The mutual interaction between the time-mean atmospheric circulation and continental-scale ice sheets

by

Johan Liakka
Geomorphological evidence of glaciations exist for the Last Glacial Maximum (∼ 20 kyr ago). At this time, both North America and Eurasia were covered by extensive ice sheets which are both absent today. However, the temporal and spatial evolution of the ice sheets from the previous interglacial up to the fully-glaciated conditions at LGM is still unresolved and remains a vexing question in climate dynamics.

The evolution of ice sheets is essentially controlled by the prevailing climate conditions. On glacial time-scales, the climate is shaped by the orbital variations of the Earth, but also by internal feedbacks within the climate system. In particular, the ice sheets themselves have the potential to change the climate within they evolve. This thesis focuses on the interactions between ice sheets and the time-mean atmospheric circulation (stationary waves). It is studied how the stationary waves, which are forced by the ice-sheet topography, influence ice-sheet evolution through changing the near-surface air temperature.

In this thesis, it is shown that the degree of linearity of the atmospheric response controls to what extent the stationary waves can reorganise the structure of ice sheet. Provided that the response is linear, the stationary waves constitute a leading-order feedback, which serves to increase the volume and deform the shape of ice sheets. If the stationary-wave response to ice-sheet topography is nonlinear in character, the impact on the ice-sheet evolution tends to be weak. However, it is further shown that the amplitude of the nonlinear topographical response, and hence its effect on the ice-sheet evolution, can be significantly enhanced if thermal cooling over the ice sheets is taken into account.
List of papers

The thesis consists of an introduction and the following four papers, which are referred to by their Roman numerals:


Papers II and IV are reprinted with permission from Springer-Verlag. The ideas to papers I, II and III were formulated by me. Johan Nilsson contributed to the writing of papers I and II, and Marcus Löfverström contributed to the development of the linear model (LUMA) in paper II. The ideas to the experiments in paper IV were formulated by Florence Colleoni, who also contributed to most of the writing. My contribution to paper IV was to perform the experiments with the climate model Planet Simulator, and to write the description about the model in the paper.
# Contents

1 Glacial climates  
  1.1 Spatial reconstructions of past ice sheets  1

2 The temporal evolution of ice sheets  4  
  2.1 Ice-sheet boundary conditions  5  
  2.2 How well do we understand the evolution of the past ice sheets?  7

3 Climate conditions  9  
  3.1 Glacial surface conditions  10  
  3.2 Feedbacks between the atmosphere and ice sheets  11

4 The stationary-wave feedback  14  
  4.1 The simplest model of topographically-forced stationary waves  15  
  4.2 Do the stationary waves warm or cool the local ice-sheet climate?  17  
  4.3 The effect of nonlinearity  18  
  4.4 The effect of diabatic cooling  19

5 Summary of the papers constituting this thesis  20  
  5.1 Paper I: The impact of stationary waves on the local ice-sheet climate  20  
  5.2 Paper II: Linear versus nonlinear response  21  
  5.3 Paper III: The effect of thermal cooling  22  
  5.4 Paper IV: Late Saalian SST  23

6 Future outlook  24

Acknowledgments  27

References  29

Appendix: corrections  35
1 Glacial climates

Perhaps the most striking evidence of former glaciations is the presence of large rocks that have travelled hundreds of kilometers from their original locations. Before the 1800s, it was still believed that these rocks were carried by extensive floods. In the beginning of the 19th century, however, more and more natural scientists in the Alps and in the Scandinavian mountains began to ascribe odd traces in the landscape to extensive glaciations (e.g. Agassiz, 1838; Forbes, 1842). At first these ideas were met with much scepticism in the scientific community. During the next few decades, however, these sceptics of "glacial theory" were gradually converted, as more and more glacial deposits were found widely across Europe and North America. By the end of the 19th century, glacial theory was completely established.

How much have we learned about glaciations since the pioneering work by the 19th century scientists? In this context the study of ocean sediments has given us much valuable insight into the temporal behaviour of glaciations. By measuring the ratio of heavy and light oxygen-isotopes in the ocean sediments, it is possible to estimate how the global ice volume has evolved. Figure 1 shows the temporal evolution global ice volume, expressed in sea-level changes, over the last 350 kyr. The data is taken from an ocean sediment record from the equatorial Pacific (Lea et al., 2002). Consistently with other records across the world oceans, this record reveals (1) that glaciations exhibit a cyclic signal of $\sim 100$ kyr, and (2) that each 100 kyr cycle is characterised by a slow ice-accumulation rate and a rapid decay. This information is valuable for understanding the global evolution of the land-based ice sheets, but it reveals no details on where the ice sheets were located. To obtain such spatial constraints of the past ice sheets we have to do like the founders of glacial theory: we have to look for traces in the landscape.

1.1 Spatial reconstructions of past ice sheets

Typical landscape features that remind of former glaciations are (1) materials (soil/rock) transported by glaciers and then deposited (moraines), and (2) scratches into the bedrock and glacial sediments by glacial ablation (stri-
Glacial climates

Figure 1: A reconstruction of the sea-level changes over the last 350 kyrs. The reconstruction is based on oxygen-isotopes measured from an ocean sediment record from the equatorial Pacific (Lea et al., 2002).

The abundance of these traces has yielded a well established view of the maximum extents of the ice sheets. The extents of the North American and Eurasian ice sheets at the Last Glacial Maximum (LGM; ∼ 20 ka (kyrs ago), see Fig. 1), are shown in the leftmost panels of Figs. 2 and 3, respectively. These figures originate from Kleman et al. (2010) (North America), and Svendsen et al. (2004) (Eurasia). In North America, the LGM ice sheets covered most of Canada and northern USA. The maximum southward extent was reached over the eastern parts of the continent, whereas Alaska in the northwest was essentially ice-free. In Eurasia, the LGM ice sheet covered the whole Scandinavia, most of the British Isles, and the northernmost parts of the continental Europe.

To understand how the ice sheets arrived at the LGM extents, it is valuable to know their extents prior to the LGM. However, assessing the pre-LGM ice extents has proven to be far more difficult than for the LGM. This is especially true for the North American ice sheet (Clark et al., 1993; Kleman et al., 2010). The main difficulty is that large parts of the traces of the pre-LGM ice sheets were destroyed by the overriding ice sheets at LGM. For the Eurasian ice sheet, the basic evolution pattern is reasonably well understood (Ljungner, 1949; Kleman et al., 1997), but the dating of the early stages is still uncertain, particularly before 40 – 50 ka which is the limit for the C¹⁴ dating method. Under these circumstances, the data set which is used by geomorphologists and glacial geologists to outline pre-LGM ice-sheet configurations is very sparse. The few traces that exist are mainly the
Glacial climates

Figure 2: Spatial reconstructions of the North American ice sheet at the LGM (left panel) and at some time between 40 and 70 ka (right panel). The blue line represents the reconstructed ice margin, and "D" denotes the suggested positions of the ice domes. The figure is modified from Kleman et al. (2010).

Glacial striations and till lineations. In particular, crosscutting striations and lineations are valuable because they allow the scientists to determine relative-age relationships (Kleman, 1990; Clark, 1993). Striations and till lineations yield information on the flow direction of the ice sheet that prevailed when they were inscribed. Using simple flow laws of the ice sheets, it is thereafter possible to outline the plausible ice-sheet extent. An example of a pre-LGM reconstruction of the North American ice sheet is shown in the right panel of Fig. 2. The uncertainty associated with dating is clearly evident; the authors estimate that the reconstructed ice sheet in Fig. 2 should have occurred at some point between 40 and 70 ka. Although uncertain, this reconstruction suggests that the ice-sheet build-up before the LGM took place mainly over the eastern parts of North America, and that the western parts, including Alaska, were ice-free.

A similar reconstruction, but for the Eurasian ice sheet, was carried out

Figure 3: Spatial reconstructions of the Eurasian ice sheet at the about 20 ka; (LGM; left panel), at about 60 ka (middle panel), and at about 140 ka (right panel). The figure is modified from Svendsen et al. (2004).
by Svendsen et al. (2004). Their reconstructed ice extents at \(\sim 60\) ka and \(\sim 140\) ka are shown in the middle and right panels of Fig. 3. As opposed to the North American ice sheet, the Eurasian ice sheet did not reach its maximum extent at LGM. Hence, all traces created by the predecessors were not destroyed, implying that the reconstructed Eurasian ice extents are more certain than those for North America. Compared to the LGM, the Eurasian ice sheet at \(\sim 60\) ka has similar size, but its location is shifted eastwards. At \(\sim 140\) ka, which is often referred to as the Late Saalian Maximum, the global ice-volume reached its maximum value during the second-last glacial cycle. At this time, the extent of the Eurasian ice sheet was substantially larger than at LGM.

The papers forming the basis of this thesis have examined climatological features that are potentially important for the temporal evolution of these ice sheets. Papers I-III deal with large-scale zonal atmospheric variations, i.e. the stationary waves, that are forced by the ice sheets themselves. The main conclusions from these studies are that the stationary waves, through decreasing the melt rates, potentially contributed to the growth of the North American ice sheet up to LGM size, and that they can possibly explain why the North American ice sheet became larger to the east. The focus of paper IV is on how the Late Saalian ice sheet could become so large compared to the LGM.

2 The temporal evolution of ice sheets

The growth and decay of ice sheets are primarily determined by the prevailing climate conditions. If the mass gain of the ice-sheet exceeds the mass loss, the ice sheet becomes larger. If the situation is reversed, on the other hand, the ice sheet decays. How the ice sheet responds to the climate conditions depends also on the features associated with the ice sheet itself, e.g. its size, areal extent, shape, albedo. In turn, these features are shaped by the climate and the processes below the ice sheets as well as by ice-sheet dynamics.

In the first part of this chapter, some of the most important features for ice-sheet evolution are reviewed. To illuminate the present understanding of the temporal evolution of the past ice sheets, the second part of this chapter contains results from two studies that performed numerical simulations of the evolution of the ice sheets through the last glacial-interglacial cycle.
The temporal evolution of ice sheets

2.1 Ice-sheet boundary conditions

To understand the temporal evolution of the past ice sheets, one common approach is to run numerical ice-sheet models with appropriate boundary conditions, and subsequently compare the model results to observational data, such as the ones in Figs. 2 and 3. However, to model the boundary conditions is not at all an easy procedure. First of all, the physics behind some of the boundary features, particularly those represented below the ice sheet, is poorly understood. Secondly, since the ice-sheet boundary conditions feed back on the ice-sheet extent, they are interrelated such that changing one boundary condition will modify the effect of the others. In the following the most common ice-sheet boundary conditions that are involved when modelling ice sheets are briefly presented. These are conceptually summarised in Fig. 4.

Below the ice sheet, one needs to consider that the weight of the ice sheet exerts pressure on the underlying bedrock, which eventually begins to sink. To first order, the amount deflection is about 30% of the overlaying ice thickness, which is the density ratio between the ice and the mantle. The associated lowering of the ice-sheet surface has implications for the interaction between the ice sheet and the climate, not the least because the atmospheric temperature generally decreases with height (cf. section 3.2).
Another important feature to consider when modelling ice sheets is associated with their thermal properties. The temperature distribution within the ice sheets influences the ice deformation rate and consequently the shape of the ice sheets. Moreover, frictional heating due to the ice deformation may, along with the geothermal heat flux, produce basal meltwater which lubricates the layer between the ice and the bed (Kleman and Glasser, 2007). If a sufficiently large area becomes lubricated, the overlying ice may start to slide over the bed. Modelling studies have revealed that the effect of basal sliding yields thinner ice sheets (see e.g. Weertman, 1972; Oerlemans, 1982; van der Veen, 1999).

The boundary condition at the ice-sheet surface is mainly determined by the annual ice-accumulation rate, which is the resultant of precipitation, evaporation, sublimation and run-off of meltwater. For the most cases (apart extreme cold conditions), accumulation is given by precipitation and ablation mainly by melting (Oerlemans and van der Veen, 1984). When modelling the melting of past ice sheets, the conventional assumption is that the annual melting is proportional to the quantity PDD (positive degree-days), which is the sum of the temperatures above the melting point during that year. Hence, for modelling the past ice sheets, two climatic variables, namely the precipitation and temperature, are particularly interesting. Ice sheets may also interact with the ocean through ice shelves. An ice shelf is sustained by a continuous mass flux from the grounded ice (i.e. the ice sheet), whereas the primary mechanisms of mass loss is through ice-berg calving and melting at the lower surface. Hence, changing the ablation of an ice shelf, due to other ocean conditions, influences the dynamics of the land-based ice sheet.

Finally, a few words on ice-sheet dynamics should be mentioned. For large ice sheets, essentially only the margins are affected by melting whereas precipitation (although at low rates) occurs everywhere. This implies that the net ice-accumulation rate is positive over most of the ice sheet, but negative at the margins. Hence, in the lack of any ice flow, the slopes of the ice sheet would continuously become steeper and steeper. However, since ice is a deformable material, gravity-induced stresses within the ice sheet give rise to velocity shears (deformation). The highest ice velocities, which occur near the underlying bed and below the slopes, will most effectively contribute to transport of ice toward the margins. This thesis does not deal explicitly with the details involved with ice-sheet dynamics, although simulations are conducted with dynamic ice-sheet models. Comprehensive reviews on ice-sheet dynamics are found in Oerlemans and van der Veen (1984) and van der Veen (1999).
2.2 How well do we understand the evolution of the past ice sheets?

In this section, the spatial distributions of modelled ice sheets through the last glacial cycle are compared to the reconstructions in Figs. 2 and 3. One of the few existing ice-sheet models that focuses on the build-up stages of the North American ice sheets during the last glacial cycle is presented in Kleman et al. (2002). In their study, the zonal variations of the mass balance were tuned such the temporal evolution of the ice sheets was in reasonable accord with the geological and geomorphological evidence. However, the emphasis here is on studies in which the temperature and precipitation as simulated by atmospheric models, have served the basis for the mass-balance calculations. The results presented here are taken from Charbit et al. (2007) and Bonelli et al. (2009) because these studies are recently published and therefore elucidate the state-of-art capability of numerical models to simulation the last glacial cycle. The ice-sheet models in both studies simulate the effects of a deformable bedrock and have a representation for basal sliding. The effect of ice shelves on the ice-sheet dynamics, however, is not accounted for in neither of the studies.

The climate conditions were deduced differently in both studies. In Charbit et al. (2007), the temperature and precipitation fields were computed by six different Atmospheric General Circulation Models (AGCMs). For each AGCM, two climate snapshots, one for LGM and one for present-day conditions, were carried out. These two snapshots were thereafter interpolated in time using the temperature record inferred from the $\delta^{18}O$ GRIP ice-core record. Note that when computing the climate snapshots, the same LGM and present-day atmospheric boundaries (sea-surface temperatures, vegetational cover, etc.) were used in all the models. Bonelli et al. (2009) used a coupled ice sheet-climate model in which the climate fields were updated every 20 years of the ice-sheet simulation. The coupled model was driven by variations of insolation and atmospheric CO$_2$ concentrations. Their climate model, named CLIMBER 2.3, describes the atmosphere, ocean, sea-ice, land-surface processes and terrestrial vegetation cover. However, especially the atmospheric representation in the model is very simplified. For example, it uses a fixed lapse-rate and the heat transports due to synoptic-scale eddies are parameterised.

The spatial distributions of the Northern Hemisphere ice sheets at $\sim 60$ ka and at LGM in Charbit et al. (2007) and Bonelli et al. (2009) are shown in Figs. 5 and 6, respectively. In Fig. 5 the results are shown for three out of the total six AGCM simulations. The modelled ice sheets should qualitatively be compared to the reconstructions of the North American and Eurasian ice
The temporal evolution of ice sheets

In Figs. 5 and 6, it is clear that some of the known prominent features of the North American and Eurasian ice sheets at ~60 ka and at LGM are captured qualitatively in the simulations. For example, in Charbit et al. (2007) (Fig. 5), the simulation with the GEN2 atmospheric model yields no ice accumulation over most of Alaska, and in the CCSR simulation the increased equatorward extent over the eastern side of the North American continent is well captured. Both these features are consistent with the reconstruction in Fig. 2. Similarly, other than too little eastward expansion of the Eurasian ice sheet at 60 ka, the model of Bonelli et al. (2009) (Fig. 6) simulates the observationally-inferred evolution of this ice sheet fairly well (cf. Fig. 3).

However, apart from the fact that one, or perhaps two, of the prominent features of the ice sheets are captured in some of the last glacial-cycle simulations, none of them yield satisfactory results at a given time for both the Eurasian and North American ice sheets. In Fig. 5, the spatial extent of the Eurasian ice sheet is poorly constrained. On the other hand, in Fig. 6, the
North American ice sheet expands over the western parts of the continent (including Alaska) instead of over the eastern parts, which is suggested by the reconstructions. In fact, in none of the simulations presented in Fig. 5 and 6, the outline of the North American ice margin, as suggested by the reconstructions, is fully captured. Hence, the models exhibit serious difficulties obtaining only little ice over Alaska along with an increased equatorward extent to the east.

Possibly, some of the modelled errors of the spatial extents of the past ice sheets are attributed to the resolution in the ice-sheet model (Marshall, 2002) and/or in the atmospheric model (Jost et al., 2005). However, the model discrepancies also illuminate an unsatisfactory understanding of the underlying physics associated with ice-sheet evolution. In particular, the evolution of the glacial climate remains highly uncertain (Pollard and PMIP, 2000). The results in Charbit et al. (2007) illustrate that the spatial extents of the North American and Eurasian ice sheets are sensitive to the employed atmospheric model. Hence, the atmospheric state, which is partly shaped by the ice sheets themselves, constitutes one of the features that requires a better understanding.

3 Climate conditions

One important message from the previous chapter is that a more accurate modelling of the past ice sheets requires a better understanding of the tem-
poral evolution of the past climate. The Earth’s climate is primarily set by the orbital configuration of the Earth according to Milankovitch theory (Milankovitch, 1930; Berger, 1978). However, the insolation at high latitudes is mainly controlled by the precessional (20 kyr) and obliquity (40 kyr) cycles, which are considerably shorter than the glacial cycles (100 kyr, see Fig. 1). Hence, the climate system responds nonlinearly to orbital forcing. This is further illuminated by the fact that the atmospheric CO\textsubscript{2} concentration, which is an internal forcing agent, exhibit a cyclic behaviour similar to that of glacial cycles (Berger et al., 1998; Shackleton, 2000). In addition, the Earth’s climate is controlled by changes in the land and ocean surface conditions, as well as by feedbacks due to the ice sheets themselves.

3.1 Glacial surface conditions

When considering the Earth’s background climate it is inevitable to emphasise the importance of the sea-surface conditions. First of all, the sea-surface conditions control the sea-ice cover, mainly by the temperature but also by the salinity. Secondly, the sea-surface temperature (SST) distribution of a certain past climate provides fixed boundaries to the atmospheric model which facilitates the procedure of finding also the atmospheric conditions. Primarily, the SST:s are maintained by the heat fluxes to the atmosphere as well as by the wind and density-driven ocean circulations. Hence, in order to obtain the SST distribution, on the basis of physics, requires information about the atmospheric state. On the other hand, as mentioned above, for computing the atmospheric state the SST:s must be known. Thus, in this context, estimating the SST:s in a past climate requires either (1) simulations with coupled atmosphere-ocean models, in which the atmospheric and oceanic states are interactively updated, or (2) usage of available proxy data.

Based on ocean sediment records, much effort has been put in to deduce the spatial SST distribution at LGM. The first reconstruction of the LGM SST:s was provided within the research project Climate: Long range Investigation, Mapping, and Prediction (CLIMAP, 1984). In the CLIMAP reconstruction, the global annual-mean SST is 3°C lower than the modern value. Although CLIMAP has been widely used as global ocean boundary-condition when modelling the LGM climate, it has also been subjected to some serious controversy. The criticism has been based on that the CLIMAP ocean surface is (1) too warm in the tropics, particularly in the tropical Pacific, and (2) too cold in the North Atlantic implying that the Nordic Seas were completely sea-ice covered even in the summer.

The criticism towards CLIMAP was confirmed in the LGM SST reconstruction that was undertaken within the recent project Multiproxy Ap-
Climate conditions

proach for the Reconstruction of the Glacial Ocean surface (MARGO; Waelbroeck et al. 2009). For the MARGO project a larger amount of proxy data was available than for CLIMAP implying a better geographical and temporal coverage. Compared to CLIMAP, the MARGO reconstruction suggests colder conditions in the tropics as well as ice-free conditions in the Nordic Seas in the summer. The importance of seasonally ice-free conditions in the North Atlantic for the mass balance of the LGM ice sheets has been recognised in Ruddiman and McIntyre (1979) and Hebbeln et al. (1994).

For glacial times other than the LGM, there are fewer SST estimates based on the ocean sediment records. Therefore, global SST reconstructions, such as CLIMAP and MARGO, have not been compiled for periods before the LGM. To construct such dataset for a glacial time with few observation-based SST records, it is reasonable to complement these reconstructions with simulated SST:s from coupled atmosphere-ocean models. In paper IV of this thesis, we used this approach to construct the first-ever estimate of the global SST and sea-ice distribution for the Late Saalian climate (140 ka; see Fig. 3).

The properties of the land surface surrounding the ice sheets influence the climate mainly through the albedo. In cold glacial conditions, such as the LGM, there is a retreat of the boreal forest in favour of tundra, which increases the terrestrial albedo (Crowley and Baum, 1997). The reduced albedo serves to decrease the near-surface temperature, which provides more favourable conditions for ice-sheet growth (see e.g. Colleoni et al., 2009).

3.2 Feedbacks between the atmosphere and ice sheets

In addition to the background climate, modelling the evolution of the past ice sheets requires careful treatments of how the ice sheet interacts with and potentially modifies the local climate. In this context, a number of early studies have pointed out that the high albedo and high elevation of ice sheets constitute two feedbacks that serve to induce further ice growth (Weertman, 1976; Källén et al., 1979; Oerlemans, 1980).

The mechanism behind the height-mass balance feedback is that the mass balance generally increases with the ice height as the ice-top temperature decreases due to the atmospheric lapse. An important implication of this feedback is that once the ice sheet is thick enough, it can sustain itself. Calculations with simple ice-sheet models yield a range of climate conditions in which this feedback gives rise to two stable steady-state solution: one with zero ice thickness and one with a large ice sheet. In fact, this is the case for the Greenland ice sheet in today’s climate. If the ice on Greenland was suddenly removed, it would not re-grow in the present climate conditions.
Climate conditions

(Oerlemans and van der Veen, 1984).

The schematic picture of the temperature-albedo feedback is that the albedos of ice and snow are higher than the surroundings. Hence, this feedback serves to reduce the temperature over the ice sheet. The relative importance of the temperature-albedo feedback has been recognised especially for the initial growth phase of ice sheet, at which point that height-mass balance feedback has not fully kicked in (Kageyama et al., 2004). A feature that could potentially decrease the effect of the albedo is dust deposited on the snow-surface of the ice sheet (Calov et al., 2005; Colleoni et al., 2009). Analysis from ice cores has revealed that the dust concentrations in the glacial atmosphere were much higher than today (Mahowald et al., 1999).

While the height and albedo of ice sheets mainly feed back on the temperature, the effect of topographically-forced vertical winds and storm tracks constitute important precipitation feedbacks. When near-surface winds impinge the slopes of ice sheets, the resulting vertical winds serve to increase the precipitation over the windward slopes (Sanberg and Oerlemans, 1983; Roe and Lindzen, 2001b; Roe, 2005). In both Sanberg and Oerlemans (1983) and Roe and Lindzen (2001b), it was shown that this mechanism acts to induce westward propagation of the midlatitude ice sheets. Roe and Lindzen used the following parameterisation of the precipitation rate $P$:

$$P = e_{\text{sat}}(T_s) \times \max[0, (a + bw')]f(w')dw',$$

where $e_{\text{sat}}$ is the saturation water vapour pressure (given by the surface temperature $T_s$ through the Clausius-Clapeyron relationship), $a$ and $b$ are constants, and $f(w')dw'$ represent a Gaussian distribution centered on the topographically-induced vertical velocity $w$, which is given by:

$$w = u \cdot \nabla h.$$
flow law to relate the strain rates to the third-power of the applied stresses. In Fig. 7a, the effect of flow-induced precipitation was switched off (i.e. $b = 0$ in Eq. (1)), and in panel b the upslope precipitation effect was included. The vertical winds were updated by an atmosphere primitive-equation model each 500 years of the ice-sheet simulation.

When the effect of upslope precipitation is suppressed (Fig. 7a), the ice sheet arrives at a zonally symmetric shape as expected. In agreement with Roe and Lindzen (2001b), flow-induced precipitation serves to increase the ice thickness near the west coast (Fig. 7b). This increase is associated with the prevailing midlatitude westerly flow in the atmospheric model. Moreover, in the present model, flow-induced precipitation serves to deform shape of the southern margin. The underlying physics is that localised precipitation anomalies induce small zonal variations of the southern ice margin. In turn, the zonal variations of the ice-margin possibly amplify the upslope precipitation anomalies. However, the results in Fig. 7 should be interpreted with caution. Firstly, the precipitation parameterisation in Eq. (1) assumes that the moisture source is infinite, which may infer too high precipitation values, especially over the interior of the continent. Secondly, because the vertical velocity is activated also on the smallest scales of the atmospheric model,
features associated with numerics, such as horizontal hyperdiffusion, become relevant.

Finally, a number of studies have revealed that the topographically-forced stationary-wave pattern at LGM changes the synoptic variability, i.e. the storm tracks. For example, on the basis of a regional climate model, Bromwich et al. (2005) found that the shift of the location of the LGM storm-tracks enhances the precipitation over the southern margin of the North American ice sheet. Furthermore, it has been found that the storm tracks over the Atlantic Ocean provide an important link between the North American and Eurasian ice sheets (Kageyama and Valdes, 2000). Obviously, for this feature, also the sea-ice margin in North Atlantic is an important factor.

4 The stationary-wave feedback

Stationary waves are defined as the time-mean atmospheric circulation. In the presence of a westerly atmospheric flow, they exist due to zonal asymmetries of the topography and the diabatic heating. Papers I-III of this thesis are based on the idea that the topography as well as the thermal properties of ice sheets have the potential to alter the stationary waves, and consequently also to affect the zonal asymmetries of the climate in which they evolve.

The present-day stationary waves in the Northern Hemisphere are shown in Fig. 8. In winter, the largest stationary-wave amplitudes (variations of the geopotential height) are found east of the largest mountain ranges: the Rockies and the Himalayas. Typically, the response to topography is associated with a ridge (trough) west (east) of the topographic barrier. In the first part of this chapter, it is shown that this pattern can be captured even with the simplest numerical circulation models. In summer, when the atmospheric circulation is weaker, the topographical effect on the waves is less pronounced. Instead the summer conditions are associated with a larger impact of diabatic heating on the stationary waves (Ting, 1994). Further, in winter there is clear link between the stationary-wave amplitude and the temperature perturbation of the lower atmosphere, illuminating that the stationary waves contribute extensively to the zonal variations of temperature. In summer, however, the temperature perturbations are to a larger extent determined by the land/sea contrasts, hence suggesting that the heating of the land surfaces are important.

The impact of the stationary waves on maintaining the great past ice sheets through the ablation was recognised already in Lindeman and Oerlemans (1987). However, the first ones to study this feedback in detail were taken by Roe and Lindzen (2001a,b). Roe and Lindzen (2001a) used a one-
dimensional plastic ice-sheet model to examine the stationary-waves. The amplitude of the stationary waves in their study was assumed to be proportional the maximum height of the ice sheet. Their main result was that the stationary-wave induced cooling (warming) substantially enhanced (reduced) the extent of the 1D ice sheet. In Roe and Lindzen (2001b), the stationary-wave feedback was examined using the thermomechanical ice-sheet model, SICOPOLIS, interactively coupled to a linear quasi-geostrophic stationary-wave model. The ice sheet evolved on the same idealised continent as in Fig. 7 from a regional-scale to a continental-scale equilibrium extent. In their study, the stationary-wave induced temperature perturbations served to increase the overall ice volume, and to strongly deform the shape of southward ice margin: over the westernmost parts of the continent the ice sheet retreated poleward whereas over the eastern parts, the ice sheet expanded further southward.

4.1 The simplest model of topographically-forced stationary waves

Because the stationary waves are defined as time-mean deviations of the zonal-mean climate, the simplest numerical stationary-wave models have no time dependence and they are linearised about a zonal-mean basic state. Furthermore, the simplest models employ the quasi-geostrophic approximation, which assumes that the wind field is primarily in geostrophic balance, and that the variations of the Coriolis parameter are small. These two assumptions constrain the quasi-geostrophic equations to synoptic-scale phenomena at the midlatitudes. Further, in stratified quasi-geostrophic models (such as the one used by Roe and Lindzen 2001b), it is assumed that variations of
stratification is much smaller than the basic-state stratification. In single-layer (barotropic) models this assumption is equivalent to small perturbations of the free-surface compared to the mean fluid depth. Making use of the β-plane approximation, i.e. that the meridional gradient of the Coriolis parameter is constant, the linear, barotropic, quasi-geostrophic potential vorticity equation takes the following form:

\[ [u] \frac{\partial}{\partial x} \nabla^2 \psi^* + \beta_0 \frac{\partial \psi^*}{\partial x} = \frac{f_0}{H} [u] \frac{\partial h^*}{\partial x} - r \nabla^2 \psi^*, \]  

(3)

where \( x \) is the zonal coordinate, \( \psi \) the streamfunction, \( u \) the zonal wind, \( H \) the mean fluid depth, \( f_0 \) the Coriolis parameter, and \( \beta_0 \) the constant meridional gradient of \( f_0 \). The square brackets denote zonal-means and the asterisks deviations from the zonal mean. The last term on the right-hand side of Eq. (3) represent linear damping with a damping time-scale \( r^{-1} \). The flow is geostrophically balanced so that \( u = k \times \nabla \psi \), and the zonal-mean flow is westerly, i.e. \( [u] > 0 \).

Equation (3) can be analysed by considering one Fourier component of the ice sheet and the streamfunction as:

\[(h^*, \psi^*) = (\tilde{h}, \tilde{\psi}) \exp(ikx) \sin(ly), \]  

(4)

where the tildes denote Fourier amplitudes, \( y \) is the meridional coordinate, \( k \) and \( l \) the zonal and meridional wavenumbers, respectively. The ansatz in Eq. (4) assumes that \( \psi^* \) vanishes at the meridional boundaries of the β channel, and that the wave amplitudes in the zonal direction are limited only by the presence of damping. Substituting Eq. (4) into Eq. (3) yields after some algebra:

\[ \tilde{\psi} = \frac{f_0}{H K^2 - K_s^2 - iR}, \]  

(5)

with \( K_s^2 = \beta/[u] \), \( K^2 = k^2 + l^2 \) and \( R = rK^2/(k[u]) \). Hence, Eq. (5) yields that the amplitude of the streamfunction is proportional to the height of the ice sheet. Furthermore, in the limit of no friction \( R \to 0 \), the streamfunction is in phase with the ice sheet for \( K > K_s \), and out-of-phase with the ice sheet for \( K < K_s \). For \( K = K_s \), the stationary-wave resonates with the topography and its amplitude is only limited by the linear damping. A realistic representation of the ice-sheet topography have contributions from all Fourier components, infering that the response is dominated by the resonant wave. This implies that the streamfunction is phase-shifted 90° relative to the ice sheet manifested by a ridge (trough) located upstream (downstream) the ice sheet. This type of streamfunction response is illustrated in Fig. 9a,b, and a similar pattern is found also near the mountain ranges in the present-day climate, at least in winter (Fig. 8).
The stationary-wave feedback

Figure 9: Topographically-forced stationary-wave responses at 950 hPa (a,c,e), and at 300 hPa (b,d,f) for a typical linear response (a,b), nonlinear response (c,d). In panels (e) and (f), the topographically-forced stationary waves are computed with a prescribed thermal cooling (Eq. (9) with $Q_s = -2 \text{ K day}^{-1}$ and $H_Q = 2 \text{ km}$), which is applied uniformly over the topography. The stationary waves are represented by the zonally asymmetric streamfunction (contours) and temperature (coloured shading). The topography (gray contour) has a Gaussian shape and its maximum height is 2500 m.

4.2 Do the stationary waves warm or cool the local ice-sheet climate?

Until now we have only considered how topographical features, such as large-scale ice sheets, influences the stationary flow-pattern in the atmosphere. In
this section, also the relation to the temperature will be discussed. In the absence of thermal damping and diabatic heating, the potential temperature \( \theta \) is a conserved quantity. Linearising the steady-state thermodynamic equation about a zonal-mean state yields:

\[
[u] \frac{\partial \theta^*}{\partial x} + v^* \frac{\partial \theta}{\partial y} + w^* \frac{\partial \theta}{\partial z} = 0,
\]

(6)

where \( w \) is the vertical wind, and \( z \) the vertical coordinate. At first, we assume that the atmosphere is neutrally stratified, i.e. \( \frac{\partial \theta}{\partial z} = 0 \). Assuming geostrophic balance, and that the perturbations vanish far upstream the ice sheet, Eq. (6) simply yields that:

\[
\theta^* = -\frac{\psi^*}{[u]} \frac{\partial \theta}{\partial y}.
\]

(7)

Because \( \frac{\partial \theta}{\partial y} < 0 \) in the Northern Hemisphere, the temperature perturbations are in phase with the streamfunction perturbations (equivalent barotropic). For a ridge (trough) over the ice-sheet, the balance in Eq. (7) yields that the temperature perturbations are associated with a warm (cold) anomaly.

For the large-scale atmospheric conditions, however, the stratification is positive implying that \( \frac{\partial \theta}{\partial z} > 0 \). Neglecting meridional temperature advection, and using Eq. (2) we obtain that:

\[
\theta^* = -h \frac{\partial \theta}{\partial z}.
\]

(8)

Hence, in the linear regime, stratification cools the local ice-sheet climate through adiabatic motions. Further, Eq. (8) reveals that the largest cooling occurs over the interior of the ice sheet. The main conclusion from this analysis is that, except for atmospheric basic-states represented by a very weak stratification and a strong meridional temperature gradient, the stationary waves dominantly cool the local climate over ice sheets. This is confirmed in Fig. 9a, in Roe and Lindzen (2001b), as well as in papers I and II of this thesis.

4.3 The effect of nonlinearity

When linearising Eq. (3), one assumes that the perturbation winds are smaller than the zonal-mean zonal wind. This implies that two terms are neglected: (1) advection of relative vorticity by perturbation winds \((u^* \cdot \nabla \zeta^*)\), and (2) perturbation winds interacting with the ice sheet \((u^* \cdot \nabla h^*)\). Are these two
The stationary-wave feedback

terms at all important? Because the perturbation winds increase with the height of the ice-sheet along the streamfunction perturbation (cf. Eq. (5)), these two nonlinear terms\(^1\) become large when the ice sheet is sufficiently high. What is "sufficient" in this context depends on features such as the background climate and the shape of the topography. Ringler and Cook (1997) found that the stationary-wave response becomes "more" nonlinear when the meridional temperature gradient is weak (e.g. summer conditions), and when the topography is elongated in the zonal direction (e.g. the Himalayas).

When the nonlinear terms are significant, the stationary-wave response can be substantially altered. Typical near-surface and upper-level nonlinear stationary-wave responses are shown in Fig. 9c and d, respectively. Compared to the linear response, the nonlinear streamfunction response is associated with a clockwise rotation of the anomalies (see also Cook and Held, 1992; Ringler and Cook, 1997). Furthermore, the nonlinear response yields weaker temperature anomalies than the linear case. The largest temperature anomalies over the ice sheet are found near the poleward margin, where the melt rates are generally small. The effect of nonlinear stationary waves on the evolution of ice sheets is examined in paper II.

4.4 The effect of diabatic cooling

In addition to topographical forcing, stationary waves are driven by zonal asymmetries of diabatic heating. Because the surface albedo of ice and snow are generally higher than those of the land and ocean surfaces, ice sheets cools the overlaying atmosphere and thereby induce zonal perturbations in the diabatic heating field. In paper III, the effect of thermal cooling on the topographically-forced stationary waves is examined.

Figures 9e and f show the near-surface and upper-level topographically-forced stationary-wave responses when they are modified by thermal cooling. The thermal cooling \(Q\) is applied uniformly over the topography and its vertical distribution takes the following form:

\[
Q = Q_s \exp\left(-\frac{(z - h)}{H_Q}\right),
\]

where \(Q_s\) is the thermal cooling at the surface, and \(H_Q\) the vertical-scale of the cooling. Again, \(z\) is the vertical coordinate, and \(h\) the height of the topography. The topographical response is calculated by taking the difference between the response from a simulation with both the ice-sheet topography

\(^1\)Because \(h\) is an independent variable, \(\mathbf{u}^* \cdot \nabla h\) is actually a linear term. However, since this is conventionally omitted in linear models, we consider it as a "nonlinear" term.
and thermal cooling and a simulation with thermal cooling and no topography. It is clear the thermal cooling amplifies the topographically-forced streamfunction as well as the temperature perturbations near the surface. The amplified response is also noted in the upper-levels. In fact the response tends to be more "linear" than for the case without thermal cooling (panels c and d), which is in agreement Ringler and Cook (1999).

5 Summary of the papers constituting this thesis

This chapter summarizes the papers constituting this thesis. The common denominator of Papers I-III is that they all examine different aspect of stationary wave-ice sheet interactions. Paper IV presents an estimate of the SST field during the Late Saalian Maximum (∼140 ka).

5.1 Paper I: The impact of stationary waves on the local ice-sheet climate

In this study, we used a linear two-layer atmospheric model in a β-plane channel to examine the influence of ice-sheet topography on the stationary waves. In particular, we considered stationary-wave induced temperature anomaly locally over ice-sheet topography, which was assumed to be governed by the plastic ice approximation. Due to the simple nature of the two-layer model, the results in this study should be interpreted qualitatively. To reduce the uncertainty associated with model parameters and geometry, we chose to monitor the impact of the stationary waves on the ice sheets using the temperature anomaly averaged over the whole ice sheet rather than to use a surface mass-balance calculation, which is sensitive to the spatial structure of the temperature anomalies.

The advantage of using a simple atmospheric model, such as the linear two-layer model, was that we could derive analytical expressions that related the stationary-wave induced temperature-anomalies to ice-sheet topography. This analysis revealed that the stationary waves induce a mean cooling over ice sheets (cf. section 4.2). In turn, this cooling increases linearly with the ice volume as long as the stationary waves are large compared to the ice sheet, which is the case when the following condition is satisfied:

\[ \lambda_x > 4\pi L. \]  

Here \( \lambda_x \) is the zonal wave-length of the stationary waves and \( L \) the half-length of the ice sheet, which was assumed to have similar extents in the
Summary of the papers

zonal and meridional directions. Hence, for a stationary-wave response that is dominated by zonal wavenumber $3$, the stationary-wave induced cooling over ice sheets increases linearly with ice volume up to about $L = 700$ km (at $50^\circ$N). For larger ice sheets, the dependence of the cooling on the ice volume becomes gradually weaker and the eventually disappears.

The linear relation between the stationary-wave induced temperature and ice volume was shown to have a strong influence on the mean temperature an ice sheet that grows and expands equatorward. In this study, we found that there is a range of intermediate-sized (between $L = 500$ km and $L = 1200$ km) ice sheets for which, as they grow, the cooling due to stationary waves and the atmospheric lapse rate completely cancel the warming effect due to the meridional temperature gradient. For these ice sheets, the area-mean temperature can decrease as they expand equatorward.

5.2 Paper II: Linear versus nonlinear response

The objective of this study was to examine the mutual interactions between ice sheets and stationary waves in more comprehensive atmospheric models than had been done in previous fully-coupled atmosphere-ice sheet studies. Prior to paper II, all studies on this topic had used either conceptual models (Roe and Lindzen, 2001a) or linear steady-state models (Roe and Lindzen 2001b, paper I). In this study, the atmospheric response was calculated using a linear as well as a fully-nonlinear dry primitive-equation (PE) model. The linear PE model has essentially the same dynamics as the linear models used in Roe and Lindzen (2001b) and in paper I, whereas the full PE model also includes the terms that are neglected in the linear approximation (cf. section 4.3). Both atmospheric models in this study were separately coupled to the 3D thermomechanical ice-sheet model, SICOPOLIS. Simulations were conducted in which a small ice mass evolved to a continental-scale equilibrium ice sheet on an idealised "North American" continent (such as in Roe and Lindzen 2001b). One important difference between Roe and Lindzen (2001b) and here was that the stationary waves in their study were calculated in annual-mean climate conditions, whereas in our study, they were calculated in summer, which is the season with the highest ablation rates.

Paper II contains two major results:

1. The stationary waves computed with the linear PE model served to substantially deform the equatorward margin of the ice sheet, whereas the atmospheric response computed with the nonlinear model had essentially no impact on the equilibrium shape of the ice sheet. Hence, the degree of linearity of the atmospheric response controlled to what
extent topographically-forced stationary waves could reorganise the structure of the ice sheets. The reason was that the linear and nonlinear stationary-wave responses yielded completely different temperature anomalies over the ice sheet (cf. Fig. 9a,c).

2. Provided that the response was linear, the shape of the equatorward margin of ice sheets was controlled by the following parameter: \( L_x / \lambda_x \), where \( L_x \) is the zonal extent of the continent, and \( \lambda_x \) the zonal wavelength of the stationary wave. If \( L_x / \lambda_x \approx 1/2 \), such as in Roe and Lindzen (2001b), there was a warm (cold) anomaly over the western (eastern) part of the continent, causing the ice sheet to retreat poleward (advance equatorward) to the west (east). This shape of the ice sheet is broadly reminiscent of the North American ice sheet at the LGM (see Fig. 2). In the linear simulations, we also obtained a \( L_x / \lambda_x \approx 1 \) response, which is associated with an increased ice-sheet extent over the central parts of the continent, and a retreat over the eastern parts.

5.3 Paper III: The effect of thermal cooling

This paper takes into account also the effect of a possible ice-sheet induced thermal cooling of the atmosphere on the topographically-forced stationary waves (cf. section 4.4). Note that this paper does not consider the direct effect of thermal cooling on ice-sheet evolution; rather it deals with the atmospheric thermal cooling over an ice sheet modifies the topographically-forced stationary waves.

This study encompasses a number of uncoupled simulations, in which I forced the fully-nonlinear PE-model from paper II with prescribed representations of the topography and the atmospheric thermal cooling. The objective of the uncoupled simulations was to obtain a physical understanding of the interaction between thermal and topographical forcing. I found that thermal cooling enhanced the topographical stationary-wave response (Fig. 9e,f). The reason for the amplified topographical response was that thermally-induced perturbation winds amplified the topographically-induced vertical velocities and hence the forcing of stationary waves; see Eq. (2).

In paper II, it was found that the nonlinear stationary-wave response, in the absence of thermal cooling, had essentially no effect on the equilibrium features of ice sheets. Here, an additional coupled simulation was carried out, in which thermal cooling was added to the calculation of the topographically-forced stationary waves. Due to the enhanced topographical stationary-wave response in the presence of thermal cooling, the ice sheet advanced further equatorward. The largest increase of the ice-sheet extent was obtained near
the continental west coast. The results of this paper suggest the possibility that the stationary waves in a glacial climate are also affected by ice-sheet features that influence the atmospheric thermal cooling, e.g., the albedo of the ice-sheet surface, which is among other things controlled by the deposition of dust.

5.4 Paper IV: Late Saalian SST

The objective of paper IV was to calculate an estimate of the prevailing SST:s at the Late Saalian Maximum. The motivation for this work is summarised in Fig. 3: how could the Eurasian ice sheet become so much larger at the Late Saalian Maximum than at the LGM? As noted in section 3.1, the LGM ocean surface conditions are constrained by a relatively large set of proxy data. For the Late Saalian, however, the number of proxy records are much less than for the LGM. In this context, it seemed reasonable to complement the scarce proxy data with SST:s derived from numerical modelling efforts. The SST:s were computed using Planet Simulator (Fraedrich et al., 2005), which is a climate model of intermediate complexity. The central part of Planet Simulator consists of a spectral AGCM with fairly simple parameterizations of the physics. In this study, the AGCM was interactively coupled to mixed-layer ocean model, which was used to calculate the SST:s, and to a thermodynamic sea-ice model.

We found that the simulated SST:s for the Late Saalian agreed reasonably with the few existing proxy records, except for the North Atlantic where the model predicted colder conditions than the geological records. The sea-ice edge in the North Atlantic reached as far south as to northern Spain and northwestern Portugal during winter, while it retreated slightly northward during summer. Also in the North Pacific, the sea-ice margin extended further southward than in the LGM reconstructions. Because the North Atlantic and North Pacific are key areas for the Eurasian ice sheet, the increased sea-ice extents in these regions reduced both the precipitation and ablation over the Eurasian ice sheet.

A comparison between the computed Late Saalian SST:s and the reconstructed LGM SST:s revealed a pronounced asymmetric SST distribution between the Northern and the Southern Hemispheres during both winter and summer. Relative to the LGM, the Late Saalian ocean surface was found to be about 4°C warmer in the Southern Hemisphere and about 4°C cooler in the Northern Hemisphere. Further analyses showed that the SST asymmetry between the hemispheres was caused by the Late Saalian Eurasian ice-sheet topography: when a simulation was performed with the Late Saalian orbital parameters but with the LGM Eurasian ice sheet, the computed SST:s re-
6 Future outlook

It is hoped that this thesis has contributed to an improved understanding of the physics that controls the mutual interplay between stationary waves and ice sheets. The present results serve to support to notion proposed by Roe and Lindzen (2001b) that stationary waves can be a main factor controlling the shape of a continental-scale ice-sheet’s equatorial margin. However, the importance of the stationary waves relative to the background geography and other internal feedbacks for determining the structure of an ice sheet has not been addressed in the present thesis and remains to some extent still an open question. To examine the stationary-wave responses in the models participating in the Paleoclimate Modeling Intercomparison Project (PMIP) could presumably give some further insight. In this context, it would be of particular interest to determine why the AGCM:s in Fig. 5 yielded such different ice-sheet extents through the last glacial cycle. One possibility is that the various ice-sheet extents are attributed to different stationary-wave patterns among the models. In this case, the different ice-sheet configurations arise because the models produces different zonal-mean climates that results in stationary waves with varying wave lengths and degree of non-linearity. Another possible cause for the spread in ice sheet configurations are the parameterizations of subgrid processes such as clouds and convection or even the numerical schemes, which differ between the models.

As demonstrated by the results in paper II, the ability of the stationary waves to feed back strongly on the ice-sheet ablation is linked to the degree of linearity of the stationary-wave response. Thus, another important and essentially open question is whether the summer stationary-wave response to ice-sheet topography in a glacial climate is linear or nonlinear in character. To my knowledge, a study of linear versus nonlinear stationary-wave responses in glacial climates has so far only been undertaken by Cook and Held (1988), who found that the winter stationary-wave response to the LGM ice-sheet topography was well approximated by linear theory. However, when studying the character of the stationary waves in another climate than the present one, a major concern is that we must almost completely rely on the output from atmospheric circulation models. This is a problem because the character of the stationary wave response to a perturbed climate seems to be highly model dependent (Brandefelt and Körnich, 2008). As an illustration of the modeling-related challenges, Ringler and Cook (1997) showed that the critical height of topography, at which linear theory becomes invalid, is sensitive to the nature
of the frictional damping near the lower boundary. In the atmosphere, this damping is a result of small-scale three-dimensional turbulence, a feature that is difficult to represent in a coarse resolution model. Possibly, improved temporal-spatial ice-sheet reconstructions, such as outlined by Kleman et al. (2010), combined with modelling can prove the key to resolve the glacial stationary-wave features.

Another essentially open question concerns the asynchronous build-up of the North American and Eurasian ice sheets. At LGM, more ice was located over North America than over Eurasia, whereas at the Late Saalian Maximum the situation was reversed (F. Colleoni, personal communication). When including prescribed ice-sheet topography in atmospheric models, such as in paper IV, it is possible to analyse how the maximum ice-sheet extents are maintained, but it reveals very little information on how the ice sheets actually arrived at that state. It remains to further examine the physics behind the zonal climate variations prior to these two maxima. In this context, ice-sheet modelling should preferably be exploited.
Future outlook
Acknowledgments

This thesis would not have been possible without the support of many people. I wish to express my gratitude to my head supervisor, Prof. Johan Nilsson who has offered me invaluable support and guidance, and whose passion for this project has constituted a great source of inspiration for me. Special thanks also to my co-supervisors Prof. Johan Kleman and Prof. Erland Källén for always willingly sharing their knowledge and experience with me. Deepest gratitude is due to Florence Colleoni for interesting scientific discussions and fruitful collaborations that I am sure will continue into the future. I also wish to thank Marcus Löfverström for joyful discussions about everything from atmospheric dynamics to disrespectful attitudes toward cyclists among the taxi drivers of Stockholm.

I am grateful to the MISU staff who has formed the basis of a superb working environment. Special thanks Heiner Körnich, Rune Grand Graversen and Anna Lewinschal for inspiring conversations. Going to work every day would not have been as enjoyable if it was not for my office mates Joe Sedlar and Hanna Corell, who have offered me much-needed breaks with relaxed and interesting conversations. Many warm thanks also to everyone who participated in the weekly innebandy sessions. I would like to convey thanks to Prof. Josep Paredes at the Department of Astronomy and Meteorology in Barcelona for providing me a desk with an inspiring view over Camp Nou during the completion of this thesis. A big hug is delivered to Nina Kirchner for both scientific and emotional support as well as for much-needed espresso breaks. I would also like to acknowledge my lovely friend Andreas Vallgren for scientific discussions and emotional support. Finally, I express my love and gratitude to my family and all friends that have made me smile through the duration of this thesis.
Acknowledgments
References


References


References


Appendix: corrections

The formulation of Eq. (1) in paper IV is incorrect. In the following, a correct description of how the Late Saalian SST:s are computed is provided. For each gridpoint in the mixed-layer ocean model, the rate of change of the temperature, $T_{mix}$, is given by:

$$\frac{dT_{mix}}{dt} = \frac{Q_a + Q_o}{\rho_w c_p h_{mix}},$$

where $Q_a$ is the heat flux from the atmosphere, $Q_o$ the oceanic heat flux within the mixed layer, $h_{mix}$ the mixed-layer depth, $\rho_w$ the density of water, and $c_p$ the specific heat capacity of water. At every time-step, $Q_a$ is governed directly from the AGCM, whereas the lack of ocean dynamics infers that $Q_o$ needs to be parameterised.

To calculate $Q_o$, a simulation with prescribed SST:s is performed from which the monthly-mean values $Q_a$ are stored. Thereafter, it is simply assumed that $Q_o = -Q_a$, i.e. for each gridpoint, the atmospheric heat flux to the ocean surface is completely cancelled by heat transports within the mixed layer. Hence, the values of $Q_o$ are tuned to fit the climate of the simulation with prescribed SST:s. Once the climate is perturbed, the monthly-mean values of $Q_o$ are kept constant throughout the simulation, implying that the new SST:s are completely determined by the changes in $Q_a$. Thus, because $Q_o$ is constant for every month, it is preferable if the climate, from which $Q_o$ is calculated, is not radically different from the perturbed climate. With regard of this, $Q_o$ in paper IV is calculated using the LGM ocean surface, along with LGM conditions for orbital forcing, CO$_2$ and vegetational cover.