Finnish Mathematics Teaching from a Reform Perspective: A Video-Based Case-Study Analysis

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This article offers a qualitative analysis of videotaped mathematics lessons taught by four teachers in a provincial university city in Finland. My study is framed not only by Finnish success on Programme for International Student Assessment (PISA) but also by the objectives of current mathematics education reform, which are consistent with PISA’s goals of measuring mathematical literacy. The analyses indicated that conceptual understanding and procedural fluency were addressed by all four teachers. However, adaptive reasoning, strategic competence, and the development of a productive disposition appear rarely. I observed few occasions where students were invited to solve authentic problems.

In this article I examine the teaching of mathematics using four classrooms in provincial Finland as case studies. It is set against the background of repeated Finnish success on the Programme for International Student Assessment (PISA), which has spawned such international interest that over “the past few years, more than 100 delegations from all over the world have beaten their way to the doors of the ministry of education in Helsinki to find out how the Finnish system works” (Crace 2003). Theoretically, the study is framed against the objectives of mathematics education reform, in general, and Jeremy Kilpatrick and colleagues’ (2001) five strands of mathematical proficiency, in particular. These align well not only with recent curricular reforms in Finland but also with PISA’s key objective, with respect to mathematical literacy, of assessing students’ application of mathematical knowledge and skills to authentic settings, both within mathematics itself and more broadly (Adams 2003). The analyses indicated that two of the five strands, conceptual understanding and procedural fluency, were addressed regularly by all four teachers, although adaptive reasoning, strategic competence, and the development of a productive disposition appeared rarely. With respect to PISA expectations, few occasions were observed in which students were invited to solve authentic problems, that is, problems typically located in a hypothetical real world from which students are expected to extract relevant
information, formulate that information mathematically before solving the problem, and relating the solution back to the original context. Finally, the data alluded to a didactic tradition, largely incommensurate with reform ambitions, whereby teachers created opportunities from which students inferred meaning that seemed unlikely to explain Finnish PISA success.

Reform in Mathematics Education

Reflecting similar shifts internationally, Finnish mathematics education has been undergoing systemic reform for several decades. The reform movement grew from an increasing awareness internationally that traditional teaching, with its emphasis on rote acquisition of procedural skills, was allowing students to leave school not only with little understanding of mathematics and its processes but also with inadequate skills for a changing labor market (Hiebert 1999). Traditional mathematics teaching is typically construed as emphasizing mathematical knowledge as facts and skills, which are learned with little understanding, frequently by rote, and then applied to familiar problems (Lloyd 1999). The outcome of such practices has been described by Giyoo Hatano and Kayoko Inagaki (1986) as routine expertise. Routine experts may be competent and efficient within a limited scope but lack the resources to “search out new solutions to problems when it is appropriate or fail to notice that a problem is significantly different from problems solved in the past” (Martin et al. 2005, 258).

Reform mathematics teaching focuses on the development of deep conceptual knowledge and connections between concepts as the means by which procedural skills are made sensible (Hiebert 1999). Reform mathematics teaching not only encourages learners to solve nonroutine problems but also provides students with opportunities for working together and for cooperating on finding the solution to those problems (Lloyd 1999). It encourages students to make links within mathematics itself and between mathematics and the real world (Cady et al. 2007). Reform mathematics valorizes the development of adaptive expertise (Hatano and Inagaki 1986), whereby learners have a sufficient and connected underlying understanding to enable them to solve unfamiliar problems (Heinze et al. 2009). They understand why the procedures they use work, can modify them, or even invent new ones. Adaptive experts are able to regulate their own learning as something dynamic rather than static (Martin et al. 2005; Verschaffel et al. 2009).

Didactically, and typically drawing on social constructivist theories of learning, reform classrooms reflect a shift from traditional teaching whereby “the teacher is in complete control and the students’ only goal is to learn operations to get the right answer” (Stipek et al. 2001, 214). In reform classrooms, teachers acknowledge that learners actively construct rather than passively receive knowledge (Cady et al. 2007; Koustourakis and Zacharos 2011) and, “through interaction, are able to challenge one another’s con-
structions in ways that facilitate the construction of increasingly shared and powerful knowledge” (Beswick 2005, 43). In such circumstances, the teacher becomes an “organiser, an enabler, and a coordinator of the communication between the pupils in the classroom” (Koustourakis and Zacharos 2011, 372). Moreover, reform-oriented teaching acknowledges that “students’ learning behaviors [are] not only . . . highly situated but also to be fundamentally constituted by interactions among (meta)cognitive, conative, and affective processes” (Op ’t Eynde and Turner 2006, 362).

In summary, reform mathematics teaching differs from traditional mathematics in several ways. It acknowledges that procedural skills are best taught alongside, possibly even subordinated to, an emphasis on deep conceptual knowledge. It requires students to reason mathematically as a key element of problem-solving as an integrated element of the learning experience. It understands that problem-solving of this nature necessitates a positive attitudinal disposition and a willingness to engage with others. Seen in these ways, the objectives of reform mathematics are well summarized by Kilpatrick et al.’s (2001) strands of mathematical proficiency, which they summarize as follows:

1. **conceptual understanding**, which refers to the student’s comprehension of mathematical concepts, operations, and relations;
2. **procedural fluency**, or the student’s skill in carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately;
3. **strategic competence**, the student’s ability to formulate, represent, and solve mathematical problems;
4. **adaptive reasoning**, the capacity for logical thought and for reflection on, explanation of, and justification of mathematical arguments;
5. **productive disposition**, which includes the student’s habitual inclination to see mathematics as a sensible, useful, and worthwhile subject to be learned, coupled with a belief in the value of diligent work and in one’s own efficacy as a doer of mathematics.

**Reform in Finnish Mathematics Education**

Traditional Finnish teaching, whereby “the teacher talks more than two-thirds of the time, and the pupils give short responses,” has changed little for more than 50 years and has vexed policy makers for several decades (Carlgren et al. 2006, 313). A commission of enquiry initiated in the late 1980s led to a decentralized curriculum emphasizing “problem solving and application of mathematical knowledge” (Kupari 2004, 11). However, despite such innovations, a government commissioned external review of Finnish teaching and learning, found little change with respect to reform-related practice. They observed “whole classes following line by line what is written
in the textbook, at a pace determined by the teacher. Rows and rows of children all doing the same thing in the same way whether it be art, mathematics or geography. We have moved from school to school and seen almost identical lessons, you could have swapped the teachers over and the children would never have noticed the difference” (Norris et al. 1996, 29).

A more recent initiative aimed at improving the quality of Finnish mathematics teaching (Kupari 2004) provided substantial in-service opportunities for teachers to develop the understanding and skills necessary for overcoming the traditional dominance of procedural competence over conceptual understanding (Desimone et al. 2005). However, Finnish teachers continue to be slow to incorporate systemic expectations of mathematical problem solving (Pehkonen 2009). Recently, university mathematicians, despite PISA success, have voiced concerns that curricular reforms have compromised the intellectual integrity of mathematics. They argue that emphases on equity and preparation for a world beyond school may have secured PISA success but are incompatible with preparation for higher mathematics (Astala et al. 2006; Tarvainen and Kivelä 2006).

Explicit and Implicit Instruction

Although framed against the expectations of mathematics education reform, this study did not set out to test hypotheses. However, while the above addressed issues previously identified as pertinent to the study, unforeseen phenomena emerged from the analysis necessitating post hoc examinations of specific literature. In the following, two such phenomena are considered; the first relates to distinctions between explicit and implicit instruction, and the second relates to the triadic exchange of classroom discourse.

On the one hand, explicit instruction typically “involves the overt, teacher-directed instruction of strategies, including direct explanation, modeling, and guided practice in the application of strategies” (Manset-Williamson and Nelson 2005, 61). From the particular perspective of mathematics teaching, a typical explicit-instruction lesson comprises a review of the previous lesson, an introduction to the new topic or explanation of how to solve the problem in question, a period during which students practice problems either as a group or on their own, and, finally, the public sharing of solutions (Kroesbergen et al. 2004). The second phase, in particular, necessitates teachers modeling explicitly each step of the process by means of scaffolded instruction (Carnine et al. 1994). Implicit instruction, on the other hand, typically occurs when teachers provide students with tasks from which meaning may be inferred with appropriate teacher support and guidance (Koike and Pearson 2005; Manset-Williamson and Nelson 2005; Martínez-Flor and Fukuya 2005). However, in the context of science teaching in general and the nature of science in particular, implicit instruction assumes that by doing science, particularly hands-on, inquiry-based activities, students will acquire an un-
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derstanding independently of teacher support (Khishfe and Abd-El-Khalick 2002; Clough 2006). Thus, implicit instruction is conventionally construed as the management of incidental learning (Radwan 2005), although there is variation, according to subject tradition, with respect to whether teachers play explicit roles in this process.

The Triadic Exchange

Finally, the triadic exchange, described as either IRF (initiation, response, feedback) or IRE (initiation, response, evaluation) (Nassaji and Wells 2000), typically accounts for around 70 percent of all classroom discourse (Wells 1993). Shown to play a key role in the coconstruction of the knowledge that a culture regards as significant (Nassaji and Wells 2000), it generally takes the form of a teacher question (initiation), followed by a student or students being invited to answer (response), before the teacher offers feedback on or an evaluation of the answer (feedback/evaluation) (Wells 1993; Hellerman 2003; Smith and Higgins 2006). However, despite concerns that closed initiating questions may lead to short factual responses and students’ guessing what the teacher is thinking (Wilson and Haugh 1995), a consensus has emerged that the educational significance of the IRF sequence lies less in the questions teachers ask than in the ways they manage feedback (Smith and Higgins 2006). For example, several studies have shown the importance of intonation when teachers affirm student responses; they show affirmation by repeating not only students’ words but also their intonations (Hellerman 2003; Skidmore and Murakami 2010). More profoundly, teachers who make connections between students’ contributions or allow student ideas to determine the direction of subsequent activity engendered more successful learning than teachers who do not (Wells 1993; Smith and Higgins 2006).

In light of the research supporting teaching reforms, this article presents an initial analysis of Finnish mathematics teaching. It is framed against Kilpatrick et al.’s (2001) five strands of mathematical proficiency to facilitate our understanding of how four provincial mathematics teachers’ practices resonate with reform objectives. In so doing, I highlight what appear to be didactic practices unique to the Finns and consider, in relation to some of the factors discussed above, the cultural construction of Finnish performance on PISA and the Trends in International Mathematics and Science Study (TIMSS).

Methods

The data derive from the European Union-funded Mathematics Education Traditions of Europe (METE) project, whose broad aim was to examine how teachers conceptualize and present mathematics to students in the age range 10–14 years in Flanders, England, Finland, Hungary, and Spain.
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The primary data set comprised video recordings of four or five successive lessons on topics agreed upon by project colleagues as representative of not only their countries’ curricula but also the transition of school mathematics from concrete and inductive to abstract and deductive. Focusing on sequences of lessons not only facilitated a clearer understanding of how teachers develop topics but also made the presentation of showpiece lessons unlikely. The four topics, each taught by teachers construed locally as effective, were percentages in grades 5 or 6, polygons in grades 5 or 6, polygons again in grades 7 or 8, and linear equations in grades 7 or 8.

Of course, four teachers are unlikely to be representative of all teachers in a given population. However, there are characteristics of Finnish teacher education that allow some confidence with respect to their standing both locally and nationally. First, for several decades, a master’s degree has been an essential prerequisite for teaching in a comprehensive school (Laukkanen 2008; Tuovinen 2008). Second, Finnish preservice teacher education, offered in only eight universities across the country (Krokfors et al. 2011), draws on a common research-based framework focused on facilitating beginning teachers’ solutions of pedagogical problems (Byman et al. 2009; Krokfors et al. 2011). Third, all teachers were working in partnership with their local teacher education department and therefore would have had “to prove they are competent to work with trainee teachers” (Sahlberg 2011, 36). Finally, the university, to which they were attached, had been nominated as a Centre of Excellence in Mathematics Teacher Education by the Higher Education Evaluation Council. Thus, acknowledging that all four teachers were not only products of such training but also qualified to work with trainee teachers, it seems reasonable to assume that they were representative of Finnish expectations of good practice.

The METE team used a single camera focused on all utterances made by the teacher and as much boardwork as possible. In all cases, the tripod-mounted camera was placed near the rear of the room. Teachers wore radio microphones while discretely placed static microphones captured as much student-talk as possible. After filming, each digital tape was compressed and transferred to CD for coding, copying, and distribution. The first two lessons of each sequence were transcribed and translated into English, enabling the production of subtitled recordings that would facilitate analysis by any project member, irrespective of nationality.

Of course, the five strands of mathematical proficiency represent learning outcomes that are not obviously inferable from videotapes focused on teachers’ actions and utterances. However, to facilitate the quantitative analyses of its data, the METE team undertook a week of live observations in each country to develop an analytical framework that all project colleagues could operationalize (Andrews 2007). Five of the inferable learning objectives included in this framework—conceptual knowledge, procedural knowledge, structural...
knowledge, problem solving, and mathematical reasoning—while not explicitly focused on the five strands of mathematical proficiency, serendipitously allowed the identification of teaching exchanges that addressed them. Also, since the English subtitles of two lessons in each sequence allowed for colleagues from two countries to code each set of lessons, Cohen Kappa coefficients were calculated to evaluate cross-national intercoder reliability. The results of this process, with Kappa coefficients typically exceeding the 0.75 typically assumed indicative of reliability, indicated that METE project colleagues were equally skilled at identifying teaching exchanges focused on the strands of mathematical proficiency.

With regard to their qualitative analyses, I viewed all videos several times, first those with subtitles and then those without, to get a feel for how the different lessons played out. After this the first video of a randomly selected sequence was scrutinized again several times for evidence of the five strands of mathematical proficiency. As incidents were observed, the lesson transcript was annotated to explicate the evidence warranting the identification of the particular strand observed. With each viewing, the annotations were refined and decisions clarified. This same process was repeated for each of the subtitled videotapes. Finally, the analyses and their respective videos were shown to two Finnish colleagues working in the author’s institution who confirmed the veracity of the interpretations made against their experiences as learners in Finland.

Results

The four Finnish teachers, here given pseudonyms, taught in different comprehensive schools in a provincial university city with a large teacher education department. All four, three male and one female, were involved in teacher education activities as part of the university’s program. Thus, as indicated above, all four were strongly grounded in the norms and values of mathematics education within their respective community. The males, Markku, Jussi, and Matti, were all in their early thirties, while the female, Sirkka, was in her late forties.

The analyses indicated that all four teachers focused considerable attention on the development of their students’ conceptual understanding and, to a slightly lesser extent, their procedural fluency. Evidence addressing either adaptive reasoning or strategic competence was rare. Consequently, the following focuses first on the development of students’ conceptual understanding, second on procedural fluency, and, finally, the remaining strands of proficiency. Interestingly, as the analyses unfolded, it became apparent that Finnish students’ learning of mathematics was informed by more than just the ways in which their teachers addressed the five strands of mathematical proficiency. For example, an interesting characteristic of case-study teachers’ practice was a tendency not to offer feedback on students’ responses to
publicly posed teacher questions. Also, despite superficial appearances to the contrary, teachers rarely made explicit the meaning they expected their students to take from public exchanges.

The Development of Students’ Conceptual Understanding

Jussi, in his introduction to linear equations, addressed the conceptual basis of equations in a variety of ways including, for example, exposition and whole-class reflection on generative tasks or exercises. He began by writing and repeating orally the following: “Definition: An equation is two expressions denoted as being of equal magnitude.” He then operationalized his definition by writing six statements on the board, some of which were equations and others not, that were publicly examined after a short period of private reflection. The first, the number 5, led to the following:

JUSSE: Is example a an equation? Think about the definition. [pause] Joonas?

JOONAS: No, because there is only one expression.

JUSSE: Yes. There is only one expression, 5, which is only a number.

In a similar vein, having written, in silence, the following, “Equation solving means finding out when an equation is true or false,” Jussi introduced his students to the idea that equations can be always true, conditionally true, or false. He asserted, but wrote nothing on the board, that “Solving (an equation) means thinking about when an equation is true. The other possibility is that it might never be true. So, then it would be false. So, it is settling between those two possibilities, which one it is.”

He operationalized these definitions and that of conditional truth, by returning to the six statements discussed earlier. For example, when asked whether “5 + 3 = 7” was true or false, Aliisa replied that it was “false,” to which Jussi commented, “yes, it is always false; we use the term identically false equation.” He then attended to \( x^2 = 8 \), asking, “What should \( x \) be to make \( f \) true—\( x \) squared equals 8?” After some prompting, Joonas volunteered “the square root of 8,” to which Jussi added, “yes, if \( x \) is the square root of 8, this is true. So, this equation is a conditional equation. This equation is true, when \( x \) is the square root of 8.”

While one might argue that the above exchanges, based on pedagogically different definitional presentations, were unambiguously focused on students’ conceptual understanding, they were, as far as the METE data extended, atypical for both Jussi and his colleagues. First, Joonas’s response incorporated an uninvited explanation. This was unusual in case-study lessons as teachers rarely sought or offered explanation or clarification. Second, with respect to the first exchange, Jussi offered an explicit definition, which was presented both orally and in writing for students, by means of an implicit invitation, to copy. While all four teachers offered many implicit opportunities
for students to copy, they rarely made explicit the function of the text students were expected to copy. Third, Jussi appeared to engage in acts of feedback commensurate with the IRF sequence. In particular, his feedback during the second exchange allowed him to refine the definition relating to equation solving. Interestingly, however, he did not expand on either *identically false equation* or *conditional equation*, leaving students to make whatever sense they could. Moreover, he neither offered nor sought comment on the falseness of \( 5 + 3 = 7 \), appearing to leave his students to confirm the meaning themselves. Thus, even within exchanges with unambiguous intentions, elements of ambiguity appeared to remain.

A second example, drawn from the introduction of Markku’s first lesson on percentages, highlighted the more conventional manner in which case-study teachers appeared to manage student responses. He wrote, very slowly and in silence, “Prosentti (=sadasosa),”\(^1\) before commenting that “there are three ways to denote one-hundredth. Write these three ways in your notebook; write them in an equation form.” After about a minute, Salla, having been invited to the board, wrote without hesitation but also slowly and deliberately, “\(1/100 = 1\% = 0.01\).” Markku then thanked Salla before asking his students for examples of one-hundredth from everyday life. Part of the ensuing conversation went as follows:

**MARKKU:** Where have you met one-hundredth? Liisa?

**LIISA:** The circumference of the earth compared with the circumference of the sun.

**MARKKU:** Yes, Simo?

**SIMO:** One hundred percent fat.

**MARKKU:** Yes, but it can also be one percent fat. Leena?

**LEENA:** I have not actually seen one-hundredth, but I have seen percentages in election results.

**MARKKU:** Yes. For instance, there are percentages in alcoholic drinks. They contain more than one percent.

Markku offered no explicit definition, although it could be argued there was an implicit definition in Salla’s tripartite equation. His reactions to students’ responses, at least to an outsider, were ambiguous. For example, when thanking Salla for her equation, it was not clear whether her response had been accepted as accurate as he neither offered nor sought further comment. His reaction, yes, to the responses of Liisa and Leena left an observer wondering what he intended his students to take from the exchange as, in both cases, he followed his utterance with a change of direction unrelated to the contributions made. Indeed, the shift of attention from election percentages to

\(^1\)Percent (=hundredth).
alcoholic drinks seemed particularly strange, especially when viewed against the ages of his students.

Such ambiguity appeared not uncommon in the discourse of case-study lessons. For example, Matti, early in his second lesson on polygons, drew his students’ attention to a triangle formed from two intersecting rays and a segment joining arbitrary points on the rays. He invited them to “give names to the vertices . . . something we have done a million times.” However, despite his prompts, students seemed reluctant to offer suggestions, leading his declaring, “I’ll do it again so that we can get started.” He rose from his seat and labeled the vertices A, B, and C, before returning to his seat and commenting:

MATTI: And now we have a closed broken line . . . name the closed broken line.
TANELI?

MATTI: ABCA.

TANELI: Correct.

Taneli’s contribution went without further comment, creating, it seems, two obvious ambiguities. Matti neither sought nor offered any clarification as to what he meant by a closed broken line or what Taneli meant by his labeling and why, for example, the A should have been repeated or whether other vertices could have been chosen as starting points. This latter point seemed particularly pertinent as later in the same exchange Matti invited his students to name the triangle and accepted ABC as correct. Thus, while he may have wished to present a clear distinction between the closed broken line ABCA and triangle ABC, he made no obvious attempt to make that distinction explicit.

Sirkka, at the start of her second lesson, began by questioning her students with respect to the material covered the previous day. The conversation went as follows:

SIRKKA: Yesterday we learned about the concept of twin circles. What does the concept of twin circles mean? Jari?

JARI: Are they identical circles?

SIRKKA: Yes, identical circles and . . . Leena?

LEENA: They have different centers.

SIRKKA: Different centers but . . . Simo?

SIMO: The same radius.

SIRKKA: The same radius.

Nothing further was said as Sirkka shifted attention to the next task, which was the construction of several pairs of twin circles, centered on the same two points, to establish that the intersections of each pair of circles lie on
the perpendicular bisector of the segment connecting the centers. She invited her students to “mark two points at the center of the page” with a “space between of about 6 centimeters.” She did the same on the board and labeled her points \( K_1 \) and \( K_2 \), before adding that “the next step is to draw twin circles, with centers \( K_1 \) and \( K_2 \) and radius 4 centimeters.” Simultaneously, she wrote, in capitals, the heading and the instructions, while her students copied. While students constructed their own twin circles, Sirkka did the same on the board. This process, with another two identically managed repetitions, took nearly 15 minutes and led to the construction of six points of intersection.

**SIRKKA:** Now, let’s have a look at the points of intersection. Do you notice anything?

**STUDENT:** They are all in a line.

**SIRKKA:** All in a line. At least it looks like that, doesn’t it?

**SIRKKA:** Does everyone agree?

**STUDENT:** Yes.

**SIRKKA:** All are on the same straight line.

Sirkka completed the exchange by instructing her students, and doing the same herself, to draw the line through the intersections and label it \( n \).

Her approach, which could be construed as a form of discovery, yielded the intended straight line. However, at no point did she discuss why the intersections were collinear. Indeed, it could be argued that her seeking a collective agreement approximated a proof by consensus, which emerged as a not uncommon feature of case-study lessons. Like Jussi, she engaged in IRF sequences, although in every instance she repeated the student response with, essentially, no further feedback. Her instructions were explicit, although she neither offered nor sought clarification as to their comprehensibility. Her use of the board, the ways in which she always undertook the same construction as her students, and her slow writing in capitals of instructions reflected not only a meticulous attention to detail but the implicit expectation that her students would copy what she had written.

Yet despite such detailed attention, several ambiguities permeated the exchanges. There was ambiguity with respect to the intersections lying on the same line. There was ambiguity in the proof by consensus, although it is always possible that such practices reflect a tradition, with which students have become familiar, whereby teachers only accept the popular vote when they know the majority has made a correct judgment. There was ambiguity in the status of the objects under scrutiny, in particular the way in which the line, \( n \), was drawn indicated that it could have been construed as a segment.
The Development of Students’ Procedural Fluency

Of course, concepts alone do not make mathematics, and a substantial proportion of time was spent on addressing students’ procedural skills. Jussi, during the second lesson on equations, having introduced the balance scale as an embodiment for solving equations with the unknown on both sides, initiated the solving of the collectively constructed equation, \(3a - 5 = 8a + 7\):

**JUSSI:** So, what could we do to make the equation? Henri.

**HENRI:** Divide by 3.

**JUSSI:** Yes, take 3\(a\) away from both sides [Jussi wrote \(-3a\) against the equation and performed the calculation.]

**JUSSI:** So, here [pointing to the left-hand side] it is \(-5\) and there [pointing to the right-hand side] it is \(5a + 7\). [He writes \(-5 = 5a + 7\).]

**JUSSI:** What could you do next to simplify this? So, tell me then, what are you supposed to do? [He points to the solution of the earlier equation and pauses for several seconds; Katja volunteers.]

**KATJA:** Take 7 away.

**JUSSI:** Yes. Let’s take 7 away from both sides. [Jussi wrote \(-7\) to the right of the equation.]

**JUSSI:** So, here it is \(5a\), and when you take away \(7\) from \(-5\)? Miika, what do you get?

**MIIKA:** 12.

**JUSSI:** Yes. 12. [He writes \(-12 = 5a\) on the board.]

The exchange continued and the solution, \(a = -12/5\), was eventually found. In this scenario Jussi engaged in a seemingly typical IRF routine focused on particular procedural characteristics of equation solving. There were other familiar elements within the exchange, not least of which was the annotation of the solution to the right of the each line of the argument. However, there were ambiguities. Henri’s reasonable initial suggestion to “divide by 3” was reformulated as “yes, take 3\(a\) away from both sides,” leading to at least two interpretations. Did Jussi mishear Henri’s suggestion and respond as though he had said “subtract 3\(a\)”? Somehow this seems unlikely, as *jakaa kolmella* (divide by 3) is unlikely to have been heard as *vähennetään kolme a* (subtract 3\(a\)). Alternatively, did Jussi hear the suggestion but decide that an acknowledgement would interfere with his intended procedure? In this latter respect, the reformulation of a response into something entirely different appeared to be a not unfamiliar aspect of lessons. Also, in repeating student responses and asserting the outcomes of these suggestions, opportunities to offer or seek clarification were left unacknowledged, leaving students to infer for...
themselves why, for example, taking $3a$ from both sides would yield the equation $-5 = 5a + 7$ or why subtracting 7 would leave $-12 = 5a$. Thus, the whole exchange incorporates a substantial number of ambiguities with respect to student comprehension. That is, throughout the exchange, Jussi paid no attention to the justification for any element of the procedure, leaving much for his students to infer.

During his second lesson on polygons, Matti, having encouraged his students to label the vertices in a diagram, invited them to count the number of triangles. This led to some disagreement as to the correct number, with students offering everything between 5 and 9. Matti asked, “How many got 8?” and, on seeing the number of hands raised, concluded, “I think 8 is the right number.” He proceeded to point out “that our triangles have names . . . using those, we could check the number of names there are,” before concluding, “let’s not check and just move on.” A similar task led to his concluding, “let’s not check because it would take an eternity to write them all on the board.”

In these exchanges, Matti seemed to encourage his students to adopt procedures, exploiting labeled vertices to identify unique triangles for the purpose of counting, without making explicit what he meant. Moreover, on every occasion where the opportunity to invoke the procedure arose, he elected very publicly to ignore it. In so doing, he left ambiguous his expectations and, as is discussed below, may have acted in ways counter to the development of his students’ productive disposition. Also, as did Sirkka earlier, Matti invoked a form of proof by consensus and, in so doing, created further ambiguity for his students; why was the answer 8?

During his second lesson on percentages, Markku commented that “from a fraction we can get a decimal by dividing the numerator by the denominator.” He invited his class to undertake the calculation for $1/6$ in their notebooks before writing the division on the board and initiating the following:

MARKKU: Well, who can start the division? Sanni.

SANNI: 6 goes into 1 zero times. [Markku writes zero above the division.]

MARKKU: Then we put the point there [Markku inserts decimal points into the calculation.]

MARKKU: Pirkka, you continue.

PIRKKA: For the sake of the point we have to add zero. 6 goes in 10 once. [Markku writes in the 1 and then performs the calculation that yields the remainder of 4, which he writes into the calculation before bringing down the zero to create 40.]

MARKKU: Markus, continue.
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Markku: 6 goes into 40 6 times. Again 4 remains there. [Markku adds 6 to the growing number on top and adds in the remainder 4 and another zero.]

Markku: I asked for an accuracy of one-hundredth. What is the answer? Aku?

Aku: 0.17.

Despite the involvement of four students, Markku completed the procedure with no commentary, neither offering nor seeking clarification. Thus, despite what appeared to be an act of explicit teaching, the manner in which it was conducted left observers with much to infer. For example, what was the significance of the added zeros? What was the meaning of the remainders, what did Aku mean, and what did Markku expect his students to infer when he said the answer was 0.17 to an accuracy of one-hundredth, and why did Markku terminate a recurrent decimal at the point he did?

The Development of Students’ Adaptive Reasoning, Strategic Competence, and Productive Disposition

I close this section with an examination of the ways in which the teachers in this case study addressed the remaining three strands of mathematical proficiency: adaptive reasoning, strategic competence, and productive disposition. All four teachers appeared to address, in similar ways, their students’ productive disposition. For example, as indicated above, they encouraged frequently, and for the most part implicitly, their students to take notes. In so doing they appeared to facilitate their students’ engagement, at least as far as note-taking would guarantee. For example, in behavior typical of her colleagues, Sirkka implicitly encouraged her students, through meticulous and deliberately slow straight-edge and compass constructions, to do the same. Indeed, whenever she wrote on the board, her students, seemingly unprompted, did the same. In a similar vein, Jussi presented, orally and in writing, a definition, which students were asked to use in deciding whether various statements were equations. Importantly, the manner in which the definition was presented—slowly and in writing on the board—created an implicit opportunity for his students to take notes. Indeed, the students in all four classrooms were frequently observed to take notes unbidden by their teacher as part, it would seem, of the everyday classroom discourse with which they were familiar. In so doing, students were inferring meaning from whatever interactions had occurred in preparation, it seemed, for later use. Interestingly, apart from the very rare occasions when a teacher offered a definition explicitly, none of the teachers ever engaged with the content of the notes taken by their students. In this regard, despite what could be construed as an implicit tradition with regard to students’ productive disposition, there was little teacher-initiated control over the quality of the notes recorded.

However, there were also occasions in all four sequences where teachers’
actions militated against students’ productive disposition. For example, Matti’s actions in relation to his triangular counting tasks seemed to prevent rather than promote a productive disposition. Similarly, Sirkka began her third lesson by asking if anyone had experienced problems with their homework. No one volunteered any difficulties, leading her to comment that, “We do not need to discuss it. We can go directly to a new topic.” Several assumptions seem to underpin such a response: that asserting they had no difficulties meant they had no difficulties, that their solutions were correct with no need of evaluation, and that nothing would be gained by comparing and contrasting solution strategies. Across all 20 lessons, evidence of teachers supporting the development of either adaptive reasoning or strategic competence was scarce. Reasoning, if encouraged at all, was through closed questions requiring factual answers, which generally went unevaluated. Indeed, the rarity with which case-study teachers evaluated student responses seems indicative of a tradition in which reasoning, if it is encouraged at all, is not encouraged explicitly.

On the few occasions where strategic competence was addressed, it was managed ambiguously and obliquely. For example, Sirkka posed several non-routine problems but rather than encourage students to engage with them individually, she always solved them on the board contemporaneously with her students. Jussi’s introduction of algebraic equations, involving an equation deliberately chosen to create cognitive conflict for his students, was managed in ways that offered little scope for students to make decisions for themselves. Even when they were encouraged to make suggestions, there was a tendency, as with Henri’s contribution to the solution of $3a - 5 = 8a + 7$, for them to be sidelined in ways that left students to infer for themselves the reasoning behind their teachers’ decisions.

Discussion

This study was undertaken in order to understand how Finnish teachers of mathematics facilitate their students’ learning in relation to the reform agenda characterized by Kilpatrick et al.’s (2001) five strands of mathematical proficiency. Although developed in the United States, the five strands reflect current Finnish curricular guidelines and so can be construed as forming an appropriate framework for analysis. The data indicate that two of the strands, conceptual understanding and procedural fluency, are addressed regularly by all four teachers in this case study, although adaptive reasoning and strategic competence appeared rarely. Consequently, and acknowledging Kilpatrick et al.’s (2001, 131) assertion that a student’s productive disposition “develops when the other strands do,” little was observed to indicate that teachers attended in observable ways to their students’ productive disposition.

Case-study teachers’ emphases on procedural fluency resonated with earlier studies highlighting the enduring nature of such goals (Carlgren et al.
2006), while the apparent failure of teachers to encourage their students to solve independently nonroutine problems accorded with Erkki Pehkonen’s observation that, after 25 years of systemic invocation and substantial investment in training, Finnish teachers had largely failed to incorporate problem solving into their repertoires (Pehkonen 2009). However, a significant difference between the teachers and their predecessors lay in their major emphasis on conceptual understanding as a learning outcome. In this respect, earlier reform-related ambitions to develop in teachers those understandings and skills necessary for overcoming the traditional dominance of procedural competence over conceptual understanding (Kupari 2004; Desimone et al. 2005) may have achieved partial success. However, the ways in which teachers managed their students’ conceptual understanding, particularly in the absence of any systematic attempts at forging connections, alluded more to the disconnected knowledge of routine expertise than the connected knowledge of the adaptive expert (Hatano and Inagaki 1986). Notions of adaptive reasoning were of such rarity as to be essentially absent. Indeed, several occasions arose in the analyzed lessons, as with Sirkka’s construction of the perpendicular bisector of the chord connecting the centers of two circles and Matti’s management of the solutions of his triangular problems, where consensus was presented as a mathematical warrant. Thus, the evidence suggests that since Norris et al.’s (1996) evaluation of Finnish classrooms, only limited progress toward systemic reform-related objectives has been achieved.

Didactically, there was no evidence of the student-centered activities typically associated with reform-related mathematics teaching. That said, it would be foolish to assume that a failure to encourage student-centered activity necessarily conflicts with reform objectives. For example, Andrews (2003) has highlighted a Hungarian tradition, which predates current reform interests (Varga 1988), in which all lessons are teacher-managed but which incorporate substantial elements of individual and collective opportunities for students to engage with problem solving, both mathematical and realistic, mathematical reasoning, and proof. In other words, at least from a European perspective, there are didactical traditions in which reform-related objectives are satisfied in teacher-centered environments. However, unlike their Hungarian colleagues, case-study teachers were never observed to act in ways that explicitly acknowledged notions of student agency as highlighted by either Finnish curricular reforms (Norris et al. 1996) or the discourse of mathematics teaching reform (Koustourakis and Zacharos 2011).

In addition to a growing awareness of the rarity of reform-related practice came an understanding that teachers’ typical practice could not be described as either explicit instruction or implicit instruction. With respect to the former, teachers were rarely seen to model explicitly each step of the procedures or scaffold the concepts they were teaching (Carnine et al. 1994; Manset-Williams and Nelson 2005). For example, when Jussi defined and opera-
tionalized equations, which involved several explicit elements, he neither offered nor sought clarification with respect to his students’ understanding of the different public exchanges. For example, when discussing the truth of $5 + 3 = 7$, his reaction to Aliisa’s “false” was to comment, “Yes, it is always false; we use the term identically false equation.” In introducing the notion of _identically false equation_, no attempt was made to clarify or seek clarification as to students’ construal of the term, indicating that students were expected to infer for themselves whatever meaning they could. During Markku’s questioning of students’ real-world experiences of percentages, it was not clear from his management of Liisa’s and Leena’s contributions what he expected his class to infer from the exchanges. Both girls had offered perspectives from which meaning was difficult to infer without some form of teacher probing, but this did not happen. Matti encouraged his students to exploit vertex labels when counting triangles but said nothing by way of explicating what he meant at the procedural level, leaving his students to infer the procedural consequences of his invocation. In summary, the typical public exchange seemed premised on an expectation that students would infer meaning from the opportunities they were given.

If explicit instruction was a rarity, then it would be reasonable to assume—particularly in light of the many occasions where students were expected to infer meaning—that implicit instruction might be the norm. In this respect, there were frequent episodes in all four sequences that could be construed as encouraging incidental learning (Radwan 2005). However, if implicit instruction entails that teachers offer appropriate support, guidance, and feedback (Koike and Pearson 2005; Manset-Williamson and Nelson 2005; Martínez-Flor and Fukuya 2005) or, as with the Nature of Science tradition, immersion in hands-on activities from which meaningful inferences might emerge (Clough 2006), then teachers did not engage in implicit instruction as typically understood.

Also, public discourse frequently fell outside conventional triadic sequence norms. On the one hand, teachers’ evaluations took the form of either the single word _yes_ or a repetition of students’ responses. In the former, as in Markku’s initial percentages episode, his _yes_ was the same, irrespective of the appropriateness of the answer, and showed none of the variation of intonation discussed in the literature (Hellerman 2003; Skidmore and Murakami 2010). In a similar vein, during his exposition on the solution of equations, Henri’s legitimate suggestion that Jussi should “divide by 3” was met by, “yes, take $3a$ away from both sides.” In other words, teachers’ use of the word _yes_ frequently left students to infer what was or was not acceptable and could not be construed as an affirmation of accuracy. When teachers repeated student responses, as during Sirkka’s revision of twin circles, they seemed to have an unambiguous affirmative intent (Hellerman 2003; Skidmore and Murakami 2010). On other occasions, as with Jussi’s discussion of
the truth of different equations, affirmation was clouded by additional definitions—identically false or conditional equations—interjected with no explanation. On the other hand, there was no evidence in any lesson of the form of feedback identified as contributing to learning; teachers were never seen to follow up a response in ways that exposed student thinking, compared or contrasted student responses, or opened up new possibilities for discussion (Smith and Higgins 2006). On a few occasions, as with Sirkka’s completion of constructions contemporaneously with her students or Markku’s conversion of a fraction to a decimal by division, teachers were seen to model procedures, but, in every case, no justifications for teachers’ actions were offered.

In sum, not once was a public conversation held that involved more than two people—the teacher and one student—engaged on the same question. Not once was a teacher seen to invite one student to comment on another’s offering or “revoice the students’ comments to clarify the students’ meanings and to focus class-room discussion” (Jacobson and Lehrer 2000, 85). Indeed, student involvement rarely went beyond responding to unproblematic closed questions and even more rarely followed an invitation to explain or justify; Joonas’s explanation with respect to whether 5 is an equation was the only observed instance of a student volunteering an explanation. This lack of mathematical warrant, exemplified in teachers neither offering nor seeking clarification, was typical of all lessons. Thus, despite curricular expectations based on a “theory of teaching which sees the teacher as facilitator and not as the source of knowledge and transmitter of information” (Norris et al. 1996, 23), the evidence indicates that the model of Finnish teaching, dominant throughout the twentieth century, in which “the teacher talks more than two-thirds of the time, and the pupils give short responses” (Carlgren et al. 2006, 313), seems little changed. Significantly, with respect to the teaching of reform-related mathematics, there is little evidence that any of the four teachers operated in ways that would promote students’ mathematical proficiency or explain Finnish students’ repeated PISA success. In short, the discourse of case-study lessons, viewed from various perspectives, seems relatively unchanged from earlier descriptions of an intelligence and emotional wasteland (Carlgren et al. 2006).

Conclusion

In closing, I am drawn to Michael Sadler’s perennially pertinent caution: “In studying foreign systems of Education we should not forget that the things outside the schools matter even more than the things inside the schools,” (Sadler 1900, quoted in Bereday 1964, 310).

The evidence above indicates that the observable characteristics of Finnish classrooms, at least as far as these case studies show, are unlikely to account for repeated PISA success. If there are explanations, then perhaps they are...
more likely to be found in an examination of factors external to individual classrooms. For example, some internal commentators have argued that Finnish educational achievement may be due to the systemic equity embedded in the comprehensive school system (Väljärvi et al. 2002; Sahlberg 2011). Others have suggested it may be due to the quality and extent of the special educational needs provision (Vislie 2003; Halinen and Järvinen 2008) that is typically focused on students’ mother tongue and mathematical skills (Hausstätter and Takala 2011). Indeed, it would be foolish to dismiss the importance of such factors in the construction of Finnish educational achievement, not least because, given an appropriate societal will, they are replicable elsewhere.

However, two other, cultural and nonreplicable, factors may also have played their parts. First, the Lutheran church, through centuries of colonial subordination, taught Finnish people “to read the Bible in their own language” and, in so doing, established the basis not only of “Finnish literature and popular literacy” but also a community with a “strong appreciation for education” (Halinen and Järvinen 2008, 80). Indeed, reading is valued so highly that not only is the Finnish library network among the densest in the world but Finns borrow more books than individuals in any other country (Sahlberg 2007). Second, the Finnish population comprises around 94 percent Finnish speakers. Of the remainder, almost all are members of the economically powerful Swedish-speaking community (Karhunen and Keloharju 2001). However, despite being taught the same curriculum in equally well-resourced Swedish-language schools (Kupiainen et al. 2009), Swedish-speaking Finns perform significantly poorer on PISA and internal Finnish assessments than Finnish-speaking Finns (Niemi and Metsämuuronen 2010). Thus, one concludes that there must be something about the Finnish-speaking community that distinguishes it from others, not least because elsewhere, the economically powerful typically achieve more highly than the rest of the population. In short, while Finnish PISA-related success may in part be due to replicable characteristics of Finnish educational policy making, it may also be due to factors unique to the population. Thus, it would be wise for the many envoys that have visited Finland to understand its success (Laukkanen 2008) to acknowledge not only that replicable policy analyzed independently of a deep cultural analysis is unlikely to prove productive but also that teachers are unlikely to change long-established practices, irrespective of their efficacy, “as long as they do not have to” (Simola 2005, 463).

References


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