USING PRACTICAL EPISTEMOLOGY ANALYSIS TO STUDY THE TEACHER’S AND STUDENTS’ JOINT ACTION IN THE MATHEMATICS CLASSROOM

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This paper aims at characterizing the teaching actions that are used in a primary school mathematics lesson, and their consequences for the learning progression. To proceed, we explore the analytical outcomes of combining two analytical frameworks, namely the Practical Epistemology Analysis of classroom discourse (Wickman & Östman, 2002; Wickman, 2004; 2006) and the triple [Meso; Topo; Chrono]-Geneses featuring didactical transactions in the Theory of Joint Action in Didactics (Sensevy, Mercier, Schubauer-Leoni, Ligozat & Perrot 2005; Ligozat & Schubauer-Leoni, 2010; Sensevy 2010). The analytic approach is exhibited through an empirical sample of a mathematical lesson about the learning of surface area with 4th grade students. Analyses are guided by two questions: 1) testing the PEA for identifying the content learnt in transactions in the case of mathematics; 2) examining how PEA may augment the MTC-Geneses description to characterize the learning progression over time within the teacher’s and students’ joint actions. This later question is crucial to understand the generalization of the students’ experience against the teaching process unfolded by the teacher.

THEORETICAL BACKGROUND

Practical Epistemology Analysis (PEA)

In the Swedish pragmatist approach to science learning, the PEA framework was developed as an analytical tool for characterizing the meaning-making process in science-classroom discourse. This approach features learning as the unfolding of purposeful action and change of habits for coping with reality (Rorty 1991). Cultural practices entail epistemologies, as implicit rules for acting adequately in social groups. In designing and carrying out classroom work, the teacher makes explicit and implicit decisions about the situations that the students will experience. Wickman (2004) suggests that as the curriculum unfolds in the teacher's and the students' actions and discourse, a practical epistemology is shaped. Hence, from the student point of view, learning content is dependent on the epistemologies developed in the classroom, as a set of epistemic and social norms that guide the selection of relevant actions to achieve a purpose. Of course, such norms are tied to the socio-historical traditions embedded in curricula. We do not aim at describing such rules and power relations per se but we seek for the connection between how classroom participants produce meaning and what meaning is produced in a specific practice.

The model of Practical Epistemology Analysis developed by P.-O. Wickman and collaborators relies upon L. Wittgenstein's notion of language-game (Wittgenstein
1967) and J. Dewey's theory of *continuity in experience* transformation (Dewey 1938/1997). For the former, meaning is a given in the socially shared rules supported by a language proper to a context. Learning is then mastering a language-game, i.e. the grammar of actions featuring a practice. For the latter, experience is continually transformed by the transactions taking place between an individual and his environment. Subjects build continuity between past and present experiences so that experience earned in a given situation becomes an instrument for understanding and dealing with the situations which follow (Dewey 1938/1997, p44). PEA is grounded on *four categories* for analyzing discourse as a transformation of experience within a language game (Wickman & Östman 2002; Wickman 2004; Wickman 2006).

(i) *Encounter*: an encounter delineates a specific situation in terms of what can be seen to meet or interact in actions and discourse. This involves human beings as the participants of the situation and the "things" that become part of the experience in this situation. These may include physical objects, signs, words, utterances, phenomena like natural facts and events as well as recalled experiences.

(ii) *Stand fast*: in an encounter, certain objects are manipulated without any questions arising about their use. Such objects and words stand fast in the encounter. What stand fast in one situation may later be questioned in another situation. Neither does stand fast necessarily imply a correct use from the observer's perspective. It simply implies that the meaning of certain words in discourse is self-evident for the participants with respect to this specific situation.

(iii) *Gaps and relations*: in an encounter participants notice gaps. They then establish difference and similarity relations to what stand fast. If participants succeed in filling a gap with relations they build continuity between past and present experience. If they fail, the gap lingers and the course of action may change direction toward another purpose.

It is important to notice that the four concepts of PEA enable a first analysis of meaning-making from the interlocutors’ perspective. From the observer's perspective (the researcher in this paper), "something" is learnt when the activity moves on, that is when there is evidence that the participants can proceed towards a purpose. Learning proceeds when people notice gaps and fill them with relations to what stand fast in encounters. This inclusive account of learning focuses on what works in the situation in order to overcome it and not solely what is right or wrong with respect to conceptual knowledge. Questioning truth is central in scientific reasoning but it is only one of several ways to proceed in everyday life practices (Habermas, 1984/1990). PEA accounts for the meanings being construed in action without prioritizing what is true / not true and what should be said or done in order to acquire the expected knowledge. This may also be understood as a methodological caution aiming at minimizing the risk of overlooking certain forms of learning just because they were not included in the definition from the outset.
The triple [Meso; Topo; &Chrono]-Geneses

The Joint Action Theory in Didactics (JATD) envisions the teaching and learning practices as a *didactical game* in which the teacher achieves his/her goal - making the students learn knowledge content- only if the students get involved and act in *a certain way* (Sensevy, 2010). The expected way of acting defines the rules of the learning progression. For the student to learn, the teacher has to design *a set of conditions* made of material and symbolic objects bound to a question, task or inquiry to be attended and featuring the students’ ends in view. This set of conditions is viewed as the "primitive" milieu (or a teaching design) from which certain meanings are construed in action. The teacher and the student(s) cannot achieve their respective purposes without paying attention to the action of the other and moreover to the object of the other's action. An "object" is anything that can be the target of attention of an individual and that can be designated by him. The meaning of an object is given to an individual by the way in which the others are prepared to act toward it (Mead 1934/1992). The teacher's action and the student's action are *joint actions* in which each participant adjusts his action to the other's line of action (Blumer 2004). In the framework of the JATD the triple [Meso; Topo; Chrono]-Geneses models the construction of a *common ground* of meanings in the joint actions that are performed by the teacher and the student(s).

(i) *Mesogenesis* - The fitting of lines of actions of the teachers and the students (or within a student group) to achieve their respective purposes generates new meanings through the relations that participants establish to the objects of their environment. In adopting this point of view, the milieu in which actions unfold, is not just the set of conditions defined in the teaching design, regulated by the teacher over time, and against which the student would play a game (i.e. the milieu in TSDM; Brousseau 1997, p55-58). It is *a constant build up of relationships* to objects in discourse and actions, i.e. a mesogenesis in Chevallard's words (1992). Mesogenesis takes up both the students’ elaboration of meanings and the teacher's elaboration of meanings on the basis of what the students produce.

(ii) *Topogenesis* – Each category of participants (teacher versus student) lives in distinctive epistemological positions within the classroom collective. They do not share the same perspective on the task at the same time. These positions are movable but they never merge. The moves in the epistemological positions (either towards a reduction of the gap or towards its deepening) feature the topogenesis. Topogenetic moves result from the *division of the activity* between the teacher and the students, but also among students themselves according to their potentialities in a specific situation.

(iii) *Chronogenesis* - The teacher knows the overall direction that learning should take on the knowledge timeline. The learning content expected by the teacher in the first place corresponds to an institutional purpose in terms of contents and values to be conveyed to the students. Such a purpose is described as pieces of knowledge to
be learnt, attitudes to be adopted, competencies to develop…etc. in curriculum texts that reconfigure outer-school socio-cultural knowledge historically construed in the human activities. The overall direction that the learning progression takes in the classroom is described by the chronogenesis. Chronogenetic moves result from the legitimating process of certain meanings made by the students in the collective and/or the introduction of new relations directly made by the teacher.

The primitive milieu designed by the teacher is continuously augmented over time by the meanings arising in the participants' fitting lines of action. Meanings are epistemologically distributed across the classroom collective between the students' position(s) and the teacher's position. Certain meanings construed at a time in the mesogenesis may be judged relevant or not by the participants with respect to 1) their potential to support the ends-in-view structured by the task (epistemic relevance); 2) the expected learning content that is the overarching goal of the teacher (didactical relevance). The outcome of the collective analysis of situated actions is the departure point for the participants to further their activities. We contend that the meanings built in the mesogenesis undergo a selective process to become part of a "supposed-to-be-shared" common ground in the classroom collective (Ligozat & Leutenegger 2008). The ongoing construction of this reference is an institutionalizing process of meanings construed in the situated actions towards a collective objectivation of knowledge in discourse.

**EMPIRICAL FINDINGS**

In the following, we attempt to use PEA for describing meanings made in the contingencies of mathematical activities with primary school students. In particular, we try the analytical categories of the PEA approach (encounter, stand fast, gaps and relations) for describing the dynamics of purposes in the joint action and the content of the mesogenesis. The chronogenetic and topogenetic moves feature how the teacher directs the students' attention towards certain relevant objects and correlations in the setting in order to achieve a mathematical task. We merely use a short excerpt of classroom discourse to highlight the analytical potentialities of combining both PEA and MTC-Geneses frameworks. The students are working in small groups, with a set of 13 geometrical shapes and with a worksheet bearing the instructions i) "rank the shapes from the smallest to the largest according to their area"; ii) "justify your ranking"). They first make conjectures about the use of the objects provided [Gap 1: what should we do?]. When the teacher comes nearby, the students call upon her for helping. The teacher tells the students to read the instruction and asks them about the meaning of the word "area". A new gap is noticed [Gap 2: what is area?]. The students suggest that "it is the shapes", i.e. a word that stand fast to them in this situation. From the teacher's perspective, the students do not manage to construe any relevant relations to the word "area" (something like "it is the surface of the shapes" or "the space lying inside the borders
of the shape" may be expected). At min 3:29, the teacher goes back to the instruction in the following way:

43. TEACHER: [...] it is written / rank from the smallest to the largest
    /// actually from the largest to the smallest-
44. KAM: well / first we know that this one is the smallest (takes
triangle H out of the set)
45. TEACHER: how do you know that it is the smallest / how can you
    prove that it is the smallest+
46. MAR: (puts square C close to triangle H [PICT2]) because this one
    is smaller
47. DIA: it is half of it-
48. MAR: (murmuring) but we can't prove it-
49. TEACHER: (looking at Mark) OK but… / this is your feelings
    OK+ / but how can you prove it+ / because here
    (points at the corner of triangle H – PICT 3) / one
    may say / it sticks out a little bit- / so how can you
    prove that it is really the smallest+
50. DIA: (getting excited) I know I know+
51. MAR: oh like this / according to their area / we've got to set in line
    (sets the base of triangle H on the same line as
    square C [PICT4])
52. DIA: no look+ / I disagree / this is smaller because this (picks up
    triangle H) is half of the square (puts H onto C
    [PICT5])
53. TEACHER: ah+ / do you think this could be a proof+
54. [silence 5 sec]
55. KAM: this / that's two / mmh / the whole square that's twice this
    one (points at H)
56. TEACHER: how did Dina do to prove you this+
57. KAM: she puts it over (points at H again)
58. TEACHER: yeah / she puts it over the square / she superimposed
    the shapes // now / you have some transparent paper /
    some square grid paper // and by using Dina's
    technique / you've got to find some tricks / that to
    prove that / a shape is larger or smaller than another
    one

Our 1sr analytic focus looks at encounters, stand fast words and gaps to describe the
dynamical structure of the joint action. The encounters delineate the relationships
developed on a small portion of time by the participants with respect to certain
objects (words, things, signs) that become parts of their environment. A new
encounter opens up when we identify some changes in the participants’ purposes
during the course of action.
- Encounter 1 (line 43-45) → Mark, Dina, Kamer and Teacher take into account shape H and shape C (among all the set), smallest (a word from the instruction sentence)

- Encounter 2 (line 46 – 57) → Mark, Dina, Kamer and Teacher take into account shape H and shape C (among all the set), know, prove, proof, stick out, set in line, half, twice, put over.

- Encounter 3 (line 58) → Mark, Dina, Kamer and Teacher take into account shape H and C (among all the set), put over, superimpose, transparent paper, square grid paper, Dina's technique, tricks, prove, larger, smaller.

In this case, changes in purposes and so the openings of new encounters are prompted on the basis of gaps indicated by the teacher. As such, they coincide with some chronogenetical moves (CM) and topogenetical moves (TM). Encounter 2 corresponds to an expansion of teaching time to attend the need to "prove" (CM); the teacher adopts a feigned "low" position supposed to give some responsibilities to the students in finding "proof" arguments (this TM analysis will be nuanced in our 2nd focus of analysis). Encounter 3 corresponds to a contraction of the teaching time with the acknowledgement of the superposition technique as a to-be-shared reference (CM); the teacher uses her institutional power to prompt a new purpose in the joint action (TM). Through the dynamics of encounters 1, 2 and 3, we get an insight of the nature of the expectations upon the student's actions: i) sum up what is known from the reading of the instruction, ii) how we get to know it, and iii) extend the use of the "put over" technique. Changes in purposes may also originate in the students' course of action when a gap cannot be filled. For instance, at the very beginning of the group work session, when the students cut off the shapes from the cardboard, one of them tried isolating the smallest shape, another tried to order the shapes according to their alphabetical letters, then they tried ordering 4 shapes on a same line. At some point, the students stopped manipulating the shapes because they could not find a way to determine what to do, or more exactly, what they are expected to do in this situation. Gap 1 [what should we do?] lingers, resulting in a change of purpose from trying to do something with the shapes to getting an explication from the teacher. Hence, encounters, stand fast words and gaps reflect the evolution of the reciprocal expectations in the teacher and students' joint action.

Our 2nd analytic focus is put on gaps and relations to describe the epistemic content built up in the mesogenesis. In bold characters, are the teacher participations; in italic characters are the students' participations.
Table 2: Gaps and epistemic relations developing in the mesogenesis

<table>
<thead>
<tr>
<th>line</th>
<th>GAPS</th>
<th>RELATIONS</th>
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<tbody>
<tr>
<td>infra</td>
<td>G1: <em>what should we do?</em> =&gt; linger</td>
<td></td>
</tr>
<tr>
<td>infra</td>
<td>G2: <em>What is area?</em> =&gt; linger</td>
<td></td>
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<tr>
<td>44</td>
<td></td>
<td>Rel-a: <em>know- smallest- (triangle H)</em></td>
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<tr>
<td>45-49</td>
<td>G3: <em>How do we know/prove [Rel-a]??</em> =&gt; filled by Mark and Dina</td>
<td>Rel b: <em>knowing – need to prove</em> =&gt; implicit Rel-c: <em>(triangle H) – half – (square C)</em> Rel-d: <em>[Rel-a] – can’t prove</em></td>
</tr>
<tr>
<td>49-52</td>
<td>G4: <em>Could C be the smallest?</em> =&gt; filled by Dina</td>
<td>Rel-e: <em>(triangle H) – sticks out – (square C) – (parallel bases)</em> Rel-f: <em>(area – (aligned bases))</em> Rel-g: <em>[Rel-a] – because- [Rel-c] – (triangle H over square C)</em></td>
</tr>
<tr>
<td>53-56</td>
<td>G5: <em>is [Rel-g] a proof??</em> =&gt; filled by Teacher</td>
<td>Rel-h: <em>(whole square – twice (triangle H))</em> Rel-i: <em>[Rel-g] – a proof</em></td>
</tr>
<tr>
<td>56-58</td>
<td>G6: <em>how did Dina proved [Rel-f]??</em> =&gt; filled by Kamer</td>
<td>Rel-j: <em>prove- put over</em> Rel-k: <em>prove – superimpose the shapes</em> =&gt; implicit</td>
</tr>
<tr>
<td>58</td>
<td>G7 (→ Gap1): <em>what should we do?</em> =&gt; (implicit); partially filled by Teacher</td>
<td>Rel-l: <em>transparent paper – square grid paper-Dina’s technique – tricks – prove – smaller/larger shapes</em></td>
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(i) Gap 4 is a challenge to Mark's belief that we can't prove that H is smaller than C (line 48; Rel-d). Mark attempts to fill Gap 4 in establishing some conditions for triangle H to be smaller than C (line 48, Rel-f). He tries an inferential relation between the word "area" for the first time at this point and "set in line". The inference is: if the criteria "according to area" (that does not stand fast) was to mean "set in line" (that stand fast in actions), then the conditions for H to be smaller than C would be warranted. Unfortunately for Mark, this inference cannot be sustained in the common ground of meanings privileged by the teacher because this is not compliant with the mathematical culture. The teacher opposes Mark’s relation with another relation (line 46; Rel-e) stemming from a change in the relative positions of the two shapes. The consequence is that in this position, triangle H may not be smaller than square C. Indeed, the order relation between H and C depends on the geometrical objects considered: side length of square C is smaller than base length of triangle H but side length of square C longer than the height of triangle H. These order relations on length are more or less salient depending on the relative position of the shapes in space (parallel bases VS aligned bases of the shape). The teacher uses spatial contingency to increase uncertainty and make the students noticing Gap 4 [Could C be the smallest?]. We have an example of a topogenetical move toward more responsibilities to the students in assessing a relation being made with respect
to the geometrical relationships among the shapes in the primitive milieu managed by the teacher (moving the shapes).

(ii) To fill Gap 4, Dina brings in a new relation that aggregates previous ones into a causal pattern (line 52; Rel-g). This relation ties in discourse the order relation [H is smaller than C] and a new pattern of the relative positions of shapes H and C in space [H over C]. From the researcher's perspective, the statement [H is half of C] is a quantification of the magnitude of the surface area of triangle H with respect to square C as a standard unit. It is an argument to prove that the surface area of H is smaller than the surface area of C. But what is the significance of Rel-g for the students? The mere thing we can do is to track any correlated relations unfolding in the participants' action and check whether these relations stand fast in furthering the activity. In eliciting Gap 5 [is Dina's utterance a proof?], the teacher tries to empower the group with the task to assess Dina's relation with respect to Gap 3 and gap 4. But the word "proof" does not stand fast to the students and Gap 5 cannot be filled by the students themselves. In noticing Gap 6, the teacher introduces a relation (Rel-i; line 56) that implicitly fills Gap 5. The responsibility given to the students is too high about a task (identifying a proof) that is out of reach of the students. The teacher subtly moves back toward high position, to manage the answers and reduce uncertainty. In this topogenetical move, the focus on "what" is proved is drifted towards "how" it is proved.

The mesogenesis is a series of ephemeral and situated encounters which are co-elaborated by the participants. Certain words and actions stand fast in these encounters. We contend that the student's experience in which epistemic relations is made is an individuated experience stemming from a collective experience made of joint actions and shared meanings. "Smallest", "largest", "set on a line" are recognized with respect to the collective experience shared in the group about ranking objects activities like sticks, blocks, etc. What stand fast describes the reference from the participants perspective. But what stand fast hic et nunc in an encounter may also remain contingent for the students. Indeed, observing the students' action in the subsequent encounters of this lesson show that the students go on with making superimpositions of shapes but they hardly draw conclusions from them, in terms of larger/smaller shapes. From this, we understand that "Dina's technique" does not make sense in the collective experience as a generalized content (or knowledge) which in turn, could be a resource for each student in further activity. Furthermore, since the word "area" was not related to this rule, the concept of surface area does not earn significance in action. From the succession of gaps highlighted by the teacher and featuring a fine-grained chronogenesis, we understand that the knowledge of the mathematical concepts (what is surface area?) and practices (knowing in mathematics is proof-based) is prioritized over the relations effectively made by the students in the encounter (ordering the shapes by sight estimation, comparing side lengths, finding a numerical ratio between the shapes). Of course, the
teacher takes up these relations (cf- Gap 6), but these relations construed in situated action remain contingent from the student's perspective. Each time that a gap is filled, it is replaced by a new one prompted by the teacher without enabling the students to proceed with the new relations made.

CONCLUDING REMARKS

We now discuss the implications of our analysis for the (re)conceptualization of PEA tools in the JATD. PEA empowered us with high resolution tools to analyse the content being learnt in classroom joint action from the participants’ perspective. From this analysis it can be seen that the basic concepts of the lesson (proof and area) do not make sense to the students as part of the purpose of ordering according to size, without being reformulated as different kinds of doings (putting side by side, superimpose). This demonstrates that the teacher, in joint action with the students, would need to construe relations between those terms which the students are supposed to learn (proof, area) and those that stand fast (putting side by side, superimpose). Here there is no evidence of this in action, and so there is no evidence to the teacher that students have learnt what proof and area means in terms of habitual ways of talking and acting mathematics. [MTC]-geneses augment the analysis in directing analysis on the overall joint action about how the relations sum up (mesogenesis), the role of the student vis-à-vis the teacher (topogenesis), and how the learning progresses over time (chronogenesis). The MTC-analysis offers a means to analyse the social control on the meanings to be learnt from an institutional perspective. Thus, the power of this combined analysis lies in its ability to elucidate the meaning-making process from the participants’ perspective (PEA-analysis), and combine this with an analysis of the consequences of the teacher managing the learning progression in certain, specified ways (MTC-genesis analysis). If teaching is organizing "signs" (words, symbols, constellation of artefacts) to make someone learn a content, learning involves making sense of such signs and forms in order to act adequately with respect to the sign-maker/organizer's purposes. But learning cannot be unilaterally controlled by the organization of signs in a teaching design, however genuine it may be. Learning is contingent on the experience of the learner and on the haphazard sequence of events developing in the joint action of the classroom (Hamza, 2010). Meanings arising in encounters are not "controlled" at their source (in the mind of the students) by the teacher but they are shaped in discourse according to a collective process of selection, aggregation and social validation and so needs close empirical examination.

REFERENCES


