Numerical modeling of Svalbard’s ice cover

Case studies and comparison to spatial reconstructions

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Front cover: Svalbard Topography in Lambert Projection.
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Preface

This Master’s thesis is Jorina Marlena Schütt’s degree project in Physical Geography and Quaternary Geology at the Department of Physical Geography and Quaternary Geology, Stockholm University. The Master’s thesis comprises 45 credits (one and a half term of full-time studies).

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Abstract

An ice cover is not only a huge ice block simply moving downwards, following the gravitational force, but consists of different components. The two main components are ice sheets (the grounded part) and ice shelves (the part floating on the ocean). Further, fast-flow features, such as ice streams are also important components not to neglect. Ice in the interior of large ice masses can be treated like highly viscous fluid which behavior is governed by the Stokes equations. The relationship between stress and strain in the fluids is formulated with the Glens flow law. This, for use in glaciology, is a non-linear equation which makes solving difficult and numerically expensive. In the 1950, when ice sheet modeling started, the limited computing power demanded simplifications of the complex dynamics and behavior of ice. The simplest and most commonly used approximation is the so called Shallow Ice Approximation (SIA) which assumes ice sheets to be much broader than thick. As this approach is so crude that it is impossible to connect the ice sheet dynamics (of the SIA) to the ice shelf dynamics (of an approximation called SSA, Shallow Shelf Approximation) there is need for more sophisticated implementations. The mentioned approximations can be upgraded to higher order approximations such that equations can be combined to a hybrid model. Even though, the Full Stokes system of equations is the implementation striven for, the hybrid models are feasible and within the computational practicable.

The reconstruction of the ice sheet history of the Eurasian Arctic has been attempted in numerous studies. The Eurasian Arctic was covered over long time periods with the Eurasian Ice Sheet, which advanced and retreated from the British Isles in the West, over Scandinavia and Svalbard in the North, to the High Russian Arctic in the East. Part of the ice sheet that covered the Archipelago of Svalbard and the Barents Sea was called Svalbard Barents Sea Ice Sheet (SBIS).

In this study, the mathematical background of ice dynamics is discussed at first. The ice code ARCTIC-TARAH which is a Bolin Centre for Climate Research spin-off from the Pennsylvania State University Ice sheet model (PSUI) is used in this study. Selected physical processes of an ice cover and their implementation are explained. This study focuses on responses to changes in ice-ocean interface parameters and interaction with subglacial sediment. For this purpose ARCTIC-TARAH was tested analytically with several input settings. A timescale of 35 000 years is chosen for the test runs aiming for equilibrium conditions. Data-based reconstructions of Svalbard’s ice
cover are compared with the modeled results. Finally, a case study on the deglaciation history of SBIS starting at the Younger Dryas up until present day conditions is performed.

The results of this study show that the simple SIA ice sheet model is not enough when aiming for a realistic ice sheet simulation: at least a coupled system of ice sheet and ice shelf is needed. The results show furthermore that the hybrid model reacts on differences in subglacial basal conditions (i.e. basal sliding coefficients) and behave differently regarding total ice volume and ice thickness. The main achievement of this study is the fairly good agreement between simulated and data-based reconstructions of Svalbard’s ice cover regarding ice extent and ice streams, which supports the applicability of the hybrid model.

However, improvements and developments to be done to the model concern external mass balance control parameters as oceanic melting and calving rate and a refinement of the response to external climate forcing.
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1 Introduction

Considerable effort has been put into reconstructing the ice sheet history of northern Eurasia. Svendsen et al. (2004), Jakobsson et al. (2013) and Ingólfssoon and Landvik (2013) review the achievements made in understanding the Svalbard Barents Sea Ice Sheet (SBIS). As new data becomes available, the SBIS is recognized as a highly dynamic and multi-domed marine ice sheet which is controlled by climate fluctuations, subglacial topography, sediment properties and changes in ocean temperature and circulation. Efforts of reconstructing the Svalbard’s glacial history are ongoing (e.g. Alexanderson et al., 2013, Noormets et al., 2013) and it is now timely to complement the improved data-based reconstructions with numerical simulations with a ‘new generation’ ice sheet model (for earlier numerical modeling of the SBIS/Eurasian North, based on the classical Shallow Ice Approximation approach, see e.g. Siegert et al. (1999) and Siegert and Dowdeswell (2004)).

Furthermore fast-flow features, such as ice streams, but also floating glacier tongues are of special interest. An ice stream which crosses the grounding line begins to float as an ice tongue and the ice begins to calve into the ocean (Rignot and Jacobs, 2002). As ice shelves and floating glacier tongues are affecting the stability of the belonging ice sheets they are of major importance. Processes at the ice margin, at the ice-ocean interface, are governed by thermal fluxes and pressure conditions (Rignot and Jacobs, 2002). These include (re-)freezing of seawater – melt water mixtures on the bottom of ice shelves and changes in ocean water circulation due to cold and dense glacial melt water. But even processes at the ice-sediment interface should be taken into account, as they probably are affecting mass balance and flow velocity of the glacier ice. To improve the understanding of these processes case studies have been undertaken, e.g. investigating the dynamics of single glacier outlets (e.g. Woodward et al., 2003) and the entire ice cap (e.g. Ottesen et al., 2007). It is important to have data-based reconstructions in order to be able to test and improve numerical modeling of ice sheet – ice shelf – systems.
2 Aim of study

In this study a ‘new generation’ ice model is tested analytically with several input settings against data-based reconstructions of the SBIS cover. ‘New generation’ ice models became available roughly a decade ago and first applications to the SBIS (Kirchner et al., 2014) and the Eurasian ice sheet (Kirchner et al., 2011b) has been undertaken. The model used for this project is the ice code ARCTIC-TARAH which is a Bolin Centre for Climate Research spin-off from the Pennsylvania State University Ice sheet model (PSUI) and which was originally designed to model Antarctic glaciations (Pollard and DeConto, 2009, 2012).

Once the model is configured for the Svalbard domain several tests can be performed addressing spatial development of the ice cover over time and glacial dynamics of the ice mass. Aiming for high resolution simulations the appropriate choice of simulation domain, discretization, projection, topography and climate forcing fields is important. A special emphasis is then placed on responses to changes in ice-ocean interface parameters and interaction with subglacial conditions.

The aim of this project is to compare modeled versus reconstructed characteristics of the Svalbard ice cover. This includes ice extent, fast-flow features, ice domes and (de-)glaciation patterns.

The research questions this study addresses are: 1/ Do the simulated ice stream locations coincide with the observed ones? 2/ What is the effect oceanic parameters have on the ice cover? 3/ What is the impact of subglacial bed conditions on ice flow patterns?
3 Background

3.1 Theoretical glaciology and numerical ice modeling

3.1.1 Continuum ice dynamics

Every closed and continuous system has to obey conservation laws as no energy, no mass and no momentum is lost or gained but converted or rearranged. This is founded upon one main demand: independency of the observer/reference system. These criteria are expressed in the conservation laws:

\begin{alignat}{2}
\text{Mass} & : & \rho + \rho \text{div} \mathbf{v} & = 0 \quad \text{(3.1a)} \\
\text{linear momentum} & : & \rho \mathbf{\dot{v}} = \text{div} \mathbf{T} + \rho \mathbf{g} & \quad \text{(3.1b)} \\
\text{internal energy} & : & \rho \dot{\varepsilon} = -\text{div} \mathbf{q} + \text{tr}(\mathbf{D} : \mathbf{T}) + \rho r. & \quad \text{(3.1c)}
\end{alignat}

The variables are \( \rho \) the density, \( \mathbf{v}(x,y,z) \) the velocity, \( \mathbf{T} \) the Cauchy stress tensor, \( \mathbf{g} \) the gravitational acceleration, \( \varepsilon \) the internal energy, \( \mathbf{q} \) the heat flux, \( \mathbf{D} = \text{sym grad} \mathbf{v} = \frac{1}{2} (\text{grad} \mathbf{v} + (\text{grad} \mathbf{v})^T) \) the strain rate tensor and \( r \) the heat radiation. The operators appearing in the equations are the standard differential operators in vector analysis: the divergence (div) is measuring the magnitude of a vector fields source or sink and the gradient (grad) measures the rate and direction of a scalar field. The strain rate tensor \( \text{sym grad} \mathbf{v} \) describes the symmetric component of the gradient of the flow velocity \( \mathbf{v} \). The dot above a symbol stands for the material time derivative of the quantity.

The second law of thermodynamics, stating that the entropy on any closed system never decreases, is complemented to the conservation laws. Those general equations hold for all materials, solids and fluids, and are further adjusted to material specific properties. The resulting equations (called field equations) build a set of equations which possesses the potential to determine all variables uniquely. For ice occurring in ice sheets and glaciers these equations need to be complemented by material specific behavior, such as incompressibility. The incompressibility constraint, \( \dot{\rho} = 0 \), yields in (3.1a):

\[ \text{div} \mathbf{v} = 0 \iff \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0. \quad \text{(3.2)} \]

The Cauchy stress tensor \( \mathbf{T} \) is often decomposed into

\[ \mathbf{T} = -p \mathbf{I} + \mathbf{T}^E, \quad \text{(3.3)} \]

where \( \mathbf{T}^E \) is the deviatoric part of the tensor, \( p \) is the scalar pressure and \( \mathbf{I} \) is the identity tensor. Applying (3.3), the balance equation of
momentum (3.1c) can be rewritten as

$$\rho \ddot{v} = - \nabla p + \text{div} T^E + \rho g,$$  \hspace{1cm} (3.4)

The acceleration $\ddot{v}$ (time derivative of the velocity $v$) in Equation (3.4) is due to the Reynolds number (quotient between internal and external forces) very small and thus neglected in calculations regarding ice masses. Furthermore, one constitutive equation describes the relation between the strain rate $\dot{D}$ and stress $T^E$ and is expressed in the so called Glen’s Flow law

$$D = A(T')f(\sigma) \ T^E = \frac{1}{2\eta} \ T^E,$$  \hspace{1cm} (3.5)

where $A(T')$ is the Arrhenius rate factor relating $T'$ (the temperature relative to pressure melting point) and the viscosity $\eta$ ($1/\eta = 2A(T')f(\sigma)$). The creep response function

$$f(\sigma) = \sigma^n \hspace{1cm} (3.6)$$

has the effective stress as its argument:

$$\sigma^2 = \frac{1}{2} \ tr(T^E)^2 = (T^E_{xx})^2 + (T^E_{yy})^2 + (T^E_{zz})^2 + \frac{1}{2} \ (T^E_{xy})^2 + (T^E_{xz})^2 + (T^E_{yz})^2). \hspace{1cm} (3.7)$$

Here, $T^E_{ij}, \ i,j \in \{x,y,z\}$, are the components of $T^E$: $T^E_{xx} (= T_{xx} + p)$, $T^E_{yy} (= T_{yy} + p)$ and $T^E_{zz} (= T_{zz} + p)$ are the normal deviatoric stresses, $T^E_{xz} (= T_{xz} = T_{zx})$ and $T^E_{yz} (= T_{yz} = T_{zy})$ vertical shear stresses and $T^E_{xy} (= T_{xy} = T_{yx})$ horizontal plane shear stress. In the purpose of ice dynamics the stress exponent $n$ in Equation (3.6) is chosen to be 3. Glen’s flow law (3.5) can be inverted using the creep response function $f$ (3.6) and the viscosity equation such that stresses are expressed in terms of strain rates:

$$T^E = 2 \eta(T',d) \ D. \hspace{1cm} (3.8)$$

The viscosity is now a function of $T'$ and $d = \sqrt{\frac{1}{2} \ tr(D)^2}$, the effective strain rate. Usually, the determination of velocities in an ice mass is of special interest. The Equations (3.2) - (3.4) and (3.8), together with the assumption of zero acceleration, yields the Stokes equation for ice flow:

$$-\nabla p + \text{div}[\eta(T',d)(\nabla v + (\nabla v)^T)] + \rho g = 0. \hspace{1cm} (3.9)$$
To define the system of partial differential equations (3.9), boundary conditions at the ice base and ice surface are added. At the interface between ice and bedrock/sediment, at the ice base, the total velocity can be restricted depending on if slip conditions are allowed ($v = 0$, no slip conditions allowed). The ice surface is assumed to be stress-free ($T \cdot n = 0$, where $n$ the outward pointing normal vector of the ice surface). Besides these dynamic boundary conditions, one requires kinematic boundary conditions. Then a transport equation e.g. for the position of the free ice surface $h(x,y,t)$ is solved:

$$\frac{\partial h}{\partial t} + v_x \frac{\partial h}{\partial x} + v_y \frac{\partial h}{\partial y} - v_z = a_s.$$  (3.10)

In this time-dependent case the velocity components $v_x$, $v_y$ and $v_z$ enter as coefficients and the accumulation-ablation function $a_s$ acts as a forcing.

### 3.1.2 Approximations to ice dynamics

The Equation (3.9) is what is known as the Full Stokes equation, the equation of motion for ice flow. To solve that system of differential equations is very complicated and numerically expensive due to the non-linearity of the flow law (remember, exponent $n=3$ in the creep response function (3.6)). To overcome this (numerical) problem approximations of the actual system of equations are used. For this, a shallowness parameter $\varepsilon$ is introduced

$$\varepsilon = \frac{[H]}{[L]},$$  (3.11)

relating the scale for vertical extension, ice thickness, $[H]$ to the scale for horizontal extension, width, $[L]$ of the ice mass. The order of these quantities is typically $[H] = 1$ km and $[L] = 1000$ km for ice sheets and $[H] = 500$ m and $[L] = 500$ km for ice shelves (e.g. Blatter et al., 2011, Kirchner et al., 2011a). This means that the shallowness parameter for both, ice sheet and ice shelf, is $\varepsilon = 10^{-3}$. Further scaling commonly used in ice sheet models (presented in e.g. Greve (1997)) are:

$$\begin{align*}
(x, y) & = [L](\bar{x}, \bar{y}), \\
z & = [H]\bar{z}, \\
t & = ([L]/[V_L])\bar{t}, \\
p & = \rho g[H]\bar{p}. \tag{3.12}
\end{align*}$$
\[
\begin{align*}
(T^E_x, T^E_y, \sigma) &= \epsilon \rho g [H] (\tilde{T}^E_{xx}, \tilde{T}^E_{yy}, \tilde{T}^E_{zz}), \\
(T^E_x, T^E_y, T^E_{xy}, T^E_{zz}) &= \epsilon^2 \rho g [H] (\tilde{T}^E_{xx}, \tilde{T}^E_{yy}, \tilde{T}^E_{xy}, \tilde{T}^E_{zz}), \\
(v_x, v_y) &= [V_L] (\tilde{v}_x, \tilde{v}_y), \\
v_z &= [V_H] \tilde{v}_z, \\
F &= [V_L]^2 / g [L].
\end{align*}
\]

where additionally to the horizontal and vertical extent also the horizontal and vertical velocities \([V_H] \) and \([V_L] \) appear. Dimensionless quantities are indicated by tilde. It is assumed that both \([H] \ll [L] \) and \([V_H] \ll [V_L] \), such that the shallowness parameter really becomes small. The scaling (3.11) and (3.12) are now put into all equations describing the thermodynamic problem of ice flow. Also operators are scaled, e.g. the operator \(dT_{xx}/dx \) becomes \(d\epsilon \rho g [H] \tilde{T}^E_{xx} / d[L] \tilde{x} = \epsilon^2 \rho g d\tilde{T}^E_{xx} / d\tilde{x} \) as \([H]/[L] = \epsilon \) (3.11). The variables are expanded into a power series, e.g. the dimensionless quantity \(\tilde{A} \) appears than as

\[
\tilde{A} = \sum_{i=0}^{\infty} \epsilon^i \tilde{A}_{(i)} = \epsilon^0 \tilde{A}_{(0)} + \epsilon^1 \tilde{A}_{(1)} + \cdots + \epsilon^n \tilde{A}_{(n)} + \cdots.
\]

Now, collecting all terms of equal order in shallowness parameter \(\epsilon \) establishes a hierarchy of models. The simplest approach is to take only terms with zeroth order coefficients of \(\epsilon \), the so called SIA (Shallow Ice Approximation) model (e.g. Greve, 1997, Kirchner et al., 2011a). Taking all first order coefficients into the equations, one establishes a FO-SIA (First Order Shallow Ice Approximation) model. Thus, starting from the zeroth order equations, taking gradually higher order coefficients of \(\epsilon \) into the system one establishes first the FO-SIA, then the SO-SIA (Second Order Shallow Ice Approximation) and continuing like this the equations become more and more complex and difficult to solve as it approaches the complete system, the Full Stokes problem. For example, the SIA equations for the momentum balance equation (3.4) are:

\[
0 = -\frac{\partial \tilde{p}_{(0)}}{\partial \tilde{x}} + \frac{\partial \tilde{T}^E_{xx(0)}}{\partial \tilde{z}}, \quad (3.14a)
\]
\[
0 = -\frac{\partial \tilde{p}_{(0)}}{\partial \tilde{y}} + \frac{\partial \tilde{T}^E_{yy(0)}}{\partial \tilde{z}}, \quad (3.14b)
\]
\[
1 = -\frac{\partial \tilde{p}_{(0)}}{\partial \tilde{z}}, \quad (3.14c)
\]

the FO-SIA equations are:

\[
0 = -\frac{\partial \tilde{p}_{(1)}}{\partial \tilde{x}} + \frac{\partial \tilde{T}^E_{xx(1)}}{\partial \tilde{z}}, \quad (3.15a)
\]
\[
0 = -\frac{\partial \tilde{p}_{(1)}}{\partial \tilde{y}} + \frac{\partial \tilde{T}^E_{yy(1)}}{\partial \tilde{z}}, \quad (3.15b)
\]
\[
0 = -\frac{\partial \tilde{p}_{(1)}}{\partial \tilde{z}}. \quad (3.15c)
\]
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and the SO-SIA equations are:

\[ 0 = - \frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial \tau_{xx}^E}{\partial \bar{x}} + \frac{\partial \tau_{xy}^E}{\partial \bar{y}} + \frac{\partial \tau_{xz}^E}{\partial \bar{z}}, \quad (3.16a) \]

\[ 0 = - \frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\partial \tau_{xy}^E}{\partial \bar{x}} + \frac{\partial \tau_{yy}^E}{\partial \bar{y}} + \frac{\partial \tau_{yz}^E}{\partial \bar{z}}, \quad (3.16b) \]

\[ 0 = - \frac{\partial \bar{p}}{\partial \bar{z}} + \frac{\partial \tau_{xz}^E}{\partial \bar{x}} + \frac{\partial \tau_{yz}^E}{\partial \bar{y}} + \frac{\partial \tau_{zz}^E}{\partial \bar{z}}, \quad (3.16c) \]

Note, the SIA equations in contrast to the SO-SIA equations do not contain other than vertical shear stresses. Note further, some stress components \( \tau_{ij}^E(k) \) in the SO-SIA equations do not have the index \( k = 2 \) as expected but are identified with the zeroth order index \( k = 0 \). This will be explained further in this section.

So far, the approximation of the Full Stokes Equations was done under the assumption, that the ice sheets are much broader than high (shallow). It was pointed out, that the shallowness parameter \( \varepsilon \) is the same for ice sheet and ice shelf. Further, ice sheets and ice shelves are at first glance not that different: huge ice masses which are moving gradually influenced by external impacts of climate (as e.g. temperature and precipitation) and prevailing gravity force. They are even obeying the same basic physical laws of conservation (Eqns. 3.1).

The main reason to differentiate ice sheet and ice shelf and to treat them separately though is that the underlying medium they are moving on: ice sheets are grounded on land while ice shelves are floating on water. To handle this problem ice sheet and ice shelf are implemented separately with the SIA (Shallow Ice Approximation) and SSA (Shallow Shelf Approximation) approaches respectively. Based on the same physical equations ice sheets and ice shelves are provided with different scaling. The strategies though are analogical. In the same manner as SIA models can be upgraded, SSA models are progressively made more sophisticated: FO-SSA and SO-SSA are the equivalents to FO-SIA and SO-SIA. The difference between SIA and SSA can be reduced to the assumption that in the shelf approximation the ice flows mainly by horizontal stretching. This is considered in the scaling of components of the stress tensor: for ice sheet flow the vertical shear stresses \( \tau_{xz}^E, \tau_{yz}^E \) are scaled with \( \varepsilon \) and the sheet parallel shear stress \( \tau_{xy}^E \) and the normal stresses are scaled with \( \varepsilon^2 \), while for ice shelf flow the vertical shear stresses are also scaled with \( \varepsilon \) are the horizontal plane shear stresses and the normal stresses scaled with \( \varepsilon^0 \). With these different scaling the SSA equations for the momentum balance equation (3.4) become:

\[ 0 = - \frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial \tau_{xx}^E}{\partial \bar{x}} + \frac{\partial \tau_{xy}^E}{\partial \bar{y}} + \frac{\partial \tau_{xz}^E}{\partial \bar{z}}, \quad (3.17a) \]
\[ 0 = - \frac{\partial \hat{p}_{(0)}}{\partial y} + \frac{\partial \hat{T}^E_{xy(0)}}{\partial y} + \frac{\partial \hat{T}^E_{yx(0)}}{\partial y} + \frac{\partial \hat{T}^E_{yz(0)}}{\partial z}, \]  
\[ \rho_{sw} - \rho = - \frac{\partial \hat{p}_{(0)}}{\partial z} + \frac{\partial \hat{T}^E_{xz(0)}}{\partial z}, \]  

where \( \rho_{sw} \) sea water density; the FO-SSA equations are:

\[ 0 = - \frac{\partial \hat{p}_{(1)}}{\partial y} + \frac{\partial \hat{T}^E_{xy(1)}}{\partial y} + \frac{\partial \hat{T}^E_{yx(1)}}{\partial y} + \frac{\partial \hat{T}^E_{yz(1)}}{\partial z}, \]  
\[ 0 = - \frac{\partial \hat{p}_{(1)}}{\partial y} + \frac{\partial \hat{T}^E_{xy(1)}}{\partial y} + \frac{\partial \hat{T}^E_{yx(1)}}{\partial y} + \frac{\partial \hat{T}^E_{yz(1)}}{\partial z}, \]  
\[ 0 = - \frac{\partial \hat{p}_{(1)}}{\partial y} + \frac{\partial \hat{T}^E_{xz(1)}}{\partial z}, \]  

and the SO-SSA equations are:

\[ 0 = - \frac{\partial \hat{p}_{(2)}}{\partial y} + \frac{\partial \hat{T}^E_{xz(2)}}{\partial z}, \]  
\[ 0 = - \frac{\partial \hat{p}_{(2)}}{\partial y} + \frac{\partial \hat{T}^E_{xy(2)}}{\partial y} + \frac{\partial \hat{T}^E_{yx(2)}}{\partial y} + \frac{\partial \hat{T}^E_{yz(2)}}{\partial z}, \]  
\[ 0 = - \frac{\partial \hat{p}_{(2)}}{\partial y} + \frac{\partial \hat{T}^E_{xz(2)}}{\partial z}, \]  

Here again, some stress components \( \hat{T}^E_{ij(k)} \) show \( k = 0 \) where \( k = 2 \) would be expected. This means actually, that the stress components can be traced across the grounding line for those entries where \( \left[ \frac{\partial \hat{T}^E_{ij(k)}}{\partial (\hat{x}, \hat{y}, \hat{z})} \right]_{(SO-SSA)} = \left[ \frac{\partial \hat{T}^E_{ij(k)}}{\partial (\hat{x}, \hat{y}, \hat{z})} \right]_{(SO-SIA)} \) for \( k = 0,2 \) as the (SO-)SIA and (SO-)SSA equations are valid on either side of the grounding line, for the grounded ice sheet and the floating ice shelf respectively. The method to get to this equivalence is called ‘matched asymptotic expansion technique’ and is explained in Schoof and Hindmarsh (2010).

In the 1950s when ice modeling started, implementations were limited by computational power. Due to this hierarchies of models were established, Schneider and Dickinson (1974) were among the first researchers supporting this idea. As the development of ice models went hand in hand with the evolution of computer capacity more and more complex simulations were run over longer and longer time periods. Substantial progress has been made especially in temporal techniques (to overcome the mismatch in time scales of climate and ice sheets) and in spatial techniques (to overcome the mismatch in model grid resolution) (Pollard, 2010). To improve the modeling of the ice mass as a whole, including ice sheet and ice shelf and finally even fast-flow features such as ice streams, equations of ice sheet and ice shelf flow need to be coupled and so called hybrid ice models be
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derived. A hybrid model is linking the equations of sheet and shelf dynamics such that both parts of the ice mass are respected and treated in one single code (cf. e.g. Pollard and DeConto, 2009, 2012). Models implementing the Full Stokes equation, of course, are an alternative as they comprise equations valid the whole ice mass.

Comparison of Equations (3.14-16) and (3.17-19) shows that zero-order equations of the shallow ice approximation are lacking other than vertical shear stresses (\(T_{xx}, T_{yy}, T_{zz}\)) and thus not capable to be combined. Normal deviatoric shear stresses (\(T_{xy}, T_{yx}, T_{yz}, T_{zy}\)) and the horizontal plane shear stresses are (\(T_{xx} = T_{yy}\)) are important components in ice sheet - ice shelf - ice stream complex and thus not negligible in hybrid ice models. It was thought that it is sufficient to apply second-order models (SO-SIA and SO-SSA; all stresses are included) when dealing with coupled sheet-shelf systems (Kirchner et al., 2011a). Nevertheless, recent studies (Ahlkrona et al., 2013) show that the Full Stokes approach is the method of choice. Higher order models (e.g. SO-SIA) do not capture all dynamics in the high accuracy of Full Stokes models. One reason for this lack in accuracy is that the SIA and higher order SIA models neglect a high viscosity boundary layer near the surface (Ahlkrona et al, 2013). The Full Stokes Problem is still with today’s computer power numerically too expensive to be run on long time scales: a Full Stokes simulation done by Seddik et al. (2012) was run for 100 years for the Greenland ice sheet. This is only a fraction of the time magnitude of \(10^4\) the hybrid model ARCTIC-TARAH is used for in this study, for instance. However, in terms of accuracy of e.g. horizontal velocity the Full Stokes problem thus is the one to strive for (Ahlkrona et al., 2013, Pattyn et al, 2008); here the momentum balance equation (3.4.):

\[
\begin{align*}
0 &= -\frac{\partial p(x,y,z,t)}{\partial x} + \frac{\partial T_{xx}^E(x,y,z,t)}{\partial x} + \frac{\partial T_{yy}^E(x,y,z,t)}{\partial y} + \frac{\partial T_{zz}^E(x,y,z,t)}{\partial z} \\
0 &= -\frac{\partial p(x,y,z,t)}{\partial y} + \frac{\partial T_{xx}^E(x,y,z,t)}{\partial x} + \frac{\partial T_{yy}^E(x,y,z,t)}{\partial y} + \frac{\partial T_{yz}^E(x,y,z,t)}{\partial z} \\
0 &= -\frac{\partial p(x,y,z,t)}{\partial z} + \frac{\partial T_{xx}^E(x,y,z,t)}{\partial x} + \frac{\partial T_{yz}^E(x,y,z,t)}{\partial y} + \frac{\partial T_{zz}^E(x,y,z,t)}{\partial z} + \rho g.
\end{align*}
\]

### 3.2 The ice code ARCTIC-TARAH: general structure and treatment of selected physical processes.

The ARCTIC-TARAH ice code which is used in this project is a Bolin Centre for Climate Research spin-off from the Pennsylvania State University Ice sheet model (PSUI) which was originally developed for Antarctic modeling (Pollard and DeConto 2009, 2012). It is a 3D hybrid
model (coupling ice sheet and ice shelf dynamics) written in Fortran designed for long-term (in the order of $10^7$ years) continental-scale applications.

The code ARCTIC-TARAH is structured with horizontal and vertical grids, of which the horizontal ones can be chosen to be a Cartesian or Spherical (longitude - latitude) coordinate system. However, in the horizontal grid the velocities ($u, v$) are calculated separately on grids staggered half the box size as the ice thickness ($h$), Figure 1(a). This is because velocity is defined as a change in quantity in time and a transition between points.

![Horizontal and Vertical Grids](image)

**Fig. 1:** (a) Horizontal grid. $h$ indicating the grid point where ice thickness is calculated and $u$ and $v$ indicating the grid points for horizontal velocity calculation. (b) Vertical grid.

The vertical grid, which is applied in this study, is divided into 10 unevenly distributed layers (more closely spaced near the upper and lower boundary of the ice to capture the differences in e.g. velocity and stress components, which are more crucial at the boundaries of the ice mass compared to inside) and the horizontal velocities are defined in-between at the midpoint of each layer (Figure 1(b)). Further, the ice thickness is scaled down to a vertical coordinate $z' \in [0,1]$, where $z' = 0$ at the ice surface and $z' = 1$ at the base:

$$z' = (h_s - z)/h,$$

(3.21)

with $h_s$ the surface elevation and $z$ the vertical coordinate axis. Ice dynamics are implemented with a combination of the vertical shear stresses from the scaled SIA equations and the residual stresses (horizontal shear and deviatoric stresses) from the scaled SSA equations described above, Sect. 3.1.1. Using Cartesian coordinates
the horizontal ice velocities are defined by \( u(x, y, z) \) and \( v(x, y, z) \). It is further assumed that \( u = u_b + u_i \). Thus, with the basal velocities at \( z = z_b \): \( u_b(x, y) \) and \( v_b(x, y) \), the internal shear velocities are defined then as \( u_i(x, y, z) = u - u_b \) such that \( u_i(x, y, z_b) = 0 \). Vertical averages are denoted with a bar: \( \bar{u} = u_b + \bar{u}_i \). Analogously for \( v_i \) and \( \bar{v} \). With this in mind and using the effective stress equation (3.7), the relation between strain and stress (Glen’s Flow law (3.5)) is rewritten for the internal shear velocities for \( u_i(x, y, z) \) and \( v_i(x, y, z) \):

\[
\frac{\partial u_i}{\partial z} = 2 \mathcal{A} \left[ \frac{T_{x}^E}{x} + \frac{T_{y}^E}{y} + \frac{T_{z}^E}{z} \right] + \frac{\partial}{\partial y} \left[ \frac{\mu h}{\mathcal{A}^{1/n}} \left( 2 \frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} \right) \right] = \rho_i gh \left( \frac{\partial h}{\partial x} + \frac{f_b}{C^{1/m}} |u_b|^2 \right) \frac{1-m}{2m} u_b
\] (3.22a)

\[
\frac{\partial v_i}{\partial z} = 2 \mathcal{A} \left[ \frac{T_{x}^E}{x} + \frac{T_{y}^E}{y} + \frac{T_{z}^E}{z} \right] + \frac{\partial}{\partial x} \left[ \frac{\mu h}{\mathcal{A}^{1/n}} \left( 2 \frac{\partial v_i}{\partial y} + \frac{\partial w_i}{\partial x} \right) \right] = \rho_i gh \left( \frac{\partial h}{\partial y} + \frac{f_b}{C^{1/m}} |v_b|^2 \right) \frac{1-m}{2m} v_b.
\] (3.22b)

The terms in the brackets are included or neglected depending on which assumptions (SIA or SSA) are made. The creep functions relate back to Equation (3.6): the affiliation of terms belonging to the SIA or SSA is based on the assumption made for respectively case: vertical shear stresses for SIA and residual stresses for SSA. The horizontal stretching equations can be derived from Glen’s Flow law (3.8) and the definition of the strain rate tensor \( D = \frac{1}{2} (\text{grad} \ v + (\text{grad} \ v)^T) \); for \( \bar{u}(x, y) \) and \( \bar{v}(x, y) \):

\[
\frac{\partial}{\partial x} \left[ \frac{2 \mu h}{\mathcal{A}^{1/n}} \left( 2 \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} \right) \right] + \frac{\partial}{\partial y} \left[ \frac{\mu h}{\mathcal{A}^{1/n}} \left( 2 \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right) \right] = \rho_i gh \left( \frac{\partial h}{\partial x} + \frac{f_b}{C^{1/m}} |u_b| \right) \frac{1-m}{2m} u_b
\] (3.23a)

\[
\frac{\partial}{\partial y} \left[ \frac{2 \mu h}{\mathcal{A}^{1/n}} \left( 2 \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{u}}{\partial x} \right) \right] + \frac{\partial}{\partial x} \left[ \frac{\mu h}{\mathcal{A}^{1/n}} \left( 2 \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right) \right] = \rho_i gh \left( \frac{\partial h}{\partial y} + \frac{f_b}{C^{1/m}} |v_b| \right) \frac{1-m}{2m} v_b.
\] (3.23b)

Equations (3.23a/b) include additionally an extra term \( X \) accounting for basal friction, \( C \) is the sliding coefficient which can be chosen appropriate to high or low friction. In the SSA assumptions is the extra term \( X \) neglected as one assumes the ice to move frictionless on the ocean. Further, \( \mathcal{A} = \int \mathcal{A} dz / h \) is the vertical mean of the Arrhenius rate factor (cf. Equation (3.6)) and \( \mu \equiv \frac{1}{2} (d^2)^{1-n} / \mathcal{A}^{1/n} \) the constitutive relation for ice rheology (\( d \) the effective strain rate (cf. Equation (3.8))). More, the vertical shear stresses \( T_{x}^E \) and \( T_{y}^E \) can be derived by solving the momentum balance equation (3.17a-c):

\[
T_{xz} = - \left( \rho_i gh \frac{\partial h}{\partial x} - \text{LHS}_x \right) z' \] (3.24a)
\[ T_{yz} = - \left( \rho_i g h \frac{\partial h}{\partial y} - \text{LHS}_y \right) \cdot z'. \] (3.24b)

The terms LHS\(_x\) and LHS\(_y\) are the left-hand sides of (3.23a) and (3.23b), respectively, and were added to account for horizontal stretching forces, which otherwise - as the shear stresses are derived from SIA momentum balance equations - are neglected.

The ARCTIC-TARAH now performs an iteration through the three systems of equations (3.22 - 3.24) aiming for converging to an appropriate solution: (3.23) is solved for \( \bar{u} \) and \( \bar{v} \) using \( u_i \) and \( v_i \) from the previous iteration, (3.24) is solved for \( T_{xz}^E \) and \( T_{yz}^E \) and (3.22) is solved for \( u_i \) and \( v_i \). When this iteration approaches its limit, only one value for the internal shear velocities is acquired for one time step under SIA conditions. Higher order models solve similar equations, where the individual components (e.g. normal and parallel shear stresses) are scaled with different powers of \( \varepsilon \) (compare Eqns (3.14-16) and (3.17-19)). However, quantities like the ice thickness are still to calculate based on forcing parameters like e.g. basal sliding, oceanic melting or precipitation. The ice thickness evolution (3.25) is part of the ice velocity iteration.

\[ \frac{\partial h}{\partial t} = - \frac{\partial (\bar{u} h)}{\partial x} - \frac{\partial (\bar{v} h)}{\partial y} + \text{MBT} \] (3.25)

Main factors entering the equation are horizontal velocities (\( u \) and \( v \)) and mass balance terms (MBT) of the different interfaces (surface, base and ocean). Selected processes and their implementation are described in the following section 3.2.1, for more detailed description of iterations performed in ARCTIC-TARAH see e.g. Pollard and DeConto (2012).

### 3.2.1 Description of selected physical processes

The ice sheet-ice shelf system is constrained by three major boundaries: lithosphere, ocean and atmosphere (cf. Fig. 2). Physical processes are characterizing each boundary:

Firstly, grounded ice is in contact with the subglacial sediment. Depending on the thermal conditions of the ice (warm-based or cold-based) the ice mass is sliding over the sediment or frozen to the ground. However, frictional forces and geothermal heat fluxes occur at this boundary. Secondly, the movement of ice floating on the ocean is assumed to be frictionless. Water temperature and salinity affect the ice mass balance through melting, re-freezing and calving. Thirdly, the atmospheric temperature and precipitation have major impact on the
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accumulation and ablation of the ice mass controlling its mass balance.

Fig. 2. Schematic illustration of the ice sheet-ice shelf system with boundaries (After Blatter et al., 2011). Under favorable atmospheric conditions is an ice sheet building up on the lithosphere. At the same time as the ice cover gets thicker and gravity driven processes transport the ice from the center of the ice mass to the margins, the lithosphere deforms under the weight of the ice. This deformation is partly reversible and the lithosphere rebounds due to the isostatic uplift after the ice masses wane. This process can continue several thousand years after the ice has disappeared completely. Geothermal heat flux is affecting the ice from below. Besides this, sidling of the ice sheet over the subglacial ground surface produces heat by friction. As the ice masses reach the ocean they continue moving into the ocean but as they are too weighty, they are still grounded. Ice, which is grounded below sea level is called marine ice. As the ice advances into progressively deeper water, buoyancy forces eventually exceed the gravitational forces and the ice starts to float. The grounding line is the boundary between grounded and floating ice. Movement of the floating ice is almost frictionless.

Processes and physics of some ice sheet/shelf control and forcing fields are not adequately implemented in current ice models. Forcing fields as air temperature, precipitation and pre-requisites as e.g. the basal topography can easily be included. Processes such as sliding, transition between sliding and floating, grounding line migration and calving are not up to full reliability included in the models, though (Blatter et al., 2011).
Basal sliding is difficult to include in ice sheet modeling due to the variety of different sliding velocities: distinguish between no sliding condition on frozen bases, slow sliding velocities on broad areas of warm based glaciers and fast sliding within ice stream features. Thus, several sliding parameters are needed to simulate the sliding within one ice sheet. The biggest problem is nevertheless, basal sliding can hardly be measured and values of basal sliding coefficients are often not more than an educated guess. For ice shelves there are other conditions, the ice shelf is gliding (almost) frictionless on the ocean and thus sliding terms are neglected (cf. Eqn 3.23). For ice sheets, the mean basal sliding velocity $v_b$ relates to the basal sliding coefficient $C$ and the $m$-th power of the basal drag $t_b^m$:

$$v_b = C t_b^m.$$  \hfill (3.26)

The power $m$ is an empirically determined constant and the $t_b^m$ can be determined e.g. using the Stokes equation for the basal velocity of a fluid over a wavy bed (Gudmundsson 1994). This is one possible parameterization of basal sliding. The basal velocity vector is always parallel to the tangential component of the basal stress vector (Blatter et al., 2011). The implementation of a sliding parameterization in a zero-order ice sheet model (SIA) has as consequence a singularity at the bed position where cold-based conditions change into warm-based conditions (or vice versa). This is because basal shear traction (related to sliding velocity) does not respond to neighboring stress vectors. To overcome the discontinuity at the ice-bed interface the implementing of a homogeneous deforming thin layer between the actual ice sheet and the underlying bed is one possibility. Here, no sliding is assumed neither between the bed and the thin layer nor between the thin layer and the ice sheet (Vieli et al., 2001). However, due to the high variability of flow velocity - over time and in space - sliding is critical to implement.

In ARCTIC-TARAH the basal sliding component is realized with the following equation:

$$v_b = C |\tau_b|^{k-1}\tau_b,$$  \hfill (3.27)

where $v_b$ the basal sliding velocity, $\tau_b$ the basal driving stress and $C$ the temperature-depending sliding coefficient. The values used in the modeling done for this project are listed in the section 4 Methods.
The **grounding line** is the borderline between the sliding grounded ice (ice sheet) and the gliding floating ice (ice shelf). One difficulty is to locate the grounding line position (possibly back and forth movement over time) (Blatter et al., 2011) both in models and in observations on site. The ice which is flowing towards the ocean will be lifted up when buoyancy forces exceed gravitational forces. Sliding turns into gliding and thermal exchange with the water instead of geothermal and friction heat has impact on the ice once it has passed the grounding line. As it is crucial to connect these two regimes in order to depict the whole system, the dynamics of the grounding line need to feed back with both, the ice sheet and the ice shelf. Basal melt and re-freezing at the ice-ocean interface as well as (partly) breakup of the floating ice and also mass changes in the accumulation area, for instance, trigger a displacement of the grounding line towards land or towards ocean. Such a displacement of the grounding line into critical areas, where the ice masses are no longer in stable position, can result in the collapse of the entire ice shelf (e.g. Schoof, 2007, Weertman, 1974, Thomas and Bentley, 1978). The dynamics of the grounding line are such small-scaled that the modeling requires a fine discretization solution (e.g. Blatter et al., 2011, Schoof, 2007).

ARCTIC-TARAH uses the grounding line migration criterion by Schoof (2007). Here (in Schoof, 2007), the coupled sheet-shelf flow model is a depth-integrated simulation for a rapidly sliding two-dimensional symmetric marine ice sheet. At the grounding line position, the ice sheet model is linked to the ice shelf model. It is assumed that the ice flux, ice thickness and longitudinal stresses are continuous at the transition to floating ice (at the grounding line). Furthermore, basal friction is neglected in ice shelves. Maintaining these assumptions, the grounding line criterion is applied to the evolving non-symmetric ice sheet and the code ARCTIC-TARAH steps through the grid points to find the positions of the grounding line. Ice thickness and ice velocities at these points and upstream of the grounding line are calculated. Additionally, the information about the position of the point is kept such that the calculated velocities are treated like shelf velocities later on in the code.

**Calving** is the process of separation of icebergs and smaller ice blocks from an ice shelf into water. Mainly weakness of the ice but as well impact from ocean and surface melt water is responsible for the breaking off of ice masses. The larger the stresses the more likely is damage (Kachanov (1999) introduces a damage variable $D$ to parameterize a material deterioration in terms of rheology properties) and the more
likely is calving, because a highly damaged zone is less viscous. As calving is a process of mass loss the calving rate $U_c$ can be defined as length loss over time:

$$U_c = \bar{v}_\perp - \frac{dL}{dt}, \quad (3.28)$$

where $\bar{v}_\perp$ the averaged vertical flow velocity, $L$ the glaciers length and $t$ the time (Benn and Evans, 2010). The notation $dL/dt$ is then the change in length $dL$ over a certain time period $dt$. Other calving laws (e.g. the water-depth calving law proposed by Brown et al. (1982), criteria such as the height-above-buoyancy calving criterion (Vieli et al., 2000) and the crevasse-depth calving criterion (Benn et al., 2007)) are established to formalize the process of calving. As the processes triggering calving are operating on small scales compared to the coarse spatial resolution ice models work on it is difficult to implement calving processes properly and a parameterization is needed to handle this (Blatter et al., 2011).

In ARCTIC-TARAH calving is simply a term (calvrate) which can be chosen to be included or not. If included the calving rate enters the code within the calculation of the total mass budget. In the setting applied in this study the mass budget is calculated with:

$$\text{budgall} = \text{budgsnow} - \text{oceanmelt} - \text{calvrate} \quad (3.29)$$

Further, in the applied settings the calving rate is assumed to be homogeneous over the whole domain (i.e. everywhere where ocean is detected) and over time. More advanced options can be chosen to be implemented in the code, though. To calculate the grid mean calving mass balance (CMB) essentially two steps are carried out. First, the divergence of the velocities $u$ and $v$ at floating ice shelf grid points is calculated: $\text{div } u = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$, $u$ the velocity vector. For this the values from the solution of the equation (3.22) under SSA assumptions is used. Second, the points at the shelf margin (adjacent to ocean) are selected and the CMB determined as a weight between two values: for thinner ice shelves weighted towards a chosen calving rate and for thicker shelves towards a value proportional to the divergence, if this is positive.

Oceanic melting and its opposite component, basal (re-)freezing, are important for the mass balance of ice shelves. As the freezing point of water decreases with increasing pressure and salinity the basal melting and freezing processes are basically controlled by the subsea cavity and properties of the ocean water. Oceanic melting is
escalating with increasing water temperatures but also with increasing salinity or pressure whereas the opposite, (decreasing temperature, salinity and pressure) hold for basal (re-)freezing. Water circulation on smaller and larger scales can have major effects on whether the ice shelf gains or loses mass at its base: Cold and fresh melt water, which emerges at the grounding line will migrate upwards along the shelf base. This in turn will lead to a re-freeze at the base of the outer ice shelf. The formation of sea ice in front of an ice shelf yields high-salinity water as salt is expelled during the freezing process. This dense water migrates underneath the ice shelf and leads to ocean melting. Even though the water is close to the freezing point the higher pressure at depth is sufficient to melt the ice at the base of the ice shelf (Benn and Evans, 2010).

ARCTIC-TARAH calculates ocean melt in two steps. First, the area of each vertical face which is in contact with ocean is detected and then the grid cell is multiplied with characteristic oceanic sub-ice melt factors. The vertical extent of any cell adjacent to ocean is given by

\[ \Delta z = \frac{\rho_i}{\rho_w} h_e \]  
for floating ice \hspace{1cm} (3.30a)

\[ \Delta z = \text{sealev} - h_b \]  
for grounded ice \hspace{1cm} (3.30b)

where \( h_b \) the bed elevation and \( h_e \) the sub-grid ice thickness (cf. Eqn. 3.30). The vertical extent \( \Delta z \) is multiplied with the length of the interface \( dx \) and with a sub-ice melt factor. The ocean rate enters, as the calving rate, the total mass budget equation (3.29). In the settings applied and discussed in this study the oceanic melting is assumed to be homogeneous over the whole domain (again everywhere where ocean is present) and over time. However, runs with changed oceanic melting parameter were performed but are not mainly subjected to in the following discussion of results.

2/ Ice streams

An ice stream is “Part of an ice sheet or ice cap in which the ice flows more rapidly and not necessarily in the same direction as the surrounding ice” (Alean and Hambrey, 2006). Ice streams are radiating out like channels from the inner parts of an ice sheet or ice cap (Benn and Evans, 2010). The interface between the fast moving ice stream and the slowly moving surrounding ice is often evident by strongly sheared and crevassed ice (Alean and Hambrey, 2006). Mainly four areas need to be studied to recognize, understand and model ice streams (Benn and Evans, 2010):
a/ ice stream onset zones: the origin of fast flow features within slowly moving ice masses
b/ ice stream beds: favorable subglacial environment for ice streams
c/ lateral shear margins: stress fields and force balance at the margin of fast flow features
d/ grounding zones between ice streams and floating ice shelves: impact of the ocean (e.g. tidal variations) on marine terminating fast flow features.

In the ice stream onset zone the ice streams arise and the slowly moving inland ice frames the fast flowing ice stream. Here two totally different regimes of shear and driving stress are encountering one another: The slowly moving ice masses are controlled by vertical shear stresses and velocity increases driving stresses. Ice streams are controlled by lateral shear stresses and the driving stress is inversely proportional to flow velocity. This discrepancy of stress fields results transverse crevasses in the onset zones that are often clearly visible on satellite images. The higher flow velocity of the ice stream is accomplished by basal motion and higher melt water production at the onset zone. High flow velocities result in increased basal melting leading to a positive feedback loop facilitating ice streams inception. Nevertheless, the high basal melt at the onset zone stands in contrast to the low basal melt underneath the ice stream itself (Benn and Evans, 2010).

At the lateral shear margins of ice streams basal environments change from low-drag, temperate to high-drag, frozen subglacial conditions. The velocity gradient near the margin is very steep. Thus, shear stresses are very large and as a consequence the lateral margins of ice streams are characterized by concentrated crevassing (Benn and Evans, 2010). Van der Veen (1999) proposed a proportionality between the width of the ice stream and the flow velocity. This means that the velocity would increase with $10^4$ when the ice stream widens with 10. This could explain long-term changes in flow velocity ranging from stagnation to notably fast flow but there are several explanations for such a behavior. Especially two theories stand out: internally regulated velocity cycles and velocity variation based on change in internal water flow patterns due to e.g. climate factors. The former claims a feedback cycle between ice geometry, basal energy balance and flow velocity, the latter between subglacial bed properties (including drainage), basal energy balance and flow velocity. Ice streams are in any case reflecting the complexity and sensitivity of external and internal changes (Benn and Evans, 2010).
The grounding zones of ice streams are similar to the grounding line, the transition between grounded ice sheets and floating ice shelves. Into the ocean terminating ice streams can have a grounding zone extending over tens of kilometers, revealing patches of grounded or lightly grounded or floating ice. Internal hydrothermal changes but even changes in oceanic forcing (e.g. high or low tide water) are affecting force balances, flow velocities and stability of the ice. A change in hydrology due to tide water can propagate far up-ice stream (at least 40 km; Gudmundsson, 2006) and is thus another sensitive factor for ice streams.

Ice streams are responsible for the main part of the discharge and calving of an ice sheet. Thus, the major impact ice streams have on the hydrology of ice sheets and ice shelves make them very important regarding behavior and stability of ice sheets (Benn and Evans, 2010).

In theory, ice streams can be modeled when using the system of equations of hybrid sheet-shelf simulations; and the Full Stokes models, of course. Ice streams should be detectible on the velocity fields. In ARCTIC-TARAH the velocities are calculated both at the bottom and the top of the ice sheet, in the two horizontal directions that easily gives the absolute velocity. The top absolute velocity is expected to be larger than the bottom velocity due to the velocity distribution in ice sheets (cf. Benn and Evans, 2010). One aim of this study is to investigate the ice stream behavior of the ARCTIC-TARAH ice code. The model is expected to catch the difference in bottom and surface velocity with the code as well as areas of exceptionally high velocities surrounded by vast areas of slow ice velocities.

3/ Interaction of glaciers with neighbored systems, lithosphere and atmosphere

Glacial isostasy is the displacement of the Earth's mantle due to the weight of an ice sheet. Beneath the ice the land is pushed down, just in front of the ice margin the surface experiences a peripheral depression and beyond this the crust raises to a forebulge (Benn and Evans, 2010). This affects the relative sea level and regional drainage system. After the ice sheet has waned, the crust adjusts itself to its former isostatic balance; except for some irreversible deformations and landforms typical for (de)glaciated areas. When attempting long-time (in the order of millennia) ice sheet modeling glacial isostasy is of importance for ice mass calculations. Surface
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elevation is decreased (impact on surface mass balance) and more ocean-free surface is exposed. The ductility of the lithosphere influences the ice sheet and thus the choice of Earth model and coupling between lithosphere and ice sheet is essential.

In this study the interference with the lithosphere is neglected. Only investigations concerning different types of subglacial sediment and bedrock are undertaken.

Average annual air **temperature** below zero and non-zero **precipitation** is needed for ice sheets to build. The larger the ice sheet gets the more impact on the regional climate it has in terms of albedo effect and increasing surface elevation. There are mainly two different wide spread approaches to the climatic forcing: the snapshot method and the matrix method. The snapshot method requires one set of parameters which is applied to the ice sheet over a certain simulation time. Then the parameters are adjusted to the conditions and used for the next time step. This includes a discontinuous, iterative adaption and to overcome this, an intermediate climate forcing is used between the adjustment steps. The other often used method, the matrix method, provides a collection of different combinations of parameters and when the ice sheet begins to develop the most appropriate set of parameters is chosen (Benn and Evans, 2010). The best solution though is claimed to be a combination of the snapshot and the matrix method (DeConto and Pollard, 2003), which moreover includes a real-time co-development of the climate parameter set besides the ice sheet evolution (Ridley et al., 2005)

The mass balance in ARCTIC-TARAH is driven by precipitation and air surface temperature fields as well as a mass balance scheme. The applied mass balance scheme in this study is the mass balance curve according to Pelto et al. (1990). Investigations on present-day glaciers and their mass balance gradients showed a clear dependence between the balance gradient and climate (Pelto et al., 1990). Studies and mapping of balance gradients on 83 present-day alpine, polar and temperate glaciers resulted in a table distinguishing between five different glacier types (polar continental, polar mix, sub-polar mix, sub-polar maritime and temperate maritime) based on winter and summer temperature and annual precipitation. Each type then is assigned a parameter specifying ablation at the margin ($A_1$), accumulation at the margin ($A_2$), decay exponent of ablation with elevation ($x_1$) and decay exponent of accumulation with elevation ($x_2$). The formula for mass balance in altitude (h [m]) is found to fit best as:

\[ \text{mass balance} = \begin{cases} 
A_1 - A_2 & \text{if } h < 0 \\
A_1 - A_2 + x_1 h & \text{if } 0 < h < x_2 
\end{cases} \]
\[ A(h) = A_1 e^{-x_1 h^2} + A_2 e^{-x_2 h^2}. \] (3.31)

A refinement of this includes a shift of the curve into colder or warmer climate conditions. By inserting a linear term in the exponent \( (h \rightarrow (h - dELA)) \) one can manually force the equilibrium line altitude to shift into lower/higher altitudes and in this way to restrict accumulation to higher elevations or to induce in lower elevations. Figure 3 shows the two mass balance configurations (applying polar mix parameters) which are applied in this study.

Fig. 3. Pelto mass balance curve, scaled with 0.3. Red: \( dELA = 0 \), no shift. Blue: \( dELA = -2.4 \), shift towards ‘colder’ conditions.

This shift mimics temperature changes, which otherwise are not included in the Pelto bass balance curve. As temperature and precipitation fields are otherwise kept constant throughout all runs of this study and a stepwise adjustment of the Pelto mass balance curve is applied the method of climate forcing is the snapshot method in this study.

### 3.3 The Eurasian Arctic and its glacial history

#### 3.3.1 The Eurasian Ice Sheet

The Barents Sea, located between Svalbard, Franz Josef Land, Novaya Zemlya and Scandinavia (Fig. 4), was several times covered by an ice sheet.
Svendsen et al. (2004) describe the Quaternary ice sheet history of the Eurasian Arctic and the development of the Eurasian Ice Sheet in very much detail from the Late Saalian (>140 ka BP; BP = Before Present) up until the Late Weichselian (25 - 15 ka BP). The Svalbard Barents Sea Ice Sheet (SBIS) was an ice sheet which developed on Svalbard and the Barents Sea and which was during the full glacial (cf. Figures 5-8) a part of the Eurasian Ice Sheet.

The maximum extent during the Late Saalian (160 -140 ka BP) is depicted in Figure 5, taken from Svendsen et al. (2004). The ice covered entire Scandinavia, Svalbard and reached out far into Russia in the East and British Isles in the West. The dotted line refers to the approximate maximum extent of the Quaternary glaciations. Note, that smaller ice caps of the Iceland, Greenland are not included in this figure.
The Early Weichselian (90–80 ka BP) ice extent of the Eurasian Ice Sheet is shown in Figure 6. The ice extent decreased compared to the Late Weichselian and embraces now only Scandinavia, parts of Svalbard, but reaches farther east over Russia. The ice cover is, nevertheless, centered over the Barents Sea.

![Fig. 6: Eurasian Ice Sheet during the Early Weichselian (90–80 ka BP), from Svendsen et al. (2004).](image)

In the Middle Weichselian (about 60 ka BP), the ice sheet had extended further south over Fennoscandia and the retreated northward over Russia, Figure 7.

![Fig. 7: Ice extent during the Middle Weichselian (approx. 60 ka BP), from Svendsen et al. (2004).](image)

The ice extent at the Last Glacial Maximum (LGM) during the Late
Weichselian (20 ka BP) is shown in Figure 8. The ice retreated further northward towards Novaya Zemlya but reaches again the British Isles.

Soon after the LGM, the ice started to withdraw, the onset of the separation of the ice covers on the land masses. The thinner the Eurasian Ice Sheet got, the less was the SBIS influenced by the emerging Scandinavian Ice Sheet. The relatively shallow (continental shelf) ocean conditions of the Barents Sea enable the built up of a persistent ice cover. Snow accumulates on the freezing water surface of the Barents Sea and the sea level drops. At the same time, as the ice thickens on Scandinavia and the islands in the north (Svalbard and Franz Josef Land) the isostatic weight of the grounded ice presses down the land masses whereas the sea floor of the Barents sea experiences a rise at the same time (Ingólfsson and Landvik, 2013). In this way is a large-scale glaciation favored in the region of the Barents Sea.

3.3.2 Svalbard’s glacial history and the SBIS

The Svalbard archipelago (76° - 81° N, 8° - 32° E) is situated between the cold water of the Atlantic Ocean and the Barents Sea and the warmer waters of the Northern Atlantic and the Norwegian and Greenland Seas, cf. Fig. 9.
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Fig. 9. Svalbard map (from CIA gov. library).

Several glaciations left traces on Svalbard and on the sea floor around Svalbard. Investigations of submarine landforms provide information about the former glaciations, the LGM in particular. Mega-scale glacial lineations (MSGL), drumlins and crag-and-tail formations which are indicating ice movement direction are mapped east of Svalbard by Dowdeswell et al. (2010). They concluded that one ice dome was situated on the easternmost Spitsbergen or southern Hinlopen Strait and another, smaller one, west of Kong Karls Land without excluding the possible presence of additional ice domes. Other studies (Figure 10), however, propose different locations for ice domes and it is an ongoing discussion whether the SBIS was single- or multi-domed. Figure 10 shows ice domes and ice movement directions resulting from different studies done by Dowdeswell et al. (2010), Hogan et al. (2010), Landvik et al. (1998), Lambeck (1995, 1996) and Hormes et al. (2011).
Fig. 10. Proposed ice sheet domes and ice flow directions under LGM and the subsequent deglaciation. The yellow polygon marks Hinlopen Strait. Modified from Ingólfssson and Landvik (2013).

The common view is that the SBIS was a highly dynamic ice sheet and ice domes shifted throughout the evolution (Svendsen et al, 2004) and Figure 10 is supporting this idea. However, ice flow directions proposed by all studies agree rather well. Main flow directions were determined by major fjords and troughs. Generally the ice was moving westward from the western part of the Archipelago and eastward from the eastern part. Only at one spot the studies of Lambeck (1995, 1996) and Landvik et al. (1998) (blue in Figure 10) are contradicting the of Dowdeswell et al. (2010) and Hogan et al. (2010) (red in Figure 10): The former propose a northward flow through the entire Hinlopen Strait beginning west of Kong Karls Land, while the latter locate an ice dome exactly in the southern opening of Hinlopen Strait and thus an ice divide at this site. Another study published by Ottesen et al. (2007) also used high-resolution bathymetric mapping to detect landforms and claims that major and fast ice flow have been in the larges fjords moving west- and northwards (Figure 11).
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Fig. 11. Ice flow regime of the Svalbard Ice Sheet during the Late Weichselian (the white solid line marks the ice extent). Major ice streams are indicated with white arrows. The color shows depths, ranging from 100m (red) to less than 1000m (dark blue). From Ottesen et al. (2007).

Studies by Ottesen et al. (2007) and Hogan et al. (2010) establish the idea that during the deglaciation the ice velocity was faster in the deeper and outer parts of the troughs and temporary calving bays appeared in deeper parts of troughs. In the shallower parts on the other hand a slower ice recession and temporary stagnation or re-advance even formed recessional moraines (Hogan et al., 2010). The Late Weichselian Svalbard Ice Sheet is better established in the western and southern regions of Svalbard than in the eastern and northern regions for the simple reason of more favorable sea ice conditions and proximity of major settlements in the former areas (Noormets et al., 2013). Glaciation curves (based on C\textsuperscript{14}, OSL, IRSL, TL and U/Th ages of sediment and rock samples) show that the glacial history and the ice sheet dynamics of the SBIS are much more complex and complicated than previously assumed (Alexanderson et al., 2013). However, recent studies suggest that the deglaciation of the SBIS
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started about 15 000 C\(^{14}\) years BP and further, that the SBIS separated into several ice parts, such that the one over Svalbard was independent from the Barents Sea ice sheet about 12 000 C\(^{14}\) years BP (see Fig. 12; Ingölfsson and Landvik, 2013).

![Fig. 12: Reconstructed ice margins of the SBIS/Svalbard ice cover during the deglaciation from the LGM: at about 15 000 C\(^{14}\) years BP (approx. 17 000 calibrated years BP) and 12 000 C\(^{14}\) years BP (approx. 14 000 calibrated years BP). From Ingölfsson and Landvik, 2013.](image)

After the onset of the deglaciation, the Svalbard ice sheet decreases further - possibly with a temporary growth during the Little Ice Age between 1550 and 1850 - until present conditions. Today about 60% or 36 600km\(^2\) of Svalbard are covered by glaciers and ice caps (Hagen et al., 1993).

3.4 Modeling the SBIS - a short review

First attempts to model the SBIS were done by Hughes in 1979. The model used ice flow lines which were estimated considering geologically determined ice sheet margins. Applying constant climate forcing the model reached equilibrium condition at each flow line. Since the SBIS was modeled as one component of the larger Eurasian ice sheet the maximum ice thickness (2,1km) was obtained over Novaya Zemlya. Later, in the 1990s, a comparable model including basal sliding and sediment was presented by Lindstrom (1990) and Isaksson (1992), respectively. These later approaches treated the Svalbard ice
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sheet separately and obtained the maximum ice thickness (∼1.7 km) over Edgeøya. With the onset of increasing modeling several statements regarding the glaciation of the area were made: Hughes (1981 and 1987) claimed that the SBIS only could initiate from a permanent sea ice; Peltier (1988) declared that the Barents Sea was necessarily covered by a large ice sheet during the LGM; Tushingham and Peltier (1991 and 1993) used a global isostatic rebound model and achieved an ice sheet distribution describing a grounded, 2.2 km thick ice cover over the Barents Sea. More complex models were applied to gain understanding of the (de-)glaciation history of the SBIS. A non-spherical layered viscoelastic fluid model used by Elverhøi et al. (1993) suggests that large parts of the Barents Sea were ice free already by 15 ka BP. A glacio-hydro-isostatic rebound model with two different initial ice extent conditions (Lambeck, 1995) reproduces relative sea level curves of the Barents Sea region. Based on the results of Elverhøi et al. (1993) and Lambeck (1995) a revised development is suggested by Lambeck (1996): the ice sheet over the Barents Sea had a thickness of approximately 3400 m and Svalbard was covered with ice until 13 ka BP. After new geological evidence had been presented by the Quaternary Environment of the Eurasian North (QUEEN) program, numerical ice sheet models had new input data. With this, new information regarding dimension and chronology of the LGM ice sheet and the deglaciation were obtained. The ice model processed by Siegert and Dowdeswell (2004) assessed the time development of the ice sheet, its mass balance and reaction to climate and glaciological production including sediments, ice bergs and melt water. Fast flowing ice streams along the western margin were recognized, occupying bathymetric troughs which transport enormous volumes of sediment to the continental margin.

The above models, which were applied to the ice covers of the Eurasian North, are using ice sheet calculations (SIA). As the SIA models are for grounded ice only, the results were achieved by artificial adjustments such as lowering of sea level. In the here presented study, a hybrid model is applied to Svalbard's ice cover. This means ice sheet and ice shelf equations are concurrently implemented in the code and an artificially lowering of the sea level is not necessary to achieve appropriate results. To demonstrate the importance of hybrid models, the results of hybrid and SIA runs with and without artificially sea level are compared. Tests concerning subglacial basal conditions are performed to get additional information about the susceptibility of hybrid and SIA models.
4 Methods

The ARCTIC-TARAH ice code was provided with a SVALBARD module which includes the topography and climate forcing fields of the simulation domain, the Svalbard archipelago. Firstly the topography, ETOPO1, was configured and downloaded from the National Oceanic and Atmospheric Administration (NOAA): the domain is extending from 76° - 81° N and from 8° - 32° E and the resulting 151 times 721 two-minute-grid is read in ARCTIC-TARAH. ETOPO1 is a global relief model of Earth’s surface integrating both, land topography and ocean bathymetry.

In summary, nine simulations were carried out, Table 1.

<table>
<thead>
<tr>
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<th>SIA</th>
<th>Sea level</th>
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<td>Run #8</td>
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| Deglaciation            | Run #9    |
| Hybrid: Sea level adjustment after Waelbroeck et al. (2002) and specified sliding parameter. |

Table 1. Nine runs performed in this study.

The hybrid model was run (runs #1 - #4) for each combination of sea level (= 0.0 m or = -50.0 m) and sediment raster (uniform subglacial sediment or spatial specified sediment). Analogously, the SIA model was run with the same four settings (runs #5 - #8). For the latter, the shelf equations were commented out in the hybrid mode, then only the sheet dynamics are treated. These eight runs started at ice free conditions and evolved for 35 000 simulated years. The time scale is not coupled to calendar years but serves as a control time for the spin-up runs #1 - #8. Lowering the sea level is the only way for SIA models to be used in long-term simulations as ice extent is restricted to areas above sea level. With this artificial adjustment ice can extent onto the continental shelf regions. The dropped sea level was in this study applied to demonstrate how massively sea level effects ice extent under SIA conditions and how the two model setups (SIA and hybrid) differ. Further, to drop the sea level is a tool to mimic glaciation conditions, when vast areas are covered with ice and the ocean water is trapped in the ice masses. The sediment raster is a simple query whether or not the sediment grid point lies above or beneath respective sea level (0m or -50m): if above, bedrock conditions are assumed and the basal sliding parameter (cf. Section

34
3.2.1) is set $C_{\text{bed}} = 10^{-10}$ to prohibit high sliding motion, if below, subglacial sediment conditions are assumed and the sliding coefficient is set $C_{\text{sed}} = 10^{-6}$ to allow sliding. In the case of no specified sediment raster, the coefficient is set $C_{\text{const}} = 10^{-8}$, to mimic moderate sliding conditions over the whole domain. The dimensionless sliding parameter is proportional to the sliding velocity, the smaller the coefficient the slower the velocity. The uniform sliding coefficient was chosen to lie between the sliding coefficients of the low friction zone (sand/silt) and the high friction zone (bedrock) to enable comparison. The different sliding options were applied in order to investigate ice flow behavior.

The following parameters were applied for all 8 runs: a constant precipitation field of 0.2 m/yr over the whole domain; mass balance parameter ocean melt 1.0 m/yr and calving rate 1.0 m/yr as well as variables of the “polar mix” for the Pelto mass balance curve $A_1 = -2.849$, $A_2 = 0.8378$, $x_1 = 8.5345 \cdot 10^{-6}$ and $x_2 = 3.0453 \cdot 10^{-8}$ (cf. Section 3.2.1), which is further scaled with 0.3 to achieve feasible ice extent. To mimic the natural changes in climate and aiming for an equilibrium in ice evolution, the mass balance was controlled with the Pelto forcing and a shift in equilibrium altitude line (ELA). A shift in ELA was performed in an alternating pattern: $d\text{ELA}_{\text{cold}} = -2.4$ forcing precipitation to accumulate even in low altitudes (for 2500 years) and $d\text{ELA}_{\text{warm}} = 0$ pushing the area of accumulation into higher altitude (for 1000 years), see Figure 3. The area of accumulation lies than above approx. -100m and +400m respectively.

Finally, the hybrid model was run to simulate a deglaciation starting from an ice cover similar to the reconstructed ice extent about 12 ka BP (during the Younger Dryas) (cf. Figure 12). The mass balance parameter $d\text{ELA}$ was taken from the GRIP temperature curve and the sea level changes were step-wise adjusted with the reconstructed sea level curve from Waelbroeck et al. (2002). Further, the spatial specified sediment raster was applied.

4.1 Generating an ice cover: hybrid model

The hybrid model was tested at first. A uniform subglacial raster over the whole domain was applied in runs #1 and #2. This setup was applied for sea level 0.0 m (run #1) and -50.0 m (run #2).
Application of a subglacial sediment raster

For the next two runs (# 3 and #4) all parameter are as in the first two runs, only a subglacial sediment raster accounting for bedrock and sediment distribution is applied. As explained before, the sliding coefficient is higher in areas below respective sea level. These simulations were performed for sea level 0.0 m (run #3) and -50.0 m (run #4).

4.2 Generating an ice cover: SIA model

Secondly, the SIA model was tested (runs #5 - #8) mainly for analytical reasons and to compare the evolving ice extent and the development of fast flow features such as ice streams between the two model setups. As the SIA does not allow any marine ice sheets or ice shelves to form, it is especially interesting in which degree these features develop. First, a spatially uniform sliding coefficient was applied (runs #5 and #6). This setup was once more applied for sea level 0.0 m (run #5) and -50.0 m (run #6).

Application of a subglacial sediment raster

The parameters are kept as in the first two runs of the SIA setup, except for the change in subglacial sediment raster. The specified sliding parameters are used as before for run #7 and #8. These runs were performed for sea level 0.0 m (run #7) and -50.0 m (run #8).

4.3 Deglaciation: Younger Dryas until present day’s ice cover

For the deglaciation run, the ice extent reached after 35 000 model years in what can be regarded as a spin-up run (run #8) was used as a starting point as it - by visual inspection - comes closest to the ice extent from 12 000 years BP found by Ingólfsson and Landvik (2013), cf. Figure 12. The model runtime was chosen to be 12 000 years, reaching from the Younger Dryas until present. To mimic the climate over the last 12 000 years, the mass balance parameter dELA was adjusted from the GRIP temperature curve (data from the temperature reconstruction presented in Vinther et al., 2009). Additionally, the reconstructed sea level curve from Waelbroeck et al. (2002) was used to adjust a stepwise sea level change over that time. Both curves are shown in Figure 13.
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Figure 13: GRIP temperature curve (Vinther et al., 2009, blue) and sea level (red) change stepwise adjusted simplifying the sea level curve from Waelbroeck et al. (2002).

Further, the spatial specified sediment raster was kept in the simulation and the oceanic melting parameter was set to 5.0 m/year.

5 Results

The following results of the settings described above refer to the Svalbard domain (Figures 14, 15 and compare also Figure 9).

Figure 14: Modeling domain Svalbard. Topography and bathymetry in
Lambert projection. Dataset downloaded from National Oceanic and Atmospheric Administration (NOAA); Amante and Eakins (2009).

The following plots in the result section are all shown in a plane xy-coordinate system. For comparison between the two coordinate representations, the Svalbard domain in a plane xy-coordinate system is shown in Figure 15.

Figure 15: Modeling domain, Svalbard. Plane xy-coordinate system. The present day coast line (border between orange and yellow areas) is for comparison included in all following ice thickness plots. Is: Isfjorden, HS: Hinlopen Strait, H: Hornsund, R: Recherchfjorden, S: Storfjorden, VM: Van Mijenfjorden and W: Wijdefjorden.

Further, the black dots and arrows in Figure 15 indicate the location of fjords later referred to: Isfjorden, Hinlopen Strait, Hornsund, Recherchfjorden, Storfjorden, Van Mijenfjorden and Wijdefjorden.

The two main runs performed within the first part of this study are the hybrid (run # 4) and SIA (run #8) runs with specified subglacial sediment raster and dropped sea level to -50.0 m as their parameters resemble the most feasible glaciation conditions. This is because it is feasible for the entire runtime to assume some ocean water to be kept in the (evolving) ice cover and to provide spatially non-uniform sliding coefficients (sediment will accumulate on the sea floor after being transported by ice). The simulation of the deglaciation from the Younger Dryas until today’s conditions is the third important run in this study (run #9). These three runs will be paid most attention to in the following result and discussion sections even though results of all runs will be shown. In all ice thickness plots is todays coastline shown with a bold black contour line.
5.1 Generating an ice cover: hybrid model

The development of the ice thickness and the ice volume [km$^3$] of the first simulation (run #1) with a uniform basal sliding coefficient and a sea level corresponding to today's coastline is shown in Figures 16 and 17.

![Fig. 16: Run #1: Ice thickness [m] after (a) 3 000, (b) 5 000, (c) 10 000 and (d) 20 000 simulations years. The color bar ranges from -50 to 1000 [m].](image)

The ice spreads over the continental shelf south-east of Svalbard and reaches to the shelf break in the north-west. Ice thicknesses range from some tens of meters over the highest mountain tops to 1000 m in the fjords and troughs.

![Fig. 17: Run #1: Ice volume [km$^3$] against time [$10^2$ years]. Blue: total ice volume; Green: grounded ice volume (left axis); Red: floating ice volume, smoothed curve (right axis).](image)
After 35,000 years of simulation the total ice volume is about $11.5 \cdot 10^{13}$ km$^3$. The floating part of the ice masses starts to build up after 2,000 years of simulation and reaches towards the end of the runtime an equilibrium of about $0.04 \cdot 10^{13}$ km$^3$, which corresponds to approx. 0.3% of the total ice volume.

The ice thickness [m] for the hybrid run with uniform basal sliding coefficient and a dropped sea level down to -50.0 (run #2) are shown in Figure 18.

![Fig. 18: Run #2: Ice thickness [m] after (a) 3,000, (b) 5,000, (c) 10,000 and (d) 20,000 years of simulation. The color bar ranges from -50 to 1000 [m].](image)

The ice thickness and extent shows essentially the same pattern as in the run before. Ice extends somewhat further southeast. This is possibly due to the additional land area, which was made available as the sea level was lowered.

The ice volume evolution looks like the one for run #1 and is not shown. After 35,000 years of simulation the total ice volume is about $13.8 \cdot 10^{13}$ km$^3$. The floating part of the ice masses starts to build up after 2,000 years of simulation and reaches at the end of the runtime an equilibrium of about $0.03 \cdot 10^{13}$ km$^3$, which corresponds to approx. 0.2% of the total ice volume.

**Application of a subglacial sediment raster**

Again, the development of ice thickness [m] and ice volume [km$^3$] for
Numerical modeling of Svalbard’s ice cover

the hybrid run with specified subglacial sediment raster and a
levelled sea level at 0.0 m (run #3) is shown in Figures 19 and 20. It
is expected that the smaller sliding coefficient (and thus slower ice
flow velocity) on land leads to a higher land ice thickness. Further
ice thickness and velocity behavior for the runs with specified
sliding coefficients is discussed in 6.1 Ice cover build-up.

![Fig. 19: Run #3: Ice thickness [m] after (a) 3 000, (b) 5 000, (c) 10 000 and (d) 20 000 simulations years. The color bar ranges from -50 to 1000 [m].](image)

The ice extent development seems delayed compared with the runs
before. The smaller sliding coefficient possibly slows the ice that
much down, that it just reaches the sea level within the modeling
time. Ice is after 20 000 years of simulation almost only on land and
in the fjords and not exceeded over the continental shelf in the
south-east or to the shelf break in the west. This means, that the
higher sliding velocity beneath sea level does not affect the ice
extent. Ice thickness ranges from tens of meters on the highest
elevations to 850 m in the fjords.
After 35 000 years of simulation the total ice volume is about $2.7 \cdot 10^{13}$ km$^3$. The floating part of the ice masses starts to build up after 2 000 years of simulation. At the end of the run time the floating ice reaches an equilibrium of about $0.005 \cdot 10^{13}$ km$^3$, which corresponds to approx. 0.2% of the total ice volume. The ice volume increases steeply in the first interval of the run time and the growth rate decreases in the second time interval.

Now, for the hybrid ice model run with sea level at -50.0 m and the specified subglacial sliding raster (run #4) the evolving ice cover (ice thickness [m]) is shown in more detail in Figure 21. Other features will be presented in more detail for this run as well.
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Fig. 21: Run #4: Ice thickness [m] for every 3 000 years of simulation, from 3 000 until 33 000. The color bar ranges from -50 to 1000 [m].

It is evident, that the ice starts spreading over the seabed south-east of Svalbard. It does not reach as far as to the shelf break in the west but is limited to the west coast of Svalbard. Ice thicknesses range from some tens of meters over the highest mountain tops to 850 m in the fjords and troughs.

The ice volume evolution from this run and the previous one (run #3) look alike and the plot for this run is thus not separately shown. After 35 000 years of simulation the total ice volume is about $3.5 \times 10^{13}$ km$^3$. The floating part of the ice masses starts to build up after 2 000 years of simulation and reaches towards the end of the runtime an
equilibrium of about $0.007 \cdot 10^{13}$ km$^3$, which corresponds to approx. 0.2% of the total ice volume.

Further, for visualization how the ice cover is built up on the topography and how the ice grows into the ocean, three cross-sections in West-East (Figures 22a-c) and three in North-South direction (Figures 23a-c) were plotted; all depict the ice cover after 21 000 years of simulation. These fjords were chosen to look closer at, as they represent the main fjords in the Svalbard Archipelago. Additionally, they extend in fairly straight West-East or North-South directions such that they can be plotted with a straight cut-through. It is expected to see the formation of marine ice sheets (ice sheets which are grounded below sea level) and ice shelves (floating ice). To locate the fjords on the map (Figure 15), the coordinates of the opening of the fjords are specified in the captions. Additionally, the location of these fjords is indicated in Figure 15. In each of the following cross section plot is the black line indicating the topography/bathymetry (see Figure 15 as well). The red line indicates the ice cover and the blue line marks the sea level.

Fig. 22a: Run #4: Isfjorden, (x = 170, y = 65).
Fig. 22b: Run #4: Van Mijenfjorden, (x = 180, y = 52).

Fig. 22c: Run #4: Hornsund, (x = 225, y = 30).

Fig. 23a: Run #4: Recherchefjorden, (x = 200, y = 48).
All cross-sections show besides the ice sheet, the marine ice sheet and the ice shelves, which are evolving where the ice flows into the ocean. The ice shelves are very small, only Isfjorden (Figure 23a) shows somewhat large ice shelf evolving.

The velocities [m/year] at the bottom and at the top of the ice sheet are shown for three time steps in Figures 24 - 26 a and b, respectively. The three time steps are at 3 000 years (Figures 24), right after the start of the modeling, at 9 000 years (Figures 25), in the middle of the build-up and at 21 000 years (Figures 26), close to the end of the simulation. Velocities are plotted on log-scale.
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Fig. 24: Run #4: Velocity [m/year] after 3 000 years of simulation. (a) bottom velocity (b) surface velocity.

Fig. 25: Run #4: Velocity [m/year] after 9 000 years of simulation. (a) bottom velocity (b) surface velocity.

Fig. 26: Run #4: Velocity [m/year] after 21 000 years of simulation. (a) bottom velocity (b) surface velocity.

The ice velocities at 3 000 years of simulation indicate higher velocities in the fjords, between 5 and 50 m/year. The inland ice is moving slower, with about 0-5 m/year. However, the ice is moving faster where topography is down sloping and slower at the top of mountain ranges and plateaus. Further, the bottom velocities are smaller than the top velocities: the differences is small at first (about 5 m/year), increases (80 m/year) and levels off towards the end of the simulation. The velocities after 9 000 years of simulation show clearly fast flow features in the fjords. Especially in the surface plots, almost all fjords (sea e.g. Isfjorden and Wijdefjorden) show high velocities from the highest part of the fjords reaching to the coast, where the area of fast flowing ice widens. After 21 000 years of simulation the high velocities (250 m/year) at Isfjorden and Hinlopen Strait are striking.
5.2 Generating an ice cover: SIA model

In the following the results of the SIA runs (runs #5 - #8) are presented.

First, the development of the ice thickness [m] and the ice volume [km³] of the first SIA simulation (run #5) with a uniform basal sliding coefficient and a sea level corresponding to today’s coastline 0.0 is shown in Figures 27 and 28.

The ice thickness ranges between some tens of meters at the highest elevations and coastal areas to 550 m in the fjords (the part of the fjords which is above sea level). The same color bar as for the hybrid runs is chosen to enable better comparison. The ice does - by construction - not flow out into the ocean.
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Fig. 28: Run #5: Total ice volume [km$^3$] against time [$10^2$ years].

After about 8 000 years of simulation the ice volume levels off with about $8.7 \times 10^{13}$ km$^3$. The grounded ice volume corresponds to the total ice volume as there is no floating ice, by construction.

The same features for the SIA run with uniform basal sliding coefficient and a dropped sea level down to -50.0 (run #6) is shown in Figures 29 and 30.

Fig. 29: Run #6: Ice thickness [m] after (a) 3 000, (b) 5 000, (c) 10 000 and (d) 21 000 years of simulation. The color bar ranges from -50 to 1000 [m].

The general ice thickness pattern is the same as in the SIA run before, but the maximum value for ice thickness lies at about 700 m
for this run.

Fig. 30: Run #6: Total ice volume [km³] against time [10² years].

After about 10 000 years of simulation the ice volume levels off with about $1.8 \times 10^{13}$ km³. Again, by construction no floating ice.

The difference between the two model setups hybrid and SIA becomes very clear when comparing runs #2 and #6. By construction the ice does not exceed into the ocean when SIA model is applied. The ice extent of the hybrid configuration can only be achieved when the sea level is artificially lowered.

Application of a subglacial sediment raster

Applying the subglacial sediment raster for the SIA runs does only have an effect on the thickness and ice velocity of the ice sheet. The sliding coefficient is chosen to be smaller above sea level, when spatially uniform sliding is applied, compared to specified sediment raster. This means, that the ice velocity should decrease and the ice thickness increase. As the ice only builds up on land, the sub sea level sliding coefficient does not play a role.

Development of ice thickness [m] and total ice volume [km³] for the SIA run with specified subglacial sediment raster and a levelled sea level at 0.0 m (run #7) is shown in Figures 31 and 32.
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Fig. 31: Run #7: Ice thickness [m] after (a) 3 000, (b) 5 000, (c) 10 000 and (d) 21 000 simulations years. The color bar ranges from -50 to 1000 [m].

As well for this run, the ice thickness evolution is the same as in the previous SIA runs (runs #5 and #6). However, the maximum value of ice thickness lies at about 550 m, like in the first SIA model simulation (run #5).

Fig. 32: Run #7: Total ice volume [km$^3$] against time [$10^2$ years].

Similar to the first SIA model run (run #5), the total ice volume reaches an equilibrium after about 8 000 years of simulation with about $8.5 \cdot 10^{13}$ km$^3$.

Now, for the SIA ice model run with sea level at -50.0 m and the
specified subglacial sliding raster (run #8) the evolving ice cover (ice thickness [m]) is shown in more detail in Figure 33.
The ice thickness evolution is similar to all previous SIA model runs and the maximum value of ice thickness is comparable to the second SIA model run (run #6), about 700m. Again, the ice is only building up on land and thus no ice shelves are present.

Figure 34 shows the ice volume [km³] development plotted against time for this run.

After about 10 000 years of simulation the total ice volume achieves an equilibrium of about $1.8 \times 10^{13}$ km³.

The cross-sections for this SIA run (#8) are shown in the following figures: three cross-sections in West-East (Figures 35a-c) and three in North-South directions (Figures 36a-c). Here as well, all cross-section depict the ice cover after 21 000 years of simulation. The fjords are the same as in the cross-sections of the hybrid run #4 (Figures 22 and 23). Comparison between the cross-sections for the respectively runs will is expected to show the importance of integrated ice sheet-ice shelf modeling. See Figure 15 to locate the fjords. As before in the cross-sections for the hybrid run #4, the
black line indicates the topography/bathymetry, the red line the ice sheet and the blue line marks the sea level.

Fig. 35a: Run #8: Isfjorden, (x = 170, y = 65).

Fig. 35b: Run #8: Van Mijenfjorden, (x = 180, y = 52).
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Fig. 35c: Run #8: Hornsund, (x = 225, y = 30).

Fig. 36a: Run #8: Recherchefjorden, (x = 200, y = 48).

Fig. 36b: Run #8: Wijdefjorden, (x = 225, y = 115).
The cross-sections show, that the ice is only staggering on land and – according to the construction of SIA models – no marine ice sheets or ice shelves are building in this configuration. The peaky surface is commented in the section 6.2.

The bottom and the top velocities [m/year] of the ice sheet are shown for three time steps in Figures 37 – 39 a and b, respectively. The three time steps are again at 3 000 years (Figures 37), 9 000 years (Figures 38) and 21 000 years (Figures 39), such that three phases of the simulation, start-up, intermediate and end are presented. The velocities are plotted on log-scale.

Fig. 37: Run #8: Velocity [m/year] after 3 000 years of simulation. (a) bottom velocity (b) surface velocity.
The velocity plots show that the ice velocity is gradually increasing with advancing simulation time. High velocities (5 m/year to max value of respectively time step) are achieved at the ledge of the ice sheet and “upstream” the fjords. The ice sheet interior has small ice velocities of about 0-5 m/year. However, ice velocities are smaller at plateaus and at the top of mountain ranges. Further, the bottom velocities are smaller than the top velocities.

5.3 Deglaciation: Younger Dryas until present day ice cover

Finally, the ice thickness [m] development under the forcing of the GRIP temperature curve (realized through the shift in Pelto mass balance curve) and under the modification in sea level according to Waelbroeck et al. (2002) is shown in Figure 40. For the forcing, the Pelto mass balance curve was shifted with different, to the GRIP temperature curve adjusted dELA terms as described in section 3.2.1/3: see Figure 3. The sea level modification is a stepwise adjustment as plotted in Figure 13. The deglaciation was started with the ice configuration at 35 000 years of simulation time from run # 8 (hybrid run with sea level at -50.0 and with spatially non-uniform sliding coefficient).
Fig. 40: Run #9: Ice thickness [m] each 1,000 years of simulation starting at 12,000 BP until 0 BP. The color bar ranges from -50 to 1,000 [m].

The maximum values of ice thickness decreases from about 850 m to 600 m. At modeled present day conditions, 0 BP (Figure 40m), the ice is mainly vanished on land except for some inland areas and at the coasts. Thick marine ice in Storfjorden, Hinlopen Strait and other larger fjords is still present.

The development of the ice volume under the applied forcing is shown in Figure 41.

Fig. 41: Run #9: Blue: GRIP temperature curve; Red: sea level curve (right axis); Blue thin: total ice volume; Dark green: grounded ice volume; Light green: floating ice volume (left axis).

The ice volume decreases from $3.8 \times 10^{13}$ km$^3$ to $2.2 \times 10^{13}$ km$^3$. The ice shelves have vanished after a 100 years. For correlation are the GRIP temperature curve and the steps of the adjusted sea level included in the Figure 41 (compare Figure 13).
6 Discussion

6.1 Ice cover build-up

The alternating shift of the Pelto mass balance curve from cold conditions (accumulation allowed in low areas) to warm conditions (accumulation only allowed in higher elevations) was applied to mimic variable climate conditions. As the first 8 runs are detached from any certain time interval, is the approach of periodically shifting warm and cold climate conditions sufficient for these simulations aiming for equilibrium ice covers. As the external forcing (e.g. temperature, precipitation, oceanic melting, calving rate) was held constant in the runs discussed in detail, the differences in ice volume and ice thickness evolution can be traced back to the different setups of sea level and basal sliding coefficients and the two different types of models. A drop of sea level exposes more land area, which is expected to be important especially for the SIA models as they only allow ice sheet building on land (above sea level). The difference in sediment (implemented through the basal sliding coefficient) is expected to have an impact on the ice velocity and ice thickness: a small sliding coefficient (e.g. bedrock) prohibits sliding and thus reduces the flow velocity and implies a slower extent of the ice as well as increased ice thickness. A larger sliding coefficient (e.g. sand/silt) in contrast is encouraging ice flow and thus implying an increased ice flow velocity and a decreased ice thickness. As only the hybrid ice model is coupling ice sheet and ice shelf, the different basal sliding configurations are expected to be more important for the hybrid model than for the SIA, when comparing the results of spatially uniform and non-uniform sliding coefficients. Further, the great advantage hybrid models provide is the ability to actually form ice shelves.

6.1.1 Ice volume evolution

The SIA runs (#5 - #8) achieve a pronounced equilibrium of the ice volume: \(8.5 \times 10^{12} \text{ km}^3\) (sea level 0.0 m (runs #5 and #7); Figures 28 and 32) after 8 000 years of simulation and \(1.8 \times 10^{13} \text{ km}^3\) (sea level -50.0 m (runs #6 and #8); Figures 30 and 34) after 10 000 years of simulation. There is only a small difference (approx. 3%) between the different sliding coefficient configurations. The ice volumes of the two sea level settings, however, differ by nearly 50%. This means, that the SIA model is much more susceptible to sea level drop than to sliding coefficients. This can be explained with the general structure of the SIA model, namely the exclusion of ice shelf building. As
expected, the ice volume is essentially depending on the available land area: a larger land area results in increased ice volume. Further, the total ice volume is comprised of grounded ice, as ice shelf building is prohibited in the SIA configuration.

The hybrid runs (#1 -#4) do not achieve an explicit equilibrium but a leveling of the total ice volume is recognizable for the two runs with spatially non-uniform sliding coefficient (runs #2 and #4). The ice volume of the floating parts of the ice mass for all hybrid runs is, however, clearly achieving an equilibrium. After 35 000 years of simulation the two runs with spatially uniform sliding coefficients (runs #1 and #2) reach an total ice volume of $11.5 \times 10^{13}$ km$^3$ and $13.8 \times 10^{13}$ km$^3$ with sea level set to 0.0 (run #1; Figure 17) and -50.0 (run #2) respectively. The difference between the two sea level settings is small (approx. 16%), where the ice volume is larger for dropped sea level. This results from the fact that the ice accumulates easiest on land and with dropped sea level is the land area extended. The ice shelves start to build at the edges of the ice cover after about 2 000 years of simulation and reach towards the end of the runtime an equilibrium of approx. $0.04 \times 10^{13}$ km$^3$ and $0.03 \times 10^{13}$ km$^3$ which corresponds to 0.3% and 0.2% of the total ice volume for sea level set at 0.0 and -50.0 respectively.

The two runs with spatially non-uniform sliding coefficients (runs #3 and #4) reach after 35 000 years of simulation an total ice volume of approx. $3.5 \times 10^{13}$ km$^3$ and $2.7 \times 10^{13}$ km$^3$ with sea level set at 0.0 (run #3; Figure 20) and -50.0 (run #4) respectively. The ice volume development of these two runs shows a pronounced leveling off. The volume increases steeply during the first approx. 10 000 years of simulation and the curve flattens in the further run. The difference between the two sea level settings is with about 23% somewhat larger than for the previous two runs. Again, the run with dropped sea level has a larger ice volume. Ice shelves begin to build at the edges of the ice cover after 2 000 years of simulation and at the end of the simulation the floating ice volume reaches it equilibrium of $0.005 \times 10^{13}$ km$^3$ and $0.007 \times 10^{13}$ km$^3$ for sea level 0.0 and -50.0 respectively, which corresponds in both cases to about 0.2% of the total ice volume.

The differences in total ice volume between the two sea level settings (approx. 16% and 23% respectively) are slightly smaller than the differences between the two sliding coefficient setting (approx. 23% and 25% respectively), which shows, that the hybrid model is somewhat more susceptible to the sliding coefficients than to the sea level. Regarding this, hybrid and SIA models react the opposite way. The hybrid model allows the ice to flow into the ocean and thus both basal
sliding coefficients (above and below sea level, low and high sliding respectively) are recognized in the runs. Apparently, the spatially different sliding conditions balance each other well enough, or even possibly increase the ice flow, such that the total ice volume decreases compared with the spatially uniform sliding conditions.

Further, it is striking, that the ice shelves form relatively early during the run and reach an equilibrium shortly after the initiation. The ice shelves are located at the edges of the ice cover and the cross-section plots (Figures 22 and 23) show that they then only appear where the bathymetry falls steeply enough to force the ice to float (due to buoyancy). The ocean parameters (oceanic melting and calving rate) are balancing the flow of ice such that the volume of floating ice becomes stable after a relatively short period of time. Test runs (not discussed), which were carried out with smaller ocean parameters resulted in much larger ice shelf extent. This shows that the model is very sensitive to oceanic forcing parameters. Time constraints didn’t allow to investigate this further. The floating ice volume of ice shelves is very small compared to the grounded ice volume of the ice sheet. This is because there are only a few small ice shelves present and also due to the fact, that the ice shelves need to be much thinner than marine ice sheets, as they are not touching the ground. The relation in thickness between ice sheet, marine ice sheet and ice shelf is visible in the cross-section plots, Figures 22 and 23.

6.1.2 Ice thickness evolution

The developing ice thickness of the SIA runs (#5 - #8) depends on the sea level setting. With a sea level at 0.0 (runs #5 and #7) the ice thickness reaches its maximum at 550 m (Figures 27 and 31). The ice is thickest in the fjords (the part above sea level, as the ice is not allowed to flow into the ocean). The ice is only a couple of tens of meter thick at the highest mountain tops and in coastal areas. The same thickness distribution pattern is resulting of a dropped sea level to -50.0 (runs #6 and #8), but the maximum ice thickness is 700 m instead (Figures 29 and 33). Even though, only a stripe which corresponds to an elevation change of 50m is added to the land area available to build an ice cover on, this additional area is enough to enable a staggering of ice about 150m higher than without this area. Thus, in SIA mode, the ice thickness is a property strongly depending on the sea level. The different sliding configurations do not appear to play an important role for this feature. Fast sliding was only
Numerical modeling of Svalbard’s ice cover allowed in areas below sea level. The ice in the SIA mode does – by construction – never reach below sea level. Thus, only the difference between the two sliding coefficients which were chosen for the setting with spatially uniform sliding \( C_{\text{const}} = 10^{-8} \) and for spatially non-uniform sliding above sea level \( C_{\text{bed}} = 10^{-10} \) are relevant. The difference is too small to affect the ice thickness, though.

The ice thickness of the hybrid runs (runs #1 - #4; Figures 16, 18, 19, 21), however, does appear to differ in the opposite manner as the SIA cover (runs #5 - #8; Figures 27, 29, 31, 33): ice thicknesses of 1000 m are in this run time only achieved where spatially uniform sliding coefficients were applied (Figures 16 and 18). Among these two runs (#1 and #2), the one with dropped sea level (run #2; Figure 18) shows the maximum ice thickness in broader areas as the one with sea level at 0.0 (run #1; Figure 16). The runs with spatially non-uniform sliding (runs #3 and #4; Figures 19 and 21) show a maximum ice thickness of 850 m, which is slightly thinner than for the previous runs. Here, the difference of maximum ice thickness depends on the different settings of sliding coefficient and not markedly on the available land area as the hybrid model allows ice regardless sea level. In the setting of uniform sliding the coefficient was chosen to be \( C_{\text{const}} = 10^{-8} \) over the whole domain. In the setting of spatially non-uniform sliding, the coefficient was chosen to be smaller (reducing sliding) \( C_{\text{bed}} = 10^{-10} \) above sea level and larger (enhancing sliding) \( C_{\text{sed}} = 10^{-6} \) below sea level. Even though the sliding coefficient thus is reducing sliding above sea level in the setting of non-uniform sliding, is the ice thickness thinner compared with the thickness resulting from uniform sliding. This can only be explained if the larger sliding coefficient below sea level balances the ice thickness and pulls even land ice into the ocean. This behavior is to investigate further.

The inland ice is for all runs up to 500 m thick, only at the highest tops of the mountains the ice thickness decreases to a couple of tens of meters. The ice is thickest (600m - 1000m) inside the fjords (e.g. Isfjorden, Wijdefjorden) and in Hinlopen Strait and at Storfjorden. Modeling studies done by Landvik et al. (1998) suggest maximum ice thickness of the Svalbard ice cover of 1000 to 1200m at full glaciation conditions (20 000 years BP). A more detailed study based on cosmogenic nuclide dating done by Gjermundsen et al. (2013) proposes ice thicknesses up to 900m in the fjords and ice surface elevations of 1350m at the highest spots (mountain elevations up to 1000-1300 m). This correlates very well to the results of this study. This shows that the hybrid model is capable to catch the data based
reconstructions. Even though it was run with artificial forcing parameters, the evolving full glacial ice cover stabilizes presenting characteristics found by data-based investigations. The ice thins out towards the margins of the ice cover. Special features like ice domes are discussed later, Section 6.3.2.

6.2 Spatial extent: Ice Sheet vs. Ice Shelf

In this study, only the hybrid model is able to produce ice shelves as the sheet equations are coupled to the shelf equations. This is not the case for the SIA models. In the ice thickness plots (Figures 16, 18, 19 and 21 (hybrid) and 27, 29, 31 and 33 (SIA)), but especially in the cross section plots (Figures 22 and 23 (hybrid) and Figures 35 and 36 (SIA)), the huge difference between the two configurations is apparent. It is evident, that if aiming for a realistic simulation the hybrid model is to choose instead of the SIA model.

The SIA model (runs #5 - #8) restricts ice buildup to the areas where the topography is above sea level and the ice cover can't exceed into the ocean; no ice shelves develop. Consequently, the ice sheet is larger for the runs with dropped sea level. The actual extent of the ice is not influenced by the sliding coefficients. The walls of the ice thickness are extremely steep (e.g. Hornsund, Figure 35c) and the ice cover is following the steep peaks of the topography (e.g. Van Mijenfjorden (Figure 35b) and Storfjorden (Figure 36c)). This is an unrealistic representation and smoothing factors should be considered in a further development of the ice code ARCTIC-TARAH. The SIA model is not applicable when aiming for a realistic ice cover simulation.

The hybrid model allows ice to extend into the ocean and the ice flows over the seabed south-east of Svalbard and north-west-wards towards the shelf break. Here, at the continental shelf break, is the abrupt drop in sea floor restricting the ice flow towards the north-west. Where the sea floor goes steeply downwards, the marine ice sheet is forced to begin to float forming an ice shelf, given that the buoyancy acts successfully against the gravitational force. Floating ice does not leave traces on the sea floor as grounded ice does. Bathymetric mapping tracks grounding zone wedges where the ice proceeded from grounded to floating ice (e.g. grounding zone wedge at Isfjorden, Figure 3 Ottesen et al., 2007). Grounding zone wedges may also mark the position of the ice margin. The different parts and features of the ice cover are illustrated in the cross-section plots (Figures 22 and 23): the thick ice sheet and marine ice sheet, the grounding line and the (compared to the ice sheet) thin ice shelf. The onset of ice
shelves is visible in all cross-section plots of the hybrid model runs. Only where a critical steepness of the bathymetry is exceeded, the marine ice sheet is forced to float and develop ice shelf. Especially in Isfjorden (Figure 22a), the ice shelf is pronounced. Figure 42 below illustrates the difference between the ice cover of the hybrid and SIA model runs (runs #4 and #8) at Isfjorden. Note especially the ice configuration at the ice-ocean interface.

![Fig. 42. Cross-section at Isfjorden. Ice cover after 21 000 years of simulation. Red: SIA run (#4), green: Hybrid run (#8). Black: topography/bathymetry, Blue: sea level.](image)

A steep bathymetry is also at the northern end of the Storfjorden plot (Figure 23c) present but no ice shelf is forming. This could possibly be due to the fact that the topography at that location is very steeply falling from 600m down to sea level and the ice - as it follows the topography - does not build a marine ice sheet. In a longer run time configuration, both a small marine ice sheet and ice shelf developing is possible. This means, that ice shelves require a moderate to flat topography/bathymetry and a steep downward sloping bathymetry to evolve from ice sheets/marine ice sheets. In this manner, Recherchfjorden (Figure 23a) has the best preconditions to build ice shelves in a longer run time. The pre-requisites and conditions for the buildup of marine ice sheets and ice shelves are discussed in grounding line movement and ice sheet/shelf stability studies. Besides complex hypotheses regarding ice sheet/shelf stability, Goldberg et al. (2009) state that the probability of a collapse to happen quickly is commonly increased with steeper bed slopes. Generally, as it was for SIA ice covers, the ice is following the topography and steep peaks of ice are evolving (e.g. Hornsund (Figure 22c), Storfjorden (Figure 23c)). Differently to the SIA, the
walls of the ice cover are not that steep, due to the possibility of the ice to flow towards the ocean. Interesting regarding the steepness of the ice cover and the possibility to grow into or to cover ocean is the comparison of the SIA and the hybrid ice covers: looking at Storfjorden (Figures 23c (hybrid) and 36c (SIA)) it becomes evident, that the ice is bridging Hinlopen Strait with hybrid settings, whereas the ice is forced to come down to sea level and to disconnect at the same place under SIA settings. This is illustrated in Figure 43 below.

Fig. 43. Cross-section at Storfjorden. Ice cover after 21 000 years of simulation. Dark green: SIA run (#4), light green: Hybrid run (#8). Black: topography/bathymetry, Blue: sea level.

The difference in the spatial extent of the ice cover between the different sea level setups with spatially uniform sliding coefficient (runs #1 and #2; Figures 16 and 18) is not very large: the extent over the seabed east of Svalbard is simply delayed in time for the run with sea level 0.0 (run #1; Figure 16) compared with the one with sea level -50.0 (run #2; Figure 18). This delay is even more pronounced for the runs with spatially non-uniform sliding (runs #3 and #4; Figures 19 and 21). The run with lowered sea level (run #4; Figure 21) shows a larger ice extent within the run time than the one with sea level at 0.0 (run #3; Figure 19) does. This difference (the ice extent evolution within the run time of 35 000 years) between the runs with either spatially uniform or non-uniform sliding coefficients, indicates that the higher subsea sliding coefficient slows the spreading down. This is opposite to the expected behavior and similar to the unexpected behavior of the ice thickness (see 6.1.2).

It is also striking, that in all runs the islands in the east and north do not show marine ice sheet development. This is possibly due to the size of the islands, they are too small to provide enough ice.
flowing from the land into the ocean, such that it is impossible to evolve marine ice sheets or even ice shelves. Prins Karls Forland in the west is - confirming this suggestion - only surrounded by marine ice sheets when ice from the mainland advanced far enough to the west and enclosed the Island (compare e.g. Figures 16 and 19). Investigations of ice flow direction at Prins Karls Forland confirm this observation (Landvik et al., 2013).

However, the modeled spatial extension fits at the northern ice margin very well together with data-based reconstructions of Svalbard’s ice cover (Figures 11 and 12). Figure 44 below, shows the ice extent of test run #2 after 20 000 years of simulation fitted on the ice extent proposed by Ingólfsson and Landvik, 2013 (see also Figure 12).

6.3 Features and characteristics of the modeled ice cover

6.3.1 Ice streams and ice velocity

All runs applying either the SIA or the hybrid configurations do show some evidence of ice streams. Fast flow features like ice streams are best visualized with velocity plots (Figures 24 - 26 (hybrid) and
The SIA ice cover however, shows only were the ice starts to flow faster above sea level, which appears to be in the higher elevated areas of the fjords. Thus, a dropped sea level amplifies the ice stream. The velocity plots (Figures 37 - 39) mainly show the high velocities (up to 160 m/year) at the edges of the ice sheet, at the steep marginal walls of the ice covers, and slow velocities (0-10 m/year) of the inland ice. The inland velocities are somewhat faster where the topography is down sloping and slower at plateaus and the highest tops of mountain ranges. However, higher ice velocities are mostly indicated at the highest spots of the fjords, e.g. Storfjorden, Wijdenfjorden and Van Mijefjorden. The enlarged Figure 37b is shown below in Figure 45. The velocity is again plotted on log-scale.

Fig. 45. Enlarged Figure 37b: Surface ice velocity [m/year] after 9000 years of simulation.

S = Storfjorden, W = Wijdefjorden and M = Van Mijenfjorden.

Maximum ice velocities at the bottom are smaller than at the surface, which is corresponds to the velocity distribution of an ice mass (cf. Section 3.2.1). There is no striking difference in ice stream formation between the two different sliding coefficient settings (Figures not shown).

The ice cover evolving in hybrid runs indicates ice streams in the openings and inside the large fjords as well as in Hinlopen Strait. The dropped sea level only increases the areas of large ice thickness (compare e.g. ice thickness Figures 16 and 18), but does probably not influence the ice stream development. The higher sliding coefficient in the spatially non-uniform setting should encourage ice streams to
flow faster compared to the uniform sliding setting (Figures to compare maximum ice velocities not shown). At the beginning of the simulation (Figure 24) the ice velocities are higher (5–60 m/year) only at the margin of the ice cover. Slow velocities (0–5 m/year) are plotted almost over the whole ice mass at all time steps. After 9 000 years of simulation (Figure 25) ice stream onsets are visible flowing towards the larger fjords, such as Isfjorden and Wijdenfjorden. Also the northern opening of Hinlopen Strait experiences higher ice velocities: maximum ice velocities are 140 m/year at the bottom and 220 m/year at the surface. The enlarged Figure 25b is shown below in Figure 46a and b. In Figure 46b the velocities are in log-scale.

![Figure 46](image1.png)

**Fig. 46.** Figure 25b: Surface ice velocity [m/year] after 9 000 years of simulation, (a) Color bar ranges from 0 to 220 (b) velocities on log-scale. I = Isfjorden, W = Wijdenfjorden and H = Hinlopen Strait.

After 21 000 years of simulation (Figure 26) Isfjorden and the northern opening of Hinlopen Strait are still indicating fast flow features. The Isfjorden ice stream is very pronounced: maximum ice velocities are 250 m/year at bottom and surface of the ice. The enlarged Figure 26b is shown below in Figure 47.

![Figure 47](image2.png)

**Fig. 47.** Figure 26b: Surface ice velocity [m/year] after 21 000 years of simulation. (a) Color bar ranges from 0 to 220 (b) velocities on log-scale. I = Isfjorden and H = Hinlopen Strait.

Comparing surface and bottom velocities further, the correct velocity distribution is caught by the ice code (higher velocities at the surface), Figure 48 below. Regarding spatial extent of fast-flow features, the surface high-velocity areas are larger compared with...
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those at the bottom. Figure 48 shows close ups of the bottom (a and c) and of the surface (b and d) velocities at the northern opening of Hinlopen Strait after 21 000 years of hybrid simulation (compare Figure 47a).

Fig. 48. Close ups of bottom and surface ice velocity [m/year] at northern opening of Hinlopen Strait after 21 000 years of simulation (compare Fig 47). (a) bottom, (b) surface velocity: the Color bar ranges from 0 to 250. (c) bottom, (d) surface velocity: log-scale.

Figure 48 (a,b) shows that the ice velocity increases from the outside (only a few meter per year) to the inside (approx. 150 m/year) of the fast-flow feature; this for both bottom and surface velocity. Interesting is the onset of the fast velocity: at the surface the higher velocity starts already further to the south-east, whereas the high velocity starts abrupt and further to the coast line at the bottom of the ice mass. When fast-flow features are triggered by ‘slippery’ bed conditions (cf. Section 3.2.1) the comparison between the velocity fields Figure 48a and b, c and d can be interpreted to mean that the onset of the high velocity is at the bottom of the ice and propagates not only downstream but also upstream and towards the ice surface. Higher ice velocities occupy a larger area at the ice surface than at the bottom, when relying on the results of this study. Further, the ice surface velocities are gaining fast flowing mass from different branches connecting to one large ice stream close to the coast (Figure 48d). Megascale glacial lineations and lateral ice stream moraines, which indicate fast-flow movement parallel to these bathymetric features, are mapped by Ottesen et al. (2007) e.g. at
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Isfjorden. Data-based ice streams locations (Figure 10; Ottesen et al., 2007) match very well with modelled ice streams in this study. In Figure 49, the fast flow areas of test run #2 after 10 000 years of simulation are colored in black and fitted on the ice flow regime map of the Svalbard Ice Sheet by Ottesen et al. (2007), Figure 11.

Fig. 49. Fast flow areas (run #2 after 10 000 years of simulation; black) on flow regime map (Fig. 10; Ottesen et al., 2007).

It is apparent that the main fast flow areas of this study (black) coincide with the fast flow features of the underlying flow regime map of Ottesen et al. (2007) (white). This is valid especially for Spitsbergen (main island of the Svalbard Archipelago, cf. Figure 9). On the map of Ottesen et al. (2007), there is no ice stream indicated in the north east (on Nordaustlandet), while there is one according to the results of this modeling study. This possibly is due to the artificial forcing applied in this study, which is not respecting locally specific conditions.

6.3.2 Ice domes

The modelled ice thickness of the hybrid model runs suggest the presence of two ice domes located at the Hinlopen Strait and at the northern end of the Storfjorden, both about 1000 m in ice thickness (e.g. Figure 18). The high ice thickness at Hinlopen Strait does well agree on the ice dome position proposed by Dowdeswell et al. (2012) and Hogan et al. (2012) (Figure 10). Looking at the surface elevation plot (not shown), the highest point of the ice cover is close to the highest point of Svalbard and no other dome-like elevation feature is
visible in the plot. A possible ice dome at Storfjorden, like the ice thickness of 1000 m in Figures 16 and 18 suggests, is not proposed by either study summarized by Ingólfsson et al. (2013). However, neither suggestion is proven to be true or to be wrong. Simulations with this artificial setup can’t catch up all through e.g. locally specific climate conditions triggered ice cover characteristics.

6.4 Simulated deglaciation from the Younger Dryas until present

The ice thickness and ice extent decrease in the beginning of the simulation relatively fast (Figure 40). On land the ice starts to vanish at first, starting from the north and from the west. The ocean is never totally ice free, marine ice sheets remain. Thin ice on land is feasible for present ice conditions on Svalbard but the thick marine ice (600m) e.g. at the northern end of Storfjorden and in Hinlopen Strait and other fjords is of course not matching the reality. This happens even though the oceanic melting coefficient increased to 5 m/year for this run.

The total ice volume decreases in this simulation from \(3,8 \times 10^{13} \text{ km}^3\) to \(2,2 \times 10^{13} \text{ km}^3\) (Figure 41), which corresponds to 58% of the original ice volume. The difference between the first approx. 2 000 years, from 12 000 BP until 10 000 BP, is the largest with almost \(0,6 \times 10^{13} \text{ km}^3\). The total amount of floating ice is gone already after some 100 years. The ice volume follows the GRIP temperature curve in a general way. After the first 2 000 years of fast reduction, the ice decrease slows down. The temperature curve is behaving similarly: the temperature is highly increasing within the first 2000 years and afterwards leveling off around 0°C. Smaller changes in temperature are not reflected in the ice sheet development, which is likely as the climate forcing was realized by a shift in Pelto mass balance curve. As we saw in previous runs, a change in sea level is in particular for the hybrid model not influencing the ice sheet and thus there is no correlation between the sea level curve and the total ice volume visible (Figure 41).

Comparing this results with those Landvik et al. (1998) came up with, one notices that the ice volumes at the starting point 12 000 years BP are not matching. In this study, the immense volume of \(3,8 \times 10^{13} \text{ km}^3\) is the basis, whereas the ice mass at that time consists of only 200 000 km³ in the model of Landvik et al. (1998). However, the deglaciation pattern differs not that much. At the end of the simulation of this study, thick ice (600m) is left at Storfjorden, Olav V Land and the southern opening of the Hinlopen Strait. At the end of the simulation, Landvik et al. (1998) show ice of about 400m thickness at the same
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locations. The main difference however is, that their model needs only 2000 years for this development whereas the model of this study with the described setting needs 12,000 years. This means, that the pattern of deglaciation is presented consistently. Ice free conditions in the ocean and thus more realistic conditions might be resulting when the deglaciation in this study is started with a smaller ice volume. It is possibly due to the huge ice volume, that it took so long time to reduce the ice cover. Applying better adjusted and refined forcing fields (e.g. temperature, precipitation, oceanic melting) shall be tested in the model of this study, ARCTIC-TARAH, to improve response and response time between external forcing and ice cover in both, spin-up and specially adapted runs.

6.5 Future work

High spatial resolution is always preferred as more details of the topography and thus the modeled results of e.g. ice thickness, spatial evolution and velocity are included. Since the domain studied is relatively small a higher resolution (1-minute grid instead of 2-minute grid) is worth testing. This is not done in this study to be able to provide simulations with longer time scale and to omit script failures. On the other hand, the relatively short run time prohibits the ice extension eastward over the Barents Sea, which could be compensated with different (ice-ocean interface) forcing in combination deforming lithosphere. To expand this short time scale into a longer time series and further, to enlarge this local study into a more regional study are second prospects. The latter especially associated with more sophisticated test regarding oceanic melting and calving rate, as runs (here not discussed) with smaller oceanic forcing parameters resulted in an enormous extent of floating ice, reaching all domain boundaries.

A second issue to be improved within a development of the ice code ARCTIC-TARAH is to include a smoothing parameter or factor, such that the ice cover does not follow the topography in the same way as it does in this study. Performance of test runs with different enhancement factors (controlling viscosity of the ice) is a possible approach to start dealing with this concern.
7 Conclusion

When choosing between a hybrid and a SIA model, the hybrid is the preferred model for coupled handling of ice sheets and ice shelves. The tests performed in this study regarding oceanic forcing and the impact of subglacial sediments exposed how the two types of numerical ice sheet models behave. The tests showed that the simplistic SIA models are susceptible to changes in sea level, while the more advanced hybrid models are more susceptible to changes in subglacial sediment/sliding coefficients than to sea level change. Further, analytical tests (not all discussed in detail) show that the model is very susceptible to changes in ice-ocean interface parameters. This answers the question posed at the beginning, asking for the impact changes in ice-ocean interface parameters and subglacial bed conditions have on the evolving ice cover. The main differences are due to the limitation of SIA models to be restricted to ice sheet dynamics. The SIA runs, however, achieve a pronounced equilibrium of total ice volume, while the hybrid runs only show a first sign of a leveling off, which is expected to become an equilibrium for longer simulation periods.

In numerical modeling, the restriction in spatial resolution and spatial extension of the modeling domain together with a long simulation time is always to concern about. In this study, the run time restriction prohibits the ice to extent further towards the north-east, east over the subsea plane and over the Barents Sea. In this way, the simulations performed for this study are short time simulations. Nevertheless, very good agreement between simulated and data-based reconstructions of Svalbard’s ice cover is achieved at the western margin. Further, location of ice streams and ice domes are corresponding to data-based suggestions. This, finally, answers the first question posed at the beginning, how well modeled ice streams coincide with data-based reconstructed ones.

The reconstruction of the ice cover from the Younger Dryas until present reflects only a general development, although the steep temperature increase at the beginning of the time interval is mirrored in the ice volume decrease. Small changes in temperature don’t have an influence under the settings used in this study. The ice thickness reaches feasible values only on land, while thick marine ice is remaining on the fjords.

For future work, a longer time scale and a refined grid are proposed. Further, functionality test regarding ice-ocean interface (including
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oceanic forcing parameters) and investigations of subglacial sliding coefficients shall be undertaken in the first place. More, the response of external forcing fields and some smoothing parameter should be considered in further development of the ice code ARCTIC-TARAH.
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