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Negotiating meaning in cross-national studies of mathematics teaching:

Kissing frogs to find princes

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Abstract

This paper outlines the iterative processes by which a multinational team of researchers developed a low-inference framework for the analysis of video-recordings of mathematics lessons drawn from Flemish Belgium, England, Finland, Hungary and Spain. Located within a theoretical framework concerning learning as the negotiation of meaning we discuss problems of linguistic and conceptual equivalence and the manner by which they were resolved. Significantly, when compared with the time-stamped codes of projects like the TIMSS video studies, we argue that the unit of analysis adopted, the episode, allowed for the distinctive patterns of a lesson to be retained for comparison with others. Also, we suggest that the framework's generic, though subject-focused, codes are amenable to adaptation to other curriculum areas, thus providing an opportunity for the comparative study of subjects not normally associated with work of this nature.

Introduction

Comparative studies of mathematics education conventionally fall into two camps. Large scale quantitative studies compare the achievement of learners in one country with those in others. Such studies are helpful in alerting a system's participants not only to its mathematical attainment relative to other systems but also, by means of test repeats, to previous performance. Small scale qualitative studies share a common characteristic of seeking insight into the ways in which mathematics is systemically conceptualised and presented to learners in different countries. Significantly, they highlight the emphases "placed on the two central roles of formal schooling systems, namely the inculcation of knowledge and skills on the one hand (the cognitive function) and on the other, the shaping of values and attitudes in preparation for the future role of citizen (the affective function)" (Osborn, 2004, p.267, author's parentheses).

Recent research has indicated that teachers behave in culturally-determined ways to the extent that differences between cultures are greater than those within (Schmidt et al, 1996; Givvin et al, 2005). This is probably not surprising, particularly when comparing Eastern and Western traditions with their respective Confucian and Socratic underpinnings (Leung, 1995; Tweed and Lehman, 2002). Various descriptions have been applied to this sense of predictability. Stigler and Hiebert (1999) discuss *cultural scripts*, Hiebert et al (2003) write of *lesson signatures* and Schmidt et al (1996) highlight *characteristic pedagogical flows* (Schmidt et al, 1996) which embody "the pedagogical strategies and approaches typical of a set of lessons" which are "enacted repeatedly in a country's classrooms" and, which, appear almost "below the conscious level for most teachers" (Cogan and Schmidt 1999, p71).

However, the identification of national patterns of didactic behaviour is no straightforward process; Stigler and Perry (1990), for example, found that every examined dimension of classroom activity differentiated the systems under scrutiny. Moreover, if an objective of comparative research is a better understanding of how learning might be managed, then such

problems are exacerbated by research implicating socio-cultural factors as predictors of systemic attainment. For example, and perhaps unsurprisingly, systems in which students have positive dispositions to school attain more highly than those with high levels of disaffection (Elliot et al, 2002; Hufton and Elliott, 2000).

Frameworks for comparing comparative studies

Although writers such as Chabbott and Elliott (2003, p.13) have categorised comparative research in ways that avoid “false dichotomies” such as “large-scale versus small scale” or “quantitative versus qualitative” many studies are categorised well by them. In general, large scale studies, which are almost always quantitative, examine the processes and outcomes of educational systems (Postlethwaite, 1988). They have breadth and are able to frame their findings within a more generalised systemic context (Kaiser 1999). Recent examples of such studies in mathematics have been the third international mathematics and science study (TIMSS) and its repeats (Beaton et al, 1996, Mullis et al, 2000, 2004) and the programme of international student assessment (PISA) and its repeat (OECD 2001, 2004). Small scale studies are classroom-focused, generally qualitative, and provide evidence for informing classroom practice (Kaiser 1999, Postlethwaite 1988). They provide a richness of data (Theisen and Adams 1990, Chabbott and Elliott, 2003) and account for the influence of context (Schmidt and McKnight, 1995, Osborn, 2004). They are inexpensive straightforwardly managed, and require little “diplomatic capital” (Chabbott and Elliott, 2003, p.23).

Another means by which comparative studies may be compared derives from the distinction between the globalisation and internationalisation of education. Globalisation has been described as an increasing awareness of the world as one (Atweh et al, 2003) with the globalisation of mathematics being an acceptance of the possibility of a curriculum to which all schools systems would subscribe (Clarke, 2003). The internationalisation of mathematics celebrates differences rather than eliminates them, aiming to identify the adaptive potential of one system's practices for another while acknowledging the culturally located traditions of both (Clarke, 2004). In short, globalisation is a process of curricular convergence, whether knowing or unknowing, while internationalisation acknowledges and celebrates diversity.

Large scale studies have been criticised from a number of perspectives. They are costly and cumbersome (Noah, 1988). They create problems of scale construction and data validity and reliability (William, 1998; Keitel and Kilpatrick, 1999; Kitchen, 2000). Moreover, with English the usual project language, there is frequent ethnocentric bias in project management and dissemination (William, 1998, Keitel and Kilpatrick, 1999). Significantly, they have been described as attempts to globalise education, to the extent that participants are unaware of the process (Keitel and Kilpatrick, 1999), and are increasingly construed as competitions with inevitable winners and losers (Schmidt and McKnight 1995; Stronach, 1999). They lack detail (Theisen and Adams, 1990) and rarely account for those cultural issues which have been implicated in educational attainment (Osborn, 2004; Brown, 1996; Shen 2001).

Small scale studies are prone to problems of reliability and unacknowledged cultural differences on the part of researchers (Theisen and Adams 1990), particularly when those involved in the work are from within the system itself (Schmidt et al 1996) and at worst, may lead to “unwarranted correlation” and “unfounded speculation” (Jenkins 2000, p.137). Such studies, in their celebration of context, accord with the processes of internationalisation and allow for the culturally coherent educational practices of one country to be analysed and assessed for its adaptive potential for another (Clarke, 2004).

Perspectives on comparative studies of mathematics education

According to Robitaille and Travers (1992, p.687) more “attention appears to have been given to international comparisons in mathematics than any other areas of the curriculum”, an assertion justified by their perception that mathematics figures in all curricula, has much content

in common and is perceived to embody an international language. We argue that the last two assumptions are not beyond challenge. As indicated above, the best known large scale comparative studies of mathematics education have been the three TIMSS, which have examined learners' mathematical competence at ages 10 and 14 in around forty countries, and the two PISA studies which have assessed the abilities of learners at age 15 to apply their mathematical knowledge. Both have been criticised from the perspectives discussed above in relation to large scale studies. Significantly, deriving from a challenge to Robitaille and Travers' (1992) assertion of common curricular content, a particular criticism is that they have globalised mathematics by creating a myth of curricular conformity (Kitchen 2000, Clarke 2003). Further, the last of Robitaille and Travers' (1992) assumptions, as will become evident, is challenged by the work we report below.

In respect of small scale studies of mathematics teaching it is interesting to note that this, too, has been dominated by the Anglophone world. Moreover, an interesting characteristic of much of this research has been the attention paid to particular educational systems by single research teams from another. For example, much US work has been dominated by Stigler and his colleagues and its attention to the mathematics teaching traditions of Japan (Jacobs et al, 1997; Stigler et al, 1987, 1996; Stigler and Perry, 1990; Stigler and Hiebert, 1999; Stevenson and Stigler, 1992). Clearly such studies do not represent the totality of small scale comparative studies although Japan remains a recurrent theme in the US (Whitman and Lai, 1990; Becker et al, 1999). Similar traditions have emerged in England. Andrews and Hatch have compared the teaching of mathematics in England and Hungary (Hatch, 1999; Andrews and Hatch, 2000, 2001; Andrews, 1999, 2003, 2006) while Pepin and Haggarty have examined France, Germany and England (Haggarty and Pepin, 2002; Pepin and Haggarty, 2002; Pepin, 1999, 2002).

More recently, several studies have focused simultaneously on several countries' mathematics education traditions and practices. Without discussing the minutiae of their respective data collection and analysis procedures, we shall summarise their key characteristics from the perspective of informing our work. Three of these emerged as supplementary studies of TIMSS. The first of these, the survey of science and mathematics opportunities (SMSO) (Schmidt et al, 1996), by means of observations undertaken by home researchers, examined the mathematics and science pedagogic traditions of six countries. Lessons were evaluated against protocols developed by a multinational team in an iterative and bottom up interrogative process intended to make explicit that which had been implicit in an observer's analysis. The study has been criticised for its small sample size – around ten mathematics lessons were observed in each country – although the "characteristic pedagogical flow" which emerged has proved an extremely helpful construct for comparative researchers.

The TIMSS video study focused attention on three countries, Germany, Japan and the US. The objectives included the development of objective measures of classroom instruction, the comparison of mathematics teaching in the three countries at the grade eight and an assessment of the feasibility of a wider-scale video study. Like the SMSO, the multinational nature of its personnel, particularly in respect of "creating a coding scheme that fairly described teaching in each country" (Stigler et al, 2000, p.92) was exploited. Data were collected from 231 schools across project countries with one grade eight mathematics lesson selected in each. Each lesson was captured with a single camera focused on what an ideal student would be doing during the lesson (Jacobs et al, 1999). The project team claimed to have overcome many of the inadequacies of both large and small comparative studies by adopting a blend of qualitative and quantitative approaches to data collection and analysis. However, the project's cycle of qualitative analyses of a small number of lessons to identify issues for systematic coding and quantitative analysis (Jacobs et al, 1999) represented a disappointing reduction and lack of effective exploitation of high quality data.

The TIMSS 1999 video study, comprising 638 lessons from seven countries including a reanalysis of the original Japanese lessons, addressed the third of the earlier project's objectives (Hiebert et al, 2003). However, the criticisms above concerning the failure to exploit the quality of the data continue to apply as lessons were again reduced to codes. Indeed, a secondary analysis (Givvin et al, 2005) parsed lessons into "increments of 10 percent of the lapsed lesson time" to provide eleven points at which codes would be applied. This, though helpful in addressing the questions they had posed, provided another example of a disappointing reduction of rich qualitative data. Indeed, in both the TIMSS video studies the use of qualitative approaches were subordinated to quantitative to the extent that all the publications of which we are aware are derive, essentially, from quantitative analyses.

The learner's perspective project (Clarke, 2002) set out to examine mathematics and science teaching in nine countries. Unlike previous studies the LPS paid explicit attention to both teachers and learners and did not, as in the manner of the TIMSS video studies, reduce the data to codes. Unusually, data were collected by means of three cameras focused on the teacher, an ideal student and the whole class. All three images were combined on a single screen which was used to stimulate discussion in subsequent interviews with different participants. In respect of mathematics, data were collected in each country for ten successive grade eight lessons for each of three teachers, from demographically different schools and assessed against local criteria as effective. However, as with the TIMSS video studies, the topics of the videotaped lessons were not specified. The project identified a number of teacher behaviours which varied in form and function in the different project countries reflecting long held traditions of and expectations for the educational process.

The mathematics education traditions of Europe (METE) project

The work described here derives from the European Union funded mathematics education traditions of Europe (METE) project; a comparative study of the teaching of mathematics in five countries: Flemish Belgium, England, Finland, Hungary and Spain. These countries represent well the social and cultural diversity of the continent and substantial variation in achievement on international tests like TIMSS and PISA. A major aim of the study was to identify the patterns of didactic behaviour, should they exist, of participant countries and assess them for their adaptive potential.

One objective was to examine how learning was structured over the duration of a topic by focusing on sequences of lessons taught on topics agreed during project discussions to be representative of all project countries' curricula. Initially data collection was to have been managed by means of live observations but events, which we outline below, initiated methodological changes leading to the main data set being derived from video recordings made by each home team. However, to ensure the elimination of cultural bias from subsequent analyses, data collection was preceded by an extensive programme of live observations which enabled the development of a descriptive framework for the analysis of videotaped lessons. This paper is concerned with the processes by which this framework was developed and the insights generated into the problems and rewards of working in an international team.

An additional objective was to focus on the teaching of mathematics in the age range 10-14 as this represents a period of significant change in the ways that mathematics is conceptualised and taught in most systems. During this period mathematics learning moves from a concrete and inductive experience towards an abstract and deductive experience. Therefore the project team decided to focus attention on the teaching of

- percentages (a topic of arithmetic applicability) in grades 5 or 6
- polygons (a routine geometrical topic) in grades 5 or 6
- polygons (a routine geometrical topic and an examine of curriculum continuity) in grades 7 or 8

- linear equations (an early topic of formal algebra) in grades 7 or 8.

Also, in the manner of the learner's perspective study (Clarke, 2002), the project focused on teachers defined by local criteria to be effective. In so doing we hoped to maximise the adaptive potential while minimising the possibility of bias in respect of teacher selection by project teams. Further, the insistence that teachers would be video-taped over four or five successive lessons provided two additional advantages over studies like the TIMSS video study. Firstly, by focusing on topics one significant variable in respect of classroom activity was controlled. Secondly, by insisting on sequences of lesson, the likelihood of teachers presenting atypical or show-piece lessons was minimised.

Theoretical framework

According to Lerman (2001) learning is a consequence of "social interactions" which "along with physical and textual interactions can cause disequilibrium in the individual, leading to conceptual reorganisation" (Lerman, 2001, 55). Traditionally teachers present tasks and invite interactions from which learning will derive, which, he suggests, is an inadequate description as learning is a consequence of planned interactions informed by hidden or unpredictable influences. He writes that a *learning activity* is "certainly the task set by the teacher but is also a function of the style of classroom interaction, the texts, the ethos of the school, the possibilities arising out of the particular mix of the actors that day... It does not exist, for the class, for the teacher, or for the individual students, before the interaction." (57). This notion of learning activity informed our work considerably.

All research is, in one way or another, conceptually-located and the development of our descriptive framework was rooted initially in the concepts that colleagues brought to the negotiating table. Lerman (1996) writes that as concepts derive their meaning from being used, their acquisition can be interpreted as the result of an individual coming to share meaning through negotiation. However, we must be careful not to assume that terms such as negotiation are unproblematic. According to Cobb and Bauersfeld (1995) negotiation is the "interactive accomplishment of inter-subjectivity" (295) or shared understanding. However, while one product of negotiation may be enhanced intersubjectivity, the nature of the negotiative process requires a prerequisite intersubjectivity that enables and sustains that negotiation (Clarke 2001). Moreover, negotiation is dependent on language which is itself intersubjective (Clarke 2001). As will be shown below, this prerequisite sense of intersubjectivity was initially missing in our work to the extent that it had to be developed over several months before significant progress could be made.

Hiebert et al (2003) have likened research to the use of a lens; a wide angle highlights similarity of practice while a close-up emphasises differences. This is a helpful metaphor, particularly if the classroom is construed as a piece of fabric whose pattern we wish to distinguish from that of other fabrics. However, it is our view that both long and short lenses offer different forms of similarity which prevent our seeing the distinctive patterns. A wide-angle lens alerts us to the existence of similarly patterned fabrics, and, we believe, all formal classrooms are similarly patterned at this level of detail. A telescopic lens highlights individual threads with a different sense of similarity; we can infer little of the fabric's pattern from the colours on an individual thread. As a team we wanted a lens that that would highlight the distinctive patterns of a fabric rather than the particular similarities yielded by long and short lenses.

Osborn (2004) has indicated that successful comparative research is dependent on the researcher satisfying four criteria. These are:

- *conceptual equivalence*: determining "whether the concepts under study have any equivalent meaning in the cultures under study" (Osborn, 2004, p.269)
- *equivalence of measurement*: "developing equivalent indicators for the concepts" as "concepts may differ in their salience" (p.269).

- *linguistic equivalence*: ensuring that the words used in different languages reflect the same meaning in each
- *sampling problems*: establishing appropriate sampling criteria is one way of securing comparability in such work.

As our work developed the significance of these issues, particularly conceptual and linguistic equivalence, became increasingly clear.

The research process

The following tells the story of a journey which started with an initial team meeting in Leuven, Belgium in February 2003 and ended in Huelva, Spain in January 2004. It charts our growing understanding of each others' perspectives on mathematics education and the impact of those perspectives on the ways in which colleagues saw and described the activities of mathematics classrooms. It charts, also, the development of a descriptive framework for mathematics classrooms which distinguished between the different fabrics of project classrooms and which all colleagues were able to make operational.

At this first meeting, to help us understand each others' context, each team gave a short presentation, framed against predetermined criteria, on its country's educational traditions. As SMSO colleagues found, "presentations planned to take 15 or 20 minutes took hours" although it was "time usefully spent in listening, questioning and interacting" (Schmidt et al, 1996, p.7) for as we talked, concepts assumed to be universally understood were found to have contextually located meanings. For example, the English used *pedagogy* as others used *didactics*; to signify an articulated theory of subject teaching rather than the more generally accepted "act of teaching and the body of knowledge, argument and evidence in which it is embedded and by which particular classroom practices are justified" (Alexander, 2004, p.10). We found, also, that the word *class* did not always refer to a group of students gathered in a *classroom*. In Hungary it meant also the totality of all students in a particular grade. Such a use may be reflection of the Hungarian use of repeat years to the extent that labels like *year group*, used in England where repeat years are unknown, may be inappropriate.

One objective for this first meeting had been the development of a descriptive framework for mathematics classroom activity which we had assumed would be derived from the literature. However, the unexpected conceptual and linguistic difficulties indicated that too little common ground was shared for us to proceed as intended. Consequently, the original plan to undertake two rounds of systematic live observations was replaced by a plan to undertake a single round of observations, allowing sufficient time for a descriptive framework which all understood to emerge, to be used with videotaped lessons filmed during the second phase of the project. We had come to understand that colleagues would not be able to describe mathematics classrooms in ways recognisable to others unless all had experienced the same, culturally located, lessons. Eventually we agreed that one week of live observations would be undertaken in each country and the following describes that process.

A single lesson was observed in a different school each morning by a team comprising at least one member from each project country. During this time a *home* observer would provide a quiet commentary to facilitate foreigners' access to the discourse. Afterwards observers returned to the host institution to discuss what had been seen and to relate the lessons' activities to the developing descriptive framework. This was accompanied by the video recording made the same morning. The first playing of the videotape, typically lasting upwards of two hours, facilitated the clarification of fact. Thus colleagues were able to refine and amend any notes made during the observations. Once colleagues were satisfied they had a clear sense as to the structure of the lesson and its activities, discussion and schedule development began. This process included a second viewing of the videotape and the episodic manner in which this occurred was to prove a pointer for later decisions regarding units of analysis.

Systematic observation of lessons and framework development

The first week of observations took place in Cambridge in June 2003. It had been agreed, as all colleagues spoke English, that this would maximise access to the classroom discourse at this early stage of the project's development. During the week the strength of colleagues' culturally-informed expectations in respect of effective teaching became increasingly apparent. Indeed, most were unable to suspend the desire to evaluate teachers' efforts against unarticulated indicators of teaching quality. However, as the meetings and discussions progressed we became more skilled at making objective statements about what was seen rather than what was missing. That said, it was clear that colleagues tended to assume that what they were seeing, despite the influence of their respective cultural lenses, was also seen by others.

By the end of the third day a set of descriptors, comprising almost four pages of activities and interactions, had emerged. These were loosely structured by the phases, or structural episodes, through which lessons were perceived to pass and included introduction, exposition, problem posing, problem solving, problem sharing, seatwork and plenary. Each of these comprised various subcategories. For example, exposition included; occasional input from students, frequent input from students, explicit conceptual development, activating prior knowledge and skills, invoking visualisation, use of mathematical terminology, use of manipulatives, use of teaching aids, use of teacher's body, explicit structural development, demonstration, modelling, process orientation and performance orientation. At this stage of its development, the framework looked, as it should, the result of open-ended discussions about observable behaviours that might typify and distinguish mathematics classrooms.

An issue to vex the team throughout the development of our framework concerned not only the ways in which teachers initiated the tasks on which students work but, importantly, the nature of the tasks themselves and the manner in which they supported the learning of different aspects of mathematics. Initially the problem posing phase included subcategories like realistic problems, quasi-realistic problems, mathematical problems, open problems, closed problems, multiple response problems, single response problems, process-oriented problems, performance-oriented problems. Some of these categories were influenced, in accordance with earlier expectations, by colleagues' understanding of the literature. Initially this seemed a principled way in which to work but it created substantial emotional and intellectual barriers for some colleagues. For example, the descriptions of realistic and quasi-realistic derived the Dutch tradition of realistic mathematics education (RME). However, for some colleagues the word realistic had an unambiguous connection with some sense of the real world. But, as Van den Heuvel-Panhuizen (2002, p.3) (her capitalisation) writes, it concerns

"... not just the connection with the real world, but ... the emphasis that RME puts on offering the students problem situations which they can imagine. The Dutch translation of the verb 'to imagine' is 'zich REALISeren'. It is this emphasis on making something real in your mind that gave RME its name."

She goes on to say that any context, real-world, formal mathematics or even fairy tale, is suitable for a problem provided it is "real in the student's mind". This sense of the imaginable as realistic was to prove problematic for some colleagues and became an issue to which we returned over several months.

During the week of English observations three classroom-based activities challenged colleagues' expectations of a mathematics lesson and focused discussion. The first concerned the notion of *coursework*. In such lessons, usually one of several consecutive lessons, students work, usually independently of both teacher and peers, on an extended task before writing an individual report summarising what had been done and learnt. According to colleagues' comments, such activity was unknown in other project countries, either as a learning opportunity or as a means of

assessment. The second concerned what is known as a *CAME* (cognitive acceleration in mathematics education) lesson. This is a stand alone lesson, separate from the daily and continuous delivery of curriculum content, in which learners work collaboratively on tasks designed to develop higher order reasoning skills. Again, colleagues indicated that stand-alone lessons were unknown in their respective contexts with several, most notably the Finnish and Hungarian, commenting that the development of mathematical reasoning took place continuously through the problems on which learners worked on a daily basis. The third concerned the *oral and mental starter*. This is an activity, frequently unrelated to the intended learning outcomes of the lesson, intended to facilitate students' mental warm-up. While the notion of a warm-up itself was not unfamiliar to project colleagues, the prominence given in both time and systemic encouragement was unfamiliar to colleagues from outside England. Such issues reminded us well of our culturally-located expectations of mathematics classroom traditions and that the script followed by individual teachers is informed by a number of factors including systemic expectations and as indicated above, unarticulated and assumed beliefs about effectiveness. They also suggested that the choice of England for the first week's observations was helpful, beyond the original expectation of linguistic accessibility, in framing our discussions. Moreover, the divergence of such single lessons from colleagues' expectations and traditions reinforced the importance of the decision to focus on sequences of lessons taught on common topics.

During the first half of the second week of observations in Joensuu, Finland, in September 2003, some of the categories developed in Cambridge were discarded for being too particular while new ones, prompted by the Finnish lessons, were proposed. However, as time progressed an awareness developed that too many insufficiently general categories were being proposed. Moreover, it was becoming increasingly clear that the intention of exposing the patterns of a fabric in ways that would distinguish one from another was being compromised by an instrument that was highlighting individual threads. This led the team to reconsider its approach, although one colleague's comment, *you have to kiss a lot of frogs to find your prince*, reminded us that the development of a set of descriptors that fulfilled our objectives would require the examination of a much larger set.

As indicated above, lessons were already being construed as passing through phases and although observations alerted us to the naivety of our assumption that all lessons comprised equivalent phases, it became increasingly clear that all lessons were episodically structured. Clarke (b) stratifies classroom discourse into six levels - the lesson, the activity, the episode, the negotiative event, the turn and the utterance. The view of the team was that the first two units would yield the image of the wide-angle lens while the last three that of the telescopic. The episode, however, would allow one fabric, or lesson, to be distinguished from another. Importantly, an episodic analysis would retain the structure, and therefore the unique characteristics, of an individual lesson in ways that time-determined codes as used in the TIMSS video studies (Stigler et al, 2000, Givvin et al, 2005) could not. At this point we switched from a search for behaviours or activities characteristic of particular phases to those which could be generically applied to any episode.

Over the next four months the process of observation and discussion was repeated in Budapest, Leuven and Huelva. The outcomes were discussed by the whole project team at periodic meetings in Leuven and refined by means of an interrogative process not dissimilar to that employed by the SMSO team; colleagues who had not been present at the observations challenged the categories proposed by those who had. The definition of an episode emerged gradually as that part of a lesson in which the teacher's observable didactic intention remained constant. Thus, for example, an episode could be short and managerial, as in the calling of a register, or long and didactic as in a period of seatwork during which learners complete exercises posed from a text book. The first set of generic categories fell into four forms which structured future developments. The mathematical focus of an episode referred to the observable

learning objectives intended by the teacher. The mathematical context referred to the location, in a realistic mathematics education sense, of the activities of an episode. Activities inducing referred to the level of learning likely to result from the activities of an episode while the didactics referred to the strategies employed by the teacher during an episode.

Over the subsequent months the activities inducing category was removed. The team believed that the cognitive demands of a task should be acknowledged but decided that determining whether or not a particular task was likely to lead to surface or deep learning was an inference beyond which it should not go. However, the mathematical focus subcategory eventually included observable learning objectives that could be implicated in surface learning – an emphasis on procedural acquisition – and deep learning – an emphasis on problem solving or reasoning. Thus, the team had moved from a high inference to a low inference categorisation. This aligned better with the objective of constructing a framework which colleagues would be able to implement straightforwardly.

Also, despite lengthy discussions based on an examination of RME-related literature and several attempts to operationalise RME-derived categories against the activities of videotaped lessons, it became increasingly clear that we were far from achieving conceptual and linguistic equivalence. It was interesting to note that despite the passion that accompanied many of our discussions, the only serious tensions occurred when we tried to use RME-based categories to describe the context of mathematical tasks. Colleagues familiar with the RME tradition were becoming frustrated at others' failure to operationalise what they perceived to be a simple set of categorisations while those unfamiliar with it felt marginalised by a process which they perceived to be forcing them to work with concepts alien to their understanding of mathematics education.

This RME-related discussion reinforced the importance of acknowledging and then suspending our culturally-informed perspectives on the nature of mathematics and its teaching. Indeed, this particular episode highlighted well societal expectations in respect of curriculum content and our burgeoning awareness that different educational traditions have different ways of "choosing, preparing and evaluating mathematical topics for teaching purposes" (Winkelmann, 1994, p.11). However, the team retained a desire to include some means of classifying the tasks on which learners work and eventually, having finally agreed that a continued emphasis on realistic mathematics education excluded too many colleagues, adopted a categorisation based on whether or not a task is located explicitly in a real world context and whether or not the data on which it depends is fabricated. Importantly, the team came to realise that concepts derived in one system may be difficult for those rooted in another to come understand and that just because something is reported in the literature does not mean it has an operational validity in systems other than that in which it was developed.

Throughout our visits we discussed regularly the vocabulary of mathematics. As we did it became apparent that the commonly-held view of mathematics as a universal language was a fallacy. Also, we came to see how the use of the vernacular may support the learning of mathematics more effectively than other linguistic traditions. As an example we draw on our discussion of geometry. One of the first things we noticed, which came as a surprise to colleagues from those two countries, was that it is only in English and Spanish that most of the vocabulary derives from Greek and Latin. The Latinate word quadrilateral, referring to a general four-sided polygon, bears little relationship to any vernacular English sense of four-ness although, due to its romance provenance, the Spanish *cuadrilátero* is clearly related to *cuatro*. In Flemish, Finnish and Hungarian the words *vierhoek*, *nelikulmio* and *négyszög* respectively refer to polygons with four angles while, in Hungarian, a general quadrilateral is an *általános négyszög* or general four angles. Other, interesting examples include the Finnish words *suunnikas* (parallelogram), which emphasises a sense of same direction and *puolisuunnikas*, or half parallelogram, for trapezium; the Hungarian use of *téglalap* (brick shape) for rectangle and *téglatest* (brick solid) for cuboid. The most consistent use of the vernacular was found in

Flanders where an octahedron is an *achtvlak*, or eight faces, whereas Greek had permeated both Finnish and Hungarian with *oktaedri* and *oktaéder*, respectively. It seems that Greek enters Flemish only when no natural or obvious vernacular word is available. So, for example, the word *prisma* is used primarily because of the need for a noun and there is no simple way to describe a prism by means of juxtaposed vernacular terms. It is our conjecture that where mathematical vocabulary derives from the vernacular, particularly in the structured manner found in countries like Hungary and Flanders, learner access to mathematics is enhanced. The use of Greek in both Spanish and English to describe polygons and polyhedra adds a layer of linguistic complexity not found in other countries.

By the end of the year at least two, sometimes three but, in respect of Hungary, four colleagues from each country had participated in the observations. Consequently, the expertise, including colleagues who had not participated in the observations but contributed to team meetings in Leuven, brought to the discussion was extensive. Eventually, following observations in Huelva, Spain, in January 2004, the team concluded that the episodes of each lesson would be coded against three broad categories of mathematics classroom activity, each of which comprised several codes perceived to be general in their applicability. The seven categories of mathematical focus, defined in Table 1, reflect a range of general mathematical learning outcomes. The four categories of mathematical context, defined in Table 2, examine the relationship between the tasks posed by teachers and aspects of either mathematics itself or the real world. The ten categories of mathematical didactics, defined in Table 3, reflect a range of teacher or teacher-instigated strategies. Importantly, all the codes addressed observable behaviours and made little expectation on the part of the observer to infer meaning. The schedule reflects a hard-won conceptual and linguistic equivalence and provides a lens of sufficient focal length for identifying the distinctive patterns of every project lesson. Indeed, as we show below, it is simple of use but has the sensitivity to highlight both similarity and difference in teachers' practices.

Using the schedule

In the year subsequent to its development the schedule was used in four of the five project countries; due to unforeseen circumstances the Finnish data has yet to be coded. However, to frame its use we offer a summary of the itinerary of the second year of the project. As indicated above, project colleagues collected data on the teaching of key mathematical topics by means of video recordings of sequences of lessons taught by teachers perceived by their local team to be representative of the better practice in that country. Defining good practice is clearly not straightforward and extensive discussion led us to the conclusion that it was so contextually-located that local conditions and traditions should hold sway in the manner of the learners' perspective study (Clarke, 2002). Thus, teachers were invited according to the recommendation of local teacher education institutions, local authority advisors, professional associations or other educational professionals. Teachers met with project colleagues who explained the nature of the project and its desire to film at least four, preferably five, successive lessons, starting with the first, on a particular topic. With the teacher's agreement, arrangements were made to visit the schools and tape the lessons. Clearly, as there was no guarantee when a particular topic might be taught in a teacher's annual programme, arrangements were frequently made several months before filming. In addition, teachers were interviewed before and after the sequence of lessons in order to ascertain their intended learning objectives, proposed didactic strategies and professional reflections.

As the coding schedule focused on the observable behaviour of teachers videographers were instructed to follow the teachers whenever they were talking and to use the zoom facility whenever circumstances demanded close attention on the teacher. They were instructed, also, to capture as much board-work as possible. In general a tripod-mounted camera was placed at the rear of the room. The teacher used a radio microphone while a telescopic microphone was placed strategically for capturing as much student-talk as possible. However, as the emphasis

was on teachers, and the manner in which they structured opportunities for learning, it was felt that lost student talk was unfortunate but not a significant threat to the integrity of data collection. In some countries, due to local decisions to augment the data obtained, a second camera was used. In general this was placed near the front of the room and focused almost exclusively on the students.

After filming, the digital videotapes were compressed and each lesson transferred to a CD ROM for copying and distributing to other project teams. Each lesson was coded by its home team against the project's schedule. Additionally, the first two lessons in each sequence were transcribed using TransTool® and then, where appropriate, translated into English. This allowed colleagues to code other countries' lessons in order to establish satisfactory levels of inter-coder reliability. Each national team was given responsibility for the analysis of different aspects of the project's data. The English team's role was to focus on the analysis of the coding sheets while each of the other teams undertook qualitative analyses of the lessons of a particular topic. To ensure satisfactory levels of inter-coder reliability, two lessons from each project team were independently coded by the English team. Cohen Kappa coefficients were calculated which showed, assuming a kappa value of 0.75 as acceptable, high levels of inter-coder reliability between the coders of England and Flanders ($\kappa = 0.877$), England and Hungary ($\kappa = 0.875$) and acceptable levels for England and Spain ($\kappa = 0.796$).

During 2004 the English team subjected the codes to a variety of descriptive and inferential analyses. Initial findings from these processes – separate and combined analyses of the mathematical foci and mathematical didactics – have been presented and an analysis of their interactions is underway. The results of the single analyses confirm not only that teachers' intended objectives vary according to nationality (Andrews et al, 2005) but also that differences in the didactic strategies they employ distinguish those in one country from those in another (Andrews and Sayers, forthcoming). Interestingly, the findings of the combined study (Andrews, 2005) not only allude to the existence of cultural scripts but also typologies of practice which transcend cultural boundaries. Moreover, unlike the TIMSS video studies, the project team has undertaken several qualitative analyses of the videotaped lessons which have yielded some interesting outcomes in respect of how different topics are conceptualised and delivered to learners (Depaepe et al, 2005; Climent and Carrillo, 2005). These analyses are also ongoing.

Concluding thoughts

It seems clear to us that the development of our descriptive framework was a learning activity in the sense described above. Each of us, through the observation and discussion of lessons in countries other than our own and the exposure of the pre-conceived notions of effective practice we brought to the process, learnt more about the teaching of mathematics and the traditions underpinning teachers' activity than we could by remaining at home. We struggled to acquire conceptual and linguistic equivalence and the intersubjectivity necessary for successful negotiation took time to establish. Lerman (1996) writes that "intersubjectivity is a function of the time and place and the goals of the activity and the actors" (137) and goes on to say that the role of the teacher is to assist the learner "in appropriating the culture of the community of mathematicians as a further social practice" (146). This describes well the processes by which our descriptive framework was developed. Our efforts were clearly focused on establishing a community of practice through, at different times for different colleagues, legitimate peripheral participation (Lave and Wenger, 1991).

As indicated throughout, language has been both barrier to and facilitator of learning. Indeed, ambiguity has dominated much of our work and reflects Clarke's (b) perception that humans interact as though they hold meanings in common but which are frequently misconstrued. In our case this may have been due to the unacknowledged and culturally-informed conceptual and linguistic perspectives that colleagues brought with them. However, the manner of our work,

focusing on the common experiences of participants, circumvented the futility of discussing "the meaning of a word or term in isolation from the discourse community of which the speaker claims membership" (Clarke 2001, p.36). As an example of mathematical ambiguity we offer the example of area. In Spanish *superficie* is used in all contexts pertaining to area and translates to surface. This is at variance with the English use of *area* to denote a measure of surface in the plane with *surface area* shifting attention to three dimensions. In Hungary *terület* refers explicitly to area in the plane with *felszín*, translating directly as surface with no connotation of area, representing the concept in three dimensions.

In describing the development of our observational schedule we have shown how our work reflected closely the processes of learning as a social construction. We have shown, also, how a collaborative and iterative approach led to a descriptive framework that achieved conceptual and linguistic equivalence, highlighted the patterns of the fabrics that are classrooms and was implementable for a multinational team drawn from diverse mathematics education traditions. Lastly, and this reflects an unexpected take on Clarke's adaptive potential, the generic nature of our schedule provides a framework for cross national studies of curriculum subjects not normally associated with such work. Thus a multinational team of history educators should be able to adapt our framework in a relatively straightforward manner to facilitate an examination of the practices of teachers of history in diverse educational contexts with, for example, causality construed as an example of a generic historical focus - the teacher is observed to encourage learners' understanding of causality as an element of historical analysis.

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References

- Alexander, R. (2004) Still no pedagogy? Principle, pragmatism and compliance in primary education, *Cambridge Journal of Education*, 34 (1), 7-33.
- Andrews, P. (1999) Looking behind the rhetoric: some new insights from Hungary, *Mathematics Teaching*, 167, 6-10.
- Andrews, P. (2003) Opportunities to learn in the Budapest mathematics classroom, *International Journal of Science and Mathematics Education*, 1 (2), 201-225.
- Andrews, P (2005) A comparative analysis of the educational objectives and didactic strategies of mathematics teachers from five European countries, *Paper presented to the biennial conference of the European Association for Research into Learning and Instruction (EARLI)*, University of Cyprus, August 2005.
- Andrews, P. (forthcoming) The curricular importance of mathematics: a comparison of English and Hungarian teachers' espoused beliefs, *Journal of Curriculum Studies*.
- Andrews, P. & Hatch, G. (2000) A comparison of Hungarian and English teachers' conceptions of mathematics and its teaching, *Educational Studies in Mathematics*, 43 (1), 31-64.
- Andrews, P. & Hatch, G. (2001) Hungary and its characteristic pedagogical flow, in: J Winter (Ed) *Proceedings of the fifth British Congress for Mathematics Education*, University of Keele.
- Andrews, P., Hatch, G. & Sayers, J. (2005) What do teachers of mathematics teach? An initial episodic analysis of four European traditions, in: D. Hewitt & A Noyes (Eds) *Proceedings of the Sixth British Congress on Mathematics Education*, University of Warwick.
- Andrews, P. and Sayers, J. (forthcoming) How do teachers of mathematics teach? A four-way international comparison, in: R. Barwell & A. Noyes (Eds) *Research in Mathematics Education*

Volume 7: Papers of the British Society for Research into Learning Mathematics (London, BSRLM)

Atweh, B., Clarkson, P., & Nebres, B. (2003) Mathematics Education in International and Global Context, in: F. Leung (Ed) *Second International Handbook of Mathematics Education: Volume 1* (Dordrecht, Kluwer) 185-229.

Beaton, A.E., Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., Kelly, D.L. and Smith, T.A. (1996) *Mathematics in the middle school years: IEA's third international mathematics and science study* (Boston MA, Boston College).

Becker, J., Sawada, T. and Shimizu, Y. (1999) Some findings of the US-Japan cross-cultural research on students problem-solving behaviour, in: G. Kaiser, E. Luna and I. Huntley (Eds), *International comparisons in mathematics education*, London, Falmer.

Brown, M. (1996) FIMS and SIMS: the first two IEA International Mathematics Surveys, *Assessment in Education: principles, policy and practice*, 3 (2), 193-212.

Chabbot, C. and Elliott, E. (Eds) (2003) *Understanding others, educating ourselves* (Washington, DC, The National Academies Press).

Clarke, D. (2001) Untangling uncertainty, negotiation and intersubjectivity, in: D. Clarke (Ed) *Perspectives on practice and meaning in mathematics and science classrooms* (Dordrecht, Kluwer) 33-52.

Clarke, D. (2002) The learner's perspective study: methodology as the enactment of a theory of practice, paper presented as part of the symposium, international perspectives on mathematics classrooms, at the *annual meeting of the American Educational Research Association*, New Orleans.

Clarke, D. (2003) International Comparative Studies in Mathematics Education, in: A. Bishop, M. Clements, C. Keitel, J. Kilpatrick, & F. Leung (Eds.) *Second International Handbook of Mathematics Education* (Dordrecht, Kluwer).

Clarke, D. (2004) Issues of voice and variation: The problematics of international comparative research in mathematics education, *Paper submitted to Discussion Group 11 of the tenth international congress on mathematics education (ICME 10): International comparisons in mathematics education*, Copenhagen, Denmark.

Climent, N. and Carrillo, J. (2005) Proyecto 'mete': Polígonos en primaria, in: A. Maz, B. Gomez and M Torralbo (Eds) *Investigación en educación matemática: noveno simposio de la sociedad Española de Educación Matemática SEIEM* (Cordoba, Universidad de Córdoba).

Cobb, P. & Bauersfeld, H. (1995) (Eds) *The emergence of mathematical reasoning: interaction in classroom culture* (Hillsdale, NJ, Lawrence Erlbaum).

Cogan, L.S. & Schmidt, W.H. (1999) An examination of instructional practices in six countries, in: G. Kaiser, E. Luna & I. Huntley (Eds) *International comparisons in mathematics education*, (London, Falmer).

Depaepe, F., De Corte, E., Op 't Eynde, P., and Verschaffel, L. (2005) Teaching percentages in the primary school: A four country comparative study, in: L. Verschaffel, E. De Corte, G. Kanselaar, and M. Valcke (Eds) *Powerful environments for promoting deep conceptual and strategic learning*, (Leuven, Leuven University Press), 147-171.

Elliott, J, Illushin, G. & Hufton, N. (2002) Educational motivation and engagement: qualitative accounts from three countries, *British Educational Research Journal*, 28 (2), 265-289.

Givvin, K., Hiebert, J., Jacobs, J., Hollingworth, H. & Gallimore, R. (2005), Are there national patterns of teaching? Evidence from the TIMSS 1999 video study, *Comparative Education Review*, 49 (3), 311-343.

- Haggarty, L & Pepin, B. (2002) An investigation of mathematics textbooks and their use in English, French and German classrooms: who gets an opportunity to learn what? *British Educational Research Journal*, 28 (4), 567-590.
- Hatch, G. (1999) It wouldn't be like that here, *Mathematics Teaching*, 168, 26-31.
- Hiebert, J., Gallimore, R., Garnier, H., Bogard Givvin, K., Hollingsworth, H., Jacobs, J., Miu-Ying Chui, A., Wearne, D., Smith, M., Kerstling, N., Manaster, A., Tseng, E., Etterbeek, W., Manaster, C., Gonzales, P. & Stigler, J. (2003) *Teaching mathematics in seven countries: results from the TIMSS 1999 video study* (Washington, National Center for Educational Statistics).
- Hufton, N. & Elliott, J. (2000) Motivation to learn: the pedagogical nexus in the Russian school: some implications for transnational research and policy borrowing, *Educational Studies*, 26 (1) 115-136.
- Jacobs, J., Makoto, Y, Stigler, J. & Fernandez, C. (1997) Japanese and American teachers evaluations of mathematics lessons: a new technique for exploring beliefs, *Journal of Mathematical Behaviour*, 16 (1), 7-24.
- Jacobs J., Kawanaka T. & Stigler J. (1999) Integrating qualitative and quantitative approaches to the analysis of video data on classroom teaching - An overview of the TIMSS Video Study, *International Journal of Educational Research*, 31 (8) 717-724.
- Jenkins, E., (2000). Making use of international comparisons of student achievement in science and mathematics, in: D. Shorrocks-Taylor & E. Jenkins (Eds) *Learning from others* (Dordrecht, Kluwer).
- Kaiser, G. (1999). International comparisons in mathematics education under the perspective of comparative education, in: G. Kaiser, E. Luna & I. Huntley (Eds.), *International comparisons in mathematics education* (London, Falmer) 140-150.
- Keitel, C. & Kilpatrick, J. (1999) The rationality and irrationality of international comparative studies, in: G. Kaiser, E. Luna and I. Huntley (Eds) *International comparisons in mathematics education* (London, Falmer).
- Kitchen, A. (2000) The need for caution when using the results of international mathematics testing to inform policy decisions in education in the member countries, in: D. Shorrocks-Taylor and E. Jenkins (Eds) *Learning from others* (Dordrecht, Kluwer).
- Lave, J. & Wenger, E. (1991) *Situated learning: legitimate peripheral participation* (Cambridge, Cambridge University Press).
- Lerman, S. (1996) Intersubjectivity in mathematics learning: a challenge to the radical constructivist paradigm, *Journal for Research in Mathematics Education*, 27 (2), 133-150.
- Lerman, S. (2001) Accounting for accounts of learning mathematics: reading the ZPD in videos and transcripts, in: D. Clarke (Ed) *Perspectives on practice and meaning in mathematics and science classrooms* (Dordrecht, Kluwer) 53-74.
- Leung, F. (1995) The mathematics classroom in Beijing, Hong Kong and London, *Educational Studies in Mathematics*, 29 (3), 297-325.
- Mullis, I., Martin, M., Gonzalez, E., Gregory, K., Garden, R., O'Connor, K., Chrostowski, S. & Smith, T. (2000) *TIMSS 1999 International mathematics report* (Boston, Boston College).
- Mullis, I., Martin, M., Gonzalez, E. & Chrostowski, S. (2004) *TIMSS 2003 International mathematics report* (Boston, Boston College).

- Noah, H. (1988) Methods of comparative education, in: T. Postlethwaite (Ed) *The encyclopaedia of comparative education and national systems of education* (Oxford, Pergamon).
- Organisation for economic co-operation and development (OECD) (2001) *Knowledge and skills for life: first results from the OECD programme for international student assessment (PISA) 2000* (Paris, OECD).
- Organisation for economic co-operation and development (OECD) (2004) *Learning for tomorrow's world: first results from PISA 2003* (Paris, OECD).
- Osborn, M. (2004) New methodologies for comparative research? Establishing 'constants' and 'contexts' in educational experience, *Oxford Review of Education*, 30 (2), 265-285.
- Pepin, B. (1999) The influence of national cultural traditions on pedagogy: classroom practices in England, France and Germany, in: J. Leach and B. Moon (Eds) *Learners and pedagogy* (London, Paul Chapman).
- Pepin, B. (2002) Different cultures, different meanings, different teaching, in: L. Haggarty (Ed) *Teaching mathematics in secondary schools* (London, Routledge Falmer).
- Pepin, B. & Haggarty, L. (2002) Making connections and seeing links: directed numbers in English, French and German textbooks, *Paper presented to the annual conference of the British Educational Research Association* (University of Exeter).
- Postlethwaite, T.N. (1988) Preface, in: T. Postlethwaite (Ed) *The encyclopaedia of comparative education and national systems of education* (Oxford, Pergamon).
- Robitaille, D. & Travers, K. (1992) International studies of achievement in mathematics, in D. Grouws (Ed) *Handbook of research on mathematics teaching and learning* (New York, Macmillan).
- Schmidt, W., Jorde, D., Cogan, L., Barrier, E., Gonzalo, I., Moser, U., Shimizu, K., Sawada, T., Valverde, G., McKnight, C., Prawat, R., Wiley, D., Raizen, S., Britton, E. & Wolfe, R. (1996). *Characterizing pedagogical flow* (Dordrecht, Kluwer).
- Schmidt, W. & McKnight, C. (1995) Surveying educational opportunity in mathematics and science: an international perspective, *Educational Evaluation and Policy Analysis*, 17 (3), 337-353.
- Shen, C. (2001) Social values associated with cross-national differences in mathematics and science achievement: a cross-national analysis, *Assessment in Education*, 8 (2), 193-223.
- Stevenson, H. & Stigler, J. (1992) *The learning gap* (London, Simon and Schuster).
- Stigler, J., Lee, S.-Y. & Stevenson, H. (1987) Mathematics classrooms in Japan, Taiwan, and the United States, *Child Development*, 58 (5), 1272-1285.
- Stigler, J. & Perry, M. (1990) Mathematics learning in Japanese, Chinese and American classrooms, in: J. Stigler, R. Shweder & G. Herdt (Eds) *Cultural Psychology: essays on comparative human development* (Cambridge, Cambridge University Press).
- Stigler, J., Fernandez, C. & Yoshida, M. (1996) Traditions of school mathematics in Japanese and American elementary classrooms, in: L. Steffe, P. Nesher, P. Cobb, G. Goldin & B. Greer (Eds) *Theories of mathematical learning* (Mahwah, NJ, Lawrence Erlbaum).
- Stigler, J. & Hiebert, J. (1999) *The teaching gap* (New York, The Free Press).
- Stigler, J., Gallimore, R. & Hiebert, J. (2000) Using video surveys to compare classrooms and teaching across cultures: examples and lessons from the TIMSS video studies, *Educational Psychologist*, 35 (2), 87-100.

- Stronach I. (1999) Shouting theatre in a crowded fire: "educational effectiveness" as cultural performance. *Evaluation*, 5 (2), 173-193.
- Theisen, G. & Adams, D. (1990). Comparative education research, in: R. Thomas (Ed) *International comparative education: practices, issues and prospects* (Oxford, Butterworth-Heinemann).
- Tweed, R. & Lehman, D. (2002) Learning considered within a cultural context: Confucian and Socratic approaches, *American Psychologist*, 57 (2), 89-99.
- Van den Heuvel-Panhuizen, M. (2002). Realistic Mathematics Education as work in progress, in: F-L. Lin (Ed) *Common sense in Mathematics education*. Proceedings of the Netherlands and Taiwan Conference on Mathematics Education (Taipei, National Taiwan Normal University).
- Whitman, N. & Lai, M. (1990) Similarities and differences in teachers' beliefs about effective teaching of mathematics: Japan and Hawaii, *Educational Studies in Mathematics*, 21 (1), 71-81.
- Wiliam, D. (1998) Making international comparisons: the third international mathematics and science study, *British Journal of Curriculum and Assessment*, 8 (3), 33-37.
- Winkelmann, B. (1994) Preparing mathematics for students, in: R. Biehler, R. Scholz, R. Strässer and B. Winkelmann (Eds) *Didactics of mathematics as a scientific discipline* (Dordrecht, Kluwer).

<p>The mathematical focus of an episode relates to the underlying objectives of a teacher's actions and decision making. There may be more than one such focus addressed within each episode of a lesson or, in fact, there may be no such focus for a particular episode.</p>	
Conceptual	The teacher is seen to emphasise or encourage the conceptual development of his or her students.
Derivational	The teacher is seen to emphasise or encourage the process of developing new mathematical entities from existing knowledge
Structural	The teacher is seen to emphasise or encourage the links or connections between different mathematical entities; concepts, properties etc.
Procedural	The teacher is seen to emphasise or encourage the acquisition of skills, procedures, techniques or algorithms.
Efficiency	The teacher is seen to emphasise or encourage learners' understanding or acquisition of processes or techniques that develop flexibility, elegance or critical comparison of working.
Problem solving	The teacher is seen to emphasise or encourage learners' engagement with the solution of non-trivial or non-routine tasks.
Reasoning	The teacher is seen to emphasise or encourage learners' development and articulation of justification and argumentation.

Table 1

<p>The mathematical context relates to the conception of mathematics underlying the tasks posed in a mathematics lesson and is located on a two dimensional grid. One axis considers whether or not the task is related explicitly to the real world or a plausible representation of it. The second axis relates to the genuineness or otherwise of the data or entities on which the task is based. In respect of the real world the issue to address is whether or not a task not only draws from the real world but feeds back into it. The data or entities on which the task is based should be considered as genuine if they are true, or derived from students' own actions.</p>	
<p>The task is explicitly related to the real-world and based on data or entities invented by the teacher.</p>	<p>An example of such a task might be the cost of decorating a hypothetical room. The task is related to the real-world but is located in a fantasy of data – the dimensions of the room, the costs of paper and so on. Another example could be the revised cost of a hypothetical pair of trousers after a reduction in a sale.</p>
<p>The task is explicitly not related to the real-world and based on data or entities invented by the teacher.</p>	<p>An invitation to solve the equation $x^2-3x+1=0$ would fall into this category. It is not based in the real world and the data or entity - the equation itself – is not the result of students' own activity. Most text-based questions or exercises would fall into this category.</p>
<p>The task is explicitly related to the real-world and based on genuine data or entities.</p>	<p>An example of this might be the testing of statistical hypotheses derived from real data collected by students. Another example might concern the measurement of desks and the cost of their manufacture. In this latter case the act of measurement, which creates genuine data, feeds back into the real world as it addresses the cost of making the desks.</p>
<p>The task is explicitly not related to the real-world and based on genuine data or entities.</p>	<p>A task in which students explore, say, the minimum value of a quadratic expression of their choice would fall into this category. The task has no explicit relation to the real world, but the data, the choice of the individual child, is genuine. Another example would an invitation to learners to measure the length of their desks for no other purpose than to practice the skills of measurement. The task is not explicitly related to the real world – because it does not feed back into it – but is located in genuine data. In such cases, the real world provides a background context for the task.</p>

Table 2

<p>The mathematical didactics refer to the teaching strategies that teachers use to facilitate their learners' ability to understand and use mathematics. With the exception of sharing, which is an explicit public act, all strategies could be seen in both public (whole class) and private (seatwork) contexts.</p>	
Activating prior knowledge	The teacher explicitly focuses learners' attention on mathematical content covered earlier in their careers in the form of a period of revision as preparation for activities to follow.
Exercising prior knowledge	The teacher explicitly focuses learners' attention on mathematical content covered earlier in their careers in the form of a period of revision unrelated to any activities that follow.
Explaining	The teacher explicitly explains an idea or solution. This may include demonstration, explicit telling or the pedagogic modelling of higher level thinking. In such instances the teacher is the informer with little or no student input.
Sharing	The teacher explicitly engages learners in a process of public sharing of ideas, solutions or answers. This may include whole-class discussions in which the teacher's role is one of manager rather than explicit informer.
Exploring	The teacher explicitly engages learners in an activity, which is not teacher directed, from which a new mathematical idea is explicitly intended to emerge. Typically this activity could be an investigation or a sequence of structured problems, but in all cases learners are expected to articulate their findings.
Coaching	The teacher explicitly offers hints, prompts or feedback to facilitate their understanding of or abilities to undertake tasks or to correct errors or misunderstandings.
Assessing or evaluating	The teacher explicitly assesses or evaluates learners' responses to determine the overall attainment of the class.
Motivating	The teacher, through actions beyond those of mere personality, explicitly addresses learners' attitudes, beliefs or emotional responses towards mathematics.
Questioning	The teacher explicitly uses a sequence of questions, perhaps Socratic, which lead pupils to build up new mathematical ideas or clarify or refine existing ones.
Differentiation	The teacher explicitly attempts to treat students differently in terms of the kind of tasks or activities, the kind of materials provided, and/or the kind of expected outcome in order to make instruction optimally adapted to the learners' characteristics and needs.