The following is the preprint version of

Marschall, G., & Andrews, P. (2015). Polish teachers' conceptions of and approaches to the teaching of linear equations to grade six students: An exploratory case study. *Research in Mathematics Education*, 17(3), 220-238.

<http://www.tandfonline.com/doi/full/10.1080/14794802.2015.1107498>

Abstract

In this paper we present an exploratory case study of six Polish teachers' perspectives on the teaching of linear equations to grade six students. Data, which derived from semi-structured interviews, were analysed against an extant framework and yielded a number of commonly held beliefs about what teachers aimed to achieve and how they would achieve them. In general, teachers' aims were procedural fluency founded on students understanding the equals sign as a relational rather than an operational entity and the balance scale as a representation supportive of students' understanding of an equation as the equivalence of two expressions. The analyses also indicated that the ways teachers proposed to conduct their lessons, whereby they pose single problems for individual work before inviting whole class sharing of solutions, resonates with the didactical traditions found in other East and Central European countries previously influenced by the Soviet Union.

Keywords

Polish mathematics teaching, linear equations, balance method

Introduction

The exploratory case study reported here was motivated by a tension in the findings of PISA 2012 (OECD, 2013a). On the one hand, Polish 15 year-old students' educational achievements, which have seen substantial rises over the lifetime of the PISA project, have been enthusiastically reported by the Organisation for Economic Cooperation and Development (OECD). For example, commenting in general rather than mathematical terms, the OECD (2010, p.6) reported that "some countries have been able to significantly improve their learning outcomes, in the case of Poland by almost three-quarters of a school year between 2000 and 2006 alone" (OECD, 2010, p.3), leading to Poland's being described as "the most rapidly improving education system in the OECD" (OECD, 2010, p.27). Moreover, Poland had one of the lowest between school variations of mathematics achievement on PISA 2012 (20% compared with an OECD average of 37%), indicating that students' opportunities to learn were less influenced by the school they attended than average (OECD, 2013b). In sum, against a variety of PISA indicators, Polish educational achievement has risen unusually rapidly and Polish schools appear to be among the most equitable in the world.

Year	2000	2003	2006	2009	2012
Mathematics	470	490	495	495	518

Table 1: Polish PISA mathematics achievement

In the particular context of mathematics, Poland's performance growth, as shown in Table 1, has essentially occurred in two phases. The first occurred between 2000 and 2003 and the second, which was greater than the first, between 2000 and 2003, occurred between 2009 and 2012, prompting the comment that Poland was one of only two countries with an accelerating positive growth in mathematics achievement (OECD, 2013a). In short, and acknowledging

the many criticisms of the PISA project itself (Meyer and Benavot, 2013), the substantial increase in Polish mathematics performance marks it as a site of potential research interest.

On the other hand, PISA 2012 (OECD, 2013a) also examined the extent to which students claimed to have been exposed to formal mathematics. On this general self-report measure Polish students achieved marginally above the international mean with respect to frequent exposure but marginally above with respect to ‘sometimes’ exposure (OECD, 2013a, p.159). However, one element of the measure of formal mathematics focused on students’ familiarity with linear equations. In this respect, Polish students claim one of the world’s lowest levels of familiarity with the topic (OECD, 2013a, p.165). This apparent lack of familiarity with such a key topic of school mathematics seems at odds with not only the overall increase in Polish students’ PISA performance but the OECD’s, albeit vague, assertion that “the estimated effect of a greater degree of familiarity with such content on performance is almost 50 points” (OECD, 2013a, p.155). In this paper, therefore, drawing on analyses of semi-structured interviews with six Polish teachers of diverse backgrounds and experience, we focus on two issues. The first, focused on trying to understand the PISA-related tensions identified above, is an investigation of Polish teachers’ perspectives on the teaching of linear equations. The second, drawing on the results of the first and the enthusiasm shown by the OECD for the improvement in Polish mathematics achievement, we consider whether Poland is a likely source of transferable pedagogical insight.

This study, therefore, is an introductory snapshot of a largely unresearched mathematics education tradition that is likely to be of growing interest internationally. In so doing we are conscious that teachers’ assertions with respect to how and why they do what they do can typically only be construed as indicators of their intentions. However, several teachers saying the same thing may reflect a common practice and address concerns with respect to belief espousal and enactment (Skott, 2009), particularly when those teachers undertook their professional development at different universities and work in different cities and schools. But first, we examine the literature with regard to the teaching and learning of linear equations. Finally, with respect to the introduction, the limited research on Polish mathematics teaching indicates two important characteristics. Firstly, the Polish mathematics curriculum remains based in an academic formalism focused on algebra, geometry and, in the later years, calculus and mathematical analysis (Zawadowski, 2009). Secondly, partly because many Polish teacher educators’ expertise was developed in Soviet universities (Pardała, 2010), much teaching, which exploits structured sets of challenging problems, continues to be formal, rigorous and located in a Soviet tradition (Zawadowski, 2009).

Issues in the teaching and learning of linear equations

When learning arithmetic, learners typically come to see the equals sign as an instruction to operate (Kaput, Carraher and Blanton, 2007). This *operational* (McNeil and Alibali, 2005) or *procedural* (Kieran, 1992) perspective frequently creates barriers when equations are introduced, not least because successful equation solving draws on a *relational* (Kieran, 1992) or *structural* (McNeil and Alibali, 2005) understanding of the equals sign as an assertion of equality between two expressions (Alibali, Knuth, Hattikudur, McNeil and Stephens, 2007). While the transition from an operational to a relational¹ conception of the equals sign may be a function of learners’ cognitive maturity (Alibali et al., 2007; Baroody and Ginsberg, 1983), focused interventions can facilitate this process (Saenz-Ludlow and Walgamuth, 1998), even after long exposure to operationally focused teaching (Jones, Inglis, Gilmore and Evans, 2013). In particular, even when textbooks encourage relational understanding, unless teachers explicitly draw attention to it, students may retain an operational perspective (McNeil et al., 2006). This latter problem is exacerbated by evidence

that teachers (albeit preservice), while able to recognise the potential of such texts to encourage a relational understanding of the equals sign, fail to recognise that many of their students hold misconceptions about it (Stephens, 2006). In sum, students typically find equation solving problematic because operational perspectives on expressions like $3x+1$ prevent their being construed as objects subject to, in relational terms, operations themselves (Kieran, 2004; Sfard, 1991).

All the above informs the extent to which students are able to respond successfully to equations-related instruction, and here it is necessary to make a distinction between arithmetical and non-arithmetical equations (Filloy and Rojano, 1989). Arithmetical equations, with the unknown in one expression only, require only a knowledge of arithmetic (Herscovics and Linchevski, 1994) and are, essentially, operational (Kieran, 1992). However, non-arithmetical equations, with unknowns in both expressions, cannot be solved by such approaches and require not only that learners “understand that the expressions on both sides of the equals sign are of the same nature” (Filloy and Rojano, 1989, p.19) but also that they are able to operate on the unknown as an entity and not a number.

Approaches to the teaching of linear equations

In the following we summarise the research on strategies for solving equations. In so doing, we focus not on studies that invoked atypical contexts to facilitate students’ use of those strategies. For example, Hewitt (2012) exploited specialist software to facilitate children’s informal approaches to the solution of linear equations. In similar vein, Pirie and Martin’s (1997) teacher used arithmagons to provide a context for generating and provoking intuitive solutions to non-arithmetical equations. In both studies, informal variants of the approaches we detail below were invoked.

Typically, research has focused on approaches to the solution of non-arithmetical equations and all, as we show, have been subjected to criticism. The most widely criticised, redistribution, reflects ‘traditional’ perspectives on equation solving. It is a rote-learned, magical (Nogueira de Lima and Tall, 2008), *change the side, change the sign* procedure (Andrews and Sayers, 2012), focused on transposing the equation so that the unknown finishes on the left hand side (LHS) and a value on the right (RHS) (Filloy and Rojano, 1989). However, the arbitrary leftwards movement of the unknown perpetuates an operational conception of the equals sign (Perso, 1996) and fails to support students’ understanding that such movement does not change the equation’s equality (Joffrion, 2005).

$$\begin{array}{c} \frac{x}{8} + 1 = 4 \\ \boxed{} + 1 = 4 \\ \frac{x}{8} = 3 \\ \boxed{} = 3 \\ \frac{}{8} = 3 \\ x = 24 \end{array}$$

Figure 1

So, what approaches have been reported that facilitate, in Skemp’s (1976) terms, relational understanding rather than instrumental understanding? Trial and improvement, where

students compare the expressions of an equation and, by means of ever-improving guesswork, derive a solution, is one. It supports an understanding of the relational nature of the equals sign and the role of the unknown in context (Knuth, Alibali, McNeil, Weinberg and Stephens, 2005). However, while it may be the first strategy invoked by students when exposed to such equations (Filloy and Rojano, 1989), it is inefficient and does not support the learning of general equation solving strategies. Another approach, limited by its applicability only to arithmetical equations, has been the cover-up method frequently exploited in Netherlands (Wijers, 2001) and the Finnish teacher in Andrews and Sayers (2012). In the latter case, as shown in Figure 1, the teacher used his chalkboard sponge to cover up those components of the equation that included the unknown in ways that reduced the equation to a series of simpler arithmetic equations.

With respect to non-arithmetical equations, an approach found in Singapore (Fong and Chong, 1995) presents each expression as a row of rectangles with those from one side of the equals sign laid on top of those from the other. Thus, $2x + 10 = 4x + 2$ would be represented as in Figure 2. Here, in essentially a cover-up manner, it is argued that $2x + 2 = 10$, after which an inverse operation yields $2x = 8$ and the solution follows. In similar vein, an approach discussed by Dickinson and Eade (2004), and shown in Figure 3, exploits the number line in a manner analogous to that of Fong and Chong. Such approaches reduce non-arithmetical equations to a series of arithmetical equations, each of which is solved intuitively. However, while they may be procedurally very helpful and offer oblique support in understanding equations as manipulable objects, the nature of those procedures may hinder students' understanding of the invariance of the solution.

Figure 2

Figure 3

Finally, in studies of teachers' unprompted approaches to equation solving, the balance scale has been the most widely reported, being the approach of choice in case studies from, inter alia, Canada (Haines, 1996), New Zealand (Anthony and Burgess, 2014), Finland, Flanders and Hungary (Andrews and Sayers, 2012). Here students manipulate, through addition or subtraction, weights on scale pans, while keeping the scales in balance. Its advocates argue that it supports an understanding of the need to do the same to both sides (Anthony and Burgess, 2014), as it helps students see the "equation as an entity rather than an instruction to achieve a result" (Warren and Cooper, 2005, p. 60). It also supports "symbolic representation, which semantically and syntactically sets the foundations for the introduction of algebraic formalisms" (Da Rocha Falcão, 1995, p. 72), a conjecture supported by the practice of the Hungarian teacher described in Andrews' (2003) study, who used a physical balance, with bags containing equal numbers of marbles alongside single marbles, to model $2x + 5 = x + 8$. Drawing on suggestions from his students he solved the equation physically before linking

the physical embodiment to diagrams subsequently drawn on the board. Finally, he made explicit the connection between the representation and the symbolic form of the solution process. Interestingly, indicating a collective pedagogy, a similar procedure was observed a decade later with a different Hungarian teacher working in different school (Andrews and Sayers, 2012).

Systematic attempts to evaluate the balance's efficacy have shown that it helps students to understand the principles of equations, solve non-arithmetical equations with understanding, particularly from the perspective of the need to do the same thing to both sides (Araya et al., 2010; Warren and Cooper, 2005). In addition, it facilitates students' acquisition of an appropriate vocabulary and a relational understanding of the equals sign (Vlassis, 2002). Critics argue that it cannot represent negatives in anything but a contrived way (Pirie and Martin, 1997), a criticism of some validity when set against invocations to imagine (Nogueira de Lima and Tall, 2008) or simulate (Anthony and Burgess, 2014) the tying of helium filled balloons to scales to counter the weight of objects in the scale pans. Furthermore, Boulton-Lewis et al. (1997) simulated the balance scale by means of a stick laid between two sets of objects, with unknowns represented by paper cups and integers by counters. Post-intervention evaluations found students preferring to use intuitive approaches rather than the concrete representations taught them, prompting the conclusion that the balance scale provoked cognitive overload. However, students were only asked to solve arithmetical equations, equations they could solve quickly by operation reversal, while the balance, itself an embodiment of an equation, was represented by an additional embodiment in the form of a stick. Thus, if there was a cognitive overload, it is not unlikely that that this was due to the imposition of an additional embodiment to represent the balance. More likely, we argue, is that students rejected this imposition for an intuitive and instantly available operation reversal. Other criticisms seem equally unwarranted. For example, the balance is thought to be unfamiliar to students living in a world of electronic scales (Pirie and Martin, 1997), an argument countered by Da Rocha Falcão's (1995, p. 80) description of the balance as "a culturally familiar artefact" and the fact that few children will not have played on a see-saw. In sum, and despite its criticisms, the balance scale remains not only the best-researched but the most used induction into equation solving.

Method

The research presented below derives from an exploratory case study focused on six teachers' perspectives on the teaching of linear equations. It aims to offer thick descriptions (Geertz, 1994) of how teachers construe their relationship with this important topic. In Stake's (2002) terms, it is an intrinsic, multiple case study. It is intrinsic in the sense that it is of interest to us, although the earlier PISA argument adds an instrumental dimension, and it is multiple because we aim to understand a particular phenomenon within a particular cultural context. As such, it is unlikely to furnish generalisations but may offer insights appropriate for later studies interested in such matters.

Fifteen state primary schools, located within and between two large Polish cities and accessible to the first author by public transport, were approached. While this physical proximity indicates their forming a convenience sample, other criteria suggest their being reasonably representative of Polish schools. For example, they offered, with respect to national tests undertaken at the end of primary education (Hörner and Nowosad, 2015), a spread of achievement comparable to national norms as well as variation in their geographical locations from rural through suburban to urban, and housing stock (Ministerstwo Edukacji Narodowej, ND).

Of the fifteen approached, four schools agreed to participate. This yielded six teachers of grade six (ages 12 and 13) students, which represents, within the context of Polish education, the final year of primary education. At this age students are taught mathematics by subject specialists who must have completed either an undergraduate course supplemented by a post graduate teaching qualification or a general teaching qualification supplemented by a programme of subject study (Hörner and Nowosad, 2015; Smoczyńska, 2014).

Details, with pseudonyms, can be seen in Table 2. Schools have been categorised according to the national test achievement quartiles produced by the Polish authorities, whereby Q₁ represents the highest achievement. Despite being small, the school sample reflected the diversity of achievement and other characteristics of the fifteen schools originally approached. For example, as shown in Table 2, the schools in which the teachers worked fell into three of the four possible achievement quartiles, with only the lowest missing. There was considerable variation in participants' teaching experience and background, with all having been trained in different teacher education institutions. Moreover, with recent OECD statistics showing that more than 85% of all Polish primary teachers are female (OECD, ND), a sample of six women should not be regarded as atypical. In sum, we argue that the diversity of both school and teacher background allows us to construe the totality as a 'telling case' (Mitchell, 1984). That is, one that serves to uncover insights into the topic under scrutiny previously unavailable or hidden (Córdova and Matthiesen, 2010, Dixon and Green, 2009).

Participant	Teaching experience	School	Location	Achievement quartile
Ewa	30 years	Sch. 1	City	Q ₁
Felicja	14 years	Sch. 2	City	Q ₁
Dorota	29 years	Sch. 3	Satellite town	Q ₂
Celina	12 years	Sch. 3	Satellite town	Q ₂
Agnieszka	7 years	Sch. 4	Satellite town	Q ₃
Barbara	18 years	Sch. 4	Satellite town	Q ₃

Table 2: Participants' details

Data derived from interviews, undertaken by the first author, exploiting a small number of open questions that were asked of all participants. It is important to add, however, that interviews were not free of supplementary questions, which emerged from teachers' responses and were posed uniquely to individuals. For example, in relation to teachers' preferred teaching approaches, supplementary questions focused on the limitations of those approaches. Thus, while the interviews were initially structured by commonly posed questions, all interviews incorporated questions that not only facilitated a guided conversation (Merriam, 1998) but also enabled the exploration of participants' equations-related understanding, motives and reasoning.

Interviews were conducted at a venue of each teacher's choosing, ensuring their being comfortable with the process. Each lasted around forty minutes, was conducted during a working school day and, with participants' permission, video recorded. At the start of the project, all participants were reassured that their participation was voluntarily, that they could withdraw from the research process at any time without reason and that they would remain anonymous in any reporting. Interviews were transcribed and translated into English. While none of the participating teachers were sufficiently proficient in English to verify the

accuracy of the translations, all agreed their Polish transcripts and permitted translations to be undertaken by the interviewer, who is fluent in both languages.

Analysis initially entailed reading and rereading all transcripts to get a feel for what teachers were saying. As this process unfolded it became clear that teachers' views were falling into categories not dissimilar to those identified in recent studies of equation solving. For example, there was an initial resonance with the four "typical teaching episodes" of Li, Peng and Song (2011, p.533), focused on "equation introduction, equation understanding, equation solving, and equation application" respectively. However, while such descriptions seemed transparent, they were not unproblematic. Firstly, Li et al. (2011) offer no indication as to the origins of their "typical teaching episodes". Secondly, none was defined and thirdly, each was operationalised against classroom excerpts focused on teaching with variation, a tradition largely unknown to European teachers. In other words, their framework seemed inappropriate for analyses of European classrooms. A second possibility lay in the four equations-related phases described by Andrews and Sayers (2012). Their analyses of sequences of equations-related lessons, one from each of Finland, Flanders and Hungary, identified four phases through which all lesson sequences passed. These they called the definition phase, whereby teachers presented their students, implicitly or explicitly, a definition of an equation; the activation phase, during which teachers revised "material covered earlier in their students' careers to both contextualise and facilitate the material that followed" (Andrews and Sayers, 2012, p. 482); the exposition phase, whereby teachers formally presented their preferred, balance scale, solution strategies; and the consolidation phase, during which teachers posed a range of exercises and problems intended to secure students' equation solving competence. Due to their resonance with our interview data, these four broad categories have been used to structure our analyses.

The definition phase

Andrews and Sayers (2012) found each of their case study teachers presenting definitions; some contextualised and some not, some explicit and some not. The six teachers reported here were no different. All commented on the importance of students' understanding what is meant by an equation. For example, Ewa said she would define an *"equation as being an equivalence of two expressions"*, while Felicija would base her definition around the argument that an *"equation is like a set of scales in equilibrium"*. In this latter respect all spoke of using balance scales to reinforce the meaning of equilibrium and how it can be maintained. In particular, Dorota, Celina and Agnieszka spoke of basing class discussions around pictures of scales in their respective textbooks. Felicija spoke of bringing a set of scales to school to illustrate different situations, although she did not go so far as to describe activities much in the manner of the Hungarian teachers discussed above, but simply spoke of posing questions like, *"what could be taken off the scales to maintain equilibrium?"* Barbara and Ewa said they would use computer-based balance simulations for collective discussion. For all six teachers, their use of the scales, albeit represented differently, was to facilitate students' understanding of an equation and its definition. In so doing, all spoke of the importance of maintaining a scale's equilibrium through an understanding that the two sides of an equation are equal.

However, to achieve these objectives, all commented on the need to ensure a range of topic prerequisites. For example, Celina commented that, *"firstly I remind students about knowns and unknowns"*, going on to add that *"before we move to solving equations, students, after the first lesson, should understand that x can be replaced by any value"*. More generally, all believed that students should be proficient with *"rational numbers"* and be able to *"collect like-terms, simplify expressions and expand brackets"*. Four teachers spoke of the need for a

good understanding of algebraic expressions and how to operate on them, as reflected in Ewa's suggestion that, *"first, we talk about algebraic expressions. Then, I lead them... onto algebraic expressions and arithmetical expressions. How are they different?"* The purpose of such preparatory material was summarised by Agnieszka, who wanted to ensure that *"these 'letters' do not come as a shock... so that they recognise that this letter is something they do not know; something they will have to work out"*.

In sum, all six spoke of ensuring their students' understanding of the relationship between the balance scale and an equation. Such beliefs were exemplified by Barbara's rhetorical questions; *"what does a balance scale look like and where does its name come from?"* and *"when on a see-saw, who is at the top and who at the bottom and why?"* All commented that this would be their students' first formal exposure to the notion of the balance scale and that this would underpin any formal definitions they give. Such views accord with the need for students to understand the equals sign relationally (McNeil and Alibali, 2005) and an equation as involving two equivalent but manipulable expressions (Alibali et al., 2007).

The activation phase

In their study, Andrews and Sayers (2012, p. 482) found all teachers, as preparation for their equations-related expositions, offering "arithmetical equations and encouraging their students to see that solutions can be found by recourse to a common-sense application of arithmetical operations". The six teachers here said they would do something similar; they would ask students to solve, by informal operation reversal means, a variety of missing number problems and simple arithmetical equations. Following those, they would present a non-arithmetical equation that students would be expected to solve by trial and improvement. While all six teachers saw this as an important phase in establishing a necessary skills base, they all thought it should be completed swiftly. As Dorota, noted:

"We do not spend one or even half a lesson on one type (of equation). It is not like we only work with the easiest examples. We cannot allow this to happen... because of the time".

Such tasks, they argued, support students' understanding of *"equality"*, the importance of *"maintaining equilibrium"* and the *"rules"* that should be followed for this to happen. All six commented on an expectation that any equations they used would exploit a range of coefficients; they would start with positive integers, in order to focus on the concept rather than provoke calculation difficulties, before introducing negative integers and eventually fractions. Underpinning this phase of their work was the development of three equations-related skills; constructing equations from simple situations, guessing' solutions to an equation and verifying whether a value satisfies an equation. In this latter respect, Ewa commented, reflecting the views of others, that:

"We are verifying whether a certain value satisfies the equation; so when substituted for the unknown it makes the equation true. That is whether the LHS and the RHS are equal".

In discussing these prerequisites, the six teachers articulated several common awarenesses. Firstly, time constrains what they do, reflecting, it seems to us, cultural norms with respect to not only what is valued but also pedagogical pace (Alexander, 2001). In this case the importance of not dwelling overlong on preparatory material. Secondly, that students should understand that equations frequently derive from and model real world problems, but translation from text to algebra is problematic (Capraro and Joffrion, 2006). Thirdly, that solution checking is an integral, but often ignored, part of the equation solving process, also with inherent difficulties (Perrenet and Wolters, 1994). Fourthly, that guesswork plays an

important role in establishing a need for more formal methods, although, despite its supporting a relational understanding of the equals sign, it is time-consuming (Knuth et al., 2005).

The exposition phase

All teachers indicated that they would structure their formal treatment of equations around the balance and, initially, arithmetical equations. Their rationale being that while students would be expected to solve arithmetical equations without any intervention, their intention would be not to introduce something beyond students' competence but something familiar to link earlier *intuitive* solutions to those derived formally. At this early stage, none of the six teachers would introduce negative coefficients, as this would compromise students' linking of intuitive and formal. In this regard, they seemed to understand, tacitly, that students' arithmetical equations-related competence is dependent on the extent to which negatives render them too abstract (Vlassis, 2002). These shared perspectives differed considerably from the three teachers discussed in Andrews and Sayers (2012), all of whom based their exposition around non-arithmetical equations.

Following this, all suggested that students would discuss equations of progressive complexity, each of which would be accompanied by a symbolic solution method before a systematic encouragement to check the solution. For example, Felicija said she would sketch something like the image in Figure 4 and initiate discussions around the meaning of the unknown, the equals sign and the necessity for maintaining the scale's equilibrium. She also commented, as did the other teachers, on the importance of highlighting the relationship between the representation of the equation and the symbolic ways of representing the actions they would take to solve it. In this latter regard, all teachers spoke of using arrows labelled with the operation. This shared emphasis on connecting the different representations of equations solving resonated with that observed in the Hungarian classrooms discussed earlier.

Figure 4

When discussing how they saw the topic progressing, only Barbara mentioned how she would use the balance with negative constants. Referring to Figure 5, she indicated that she would ask questions like:

“What is the difference between the two situations presented in the two diagrams? What has happened, what equation describes the second situation? What would you have to do to get from the equation describing the first set of scales to the equation representing the second set of scales?”

When asked about concerns, particularly with respect to the use of balloons (Nogueira de Lima and Tall, 2008), over the balance's inability to represent negatives (Pirie and Martin,

1997), she commented that she had found it successful in the past, that it had created little, or no, student difficulty and so continued to use it. She mentioned that it is essential for students to understand the function of the initial arrow but, otherwise, did not consider the issue problematic. Interestingly, while Barbara's arrow has a symbolic resonance with the use of balloons, her view was that it fitted more closely with the number line representation of negatives with which her students were familiar. In related vein, no other teacher used the balance with negative constants, arguing that by the time negative constants are introduced the model should be redundant and procedures internalised, reflecting again the Hungarian teacher' observed earlier. However, when asked during interview to consider Vlassis' (2002) findings that when negatives are introduced students cease to view equations as concrete but problematically abstract, their responses were similar to those of Barbara; by the time students experience negatives they have typically internalised the procedures embodied in the balance.

Figure 5

After these introductory examples, teachers' approaches diverged minorly. Agnieszka and Barbara would encourage students to draw scales and then use an arrow method, shown in Figure 6, to document their progress. After several examples Agnieszka would encourage students to stop drawing scales and focus and imagine what would happen before transforming those imagined actions into written symbols. The remaining four teachers would shift directly to the arrow representation as a framework for structuring the learning of an algorithm, referring constantly to the need to ensure students performed the same operations to both sides of the equation. For example, Celina said, in relation to Figure 7, that:

"I would subtract the same (or add) both from one side and the other side. And I always refer back to the equilibrium, so it does work out. And that's how I teach it".

In this respect, all indicated a belief that the arrow diagrams were transitional tools to support the establishment of students performing the same operation to both sides.

$$\begin{array}{ccc}
 3x + 2 = 8 & & \\
 \downarrow -2 & \downarrow -2 & \\
 3x = 6 & & \\
 \downarrow :3 & \downarrow :3 & \\
 x = 2 & &
 \end{array}$$

Figure 6

However, the shift towards an algorithm was discussed differently by the four teachers. Dorota and Felicija spoke of introducing a set of rules for memorisation. In this respect,

Felicija commented that the rule she taught was the one she had learned as a child, which she believed was effective. After establishing the balance, she would introduce an algorithm through the memorization of three steps. These are:

“Step one; brackets, we will do everything to eliminate brackets. Step two; get the unknowns on the LHS, numbers on the RHS [An action she described as ordering]. Step three, reduction [of like terms] and getting rid of the coefficient of x”.

$$\begin{array}{l} 3x + 2 = 8 \quad / -2 \\ 3x = 6 \quad / :3 \\ x = 2 \end{array}$$

Figure 7

Dorota’s rules were also focused on getting the unknown to the LHS and a single number to the RHS. For Ewa, the introduction of an algorithm typically arose in response to students’ parents’ inappropriate interventions at home, whereby children are given a redistribution algorithm before they have acquired the necessary conceptual understanding for using it appropriately. In her view, students are rarely able to use their parents’ recipes properly, particularly when non-arithmetical equations are involved. Consequently, she sees her interventions as attempts to reconcile parent’s rote taught rules and her aim of establishing algorithmic fluency premised on understanding. Interestingly, such approaches, while redolent of the much criticised and typically rote learnt redistribution approach (Filloy and Rojano, 1989), was construed by neither Dorota, Felicija nor Ewa as ‘magical’ (Nogueira da Lima and Tall, 2008) but warranted by their belief that the necessary prior deep conceptual understanding had been established. In all cases, however, teachers discussed how they would not, by choice, introduce redistribution, which four of the six regarded as being necessary for that procedural fluency, until they were confident that their students were conceptually ready for it. The arguments of the four typically went along the following lines; once the balance has secured students’ conceptual understanding of equations and equation solving, time-demands necessitate a rapid shift to an algorithm for solving any problem efficiently and automatically.

The consolidation phase

All three of Andrews and Sayers’ (2012, p.484) teachers, over a period of two or three lessons, “provided various opportunities for students to consolidate earlier learning and further develop equations-related conceptual and procedural understandings”. The six teachers of this study spoke of doing something similar, saying that they would spend two, three or even four lessons working through non-arithmetical equations-related problems of increasing complexity, which may involve multiple brackets and more complex algebraic fractions. In general, they argued, this would entail students working on the same problem simultaneously, first individually and then as a whole class with selected individuals presenting solutions at the board. In addition, all six spoke of wanting to prepare students to solve equations derived from word problems, which they regarded as the most important objective for their lesson sequence.

Interestingly, while Li et al. (2011) suggest that word problems would underpin their variation-based lesson sequence, the teachers here see them as the conclusion to theirs. By way of contrast, the Finnish teacher of Andrews and Sayers’ (2012) study made no concession to word problems, while their teacher from the Dutch-speaking region of Belgium

exploited just one, to introduce the whole topic, as with Vlassis' (2002) interventions in the French-speaking region of Belgium. The only teacher of their study to exploit multiple word problems was from Hungary, who, like the teachers reported here, based much of her consolidation phase around them.

Acknowledging students' difficulties with word problems, which they believed typically derived from poor reading comprehension or failures to determine what the unknown should represent, Agnieszka, Barbara and Felicija commented that, while spending as much time as possible on such tasks, they would not be disheartened if students failed to master the topic, as they would get further opportunities at the gymnasium. However, Ewa, also aware that students would revisit the topic, aimed to prepare them for this encounter, believing it is her responsibility to make students' later learning easier. That being said, Dorota and Celina, spoke of the procedures they would teach to help their students derive equations from word problems. For Dorota, this would entail recognizing the unknown, writing the equals sign and then deciding what needs to be written either side of it. Her belief is that centralizing the equals sign makes it simpler to tackle the construction of the equation. Celina's instructions were more detailed. She commented that by first looking at the final sentence of a problem's statement students should be able to see what needs to be found. This would be followed by noting down all extracted information and constructing diagrams to help visualise the problem. Her final stage would be the setting up of the equation, which, as with Dorota, should start with the writing of the equals sign first.

Finally, an important element of equation solving, which all teachers emphasised, is solution-checking. Typically, students would be expected to substitute the solution into each expression and compare. This process, which would be written beneath the solution, further supports students' understanding of both the relational nature of the equals sign and the unknown (Wagner and Parker, 1993). For all six teachers, verification must take place as, despite students' reluctance, it is an essential element of the habituated procedure for solving equations and, as with other elements, resonated with the practices of the Hungarian teachers described earlier.

In sum, all spoke of consolidating their students' competence against increasingly complex context-independent equations before starting on word problems. A key element of this process was the way in which students would work simultaneously but independently on a problem before solutions would be shared publicly. Such an approach, similar to that found in Hungary (Andrews, 2003; Andrews and Sayers, 2012), reflects teachers' belief in the primacy of the class as the unit of instruction. Also, when dealing with word problems, the placing of the equals sign at the heart of the equation-deriving process highlights its relational rather than operational properties and, teachers argue, facilitates students being able to construct equations.

Discussion

In this paper we have explored how six grade six Polish teachers, educated and working in a range of cities and schools, propose to teach linear equations. The analyses, appropriately structured by an extant framework, highlighted, with minor variations, what appeared to be commonly held beliefs focused on a relational understanding of the equals sign and the exploitation of the balance scales. Thus, acknowledging cautions with regard to the relationship between espoused and enacted beliefs (Skott, 2009), if a group of teachers with diverse backgrounds and differing experiences present a common pedagogical narrative then it is not unreasonable to conjecture that the enactment of that narrative may resonate with it. Moreover, if understanding the equals sign as a relational symbol of equivalence leads to the successful solving of non-arithmetical equations (Knuth, Stephens, McNeil and Alibali,

2006), then project teachers' emphases would seem likely to provide an appropriate foundation for later algebraic study. Also, with respect to the balance, all teachers agreed that it supports students' learning of equations-related symbolism and provides a strong conceptual basis for later procedural fluency. More prosaically, it provides students with a form of instant support, as reflected in Barbara's comment that *"I emphasise to students that they can always help themselves by drawing scales"* and, as noted by Ewa, *"especially for the weaker students... because they can see it and... feel what it is all about"*. Thus, all implicitly rejected the concerns expressed by Pirie and Martin (1997) about the balance's irrelevance to modern students. In short, if the six teachers reported here are typical of all Polish teachers, and we return to the issues of typicality below, then Polish students' lack of familiarity with linear equations (OECD, 2013a) seems difficult to understand, not least because they would have been exposed to a thorough grounding in the concepts and procedures supportive of equation solving competence.

That being said, Vlassis' (2002) finding that the introduction of negative coefficients transforms a concretely accessible equation to something too abstract for students to manage was less convincingly dismissed. Teachers' arguments that they would not introduce negatives until students had internalised both the balance and its relationship to any procedure concerning, say, doing the same to both sides, seemed over-confident. However, the belief was shared and reflected evidence from Hungary that students taught similarly can manage confidently equations with negative coefficients (Andrews and Sayers, 2012). In short, it is not inconceivable that the extensive attention paid to the relational role of the equals sign and the continued use of the balance warrants their confidence. Furthermore, in contrast to Newton, Star and Lynch's (2010, p.283) argument that flexibility in algebraic problem solving "develops slowly from knowledge of multiple procedures", the six teachers of this study reject *more is better*. Their commonly-held belief is that the balance supports all aspects of students' understanding and leaves no need for alternatives. Indeed, as Ewa noted, just two approaches would be *"enough for students to get lost completely"*.

With respect to algorithmic efficiency, all except Barbara spoke of encouraging students to gather the unknowns to the LHS, irrespective of any awkward consequences concerning negative coefficients in the manner described by Filloy and Rojano, 1989). Such behaviours were justified on the basis of efficiency and the importance of students acquiring an agreed and, they argued, the important habituated procedure that could be instantly invoked in much the same way as Anthony and Burgess' (2014) New Zealand case study teacher. Aware of possible misconceptions, particularly related to the arbitrary movement to the left (Perso, 1996; Joffrion, 2005), both Dorota and Felicija mentioned that students know they can swap the LHS and RHS round because they understand that scale pans can be swapped without upsetting the balance. Indeed, despite a similarity to the much criticised redistribution approach, the rule was warranted by teachers' beliefs that a necessary prior deep conceptual understanding had been established. In addition, all teachers' argued that students should solve equations of increasing complexity involving numerical fractions, brackets and algebraic fractions. Finally, a key element, also reflecting case studies from other countries (Andrews and Sayers, 2012; Anthony and Burgess, 2012), was a common emphasis on solution checking.

Finally, project teachers' espoused lesson structures resonated closely with that of the typical Russian lesson comprising several connected tasks, each of which would entail much teacher questioning and explaining, oral work, independent and collective working on the same problem leading to the recording of collectively agreed outcomes (Wilson, Andrew and Sourikova, 2001). Moreover, many of these Russian tasks are word problems, a long-standing tradition (Karp, 2006), with even second grade students being expected to "represent

schematically the internal quantitative relationships in word problems, and to write equations that express these relationships symbolically” (Cai et al., 2005, p.8). In other words, the Polish teachers of this study, as predicted by Zawadowski (2009), privileged teaching approaches commensurate with those found in former Soviet satellites like Hungary (Andrews, 2003, Andrews and Sayers, 2012), Estonia (Tuul, Ugaste and Mikser, 2011) and Russia itself, indicating that elements of the Soviet tradition persist nearly three decades after the fall of the Berlin Wall. This latter point seems particularly salient as both Ewa and Dorata, who would have taught during the Soviet era, continue to espouse beliefs and mathematical values commensurate with those earlier times.

So, acknowledging the OECD’s PISA-driven promotion of Poland as a model for other nations, does this small-scale study present Poland as a site of research interest for those interested in identifying transferable pedagogical insights? Our answer is a tentative, yes. All six teachers argued, independently, for a similar student experience based on establishing and exploiting a relational understanding of the equals sign, a consistent use of the balance scale, complex equations, including word problems, and an avoidance of the cognitive confusion that stems from teachers’ idiosyncratic use of too different approaches. In general terms, these commonly held beliefs seemed largely commensurate with what the literature has reported. However, a case comprising six teachers, however ‘telling’ (Mitchell, 1984), can provide little more than a justification for further research, particularly from the perspective of the relationship between current Polish practice and the Soviet tradition.

Finally, we consider the limitations of this exploratory case study. Firstly, as indicated above, our teacher sample was small, although we argue this was ameliorated by the diversity of their backgrounds and professional experiences. Secondly, all teachers referred to their preferred textbook, which, coincidentally, was the same in each case. In such a circumstance it is not unreasonable, in light of evidence that an overreliance on textbooks may compromise what is taught (Schoenfeld, 2007), to ask whether we can be confident that their, essentially, unanimous espousals were not mere repetitions of the textbook’s recommendations? Our response is two-fold. On the one hand our perception is that these teachers were neither overly reliant nor unable to make their own didactical decisions. For example, their use of expressions like “*I believe*”, “*I am convinced*”, and “*personally I prefer*” seem to go beyond a response conditioned by the text. On the other hand, we cannot dismiss that possibility. In this respect, however, the long-standing (Wojdon, 2015) and continuing (Dąbrowski and Wiśniewski, 2011; (Grodzińska-Jurczak, 2004) regulation of school textbooks suggests that such conditioned beliefs may reflect a systemic perspective on equations and equation solving. Indeed, of the nine textbooks currently authorised (Ministerstwo Edukacji Narodowej), the one used by project teachers dominates the market. Thirdly, had resources been available to us, we would have observed the same teachers teaching the sequences they described. This would have allowed us to explore the relationship between espoused beliefs and enacted practice and address concerns that an espousal is no guarantee of its enactment (Skott, 2009). Fourthly, we began by tentatively linking Polish mathematics education to a Soviet-influenced mathematical formalism (Zawadowski, 2009) underpinned by the training of many Polish teacher educators in Russian universities (Pardała, 2010). The evidence of this study and a rather sparse literature² provide some support for the conjectured didactical tradition common across former Soviet states. However, without corroborative evidence from appropriately framed comparative studies, this remains a conjecture and, therefore, a limitation of this study.

Notes

1. In the interests of consistency, we refer to operational and relational perspectives on the equal sign.
2. See, for example, the discussions on the teaching of linear equations in the Czech Republic (Novotná and Hošpesová, 2010) and Hungary (Andrews and Sayers, 2012).

References

- Alexander, R. (2001). Border crossings: Towards a comparative pedagogy. *Comparative Education*, 37(4), 507-523.
- Alibali, M., Knuth, E., Hattikudur, S., McNeil, N., & Stephens, A. (2007). A longitudinal examination of middle school students' understanding of the equal sign and equivalent equations. *Mathematical Thinking and Learning*, 9(3), 221-247.
- Andrews, P. (2003). Opportunities to learn in the Budapest mathematics classroom. *International Journal of Science and Mathematics Education*, 1(2), 201-225.
- Andrews, P., & Sayers, J. (2012). Teaching linear equations: Case studies from Finland, Flanders and Hungary. *Journal of Mathematical Behaviour*, 31(4), 476-488.
- Anthony, G., & Burgess, T. (2014). Solving linear equations. In F. Leung, K. Park, D. Holton, & D. Clarke (Eds.), *Algebra teaching around the world* (pp. 17-37). Rotterdam: Sense Publishers.
- Araya, R., Calfucura, P., Jiménez, A., Aguirre, C., Palavicino, A. M., Lacourly, N., et al. (2010). The effect of analogies on learning to solve algebraic equations. *Pedagogies: An International Journal*, 5(3), 216-232.
- Baroody, A., & Ginsburg, H. (1983). The effects of instruction on children's understanding of the 'equals' sign. *The Elementary School Journal*, 84(2), 199-212.
- Meyer, H.-D. & Benavot, A. (2013) *Who succeeds at PISA and why? The role of international benchmarking in the emerging global education governance system*. Oxford: Symposium.
- Boulton-Lewis, G., Cooper, T., Atweh, B., Pillay, H., Wilss, L., & Mutch, S. (1997). Processing load and the use of concrete representations and strategies for solving linear equations. *Journal of Mathematical Behavior*, 16(4), 379-397.
- Cai, J., Lew, H.-C., Morris, A., Moyer, J., Ng, S., & Schmittau, J. (2005). The development of students' algebraic thinking in earlier grades: A cross-cultural comparative perspective. *ZDM*, 37(1), 5-15.
- Capraro, M., & Joffrion, H. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? *Reading Psychology*, 27, 147-164.
- Córdova, R., & Matthiesen, L. (2010). Reading, writing, and mapping our worlds into being: Shared teacher inquiries into whose literacies count. *The Reading Teacher* 63(6): 452-463.
- Da Rocha Falcão, J. (1995). A case study of algebraic scaffolding: From balance scale to algebraic notation. In L. Meira & D. Carraher (Eds.), *Proceedings of the 19th International Conference for the Psychology of Mathematics Education* (Vol. 2, p.66-73). Federal University of Pernambuco, Brazil.
- Dąbrowski, M., & Wiśniewski, J. (2011). Translating key competences into the school curriculum: Lessons from the Polish experience. *European Journal of Education*, 46(3), 323-334.

- Dickinson, P., & Frank, E. (2004). Using the number line to investigate the solving of linear equations. *For the Learning of Mathematics*, 24 (2), 41-47.
- Dixon, C., & Green, J. (2009). Focus on policy: How a community of inquiry shapes and is shaped by policies: The Santa Barbara classroom discourse group experience as a telling case. *Language Arts* 86(4): 280–289.
- Filloy, E., & Rojano, T. (1989). Solving equations: The transition from arithmetic to algebra. *For the learning of the mathematics*, 9(2), 19-25.
- Fong, H., & Chong, T. (1995). Solving algebraic word problems. *Mathematics Teaching*, 151, 34-35.
- Geertz, C. (1994). Thick description: Toward an interpretive theory of culture. In M. Martin & L. McIntyre (Eds.), *Readings in the philosophy of social science* (pp. 213-232). London: Bradford Books.
- Grodzińska-Jurczak, M. (2004). Forum: Environmental education in Poland: Its present status. *International Research in Geographical and Environmental Education*, 13(3), 251-257.
- Haimes, D. (1996). The implementation of a 'function' approach to introductory algebra: A case study of teacher cognitions, teacher actions, and the intended curriculum. *Journal for Research in Mathematics Education*, 27(5), 582-602.
- Herscovics, N., & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics*, 27(1), 59-78.
- Hewitt, D. (2012). Young students learning formal algebraic notation and solving linear equations: are commonly experienced difficulties avoidable? *Educational Studies in Mathematics*, 81(2), 139-159.
- Hörner, W., & Nowosad, I. (2015). Poland. In W. Hörner, H. Döbert, L. R. Reuter, & B. von Kopp (Eds.), *The Education Systems of Europe* (pp. 621-644): Springer International Publishing.
- Joffrion, H. (2005). *Conceptual and Procedural understanding of algebra concepts in the middle grades*. Master's Thesis, Texas A&M University.
- Jones, I., Inglis, M., Gilmore, C., & Evans, R. (2013). Teaching the substitutive conception of the equals sign. *Research in Mathematics Education*, 15(1), 34-49.
- Karp, A. (2006). The Cold War in the Soviet School. *European Education*, 38(4), 23-43.
- Kaput, J., Carraher, D., & Blanton, M. (2007). *Algebra in the early grades*. Mahwah, NJ: Erlbaum.
- Kieran, C. (1992). The learning and teaching of school algebra. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390–419). New York: Macmillan.
- Kieran, C. (2004). Algebraic thinking in the early grades: What is it? *The Mathematics Educator*, 8(1), 139-151.
- Knuth, E., Alibali, M., McNeil, N., Weinberg, A., & Stephens, A. (2005). Middle school students' understanding of core algebraic concepts: Equivalence and variable. *ZDM*, 37(1), 68-76.

- Knuth, E., Stephens, A., McNeil, N., & Alibali, M. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education*, 37(4), 297-312.
- Li, J., Peng, A., & Song, N., (2011) Teaching algebraic equations with variation in Chinese classroom. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 529- 556). New York: Springer.
- McNeil, N., & Alibali, M. (2005). Why Won't You Change Your Mind? Knowledge of Operational Patterns Hinders Learning and Performance on Equations. *Child Development*, 76(4), 883-899.
- McNeil, N., Grandau, E., Knuth, E., Alibali, M., Stephens, A., Hattikudur, S., & Krill, D. (2006). Middle-school student's understanding of the equal sign: The books they read can't help. *Cognition and Instruction*, 24(3), 367–385.
- Merriam, S. (1998). *Qualitative research and case study applications in education*. San Francisco: Jossey-Bass.
- Ministerstwo Edukacji Narodowej (National Ministry of Education) (ND). Retrieved from <http://men.gov.pl/> September 1, 2015
- Mitchell, J. (1984). Typicality and the case study. In R. Ellen (Ed.), *Ethnographic research: A guide to general conduct* (pp. 238 - 241). London: Academic Press.
- Newton, K., Star, J., & Lynch, K. (2010). Understanding the development of flexibility in struggling algebra students. *Mathematical Thinking and Learning*, 12(4), 282-305.
- Nogueira de Lima, R., & Tall, D. (2008). Procedural embodiment and magic in linear equations. *Educational Studies in Mathematics*, 67(3), 3-18.
- Novotná, J., & Hošpesová, A. (2010). Linking in teaching linear equations - forms and purposes: The case of the Czech Republic. In Y. Shimizu, B. Kaur, R. Huang, & D. Clarke (Eds.), *Mathematical tasks around the world* (pp. 103-118). Rotterdam: Sense.
- OECD (2004). *Learning for tomorrow's world: First results from PISA 2003*. Paris, OECD.
- OECD. (2010). *The high cost of low educational performance: The long-run economic impact of improving PISA outcomes*. Paris: OECD.
- OECD. (2013a). *PISA 2012 Results: What students know and can do - Student performance in mathematics, reading and science*. Paris: OECD.
- OECD (2013b). *PISA 2012 Results: Excellence through equity: Giving every student the chance to succeed*. Paris, OECD.
- OECD (ND) <http://stats.oecd.org/>
- Pardała, A. (2010). The traditions and development of mathematics education: The case of Poland. In A. Karp and B. Vogeli (Eds.), *Russian mathematics education: History and world significance* (pp. 327-337). Singapore: World Scientific.
- Perrenet, J., & Wolters, M. (1994). The art of checking: A case study of students' erroneous checking behavior in introductory algebra. *Journal of Mathematical Behavior*, 13(3), 335-358.
- Perso, T. (1996). Teaching equation solving – Again? *The Australian Mathematics Teacher*, 52(1), 19-21.

- Pirie, S., & Martin, L. (1997). The equation, the whole equation and nothing but the equation! One approach to the teaching of linear equations. *Educational studies in Mathematics*, 34(2), 159-181.
- Sáenz-Ludlow, A., & Walgamuth, C. (1998). Third graders' interpretations of equality and the equal symbol. *Educational Studies in Mathematics*, 35(2), 153-187.
- Schoenfeld, A. (2007). Problem solving in the United States, 1970–2008: research and theory, practice and politics. *ZDM*, 39(5), 537-551.
- Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, 22(1), 1-36.
- Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-26.
- Skott, J. (2009). Contextualising the notion of ‘belief enactment’. *Journal of Mathematics Teacher Education*, 12(1), 27–46.
- Smoczyńska, A. (Ed.) (2014). *The system of education in Poland*. Warsaw: The Polish Eurydice Unit.
- Stake, R. (2002). Qualitative case studies. In N. Denzin & Y. Lincoln (Eds.), *The Sage handbook of qualitative research* (pp. 443-466). London: Sage.
- Stephens, A. (2006). Equivalence and relational thinking: Preservice elementary teachers' awareness of opportunities and misconceptions. *Journal of Mathematics Teacher Education*, 9(3), 249-278.
- Tuul, M., Ugaste, A., & Mikser, R. (2011). Teachers' perceptions of the curricula of the Soviet and post-Soviet eras: A case study of Estonian pre-school teachers. *Journal of Curriculum Studies*, 43(6), 759-781.
- Vlassis, J. (2002). The balance model: Hindrance or support for the solving of linear equations with one unknown. *Educational Studies in Mathematics*, 49(3), 341-359.
- Wagner, S., & Parker, S. (1993). Advancing algebra. In P. Wilson (Ed.), *Research ideas for the classroom: High school mathematics* (pp. 119–139). New York: Macmillan.
- Warren, E., Cooper, T. (2005). Young children's ability to use the balance strategy to solve for unknowns. *Mathematics Education Research Journal*, 17(1), 58-72.
- Wijers, M. (2001). How to deal with algebraic skills in Realistic Mathematics Education. In H. Chick, K. Stacey, J. Vincent and J. Vincent (Eds.), *Proceedings of the 12th ICMI Study Conference: The future of the teaching and learning of algebra* (Vol. 2, pp. 649-654). University of Melbourne.
- Wilson, L., Andrew, C., & Sourikova, S. (2001). Shape and structure in primary mathematics lessons: a comparative study in the North-east of England and St Petersburg, Russia - some implications for the daily mathematics lesson. *British Educational Research Journal*, 27(1), 30-58.
- Wojdon, J. (2015). The system of textbook approval in Poland under communist rule (1944-1989) as a tool of power of the regime. *Paedagogica Historica*, 51(1-2), 181-196.
- Zawadowski, W. (2009). *Wpływ współpracy międzynarodowej na nauczanie matematyki w Polsce* (The impact of international cooperation on the teaching of mathematics in Poland). XV Konferencja Diagnostyki Edukacyjnej. Kielce 2009.