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Assertion, Inference, and Consequence*

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1. Consequence and Inference

For some authors, at least in some contexts,¹ the distinction between *inference* and *consequence* is minimal. An inference can then be regarded as an ordered pair $\langle \Gamma, \phi \rangle$, where Γ is a set of sentences or propositions and ϕ is a sentence or proposition.² And then an inference $\langle \Gamma, \phi \rangle$ can be said to *valid* just in case ϕ is a consequence of Γ (analogously for *logically valid* and *logical consequence*).

For some other authors, the distinction between inference and consequence is of crucial importance. This is in particular the case in the intuitionistic-friendly proof-theoretic tradition, including Dag Prawitz, Per Martin-Löf, and Göran Sundholm. Here it is stressed that an inference is an *act* in which a cognitive subject advances from judgments of premises to a judgment of the conclusion. It is clear this was also Frege's view. Frege used the vertical *judgment* stroke to indicate that the content expression following the stroke is *asserted* and therefore judged to be true.³ I shall follow this second tradition in separating inference from con-

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¹See e.g. Thomason 1970, 92. This excellent book was my textbook for learning elementary logic.

²There are a number of possible complications to this simple picture, most of which I will ignore in this paper. I shall not, for instance, be concerned with multiple-conclusion inferences. I shall later be concerned with inferences where premises may be entire arguments

³For a recent exposition of Frege's view, the significance of the judgment stroke, and logic as con-

sequence, and I shall treat public inferences as acts in which the conclusion is asserted (categorically or conditionally; see below). But even if we agree that this distinction is important, it does not immediately follow that it matters much for logic. For over and above individual inferences as acts we also have inferences as *types*: any two subjects who infer ϕ from Γ make inferences of the same type. Then there is a relation that holds between a set of sentences or propositions, and/or arguments (in case we have premises that are arguments), on the one hand and a sentence or proposition on the other, just in case there is an inference type with the former as premises and the latter as conclusion. For ease of exposition we can identify the inference type with this relation. We may then ask whether this inference type is [logically] valid just in case ϕ is a [logical] consequence of Γ . If this does in fact hold, the study of valid inference is virtually the same as the study of consequence.

On Martin-Löf's conception, the two relations come apart immediately, for they do not relate the same kinds of content. The consequence relation holds between propositions while the inference relation holds between *judgments*. In the simplest case, a judgment (type)

$$a : A$$

says that a is a proof of A , or belongs to the type A . A is a proposition. So regardless of other differences between the relations of consequence and inference, their proper contents are not the same.⁴ In the present context I shall nevertheless proceed as if the contents can be treated as the same, at least for the reason that this more easily allows me to focus on other differences.

When the difference between inference and consequence is emphasized, the crucial difference, or at least one crucial difference, between the holding of the logical consequence relation and the logical validity of an inference is that whereas the former is a matter of purely semantic relations, the latter concerns the *means* or *method* for advancing from premises to conclusion. In a very general sense of *method* we want to say that a subject makes a *correct* inference only if she applies

cerning inference, see Smith 2009. Sundholm (1998) points out that this was also Bolzano's perspective.

⁴See e.g. Martin-Löf 1987; Martin-Löf 1996.

some adequate method for moving from premises to conclusion. It is not enough that the conclusion *in fact* is a consequence of the premises, for the subject might have selected this conclusion just by luck. That is, it might be by luck that the subject picked a conclusion that in fact is a consequence of the premises. In analogy with Plato's requirement that knowledge must involve something more than true belief, a correct inference must involve something more than inferring something that in fact follows from the premises. The requirement of a method corresponds to the requirement of justification in the case of knowledge.

To fix notation: use

$$(\mathcal{C}) \quad A_1, \dots, A_n \models B$$

to say that B is logical *consequence* of $A_1, \dots, A_n$. Similarly, let's use

$$(\mathcal{V}) \quad A_1, \dots, A_n \vdash B$$

to say that the *inference* from $A_1, \dots, A_n$ to B is logically *valid*.

In the case of $(\mathcal{C})$ we mean that it is logically guaranteed that the conclusion is *true* provided the premises are true. In the latter case we mean that it is logically guaranteed that an *assertion* of the conclusion is or *would be* correct provided assertions of the premises are or *would be* correct. That is, there is a method such that it is logically guaranteed that if assertions of the premises are correct, and the subject infers the conclusion from the premises by this method, then the conclusion is correct as well.

For the concept of logical consequence, it is natural to follow the traditional idea of varying the *interpretations* of the non-logical vocabulary (which we assume is given). I shall leave the concept of an interpretation primitive, rather than e.g. give a model theoretic reconstruction in the Tarskian tradition (cf. Tarski 1936). We have for each i a relation

$$A_1, \dots, A_n \models_i B$$

that holds iff B is a consequence of $A_1, \dots, A_n$ under interpretation i . I shall count B as a consequence of $A_1, \dots, A_n$ under interpretation i iff B is true under i if $A_1, \dots, A_n$ all are true under i , whatever the world is like (or “in all possible worlds”).

Then we say that B is a *logical* consequence of $A_1, \dots, A_n$ iff it is a consequence of $A_1, \dots, A_n$ under all interpretations of the non-logical vocabulary:

$$(\mathcal{J}) \quad A_1, \dots, A_n \models B \quad \text{iff} \quad \text{for all } i, A_1, \dots, A_n \models_i B$$

When we turn to validity of inferences we need to relativize both to interpretation and to method. We consider a set M of *methods* for inferring. I shall say that a method m *supports* an inference just in case that inference is in accordance with m , or justified by m in some way. In case m simply is an inference *rule*, then m supports an inference just in case it is an instance of the rule schema. We shall require of a method m that it has an unbounded class of possible instances, and that it be *decidable* whether a particular inference is an instance or not of a particular method m . Without such a requirement, the concept of validity can easily collapse into the concept of consequence.

For each method $m \in M$ we have a relation

$$A_1, \dots, A_n \mid_i^m B$$

which holds if B , under interpretation i of the non-logical vocabulary, can be validly inferred from $A_1, \dots, A_n$ by m . We shall say that B can be validly inferred from $A_1, \dots, A_n$ by method m iff B is true if $A_1, \dots, A_n$ all are true, whatever the world is like (or “in all possible worlds”) *and* it holds that m supports the inference.⁵ From here we can go on to vary the interpretation: An m -inference is *logically* valid if valid under all interpretations:

$$(\mathcal{V}_i) \quad A_1, \dots, A_n \mid^m B \quad \text{iff} \quad \text{for all } i, A_1, \dots, A_n \mid_i^m B$$

And finally, we can quantify out the method: An inference is logically valid iff for some m , it is a logically valid m -inference.

$$(\mathcal{V}_m) \quad A_1, \dots, A_n \vdash B \quad \text{iff} \quad \text{for some } m \in M, A_1, \dots, A_n \mid^m B$$

On this picture, what distinguishes consequence from validity is the require-

⁵We could also consider the possibility that it depends on what the world is like whether m supports the inference or not. This is reasonable when we consider material forms of inference, which depend on empirical regularities, but is not of significance when we consider logical validity.

ment of a method. In formal logic, the method is taken from a deductive system; it is an axiom, or a basic or derived formal rule of inference. In this case, for the cognitive subject, it is enough to have learned the deductive system, and to deliberately employ it. No further justification can be required.⁶

When we move outside formal systems of deduction, the situation changes. The subject may still learn or adopt some particular rule of inference, but now the rule isn't valid by stipulation. The subject must somehow on her own be convinced that the rule is valid. What is the nature of the justification that the subject needs in order that her assertions be correct when she makes them on the basis of an inference according to a rule she has accepted?

At this point, we move into the epistemology of logic. In the rest of this paper I shall contrast and discuss two perspectives on method justification. One perspective is rather *internalist*, in the epistemological sense that justification is internal to the psychology of the subject. This perspective has recently been developed by Dag Prawitz. The other perspective, which I shall propose, is more *externalist* in character. I shall first set out Prawitz's views and briefly discuss them. Then I shall summarize my own views on assertion, and extend them from assertion to inference. Finally, the two perspectives will be compared.

2. Prawitz on inference and grounds

In a recent paper Dag Prawitz (2009) raises the question how a subject acquires *grounds* for a judgment by means of an inference. Judgments are mental acts. Having grounds for a judgment is connected both to *knowing* that which is judged and to *correctly asserting* it in a public act. As Prawitz does, I shall leave the question of knowledge to the side, since the concept of knowledge itself is such a big and complicated topic. But although Prawitz does not explicitly discuss speech acts, it will facilitate comparison with the contrasting view, which explicitly concerns assertion, if the two qualifications of judgment and assertion can be connected. I

⁶If the description above is understood abstractly, it should be neutral between e.g. classical and intuitionistic logic. It should be acceptable to an intuitionist provided *truth* is understood as *existence of proof*, *interpretation* is taken as *choice of rule system for atomic formulas*, and the logical particles of the meta-language themselves are understood intuitionistically. See e.g. Prawitz 1985; 1987.

propose the following principle,

(Gr) An assertion is adequate, or *correct*, iff the speaker has adequate *grounds* for what is asserted.

Prawitz leaves out the qualification ‘adequate’ when speaking of grounds, and I shall do so as well: grounds, in the current sense, are adequate grounds. Prawitz’s discussion is restricted to deductive inference, where adequate grounds are conclusive grounds, and so by ‘grounds’ Prawitz does mean conclusive grounds. I shall also take for granted that when something is a ground for a judgment, it is equally well a ground for an assertion where that judgment is made public. We can then connect having grounds with correctly asserting by means of (Gr). As far as I know, Prawitz himself would endorse (Gr).

Prawitz sets up the problem as follows. A person has grounds for the premises, the inference is valid, and the person gets grounds for the conclusion. How? The question is what ingredient is missing in the following inference:

- (a) There is a valid inference J from judgment A to a judgment B
- (b) The agent P has grounds for A
- (c) ?
- (d) Hence, P has grounds for B

It is not enough that J is *in fact* valid, for if P has no awareness of its validity, P does not have grounds for B . What is the missing premise (c)?

Prawitz first considers the option

- (c_k) The agent P knows that the inference J from A to B is valid.

This is quickly rejected. One reason is that it will lead to a regress: if we first have to establish the validity of an inference before we can legitimately use it, and this establishing itself proceeds by inference, we will never be able make any inference at all.⁷ At the opposite end one might require simply

⁷Prawitz also observes that this regress is different from the Lewis Carroll paradox, which rather entails that (c_k) isn’t *sufficient* for getting grounds for the conclusion.

(c) P infers B from A .

Whether this is enough, according to Prawitz, depends on what it is to infer. It must amount to more than exhibiting some kind of public behavior, such as saying ‘ A , hence B ’. For P may be challenged to present his grounds for B , and if he can’t defend the inference he must retract his assertion of B .

Then what is it to perform an inference? On Prawitz’s view, we need a conception of the validity of an inference according to which a person with the required level of understanding gets grounds for the conclusion from the grounds for the premise when the inference is valid. This condition isn’t met by model theoretic or modal conceptions of validity. Prawitz says

Why should the additional fact that the inference is truth-preserving also when the content of the non-logical expressions is varied be relevant to [the] question whether the agent can see that the proposition affirmed in the *actual* conclusion (where the content is not varied) to be true? The same can be said of validity defined in terms of necessary truth preservation, if the necessity is understood ontologically. Why should the fact that an inference is truth preserving in *other* possible worlds help the agent to see that the proposition affirmed in the conclusion is true in the *actual* world? It is difficult to see how anything but *knowledge* of this fact could be relevant here (and if knowledge is assumed, it is sufficient to know that the truth of the propositions in the premisses materially implies the truth of the proposition in the conclusion – i.e. no variation of content is needed). Therefore, there seems to be little hope that one can find an appropriate condition (c) when validity of inference is defined in the traditional way (Prawitz 2009, 7-8).

As an alternative, Prawitz proposes that validity should be characterized in terms of *operations on grounds*. To *perform* an inference, according to Prawitz, is to perform an operation on grounds: we perform an operation on the grounds for the premise, and the result is a ground for the conclusion. These twin ideas obviously fit well together. If an inference (type) is valid, then there is an operation that actually transforms grounds for the premise to a ground for the conclusion. And the subject who performs the right operation on grounds for the premise apparently also thereby comes into possession of a ground for the conclusion.

What are these operations on grounds? One example is the operation of *conjunction grounding*, $\&G$:

($\&G$) α is a ground for the conjunction $p \& q$ if and only if $\alpha = \&G(\beta, \gamma)$ for some β and γ such that β is a ground for p and γ is a ground for q .

We are not told exactly what this operation amounts to (not e.g. that it is the operation of pair formation), only that such an operation exists.

($\&G$) is an example of an operation where the arguments are *closed* grounds. Some inferences, however,⁸ are made from assumptions, and may also be made with sentences that contain variables (open singular terms), and hence do not express propositions. To cover these cases Prawitz also introduces what he calls *open* or *unsaturated* grounds (Prawitz 2009, 11). These are contrasted with *closed grounds*. In an extended sense both are called grounds (although of course only closed grounds can be conclusive). Prawitz gives this general characterization of open grounds (with respect to assumptions):

An unsaturated ground is like a function and is given with a number of open argument places that have to be filled in or saturated by closed grounds so as to become a closed ground. Something is a ground for an assertion of A under the assumptions $A_1, A_2, \dots$, and A_n if and only if it is an n -ary unsaturated ground that becomes a closed ground for A when saturated by closed grounds for $A_1, A_2, \dots$, and A_n . Writing $\alpha(\xi_1, \dots, \xi_n)$ for the unsaturated ground and $\alpha(\beta_1, \dots, \beta_n)$ for the result of saturating it by closed grounds β_i for A_i , the condition for $\alpha(\xi_1, \dots, \xi_n)$ to be a ground for A under the assumptions $A_1, A_2, \dots$, and A_n is thus that $\alpha(\beta_1, \dots, \beta_n)$ is a closed ground for A (Prawitz 2009, 11).

An example of an operation that closes unsaturated grounds is *implication grounding*, $\rightarrow G$. It applies to a 1-ary unsaturated ground $\beta(\xi^p)$ for judging q to be true under the assumption that p is true (that the ground is typed by the assumption is indicated by the superscript). The result of applying this operation to the open ground $\beta(\xi^p)$, which is $\rightarrow G(\beta(\xi^p))$, yields thus a closed ground for $p \rightarrow q$.

⁸This holds on Prawitz's view (which I share), in the tradition from Gentzen. On the views of Frege and Martin-Löf, all inferences are categorical.

In this perspective, an inference (type) from premises $A_1, \dots, A_n$ to a conclusion B is *valid* iff there are grounds $\alpha_1, \dots, \alpha_n$ for the premises, and a grounding operation ϕ that applies to these grounds such that $\phi(\alpha_1, \dots, \alpha_n)$ is a ground for B (Prawitz 2009, 15).

The operations characterized correspond closely to the operations on proofs in the Brouwer-Heyting explanations of the logical constants, as later developed by Prawitz and Michael Dummett to concern *direct* or *canonical* proofs.⁹ There, a canonical proof of a conjunction is a pair of proofs, of the respective conjuncts. A canonical proof of an implication $p \rightarrow q$ is a function that when applied to a proof for p gives as value a proof of q .

A main point of Prawitz's discussion of grounds, however, is that it does not solely apply to formal systems of deduction, but to informal deductive reasoning as well. Then a question arises concerning the conditions under which a speaker is in possession of grounds. Prawitz writes:

It remains to say something about what it is for an agent to be in possession of a ground for the conclusion. As already said above, it means basically to have made a certain construction in the mind of which the agent is aware, and which she can manifest by naming the construction. Regardless of whether the construction is only made in the mind or is described, it will be present to the agent under some description, which will normally contain descriptions of a number of operations. It is presupposed that the agent knows these operations, which means that she is able to carry them out, which in turn means that she is able to convert the term that describes the ground to canonical form. Furthermore the agent is presupposed to understand the assertion that she makes and hence to know what kind of ground she is supposed to have for it. It follows that when an agent has got in possession of a ground for a judgment by making an inference, she is aware of the fact that she has made a construction that has the right canonical form to be a ground for the assertion that she makes (Prawitz 2009, 18).

The crucial part is the last two sentences: The agent is “presupposed to understand the assertion that she makes and hence to know what kind of ground she is

⁹See Dummett 1975, Prawitz 1974.

supposed to have for it". How does this differ from the rejected condition that the agent knows that the inference performed is valid? It appears to differ in that the object of knowledge is that the operation is of the right kind. Is this a crucial difference? It may be claimed that this particular knowledge of the appropriateness of the operation is primitive, not inferential. But even if this is right, it involves three controversial assumptions: first, that the meaning of logical concepts are constituted by grounding operations, second that a subject who understands an expression of that concept knows what operation constitutes that meaning, and third, that a subject appears to perform a non-elementary inference correctly either doesn't in fact make the inference correctly or else is able to break it down to inferences that are elementary. That these assumption are controversial does not, of course, entail that they are false. I shall return to this question in the concluding section.

3. The information account of assertion

In this section I shall not so much discuss the concept of assertion and various accounts of assertion as simply set out the account I myself favor. In the following section I shall extend it to cover inference.

The information account can be summarized as follows:¹⁰

(IA) An utterance *u* is an assertion iff *u* is *prima facie* informative.

To spell it out we need to say something about information, about informativeness, and about what it is to be *prima facie*. First a couple of words about giving information. Given sufficient nomic regularity, effects typically give information about their causes, such as smoke about fire, or about regular other effects of their causes, such as a barometer about an imminent change of weather. But we can also get information about facts where there is no causal connection: a pocket calculator can give information about mathematical facts. In order to cover all these cases we cannot analyze information just in terms of causal processes or regularities. Instead we shall take the performance of the general *method*, not its applica-

¹⁰The account was first sketched in Pagin 2008 and has been worked out in detail in Pagin forthcoming 2010.

tion in a particular case, as the basis.

We operate with a function μ between a domain E of possibly information-giving events or states of affairs and a domain S of propositions, about the truth of which information is given. We also have a *method* or type of process π that is to secure information by selecting or generating elements in E . Then we can use the idea that among the elements in E , all or almost all elements e in E that are generated or selected by means of π are such that $\mu(e)$, a proposition in S , is true.

In the calculator case, we have a large domain of possibly information-giving states of affairs consisting of the cartesian product of the set of possible input displays and the set of possible output displays, and under the intended function, most possible input-output pairs will be mapped on false mathematical propositions. The circuitry processing method π is such that for a given input display (such as the sequence $\langle '5', '+', '3' \rangle$), it pairs a particular output display ('8'). The intended function μ will map that pair on a mathematical sentence or proposition. Those pairs that are in fact generated by π will, in almost all cases, be mapped on true propositions. Hence the circuitry process π reliably selects, with respect to the relevant function μ , those pairs that are mapped on true propositions.

We shall say in general that a method or process type π is μ -reliable, meaning that π reliably selects events or facts in the domain of μ whose μ images are true, i.e. *independently* true. This means that if $e \in \pi$ and $\mu(e) = q$, then it is not also the case that π causes q to be true. Making use of this idea we could define the information relation along the following lines:

- (I) An event e gives *information* that q iff it is true that q and there are disjoint domains E and S , a surjective function $\mu : E \rightarrow S$ and a μ -reliable process type π such that $e \in \pi$ and $\mu(e) = q$.

Some information-giving states or events are special because they *represent* the state they give information about. For instance, we can get visual information about a state of affairs from a picture that represents it, auditory information from a sound or sound production representing an earlier auditory event, and information of any kind from a linguistic utterance whose content represents what it gives

information about. I shall say that the events or states of the latter kind, those that represent what they give information about, are *informative*. In these cases we can think of μ as a *semantic* function.

To say that an utterance is *prima facie* informative is to say that it has certain properties such that *by default* speakers and hearers take utterances with those properties to be informative. That is, by default, the hearer takes the proposition to have been asserted in part *because* it is true. There are good reasons not to require very sophisticated theoretical attitudes by speakers and hearers. Instead, the idea is to relate credences of speakers and hearers to the production of, and reaction to, utterances, respectively. For an utterance that has the typical characteristics of assertions (mainly grammatical and intonational), the default reaction of a hearer is to increase credence in the asserted proposition, as well as to end with a credence above indifference (i.e. with at least a weak belief). The condition for a hearer can be set out as

$$\begin{aligned} \text{(BF)} \quad & \text{i) } C_X(p|A^X(p)) > 0.5 \\ & \text{ii) } C_X(p|A^X(p)) > C_X(p) \end{aligned}$$

Here $C_X(p|A^X(p))$ is the conditional credence of hearer X in proposition p , given that it has been “asserted” to X , a sentence that means that p has been uttered to X with the surface characteristics typical of assertions.

For the speaker, on the other hand, what matters is the strength of belief in the proposition *just before* the assertion is made, not after. We take *outright belief* to be credence strong enough to act on, and in particular strong enough to motivate a disposition to assert. This suggests the following simple condition for the intention-forming mechanism of the speaker:

$$\text{(IF)} \quad A_Y(p) \rightarrow C_Y(p) > \epsilon_Y,$$

Here ϵ_Y is the threshold for outright belief of Y . So (IF) is the condition that the credence of speaker Y in proposition p , if Y makes an assertoric utterance of a sentence that means that p , is above the threshold for outright belief.

In both cases, speaker and hearer, there may be circumstances that overrule

the default conditions. The speaker may want to make it just *seem* that the utterance is informative, and usually in such a case there is a desire to mislead the hearer in some way or other. The hearer may suspect the speaker of dishonesty, may have independent reasons to believe in what the speaker says, or to disbelieve what the speaker says, or to doubt the reliability of the speaker. In all such cases, the default conditions are overruled. It is still true in these cases that the utterance *seems* informative, in having the surface properties typical of informative utterances.

These are the basic elements of the information account. The next task is to extend the idea of informativeness to inference.

4. Assertion and inference

The basic idea of extending the account to inference is to take assertion *adequacy* with respect to inference validity

$$(\mathcal{V}) \quad A_1, \dots, A_n \vdash B$$

to amount to *informativeness*. That is, an inference is *valid* just in case it is guaranteed that if assertions of the premises are informative, then so is an assertion (by the same speaker, in the same context) of the conclusion.

Intuitively, if speaker Y asserts premises $A_1, \dots, A_n$ and each of these assertions is in fact informative, which entails that the premises are true, and from these premises Y correctly infers B , then the assertion of B is informative as well. The inference is guaranteed to be truth preserving. So if Y has reliably selected the premises because of their truth, and Y has selected B on the basis of $A_1, \dots, A_n$, then Y has selected B as well because its truth.

If we abstract away the truth of the premises, we can still say that the assertion of the conclusion is *conditionally* informative: if the inference is valid, the assertion of the conclusion is informative *on condition* that assertions of the premises are informative. Note that this is a property of an assertion of the conclusion that does not depend on the status of assertions of the premises.

Conditional informativeness, in this sense, is intuitively a property of an assertion that is based on an inference from asserted premises, in case the inference is valid. In this context, however, we are interested in *defining* validity, and so move in the opposite direction. Therefore, we shall first define what it is for a *inference method* to be information preserving:

(IPM) An inference method m is *information preserving* in an inference from $A_1, \dots, A_n$ to B , under an interpretation i ,

$$A_1, \dots, A_n \mid_i^m B,$$

iff it holds that, whatever the world is like, if assertions of $A_1, \dots, A_n$ are informative, and B is inferred from $A_1, \dots, A_n$ by m , then so is the assertion of B .

Since informativeness entails truth, if an assertion of the conclusion is informative, it is also true. It does not immediately follow that any information preserving method is also truth preserving, since it is left open that if the premises are true but assertions of them are not informative, the conclusion can fail to be true. It might indeed be the case, for some method of inference, that it is truth preserving only in those special cases where the premises themselves are informatively asserted or assertible. A case in point could be the inference

$$(1) \quad \frac{A}{\text{It is informatively assertible that } A}$$

This inference is clearly not truth preserving. An assertion of the premise of (1) may be true but not informative, and in that case the inference may have a true premise and a false conclusion. Is it information preserving? It might seem so, since it is guaranteed that if the premise assertion is informative, then the conclusion is true. But being true is not enough: the conclusion must also be informative. This means that there is a reliable method π that selects an utterance of the conclusion of (1) just in case the utterance of the premise is informative. But if the method is nothing but inferring the corresponding conclusion from any premise

that has been asserted, then the method is *not* reliable. Hence, the conclusion is not informative, and the inference not information preserving.

In order to avoid this outcome, we must restrict the *application* of the method, and hence also of the inference: it must be applicable only in those cases where the premise assertion in fact *is* informative. But that is not possible either. Remember that we required of a *method* in this context that it be applicable to an unbounded number of instances, and that it be *decidable* whether or not it applies to a particular case. From an externalist point of view, there is reason to think that the property of *being informative* is not decidable. The analogy in epistemology is the KK principle: if a subject knows that *A*, then she knows that she knows that *A*. From an internalist perspective in epistemology, the KK principle has some plausibility, but from an externalist perspective, it is rejected: that your belief that *A* is reliable does not entail that your belief *that* your belief that *A* is reliable is *itself* reliable. Since informativeness is not a decidable property, we cannot restrict the applicability of the rule to premises that are informatively asserted or assertible.

The conclusion generalizes. If any rule of inference is information preserving but not truth preserving, then, since the informativeness of the conclusion entails the truth of the conclusion, the truth of the conclusion will depend on the informativeness of the assertion of the premise, or of one of the premises. Hence, to get a reliably true conclusion, the inferer must reliably select informative premises. Because of this, we can repeat the reasoning above for the general case. Without a method for selecting informative premises, the rule of inference cannot be information preserving if it is not truth preserving.

We move from information preservation to *logical* information preservation in the usual way, by varying the interpretation:

(LIPM) An inference method *m* is *logically information preserving* in an inference from $A_1, \dots, A_n$ to *B*,

$$A_1, \dots, A_n \mid^m B,$$

iff it holds that, for any interpretation *i*, $A_1, \dots, A_n \mid_i^m B$.

Could there be a method m that is logically information preserving but not logically truth preserving? Here is an argument that this cannot happen. Assume that m is information preserving under any interpretation of the non-logical vocabulary. Assume further that m is not logically truth preserving, i.e. that under some interpretation i , in some world w , $A_1, \dots, A_n$ are asserted by Y , they are true but B is false, and that in world w' that differs only insofar as the assertions of $A_1, \dots, A_n$ by Y are informative, under the same interpretation i , and in w' B is true. But in that case, the truth of B in w' must itself depend on the informativeness of the assertions of the premises. And then the truth of B in w' under i cannot depend only on the interpretation of the logical vocabulary. Hence, there is another interpretation i' of the non-logical vocabulary under which $A_1, \dots, A_n$ are asserted and true, but B is false. But then the assertion of B under i' isn't informative. And hence m isn't logically information preserving. Contradiction.

The conclusion of the argument is that logically information preserving methods are *sound*: when an inference is supported by a logically information preserving method, the conclusion is a logical consequence of the premises. So there are good informal reasons for thinking that the idea of logical information preservation is a reasonable approach to logical validity of inference.

As in Prawitz's approach by reference to having grounds, we take the last step by existential quantification:

(LV) An inference from $A_1, \dots, A_n$ to B is logically *valid*,

$$A_1, \dots, A_n \vdash B,$$

iff there is a method m such that $A_1, \dots, A_n \vdash^m B$.

If now it holds that there is a set of methods M such that a) it is decidable for any relevant inference whether it is supported by a method $m \in M$, and b) whenever B is a logical consequence of $A_1, \dots, A_n$ there is a logically information preserving method $m \in M$ that supports the inference from $A_1, \dots, A_n$ to B , then we also have *completeness*. Since e.g. second-order logic is known not to be axiomatizable, completeness is not to be expected.

So far we have only considered the case where the premises are asserted. To have a complete account we also need to cover the case where an inference is made on the basis of premises that are merely *assumed*, including cases where the premises have free variables. Here I shall leave aside the case of free variables.

In case we infer B from premises $A_1, \dots, A_n$ that are not asserted but merely assumed, the conclusion isn't asserted either. What kind of speech act is the utterance of B ? It is not an assertion, but neither a mere assumption. It is natural to characterize it as "conditionally assertoric".

We call standard assertions "categorical". Then we add to the concept of a categorical assertion the concept of a *conditional assertion*. The idea is that the utterance of a sentence expressing an inferred proposition is a conditional assertion in the sense that the utterance is a categorical assertion *on condition* that the premises have been asserted as well. That is, in the inference

$$\frac{A_1, \dots, A_n}{B}$$

B is conditionally asserted. It is also categorically asserted in case $A_1, \dots, A_n$ are themselves categorically asserted. If at least one of the premises is not categorically asserted, then neither is the conclusion.¹¹ The present idea of validity is still applicable. For a method m is information preserving just in case *whenever* the premises are informatively asserted, then if the conclusion is asserted as inferred by m , it is informatively asserted.

We also need to cover the case when assumptions are discharged. To give the basic idea, consider a simple case where an assumption C is discharged at an inference step j :

$$\frac{[C]_j \quad \vdots \quad \frac{A}{B}}{B} j$$

Here A is derived in a number of inference steps, but depends on C . So A is conditionally assertoric. C is merely assumed, so A is not categorically asserted. In the

¹¹This concept of conditional assertion should be kept distinct from another idea by the same name. According to that conception, a sincere utterance of a conditional, *if p, then q*, is a conditional assertion in the sense of constituting a categorical assertion of the consequent in case the antecedent is true. Cf. Belnap 1973, Milne 1997, Pagin 2007.

discharging step j , the premise is in a sense the entire derivation of A from C . We can characterize B as conditionally assertoric in this sense: B is assertoric on condition that: A is assertoric on condition that C is assertoric. Since it does hold that A is assertoric on condition that C is assertoric, the condition for B is met. Hence, B is categorically assertoric.

Analogously, an assertion of B is informative on condition that: an assertion of A is informative on condition that an assertion of C (by the same subject in the same context) is informative. This condition is met when the inference steps are valid, and hence an assertion of B is informative, e.g. in case B is $C \rightarrow A$.

To complete the sketch we would need to generalize to an arbitrary number of premises, as well as to premises with free variables. But I shall leave that out.

5. Two perspectives

From a formal point of view, the present account in terms of informativeness and Prawitz's account in terms of having grounds might look similar. Indeed, all the *operations* suggested by Prawitz could be accommodated as *methods* in the present framework. The opposite does not hold, however. I shall consider two problems (or what I take to be problems) for Prawitz's approach, and also one problem, or what looks like a problem from the internalist perspective, for the present approach.

The first problem for Prawitz concerns the requirement of competence on ordinary speakers. Consider a commonly accepted and intuitive form of inference, like the *disjunctive syllogism*:

$$\frac{A \text{ or } B \quad \text{not } B}{A}$$

This is a form of inference that has been performed by just about anyone who has ever used disjunctive statements (I think). It is not an elementary form of inference in the sense that it only depends on one main operator/particle. Rather, two logical particles are involved, *or* and *not*. So it is different from the ordinary introduction and elimination rules for disjunction and negation respectively, which are elementary in this sense.

The disjunctive syllogism can be justified, classically as well as intuitionistically, from the elementary rules for disjunction and negation. Should we say that only the elementary inferences are *constitutive*, in Prawitz's sense, of the propositions expressed by sentences of disjunctive or negative form? The consequence would not be so good for the logical practice of most ordinary speakers. Few speakers other than those with logical training (I think) are able to justify the disjunctive syllogism from the elementary rules. So they cannot break this complex inference down into more elementary steps. It would follow that their application of the disjunctive syllogisms are not really correct inferences. They perform an operation that is neither constitutive nor justified from constitutive operations. On this picture, such speakers lack grounds, in Prawitz's sense, even if they have grounds for the premises.

The alternative would be to say the disjunctive syllogism really *is* constitutive for disjunctive and negative propositions. But then these propositions are allowed to be overdetermined by constitutive rules. Where do we draw the line between constitutive and non-constitutive rules? Do the rules become constitutive by being commonly used? What shall we do when similar situations arise with other rules? It seems to me hard to answer such questions, and maybe there isn't even any principled answer to be given. But then it would seem that Prawitz faces the dilemma of either having to count most ordinary reasoners as not having grounds in many of their logically correct inferences or having to agree that whatever is commonly accepted, and correct, is constitutive.

Related questions concern principles that are classically but not intuitionistically valid, like the rule of double negation elimination. What should one say, in particular from an intuitionistic point of view, about those who rely on it, since it cannot be justified in a way that is acceptable to intuitionists?

The second problem concerns the accessibility to reflection that is part of the internalist picture. Recall that Prawitz writes

It follows that when an agent has got in possession of a ground for a judgement by making an inference, she is aware of the fact that she has made a construction that has the right canonical form to be a ground for the asser-

tion that she makes (Prawitz 2009, 18).

This requirement has the consequence that if a speaker is aware that she has grounds for the premises, and requires a ground for the conclusion by means of the inference, she will also be aware that she has a ground for the conclusion. But now there seems to be a dilemma. One horn of the dilemma is the result of a thoroughgoing internalism. On that position, if a speaker does have a ground, she is also aware, and hence knows, that she has a ground. But then we have reason to think that there is an operation of *awareness grounding* (call it *Aw-G*) that consists in making what one is aware of in the preceding inference explicit. If such an operation exists, then the argument

$$(2) \quad \frac{\frac{A}{\text{There is a ground for } A} \text{ Aw-G}}{A \rightarrow \text{there is a ground for } A} \rightarrow G$$

should be regarded as valid. The first step employs the awareness grounding operation. Given the requirement of awareness on the part of a subject who makes an inference, this inference step is well motivated. It is not of course a *logical* step. Whether or not it can be regarded as deductive requires a more general characterization of this concept. The last inference is just implication introduction, here labeled with the name of the implication grounding operation.

All in all, given the requirement of awareness, the argument (2) seems valid, by the internalist standards: whenever there is a ground for *A*, there is also a ground available for the claim that there is a ground for *A*. The conclusion is clearly false, however, since it is not the case that whenever *A* is true, there is a ground for *A*. So, with a rule for awareness grounding, the system of inference rules isn't sound. This seems to me to be a problem in general for an internalist approach to inference.

The other horn of the dilemma results from giving up the claim that speakers who have grounds are aware of them. But this step invites externalism: whether a speaker has a ground may depend on external circumstances of which she is not aware. Exactly how Prawitz should avoid both horns by finding some middle position is not so clear.

In the opposite direction, the internalist can accuse the externalist of having too lax standards for adequacy of assertions. Suppose Y believes in some complex rule of inference R from $A_1, \dots, A_n$ to B that is in fact logically truth preserving, although Y is unable to prove it and nobody else has succeeded or even tried. Inferring by R is then in fact a reliable inference method, and so it is information preserving. Therefore, applying R to premises $A_1, \dots, A_n$ that have been informatively asserted renders Y 's assertion of B informative as well. It can now be objected that Y 's assertion of B is not intuitively adequate, since Y does not have the appropriate grounds for the assertion. It is based on an unjustified principle.

There certainly is some force to this objection, but I think the objection can be met. I would readily concede that Y in this case lacks the personal justification needed for having made a correct assertion, i.e. an assertion that is correct in the subjective as opposed to the objective sense.¹² I would say, however, that when it comes to inference *validity* it is the objective sense of correctness that matters, and in the present approach this would amount to information preservation. Y 's assertion of B is indeed informative; the conclusion B is reliably selected by Y for asserting at least in part because of its truth. It is reliably selected in part because Y relies on a rule that is logically truth preserving.

Still, I would agree that the assertion isn't correct in the subjective sense, since Y lacks the proper personal justification. I agree with Prawitz that in this case, Y lacks a *ground* for the assertion. But I would conceive of what is missing in terms that are internal to the present approach. I would say that Y hasn't selected R *itself* by means of any method that is itself reliable in selecting inference methods. Then, although Y 's assertion of B is informative, the corresponding assertion

(3) My assertion of B is informative

is *not* informative. In order that it be informative, R must be information preserving, and an assertion by Y that R is information preserving would not be informative, since Y hasn't employed any reliable method for selecting R . That one has selected the (information preserving) rule by means of a reliable method seems to

¹²On the distinction between subjective and objective correctness, cf. Dummett 1976, 48-52.

me to be a condition that is both necessary and sufficient for subjective correctness. For instance, it seems sufficient in the case of the disjunctive syllogism: even though it is not an elementary form of inference, the normal speaker has an intuitive grasp of its validity from the mere general understanding of disjunctive and negative propositions. To the extent that intuitive grasp is a reliable method, this is sufficient for being personally justified in employing the rule.¹³ The upshot is that an externalist conception such as information preservation is better adapted to the concept of inference validity, and that the idea of reliable selection can be employed on the level of accepting methods of inference in order to characterize the notion of subjective justification.

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¹³Of course, given the kind of logical mistakes ordinary speakers tend to make, like the fallacy of *affirming the consequent*, one may wonder whether intuitive grasp is a reliable method for rule selection, but that is another question.

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