F(\alpha)$$

$$(3) \quad e_2(\alpha) p_{F2}(\alpha) c_2(\alpha) = m_H(\alpha) + e_2(\alpha) m_F(\alpha) - (1 + r_H) B_H - (1 + r_F) e_2(\alpha) \\ + e_1(\alpha) p_{F1}(\alpha) y_{H1}(\alpha) - T(\alpha)$$

$$(4) \quad M_H, M_F, c_1(\alpha), c_2(\alpha), m_H(\alpha), m_F(\alpha) \geq 0$$

all α running from 1 to S, where:

$u(\cdot)$ = von Neumann-Morgenstern strictly concave (risk averse) utility function and δ is the subjective discount factor

α = state of nature (there are S such states)

M_H = domestic money holdings after the completion of financial transactions at the beginning of the first period

M_F = foreign money holdings (there is only one type of foreign money) after the completion of financial transactions at the beginning of the first period

B_H = domestic currency denominated borrowing (B_H may be positive or negative) at the beginning of the first period, to be repaid at the beginning of the second period

B_F = foreign currency denominated borrowing ($B_F \geq 0$) at the beginning of the first period, to be repaid at the beginning of the second period

All the above decision variables are chosen before the individual knows the state of nature. Now we list the decision variables that are chosen after the resolution of uncertainty.

$c_t(\alpha)$ = consumption in period t when state α realizes, $t = 1, 2$

$m_H(\alpha)$ = home money carried over from period 1 to period 2 in state α ; this is the component of money holdings termed by Hicks "money to hold" (Hicks (1967))

So far we have listed the individual's decision variables (except for α and u). In what follows we list variables that are exogeneous to the individual, but not necessarily to the economy.

- M = the individual's endowment of home money; he has no endowment of foreign money nor past debts or claims (more about this will be said at a later state)
- e_0 = exchange rate (home currency price of a unit of foreign currency) at the beginning of the first period; this exchange rate is used for financial transactions before the resolution of uncertainty
- $e_t(\alpha)$ = exchange rate in period t in state α ; $e_1(\alpha)$ is used in currency markets that reopen in period 1 after the resolution of uncertainty, while $e_2(\alpha)$ is used both at the beginning and during period 2, because, due to lack of uncertainty at the beginning of period 2 about variables during period 2, arbitrage will equalize the two exchange rates
- $p_{Ft}(\alpha)$ = foreign currency price of goods in period t state α ; these prices are exogeneous to both the individual and the home country
- r_H = nominal interest rate on home currency denominated loans; this interest is state independent
- r_F = nominal (state independent) interest rate on foreign currency denominated loans
- $y_{Ht}(\alpha)$ = the home country's output in period t state α on which the individual has a claim
- $T(\alpha)$ = taxes, measured in home currency, which the individual has to pay at the beginning of period 2

Equation (1) describes the individual's constraint on financial transactions at the beginning of the first period. His endowment of home money plus borrowing both in terms of home and foreign currency, determine his liquidity as of the beginning of period one. Since at this instant of time the currency market is operating, he can choose any desired mixture of monies (M_H and M_F). When a state α realizes, the individual obtains information about commodity prices, exchange rates, etc. Given this composition of monies he makes a capital loss or gain (depending on the relationship between $e_1(\alpha)$ and e_0). Then, since the currency market reopens, he can change the composition of his monies, say to $M_H^1(\alpha)$ and $M_F^1(\alpha)$, such that $M_H^1(\alpha) + e_1(\alpha)M_F^1(\alpha) = M_H + e_1(\alpha)M_F$. Now he can use part or all of $M_H^1(\alpha)$ to buy home goods and part or all of $M_F^1(\alpha)$ to buy foreign goods. However, due to free trade in the currency market, the home currency price of goods has to be $e_1(\alpha)p_{F1}(\alpha)$, i.e., purchasing power parity has to be satisfied.^{4/} In this case the individual is indifferent between buying home or foreign goods and his relevant constraint becomes (2), where $c_1(\alpha)$ is his total consumption of goods in period 1 state α , $m_H(\alpha)$ is the amount of home money not spent during the period and $m_F(\alpha)$ is the amount of foreign money not spent during the period. Given the aggregate of money to hold $m_H(\alpha) + e_1(\alpha)m_F(\alpha)$, the individual is free to choose its composition. The components of money to hold are transferred to the second period thereby augmenting his liquidity at the beginning of the second period.^{5/}

At the beginning of the second period the individual's total liquidity is determined by his monies transferred from the first period [$m_H(\alpha)$ and $m_F(\alpha)$], by his debt repayments [$(1+r_H)B_H$ and $(1+r_F)B_F$], by his tax payments $[T(\alpha)]$, and by his first-period income being paid to him at the beginning of the second period $[e_1(\alpha)p_{F1}(\alpha)y_{H1}(\alpha)]$. His income is paid out to him in terms of home

currency, because local firms sell goods for local money and $e_1(\alpha)p_{F1}(\alpha)y_{H1}(\alpha)$ is the value of first-period sales to which the individual has claims. Thus, his aggregate liquid position sums up to the right-hand-side of (3). This can again be divided into home and foreign money. However, since the second period is the last one, all money is spent during this period on goods. Also, since there is no uncertainty during period 2, the reopening of currency markets during period 2 cannot bring about an exchange rate different from the one existing at the beginning of the period. In fact, without uncertainty during a period there is no need to reopen currency markets. As a result purchasing power parity holds with $e_2(\alpha)$ used to convert foreign prices into domestic prices. The individual is again indifferent between domestic and foreign goods and he, therefore, cares only about his aggregate consumption $c_2(\alpha)$, which he can obtain by spending all his beginning of period monies on goods. This explains constraint (3). Constraint (4) expresses the fact that money holdings and consumption levels cannot be negative. No nonnegativity constraints are imposed on borrowing (B_H and B_F), which means that the individual may borrow as well as lend. He is, however, constrained to repay all debts (no bankruptcies).

Assuming that the representative individual coincides with the private consumer sector and that the government does not purchase goods or factors of production, the term, $c_t(\alpha) - y_{Ht}(\alpha)$ represents the economy's deficit in the trade account during period t in state α . $y_{H2}(\alpha)$ does not appear directly in the consumer's constraints because the money value of this real income is paid out with a lag and he is not able to use it. This problem results from the finiteness of the horizon and it disappears in an infinite horizon extension of the present framework. We do, however, confine attention to the

finite horizon case in order to avoid technical complications. Nevertheless, equilibrium values of exchange rates and taxes, and therefore the purchasing power of money do depend on $y_{H2}(\alpha)$ as will become apparent later on.

Firms are rather passive in our formulation-- they sell output for money and distribute the proceeds from sales to consumers who are the claimants on their revenue.

In the formulation of the consumer problem we have not referred to exchange rate regimes. Our formulation is general enough to encompass a variety of exchange rate regimes. The specific features of the decision problem that are associated with an exchange rate regime will be discussed at a later stage. Suffice is to mention only that the taxes $T(\alpha)$ are needed in a fixed exchange rate regime in order to enable the government to stabilize the exchange rate with an intertemporally balanced budget (on this see also Helpman and Razin (1979) and Helpman (1979)).

We now assume that $u(\cdot)$ is defined only on nonnegative values of consumption. In this case the decision problem can be rewritten as the maximization of:

$$\begin{aligned}
 (5) \quad & Eu\{[M_H/e_1(\alpha) + M_F - m_H(\alpha)/e_1(\alpha) - m_F(\alpha)]/p_{F1}(\alpha)\} + \\
 & + \delta Eu\{[m_H(\alpha)/e_2(\alpha) + m_F(\alpha) - (1 + r_H)B_H/e_2(\alpha) - (1 + r_F)B_F + \\
 & + e_1(\alpha)p_{F1}(\alpha)y_{H1}(\alpha)/e_2(\alpha) - T(\alpha)/e_2(\alpha)]/p_{F2}(\alpha)\}
 \end{aligned}$$

subject to (1) and (4). E is the expectations operator. The first order conditions for this problem are:

$$(6) \quad B_H: - (1 + r_H) \delta E \lambda_2(\alpha) / e_2(\alpha) + \mu = 0$$

$$(7) \quad B_F: - (1 + r_F) \delta E \lambda_2(\alpha) + \mu e_O = 0$$

$$(8) \quad M_H: E \lambda_1(\alpha) / e_1(\alpha) - \mu \leq 0$$

$$(9) \quad M_F: E \lambda_1(\alpha) - \mu e_O \leq 0$$

$$(10) \quad m_H(\alpha): - \lambda_1(\alpha) / e_1(\alpha) + \delta \lambda_2(\alpha) / e_2(\alpha) \leq 0, \quad \alpha = 1, 2, \dots, S$$

$$(11) \quad m_F(\alpha): - \lambda_1(\alpha) + \delta \lambda_2(\alpha) \leq 0, \quad \alpha = 1, 2, \dots, S$$

plus the complementary slackness conditions. $\lambda_t(\alpha) = \{du[c_t(\alpha)]/dc_t(\alpha)\}/p_{Ft}(\alpha)$ = period t state α marginal utility of nominal spending, measured in foreign currency, and μ = the Lagrangian multiplier of constraint (1).

The interpretation of the first-order conditions is rather straightforward. The multiplier μ represents the expected marginal utility of home currency available at the beginning of the first period. Let IS (Israeli Shekel) be the home currency. If IS1 is invested in a bond denominated in IS's, it will provide $IS(1+r_H)$ at the beginning of the second period in every state α , or the equivalent of $(1+r_H)/e_2(\alpha)$ units of foreign currency in every state α . Evaluating this return by means of $\lambda_2(\alpha)$ -- the marginal utility of foreign currency at the beginning of the second period in state α -- taking the expectation and discounting to the first period by means of the subjective discount factor δ yields $(1+r_H)\delta E \lambda_2(\alpha)/e_2(\alpha)$. This has to be equal to the marginal expected utility of IS's at the beginning of the first period, i.e., μ . Condition (6) represents precisely this cost benefit calculation. A similar interpretation can be given to (7).

Apart from being invested in bonds at the beginning of the first period, an Israeli Shekel can be kept awaiting use during the first period. If kept, it is available for use in every state α . Its foreign currency equivalent during period 1 is $1/e_1(\alpha)$ which translates into utility by means of $\lambda_1(\alpha)$. Hence, the expected marginal utility of an IS kept is $E\lambda_1(\alpha)/e_1(\alpha)$, and it is not worth keeping if $E\lambda_1(\alpha)/e_1(\alpha) < \mu$. At the optimum one cannot have negative money holdings. Therefore either $E\lambda_1(\alpha)/e_1(\alpha) < \mu$ and $M_H = 0$ or there is equality and $M_H \geq 0$. This is condition (8) when coupled with its complementary slackness condition. A similar interpretation can be given to (9).

Finally, consider the meaning of (10) [a similar interpretation applies to (11)]. After a state α realizes, the individual may spend an IS on goods or he may transfer it to the second period. If he spends it on goods he gains in utility terms $\lambda_1(\alpha)/e_1(\alpha)$ and this is the alternative cost of a forward transfer. If the IS is transferred forward, the second period utility gain is $\lambda_2(\alpha)/e_2(\alpha)$, which after discounting has to be compared with the cost. Since negative transfers cannot be made, then either marginal costs exceed marginal benefits and no transfer is made, or transfers are made to the point at which marginal costs equal marginal benefits. It is easy to see from (10) and (11) that if the individual decides to transfer money to the second period he will transfer home money when $e_1(\alpha) > e_2(\alpha)$ and foreign money when $e_1(\alpha) < e_2(\alpha)$. In the limiting case $e_1(\alpha) = e_2(\alpha)$ he is indifferent between transfers of home and foreign money.

4. The Foreign Country

We assume that the foreign country is very large compared to the home country. As a result there are economic variables that are not affected by the behavior of the small country. In particular foreign currency commodity prices $p_{Ft}(\alpha)$ are determined in the foreign country and the home country faces infinitely elastic demands and supplies at these prices (in case of sales taking place in terms of home money the foreign demand for home goods is infinitely elastic at $e_t(\alpha)p_{Ft}(\alpha)$ due to purchasing power parity). In addition, the interest rate on foreign currency denominated debt, r_F , is determined by the foreign country; the home country's demand or supply of foreign currency denominated debt cannot affect this interest rate.

In a deterministic environment the specification of foreign prices and interest rates provides all the information about the foreign country that is required for the purpose of analyzing problems pertaining to the small country. Purchasing power and interest rate parities provide in this case the links between foreign and domestic prices and foreign and domestic interest rates. These parity conditions are derivable from optimizing behavior. In an uncertain environment with complete markets it suffices to specify state contingent present value foreign currency prices of each period consumption that prevail in the foreign country. Then using these prices appropriate parity condition can be used to link foreign prices and interest rates to domestic prices and interest rates. However, in the absence of complete markets no such simple links exist, and depending on the structure of financial markets there are specific links which have to be satisfied. Since we do not have complete

markets, it is necessary to specify precisely what links exist between foreign and domestic asset prices that are consistent with foreigners' optimizing behavior. For this purpose we assume that the smallness of the home country is reflected in the fact that foreign marginal utility of foreign currency spending in each period t and every state α , denoted by $\lambda_t^*(\alpha)$, is independent of decisions undertaken by the small country. In addition, the foreigners' subjective discount factor δ^* is constant.

Now assume that foreigners solve a decision problem which is similar to the decision problem solved by a home country resident, except that foreigners are endowed only with foreign money at the beginning of the first period and they receive income from foreign firms. Then conditions such as (6)-(11) apply to the foreign country. Denoting by asterisks foreign decision variables, the following conclusions can be drawn. From (6) and (7), applied to the foreign country:

$$(12) \quad (1+r_H) = (1+r_F) \frac{E \lambda_2^*(\alpha)/e_0}{E \lambda_2^*(\alpha)/e_2(\alpha)}$$

This is the interest rate parity condition in the present context. Observe that in the absence of exchange rate uncertainty in the second period this condition reduces to the deterministic interest rate parity condition. Given (12) the home country can borrow and lend in terms of either currency as much as it wants.

We assume that foreigners hold foreign money at the beginning of the first period. This means that they are willing to hold also home money at the beginning of the first period if [combine (8) with (9), using equalities]:

$$e_o = \frac{E\lambda_1^*(\alpha)}{E\lambda_1^*(\alpha)/e_1(\alpha)}$$

In case of certainty this requires $e_o = e_1$, i.e., the exchange rate does not change during the period. The important lesson from this condition is that foreigners hold home money only if a particular link exists between e_o and the distribution of $e_1(\alpha)$, with the link determined by foreigners' marginal utilities of spending. Generally, (8) and (9), applied to foreigners, impose a restriction on permissible distributions of $e_1(\alpha)$ given e_o , which is:

$$(13) \quad e_o \leq \frac{E\lambda_1^*(\alpha)}{E\lambda_1^*(\alpha)/e_1(\alpha)} \quad \text{with } M_H^* = 0 \quad \text{if strict inequality holds.}$$

The bigness of the foreign country is reflected in the fact that when (13) is satisfied with equality, foreigners have an infinitely elastic demand for home money.

Applying (11) to foreigners yields:

$$(14) \quad -\lambda_1^*(\alpha) + \delta^* \lambda_2^*(\alpha) \leq 0, \quad \alpha = 1, 2, \dots, S$$

If strict inequality holds in (14), $m_F^*(\alpha) = 0$, but this is irrelevant from the point of view of the home country. What is, however, relevant is the application of condition (10) to the foreign country, which reads:

$$(15) \quad -\lambda_1^*(\alpha)/e_1(\alpha) + \delta^* \lambda_2^*(\alpha)/e_2(\alpha) \leq 0 \quad \text{with } m_H^*(\alpha) = 0 \quad \text{if strict}$$

inequality holds, $\alpha = 1, 2, \dots, S$

When (15) holds with equality, foreigners have an infinitely elastic demand for home money to hold. In addition, (14) combined with (15) impose restrictions on permissible exchange rate movements. In particular, if (14) holds with equality in a state α , which is the case when foreigners wish to transfer foreign money from period 1 to period 2 in state α , then $e_1(\alpha)$ has to be larger than or equal to $e_2(\alpha)$ for (15) to hold.

Conditions (12)-(15) together with the values of commodity prices $p_{Ft}(\alpha)$ and the interest rate r_F represent the constraints imposed by the foreign country. These constraints have to be taken into account in every definition of an equilibrium.

5. Floating Exchange Rate

In the floating exchange rate regime we assume that there is no governmental intervention. By this we mean that there are no open market operations, no taxes, and no interventions in the foreign exchange market. As a result, all domestic private plus foreign excess demands have to be zero with $T(\alpha) = 0$, $\alpha = 1, 2, \dots, S$. In addition to (12)-(16), the equilibrium conditions are:

(where M_H , B_H , B_F and $m_H(\alpha)$ solve the home country's consumer problem):

$$(16) \quad M_H + M_H^* = M$$

$$(17) \quad B_H + B_H^* = 0$$

$$(18) \quad e_1(\alpha) p_{F1}(\alpha) y_{H1}(\alpha) = M - m_H(\alpha) - m_H^*(\alpha), \quad \alpha = 1, 2, \dots, S$$

$$(19) \quad e_2(\alpha) p_{F2}(\alpha) y_{H2}(\alpha) = M, \quad \alpha = 1, 2, \dots, S$$

Condition (16) represents equilibrium in the domestic money market at the beginning of the first period. Observe, however, that $M_H^* = 0$ if the inequality in (13) is strict. Condition (17) represents equilibrium in the domestic bond market. However, given (12), it imposes no restriction on B_H . Conditions (18) and (19) represent equilibria in domestic commodity markets. The value of sales in the first period (state α) is represented by the left-hand-side of (18), while the right-hand-side represents aggregate domestic money to spend, and the two have to be equal to each other. The right-hand-side of (18) represents aggregate domestic money spent on goods in period 1, because it is the difference between the existing domestic stock of money and private holdings of home money that are transferred from the first to the second period

by both home residents and foreigners. If (15) holds with strict inequality, $m_H^*(\alpha) = 0$. In the second period the demand for money to hold is zero, because it is the last period. Therefore all money is spent on goods. This is represented by (19). Due to a passive monetary policy, available home money in period 2 equals M .

6. Fixed Exchange Rate

In order to compare fixed with floating exchange rate regimes, we need to set them up on the same footing. For this purpose we assume that in the fixed exchange rate regime the pegging country faces foreign variables and constraints that are the same as in a floating exchange rate regime. In particular, the conditions specified in Section 4, (12)-(15), have to be satisfied. In a fixed exchange rate regime, $e_0 = e_1(\alpha) = e_2(\alpha) = e$, $\alpha = 1, 2, \dots, S$. As a result, the interest parity condition (12) implies equality between the domestic and foreign interest rates.

$$(20) \quad r_H = r_F$$

This result stems from the fact that with the exchange rate being constant over time domestic credit is a perfect substitute for foreign credit.

Condition (13) is trivially satisfied with equality. This implies that foreigners have an infinitely elastic demand for domestic money at the beginning of the first period.

The constancy of the exchange rate makes (15) equivalent to (14). Since (14) is always satisfied, it implies that (15) is also satisfied. The implication is that in those states in which foreigners wish to transfer money from period 1 to period 2, they are indifferent between transfers of foreign and domestic money. As a result, in those states foreign demand for domestic money to hold is infinitely elastic. On the other hand, in those states in which foreigners do not wish to transfer foreign money from period 1 to period 2 they also do not wish to make transfers of domestic money.

We assume that the home country's government engages in a one-sided peg by replacing the foreign exchange market. It stabilizes the exchange rate by borrowing foreign currency whenever it is required to sell foreign currency and it lends foreign currency whenever it is required to buy foreign currency. In the absence of uncertainty this regime yields the same allocation of real resources as a floating exchange rate regime. In this respect the two regimes are on the same footing. Our purpose is to investigate differences that arise between regimes as a result of uncertainty elements and imperfect capital market

Let us start by defining an equilibrium in the one-sided peg regime. For this purpose we need to define some of the government's decision variables. Let:

e = the fixed level of the exchange rate

X_0 = net increase in the stock of home money at the beginning of the first period that results from the exchange rate stabilization operations

$X_1(\alpha)$ = net increase in the stock of home money during the first period in state α on account of exchange rate stabilization

$X_2(\alpha)$ = net increase in the stock of home money at the beginning of the second period in state α on account of exchange rate stabilization

We assume a passive monetary policy (as in the floating exchange rate regime) so that all changes in the domestic money supply stem from the exchange rate stabilization operations.

Analogously to (16)-(19), the equilibrium conditions in the floating exchange rate regime, we have the following equilibrium conditions in the fixed exchange rate regime (in order to save on notation, we do not introduce

regime-specific notation for equilibrium variables):

$$(21) \quad M_H + M_H^* = M + X_O$$

$$(22) \quad B_H + B_H^* = 0$$

$$(23) \quad ep_{F1}(\alpha)y_{H1}(\alpha) = M + X_O + X_1(\alpha) - m_H(\alpha) - m_H^*(\alpha), \quad \alpha = 1, 2, \dots, S$$

$$(24) \quad ep_{F2}(\alpha)y_{H2}(\alpha) = M + X_O + X_1(\alpha) + X_2(\alpha), \quad \alpha = 1, 2, \dots, S$$

The meaning of these conditions is the same as the meaning of (16)-(19) and we shall not reiterate it. It should only be observed that in the present regime there are monetary injections which result from the stabilization of the exchange rate and they change the domestic stock of money. As a result, in (21), (23) and (24), we have the initial stock of money, M , plus appropriate monetary injections. In (22) there is no component of governmental borrowing, because the government does not engage in open market operations.

Since the government is active in the present regime, we need to specify its budget constraints. The most important aspect of these constraints is that all foreign debts have to be repaid. In order to make good its obligations it is allowed to impose taxes in the second period so that now $T(\alpha)$ need not be zero in all states.

Let B_F^g be the government's foreign borrowing at the beginning of the first period, to be repaid (including interest) at the beginning of the second period. B_F^g may be positive or negative. It may turn negative if foreigners buy home money, since for home money they pay with foreign money. Also, let M_F^g be the quantity of foreign money in the hands of the government after the completion of the financial transactions at the beginning of the first period. M_F^g represents what is usually called official reserve holdings. Then:

$$(25) \quad M_F^g = B_F^g + (B_F - M_F) + (M_H^* - B_H^*)/e \geq 0$$

Foreign money holdings by the local government are determined by three factors: (a) Its foreign currency borrowing, B_F^g ; (b) Net sales (which may be negative) of foreign currency by domestic residents. Since domestic residents start with no foreign currency, their net sales equal their foreign borrowing minus the quantity of foreign money that they leave in their possession, $B_F - M_F$; and (c) Net sales of foreign money by foreigners, which stem from their net purchase of domestic money. Since foreigners start with no domestic money, their net purchase of domestic money equals the difference between their desired holdings of home money and their borrowing of home money. Thus, their net sales of foreign currency equal $(M_H^* - B_H^*)/e$. There is a nonnegativity restriction on M_F^g because money cannot be held in negative quantities.

As a result of the above described operations there are changes in the domestic stock of money. The government prints home money (or absorbs) in order to cover its net purchases of foreign money. Its net purchases of foreign money equal the difference between the level of its reserve holdings and its foreign borrowing. Hence,

$$(26) \quad X_O = e(M_F^g - B_F^g)$$

The government's reserve holdings have to be sufficiently high in order to enable exchange rate stabilization during period 1. This means that the level of reserves should be at least as high as the expected net sales of foreign currency required in every state α , because during period 1 no borrowing can take place. This constraint can be expressed as follows:

$$(27) \quad M_F^g \geq p_{F1}(\alpha)c_{F1}(\alpha) + m_F(\alpha) - M_F - [p_{F1}(\alpha)c_{H1}^*(\alpha) + m_H^*(\alpha)/e - M_H^*/e], \alpha = 1, 2, \dots$$

where $c_{F1}(\alpha)$ stands for domestic consumption of foreign goods in period 1 state α (i.e., domestic imports) and $c_{H1}^*(\alpha)$ stands for foreign consumption of home goods in period 1 state α (i.e., domestic exports).

The right-hand-side of (27) represents the net excess demand for foreign currency with which the domestic government is faced during period 1 in state α . It has two components-- a domestic and a foreign component. The domestic excess demand of the private sector equals the demand for foreign currency needed for purchases of foreign goods, $p_{F1}(\alpha)c_{F1}(\alpha)$, plus the demand for money to hold, $m_F(\alpha)$, minus the quantity of foreign money in the hands of the private sector that was acquired at the beginning of the first period before the realization of a state α , M_F . The foreign component stems from foreigners excess demand for home money which is paid for with foreign money; this is represented by the terms in the square bracket on the right-hand-side of (27). The interpretation of the foreigners' excess demand for home money is similar to the interpretation of domestic residents' excess demand for foreign money.

In equilibrium $c_{F1}(\alpha) - c_{H1}^*(\alpha) = c_1(\alpha) - \gamma_{H1}(\alpha)$, i.e., imports minus exports equal consumption minus output -- this is the real deficit in the trade account. Using this relationship, and defining $m_F^g(\alpha)$ as the idle reserves in state α ; i.e., the quantity of foreign reserves that remain in the hands of the government at the end of the first period (after the completion of all first period transactions), (27) can be rewritten as

$$(28) \quad M_F^g = m_F^g(\alpha) + m_F(\alpha) - m_H^*(\alpha)/e - M_F + M_H^*/e + p_{F1}(\alpha)[c_1(\alpha) - \gamma_{H1}(\alpha)], \alpha = 1, 2, \dots$$

$$(29) \quad m_F^g(\alpha) \geq 0$$

In order to complete the description of the flow of funds during the first period it remains to determine the monetary injection that takes place as a result of exchange rate stabilization operations. Since the government starts with M_F^g units of foreign reserves and it ends up with $m_F^g(\alpha)$ units of idle reserves, it means that during the period it has acquired $m_F^g(\alpha) - M_F^g$ units of foreign currency. These acquisitions are paid for with newly created home money, so that the monetary injection during period one is

$$(30) \quad X_1(\alpha) = e[m_F^g(\alpha) - M_F^g], \quad \alpha = 1, 2, \dots, S$$

It is now straightforward to verify that the first period equilibrium condition in the home goods market, (23), is satisfied as a result of the monetary injections that result from exchange rate stabilization operations. This is done by substituting on the right-hand-side of (23) equations (26), (30), (28), (22), (1) and (2), where in (1) and (2) e is used instead of e_0 and $e_1(\alpha)$.

Now let us consider the last period constraint on the government. The government's acquisition of foreign currency equals $(1+r_F)B_F^g - m_F^g(\alpha)$ and it collects taxes $T(\alpha)$. Hence, in order to balance its budgets, the monetary injection has to be:

$$(31) \quad X_2(\alpha) = e[(1+r_F)B_F^g - m_F^g(\alpha)] - T(\alpha), \quad \alpha = 1, 2, \dots, S'$$

However, its taxes cannot be arbitrary, for they have to produce the exact absorption needed to sustain equilibrium at the chosen exchange rate. Combining the last period equilibrium condition in the commodity market (24) with (26), (30) and (31), we calculate the required taxes as:

$$(32) \quad T(\alpha) = M + r_F e B_F^g - e p_{F2}(\alpha) y_{H2}(\alpha), \quad \alpha = 1, 2, \dots, S$$

A special feature of this tax formula is that it depends in no direct way on decisions of the private sector. As a result, if we substitute (32) into (3), the appropriate version of the consumer decision problem will not be changed.

It is instructive to consider at this point the market for foreign exchange, and to see that the above described tax structure assures also clearing of the foreign exchange market. In the second period all financial transactions are carried out at the beginning of the period because there is no more uncertainty. The domestic private sector has an excess demand for foreign currency which equals $p_{F2}(\alpha) c_{F2}(\alpha) - m_F(\alpha) + (1+r_F) B_F$, where $c_{F2}(\alpha)$ stands for second period imports. Foreigners have an excess demand for home money that equals $e p_{F2}(\alpha) c_{H2}^*(\alpha) - m_H^*(\alpha) + (1+r_H) B_H^*$ for which they pay with foreign currency. In order to repay its debts, the government needs $(1+r_F) B_F^g$ units of foreign currency. This means that $(1+r_F) B_F^g - m_F^g(\alpha)$ has to be acquired in the market (remember that $m_F^g(\alpha)$ represents idle reserve holdings). Hence, the excess demand for foreign currency, E_F , is:

$$(33) \quad E_F = (1+r_F) B_F^g - m_F^g(\alpha) + p_{F2}(\alpha) c_{F2}(\alpha) - m_F(\alpha) + (1+r_F) B_F - [e p_{F2}(\alpha) c_{H2}^*(\alpha) - m_H^*(\alpha) + (1+r_H) B_H^*]$$

We want to show that $E_F = 0$. For this purpose, first combine (1)-(3), using $e_0 = e_1(\alpha) = e_2(\alpha) = e$, with (32), to obtain:

$$(34) \quad \sum_{t=1}^2 p_{Ft}(\alpha) [c_t(\alpha) - y_{Ht}(\alpha)] = -r_F (B_F^g + B_F + B_H/e)$$

Now, using $c_{F2}(\alpha) - c_{H2}^*(\alpha) = c_2(\alpha) - y_{H2}(\alpha)$, (20), (25) and (28), (33) and (34) imply $E_F = 0$, i.e., clearance of the foreign exchange market.

We are now in a position to formulate the effective constraints which are faced by the economy. One set of constraints applies to the private sector. This set is represented by the fixed exchange rate equivalent of constraints (1) - (4). Due to the fact that home and foreign currency denominated debts are perfect substitutes, [see (20)], we may aggregate them into a single debt variable $B = B_H/e + B_F$ and write the constraints (1)-(4) as:

$$(35) \quad M_H + eM_F = M + eB$$

$$(36) \quad ep_{F1}(\alpha)c_1(\alpha) = M_H + eM_F - m_H(\alpha) - em_F(\alpha)$$

$$(37) \quad ep_{F2}(\alpha)c_2(\alpha) = m_H(\alpha) + em_F(\alpha) - (1+r_F)eB + ep_{F1}(\alpha)y_{H1}(\alpha) - T(\alpha)$$

$$(38) \quad M_H, M_F, c_1(\alpha), c_2(\alpha), m_H(\alpha), m_F(\alpha) \geq 0$$

all α running from 1 to S.

The second set of constraints concerns the government. They can be summarized as follows: Initial reserve holdings and idle reserves have to be nonnegative, and foreign debts have to be repaid, which amounts to imposing taxes according to (33). Using (25), (22), (28), (31) and the definition of B, i.e., $B = B_H/e + B_F$, the first two constraints can be represented as:

$$(39) \quad eB_F^g + eB - eM_F + M_H^* \geq 0$$

$$(40) \quad eB_F^g + m_H(\alpha) + m_H^*(\alpha) + ep_{F1}(\alpha)y_{H1}(\alpha) - M \geq 0, \alpha = 1, 2, \dots, S$$

Constraint (39) represents the requirement that initial reserves be nonnegative while (40) represents the requirement that idle reserves be nonnegative in every state α .

It should be observed at this stage that the government is free to choose the foreign variables M_H^* and $m_H^*(\alpha)$ subject to the foreign constraints (13) and (15) with e replacing e_0 , $e_1(\alpha)$ and $e_2(\alpha)$. Since (13) is satisfied with equality in the fixed exchange rate regime, it means that foreigners have an infinitely elastic demand for home money. As a result, the government can sell to foreigners as much home money as it needs in order to build up the desired stock of initial reserves. Hence (39) is not a binding constraint [observe that M_H^* appears only in (39)].

The degree of freedom that exists in the choice of foreign holdings of domestic money at the beginning of the first period does not exist in the choice of foreign transfers of domestic money from period 1 to period 2. With a fixed exchange rate, condition (15) holds with equality only if foreigners wish to transfer foreign money from the first to the second period, i.e., when (14) holds with equality. However, with a positive interest rate it is never optimal to do it in all states.^{6/} Therefore there must be some states in which foreigners refuse to hold positive amounts of $m_H^*(\alpha)$ so that in these states the local government is constrained to choose $m_H^*(\alpha) = 0$. In these states constraint (40) cannot be dispensed with on a priori grounds. On the other hand, in a state α in which foreigners are willing to transfer money from period 1 to period 2, their demand for domestic money to hold is infinitely elastic, so that the government can choose every nonnegative $m_H^*(\alpha)$. In such states constraint (40) is ineffective [observe that $m_H^*(\alpha)$ appears only in (40)]. Let S^* denote the set of states in which foreigners refuse to transfer money from period 1 to period 2.

$$(41) \quad S^* = \{\alpha \mid \lambda_1^*(\alpha) > \delta^* \lambda_2^*(\alpha)\}$$

Then the effective set of constraints on the government, including the tax formula (38), can be represented by:

$$(42) \quad eB_F^g + m_H(\alpha) + e p_{F1}(\alpha) y_{H1}(\alpha) - M \geq 0, \quad \alpha \in S^*$$

$$(43) \quad T(\alpha) = M + r_F eB_F^g - e p_{F2}(\alpha) y_{H2}(\alpha), \quad \alpha = 1, 2, \dots, S$$

It is seen from (35)-(38) and (42)-(43) that the government's actions affect individual choices, and private choices affect the government's feasible policy variables. Therefore, we need to specify the government's objective function (the private sector's objective function was defined as expected utility maximization). We assume that the government wishes to maximize the private sector's expected utility level.

For the present purpose, it is not sufficient to define an objective function for the government; we also need to specify what it is allowed to do. We choose to distinguish between two alternatives. The first alternative, which we call the ordinary fixed exchange rate regime, is characterized by a two stage process in which the government chooses the exchange rate, its foreign borrowing and the above defined lump-sum taxes in the first stage, and the private sector optimizes subject to the government's choices. The second alternative, which we call the optimal fixed exchange rate regime, involves maximization of expected utility subject to (35)-(43) jointly. As we will show, in order to support the resulting allocation of this optimization problem the government is required to introduce additional policy tools.

It is worth pausing at this point in order to observe that the choice of the exchange rate level e (as long as e is positive and finite) does not

affect the feasible allocations of consumption in both the ordinary and the optimal fixed exchange rate regimes. The proof is straightforward by simple manipulations of the constraints. The implication is that the equilibrium consumption levels are also independent of the exchange rate level. For this reason, we restrict attention to governmental choices of taxes and foreign debt, holding the exchange rate at a predetermined level.

7. Ordinary versus Optimal Fixed Exchange Rate

The ordinary fixed exchange rate regime is the regime which is commonly referred to as the fixed exchange rate regime. In this regime the government chooses an exchange rate level and engages in exchange rate stabilization. Since we deal with a one-sided peg, it also imposes lump-sum taxes in order to make good its obligation to repay debts. No other policy measures are used by the government. The private sector optimizes subject to its constraints which are affected by the tax structure. In the present case the first order conditions for the private sector are conditions (6)-(11), with $e_0 = e_1(\alpha) = e_2(\alpha) = e$. The solution to the private sector's decision problem yields demand functions or correspondences from which we single out for the present purpose the demand for domestic money to hold. These demands depend on all variables which are exogeneous to the individual, in particular on taxes. Suppressing all exogeneous variables except for taxes, we have:

$$(44) \quad m_H(\alpha) = \tilde{m}_H[\alpha, T(1), T(2), \dots, T(S)]$$

where $\tilde{m}_H(\cdot)$ is the demand correspondence for domestic money to hold in state α . This is a correspondence because in a fixed exchange rate regime home money and foreign money are perfect substitutes.

The solution to the private decision problem enables one to define a function which describes the maximal expected utility level as a function of the exogeneous variables. Suppressing all exogenous variables except for taxes, we use:

$$V[T(1), T(2), \dots, T(S)]$$

to denote the maximal expected utility level attainable under the tax vector $T(\alpha)$, $\alpha = 1, 2, \dots, S$.

Now, using (42) and (43), we represent the government's decision problem in the ordinary fixed exchange rate regime as:

(45) Choose B_F^g and $T(1), T(2), \dots, T(S)$ to maximize $V[T(1), T(2), \dots, T(S)]$ subject to (42)-(44)

The solution to problem (45) is easily characterized. Using the Envelope Theorem, one finds immediately that $V(\cdot)$ is declining in each one of its arguments. Since the interest cost of public foreign debt is passed to the private sector through taxes, $T(\alpha)$ increases with B_F^g for every α . Hence, the government's objective is to minimize B_F^g subject to (42)-(44). This implies that, when B_F^g is optimally chosen, (42) is satisfied with equality in at least one $\alpha \in S^*$. Since consumers are indifferent between $m_H(\alpha)$ and $m_F(\alpha)$, optimality requires to have $m_F(\alpha) = 0$ in those states in which (42) is binding thus having $m_H(\alpha)$ maximal to enable lowest B_F^g 's.

The nature of problem (45) reveals that the ordinary fixed exchange rate regime is not the most efficient fixed exchange rate regime. This is so because the government generates an externality through its exchange rate stabilization operations and tax policy. This can be seen as follows. When consumers choose $m_H(\alpha)$, they do not take into account the fact that their choice affects the government's constraint on exchange rate stabilization operations, (42), which may require an adjustment in government debt, thus feeding back into the private sector through tax changes which stem from the dependence of taxes on servicing of the government's foreign debt.

The optimal fixed exchange rate regime allocation is obtained by maximizing expected utility subject to (35)-(38) and (42)-(43). The first order conditions of this problem reveal the need for differential pricing of $m_H(\alpha)$ and $m_F(\alpha)$. In particular, in the states in which (42) is binding, the marginal value of $m_H(\alpha)$ is larger than that of $m_F(\alpha)$. This means that in those states, interest should be paid to domestic residents on domestic money to hold. Generally, state dependent interest rates applied to domestic money to hold (with some rates being zero) is a policy that supports the optimal allocation. We have, therefore, a revised optimal quantity of money rule.

8. Fixed vs. Floating Exchange Rate Regime

The comparison of exchange rate regimes is a complicated problem. In the present framework, the added complexity stems from the incompleteness of the financial markets. Remember that in the absence of uncertainty fixed and floating exchange rate regimes yield the same allocation of resources and the same welfare levels. In the presence of uncertainty and incomplete markets, as we have here, different exchange rate regimes provide different risk-sharing and risk-spreading opportunities. Therefore, in a way, the question of whether one exchange rate regime is preferred to another embodies also the question which regime provides better opportunities of risk spreading.

Risk is traded by means of financial assets. The structure of returns on these assets generates the risk-spreading opportunities. However, the structure of returns on existing assets depends on the exchange rate regime. In a fixed exchange rate regime domestic bonds have the same structure of real returns as do foreign bonds because in this regime domestic prices are completely correlated with foreign prices. It is not so in a floating exchange rate regime in which due to exchange rate fluctuations domestic bonds have a different structure of real returns than foreign bonds. Also, the fixed exchange rate regime has a special feature which arises from the necessity to engage in a state dependent absorption policy. The tax liabilities play the role of a financial asset as far as risk-spreading opportunities are concerned, except that this 'asset' cannot be traded.

It is well known that in systems with incomplete markets it is difficult to provide welfare rankings (see Hart (1975)), and we have not been successful

in deriving an unqualified result regarding the most preferred exchange rate regime. We have, however, been able to identify circumstances in which a floating exchange rate regime provides a higher expected utility level than a fixed exchange rate regime. Under these circumstances the floating exchange rate regime is at least as good as the optimal fixed exchange rate regime, which implies that it strictly dominates the ordinary fixed exchange rate regime. In the remaining part of this section we identify those circumstances.

From the concavity of $u(\cdot)$, we have:

$$\begin{aligned} \Delta &\stackrel{\text{def}}{=} E \{u[c_1(\alpha)] + \delta u[c_2(\alpha)] - u[c_1^f(\alpha)] - \delta u[c_2^f(\alpha)]\} \\ &\leq E[\lambda_1^f(\alpha) p_{F1}(\alpha) c_1(\alpha) + \delta \lambda_2^f(\alpha) p_{F2}(\alpha) c_2(\alpha)] \\ &\quad - E[\lambda_1^f(\alpha) p_{F1}(\alpha) c_1^f(\alpha) + \delta \lambda_2^f(\alpha) p_{F2}(\alpha) c_2^f(\alpha)] \end{aligned}$$

where superscript f denotes equilibrium variables in a floating exchange rate regime while variables without superscripts denote variables in a fixed exchange rate regime. However, using (2) and (3)

$$\begin{aligned} v^{\text{fdef}} &= E[\lambda_1^f(\alpha) p_{F1}(\alpha) c_1^f(\alpha) + \delta \lambda_2^f(\alpha) p_{F2}(\alpha) c_2^f(\alpha)] \\ &= E[-\lambda_1^f(\alpha)/e_1(\alpha) + \delta \lambda_2^f(\alpha)/e_2(\alpha)] m_H^f(\alpha) + E[-\lambda_1^f(\alpha) + \delta \lambda_2^f(\alpha)] m_F^f(\alpha) \\ &\quad + E[\lambda_1^f(\alpha)/e_1(\alpha)] M_H^f + E\lambda_1^f(\alpha) M_F^f - (1 + r_H) \delta E[\lambda_2^f(\alpha)/e_2(\alpha)] B_H^f \\ &\quad - (1 + r_F) \delta E\lambda_2^f(\alpha) B_F^f + \delta E\lambda_2^f(\alpha) p_{F1}(\alpha) \gamma_{H1}(\alpha) e_1(\alpha)/e_2(\alpha) \end{aligned}$$

which yields, after the application of (6) - (11), the complementarity slackness conditions, (1), and the assumption that in a floating exchange rate equilibrium

domestic residents are willing to hold foreign money at the beginning of the first period:

$$(47) \quad v^f = \mu^f M + \delta E \lambda_2^f(\alpha) p_{F1}(\alpha) y_{H1}(\alpha) e_1(\alpha) / e_2(\alpha)$$

Combining (34) and (47) with (46), we obtain:

$$(48) \quad \begin{aligned} \Delta \leq & E[\lambda_1^f(\alpha) - \delta \lambda_2^f(\alpha)] p_{F1}(\alpha) [c_1(\alpha) - y_{H1}(\alpha)] \\ & + E[\lambda_1^f(\alpha) - \delta \lambda_2^f(\alpha) e_1(\alpha) / e_2(\alpha)] p_{F1}(\alpha) y_{H1}(\alpha) \\ & + \delta E \lambda_2^f(\alpha) [p_{F2}(\alpha) y_{H2}(\alpha) - r_F (B_F^g + B)] - \mu^f M \end{aligned}$$

Now, substituting (35) - (36) into (42), we obtain:

$$(49) \quad p_{F1}(\alpha) [c_1(\alpha) - y_{H1}(\alpha)] \leq B_F^g + B - m_F(\alpha), \quad \alpha \in S^*$$

where S^* is the set of states in which foreigners' money to hold equals zero. Assuming that $S^* = S$, i.e., foreigners do not use money as a store of value (in a fixed exchange rate regime), one can combine (49), (11) and the constraint $m_F(\alpha) \geq 0$ to obtain via (48):

$$(50) \quad \begin{aligned} \Delta \leq & E[\lambda_1^f(\alpha) - (1 + r_F) \delta \lambda_2^f(\alpha)] (B_F^g + B) \\ & + E[\lambda_1^f(\alpha) - \delta \lambda_2^f(\alpha) e_1(\alpha) / e_2(\alpha)] p_{F1}(\alpha) y_{H1}(\alpha) \\ & + \delta E \lambda_2^f(\alpha) p_{F2}(\alpha) y_{H2}(\alpha) - \mu^f M \end{aligned}$$

Our assumption that in a floating exchange rate equilibrium domestic residents are willing to hold foreign money at the beginning of the first period implies via (7) and (9) that the first term on the right-hand-side of (50) is zero. The using (8) and (19), we obtain:

$$(51) \quad 0 \leq E[\lambda_1^f(\alpha)/e_1(\alpha) - \delta \lambda f(\alpha)/e_2(\alpha)][e_1(\alpha)p_{F1}(\alpha)y_{H1}(\alpha) - M]$$

Due to (10), the expression in the first square bracket on the right-hand-side of (51) is nonnegative while due to (18) the expression in the second square bracket on the right-hand side of (51) is nonpositive. Hence, $\Delta \leq 0$, implying that the allocation obtained in the floating exchange rate regime is weakly preferred to the allocation obtained in the fixed exchange rate regime. Observe however, that this implies that the floating exchange rate regime allocation is strictly preferred to the ordinary fixed exchange rate regime allocation because the information on the fixed exchange rate regime that we have used in the proof holds also in the optimal fixed exchange rate regime. Thus, the floating regime is preferred to the optimal fixed exchange rate regime and it is, therefore, strictly preferred to the ordinary fixed exchange rate regime. One should only remember that this has been proved to be the case if the following assumptions are satisfied:

- (a) Foreigners do not use money as a store of value; and
- (b) In the floating exchange rate equilibrium domestic residents are willing to hold foreign money at the beginning of the first period.

On the other hand one should also remember that it was assumed that in the fixed exchange rate regime the government engages in utility maximizing foreign debt policies and non-distortionary tax policies. Clearly, if the government does not pursue these policies, the case for a floating exchange rate regime becomes even stronger.

F O O T N O T E S

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1. This statement needs to be qualified with regard to Lucas (1980) and Lapan and Ender (1980). Lucas finds that fixed and floating exchange rates produce the same real allocation, and therefore the same expected utility levels, even though his model contains imperfections in financial markets. This result stems from assumed sufficient symmetry between countries. His study represents the first attempt to derive a neutrality result in the presence of imperfections. Lapan and Enders, on the other hand, argue that expected utility is higher in the fixed exchange rate regime. However, their comparison is based on a model in which there is a built-in asymmetry in the exchange rate regimes which allows international capital flows in the fixed exchange rate regime but does not allow them in the floating exchange rate regime. We have discussed the nature of this common misspecification in Helpman and Razin (1979). It is easy to show that in the Lapan and Enders model there exists a floating exchange rate equilibrium which is identical to the fixed exchange rate equilibrium with regard to all real variables, if one allows in the floating exchange rate regime holdings of both home and foreign money, which amounts to permitting capital movements.

2. Persson (1980) has analyzed differences in allocations that result from cooperative pegs in currency unions.
3. It is not essential to have each currency denominated bond being traded against its currency of denomination, because at this instant of time monies can be traded for each other.
4. If we were to assume that the currency market does not reopen during period 1 after the realization of a state of nature, then purchasing power parity, using either e_0 or $e_2(\alpha)$ to translate foreign prices into home currency units would not hold. This specification is used in Lucas (1980).
5. Clearly, since $e_2(\alpha)$ is known after the realization of α during period one, then if he chooses to transfer money forward the individual will choose to transfer home money when $e_2(\alpha) < e_1(\alpha)$ and foreign money when $e_2(\alpha) > e_1(\alpha)$.
6. This can be seen as follows. Suppose foreigners transfer foreign money from period 1 to period 2 in all states α , then $\lambda_1^*(\alpha) = \delta * \lambda_2^*(\alpha)$ for all α . Applying conditions (7) and (9) to foreigners, with $e = e_0 = e_1(\alpha) = c_2(\alpha)$ and requiring foreigners to hold foreign money in the beginning of the first period, which amounts to requiring (9) to hold with equality, we get $E\lambda_1^*(\alpha) = (1 + r_F) * E\lambda_2^*(\alpha)$. These two conditions are inconsistent for $r_F > 0$.

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