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TAX SMOOTHING VERSUS TAX SHIFTING

by

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Tax Smoothing versus Tax Shifting*

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Abstract

Household-specific growth rates of the tax base imply that the timing of tax collections determines the distribution of tax burdens and wealth across households. Changes in financial policy do not only shift taxes across generations, but also within cohorts. Institutional deficit constraints settle tax shifting conflicts in favor of individuals with high income growth. With distortionary taxes, policy makers trade off the wealth effects of financial policy and the efficiency cost of household-specific deadweight burdens. I apply the incidence analysis of financial policy to two examples: The financing of the German unification, and the timing of tax collections over the U.S. business cycle.

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1 Introduction

In a seminal paper, Barro (1974) argued that the Ricardian equivalence proposition holds as a first-order approximation because households can neutralize the principal effects of a marginal change in financial policy. Barro, and a large subsequent literature, concluded that second-order considerations, in particular the cost of tax induced distortions, determine the optimal timing of tax collections (Barro, 1979).\(^1\) In this paper, I discuss a principal effect of financial policy that the private sector does not neutralize, and therefore a first-order determinant of optimal financial policy: its direct impact on the wealth distribution.

Household-specific growth rates of the tax base imply that the timing of tax collections determines the cross sectional distribution of tax burdens. Under a proportional income tax, for example, budget deficits and subsequent surpluses replicate a wealth tax that is progressive in the growth rate of income. Changes in financial policy shift the tax burden by altering that progression. Household-specific growth rates of the tax base break Ricardian equivalence although markets are complete.\(^2\) While the private sector could, in principle, sterilize the effects of a policy change, ex ante heterogeneous households choose not to do so. The non-neutrality is solely due to the presence of individual budget constraints; it disappears as soon as households trade behind the Veil of Ignorance.\(^3\) Tax shifting arises under broad conditions. Most prominently, it arises in an overlapping generations structure that gives rise to the well known intergenerational distributive effects of government debt (cf. Diamond, 1965; Blanchard, 1985; Auerbach, Gokhale and Kotlikoff, 1991). In effect, tax shifting consequences are inherent in financial policy choices, even if intergenerational transfers are operative (as they are according to Barro (1974)), such that intergenerational distributive effects of government debt are absent. Budget deficits or surpluses can significantly shift the tax burden within generations, even if having no net effect on the generational accounts.

I embed the analysis in a framework with distortionary taxes and operative intergenerational transfers. Lucas and Stokey (1983) have shown for this case that in a representative agent economy, Ramsey’s (1927) findings translate into a tax smoothing prescription according to which the optimal financial policy smooths deadweight burdens across time and states of nature (cf. also Bohn, 1990; Chari, Christiano and Kehoe, 1991). I extend Lucas and Stokey’s (1983) framework by introducing intragenerational heterogeneity in the form of household-specific growth rates of labor productivity. The resulting framework resembles the many-person-Ramsey setup of Diamond and Mirrlees (Diamond and Mirrlees (1971a; 1971b), Diamond (1975)). It is not isomorphic, however, since the government is restricted to a uniform (within each period) labor income tax schedule. An optimal financial policy now trades off the benefits and cost of tax smoothing and tax shifting. For given paths of government spending and aggregate productivity and for a given governmental objective function, the optimal sequence of budget deficits depends on the productivity profiles of all groups in the economy. If policy makers are, for example, concerned about inequality, they should levy income taxes less pro cyclically than suggested by the tax smoothing view since the poor face a more pro cyclical income path than the rich (cf. Castaneda, Díaz-Giménez and Rios-Rull, 1998). In the long run, they should furthermore impose a relatively front loaded tax profile if regional disparities in per capita income are

\(^1\)I use the terms “financial policy” and “timing of tax collections” as synonyms.

\(^2\)Cf. Bernheim (1987), Bernheim and Bagwell (1988), and Elmendorf and Mankiw (1999) for discussions of the Ricardian equivalence proposition.

\(^3\)Household-specific budget constraints can be interpreted as a form of market incompleteness (Geanakoplos, 1990, p.3). Ricardian equivalence holds if, in this broad sense, markets are complete (cf. also Gale, 1990).
expected to narrow. Whereas deficit data for the U.S. suggest that such a policy has actually been implemented by Congress,\(^4\) Germany’s financing of the unification does not conform with this prescription.

To explain the rising debt quotas in many OECD countries, several political economy models have stressed the role of intragenerational conflicts on government spending (cf. for an overview Alesina and Perotti, 1995). In these models, strategic interaction between interest groups typically implies that third best financial policies are implemented. Institutional constraints like a balanced budget (BB) requirement can therefore lead to Pareto improvements. In contrast, I assume that the government implements a constrained optimal allocation. The political conflict reduces the struggle about the distribution of tax and deadweight burdens. Unlike the former models, this framework can easily rationalize why some, but not all, groups within a generation are in favor of a BB requirement: Ceteris paribus, individuals with a rising income path prefer high contemporaneous and low future tax rates, whereas individuals with a downward sloping income profile favor public debt. This prediction of the model matches the results from opinion polls on the attitude towards a BB requirement (Blinder and Holtz-Eakin, 1984).\(^5\)

The paper is related to Bassetto’s (1999) recent work, which assumes that only one group of households is taxed such that the tax burden cannot be shifted.\(^6\) Since taxes are distortionary, a change in financial policy affects the deadweight burden borne by the taxpayers. At the same time, it shifts wealth between taxpayers and non tax paying “rentiers”, if the timing of tax collections affects the interest rate. The extent of tax smoothing can therefore indirectly determine the wealth distribution. In my model, financial policy affects the wealth distribution both directly and indirectly.\(^7\)

In Section 2, I proceed with a simple two-period model that clarifies the trade off between tax smoothing and tax shifting. The main model in Section 3 analyzes the wealth and welfare effects of financial policy in general equilibrium and compares them to the Ricardian benchmark in a “complete markets” economy where households act behind the Veil of Ignorance. Section 4 applies the findings to two examples: The results suggest that the deficit financing of the German unification favored the West; and that the optimal financial policy over the U.S. business cycle, taking the income distribution dynamics explicitly into account, sharply differs from the conventional policy prescription of smooth tax rates and pro cyclical tax collections. Section 5 discusses the structure of public debt that renders the welfare maximizing policy time consistent. Section 6 concludes.

\(^4\)The residual (actual minus predicted) budget-deficit-to-GNP ratio from a pure tax smoothing model (Barro, 1986, Table 4) is negatively correlated with the dispersion of per capita income across U.S. states (which declined over time). I assume that the government’s objective function aggregates the welfare of states. I measure income inequality across states by the relative per capita personal income of rich versus poor U.S. states where rich (poor) is defined as above (below) average in 1955. I use annual income data from http://www.bea.doc.gov/bea/regional/spi. Allowing for a structural break after 1943, the two series are negatively (−0.52, or −0.65 for five year averages) correlated between 1929 and 1983.

\(^5\)Variables associated with the income profile of the respondents influenced their attitude in the expected direction whereas the level of income had no effect.

\(^6\) Cf. also Fullerton and Rogers (1993) who analyze the lifetime tax burden of heterogeneous groups due to different kinds of taxes.

\(^7\) My emphasis on direct intragenerational wealth effects contrasts with Bernheim (1987) who dismisses them as being of “second-order importance” (p. 271). Bernheim’s assessment derives from his interest in the validity of Ricardian equivalence on the aggregate level, combined with his assumption that heterogeneous households exhibit similar propensities to consume out of their wealth.
2 The Trade Off Between Tax Smoothing and Tax Shifting

The trade off between tax smoothing and tax shifting can be illustrated in a deterministic two-period setting, \( t = 0, 1 \). I assume the following: The economy is small and open such that the gross rate of interest is exogenously given, at \( \beta^{-1} \), where \( \beta \) is the discount factor of households and the government. Two groups of households, \( a \)- and \( b \)-types with fractions \( \eta \) and \( 1 - \eta \), respectively, derive logarithmic utility from consumption. In each period \( t \), type \( i \), \( i = a, b \), receives an endowment \( c_i^t \), that can be used for consumption \( c_i^t \) or be traded on the international capital market. Households and the government face no borrowing constraints. Tax collections involve a deadweight burden \( h(\tau_t) \) that is strictly convex in the contemporary, uniform tax rate and satisfies \( h(0) = h'(0) = 0 \) (cf. Barro, 1979). (Later, I will endogenize the efficiency cost of taxation by explicitly modeling the labor leisure tradeoff of the households.) The government maximizes a weighted sum of household welfare, where the per capita weights are given by \( \theta^a \) and \( \theta^b \). An objective function of this form applies for example under a utilitarian social planner, if the median voter is decisive, or if the government maximizes the welfare of one type subject to a reservation utility requirement with respect to the other type.

A household of type \( i \) maximizes \( \ln(c_i^0) + \beta \ln(c_i^1) \), subject to its intertemporal budget constraint

\[
\sum_{t=0}^{\infty} p_t[c_i^t - u_i^t(1 - \tau_t - h(\tau_t))] = 0.
\]

Since \( \beta^{-1} \) equals the gross rate of interest \( p_0/p_1 \), the utility of type \( i \) is given by a constant plus \((1 + \beta) \ln(W^i)\), where \( W^i \) denotes wealth, i.e. the discounted sum of endowments after taxes and deadweight burdens,

\[
W^i \equiv \sum_{t=0}^{\infty} p_t u_i^t(1 - \tau_t - h(\tau_t)).
\]

Since the economy is open, it faces no period by period resource constraint. The government’s problem therefore consists of choosing \( \tau_0 \) and \( \tau_1 \) such as to maximize \( \theta^a \eta \ln(W^a) + \theta^b (1 - \eta) \ln(W^b) \), subject to the intertemporal budget constraint

\[
\sum_{t=0}^{\infty} \beta^t \left[ \tau_t (\eta u^a_t + (1 - \eta) u^b_t) - \delta_t \right] = 0,
\]

where \( \delta_t \) denotes the exogenous resource requirement of the government. Let \( \theta \equiv \theta^a / \theta^b \), \( W \equiv W^a / W^b \), and \( u_t \equiv u^a_t / u^b_t \). The optimal tax profile is then implicitly characterized by the first-order condition

\[
1 + h'(\tau_0) = \frac{\theta \eta W_0 + (1 - \eta) W(\tau_0, \tau_1)}{\theta \eta W_0 + (1 - \eta) W(\tau_0, \tau_1)} \frac{\eta u_0 + 1 - \eta}{\eta u_1 + 1 - \eta}.
\]

(1)

(The notation emphasizes that the relative wealth level is a function of the tax rates.)

Under the representative agent assumption, the two groups have the same endowments, \( u_0 = u_1 = 1 \), and equal wealth, \( W = 1 \). If taxes are furthermore non distortionary such that \( h'(\cdot) \equiv 0 \), (1) reduces to an identity that holds independently of the intertemporal tax profile. This is the Ricardian equivalence result. With distortionary taxes and a representative agent (1) requires the optimal tax profile to be flat, \( \tau_0 = \tau_1 \), in accordance with the tax (rate) smoothing result of Barro (1979).
Suppose now that the two groups face different endowment paths (and that $0 < \eta < 1$). In the absence of deadweight losses, the choice of tax profile has no impact on the total amount of resources available in the economy. It does, however, potentially affect the wealth distribution. Condition (1) is now satisfied for either $w_1 = w_0$ or $\theta = W(\tau_0, \tau_1)$. In the former case, where the relative endowments do not vary over time, the tax profile is incapable of shifting the tax burden because the ratio of tax payments from $a$- and $b$-types is fixed at $w_0$. If $w_1 \neq w_0$, this ratio is not fixed. The optimal tax profile then maps the two endowment streams into the socially preferred distribution of tax burdens, which equalizes the social marginal benefit of wealth across types.

In the case of heterogeneous households and tax distortions, the tax smoothing and shifting motives interact. Condition (1) implies

$$\tau_1 - \tau_0 \begin{cases} > 0 \\ = 0 \text{ if } (w_1 - w_0)(1 - \theta/W(\tau_0, \tau_1)) > 0 \\ < 0 \end{cases},$$

such that the socially optimal tax profile is flat, $\tau_0 = \tau_1 = \tau$, only if either $\theta = W(\tau, \tau)$ or $w_1 = w_0$. In the first case, a flat tax profile happens not only to minimize deadweight burdens but also to fix the wealth ratio at the socially preferred level. Thus, there are no incentives for the government not to smooth taxes. In the second case, redistribution is not feasible because both types are symmetrically affected by the tax profile. The social welfare maximum then results under the flat tax profile that minimizes deadweight burdens.

In general, tax smoothing is suboptimal. Take for example the case where $a$-types face a more quickly growing endowment path than $b$-types and, according to the government’s welfare weights, $a$-types would be too poor relative to $b$-types if the tax profile were flat: $w_1 > w_0$ and $\theta > W(\tau, \tau)$. Under equal tax rates, the left-hand side of (1) is then smaller than the right-hand side. To satisfy the optimality condition, the government imposes a higher tax rate in period 0 (when the tax base of $a$-types is comparatively low) and a lower tax rate in period 1 (when it is comparatively high). This negatively sloped tax profile improves the relative wealth position of $a$-types by shifting part of their tax burden to $b$-types. Both the extent of redistribution and the total deadweight burden increase with the steepness of the tax profile. The optimal slope between $\tau_0$ and $\tau_1$ therefore depends on the convexity of $h(\cdot)$.

We have seen that in this economy, a unique optimal financial policy exists even in the absence of efficiency costs of taxation or any market incompleteness (as long as $w_0 \neq w_1$). This result generalizes to richer settings (subject to proportional taxation) as long as the number of “relevant” time periods does not exceed the number of groups.\footnote{The requirement that the number of periods ($T$) does not exceed the number of groups ($N$) is not as restrictive as it might appear. An overlapping generations structure, for example, can be interpreted as involving $N = T$ generations who live throughout the $T$ periods but are economically active in two periods each only. An infinite horizon economy with a continuum of types constitutes another example. Even if $T > N$ but the growth rates of the tax base are equal across types from a certain time on, $T_1 + 1$ say, a unique optimal financial policy over the first $T_1$ periods generally exists if only $T_1 \leq N$. Such a condition might be satisfied if income paths can only be predicted over short horizons or if heterogeneous groups tend to disintegrate quickly due to high social mobility.} We have also seen that in the presence of tax distortions, the distributive objective interacts with the objective to minimize excess burdens. Does this interaction between tax smoothing and tax shifting rely on the assumption that the government is constrained to impose proportional taxes? It does not. Consider the case where the average tax rate faced by household $i$ in period $t$ is given by $\tau_t \pi_t(w_i^t)$, where $\pi_t(\cdot)$ is some continuously differentiable function of the contemporary endowment.\footnote{With proportional taxes we had $\pi_t(\cdot) \equiv 1$. $\pi_t(\cdot)$ provides policy makers with a flexible static distributive
deadweight burden is once more given by a strictly convex function $h(\cdot)$ of the marginal tax rate, $\tau_t(\pi_t(w_t) + w_t\pi_t'(w_t))$. The optimal $\tau_t$ profile is then characterized by a condition similar to (1),

$$\frac{\partial \eta(1 + h'(\cdot)\nu_0^\delta)w_0\pi_0 + (1 - \eta)W(\tau_0, \tau_1)(1 + h'(\cdot)\nu_0^\delta)}{\partial \eta(1 + h'(\cdot)\nu_1^\delta)w_1\pi_1 + (1 - \eta)W(\tau_0, \tau_1)(1 + h'(\cdot)\nu_1^\delta)} = \frac{\eta\pi_0 + 1 - \eta}{\eta\pi_1 + 1 - \eta},$$

(2)

where $\pi_t^i \equiv \pi_t(w_t)$, $\pi_t \equiv \pi_t^a/\pi_t^b$, and $\nu$ denotes the ratio of the marginal and the average tax rate. Equation (2) makes it clear that the introduction of a non linear tax schedule modifies the trade off between tax smoothing and tax shifting, but does not eliminate it. The condition differs from (1) in two respects: First, $w_t$ is replaced by $w_t\pi_t$ because the tax base relative to $\tau_t$ depends on the endowment and the progression of the tax schedule. Second, tax rates and deadweight burdens differ across types because of the progression. Due to these changes, the tax shifting channel operates not only if the tax bases grow at different rates but also if their levels differ. The fundamental trade off between tax smoothing and tax shifting remains intact, albeit in a more complicated form.

Although this simple framework captures the central features of tax smoothing versus tax shifting, it is not fully satisfactory. Without a thorough treatment of the general equilibrium repercussions, an endogenous determination of the efficiency cost of taxation, and a clear distinction between their wealth and their welfare effects, the incidence analysis of financial policy remains incomplete. I now turn to a model without those deficiencies.

3 The Incidence of Financial Policy

3.1 The Model

Structure of the Economy The economy is closed. As in Lucas and Stokey (1983), it consists of a government and a continuum of households of measure one. Households live from period 0 to period $T$. Both the government and the households possess perfect information on the joint distribution of all relevant exogenous variables. I denote a realization of the vector of these exogenous variables at time $t$ by $\epsilon_t$ and a specific history of realizations between dates $r$ and $s$, $\{\epsilon_{t}^s\}_{t=r}^s$, by $\epsilon_t^s$. In the case of $r = 0$, I write $\epsilon^t$. Realizations of $\epsilon_t$ between dates $r$ and $s$ are distributed according to the distribution function $F_t^s(\epsilon_t^s)$, with density (or, if applicable, probability) $f_t^s(\epsilon_t^s)$. Contracts are written at time 0 after $\epsilon_0$ has been observed.

The population is split into two groups: Type $a$ households amount to a fraction $\eta$ of the consumers ($0 < \eta < 1$), type $b$ households to a fraction $1 - \eta$. The welfare of a household is defined to be the expected value of the discounted (by factor $\beta^t$) sum of felicity functions. The latter are given by $u(c_t^i, x_t^i) \equiv \ln(c_t^i) + \gamma^a \ln(x_t^i)$ for $a$-types and $u(c_t^i, x_t^i) \equiv \ln(c_t^i) + \gamma^b \ln(x_t^i)$ for $b$-types, where $c_t^i$ and $x_t^i$ denote type $i$’s consumption at time $t$ of the single good and leisure, respectively, and $\gamma^i > 0$, $i = a, b$. The logarithmic utility assumption is not important for the results of the paper. It simplifies the equilibrium conditions by fixing the expenditure shares and by inducing fixed ratios of consumption across types. Each household is endowed with one unit of time per period. Production is linear in labor with productivities $w_t^i, i = a, b; w_t^i \in \epsilon_t, i = a, b$. For notational convenience, I define (again) $w_t \equiv w_t^a/w_t^b, \gamma \equiv \gamma^a/\gamma^b, c_t \equiv c_t^a/c_t^b, x_t \equiv x_t^a/x_t^b$.

mechanism. With just two types of households and a sufficiently non linear tax schedule, no efficiency-equity trade off arises because the government can set marginal tax rates for both types equal to zero. With two types only, a meaningful analysis of the trade off between tax smoothing and tax shifting therefore needs to take the functions $\pi_t(\cdot)$ as given.

The arguments are not affected if $T$ is replaced by $\infty$. 

6
Vector $e_t$ contains the government’s resource requirement $g_t$ financed by taxes and budget deficits. Because the government only observes a household’s labor income, but not its type, productivity, or labor supply, it must resort to a labor income tax schedule. The important implication for this paper is that the government cannot differentiate the time profile of tax rates across individuals independent of their labor income. For simplicity, I assume a proportional labor income tax such that average and marginal tax rates are identical. This renders the structure of the government’s problem similar to the many-person-Ramsey setup (cf. Diamond and Mirrlees (1971a; 1971b), Diamond (1975)); the crucial difference is the requirement that tax rates on sales of labor services by different household types be uniform.

Households behave competitively. They take the sequences of labor productivity, prices of the consumption good \{\(p_t(e^t)\)\}_{t=0}^T, and tax rates \{\(\tau_t(e^t)\)\}_{t=0}^T as given and plan consumption and leisure \(e^t, x^t\) as well as the holdings of contingent claims in order to maximize expected utility. All endogenous variables at time \(t\) are functions of \(e^t\). To simplify the notation, I write these functions without their argument.

A competitive equilibrium in this economy consists of a contingent tax and debt plan, a contingent price sequence, and contingent consumption and leisure choices satisfying the economy’s resource constraints,

\[
e_t \equiv \eta w^a_t + (1 - \eta)w^b_t - g_t = \eta(e^a_t + u^a_t x^a_t) + (1 - \eta)(e^b_t + u^b_t x^b_t), \quad \forall e^t, t = 0, 1, 2, \ldots, T,
\]

the budget constraints of households and the government, and that correspond with utility maximization on the part of consumers. I assume that government expenditure is always feasible, \(e_t > 0, \forall e^t, t = 0, 1, 2, \ldots, T\), and that equilibria are interior. In equilibrium, a temporary government budget deficit is matched by savings of the private sector. The government’s intertemporal budget constraint is automatically satisfied, whenever the households’ budget constraints and the aggregate resource constraints hold.

The weighted-by \(\theta^t \eta\) and \(\theta^t (1 - \eta)\)-sum of the welfare of the two types defines the government’s objective function. Among the contingent tax plans that result in a competitive equilibrium, the government chooses one that maximizes social welfare. At present, I neglect issues of time consistency and assume that the government is able to commit to this ex ante optimal policy.

**Households’ Problem** A household of type \(i\), \(i = a, b\), solves the problem

\[
\max_{\{e^i_t, x^i_t\}_{t=0}^T} \sum_{t=0}^T \beta^t \int \ln(e^i_t) + \gamma^i_t \ln(x^i_t) \, dF^i_0(e^i_t|e_0)
\]

s.t. \[
\sum_{t=0}^T \int p_t(1 - \tau_t)w^i_t d\epsilon^t = \sum_{t=0}^T \int p_t[e^i_t + (1 - \tau_t)w^i_t x^i_t] d\epsilon^t.
\]

\[\text{[12]}\text{Cf. Atkinson and Stiglitz (1976).} \]

\[\text{[13]}\text{In each state of each period, there are three goods: The consumption good, hours supplied by a-types, and hours supplied by b-types. Different household types value different sets of goods and are endowed with different goods. The restriction that the government imposes equal tax rates on both types (captured by equation (7) below) amounts to a uniformity requirement that is not present in the standard many-person-Ramsey setup. Moreover, due to the static technology that prevents intertemporal substitution in production, a certain allocation does not pin down relative producer prices across time and states of nature. Since households receive no lump sum income, this implies that the tax rate on purchases of the consumption good can be normalized to zero.} \]

\[\text{[14]}\text{Under the additional assumption } \theta^a = \theta^b, \text{ the Benthamite social welfare function results.} \]

\[\text{[15]}\text{I assume that no debt is outstanding at the beginning of period 0.} \]
The budget constraint requires household wealth, the market value of productivity endowments after taxes, to equal the market value of goods and leisure consumption. The first-order conditions of this problem define the household’s consumption and leisure choices as functions of productivities, tax rates, and prices.\textsuperscript{16} To reduce the number of variables, I substitute out prices and tax rates to find the implementability constraints:

\begin{equation}
\sum_{t=0}^{T} \beta^t \int 1 - \gamma^a \frac{1 - x_t^a}{x_t^a} \, dF_0^a(c^t | \epsilon_0) = 0,
\end{equation}

\begin{equation}
\sum_{t=0}^{T} \beta^t \int 1 - \gamma^b \frac{1 - x_t^b}{x_t^b} \, dF_0^b(c^t | \epsilon_0) = 0,
\end{equation}

\begin{equation}
\frac{c_t^a}{c_t^b} = \frac{c_0^a}{c_0^b} = c, \quad \forall \epsilon^t, t = 1, \ldots, T,
\end{equation}

\begin{equation}
\frac{x_t^a w_t^a}{x_t^b w_t^b} = c, \quad \forall \epsilon^t, t = 0, 1, 2, \ldots, T.
\end{equation}

Equations (4) or (5) correspond to the single implementability constraint arising in a representative agent setup. They combine the budget constraint with the static and dynamic optimality conditions. Conditions (6) and (7) capture the restrictions that both types of households face the same prices and marginal tax rates. (4)–(7) simplify to

\begin{equation}
\frac{\gamma^b}{1 + \gamma^a} \sum_{t=0}^{T} \frac{1}{\beta^t} \int \frac{w_t}{x_t^b} \, dF_0^a(c^t | \epsilon_0) = c,
\end{equation}

\begin{equation}
\frac{1 + \gamma^b}{\gamma^b} \sum_{t=0}^{T} \beta^t = \sum_{t=0}^{T} \frac{1}{x_t^b} \, dF_0^b(c^t | \epsilon_0),
\end{equation}

\begin{equation}
\frac{c_t^a}{c_t^b} = \frac{c_0^a}{c_0^b} = c, \quad \forall \epsilon^t, t = 0, 1, 2, \ldots, T,
\end{equation}

\begin{equation}
x_t^a = x_t^b c^t / w_t, \quad \forall \epsilon^t, t = 0, 1, 2, \ldots, T.
\end{equation}

Equations (8)–(11) represent all equilibrium restrictions implied by the households’ optimizing behavior.

**Constraints of the Government’s Problem** When choosing the tax profile, the government must take its own budget constraint, the implementability constraints (8)–(11), and the resource constraints into account. Given the latter, one of the three budget constraints is redundant.

Substituting for $c_t^a$ and $x_t^a$ (from (10) and (11)) in the resource constraint, results in

\begin{equation}
c_t^b = \frac{e_t - x_t^b w_t^b (\eta c + 1 - \eta)}{\eta c + 1 - \eta}, \quad \forall \epsilon^t, t = 0, 1, 2, \ldots, T.
\end{equation}

Without further restrictions on financial policy, the government faces a single intertemporal budget constraint,

\begin{equation}
\sum_{t=0}^{T} \int p_t [g_t - \tau_t (\eta w_t^a (1 - x_t^a) + (1 - \eta) w_t^b (1 - x_t^b))] \, dc^t = 0,
\end{equation}

\textsuperscript{16} $\beta c_t^a / c_t^b (c^t | \epsilon_0) = p_t / p_0, \forall \epsilon^t, t = 0, 1, 2, \ldots, T; w_t (1 - \tau_t) / c_t^a = \gamma_t / x_t^a, \forall \epsilon^t, t = 0, 1, 2, \ldots, T; and the budget constraint.
or, using again the households’ first-order conditions to substitute out prices and tax rates,

\[
\sum_{t=0}^{T} \beta^t \int s_t \, dF_0^t(\epsilon^t | \epsilon_0) = 0, \tag{13}
\]

\[
s_t \equiv \eta \epsilon^t (1 + \gamma^a) + (1 - \eta) (1 - \gamma^b) - \gamma^b (\eta w_t + 1 - \eta) / x_t^b.
\]

Under institutional restrictions on government financing, (13) is replaced by tighter constraints.\(^\text{17}\) A balanced budget (BB) requirement constitutes an important special case among such institutional restrictions. In a stochastic environment, BB policies can take two forms: A strict BB rule requires tax revenue to equal government expenditure in each state of each period, i.e.

\[
s_t = 0, \forall \epsilon^t, t = 0, 1, 2, \ldots, T. \tag{14}
\]

A weak BB rule only requires government debt to be constant over time. It leaves the government with the option to trade contingent claims across different states of a given period.

Under a strict BB rule, the allocation satisfies (8)–(11), (12), and (14) and involves no degrees of freedom. Under no BB rule, it satisfies (8)–(11), (12), and (13) and does involve degrees of freedom. I will first solve for the former allocation, because it serves as a reference point for the analysis of policy changes. Afterwards, I will solve for the latter allocation as a function of the government’s free policy instruments. This, in turn, will allow me to characterize the optimality conditions with respect to those instruments.

**Allocation under a Strict BB Rule** To derive the allocation under a strict BB policy (9) and (14) are solved for \(c_t^b, \forall \epsilon^t, t = 0, 1, 2, \ldots, T; \) (12) for \(c_t^d, \forall \epsilon^t, t = 0, 1, 2, \ldots, T; \) and finally (10) and (11) for \(c_t^b, x_t^b, \forall \epsilon^t, t = 0, 1, 2, \ldots, T; \)\(^\text{18}\)

\[
\xi = \frac{1 + \gamma^b B - (1 - \eta) \Omega}{1 + \gamma^a} 
\]

\[
x_t^b \equiv \frac{\gamma^b \eta w_t + 1 - \eta}{1 + \gamma^b} \Omega, \forall \epsilon^t, t = 0, 1, 2, \ldots, T, \tag{16}
\]

\[
c_t^d = \frac{(\eta u_t^d + (1 - \eta) u_t^d) (1 + (1 - \eta) \frac{\Omega}{B} \frac{\gamma^d - \gamma^b}{1 + \gamma^b}) - g_t (1 + \gamma^a)}{(1 + \gamma^b) (1 + \gamma^a)} \Omega, \forall \epsilon^t, t = 0, 1, 2, \ldots, T; \tag{17}
\]

\[
x_t^d = x_t^b \xi, \forall \epsilon^t, t = 0, 1, 2, \ldots, T;
\]

\[
x_t^b = x_t^b \xi, \forall \epsilon^t, t = 0, 1, 2, \ldots, T;
\]

\[
\Omega \equiv \sum_{t=0}^{T} \beta^t \int (\eta w_t + 1 - \eta)^{-1} dF_0^t(\epsilon^t | \epsilon_0), \quad B \equiv \sum_{t=0}^{T} \beta^t.
\]

Under a strict BB rule, the allocation is fully determined by the aggregate equilibrium conditions and the households’ optimizing behavior. The government’s optimal taxation program is trivial, as it involves no degrees of freedom.

**Allocation under no BB Rule** In the absence of a BB rule, the government’s choices with respect to \(c \) and \( x_t^b, \forall \epsilon^t, t = 0, 1, 2, \ldots, T, \) are only restricted by (8) and (9) (since (13) is

\(^\text{17}\)In that case, one of the households’ budget constraints—not the government’s budget constraint—is redundant.  
\(^\text{18}\)For any variable \(q_t \), let \(\bar{q}_t \) denote the value under a strict BB policy.
redundant). The associated degrees of freedom correspond with the government’s flexibility to shift tax collections across time and states of nature. Without loss of generality, I choose $x_{i}^{b}, \forall \epsilon^{t}, t = 1, 2, \ldots, T$, to represent the policy instruments. (Since tax rates have been substituted out, the “real” policy instruments are no longer present in the equation system. However, a specific sequence of $x_{i}^{b}$’s directly corresponds with a sequence of these “real” instruments.) To derive the allocation under no BB rule, solve (9) for $x_{i}^{b}$ (given the values of the policy instruments $x_{i}^{b}, \forall \epsilon^{t}, t = 1, 2, \ldots, T$); (8) for $c$; (12) for $c_{t}^{b}, \forall \epsilon^{t}, t = 0, 1, 2, \ldots, T$; and finally (10) and (11) for $c_{t}^{b}, x_{i}^{b}, \forall \epsilon^{t}, t = 0, 1, 2, \ldots, T$:

$$x = \text{vector of policy instruments } x_{i}^{b}, \forall \epsilon^{t}, t = 1, 2, \ldots, T,$$

$$x_{i}^{b}(x) = \left(1 + \gamma^{b} \frac{1 + \gamma^{b}}{1 + \gamma^{a}} B - \sum_{t=1}^{T} \beta^{t} \int \frac{1}{x_{i}^{b} dF_{t}^{b}(\epsilon^{t} | \theta)} \right)^{-1},$$

$$c(x) = u_{0} + \frac{\gamma^{b} \sum_{t=1}^{T} \beta^{t} \int \frac{w_{t} - u_{0}}{x_{i}^{b} dF_{t}^{b}(\epsilon^{t} | \theta)}}{1 + \gamma^{a} B},$$

$$c_{t}^{b} = \frac{e_{t} - x_{i}^{b} u_{0}^{b} \eta(x) (\gamma + 1 - \eta)}{\eta(x) + 1 - \eta}, \forall \epsilon^{t}, t = 1, 2, \ldots, T,$$

$$c_{t}^{b} = \frac{e_{t} - x_{i}^{b} u_{0}^{b} \eta(x) (\gamma + 1 - \eta)}{\eta(x) + 1 - \eta},$$

$$c_{t}^{b} = c_{t}^{b}(c(x)), \forall \epsilon^{t}, t = 0, 1, 2, \ldots, T,$$

$$x_{i}^{b}(x) = x_{i}^{b}(c(x) \gamma / w_{t}), \forall \epsilon^{t}, t = 1, 2, \ldots, T,$$

$$x_{i}^{b}(x) = x_{i}^{b}(c(x) \gamma / w_{t}).$$

**Redistribution** Equation (19) captures the tax shifting result in general equilibrium. It shows that changes in financial policy affect relative wealth\(^{19}\) if $w_{t}$ varies over time or states. Consider, for example, an increase in $x_{i}^{b}$ accompanied by a decline in $x_{0}^{b}$ (given by (18)). This policy change implies an increase of $x_{i}^{b} / x_{0}^{b}$ and a decline of $x_{0}^{b} / x_{i}^{b}$, $s = 1, \ldots, t - 1, t + 1, \ldots, T$.\(^{20}\) In equilibrium, the higher $x_{i}^{b} / x_{0}^{b}$ and lower $x_{0}^{b} / x_{i}^{b}$ ratios are associated with a fall of $p_{t}(1 - \tau_{t})$ and an increase of $p_{0}(1 - \tau_{0})$, relative to each $p_{t}(1 - \tau_{t})$, because households choose fixed expenditure shares for their leisure consumption. This change in the relative market values of productivity weighted time endowments benefits those households experiencing a comparatively high (low) relative productivity in period 0 ($t$).

The wealth shift operates through two channels. First, tax rates change, which alters after tax productivities and wealth in the same way as in the model of Section 2. Second, demand responses then imply, that prices adjust in order to equilibrate markets. These price changes affect wealth by altering the market values of after tax productivities. Equation (19) reports the compounded relative wealth effect due to these two channels.

Note that the resource constraint plays no role in determining $c$. For a given sequence $\{x_{t}^{b}\}_{t=1}^{T}$, the implementability constraints alone pin down the wealth distribution. This specific feature of equilibrium is due to the logarithmic utility assumption and the absence of an exogenous income stream. These ingredients imply that the equilibrium conditions are block recursive: The policy instruments together with conditions (8), (9), and (11) determine $\{x_{t}^{b}\}_{t=0}^{T}, i = a, b$, and $c$. The resource constraint and (10) then fix $\{c_{t}^{b}\}_{t=0}^{T}, i = a, b$, and the households’ first

\(^{19}\) Relative wealth equals $c(1 + \gamma^{a})/(1 + \gamma^{b})$.

\(^{20}\) For notational convenience, I abstract from uncertainty.
order conditions pin down \( \{p_t/p_0\}_{t=1}^T \) and \( \{\tau_t\}_{t=0}^T \). Alternatively, households’ leisure demands and the definitions of the households’ wealth link \( c_i \), \( \{x_i\}_{t=0}^T \), \( i = a, b \), and the relative prices of productivity endowments \( \{p_t(1-\tau_t)/p_0(1-\tau_0)\}_{t=1}^T \). Given these values, the resource constraint and households’ demands for goods determine \( \{c_i\}_{t=0}^T \), \( i = a, b \), \( \{p_t/p_0\}_{t=1}^T \), and \( \{\tau_t\}_{t=0}^T \).

**Optimal Financial Policy** The optimal policy maximizes social welfare, subject to the constraints governing the allocation under no BB rule. In equilibrium, social welfare can be expressed without explicit reference to \( c_i^t \) and \( x_i^t \) because (using (10) and (11))

\[
u(c_i^t, x_i^t) = v(c_i^t, x_i^t) + (\gamma_a - \gamma_b) \ln(x_i^t) + (1 + \gamma_a) \ln(c) + \gamma_b \ln(\gamma/w_t).
\]

The government’s program therefore reads\(^{21}\)

\[
\max_{\{x_i^t\}_{t=1}^T} \sum_{t=0}^T \beta^t \int \left( \theta^a \eta + \theta^b (1 - \eta) \right) \left[ \ln(c_i^t) + \gamma_b \ln(x_i^t) \right] + \theta^b \eta \left( (\gamma_a - \gamma_b) \ln(x_i^t) + (1 + \gamma_a) \ln(c) + \gamma_a \ln(\gamma/w_t) \right) dF_0^t(c_i^t|\epsilon_0)
\]

s.t. \((18), (19), (20), (21)\).

I substitute \((18)–(21)\) into the government’s objective function and differentiate with respect to the policy instruments.\(^ {22}\) A typical first-order condition with respect to \( x_i^t \) takes the form

\[
\sum_{j=1}^3 D_{jt} = 0,
\]

where

\[
D_{1t} = \left( \theta^a \eta + \theta^b (1 - \eta) \right) \left[ \beta^t t_0^a(c_i^t|\epsilon_0) \left( \frac{1}{c_i^t} \frac{\partial c_i^t}{\partial x_i^t} + \frac{\gamma_b}{x_i^t} \right) \right] + \left( \frac{1}{c_i^t} \frac{\partial c_i^t}{\partial x_i^t} \right) \frac{\partial x_i^t}{\partial x_i^t},
\]

\[
D_{2t} = \left( \theta^a \eta + \theta^b (1 - \eta) \right) \left[ \sum_{s=0}^T \beta^s \int \frac{\partial c_i^s}{\partial c} dF_0^s(c_i^s|\epsilon_0) \right] + \theta^b \eta \left[ (1 + \gamma_a) B \frac{1}{c} \right] \frac{\partial c}{\partial x_i^t},
\]

\[
D_{3t} = \theta^b \eta \left( (\gamma_a - \gamma_b) \left( \beta^t t_0^a(c_i^t|\epsilon_0) \frac{1}{x_i^t} + \frac{1}{x_i^t} \frac{\partial x_i^t}{\partial x_i^t} \right) \right).
\]

Terms \( D_{1t}, D_{2t}, \) and \( D_{3t} \) summarize the social welfare implications of a change in \( x_i^t \) in general equilibrium. Consider first \( D_{1t} \): A marginal increase in leisure provides utility \( (\gamma_b/x_i^t) \) but goes hand in hand with a decrease in goods consumption (due to the resource constraint), thereby negatively affecting utility \( (1/c_i^t \partial c_i^t/\partial x_i^t) \). Furthermore, the change in \( x_i^t \) must be accompanied by a variation in \( x_i^0 \) (and therefore \( c_i^0 \)) to be implementable. \( D_{1t} \) accounts for these four effects, holding the wealth distribution constant. \( D_{2t} \) measures the social welfare effect due to the impact of the policy change on the wealth distribution. \( D_{3t} \) corrects for differences in household preferences for leisure.

The partial derivatives in \( D_{1t}, D_{2t}, \) and \( D_{3t} \) (following from (18)–(21)) are given by

\[
\frac{\partial c_i^t}{\partial x_i^t} = \frac{u_i^t(\eta \gamma + 1 - \eta)}{\eta \gamma + 1 - \eta} = -\frac{\gamma_b}{1 - \tau_t x_i^t} \frac{\gamma_a + 1 - \eta}{\eta \gamma + 1 - \eta}, \forall \epsilon^t, t = 0, 1, 2, \ldots, T, \tag{23}
\]

\[
\frac{\partial c_i^t}{\partial c} = -\frac{\eta t (1 - \eta) \mu_i^t x_i^t (1 - \gamma)}{(\eta \gamma + 1 - \eta)^2} = \cdots
\]

\(^{21}\)In stating the government’s program, I neglect the inequality constraints in (3).

\(^{22}\)In Appendix A.1, I discuss a generalized (with respect to the utility functions) version of (22) and compare the first-order conditions of that problem with those found by Lucas and Stokey (1983).
\[ \frac{\partial c_t}{\partial x_t^b} = -u_t^b \]  

The marginal rate of transformation between leisure and consumption on the aggregate level, \(-u_t^b\), and as perceived by an individual household, \(-u_t^b(1-\tau_t)\), therefore only differs for \(\tau_t \neq 0\). \(D_U\) simplifies to

\[ (\theta^a + \theta^b(1-\eta)) \left[ \beta^t f_t^b(c_t|x_0) \gamma_t^b \left( \frac{x_t^b}{x_0} \right) \left( 1 - \frac{1}{1-\tau_t} \right) + \frac{\gamma_t^b}{x_0^b} \left(1 - \frac{1}{1-\tau_0} \right) \frac{\partial x_0^b}{\partial x_t^b} \right], \]

which represents a weighted sum of tax distortions. If \(\tau_t = \tau_0 = 0\), the welfare effect from a small change in \(x_t^b\) and, correspondingly, \(x_0^b\) is zero. If, however, either \(\tau_t \neq 0\) or \(\tau_0 \neq 0\), the government can potentially improve welfare by adjusting the tax rates such as to reduce the total deadweight burden. Substitution of the equilibrium values under a BB in particular implies that a marginal increase of \(x_t^b\) around the BB allocation improves welfare, if \(\tau_0 > \tau_t\). \(D_U\) therefore captures the marginal social welfare effect of tax smoothing.

Suppose next that households differ with respect to their preferences, \(\gamma \neq 1\), but relative productivities are constant, \(w_t = w_t, \forall \epsilon^t, t = 0, 1, 2, \ldots, T\), such that relative wealth is fixed and \(D_{2t} = 0\). The aggregate marginal rate of transformation between \(b\)-types’ leisure and consumption,

\[ \frac{\partial c_t^b}{\partial x_t^b} = \frac{-u_t^b \eta c_{\gamma - 1}}{\eta c_{\gamma - 1}}, \]

now depends on the composition of the population and differs from that perceived by an individual household, \(-u_t^b(1-\tau_t)\), even if the tax rate is zero. \(D_U\) reads

\[ (\theta^a + \theta^b(1-\eta)) \left[ \beta^t f_t^b(c_t|x_0) \gamma_t^b \left( \frac{x_t^b}{x_0} \right) \left( 1 - \frac{1}{1-\tau_t} \right) \frac{\eta c_{\gamma + 1} - 1}{\eta c_{1 - \gamma} - 1} \frac{\partial x_0^b}{\partial x_t^b} \right], \]

The same discussion applies under the assumption that households have identical preferences and face different productivities, with a fixed ratio, \(w_t = w\), \(\forall \epsilon^t, t = 0, 1, 2, \ldots, T\).
and $D_{3t}$ corrects for the fact that $a$-types derive different marginal utility from leisure than $b$-types. If the government behaved fully in the interest of $b$-types ($\theta^b = 0$), it would set $D_{1t}$ equal to zero. In the opposite case ($\theta^a = 0$), it would set a modified expression $D'_{1t}$ equal to zero, where $D'_{1t}$ has $\gamma^b$ in the first and third term of $D_{1t}$ replaced by $\gamma^a$ to take the different marginal utility of leisure of $a$-types into account. Around the BB allocation, a rise in $x_l^t$ improves the welfare of both types, if $\pi_0 > \pi_t$. Off the BB allocation, however, the direction of an optimal policy change generally depends on the welfare weights. Although the government can still not affect the wealth distribution ($c$ is fixed), it can affect relative welfare because financial policy imposes type-specific deadweight burdens. “The” tax smoothing policy is no longer defined. The choice of financial policy involves a normative judgment, even if it does not redistribute wealth.

Suppose finally that households have identical preferences but relative productivities vary over time or states: $\gamma = 1$, $w_i \neq w_l$ for some $\epsilon^i, \epsilon^l$. Then, $D_{3t} = 0$ and $D_{1t}$ accounts for the symmetric welfare effect on households due to changes in the deadweight burden. With $w_l$ fluctuating, the government can now influence the wealth distribution ($\partial c/\partial x_l^t \neq 0$). Since the social welfare effect of redistribution generally differs from zero, the government takes advantage of this possibility. The tax smoothing prescription for optimal policy does not apply, not even locally around the BB allocation.

In the general case ($\partial c/\partial x_l^b \neq 0$ and $\gamma \neq 1$), the different channels interact and financial policy shifts both wealth and deadweight burdens. A simple decomposition of the resource constraint offers an alternative perspective of this interaction. Define the fiscal burden of $a$- and $b$-types in period $t$, $c_{t}^{a_l}$ and $(1 - \kappa_t)g_t$, to be the amounts of government expenditure in period $t$ produced by $a$-types and $b$-types, respectively:

$$c_{t}^{a_l} = \eta w_t^a - \eta (c_t^a + w_t^a x_t^a),$$
$$c_{t}^{b_l} = \eta w_t^b - (1 - \eta) (c_t^b + w_t^b x_t^b) - (1 - \eta)(c_t^b + w_t^b x_t^b).$$

The ratio of the two total fiscal burdens at market prices, $\rho \equiv \sum \int p_t c_{t}^{a_l} dF / \sum \int p_t (1 - \kappa_t) g_t dF$, provides a summary measure of the relative incidence of taxation. Note that total goods and leisure consumption at time $t$ (the sum of the right most expressions in the equations above) is fixed because government expenditure and productivities are exogenous. Financial policy can therefore affect $\kappa_t$, only if it alters the ratio of total consumption across types. However, this ratio is given by

$$\frac{c_{t}^{a_l} + w_t^a x_t^a}{c_{t}^{b_l} + w_t^b x_t^b} = \frac{c_t^a + \gamma w_t^a x_t^a}{c_t^b + w_t^b x_t^b},$$

so that policy cannot affect $\kappa_t$ unless it either changes the wealth distribution or preferences differ. Moreover, if $\kappa_t$ is unaltered by policy changes, the same holds true for $\rho$.

### 3.2 Eliminating the Distributive Channels

Financial policy affects relative wealth and welfare by shifting tax obligations and deadweight burdens and by altering the market value of endowments. I conclude this section by briefly discussing three modifications of the model, which eliminate some or all of these channels.

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24 If $\gamma = 1$ and $\partial c/\partial x_l^b = 0$, then it must be the case that relative productivities are constant and equal to $c$. This, in turn, implies that $\kappa_t = \kappa = \eta c / (1 - \eta + \eta c)$ and $\rho = \eta c / (1 - \eta)$. 

13
No Tax Shifting  Bassetto (1999) analyzes the indirect wealth effects of financial policy that arise due to its impact on prices. He considers a setup where taxes are levied on the labor income of one group only. The other group, “rentiers”, earns income according to some exogenous endowment stream, \{yt\}_{t=0}^T say, and pays no taxes such that tax shifting is impossible. Bassetto’s setup is a special case of my model, in the following way: Let \(a\)-types be the “rentiers” and let \(\{wt\}_{t=0}^T = \{0\}_{t=0}^T\), such that rentiers do not work. Since only \(b\)-types pay taxes, condition (7) no longer binds. The implementability constraints (5) and (6) continue to hold whereas (4) and (12) are replaced by

\[
\begin{align*}
    Bc & = \sum_{t=0}^T \beta^t \int \frac{\eta}{c_t} dF_0^b(\epsilon|c_0), \\
    c^b_t & = \frac{c_t' - (1 - \eta)u^b_t x^b_t}{\eta c + 1 - \eta}, \forall \epsilon^t, t = 0, 1, 2, \ldots, T, \\
    c^t_t & \equiv \eta c_t + (1 - \eta)u^b_t - g_t, \forall \epsilon^t, t = 0, 1, 2, \ldots, T.
\end{align*}
\]

Due to the exogenous income stream, the wealth distribution does now not only depend on the implementability constraints but also on the resource constraint. The government’s first-order conditions are once more given by \(\sum_{j=1}^{\gamma^a} D_{jt} = 0\), where \(\gamma^a\) is set equal to zero and the partial derivatives \(\partial c/\partial x^b_t\), \(\partial c^b_t/\partial x^b_t\), and \(\partial c^b_t/\partial c\) in \(D_{yt}\) and \(D_{2t}\) are adjusted according to (27) and (28).

No Price Effects  In a small open economy, the terms of trade are exogenous. Policy induced price effects on the wealth distribution are therefore absent. Relative to the main model, only the resource constraint (12) changes. It is replaced by

\[
\sum_{t=0}^T \int p_t c_t d\epsilon^t = \sum_{t=0}^T \int p_t (c_t'((\eta c + 1 - \eta) + u^b_t x^b_t((\eta c + 1 - \eta)))d\epsilon^t,
\]

subject to the restriction \(p_t c^b_t = \beta^t f^b_0(\epsilon|c_0)p_t c^b_t\), \(\forall \epsilon^t, t = 1, 2, \ldots, T\). The value of \(c^b_t\) no longer solely depends on \(c\) and the contemporaneous value of \(x^b_t\), but on the complete sequence \(\{x^b_t\}_{t=0}^T\). The constraints of the government’s program are then given by (18), (19), and modified versions of (20) and (21). The first-order conditions change in two ways: In addition to the two (modified) deadweight burden expressions, \(D_{yt}\) contains new terms that capture the impact of \(x^b_t(\epsilon^t)\) on \(c^b_t(\epsilon^t), \epsilon^t \neq \epsilon^t\). Moreover, the modified resource constraint results in different expressions for the partial derivatives \(\partial c^b_t/\partial x^b_t\) and \(\partial c^b_t/\partial c\).

Full Insurance  Consider the situation where households write contracts before learning about their own type. Behind this Veil of Ignorance (Rawls, 1971), households do not only share the risks due to the uncertain future path of \(\epsilon_t\), but also the risk of being assigned a specific type. Consumers maximize expected utility

\[
\sum_{t=0}^T \beta^t \int \eta u(c_t', x_t') + (1 - \eta) v(c^b_t, x^b_t) dF_0^b(\epsilon|c_0)
\]

subject to the intertemporal budget constraint

\[
\sum_{t=0}^T \int p_t (1 - \tau_t)[\eta u_t^a + (1 - \eta)u^b_t]d\epsilon^t = \sum_{t=0}^T \int p_t [\eta c_t' + (1 - \eta)c^b_t + (1 - \tau_t)(\eta u_t^a x_t' + (1 - \eta)u^b_t x^b_t)]d\epsilon^t,
\]

14
where $c_t^i, x_t^i, i = a, b$, denotes consumption of the good and leisure of a household that turns out to be of type $i$. The implementability constraints arising from this program differ from those in the main model in two ways: The two budget constraints are replaced by the single one, and $c = 1$. Note that the elimination of type-specific budget constraints can be interpreted as eliminating a fundamental form of market incompleteness (Geanakoplos, 1990, p. 3). Ricardian equivalence holds if, in this broad sense, markets are complete: For the ex ante welfare effect of a marginal increase in $x_t^b$ now resembles, not surprisingly, the one in the representative agent framework (cf. the expression for $D_{yt}$ on page 12). It is given by

$$\left(\eta \gamma^a + (1 - \eta) \gamma^b \right) \left[ \beta^t f^t(t^t | t^0) \frac{1}{y_t} \left(1 - \frac{1}{1 - \tau t} \right) + \frac{1}{x_t^b} \left(1 - \frac{1}{1 - \tau_0} \right) \frac{\partial x_t^b}{\partial x_t^a} \right],$$

a weighted sum of tax distortions.\(^{25}\) Around the BB allocation, a marginal rise in $\tau_t$ increases the ex ante welfare of households, if $\tau_0 > \tau_t$.

Behind the Veil of Ignorance, the private sector behaves as a normative representative agent and the optimal policy amounts to tax smoothing. If households are heterogeneous and individual budget constraints bind, this is no longer true. A financial policy that is optimal with respect to a hypothetical representative consumer with “average endowments” and “average preferences” is generally inadequate. Primarily, it is not feasible. Even if it were feasible, it would neglect the fact that changes in financial policy have distributive effects.

4 Examples

4.1 Financing the German Unification

In the early 1990s, Germany faced a sudden, supposedly temporary increase in government expenditures relative to GDP.\(^{26}\) This increase did not only result from strong public investment in and transfers to the “Neue Länder” but also from transfers to the Soviet Union, loans to Eastern European countries, and contributions to the financing of the Gulf war. In accordance with the tax smoothing view, the government argued in favor of deficits and relatively small tax increases in order to finance the expenditure spike.\(^{27}\) The parliament endorsed this strategy and approved a quickly rising debt quota. Since productivity in the East was to catch up with the Western level, this choice of a flat tax profile implied a more equal distribution of the total tax burden between East and West Germans than a front loaded profile. As a result, West Germans pay lower taxes than they would have paid without the high budget deficits in the 1990s.\(^{28}\)

To quantify this effect, I apply a calibrated version of the model in Section 3. Thereby, I abstract, first, from financial market imperfections, especially borrowing constraints. If East Germans had faced liquidity constraints, the government’s potential to increase their welfare by a front loaded tax profile would have been severely restricted. High car sales in East Germany in the early 1990s suggest, however, that liquidity constraints were not binding for many households. Second, I disregard the effects of financial policy on the generational accounts.

\(^{25}\) A parallel result holds for general utility functions. Ex post, the two welfare effects differ if $\gamma \neq 1$.

\(^{26}\) For a detailed discussion of several economic aspects of the German unification, cf. Sinn and Sinn (1992).

\(^{27}\) Cf., for example, the speech of finance minister Theo Waigel to the German parliament, March 12, 1991.

\(^{28}\) Tax schedules in East and West Germany are not strictly uniform but their underlying time profiles are tightly connected. Although implementing different tax profiles in East and West Germany would have been advantageous, such a policy would have been politically infeasible.
Equivalently, I interpret the fact that Germans leave bequests as evidence for operative intergenerational altruism. Finally, I neglect migration. This is irrelevant as long as the productivity profile of a household is person specific.

I simulate a deterministic economy lasting for six decades, from 1991 to 2050. I assume that by 2030, East German productivity—which equals 40 percent of the Western value in 1991—will have reached the Western level; that the government expenditure-to-GDP ratio will have converged to 40 percent; and that after 2030, Germany will move along a balanced growth path. In Appendix A.2, I discuss details of the calibration.

Figure 1 illustrates the optimal financial policy under different welfare weights for West Germans: \( \theta^w = 0.3, \theta^e = 0.5, \) and \( \theta^e = 0.7 \). The weight for East Germans is given by \( \theta^e = 1 - \theta^w \). Under \( \theta^w = 0.3 \), the government values the welfare of East Germans higher (by a factor > 2) than the welfare of West Germans. Since East Germans are poorer than West Germans, redistribution from the latter to the former is a prior objective. The government achieves this objective by setting high tax rates at an early stage (50 percent during the first decade) when West Germans enjoyed strong productivity advantages. Indeed, the tax-shifting motive is so pronounced that the optimal policy approximately follows a BB rule. Tax rates in the first decade are sufficiently high to finance the government expenditure spike to more than 100 percent. After 2000, the rates decline sharply and converge to the long run expenditure-to-GDP ratio. The optimal policy under \( \theta^e = 0.7 \) stands in stark contrast to this “close-to-BB” policy. If the government values the welfare of West Germans higher than the welfare of East Germans, redistribution is relatively unimportant and the government’s major objective is to minimize deadweight burdens. The optimal policy is then characterized by a smooth tax profile (tax rates around 43 percent), similar to the one at which the German government actually aimed. High deficits in the first two decades (with an initial deficit quota relative to government expenditure of 8.7 percent, close to the one in the data) are followed by surpluses. Under balanced welfare weights \( \theta^e = 0.5 \), tax rates initially slowly decline but rise again after 2020. Moderate initial deficits are followed by small surpluses.

Although consumption and leisure paths do not strongly differ between the three policies, the implied welfare differences are considerable. The move from the close-to-BB policy to the policy of smooth tax rates reduces the welfare of East Germans to the same extent as a permanent reduction of their consumption by 1.1 percent or a reduction of 4.6 percent throughout the first decade. Significantly higher (two digit) welfare costs result under less conservative assumptions regarding the initial productivity difference, with a shorter time horizon, or a low utility weight on leisure. These costs arise because a smooth tax profile hurts East Germans more through the tax-shifting channel than it benefits them through the improvement in their intertemporal terms of trade (i.e., the lower interest rates associated with a tax-smoothing policy).\(^{20}\)

Since the model abstracts from the effects of tax policy on both (human) capital accumulation and income from initially outstanding asset holdings, one might wonder whether consideration of these aspects may reverse the result. The opposite is likely to be the case. Consider first the unmodeled effect of tax policy on capital accumulation and thereby labor productivity. Economic theory suggests that private investment responds less to the present income tax rate than to expected future tax rates. Under the close-to-BB policy, tax rates are slightly higher in the second decade but significantly lower in all later decades. It is therefore unlikely that

\(^{20}\)The graph displays only the first four decades. After 2030, the economy moves along a balanced growth path: Productivity, government expenditure and consumption grow at constant positive rates; tax rates and labor supply are constant.

\(^{20}\)Moreover, the intertemporal price effect vanishes if Germany is modeled as a small open economy.
the close-to-BB policy would have discouraged investment relative to the alternative policies with smoother tax rates. With respect to the second issue, note that an increase in the interest rate in the first decade (as associated with the close-to-BB policy) would have devalued initially outstanding long term bonds. This effect would have harmed West Germans much more than East Germans, since only the former held such assets. The general picture arising from the simulation—that the policy of smooth tax rates channeled resources from the poorer East to the richer West—therefore appears robust.

Judged by the government’s intentions and by the deficit quota in the 1990s, Germany finances the unification by a policy of smooth tax rates, i.e. a policy distinctly favoring West Germans. This “Western” bias on the financing side sharply contrasts with the “Eastern” bias on the expenditure side, as manifested by large transfers to the “Neue Ländler” (cf. for example Schwinn (1997, Table 2.4)). This suggests that the government implemented a constrained inefficient policy in the sense that it did not simultaneously optimize both government expenditure and revenues. The social cost of administrative effort, fraud, etc. associated with payments to East Germany could have been reduced if the latter had been partially replaced by a more front loaded tax profile. Several alternative explanations to the opposing policy biases are conceivable. The government might, for example, have tried to undo some of the well publicized transfers to East Germany by less transparent tax shifting. A possible rationale for such behavior could be that the government acted on behalf of (some) West Germans, but sought to gain votes from East Germans.\textsuperscript{31} More likely, however, the authorities were simply not aware of the policy’s distributive consequences. They erroneously considered Germany to be inhabited by a representative household (along the dimensions relevant for financial policy) and, accordingly, chose a smooth tax profile on efficiency grounds. The intention to minimize the deadweight burden led to an unintended redistribution from East to West.

4.2 Taxation over the Business Cycle

Barro (1979) suggested that, absent new information, tax rates should optimally be kept constant over the business cycle. He derived this martingale property under assumptions parallel to those in the representative agent version of the model in Section 2. Although Lucas and Stokey’s (1983) analysis demonstrated that this tax-rate-smoothing result applies only as a very special case of

\textsuperscript{31}This argument requires heterogeneity among West Germans (that is unrelated to the issues discussed here) which induces the government to seek the support of East Germans.
the general deadweight-burden-smoothing result, the conventional wisdom still holds that, for all practical purposes, optimal financial policy involves labor tax rates that fluctuate very little (Chari et al., 1991; Chari, Christiano and Keloh, 1994).

My objective here is to revisit the question of optimal taxation over the business cycle, taking the dynamics of the income distribution explicitly into account. For this purpose, I construct a stylized representation of “the” post war U.S. business cycle that captures the standard deviation and skewness of output, the average income shares of the lower sixty percent and the upper forty percent of the income distribution, and the correlation between government spending and output. I then compute the optimal tax profile over the cycle in either of two scenarios: In the first scenario, I assume relative productivity to vary over the cycle such that the income share of the rich (the poor) is negatively (positively) correlated with output, as is the case in the data (cf. Castañeda et al., 1998). As a benchmark, and to compare the results to those in the literature, in the second scenario I assume relative labor productivity to remain constant over the cycle at its average value. Details can be found in Appendix A.2.

Not surprisingly, I replicate the conventional wisdom in the second scenario: With constant relative productivity, the optimal tax profile over the cycle is essentially flat (the coefficient of variation of the tax rate is less than 0.3 percent) and tax revenue is pro-cyclical. In the first scenario, with time varying relative productivity (and a Benthamite social welfare function), the optimal financial policy is reversed: Not only is the optimal tax rate counter-cyclical but it is much more volatile (the coefficient of variation increases to 5 percent). While the optimal tax rate equals 18.9 percent in very good times (when the income share of the rich is comparatively low), it increases to 19.7 percent in bad times and 21 percent in normal times (when the income share of the rich is comparatively high). This relative increase by over ten percent implies that the optimal path of tax collections is also counter-cyclical.

The example clearly shows that the first-order distributive effects of financial policy easily dominate the second-order efficiency considerations that govern optimal policy in the representative agent setup on which most of the literature has focused. Both under a normative and a positive perspective, it is therefore crucial to account for tax shifting incentives.\footnote{Obvious, this conclusion is not limited to the particular examples discussed here. Even the case for deficit financing of a war might be significantly weakened if the relative income position of the rich improves during the war, for example because they are less likely to be drafted.}

5 Time Consistency

Lucas and Stokey (1983) showed that in a representative agent economy, the government can commit to the ex ante optimal financial policy by choosing the maturity structure of public debt in an appropriate way.\footnote{The discussion presumes that the government must honor outstanding government debt.} Bassetto (1999) demonstrates that the same is true in his model with a “taxpayer” and a “rentier”, as long as the government can adjust any one of the bilateral debt positions in the economy after having observed the others. In Appendix A.3, I show that a similar condition applies in the setup considered here.\footnote{Cf. Rogers (1986) for a discussion of time inconsistency in a framework with heterogeneous households and labor and capital taxes.}

In the representative agent setting, the government faces a single implementability constraint. The possibility of time inconsistency arises because the household’s optimal response to a distortionary tax ex post differs from its ex ante response. The optimal tax profile itself therefore also changes over time. In order to commit to a specific profile, the government needs
to influence the constraints subject to which it reoptimizes in later periods. This can be done
by employing ex ante neutral devices that are non neutral—along the relevant margins—ex post.
To counterbalance all ex post incentives, the government needs as many independent devices
of that sort as there are tax rates to be committed to. The maturity structure provides these
devices because it determines the extent to which a change in the allocation translates into a
change of the value of outstanding government debt (Persson and Svensson, 1986).

With heterogeneous households, the government faces multiple implementability constraints.
Not only is it prohibited from directly transferring resources between the private sector and the
government, but it is also prohibited from directly transferring resources across types. With
the households’ optimal response to a distortionary tax profile changing over time, the govern-
ment’s reoptimization along the intertemporal tax smoothing margin and the cross sectional tax
shifting margin would generally result in ex post policy choices that differ from the Ramsey out-
come. To avoid time inconsistency, the government needs to employ a commitment device that
counterbalances both these differential ex post incentives. An appropriately chosen maturity
structure of government debt can once more serve as such a device. Besides other factors, this
“optimal” public maturity structure depends on the maturity structure of all privately issued
bonds, a dependence arising because the welfare effect of a variation in the term structure due to
a policy change ex post depends on the total exposure of a household to the different maturities.

6 Conclusion

Heterogeneous income profiles turn financial policy into a powerful distributive mechanism. One
implication of this mechanism, the intergenerational wealth effect of government debt, has at-
tracted considerable attention in the macroeconomic literature. The general tax shifting princi-
ple has gone nearly unnoticed, though. In my view, this focus on intergenerational wealth effects
has been too narrow. Since different generations within the same family are much more likely to
be altruistically linked than members of different families, the incentives for intragenerational
tax shifting should be at least as strong as for its intergenerational counterpart.

The welfare implications of intragenerational tax shifting are significant. In particular, they
are significant relative to the second-order welfare effects of tax distortions that have generally
been considered as central in the presence of operative intergenerational transfers. The welfare
implications of intragenerational tax shifting can, among other things, explain why certain
groups support constitutional restrictions on financial policy, whereas others do not, without
having to rely on the assumption of intergenerational conflict or an inefficient political process.

Future research should further assess the quantitative importance of the tax shifting hypo-
thesis. It should also address the interaction between tax shifting considerations and other forms
of conflict in the political arena. Progress along those lines promises valuable new insights into
the conduct of financial policy.

A Appendix

A.1 The Government’s Program under General Conditions

A solution to the utility maximization problems of $a$- and $b$-types is characterized by the first-
order conditions

$$u_a(c^a_t, x^a_t) = u_b(c^b_t, x^b_t), \forall t, t = 0, 1, 2, \ldots, T,$$
\[
\beta^t \frac{u_c(c_t^e, x_t^e)}{u_c(c_0^e, x_0^e)} f_0^c(\epsilon^t | \epsilon_0) = \frac{p_t}{p_0}, \quad \forall \epsilon^t, t = 0, 1, 2, \ldots, T,
\]
\[
\sum_{t=0}^{T} \int p_t[c_t^e - u_t^e(1 - \tau_t)(1 - x_t^e)] d\epsilon^t = 0,
\]
\[
v_c(c_t^b, x_t^b)u_t^b(1 - \tau_t) = v_x(c_t^b, x_t^b), \quad \forall \epsilon^t, t = 0, 1, 2, \ldots, T,
\]
\[
\beta^t \frac{v_c(c_t^b, x_t^b)}{v_c(c_0^b, x_0^b)} f_0^c(\epsilon^t | \epsilon_0) = \frac{p_t}{p_0}, \quad \forall \epsilon^t, t = 0, 1, 2, \ldots, T,
\]
\[
\sum_{t=0}^{T} \int p_t[c_t^b - u_t^b(1 - \tau_t)(1 - x_t^b)] d\epsilon^t = 0.
\]

Substituting out prices and tax rates reduces these conditions to the implementability constraints:

\[
\sum_{t=0}^{T} \beta^t \int u_c(c_t^e, x_t^e)c_t^e - u_c(c_0^e, x_0^e)[1 - x_t^e] dF_0^c(\epsilon^t | \epsilon_0) = 0,
\]

(30)

\[
\sum_{t=0}^{T} \beta^t \int v_c(c_t^b, x_t^b)c_t^b - v_c(c_0^b, x_0^b)[1 - x_t^b] dF_0^c(\epsilon^t | \epsilon_0) = 0,
\]

(31)

\[
\frac{u_c(c_t^e, x_t^e)}{u_c(c_0^e, x_0^e)} - \frac{u_c(c_t^b, x_t^b)}{u_c(c_0^b, x_0^b)} = 0, \quad \forall \epsilon^t, t = 1, 2, \ldots, T,
\]

(32)

\[
\frac{u_x(c_t^b, x_t^b)}{u_x(c_0^b, x_0^b)} - \frac{u_x(c_t^e, x_t^e) u_t^b}{u_x(c_0^e, x_0^e) u_t^b} = 0, \quad \forall \epsilon^t, t = 0, 1, 2, \ldots, T.
\]

(33)

The government faces the intertemporal budget constraint

\[
\sum_{t=0}^{T} \int p_t[g_t - \tau_t(\eta u_t^e(1 - x_t^e) + (1 - \eta) u_t^b(1 - x_t^b))] d\epsilon^t = 0.
\]

Substituting out prices and tax rates leads to the equivalent representation

\[
\sum_{t=0}^{T} \beta^t \int v_c(c_t^b, x_t^b)[g_t - \phi_t \Psi_t] dF_0^c(\epsilon^t | \epsilon_0) = 0
\]

(34)

with

\[
\phi_t \equiv 1 - v_x(c_t^b, x_t^b)/(u_t^b v_c(c_t^b, x_t^b)),
\]
\[
\Psi_t \equiv \eta u_t^b (1 - x_t^b) + (1 - \eta) u_t^b (1 - x_t^b).
\]

The government maximizes the social welfare function subject to the implementability constraints, the government budget constraint, and the aggregate resource constraints. This program reads (with multipliers in front of the restrictions)\footnote{In stating the government's program, I neglect the inequality constraints in (3). The multipliers are given in normalized form.}
max \[ T \sum_{t=0}^T \beta^t \int \theta^t \eta u(c^a_t, x^a_t) + \theta^t (1 - \eta) v(c^b_t, x^b_t) \, dF^*_0(\epsilon^t | \epsilon_0) \]
\[ \text{s.t.} \]
\[ [\mu^a_t(\epsilon^t) \beta^t f^*_0(\epsilon^t | \epsilon_0)] \quad (3), \]
\[ [\lambda^a \eta] \quad (30), \]
\[ [\lambda^b (1 - \eta)] \quad (31), \]
\[ [\lambda^a_t(\epsilon^t) \beta^t f^*_0(\epsilon^t | \epsilon_0)] \quad (32), \]
\[ [\lambda^b_t(\epsilon^t) \beta^t f^*_0(\epsilon^t | \epsilon_0)] \quad (33), \]
\[ [\lambda^a] \quad (34). \]

The policy instruments in this program are given by \( c^a_t, x^a_t, i = a, b, \forall \epsilon^t \), \( t = 0, 1, 2, \ldots, T \). One of the three budget constraints is redundant, and one of the three respective multipliers can therefore be normalized to zero. (Under a BB policy, one of the two budget constraints of the private sector is redundant.)

The constrained ex ante optimal tax plan satisfies (3), (30), (31), (32), (33), \( \lambda^0 = 0 \), and\(^36\)

\[ \eta \{ u_{ct} \theta^t + \lambda^a [u_{ct} + u_{ct} c^a_t + u_{ct} [x^a_t - 1] - \mu_t] \\
+ \lambda^b [u_{ct} v_{ct} - u_{ct} w_{ct} v_{xt}] \\
\} = 0, \forall \epsilon^t \], \( t = 1, 2, \ldots, T \), \( (36) \)

\[ \eta \{ v_{ct} \theta^t + \lambda^a [v_{ct} + v_{ct} c^a_t + v_{ct} [x^a_t - 1] - \mu_t] \\
+ \lambda^b [-v_{ct} v_{ct} + \lambda^0 [u_{ct} v_{ct} - u_{ct} v_{xt} w_{ct}] \\
\} = 0, \forall \epsilon^t \], \( t = 1, 2, \ldots, T \), \( (37) \)

\[ \eta \{ u_{xt} \theta^t + \lambda^a [u_{ct} c^a_t + u_{xt} [x^a_t - 1] + u_{xt} - \mu_t w^a_t] \\
+ \lambda^0 [u_{xt} v_{ct} - u_{ct} v_{xt} w_{ct}] \\
\} = 0, \forall \epsilon^t \], \( t = 1, 2, \ldots, T \), \( (38) \)

\[ (1 - \eta) \{ v_{ct} \theta^t + \lambda^a [v_{ct} c^a_t + v_{xt} \phi_t v_{xt} - \mu_t w^a_t] \\
+ \lambda^b [v_{ct} v_{ct} - u_{ct} v_{xt} w_{ct}] \\
\} = 0, \forall \epsilon^t \], \( t = 1, 2, \ldots, T \), \( (39) \)

(36)–(39) only hold for \( t > 0 \). The first-order conditions with respect to \( c^0_t, c^a_t, x^0_t, x^a_t \) involve modified expressions for the term multiplying \( \lambda^t(\epsilon^t) \). The first-order condition with respect to \( c^0_t \) for example contains

\[ -u_{c0} \sum_{t=1}^T \beta^t \int \lambda^0_t v_{ct} dF^*_0 \]

instead of the \( \lambda^t(\epsilon^t) \) term in (36). Parallel modifications apply in the other cases.

The first lines of (36) and (38) correspond with the first-order conditions in a setting with a representative agent, cf. Lucas and Stokey (1983, p. 62).\(^37\) They summarize the marginal effect on social welfare due to the presence of the \( a \)-types: An increase in \( c^a_t \) or \( x^a_t \) benefits these households, affects their implementability constraint, and requires resources. With \( \eta < 1 \), additional restrictions apply: Changes in \( c^b_t \) or \( x^b_t \) benefit the \( b \)-types, affect their implementability

\(^36\) \( u_{ct} \) stands for \( u(c^a_t, x^a_t) \) etc.

\(^37\) If all households are identical, \( \eta = 1 \), the constraints (31), (32), and (33) become obsolete.
constraint, and require resources. Furthermore, the conditions of equal intertemporal marginal rates of substitution (multiplier $\lambda^a_t$) and marginal tax rates (multiplier $\lambda^b_t$) across types become binding.

Under a weak BB policy, the government faces separate budget constraints for each period and (34) is replaced by

$$\int v(c^e_t, x^e_t) [g_t - \varphi_t \Psi_t] dF^g_t(e^t|e^{t-1}) = 0, \quad t = 0, 1, 2, \ldots, T. \quad (40)$$

Accordingly, the last constraint in (35) changes to $[\lambda^2_t(e^{t-1}) \beta^t]$ (40) where the multipliers $\lambda^a_t(e^{t-1})$, $t = 0, 1, 2, \ldots, T$, are different from zero. Since the government's budget constraints are now more restrictive than the ones of the households, either (30) or (31) become redundant. The first-order conditions to the government's program are then given by (3), (30) or (31), (32), (33), (40), and (36)–(39) subject to the modification that the multiplier $\lambda^a_t$ in the latter four is replaced by $\lambda^2_t(e^{t-1})$ and that either $\lambda^a_t$ or $\lambda^b_t$ is equal to zero.

Under a strict BB policy, the government faces separate constraints for each state in each period. Condition (34) is replaced by

$$g_t - \varphi_t \Psi_t = 0, \quad \forall e^t, t = 0, 1, 2, \ldots, T. \quad (41)$$

The last constraint in (35) changes to $[\lambda^2_t(e^t) \beta^t f^g_t(e^t|e^t_0)]$ (41). The first-order conditions to the government's program are then given by (3), (30) or (31), (32), (33), (41), and (36)–(39). Three modifications apply with respect to the latter four equations: The multiplier $\lambda^a_t$ is replaced by a time and state dependent multiplier $\lambda^2_t(e^t)$; either $\lambda^a_t$ or $\lambda^b_t$ are equal to zero; and the terms multiplying $\lambda^2_t(e^t)$ are no longer the ones given in (37), (38), and (39), but $-\Psi_t \varphi_{ct}$, $\varphi_{tt} \rho \mu^a_t$, and $-\Psi_t \varphi_{xt} + \varphi_t (1 - \eta) w^b_t$, respectively.

The expressions multiplying $\lambda^2_t$ illustrate the effects of marginal changes in $c^e_t$, $x^a_t$, and $x^b_t$ on the government's budget constraint. Variations in $x^a_t$ and $x^b_t$ affect the implicit tax rate $\varphi_t$, and variations in $c^e_t$ affect the state prices. In the absence of a BB rule, the multipliers on the resource constraints and the implementability constraints fully capture the value of these effects and $\lambda^2_t$ equals zero. Under a weak BB rule, the constraints on the government's budget become more restrictive and the shadow cost of the "tax base", "tax rate", and "state price effect" are measured by $\lambda^2_t$. Under a strict BB rule, the market prices for insurance and savings no longer affect the government budget constraint and the state price effect disappears.

A.2 Notes on the Calibration

A.2.1 Financing the German Unification

Sample The simulation covers the years 1991–2050. To simplify the numerical procedures, this range is divided into six intervals of 10 years each. All variables in the simulation represent ten-year averages. I assume that the variables converge to their balanced growth path values throughout the first four intervals. From 2031, productivity, consumption and government expenditure grow at constant positive rates whereas tax rates and labor supply remain constant.

Labor Productivity I set $w^a_{1991}$ to 1. I approximate relative productivities by 0.8 times the ratio of West to East German per capita GDP. Multiplication by 0.8 is to guarantee

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a conservative estimate of this ratio, reducing the potential for tax shifting. I assume that productivity in the West grows at an annual rate of 1.5 percent, whereas productivity in the East converges to the Western one:

\[ w_t^a = 1.015 \ w_{t-1}^a, \ t = 1992, \ldots, 2030, \]
\[ w_t = w_{t-1} \left( \frac{1}{w_{t-1}} \right)^{0.1}, \ t = 1999, \ldots, 2030. \]

The path of \( w_t^a \) follows directly. In the simulation, I use ten year averages of these generated series. Relative productivity equals \( w_{1990s} = 2.0359, w_{2000s} = 1.3715, w_{2010s} = 1.1153, \) and \( w_{2020s} = 1.0387. \)

**Government Expenditure** In the model, \( g \) represents public consumption. In the data, transfers and investment outlays constitute an important component of public spending. For simplicity, I do not distinguish between these components. I assume that the utility function is additively separable in public consumption, investment, and/or transfers and that transfers are non marketable (do not enter the household’s budget constraint). These assumptions are consistent with the fact that this paper as well as the relevant literature focus on the welfare effects of the financing side of fiscal policy.

I extrapolate the government expenditure-to-GDP ratio, \( R_t \) say, under the assumption that it converges to 40 percent, subject to the following law of motion:

\[ R_t = R_{t-1} \left( \frac{0.4}{R_{t-1}} \right)^{0.1}, \ t = 1999, \ldots, 2030. \]

I find \( R_{2000} = 0.4026 \) from a maximum of \( R_{1995} = 0.5610. \) \( R_t \) times the model’s production level under a BB policy represents government expenditure. In the simulation, I use ten year averages of this generated series, namely: \( g_{1990s} = 0.3151, g_{2000s} = 0.3429, g_{2010s} = 0.3884, \) and \( g_{2020s} = 0.4477. \)

**Parameters** \( \eta = 64074/79984 \approx 0.8, \beta = 0.985, \) implying a risk free annual rate of return of 3 percent. \( \gamma^a = \gamma^b = 0.5, \) implying a steady state labor supply of 2/3.

### A.2.2 Taxation over the Business Cycle

I represent the (normalized) business cycle by a deterministic sequence of three time intervals: five periods of “normal” times, four periods of a “boom”, and two periods of a “slump”. I interpret each period as about one half year to capture the fact that the average post war business cycle (1945–1991, corresponding to the years for which I have data on the income distribution, see below) consisted of an expansion of about 4.5 years and a contraction of about one year (NBER business cycle database).

Define \( y_t \) as output if employment is at its steady state value. Let \( \hat{x}_t \) denote the percentage deviation of \( x_t \) from its mean or trend value \( \bar{x}_t \). I choose levels of \( y_t \) for each of the three intervals, such that the standard deviation and skewness of \( y_t \) (0.0233 and −0.4170, respectively) are close to the standard deviation and skewness of GDP \( \gamma (0.0217 \text{ and } -0.4579, \) respectively; trend is computed based on HP-filter (400) after taking logs; semi-annual data from 1947 to 1986).

---

Castañeda et al. (1998) report the average income shares of the quintiles of the income distribution as well as the cross correlations between (percentage trend deviations of) output and the income shares for the years 1948-1986. I synthesize these measures into those for two income shares, the ones for the lower sixty and the upper forty percent of the income distribution. (This implies $\eta = 0.6$.) I choose to split the population into those two groups since the income shares of the lower sixty percent are pro cyclical, whereas the shares of the upper forty percent are counter cyclical. I then construct productivity sequences for these two groups such that the means of the implied income shares at steady state employment match the mean income shares in the data and the shares’ covariances with $y_t$ match the covariances with output in the data (covariances of percentage deviations of output with steady state employment shares: $1.4 \cdot 10^{-4}$ and $-7.5 \cdot 10^{-5}$).

Since government spending is essentially a-cyclical (Cooley and Prescott, 1995), I choose $g_t$ to equal $g = 0.2 \text{ mean}(y_t)$. Finally, $\beta = 0.985^{0.5} = 0.9925$ and $\gamma^b = 1/2$ as in the previous example.

Table 1 reports the exogenous variables as well as model predictions in the two scenarios.

<table>
<thead>
<tr>
<th>Scenario one: relative productivity varies over time</th>
<th>Scenario two: relative productivity constant over time</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exogenous variables</strong></td>
<td><strong>Ramsey policy</strong></td>
</tr>
<tr>
<td>$g_t$</td>
<td>$e_i + g_i$</td>
</tr>
<tr>
<td>normal</td>
<td>0.2000</td>
</tr>
<tr>
<td>boom</td>
<td>0.2000</td>
</tr>
<tr>
<td>slump</td>
<td>0.2000</td>
</tr>
</tbody>
</table>

A.3 Time Consistency

At any point in time, the government’s program is isomorphic to a static problem since financial markets are complete. A sequence of optimal policies over time, however, need not necessarily represent the continuation of the initial optimal policy (Kydland and Prescott, 1977). At the beginning of period 1, for example, the households’ consumption, work, and savings decisions from period 0 as well as the government’s choice of maturity structure for debt issued in period

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40 The difference between output at steady state employment and output at the endogenous household labor supply is small, such that the covariances are not significantly altered when the endogenous labor supply is accounted for.
0 are irrevocable. The government now takes these variables as given and might therefore want to revise its initial policy.

To keep track of the budget constraints over time, it is necessary to explicitly introduce the amounts of contingent claims held by the households. Following Lucas and Stokey (1983), I denote by \( \{s_t \}_{t=s}^T \) the sequence of government issued contingent claims that are held by type \( i \) at the beginning of period \( s \) and promise payment in period \( t \) (and state \( \epsilon(t, \epsilon^{-1}) \)). Similarly, \( \{d_t^i\}_{t=s}^T \) denotes the sequence of privately issued contingent claims. Consistency requires \( \eta_t d_t^i + (1 - \eta_t) s_t^i = 0, \forall \epsilon(t, \epsilon^{-1}, t = s, s + 1, \ldots, T) \).

The following discussion applies to general utility functions. It assumes, first, that the implementability constraints which state that all households face the same prices and tax rates,

\[
\frac{u_c(c_t^i, x_t^i)}{v_c(c_t^i, x_t^i)} = \frac{u_c(c_t^0, x_t^0)}{v_c(c_t^0, x_t^0)} \equiv \frac{1}{c}, \forall \epsilon^t, t = 1, 2, \ldots, T;
\]

\[
\frac{u_x(c_t^i, x_t^i)}{v_x(c_t^i, x_t^i)} = \frac{u_x(c_t^0, x_t^0)}{v_x(c_t^0, x_t^0)} \equiv \frac{1}{x}, \forall \epsilon^t, t = 1, 2, \ldots, T;
\]

can be solved for functions \( \theta^o(c_t^i, x_t^i, c; w_t) \) and \( \bar{\theta}^o(c_t^i, x_t^i, c; w_t), \forall \epsilon^t, t = 0, 1, 2, \ldots, T \). Second, it assumes that these functions allow for solving the resource constraint

\[
e_t = \eta_t (\theta^o(\cdot) + u_t^i \bar{\theta}^o(\cdot)) + (1 - \eta_t) (\bar{\theta}^o(\cdot) + u_t^i \theta^o(\cdot)), \forall \epsilon^t, t = 0, 1, 2, \ldots, T,
\]

for functions \( \theta^o(\cdot), \bar{\theta}^o(\cdot), \forall \epsilon^t, t = 0, 1, 2, \ldots, T \), where \( h_t \equiv (x_t^i, c; w_t^i, u_t^i, g_t, \eta_t) \). (In the main model, these conditions were trivially satisfied.) Consequently, utilities and marginal utilities are also functions of \( h_t \). I denote these functions by \( u(h_t), v(h_t) \) etc.

I first derive the equations characterizing an interior optimal solution to the government’s program as of time \( r \). I compare them to the optimality conditions from the subsequent program at time \( s = r + 1 \). The policy is time consistent between \( r \) and \( s \), if the government can issue debt in period \( r \) with a maturity and ownership structure \( \{s_t^i, d_t^i\}_{t=s}^T \), such that the optimal policy as of time \( s \), given this structure, is a continuation of the one chosen in period \( r \). If this is the case, the government can, by induction, always commit to its ex ante optimal policy.

The constraints of the government’s program as of time \( r \) are given by the reduced form implementability constraints that—in contrast to the earlier representation—now incorporate the resource constraint:

\[
\sum_{t=r}^T \beta^t \int u_c(h_t)(\theta^o(h_t) - r d_t^o - r d_t^i) - u_x(h_t)[1 - x^o(h_t)] dF_t^c(\epsilon^t | \epsilon^r) = 0, \quad (42)
\]

\[
\sum_{t=r}^T \beta^t \int v_c(h_t)(\bar{\theta}^o(h_t) - r d_t^o + r d_t^i) - v_x(h_t)[1 - x^o(h_t)] dF_t^c(\epsilon^t | \epsilon^r) = 0. \quad (43)
\]

(For convenience, I here assume that \( \eta = 0.5 \).) The government’s problem,

\[
\max_{\{x_t^i\}_{t=r}^T, c} \sum_{t=r}^T \beta^t \left( \theta^o(\epsilon_t^i) + \bar{\theta}^o(\epsilon_t^i) \right) u(h_t) + \theta^o(\epsilon_t^i) \bar{\theta}^o(\epsilon_t^i) v(h_t) dF_t^c(\epsilon^t | \epsilon^r)
\]

s.t. \( [\lambda^o] \) (42), \( [\lambda^\theta] \) (43),

implies the first-order conditions (42), (43), and

\[
m_1(h_t) + \lambda^o[m_2(h_t) + m_3(h_t)(r d_t^o + r d_t^i)] + \lambda^\theta[m_4(h_t) + m_5(h_t)(r d_t^o - r d_t^i)] = 0,
\]

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\[ \forall \epsilon^t | \epsilon^r, t = r, r + 1, \ldots, T, \]
\[ \sum_{t=r}^{T} \beta^t \int m_0(h_t) dF^s_t(\epsilon^t | \epsilon^r) + \lambda^a \sum_{t=r}^{T} \beta^t \int m_1(h_t) + m_2(h_t)(r \beta^s_t + r d^s_t) dF^s_t(\epsilon^t | \epsilon^r) \]
\[ + \lambda^b \sum_{t=r}^{T} \beta^t \int m_3(h_t) + m_4(h_t)(r \beta^s_t - r d^s_t) dF^s_t(\epsilon^t | \epsilon^r) = 0, \]

where \( m_j(\cdot), j = 1, 2, \ldots, 10, \) are some functions of \( h_t. \) Denote the system of equations (42)–(45) that holds at time \( r \) by \( E^r. \) Given \( \{r b^a_t, r b^b_t, r d^s_t\}_{t=r}^{T}, r E \) determines \( \{s x^b_t\}_{t=s}^{T}, s, c, r \lambda^a, r \lambda^b \) subject to these values and an updated maturity structure of privately issued claims, \( \{s d^b_t\}_{t=s}^{T}, \) the government chooses the maturity structure \( \{s b^a_t, s b^b_t\}_{t=s}^{T}. \) Given \( \{s b^a_t, s b^b_t, s d^b_t\}_{t=s}^{T}, \) \( s E \) determines in the next period \( \{s x^b_t\}_{t=s}^{T}, s, c, s \lambda^a, s \lambda^b \). The government can commit to the optimal policy as of time \( r, \) if there exists \( \{s b^a_t, s b^b_t\}_{t=r}^{T}, s, c \) such that \( \{r x^b_t\}_{t=s}^{T}, r \lambda^a, r \lambda^b \) (43) in \( s E^R. \) The government can therefore commit to its optimal policy.\(^4\)

With heterogeneous agents, the maturity structure of government debt needs to counterbalance the differential incentives along the tax smoothing and the tax shifting margin. Moreover, it must take into account that \( \{s d^b_t\}_{t=s} \) no longer equals \( \{0\}. \) Assume, for example, that \( \{s b^b_t\}_{t=s}^{T} = \{0\} \) and that privately issued debt (satisfying (42) as of \( t = s \)) is given by a particular sequence \( \{s d^b_t\}_{t=s}^{T}. \) \( (44) \) in \( s E \) can then be solved for \( s b^b_t(\lambda^a, \lambda^b, h_t, s d^b_t), \) \( \forall \epsilon^t | \epsilon^r, t = s, s + 1, \ldots, T. \) Substituting these functions into (43) and (45) in \( s E \) results in two equations in the two unknowns \( s \lambda^a, s \lambda^b. \) A solution to these equations implicitly defines the maturity structure \( \{s b^b_t\}_{t=s}^{T} \) that allows the government to commit to its optimal tax plan as of time \( r. \)

References


\(^4\)If the government could costlessly appropriate resources, the resource constraints (incorporated in \( v(h_t) \)) would be the only constraints to bind and \( \lambda^b = 0. \) Time consistency would then be guaranteed independent of the maturity structure.


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