KINEMATIC ANALYSIS OF NUCLEAR SPIRALS: FEEDING THE BLACK HOLE IN NGC 1097

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ABSTRACT

We present a harmonic expansion of the observed line-of-sight velocity field as a method to recover and investigate spiral structures in the nuclear regions of galaxies. We apply it to the emission-line velocity field within the circumnuclear star-forming ring of NGC 1097, obtained with the GMOS-IFU spectrograph. The radial variation of the third harmonic terms is well described by a logarithmic spiral, from which we interpret that the gravitational potential is weakly perturbed by a two-arm spiral density wave with an inferred pitch angle of $52^\circ \pm 4^\circ$. This interpretation predicts a two-arm spiral distortion in the surface brightness, as hinted by the dust structures in central images of NGC 1097, and predicts a combined one-arm and three-arm spiral structure in the velocity field, as revealed in the non-circular motions of the ionized gas. Next, we use a simple spiral perturbation model to constrain the fraction of the measured non-circular motions that is due to radial inflow. We combine the resulting inflow velocity with the gas density in the spiral arms, inferred from emission-line ratios, to estimate the mass inflow rate as a function of radius, which reaches about $0.011 M_\odot \text{ yr}^{-1}$ at a distance of 70 pc from the center. This value corresponds to a fraction of about 4.2×10^{-3} of the Eddington mass accretion rate onto the central black hole in this LINER/Seyfert1 galaxy. We conclude that the line-of-sight velocity can not only provide a cleaner view of nuclear spirals than the associated dust, but that the presented method also allows the quantitative study of these possibly important links in fueling the centers of galaxies, including providing a constraint on the mass inflow rate as a function of radius.

Key words: galaxies: active – galaxies: individual (NGC 1097) – galaxies: kinematics and dynamics – galaxies: nuclei – galaxies: structure

Online-only material: color figures

1. INTRODUCTION

Gas transport to the centers of galaxies is still mainly an unsolved problem (e.g., Martini 2004). Since most of the gas is residing in a rotating disk well beyond the center, it is essentially a problem of angular momentum transport. Proposed transport mechanisms range from galactic interactions and bars to nuclear bars and spirals to stellar mass loss and disruption near the central black hole (BH). This range in scales also closely represents a range in decreasing mass inflow rates, which in turn might be correlated with activity in the galactic nucleus (active galactic nucleus (AGN)), ranging from quasar, Seyfert, LINER to quiescent galaxies. However, not only is it challenging to (observationally) establish fueling mechanisms down to a few parsec from the center, time delays between changes in the mass inflow rate and the onset of nuclear activity further complicate linking them.

It is also likely that multiple fueling mechanisms are important and act together. Large-scale bars are efficient at transporting gas inward (e.g., Athanassoula 1992), but the presence of an inner Lindblad resonance (ILR) will cause the gas to pile up in a nuclear ring, often clearly visible due to the intense star formation (see e.g., Figure 1). The gas might continue further inward through nested bars (e.g., Shlosman et al. 1989; Englmaier & Shlosman 2004), although dynamical constraints on a double-barred system may prohibit inflow down to the center (Maciejewski et al. 2002). Moreover, observational evidence of nuclear bars is scarce (e.g., Regan & Mulchaey 1999;

Martini & Pogge 1999), but should be interpreted with care since the prominent dust lanes in the main bar might be absent in secondary bars (e.g., Shlosman & Heller 2002).

On the other hand, nuclear spirals seem to be commonly observed, both in active and quiescent galaxies (e.g., Laine et al. 1999; Pogge & Martini 2002), ranging from flocculent to grand-design nuclear spirals (e.g., Martini et al. 2003). Whereas the former are suggested to form by acoustic instabilities (e.g., Elmegreen et al. 1998), the grand-design nuclear spirals are thought to be the result of gas density waves (e.g., Englmaier & Shlosman 2000) or shocks (e.g., Maciejewski 2004a, 2004b) induced by the non-axisymmetric gravitational potential of a large-scale bar. The latter two might be connected in the sense that the bar-driven spiral shocks trigger the gas density waves throughout the disk (Ann & Thakur 2005), which in turn seem to be necessary for the nuclear spiral to be long-lived (Englmaier & Shlosman 2000). The inward extended gas inflow through a nuclear spiral not only depends on the torque of the large-scale bar, but also on the gas having high enough sound speed so as to lose angular momentum (e.g., Englmaier & Gerhard 1997; Patsis & Athanassoula 2000), as well as on the presence of a central mass concentration such as a supermassive BH to overcome a closer-in (inner) ILR (e.g., Fukuda et al. 2000; Ann & Thakur 2005).

The deviations in the gas density due to nuclear spirals are typically only a few percent (Englmaier & Shlosman 2000), which makes direct imaging very difficult. Instead, the obscuration due to dust thought to be associated with the gas overdensities is often employed, but when the extinction is small or the dust is not well mixed with the gas, it can lead to unclear

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Figure 1. Left: VLT/VIMOS color composite image of NGC 1097, showing the large-scale spiral arms and bar with prominent dust lanes reaching down to the circumnuclear ring (credit: European Southern Observatory). Middle: VLT/NACO adaptive optics color composite image of the circumnuclear ring region (credit: European Southern Observatory), with the footprints of the observations with the GMOS-IFU spectrograph. Right: *HST*/ACS structure map ($20'' \times 20''$) of the same circumnuclear region with the wavelet map of Lou et al. (2001) overplotted in color, with increasing intensity from blue to red. The solid magenta curves show the two-arm nuclear spiral with pitch angle 52° , which we predict based on the three-arm spiral structure in the velocity field. The dashed magenta curves indicate the two additional arms in case an $m = 4$ spiral perturbation with the same pitch angle would be present (see Section 4.1 for further details).

(A color version of this figure is available in the online journal.)

or even missed detections of nuclear spirals. At the same time, nuclear spirals induce non-circular motions in the gas, with resulting deviations in the observed velocity field that can be a significant fraction of the underlying circular velocity. If the gas is ionized, the kinematics might be inferred from emission lines, which in general can be detected more easily and at higher spatial resolution in the optical, than can be tracers of the molecular or atomic gas such as CO and H I at radio wavelengths. Moreover, while imaging typically only yields a detection of the nuclear spiral, the observed non-circular motions in combination with the gas (over)density inferred from the simultaneously measured (line) fluxes might be used to derive an estimate of the gas mass in/outflow rate.

An alternative approach to get a constraint on the gas flow rates is to compute the gravitational torques from the observed surface brightness (SB) using a mass-to-light ratio conversion calibrated against (circular) velocity measurements (García-Burillo et al. 2005), or against stellar population models fitted to color measurements (Quillen et al. 1995). In these cases, various assumptions are made, most importantly that stellar light is a clean (once corrected for dust obscuration) and direct (mass follows light) tracer of the underlying gravitational potential. Even so, this approach shows that gravitational torques due to non-axisymmetric structures are very efficient at transporting gas inward while overcoming dynamical barriers such as the corotation resonance. However, when the resulting predicted mass inflow rates as functions of radius are compared with the non-circular motions in the observed gas velocity fields, the correlation between both is often not evident (Haan et al. 2009).

The main difficulty with the non-circular motions is to identify the fraction that is due to pure radial flow. Gas on closed elliptic orbits not only contributes to non-circular motions (or elliptic streaming) in the azimuthal direction, but also in the radial direction (Wong et al. 2004). However, since closed elliptic orbits are applicable strictly only to collisionless (stellar) orbits, we expect that as a result of shocks and other dissipational effects the angular momentum of the gas will change, leading to in/outflow. While the measured non-circular motions are often

taken to be directly representative of the radial flow velocities (e.g., Storchi-Bergmann 2007), only a fraction of them are expected to be truly radial in/outflows.

The goal of this paper is two-fold: (1) to show that harmonic expansion of gas velocity fields provides a clean way to detect nuclear spirals and to estimate their pitch angles and (2) to use simple perturbation models to constrain the fraction of measured non-circular motions that is due to radial flow, and to estimate the corresponding mass inflow rate. In Section 2, we present the harmonic analysis, and in Section 3 we apply it to the observed emission-line velocity field within the circumnuclear star-forming ring of NGC 1097. We fit the third harmonic terms as a three-arm logarithmic spiral structure consistent with a weak two-arm spiral perturbation of the gravitational potential. We then use the perturbation model derived in the Appendix to constrain the inflow rate as a function of distance from the center of NGC 1097. In Section 4, we discuss the corresponding spiral distortion in the SB and possible additional non-circular motion contributions to the velocity field. Finally, we link the estimated mass inflow rate to the accretion onto the central BH of NGC 109. We summarize and draw our conclusions in Section 5.

2. HARMONIC ANALYSIS

Maps of the SB, line-of-sight velocity (V), velocity dispersion (σ), and higher-order velocity moments (often expressed in terms of the Gauss–Hermite moments $h_3, h_4, \dots$) of nearby galaxies generally show organized, periodic features, which can be suitably studied by means of a harmonic expansion (e.g., Schoenmakers et al. 1997; Wong et al. 2004; Fathi et al. 2005; Krajnović et al. 2006; Spekkens & Sellwood 2007).

2.1. Harmonic Expansion

We start by dividing a map into a number of elliptic annuli, each with a different semimajor axis length R , but we assume all have the same flattening q and share the same angle ψ_0 and

center (x'_0, y'_0) , so that in terms of the Cartesian coordinates on the map (i.e., on the plane of the sky)

$$\begin{aligned} x' &= x'_0 + R \cos \psi \cos \psi_0 - q R \sin \psi \sin \psi_0, \\ y' &= y'_0 + R \cos \psi \sin \psi_0 + q R \sin \psi \cos \psi_0. \end{aligned} \quad (1)$$

If the map is aligned such that x' is pointing north and y' is pointing west, then $\psi_0 = \pi/2 + \Gamma$, where Γ is the common (observational) definition of the position angle of the major axis of the galaxy, measured from north through east. Next, we extract the profiles along each of the annuli and describe them by a finite number of harmonic terms n ,

$$P = c_0(R) + \sum_{m=1}^n c_m(R) \cos m\psi + s_m(R) \sin m\psi. \quad (2)$$

There are various ways to obtain the set of best-fit ellipses, but given the highly nonlinear nature of the optimization for the best-fit parameters, this is commonly done in a stepwise, iterative process. We follow a similar approach as described in Krajnović et al. (2006)—based on the usually adapted procedure in photometry (Jedrzejewski 1987)—which we briefly summarize.

For (early-type) galaxies, the maps of SB, σ , and even higher-order velocity moments ($h_4, h_6, \dots$) each are well approximated to first order by a function that is constant along (concentric, equally flattened and orientated) ellipses and only varies as a function of the semimajor axis length of these ellipses. Similarly, V and odd higher-order velocity moments ($h_3, h_5, \dots$) each are well described by a similar function, but one that has an additional cosine variation along the ellipses. In these cases, the harmonic expansion in Equation (2) truncates after the third-order term ($n = 3$). The parameters of the best-sampling ellipses for even velocity moments can then be obtained by minimizing $\chi^2 = \sum_{m=1}^3 (c_m^2 + s_m^2)$, and without c_1 for odd velocity moments. We first perform the minimization for a grid of (fixed) q and ψ_0 values, where we might use external constraints such as the inclination i and measured position angle Γ together with an initial estimate of the center (x'_0, y'_0) . Next, starting from the best-fit grid pair, we optimize for all four parameters.

We sample the semimajor axis lengths as $R = R_1[k + (1 + g)^{(k-1)}]$ for $k = 1, 2, 3, \dots$, with the initial R_1 depending on the spatial resolution; for the geometric increase factor we take $g = 0.1$. We achieve this sampling, together with uniform sampling in ψ , by bilinear interpolation of the observed map. When fitting the observed line-of-sight velocity field shown in Figure 2, we assume all motions are within the equatorial plane. We divide all harmonic terms by $q = \sin i$ to take into account the projection effect. For further details, including error estimates, see Fathi et al. (2005).

2.2. Spiral Structure

Once we have obtained the set of best-fitting ellipses, we fit the harmonic expansion of Equation (2) to each of the corresponding profiles to obtain the harmonic terms c_m and s_m (up to higher order $n > 3$), as functions of R . The difference with the above expansion up to and including $n = 3$ reveals the deviations from the latter smooth model: for example, boxiness and diskiness in the SB map, and non-circular motion in the residual V map.

In the case of a spiral structure as observed in NGC 1097, both through the dust in the SB and in the residual V map (Fathi et al. 2006), the deviations are more naturally described as a (radially varying) offset in the angle ψ than through a

complex combination of variations in the amplitudes c_m and s_m . Henceforth, we rewrite the harmonic expansion of Equation (2) (in a mathematically equivalent expression) as

$$P = K_0(R) + \sum_{m=1}^n K_m(R) \cos(m[\psi - \psi_m(R)]), \quad (3)$$

where the amplitudes K_m and phase shifts ψ_m are related to the coefficients c_m and s_m ($s_0 = 0$) by

$$K_m^2 = c_m^2 + s_m^2 \quad \text{and} \quad \tan(m\psi_m) = \frac{s_m}{c_m}. \quad (4)$$

For example, the V map of NGC 1097 in Figure 2 reveals, after subtracting the best-fit circular motion $K_0 + K_1(R) \cos \psi$ (with $K_1 = c_1$, since we set $\psi_1 = s_1 = 0$), what seems to be a three-arm spiral structure, $K_3(R) \cos(3[\psi - \psi_3(R)])$. Here, $\psi_3(R)$ traces the spiral arms as a function of radius R .

2.3. Weakly Perturbed Gravitational Potential

An axisymmetric gravitational potential in a frame that co-rotates with a weak perturbation of the harmonic number m can be written in terms of the polar coordinates as

$$\Phi(R, \phi) = \Phi_0(R) + \Phi_m(R) \cos(m[\phi - \phi_m(R)]), \quad (5)$$

where ϕ_m is the phase of the perturbation. Through Poisson's equation it follows that the corresponding surface mass density exhibits a harmonic m distortion.

To derive the line-of-sight velocity, we follow Schoenmakers et al. (1997). As described in the Appendix, we extend their collisionless analysis by including radial damping in the equations of motion to take into account the dissipative nature of gas. We assume that the gas moves on closed-loop orbits in the equatorial plane, which we observe at an inclination i away from its normal and at an (azimuthal) angle ϕ_{los} . Given a point (R, ψ) in the equatorial plane, the projection of the azimuthal and radial velocity onto the corresponding line-of-sight yields

$$V_{\text{los}} = \sin i [v_\phi(R, \psi) \cos \psi + v_R(R, \psi) \sin \psi], \quad (6)$$

where $\psi = \phi - \phi_{\text{los}} + \pi/2$ is zero on the line of nodes (see also Figure 1 of Schoenmakers et al. 1997). To first order the solutions of the equations of motion in the perturbed gravitational potential of Equation (5) yield

$$v_R(R, \psi) = v_c(R)[c_R \cos m\psi + s_R \sin m\psi], \quad (7)$$

$$v_\phi(R, \psi) = v_c(R)[1 + c_\phi \cos m\psi + s_\phi \sin m\psi], \quad (8)$$

where $v_c^2 = R d\Phi_0/dR$ is the circular velocity and c_R, s_R, c_ϕ , and s_ϕ are functions of R given in the Appendix. Substituting these solutions into Equation (6), we obtain

$$\begin{aligned} V_{\text{los}} &= V_\star \cos \psi \\ &+ c_{m-1} \cos(m-1)\psi + s_{m-1} \sin(m-1)\psi \\ &+ c_{m+1} \cos(m+1)\psi + s_{m+1} \sin(m+1)\psi, \end{aligned} \quad (9)$$

with $c_{m\pm 1} = V_\star(c_\phi \mp s_R)/2$ and $s_{m\pm 1} = V_\star(s_\phi \pm c_R)/2$, and $V_\star \equiv v_c(R) \sin i$ is the circular velocity in projection. We thus find, as concluded before by Schoenmakers et al. (1997) and already qualitatively inferred by Canzian (1993), that if the gravitational potential has a perturbation of the harmonic number m , the line-of-sight velocity field contains an $m-1$ and an $m+1$ harmonic term.

Figure 2. First panel shows the line-of-sight [N II] emission-line velocity field within the circumnuclear ring in NGC 1097, obtained with the GMOS-IFU spectrograph. The other panels show the results of applying the harmonic expansion of Section 2.1: the best-fit circular motion, the remaining non-circular motion, and the harmonic reconstruction from the sum of the latter two, taking into account the different (linear) velocity scale. The latter is indicated by the color bar on the right-hand side of each map, with the limits given below each bar. In the third panel, the magenta circles illustrate the three-arm logarithmic spiral with pitch angle 63° , derived from the third harmonic coefficients (see Figure 3 below). This is consistent with a weak two-arm spiral perturbation in the gravitational potential, which at the same time would also give rise to a one-arm logarithmic spiral with pitch angle 33° , indicated by the magenta triangles. Since both logarithmic spirals contribute in a different way it is not surprising that neither accurately traces the spiral structure in the non-circular motions (see Section 3.1 for further details).

(A color version of this figure is available in the online journal.)

2.4. Pitch Angle

How loosely or tightly wound a spiral is can be quantified via its pitch angle, ζ , which, at a given radius, R , measures the angle between the tangent of the spiral arm and a circle with radius R in the plane of the disk. Inverting the phase shift, $\phi(R)$, of a spiral, we can parameterize an arm of the spiral as

$$x = R(\phi) \cos \phi \quad \text{and} \quad y = R(\phi) \sin \phi. \quad (10)$$

One can then show that the pitch angle ζ is given by

$$\cot \zeta = \frac{d\phi}{d \ln R}, \quad (11)$$

which is positive (negative) if the spiral curves anti-clockwise (clockwise) with increasing radius. The smaller the pitch angle, the more tightly the spiral is wound, with $\zeta = 0$ a circle, while $\zeta = \pm\pi/2$ corresponds to a straight line. A specific case that is often encountered in nature is that of a logarithmic spiral

$$\phi(R) = \frac{1}{b_0} \ln \frac{R}{a_0} \quad \Leftrightarrow \quad R(\phi) = a_0 \exp(b_0 \phi), \quad (12)$$

with constants a_0 and b_0 . From Equation (11), we find the well-known property that the logarithmic spiral has a constant pitch angle $\zeta = \tan^{-1} b_0$.

In the case of an $(m \pm 1)$ -spiral in the line-of-sight velocity field, the corresponding pitch angle, $\zeta_{m \pm 1}$, follows directly from the phase shift, $\psi_{m \pm 1}(R)$, in the harmonic expansion in Equation (4). The pitch angles ζ_{m-1} and ζ_{m+1} generally take different values and bracket the pitch angle ζ_m of the m -spiral

perturbation in the gravitational potential that caused them. To show this, we start from Equations (A12) and (A13) in the Appendix and rewrite the coefficients $c_{m \pm 1}$ and $s_{m \pm 1}$ as

$$\begin{aligned} c_{m \pm 1} &= K_{m \pm 1} \cos(m\varphi_m - \theta_{m \pm 1}), \\ s_{m \pm 1} &= K_{m \pm 1} \sin(m\varphi_m - \theta_{m \pm 1}), \end{aligned} \quad (13)$$

so that after substitution into Equation (4) we obtain

$$m\varphi_m - \theta_{m \pm 1} = (m \pm 1)\psi_{m \pm 1}, \quad (14)$$

where $\varphi_m = \phi_m - \phi_{\text{los}} + \pi/2$. This links the phase shift $\phi_m(R)$ of the m -spiral perturbation in the gravitational potential with the phase shifts $\psi_{m \pm 1}(R)$ of the $(m \pm 1)$ -spirals in the line-of-sight velocity field. The corresponding pitch angles are related as

$$m \cot \zeta_m = (m \pm 1) \cot \zeta_{m \pm 1} - d\theta_{m \pm 1}/d \ln R. \quad (15)$$

In general, $K_{m \pm 1}$ and $\theta_{m \pm 1}$ depend in a rather complex way on the gravitational potential, but as we show in the Appendix they possess some generic properties.

First, the amplitude K_{m+1} is larger (smaller) than the amplitude K_{m-1} outside (inside) the corotation radius R_{CR} , and equal to it at R_{CR} . This implies a transition in the line-of-sight velocity field at R_{CR} , going from the $(m+1)$ -spiral dominating outside R_{CR} to the $(m-1)$ -spiral dominating inside R_{CR} (see also Figure 5). This is also concluded by Schoenmakers et al. (1997) for the collisionless case and earlier by Canzian (1993) for the less general case of a tightly wound spiral in the linear density-wave theory (see also Canzian & Allen 1997). However, the dissipational nature of the gas, which we model in the

Appendix via radial damping (cf. Wada 1994), can alter the relative amplitudes of the harmonic terms (see also Section 3.2).

Second, $\theta_{m\pm 1}$ typically varies much less with radius than the spiral phase shifts, so that $d\theta_{m\pm 1}/d \ln R$ in Equation (15) is relatively small. As a result, we can estimate the pitch angle ζ_m of the m -spiral perturbation in the gravitational potential from the pitch angles ζ_{m-1} and/or ζ_{m+1} of the $(m-1)$ -spiral and $(m+1)$ -spiral in the observed line-of-sight velocity field, without constructing a full dynamical model (see also the last two panels of Figure 5).

2.5. Radial Flow Velocity

Nuclear spirals in principle provide a mechanism to transport gas from kpc scales, where it often stalls inside a nuclear ring, into the center of the galaxy. Still, as mentioned in Section 1, if a nuclear spiral (or a nuclear bar) is due to gas moving on closed elliptic orbits this results in non-circular motions, also referred to as elliptic streaming, but not necessarily in net inflow toward and/or outflow away from the center. However, unlike stars, gas is not collisionless and its orbits interact and exchange angular momentum leading to net radial flows (e.g., Wada 1994).

Henceforth, in the analytic models in the Appendix, we assume a weak perturbation in the gravitational potential causing gas to deviate from circular onto elliptic orbits, while taking into account its dissipative nature via radial damping. These analytic spiral models are an extension of the analytic bar models introduced by (Wada 1994), who showed that they describe well the gas behavior seen in hydrodynamical simulations. The radial damping causes the gas to lose/gain angular momentum inside/outside the corotation radius, which nicely matches the angular momentum transfer due to the torque from the bar potential in numerical simulations. The amount of radial damping, controlled through the dimensionless parameter,⁶ λ , thus provides a constraint on the amount of net radial flow that is needed to explain the observed non-circular motions in addition to elliptic streaming.

In this way, a rather straightforward estimate of the net radial flow velocity, v_{flow} , can be obtained by comparing the radial velocity v_R (Equation (A5)) of the analytic model that includes radial damping ($\lambda > 0$) with the analytic model without radial damping ($\lambda = 0$). At a given radius R , the maximum radial velocity is given by

$$v_{R,\text{max}} = m (\Omega - \Omega_p) R \left(\frac{A^2 + B^2}{\Delta^2 + \Lambda^2} \right)^{1/2}. \quad (16)$$

Here, A and B only depend on the weak perturbation as given in Equation (A2), while

$$\Delta = \kappa^2 - m^2(\Omega - \Omega_p)^2, \quad \Lambda = 2\lambda\kappa m(\Omega - \Omega_p), \quad (17)$$

are functions of the angular frequency $\Omega(R)$ and the epicycle frequency $\kappa(R)$ of the axisymmetric gravitational potential, as well as the harmonic number m and pattern speed Ω_p of the weak perturbation. Since without radial damping ($\Lambda = 0$) all radial motion is due to elliptic streaming, we subscribe a fraction $|\Delta|/(\Delta^2 + \Lambda^2)^{1/2}$ of the radial velocity to elliptic streaming, leaving as an estimate of the radial flow velocity

$$v_{\text{flow}} = \left(\frac{\Lambda^2}{\Delta^2 + \Lambda^2} \right)^{1/2} v_R. \quad (18)$$

⁶ Note that our λ is the same as the dimensionless parameter Λ in Wada (1994), while we define $\Lambda = 2\lambda\kappa m(\Omega - \Omega_p)$.

Even though the analytic models neglect possible nonlinear effects, they can capture most features of observed non-circular motions (as we show next in the case of NGC 1097), and at the same time provide an estimate of the fraction of the observed non-circular motions that is due to net radial flow in addition to elliptic streaming. Note that radial flow here does not mean that the gas is following pure radial orbits with zero angular momentum, which would contribute only to the harmonic term s_1 (e.g., Wong et al. 2004). Instead, the gas is expected to gradually spiral inward/outward as it has both azimuthal and radial velocity components. In case the angular momentum loss/gain is driven by a weak gravitational potential perturbation with the harmonic number m , this results in a contribution to $c_{m\pm 1}$ and $s_{m\pm 1}$.

3. NUCLEAR SPIRAL IN NGC 1097

We apply the above harmonic analysis to the observed emission-line velocity field within the circumnuclear star-forming ring of NGC 1097. We recover in the non-circular motions a spiral structure and infer its pitch angle directly from the harmonic components. Next, we use a spiral perturbation model to constrain the radial inflow velocity and combine this with the gas density in the nuclear spiral to estimate the mass inflow rate as a function of distance from the center of NGC 1097.

3.1. Non-circular Motions

NGC 1097 (ESO 325–58) is a nearby (distance 14.5 Mpc, so $1'' \simeq 70$ pc) LINER/Seyfert 1 host with a strong, $\simeq 16$ kpc long, bar and a $\simeq 0.7$ kpc in radius circumnuclear star-forming ring (Figure 1). In this Sb galaxy dust can be traced within the large-scale spiral arms out to a (outer Lindblad resonance, OLR) radius of $\simeq 14$ kpc, in prominent lanes along the bar, and continuing within the nuclear ring as a spiral structure down to $\lesssim 3.5$ pc from the center (Lou et al. 2001; Prieto et al. 2005). Non-circular motions associated with this nuclear spiral structure (Fathi et al. 2006; Davies et al. 2009), indicate a possible mechanism to drive gas from kpc scales down to a few pc from the center, where a double-peaked broad H α emission profile (Storchi-Bergmann et al. 1993) indicates the presence of a supermassive BH.

Fathi et al. (2006) describe in detail the observations and reduction of the two-dimensional spectroscopy of NGC 1097 obtained with the GMOS-IFU on the Gemini South Telescope (GS-2004B-Q-25, PI: Storchi-Bergmann). Three pointings within the nuclear ring region (Figure 1) provided 1500 individual spectra covering 5600–7000 Å at a velocity resolution of 85 km s^{−1} and with a spatial sampling of 0''.1.

In the first panel of Figure 2, we present the [N II] emission-line velocity field covering the inner 0.5×1.0 kpc. A systemic velocity of $v_{\text{sys}} = 1188$ km s^{−1} (equivalent to a constant c_0 term) has been subtracted. For an inclination $i = 35^\circ$ (flattening $q = 0.82$; Fathi et al. 2006) and a position angle of $\Gamma = 141^\circ$, the other panels show the results of applying the harmonic expansion of Section 2.1. From left to right: the best-fit circular motion $V_{\text{circ}} = c_1 \cos \psi$, the remaining non-circular motion, and the harmonic reconstruction from the sum of the latter two.

In Figure 3, the first two panels show the adopted position angle and systemic velocity. The next four panels show the coefficients c_m and s_m (in km s^{−1}) for the first three harmonic terms, as a function of the (deprojected) radius R . The bottom three panels show the phase shifts ψ_m (in degrees) for the alternative formulation of the harmonic expansion given in Equation (3) as functions of the (natural) logarithm of R .

Figure 3. Harmonic expansion of the observed line-of-sight velocity field of NGC 1097. After fixing the position angle Γ and systemic velocity V_{sys} to their mean values indicated by the dotted line in, respectively, the top left and top middle panels, the harmonic coefficients c_m and s_m are extracted as functions of radius R (in arcsec). The coefficients have been divided by $q = \sin i$ to correct for the inclination $i = 35^\circ$. The dotted vertical line in each panel indicates the radius $R = 1''$, within which the measurements are less accurate due to possible contamination from the nuclear source and presence of significant amounts of dust. In the bottom panels, the phase shifts ψ_m defined in Equation (4) are plotted against $\ln R$, so that a spiral structure shows up as a smooth variation, which becomes linear in the case of a logarithmic spiral as shown in Equation (12). In this way, the bottom-right panel shows that the structure in the non-circular motions is consistent with a three-arm logarithmic spiral with indicated pitch angle ζ_3 from the slope of the thick solid linear fit. The indicated estimate of the pitch angle ζ_2 of the inferred two-arm spiral perturbation in the gravitational potential follows from Equation (15), if we neglect the radial variation in the additional term θ_3 . The latter is indeed expected to be small as indicated by the dot-dashed curve, which shows θ_3 based on an analytical two-arm spiral perturbation model. Taking into account this term yields the model prediction for ψ_3 indicated by the dashed curve, which is nearly indistinguishable from the fitted thick solid line. Similarly, in the bottom left panel, the dot-dashed and dashed curves show the model prediction of θ_1 and ψ_1 , respectively. In line with the measurements, ψ_1 is small because the c_1 coefficient in $\tan \psi_1 = s_1/c_1$ includes the dominating circular velocity v_c . The latter is shown as a black dashed curve in the top-right panel, whereas including the non-circular motions due to the modeled two-arm spiral perturbation, yields c_1 indicated by the blue dashed curve. Similarly, the model prediction for s_1 is given by the red dashed curve in the middle-left panel, and those for c_3 and s_3 by the blue and red dashed curves in the middle-right panel. (See Section 3.2 and the Appendix for further details on the spiral perturbation model.)

(A color version of this figure is available in the online journal.)

We interpret the non-circular residual motion (third panel of Figure 2) as a three-arm spiral structure. This interpretation is supported by the significant amplitudes of c_3 and s_3 , and in

particular by the smooth variation of the corresponding phase shift ψ_3 with radius. Excluding the uncertain measurements within $R = 1''$ (vertical dotted line) and the single measurement

at the edge of the map, the remaining measurements indicated by solid circles show a linear relation between $3\psi_3$ and $\ln R$. This is consistent with a logarithmic spiral defined in Equation (12), with fitted $a_0 = 1''.82 \pm 0''.33$ and $b_0 = -1.95 \pm 0.27$. The slope provides a direct and robust measurement of the pitch angle of $\zeta_3 = 63^\circ \pm 3^\circ$.

In the third panel of Figure 2, this three-arm logarithmic spiral with pitch angle 63° is plotted with magenta circles on top of the non-circular motions. If this three-arm spiral in the velocity field is due to a two-arm spiral perturbation in the gravitational potential as we argue below, it should also give rise to a one-arm logarithmic spiral in the velocity field. Combining Equation (12) and the right-hand side of Equation (14), we find $\cot \zeta_1 = \cot \zeta_3$ (and equal a_0). This means a pitch angle 33° for the one-arm logarithmic spiral, resulting in the magenta triangles in the third panel of Figure 2. The contribution of the one-arm spiral to the non-circular motions is partly absorbed into the circular motions since the c_1 coefficient is indistinguishable from the circular velocity contribution. Moreover, as can be seen from Figure 3, the remaining s_1 coefficient contributes in a different way than the combined s_3 and c_3 terms, so that it is not surprising that neither the one-arm nor the three-arm spiral alone traces the spiral structure in the non-circular motions. This is even aside from possible additional contributions, or contaminations in this case, from other (even) harmonic terms, as discussed below in Section 4.2.

The positive sign of the pitch angle indicates that the nuclear spiral is curved anti-clockwise with increasing radius, equivalent to the spiral arms extending outward of the large-scale bar, as can be seen in the left panel of Figure 1. Since the global rotation in NGC 1097 is clockwise from the observed velocity field, and the northeast is the “far side” from being more obscured, it follows that both the large-scale and nuclear spirals are trailing.

3.2. Two-arm Spiral Perturbation

The above three-arm spiral structure in the non-circular motions is consistent with a perturbation in the gravitational potential due to a $m = 2$ harmonic spiral (see also Section 2.3). From Equation (15) it then follows that the pitch angle ζ_2 of this two-arm spiral perturbation in the gravitational potential follows from the measured pitch angle ζ_3 of the three-arm spiral in the velocity field as $2 \cot \zeta_2 = 3 \cot \zeta_3 - d\theta_3/d \ln R$. We show below that, as expected, the latter term is small, so that for a measured $\zeta_3 \simeq 62^\circ \pm 3^\circ$, we calculate $\zeta_2 \simeq 52^\circ \pm 4^\circ$.

As illustrated in Figure 5, the true value of ζ_2 might differ slightly depending on the details of the gravitational potential perturbation. We construct a model for the nuclear spiral in NGC 1097 to estimate the latter difference as well as to constrain the fraction of the observed non-circular motions that is due to net radial inflow in addition to elliptical streaming. We use the analytic solutions of the Appendix for gaseous orbits in an axisymmetric gravitational potential, $\Phi_0(R)$, that is weakly perturbed by a logarithmic $m = 2$ spiral with pitch angle ζ_2 . These analytic spiral models are based on linearized equations of motion under the epicycle approximation, but are not restricted to tightly wound spirals (e.g., Lin et al. 1969; Canzian & Allen 1997). In this way, we show below that the observed loosely wound nuclear spiral in NGC 1097 is still consistent with a density wave, and not necessarily driven by shocks as suggested by Davies et al. (2009). Note that these simple models assume the existence of a weak perturbation in the gravitational potential without specifying neither how the perturbation arises nor how it is maintained. To include potential driving mechanism such as

the large-scale bar requires more sophisticated models, which is beyond the goals and scope of this paper.

We adopt the power-law model (Evans & de Zeeuw 1994) with axisymmetric gravitational potential

$$\Phi_0(R) = \begin{cases} v_0^2 \frac{2^{\beta/2}}{\beta} \left[1 - \left(1 + \frac{R^2}{R_c^2} \right)^{-\beta/2} \right] & \beta \neq 0, \\ v_0^2 \frac{1}{2} \left[1 - \ln \left(1 + \frac{R^2}{R_c^2} \right) \right] & \beta = 0, \end{cases} \quad (19)$$

and corresponding circular velocity

$$v_c(R) = v_0 2^{\beta/4} \frac{R}{R_c} \left(1 + \frac{R^2}{R_c^2} \right)^{-(1/2+\beta/4)}, \quad (20)$$

so that $v_c(R_c) = v_0/2$ at the core radius R_c . The parameter β controls the logarithmic gradient of the rotation curve at large radii: $\beta < 0$ rising, $\beta = 0$ flat, and $\beta > 0$ falling. The three gravitational potential parameters v_0 , R_c , and β are set by comparing the corresponding circular velocity $v_c(R)$ with the measured radial profile of c_1 , taking into account the non-circular contribution due to the $m = 2$ spiral perturbation. In the third panel of Figure 3, the black dashed curve shows $v_c(R)$ for $v_0 = 275 \text{ km s}^{-1}$, $R_c = 1''.2$, and $\beta = -0.6$. Except for the three values within $R = 1''$, this simple power-law model is a good representation of the c_1 measurements, once the non-circular contribution derived below is added, as indicated by the blue dashed curve. The advantage of such a simple analytic representation of the gravitational potential is that it makes all subsequent calculations concerning the perturbation very convenient.

For the amplitude of the gravitational potential perturbation, we assume $\Phi_2(R) = \epsilon_p \Phi_0(R)$, with constant strength ϵ_p , while the phase shift is given by $\phi_2(R) = \cot \zeta_2 \ln(R/a_0)$. Here, $a_0 = 1''.8$ from the above fit to the $m' = 3$ harmonic terms in the line-of-sight velocity field, while the corresponding approximation $\zeta_2 = 52^\circ$, is taken as the initial value for the pitch angle. The three additional free parameters are the pattern speed, Ω_p , of the spiral perturbation, the azimuthal viewing angle, ϕ_{los} , and finally the amount of radial damping, λ . Equations (A12) and (A13) then provide predictions for the non-circular motion contribution in terms of the harmonic coefficients c_1 , s_1 , c_3 , and s_3 , which we compare with the corresponding measured radial profiles for NGC 1097 in Figure 3.

The flattening of the H I rotation curve in the outer parts of NGC 1097 (Sofue et al. 1999) implies a nearly constant circular velocity $v_c \simeq 300 \text{ km s}^{-1}$. With the angular and epicycle frequencies approximately given by $\Omega \simeq \kappa/\sqrt{2} \simeq v_c/R$, the OLR being at $R_{\text{OLR}} \simeq 14 \text{ kpc}$ yields a pattern speed $\Omega_p \simeq \Omega + \kappa/2 \simeq 35 \text{ km s}^{-1} \text{ kpc}^{-1}$. The latter places the corotation radius, $R_{\text{CR}} \simeq v_c/\Omega_p \simeq 8.6 \text{ kpc}$, or $\sim 10\%$ beyond the extent of the large-scale bar, consistent with numerical simulations of “fast bars” (e.g., Athanassoula 1992; Debattista & Sellwood 2000), and measured pattern speeds in similar galaxies (e.g., Aguerri et al. 2003; Gerssen et al. 2003; Rautiainen et al. 2008). Assuming that the two-arm nuclear spiral as a gas density wave is being driven by the large-scale bar (Englmaier & Shlosman 2000), we adopt the same value for the pattern speed of the perturbation. Since we consider the harmonic coefficients well within the corotation radius, they are not sensitive to and hence do not constrain Ω_p .

In contrast, a small change in the azimuthal viewing angle already causes a significant radial offset in the harmonic

coefficients, so that we need $\phi_{\text{los}} \simeq (1.05 \pm 0.05)\pi/2$. Next, too little radial damping results in too small amplitudes of the coefficients c_3 and s_3 with respect to s_1 and the non-circular contribution to c_1 . Specifically, since s_1 and s_3 are of similar amplitude and shape over most of the radial range, significant radial damping with $\lambda > 1$ is needed. As described in Section 2.5 and discussed below in Section 3.3, this implies that net radial flow makes up most of the intrinsic radial velocity. Matching the amplitudes of the harmonics terms yields a strength of the gravitational potential perturbation of $\epsilon_p \simeq 0.15$. Finally, values for the pitch angle of the spiral perturbation that are in the range of $\zeta_2 \simeq 52^\circ \pm 4^\circ$, approximated above from the pitch angle ζ_3 of the three-arm spiral in the velocity field, yield predictions for the harmonic coefficients that are consistent with the measured harmonic coefficients. The effect of the additional term $d\theta_3/d\ln R$ is indeed small, so that $2\cot\zeta_2 \simeq 3\cot\zeta_3$ provides a robust measurement of ζ_2 .

This is further illustrated in Figure 3, where in addition to the measured harmonic coefficients, we show with dashed curves the predictions of the above two-arm spiral perturbation model with pitch angle $\zeta_2 = 52^\circ$, azimuthal viewing angle $\phi_{\text{los}} = 94.5^\circ$, and radial damping parameter $\lambda = 2$. Both θ_1 and θ_2 , shown as dot-dashed curves in, respectively, the bottom left and bottom right panel, indeed vary only little with radius. Similarly, the dashed curve in the bottom right panel shows the predicted relation between $3\psi_3$ and $\ln R$ while taking into account θ_3 , which is nearly indistinguishable from the fitted solid line. The predicted harmonic terms match well the c_3 and s_3 measurements in the middle right panel, and rather well the s_1 measurements in the middle left panel, except for those at larger radii. However, in particular, the last three points in radius are less certain because the corresponding ellipses to extract these measurements are not fully covered (see also the second panel of Figure 2), and they might well be disturbed by the nuclear ring. The blue dashed curve in the top right panel is the spiral model prediction for c_1 , which apart from the innermost measurements, nicely traces the measured rotation curve.

Due to the significant non-circular motion contribution, the latter is different from the circular velocity of the (power-law) axisymmetric gravitational potential shown as the black dashed curve. The corresponding $\Omega(R) - \kappa(R)/2$ black dot-dashed curve is still well above the pattern speed $\Omega_p = 35 \text{ km s}^{-1} \text{ kpc}^{-1}$ (solid horizontal line) at radius $\sim 8''$, which places the ILR well beyond the nuclear ring radius of $\simeq 0.7 \text{ kpc}$. If indeed the radius of nuclear rings is set by the location of the ILR (e.g., Buta & Combes 1996), this indicates that the nuclear ring in NGC 1097 has migrated inward. Inward migration has also been suggested for the nuclear ring in NGC 4314 (Benedict et al. 2002), has been seen in hydrodynamic simulations (e.g., Fukuda et al. 2000; Regan & Teuben 2003), and might be a consequence of shepherding of the gas ring by star clusters that formed in it (van de Ven & Chang 2009).

3.3. Mass Inflow Rate

We found that a spiral perturbation model with only elliptic streaming, in which gas is moving on closed elliptic orbits without radial damping, cannot explain the observed non-circular motions in NGC 1097. In Section 2.5, we showed that the amount of radial damping required, provides an estimate of the radial flow velocity v_{flow} in terms of the radial velocity v_R of the spiral model which matches the observed harmonic terms. Since in the case of NGC 1097 a large amount of radial damping $\lambda > 1$ is needed, the ratio $v_{\text{flow}}/v_R \simeq |\Lambda|/\sqrt{\Delta^2 + \Lambda^2}$ in

Equation (18) is close to unity. In other words, the contribution to the radial velocity in the nuclear spiral is predominantly due to net radial flow, which is directed inward to the center of NGC 1097, as we are well within the corotation radius. For the same spiral model as shown in Figure 3 with the dashed curves, the corresponding radial inflow velocity v_{inflow} is shown in the top left panel of Figure 4 with a solid curve. Even though nearly identical to the radial velocity v_R , it is still only a fraction of the total non-circular motion (dashed curve), which also includes the azimuthal velocity.

To turn this radial inflow velocity into a constraint on the mass inflow rate, we furthermore need to know the gas density in the nuclear spiral, as well as the geometry or the area through which the gas is flowing. The flux ratio of the $[\text{S II}]\lambda\lambda 6716.4, 6730.8$ emission-line doublet included in the spectral range of the GMOS-IFU observations allows us to constrain the mean electron density n_e over the observed field. Taking all values together yields an average flux ratio of 1.003 ± 0.031 , which, adopting the prescription by Shaw & Dufour (1994) and using the atomic parameters compiled by Mendoza (1983) and Osterbrock (1989), corresponds to $n_e = 600 \pm 77 \text{ cm}^{-3}$ assuming a mean electron temperature of 10^4 K . Even though the flux ratio map might indicate a spiral structure similar to that in the non-circular motions, the signal-to-noise is not high enough to quantify the gas (over)density in the spiral arms from it. Instead, we apply the same elliptic annuli used to extract the harmonic coefficients to the flux ratio map. From the distribution within each annulus, we derive n_e as a function of radius, resulting in the dashed curve in the top right panel of Figure 4 (divided by 10 for illustrative purposes). Next, we assume that the width of the distribution, indicated by the solid curve, is driven by the density wave contrast, and hence yields an estimate of the electron (over)density Δn_e within the spiral arms. Assuming a 50% lower (higher) electron temperature of $0.5(1.5) \times 10^4 \text{ K}$ leads to an increase (decrease) in the electron densities by a factor of about 20%. In all cases, the density contrast is of the order of 10%, in agreement with hydrodynamic simulations (e.g., Englmaier & Shlosman 2000; Maciejewski 2004b) and K -band imaging presented by Davies et al. (2009). Using the proton mass and a factor 1.36 to account for the presence of helium, we convert Δn_e to a gas mass overdensity $\Delta\rho_{\text{gas}}$.

For the geometry, we assume in agreement with the spiral perturbation model that both the radial flow velocity and gas mass overdensity, at each radius R in a disk with scale height h , vary as a sinusoidal function in the azimuthal angle ψ . Integrating over the ψ values for which the radial flow velocity is positive, the mass inflow rate then reduces to

$$\dot{M} = m v_{\text{inflow}} \Delta\rho_{\text{gas}} \pi R^2 \frac{h}{R} \frac{1}{4m}, \quad (21)$$

for gas of density $\Delta\rho_{\text{gas}}$ flowing in the m spiral arms toward the center at a velocity v_{inflow} . To estimate the scale height h , we start from a marginally stable disk with Toomre's (1964) $Q \simeq 1$, and substitute into $Q = c_s \kappa / \pi G \Sigma_{\text{gas}}$ a constant sound speed $c_s \simeq 10 \text{ km s}^{-1}$, the epicycle frequency κ of the power-law axisymmetric gravitational potential, and $\Sigma_{\text{gas}} \simeq \rho_{\text{gas}} h$, with ρ_{gas} inferred from the average electron density n_e . The dashed curve in the bottom left panel of Figure 4 shows h/R as a function of radius. Clearly, toward the center this leads to an unrealistically high scale height, so that we constrain h/R to be not larger than the relative width $1/(2m)$ of a spiral arm, resulting in the solid curve. Finally, in the bottom right panel of Figure 4, we

Figure 4. Estimate of the mass inflow rate as a function of distance R (in pc) from the center of NGC 1097. Top left: The solid curve shows the radial inflow velocity inferred from a spiral perturbation model matched to the harmonic expansion of the velocity field. The radial inflow velocity is only a fraction of the non-circular motions indicated by the dashed curve. Top right: the solid circles show the electron density Δn_e in the nuclear spiral arms estimated from the variation in the flux ratio of the [S II] $\lambda\lambda 6716.4, 6730.8$ emission-line doublet. This (over)density is typically a factor of 10 smaller than the average electron density n_e as indicated by the dashed line. Bottom left: the solid circles show the adopted scale height h , assuming a marginally stable disk resulting in the dashed line, but with the relative width $R/(2m)$ of the $m = 2$ spiral model in the equatorial plane as an upper limit. Bottom right: combining the solid-curve values of the first three panels into Equation (21) results in the shown mass inflow rate (in $M_\odot \text{ yr}^{-1}$). The upper dashed horizontal line indicates the Eddington accretion rate onto the central BH in NGC 1097 with a mass $M_{\text{BH}} = 1.2 \times 10^8 M_\odot$ based on the central stellar velocity dispersion. The lower dotted horizontal line at a factor 0.01 of the Eddington accretion rate indicates the approximate transition from a Seyfert 1 to a LINER AGN, and is thought to be the rate below which mass accretion becomes radiatively inefficient. The dotted vertical line in each panel indicates the radius $R = 1'' (\simeq 70 \text{ pc})$ down to which the measurements are reliable. (See Sections 3.3 and 4.3 for further details.)

present the mass inflow rate $\dot{M}$ (in $M_\odot \text{ yr}^{-1}$) as a function of radius.

4. DISCUSSION

We have argued for a two-arm spiral perturbation in the gravitational potential as the source of the three-arm spiral structure in the velocity field of NGC 1097. We verify the implied two-arm spiral distortion in the SB and discuss possible causes for contributions from additional harmonic terms. Finally, we link the mass inflow rate to the accretion onto the central BH.

4.1. Two-arm Spiral Perturbation?

An $m = 2$ spiral perturbation in the gravitational potential that would explain the three-arm spiral structure in the velocity field of NGC 1097 implies an $m = 2$ spiral distortion in the surface mass density and hence a two-arm spiral structure in the SB.

Even if the latter distortion is too weak to measure directly, it might show itself through obscuration by correlated dust. Indeed, structure maps of NGC 1097 (e.g., Pogge & Martini 2002; Martini et al. 2003; Fathi et al. 2006) show spiral-like features, but they are not evidently a two-arm nuclear spiral. The structure map in the right panel of Figure 1 is based on the Richardson–Lucy image restoration technique (Snyder et al. 1993), using a multi-step convolution of the *Hubble Space Telescope*/Advanced Camera for Surveys (*HST*/ACS) high-resolution camera FR656N image with a two-dimensional point-spread function model constructed using Tiny Tim (Krist & Hook 1997). We have overlaid the structure map with the wavelet map from (Lou et al. 2001, their Figure 1), which does seem to be consistent with a two-arm spiral structure, but additional spiral features cannot be ruled out.

The solid magenta curves show the two-arm spiral with pitch angle $\zeta_2 = 52^\circ$, which we inferred from the logarithmic spiral fitted to the $m = 3$ harmonic terms (Section 3.2). The two

open arms trace well the spiral structures in both the wavelet and structure maps, except closer to the nuclear ring when, at least in the northern part, a more tightly wound spiral seems needed. The dashed magenta curves indicate the two additional arms in the case of an $m = 4$ spiral perturbation with the same pitch angle. Such higher-order *even* harmonic terms can result from nonlinear coupling of modes, and—although smaller in amplitude than the $m = 2$ spiral perturbation—might give rise to possibly additional spiral features. Furthermore, a spiral perturbation driven by the large-scale bar is not necessarily restricted to an $m = 2$ harmonic term.⁷ Still, as long as the perturbation is bi-symmetric all resulting harmonic terms are even.

Nonetheless, Prieto et al. (2005) note in high-resolution VLT/NACO infrared images a central spiral with a three-arm symmetry, though one of the three arms does not seem to continue toward the nuclear ring, but instead splits into a number of spiral filaments. Davies et al. (2009) show the inner $4'' \times 4''$ of the NACO J -band residual image from Prieto et al. (2005) together with their SINFONI K -band residual image (their Figure 1). While the third arm is already weaker in the J band (in particular when taking into account the narrow intensity scaling that is saturating the two strong arms), it nearly disappears in the K band. Also, the residual flux distribution of (warm) H_2 , as traced by the $2.12 \mu\text{m}$ 1–0 S(1) line (their Figure 3), reveals two strong arms. Nevertheless, Davies et al. (2009) claim a (weak) third arm in the stellar and gas density to give rise to the *two*-arm spiral structure they argue to see in the residual H_2 velocity field (their Figure 5). However, when the inner $2''$ of our [N II] non-circular motions (third panel of Figure 2) are overlaid by their H_2 residual velocity field (as in their Figure 6), we believe that the (observed and predicted) *three*-arm spiral structure is traced inward by the H_2 kinematics. This apparent agreement is not obvious as their near-infrared observations are less affected by dust than our optical measurements, but also because it is not evident that (warm) H_2 emission and ionized emission trace the gas kinematics in the same way. Even so, we find that the data presented by Davies et al. (2009) do not contradict our interpretation.

As indicated by Davies et al. (2009), the kinematics of the stars do not show any significant deviations from axisymmetry (see their Figure 2). Indeed, it is expected that the bulge stars are dominating the stellar kinematics, and that the perturbation is only visible in the intensity due to dust extinction in the equatorial plane. Since the stability of the gas in the equatorial plane inhibits self-amplification (see also Davies et al. 2009), the observed spiral pattern in the gas is most likely due to a weak spiral perturbation in the total, stellar-dominated, gravitational potential. The most natural driver of this perturbation is the large-scale bar, which might induce spiral shocks in the gas (e.g., Maciejewski 2004a, 2004b). Davies et al. (2009) argue for these shocks to be present in the inner region of NGC 1097, based on a large amplitude of the radial motions with respect to the velocity dispersion of the gas. However, as discussed above (see also the top left panel of Figure 4), only part of the observed non-circular motions might be radial motions, and shocks might just be the trigger to create long-lived gas density waves (Englmaier & Shlosman 2000; Ann & Thakur 2005). Whether spiral shocks and/or bulge stars ionize the perturbed

gas, the spiral perturbation can be traced through non-circular motions in the observed ionized gas kinematics.

In this way, we find a three-arm structure in the [N II] non-circular motions, which we believe is consistent with the residual H_2 velocity field, as well as with two spiral arms visible in the residual H_2 flux, K -band image, and J -band image, presented by Davies et al. (2009). All this leads to our interpretation of a weak *two*-arm spiral perturbation in the gravitational potential driven by the large-scale bar; though further modeling is needed to find out if for example the spiral pattern in the gas is induced by shocks or by long-lived density waves. Instead, the explanation by Davies et al. (2009) of a *three*-arm spiral perturbation in the gravitational potential requires quite a special driving mechanism, such as nonlinear interactions between the large-scale bar and a nuclear bar or a (dark) massive orbiting compact object, which so far have not been demonstrated in models. In addition, observed structure in addition to that expected from a two-arm spiral perturbation in the gravitational potential, including a possible weak third arm, might be the result of asymmetric dust obscuration, as we discuss next.

4.2. Additional Non-circular Motion?

The coefficients c_m and s_m , that provide the phase shift $\psi_m(R)$, may be “contaminated” by additional contributions to the non-circular motion, but in general they are not expected to result in a smooth variation of the phase shift with radius, as seen for a spiral structure. An exception is the $m = 1$ harmonic term, since the measured coefficient c_1 also incorporates the circular velocity as $v_c \sin i$, which in general dominates over the non-circular motions c_1^{nc} and s_1 . This results in a phase shift ψ_1 that is everywhere close to zero, and due to the degeneracy in c_1 there is little hope of constraining a two-arm spiral perturbation from its contribution to the $m = 1$ harmonic term in the velocity field. However, we can predict its pitch angle ζ_1 and phase shift ϕ_1^{nc} from the (logarithmic) spiral inferred from the contribution to the $m = 3$ harmonic term: $\tan \zeta_1 = 3 \tan \zeta_3$ and $\psi_1^{\text{nc}}(R) = 3\psi_3^{\text{nc}}(R)$. As a result, we might use $c_1^{\text{nc}} = s_1 \cot \psi_1^{\text{nc}}$ as an estimate of the non-circular contribution to c_1 . The effect is shown in the top right panel of Figure 3, where the black dashed curve is v_c of the (power-law) axisymmetric potential, which after taking into account c_1^{nc} yields the blue dashed curve that matches the measured c_1 coefficients indicated by the blue diamonds. This provides a novel way to correct for non-circular motions, which otherwise might, for example, lead to an underestimation of the (inner) slope of the mass distribution (e.g., Hayashi & Navarro 2006).

Figure 3 shows that, besides $m = 1$ and $m = 3$ harmonic terms in the velocity field expected from a $m = 2$ perturbation in the gravitational potential, the non-circular motions also seem to contain an $m = 2$ harmonic term. There are several effects that might (partly) cause this additional contribution.

As shown by Schoenmakers et al. (1997, their Equation (7)), an error in the (kinematic) center results in a spurious $m = 0$ and $m = 2$ contribution to the line-of-sight velocity as

$$\delta V_{\text{los}} = V_* \left[(1 + \alpha) \frac{\delta x'}{2R} - (1 - \alpha) \left(\frac{\delta x'}{2R} \cos 2\psi + \frac{\delta y'}{2R} \sin 2\psi \right) \right]. \quad (22)$$

We see that the effect on c_2 and s_2 vanishes if $\alpha = 1$, i.e., when the circular velocity curve increases linearly with radius, $v_c \propto R$. As in most galaxies, the latter is also the case in

⁷ For example, the axisymmetric power-law potential in Equation (19), being perturbed by replacing the radius R by $R^2 = x^2 + (y/q)^2$ with $q < 1$, creates besides $m = 2$ also higher-order even harmonic terms.

the inner region of NGC 1097, but still c_2 is significantly positive within the central $\lesssim 2''$. Not surprisingly, varying the kinematic center (in the process of finding the best-fit set of ellipses as described in Section 2.1) does not remove the $m = 2$ harmonic contribution, but in contrast makes the overall fit worse. This makes a significant effect due to an incorrect center unlikely.

Based on the analysis in Section 2.3, we expect a similar contribution of both $m = 0$ and $m = 2$ harmonic terms to the velocity field from an $m = 1$ distortion of an axisymmetric distribution. It is unlikely that the gravitational potential itself is lopsided since the stars that dominate in mass do not show any such signature. Also, the nuclear spiral itself is expected to be bi-symmetric if it is indeed a gas density wave driven by the large-scale bar. Still, the phase shift $2\psi_2$ in the bottom middle panel of Figure 3 seems to vary smoothly, and even close to linearly as a function of $\ln R$ within the central $\lesssim 2''$ where prominent dust features are present. Moreover, the amplitude of the slope is similar to the linear relation of $3\psi_3$ versus $\ln R$ in the bottom right panel, but with a negative instead of a positive sign. This is consistent with a logarithmic spiral with the same pitch angle but orientated clockwise instead of anti-clockwise, i.e., leading instead of trailing. In principle, both leading and trailing nuclear spirals can exist, as shown by Wada (1994), but in their simulations the smaller leading spiral dissolves before the long-lived larger trailing spiral fully emerges. Alternatively, the spiral-like contribution to the $m = 2$ harmonic term, appearing like a “negative image,” might result from an asymmetric dust obscuration, mimicking a lopsided distortion.

The often prominent dust lanes along the leading edges of bars in galaxies are associated with shocks in the gas streaming along the length of the bar (e.g., Athanassoula 1992), which in turn lead to velocity jumps across the dust lanes (e.g., Mundell & Shone 1999). Moreover, numerical models of dust lanes (e.g., Gerssen & Debattista 2007) as well as analytical models of diffuse disks (e.g., Valotto & Giovanelli 2004) show that dust extinction can have a significant effect on the velocity along the line of sight. Henceforth, we expect the dust and possible shocks associated with the spiral features to distort the velocity field, but the modeling required to understand the specific effects on the non-circular motion is beyond the scope of this paper.

4.3. Feeding the Central Black Hole?

The Eddington accretion rate onto a central BH

$$\dot{M}_{\text{Edd}} = 2.2 M_{\odot} \text{ yr}^{-1} \left(\frac{\epsilon}{0.1} \right)^{-1} \left(\frac{M_{\text{BH}}}{10^8 M_{\odot}} \right), \quad (23)$$

adopting $\epsilon = 0.1$ for the radiative efficiency, and a mass $M_{\text{BH}} = 1.2 \times 10^8 M_{\odot}$ for the central BH in NGC 1097—based on the measured central stellar velocity dispersion of $\sigma_{\star} = 196 \pm 5 \text{ km s}^{-1}$ (Lewis & Eracleous 2006) substituted into the $M_{\text{BH}}-\sigma_{\star}$ relation (Tremaine et al. 2002)—yields $\dot{M}_{\text{Edd}} \simeq 2.7 M_{\odot} \text{ yr}^{-1}$. This value is indicated in the bottom right panel of Figure 4 by the upper dashed horizontal line. The lower dotted horizontal line at $\dot{M} = 0.01 \dot{M}_{\text{Edd}}$ is the approximate transition from a Seyfert1 to a LINER AGN (see for a review Ho 2005) and thought to be the rate below which mass accretion becomes radiatively inefficient (see for reviews Quataert 2001; Narayan 2005).

Whereas NGC 1097 is typically classified as a LINER galaxy, monitoring of the nucleus reveals evolution in its activity

up into the Seyfert1 regime (Storchi-Bergmann et al. 2003). Nemmen et al. (2006) find that the observed optical to X-ray spectral energy distribution in the nucleus of NGC 1097 is consistent with an inner radiatively inefficient accretion flow plus outer standard thin disk, with a mass accretion rate of $\dot{M} \simeq 6.4 \times 10^{-3} \dot{M}_{\text{Edd}}$. We find a mass inflow rate, down to $\dot{M} \simeq 0.011 M_{\odot} \text{ yr}^{-1} (\simeq 4.2 \times 10^{-3} \dot{M}_{\text{Edd}})$ at a distance $R = 1'' (\simeq 70 \text{ pc})$ from the center, where the gas kinematics are still accurately measured and well described by the two-arm spiral perturbation model. These constraints are obtained at very different scales (tenths versus tens of pc), and the mass inflow rate and the onset of nuclear activity are not necessarily linked in time. Even so, it is encouraging that we obtain comparable values.

Our mass inflow rate is significantly lower than $\dot{M} \sim 0.6 M_{\odot} \text{ yr}^{-1}$ estimated by Storchi-Bergmann (2007) at a distance $R = 100 \text{ pc}$. In the latter estimate, a non-circular motion of 50 km s^{-1} is adopted for the inflow velocity, whereas we find from our spiral model that even though the contribution of the radial inflow dominates over elliptic streaming, it is still only a fraction ($\simeq 13 \text{ km s}^{-1}$ at 100 pc) of the non-circular motions. Moreover, the estimated electron density of $\sim 500 \text{ cm}^{-3}$ is similar to the average density we derive directly from emission-line ratios, but more than an order of magnitude higher than the expected (over)density in the spiral arms ($\simeq 43 \text{ cm}^{-3}$ at 100 pc). Finally, instead of assuming a fixed opening angle of 20° , we incorporate the geometry from the spiral model, and allow for the scale height to vary, in order for the disk to remain marginally stable. In this way, we obtain an estimate of the mass inflow rate of a similar value as the expected mass accretion rate onto the central BH in NGC 1097, without having to invoke an unclear filling factor (e.g., Storchi-Bergmann et al. 2007). Even so, the mass inflow rate might be even further constrained, in particular by measuring emission-line ratios at higher signal-to-noise to obtain a more accurate estimate of the (over)density, as well as using other emission lines to break the degeneracy with temperature.

5. SUMMARY AND CONCLUSIONS

We presented harmonic expansion of the line-of-sight velocity field as a suitable method to identify and quantify possible structures in the non-circular motions, including nuclear spirals. We confirmed earlier findings (Canzian 1993; Schoenmakers et al. 1997) that a weak perturbation in the gravitational potential of harmonic number m causes the SB to also exhibit an m distortion, but leads to $m - 1$ and $m + 1$ harmonic terms in the velocity field. In the case of an m -arm spiral perturbation in the gravitational potential, we found that the corresponding $(m + 1)$ -arm and $(m - 1)$ -arm spirals in the velocity field are respectively less and more tightly wound, with pitch angles approximately related as $m \cot \zeta_m \simeq (m \pm 1) \cot \zeta_{m \pm 1}$. In the Appendix, we derived an analytic perturbation model, which allows for a simple estimate of the fraction of the measured non-circular motions that is due to radial flow.

We applied this method to the emission-line velocity field within the circumnuclear star-forming ring of NGC 1097, obtained with the GMOS-IFU spectrograph. The radial variation of the resulting $m = 3$ harmonic terms can be fitted with a logarithmic spiral with a pitch angle $\zeta_3 = 63^\circ \pm 3^\circ$. We linked these $m = 3$ harmonic terms in the velocity field to a weak perturbation of the gravitational potential due to a two-arm nuclear spiral with an inferred pitch angle $\zeta_2 \simeq 52^\circ \pm 4^\circ$. This predicts a two-arm spiral distortion in the SB, as hinted

by the dust structure in central images of NGC 1097, although additional spiral structure might be present as a result of higher-order even harmonic terms. Furthermore, this two-arm spiral perturbation of the gravitational potential adds a combined $m = 1$ and $m = 3$ spiral structure to the velocity field, as revealed in the non-circular motions of the ionized gas at the center of this galaxy. We argued that it is unlikely that also the presence of $m = 2$ harmonic terms in the velocity field is due to lopsidedness in the gravitational potential nor in the nuclear spiral itself, which we expect to be due to long-lived density waves in the gas driven by the bi-symmetric large-scale bar. Instead, we postulated that an asymmetric dust obscuration mimics a lopsided distortion and gives rise to the additional even harmonic terms in the velocity field.

To match the measured odd harmonic terms in the velocity field of NGC 1097, a spiral perturbation model with a large amount of radial damping is required. This indicates both strong dissipation and, that, in addition to elliptic streaming, a significant fraction of the non-circular motions is due to radial inflow. We combined the inferred radial inflow velocity with the gas density in, and the geometry of, the spiral arms to estimate the mass inflow rate as a function of radius. We calculated the gas density from the variation in the flux ratio of the [S II] $\lambda\lambda 6716.4, 6730.8$ emission-line doublet, and we incorporated in the geometry the scale height assuming a marginally stable disk. The resulting mass inflow rate decreases to $\dot{M} \simeq 0.011 M_\odot \text{ yr}^{-1}$ at a distance of $R = 70$ pc from the center, down to where our measurements are still reliable. We showed that this corresponds to $\dot{M} \simeq 4.2 \times 10^{-3} \dot{M}_{\text{Edd}}$, where the latter Eddington accretion rate is onto a BH in NGC 1097 with a mass $M_{\text{BH}} = 1.2 \times 10^8 M_\odot$ based on the central stellar velocity dispersion. This mass inflow rate is consistent with the AGN in NGC 1097 varying between LINER and Seyfert1, but comparison with mass accretion models is hampered by the (still) very different physical scales as well as possible time delay between mass inflow and the onset of nuclear activity.

We conclude that the line-of-sight velocity can not only provide a cleaner view on nuclear spirals than the associated dust does, but that the presented method also allows one to quantitatively study these possibly important links in fueling the centers of galaxies, including a constraint on the mass inflow rate as a function of radius.

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APPENDIX

GASEOUS ORBITS IN A WEAKLY PERTURBED GRAVITATIONAL POTENTIAL

We derive analytic solutions of gaseous orbits in a weakly perturbed gravitational potential (for modeling of a strong perturbation see, e.g., Spekkens & Sellwood 2007). We follow the treatment of Binney & Tremaine (1987, p. 146–148) for a weak harmonic perturbation. Like Schoenmakers et al. (1997), we include an additional phase shift $\phi_m(R)$ as in Equation (5) to accommodate for a spiral perturbation. Since gas at the same spatial location has the same velocity, we look for closed-loop orbits that do not intersect themselves. However, to take into account the dissipative nature of gas, we include, like Wada (1994), radial damping, so that the first-order equation of motion becomes

$$\ddot{R}_1 + 2\lambda\kappa_0\dot{R}_1 + \kappa_0^2 R_1 = -R_0(A \cos \eta + B \sin \eta), \quad (\text{A1})$$

with $\eta = m[\phi_0(t) - \phi_m(R_0)]$ and where λ controls the amount of radial damping. The subscript zero refers to zeroth order with constant R_0 and $\phi_0(t) = (\Omega_0 - \Omega_p)t$, with Ω_p being the pattern speed of the perturbation. We have introduced the epicycle frequency, $\kappa^2 = 2\Omega^2(1 + \alpha)$, and angular frequency, $\Omega = v_c/R$, with $\alpha = d \ln v_c / d \ln R$ being the logarithmic slope of the circular velocity, $v_c^2(R) = d\Phi_0(R)/d \ln R$, of the axisymmetric part of the gravitational potential. Furthermore,

$$A = \frac{\Phi_m}{R^2} \left[\frac{2}{1 - \omega} + \frac{d \ln \Phi_m}{d \ln R} \right], \quad B = \frac{\Phi_m}{R^2} m \cot \zeta_m, \quad (\text{A2})$$

where we have introduced $\omega = \Omega_p/\Omega$, and ζ_m is the pitch angle as defined in Equation (11). Solving Equation (A1) we find that the solution for closed-loop orbits is

$$R = R_0[1 - (a \cos \eta + b \sin \eta)], \quad (\text{A3})$$

$$\phi = \phi_0 + [2(a \sin \eta - b \cos \eta) - \xi \sin \eta]/[m(1 - \omega)] \quad (\text{A4})$$

$$v_R = v_c \{m(1 - \omega)[a \sin \eta - b \cos \eta]\}, \quad (\text{A5})$$

$$v_\phi = v_c \{1 + (1 + \alpha)[(a - \xi) \cos \eta + b \sin \eta]\}, \quad (\text{A6})$$

with α appearing in the last line because of a first-order conversion from the guiding center (R_0, ϕ_0) to a point (R, ϕ) in the observed velocity field. Moreover,

$$a = \frac{A\Delta - B\Lambda}{\Delta^2 + \Lambda^2}, \quad b = \frac{B\Delta + A\Lambda}{\Delta^2 + \Lambda^2}, \quad \xi = \frac{1}{\kappa^2} \frac{\Phi_m}{R^2} \frac{2}{1 - \omega}, \quad (\text{A7})$$

where we have defined $\Delta = \kappa^2 - m^2(\Omega - \Omega_p)^2$ and $\Lambda = 2\lambda\kappa m(\Omega - \Omega_p)$. The orbit solutions have a singularity at the corotation radius where $\Omega = \Omega_p$ ($\omega = 1$), because the adopted epicycle approximation breaks down. Without radial damping ($\lambda = \Lambda = 0$), the (collisionless) orbit solutions also have singularities when $\Delta = 0$, i.e., at the Lindblad resonances given by $\Omega - \kappa/m = \Omega_p$.

Next, we assume to first order $\phi_0 \approx \phi$ and replace it by $\psi = \phi - \phi_{\text{los}} + \pi/2$ which is zero along the line of nodes. We also define $\varphi_m = \phi_m - \phi_{\text{los}} + \pi/2$, so that $\eta = m[\psi - \varphi_m(R)]$. This allows us to recast the above expressions for v_R and v_ϕ in multiple angles of ψ as in Equations (7) and (8), with

$$c_R = -m(1 - \omega)[a \sin(m\varphi_m) + b \cos(m\varphi_m)], \quad (\text{A8})$$

$$s_R = m(1 - \omega)[a \cos(m\varphi_m) - b \sin(m\varphi_m)], \quad (\text{A9})$$

$$c_\phi = (1 + \alpha)(a - \xi) \cos(m\varphi_m) - (1 + \alpha)b \sin(m\varphi_m). \quad (\text{A10})$$

$$s_\phi = (1 + \alpha)(a - \xi) \sin(m\varphi_m) + (1 + \alpha)b \cos(m\varphi_m). \quad (\text{A11})$$

Figure 5. Axisymmetric logarithmic potential perturbed by a weak $m = 2$ logarithmic spiral with pitch angle $\zeta_2 = 50^\circ$. The top panels show for an inclination $i = 30^\circ$, projected maps on the sky plane (x' , y') of the line-of-sight velocity, non-circular motions, and separately, the contribution of the $m - 1$ and $m + 1$ harmonic terms. The first panels in the middle and bottom rows show the *intrinsic* maps in the equatorial plane (x , y) of, respectively, the azimuthal and radial velocities due to the spiral perturbation. The overplotted open circles and stars trace the maxima as a function of radius, resulting in the radial profiles shown in the second panel in the bottom row. The second panel in the middle row shows the radial profiles of the circular velocity v_c of the axisymmetric potential, while the dotted curve shows c_1 , which includes the contribution from the spiral perturbation. Moreover, the falling dashed curve is the angular frequency Ω , which intersects the horizontal dashed line of the assumed pattern speed Ω_p , at the corotation radius R_{CR} . The latter is indicated in all panels either by a vertical dashed line or a dashed ring on the maps. The third and fourth panels in the middle row show the radial profiles of the harmonic coefficients $c_{m\pm 1}$ and $s_{m\pm 1}$, divided by $\sin i$ to correct for inclination. The third and fourth panels in the bottom row show the corresponding phase shifts $\phi_{m\pm 1}$, defined in Equation (4), against $\ln R$. In this way, a spiral structure shows up as a smooth variation, which becomes nearly linear because the assumed spiral perturbation is logarithmic as in Equation (12). The slopes of the fitted solid lines yield the indicated values of the pitch angles $\zeta_{m\pm 1}$ of the $(m \pm 1)$ -arm spirals shown in the corresponding projected maps in the third and fourth panels in the top row. In turn, based on Equation (15), both values provide an estimate as indicated of the pitch angle ζ_2 of the $m = 2$ spiral perturbation. The mild difference with the input value $\zeta_2 = 50^\circ$ is due to the radial variation of the additional terms $\theta_{m\pm 1}$ indicated by the dot-dashed lines.

(A color version of this figure is available in the online journal.)

Substitution in Equation (6) results in the expression for the line-of-sight velocity in Equation (9), where the harmonic coefficients are given by

$$c_{m\pm 1} = A_{\pm} \cos(m\varphi_m) - B_{\pm} \sin(m\varphi_m) \quad (\text{A12})$$

$$s_{m\pm 1} = A_{\pm} \sin(m\varphi_m) + B_{\pm} \cos(m\varphi_m), \quad (\text{A13})$$

with

$$\begin{aligned} A_{\pm} &= \frac{1}{2} V_{\star} \{ [(1 + \alpha) \mp m(1 - \omega)] a - (1 + \alpha) \xi \}, \\ B_{\pm} &= \frac{1}{2} V_{\star} [(1 + \alpha) \mp m(1 - \omega)] b. \end{aligned} \quad (\text{A14})$$

Substituting $A_{\pm} = K_{m\pm 1} \cos \theta_{m\pm 1}$ and $B_{\pm} = K_{m\pm 1} \sin \theta_{m\pm 1}$, with

$$K_{m\pm 1}^2 = A_{\pm}^2 + B_{\pm}^2, \quad \tan \theta_{m\pm 1} = B_{\pm} / A_{\pm}, \quad (\text{A15})$$

we arrive at the form given in Equation (13).

In Figure 5, we present an example of a weakly perturbed axisymmetric logarithmic potential defined in Equation (19), with $v_0 = 250 \text{ km s}^{-1}$, $R_c = 1.0 \text{ kpc}$ and $\beta = 0$. The amplitude of the perturbation is a factor $\epsilon_p = 0.02$ times the axisymmetric logarithmic potential, while the angular dependence is due to a logarithmic spiral defined in Equation (12), with pitch angle $\zeta_2 = 50^\circ$. This results in non-circular motions which contribute a one-arm and a three-arm spiral structure to the observed line-of-sight velocity field as shown in Figure 5 for an adopted inclination of $i = 30^\circ$.

In general, $K_{m\pm 1}$ and $\theta_{m\pm 1}$ in Equation (A15) above depend in a rather complicated way on the gravitational potential. However, in the case of the linear spiral density wave theory (Shu et al. 1973; Canzian & Allen 1997) the expressions reduce to

$$K_{m\pm 1} = \frac{1}{2} v_m \sin i \left[\frac{\kappa}{2\Omega} \mp \frac{m(\Omega - \Omega_p)}{\kappa} \right],$$

$$\tan \theta_{m\pm 1} = \pm \cot \zeta_m, \quad (\text{A16})$$

where the constant amplitude v_m measures the strength of a *tightly* wound spiral without radial damping. In this case, $\cot \zeta_m \gg 1$ and $\lambda = \Lambda = 0$, so that $b = B/\Delta \gg a = A/\Delta$, and $b \gg \xi$ in Equation (A7), and hence $\tan \theta_{m\pm 1} = B/A \propto \cot \zeta_m$ and $K_{m\pm 1} \simeq B_{\pm}$. The expression for $B_{\pm}$ in Equation (A14) is proportional to the reduced expression for $K_{m\pm 1}$ in Equation (A16), since by substituting $\kappa^2 = 2\Omega^2(1 + \alpha)$ and $\omega = \Omega_p/\Omega$, the term in the square brackets in Equation (A16) is proportional to $[(1 + \alpha) \mp m(1 - \omega)]$ in Equation (A14).

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