The variance based efficiency test of the OMX index option market

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Abstract

In this paper, followed by the market efficiency definition of the absence of arbitrage opportunity in the market, we test the market efficiency of the OMXS30 index option market. We first check the arbitrage opportunity by examining the boundary conditions and the Put Call Parity that must be satisfied in the market. Then a variance based efficiency test is performed by establishing a risk neutral portfolio and re-balancing the initial portfolio in different trading strategies. In order to choose the most appropriate model for option price and hedging strategies, we calibrate several most applied models, i.e. the Black Scholes, Merton, Heston, Bates, and Affine Jump Diffusion models. Our results indicate that the Affine Jump Diffusion model significantly outperforms other models in the option price forecast and the trading strategies. Both the boundary and the Put Call Parity tests and the dynamic hedging strategy give evidence that no significant abnormal returns can be obtained in the OMXS30 option market, thereby supporting the market efficiency.

JEL classification: G14, G12, C52
Keywords: Efficiency Test, Options and Futures markets, Stochastic Volatility with Jumps Models, Mean-variance Hedge

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1 Introduction

The index option market has been one of the most successful of many innovative financial instrument introduced over the last few decades. With the fast growing of the index options in the world, the Swedish OMXS30 index was introduced on September 30, 1986, consisting of 30 most traded Swedish stocks on the Stockholm Stock Exchange. The strong success of this contract convinced the Swedish authorities to introduce the futures and the options contract on the OMX indexone year later. Nowadays, the OMXS30 index options and futures are the essential tools for risk hedging in the Nordic market. The purpose of this paper is to empirically investigate whether the OMXS30 options market is efficient.

Efficiency is of the utmost importance for the functioning and the development of financial markets. In earlier studies, most of the efficiency tests focus on stock options in the U.S. market (see e.g., Gould & Galai 1974, Klemkosky & Resnick 1980). Later on, the index option markets efficiency tests become a popular topic (see e.g., Klemkosky & Resnick 1980, Evnine & Rudd 1985, Kamara & Miller 1995, Ackert & Tian n.d.*a*, Bharadwa & Wiggins 2001). Meanwhile, the introduction and fast growing index options in Europe have also called for the attention of empirical research to these markets. For example, Cavallo & Mammola (2000), Brunetti & Torricelli (2005), Cassese and Guidolin (2006) on the Italian index (Mib30), Capelle-Blancard & Chaudhury (2001), Deville (2004), Deville & Riva (2007) on the French index (CAC40), Mittnik & Rieken (2000) on the German Index (DAX), Martikainen & Puttonen (1996) on the Finnish stock market, Corredor & Santamaria (2002) on the Spanish stock market, Chesney et al. (1995) on the Swiss index option market. The market efficiency of the option index has been concluded in most European countries.

Further, all the above mentioned papers have done the efficiency tests followed by the efficiency definition of the absence of arbitrage opportunity in the market, and the efficiency tests of the financial markets have been carried out by two methods. The first is the model-free boundary test and the Put-Call-Parity (PCP) test introduced by Stoll (1969) and extended by others, e.g., Merton (1973), Klemkosky & Resnick (1979)), which investigate the lower and upper boundary and no-arbitrage relationships that must be held between the prices of a call and a put written on the same underlying asset and having the same strike and time to maturity. The second efficiency test approach is a specific form of test, i.e., a dynamic hedging strategy usually based on a specific option pricing model (see e.g., Black & Scholes 1973, Galai 1977, Macbeth & Merville 1979, Xu & Taylor 1995, Cavallo & Mammola 2000). In this test, the Black Scholes (BS hereafter) model (Black & Scholes 1973) is the commonly applied model. For example, using an implied volatility of the Black Scholes model, Harvey & Whaley (1992) conduct an S&P 100 index option market efficiency analysis of call and put near-the-money options each day and delta-hedged using the S&P 500 futures contract. Cavallo & Mammola (2000) test the efficiency of the Italian MIO30 index option market using implied volatility and the dynamic delta hedging strategy. They show that the hedging strategy based on the implied volatility method fails to make significantly positive profits after considering transaction costs.

However, according to Hull (2002), a number of problems relate to carrying out empirical efficiency tests based on the BS model. The first problem is that any statistical hypothesis about how options are priced has to be a joint hypothesis to the effect that i) the model is valid and ii) markets are efficient. To distinguish between the two hypotheses of market efficiency and model validity, one of the two has to be taken as an assumption. Therefore, it is crucial to choose the most valid model in the dynamic hedging strategy test. A second problem concerns the choice of the best estimate of stock price volatility. Because the volatility is considered to be a proxy for risk in the financial and economic fields, it has important influences on monetary policy making, asset allocation decisions and risk management.

In this paper, we attempt to overcome the above difficulties in several substantive ways. First, the first part of the current work performs the boundary conditions and the put- call parity (PCP) tests, which do not rely for its validity on the option pricing model. We examine the the boundary conditions and the PCP by taking into account the bid and ask spread and the transaction costs. We divide the investors into three groups according to different levels of transaction costs. Following Cavallo & Mammola (2000), we consider the bid and ask spread by assuming that investors buy at ask prices and sell at bid prices. Second, in order to choose the most valid model and the best estimate of the stock price volatility, we examine several most applied option pricing models in the contemporary finance literature, i.e., the Affine Jump Diffusion (AJD) model introduced by Duffie et al. (2000), the Merton (Merton 1976), the Heston (Heston 1993), the Bates (Bates 1996) and the Black Scholes (Black & Scholes 1973) models. We therefore have the opportunity to check if the most advanced model can improve the option price forecasts and the hedging results.

The calibrations of the stochastic volatility with jump models are difficult and time consuming. The most common calibration technique is the version of daily least square estimation, which gives good in-sample predictions but is also known to be notoriously non-robust to outliers giving bad parameter estimates (see e.g., Cont & Tankov 2004). We calibrate all models with the unscented Kalman filter which has been proven to be much faster and more accurate than the least square method (see e.g., Lindström et al. 2008). Further, as the characteristic functions of the closed form of these models are available, the inverse fourier transform method has been employed to calculate the closed form solution of the option price of various selected models. The Fourier transform based method is both fast and accurate if we know the characteristic function for the log stock price in the closed form.

In the dynamic trading strategy test, we search for the arbitrage opportunity with different hedging strategies and with different hedging weights in the OMX indexoption market. We consider two different trading strategies. For the first trading strategy, we assume that an investor sells one contract of a call/put option when it is first issued in the market. A risk neutral self-financing portfolio will be built up by buying/selling a certain amount of OMXS30 index futures according to the calculated hedging weights to hedge the call/put options. The rest of the money is put into a bank account. This portfolio is maintained until the maturity of the option. Finally, we get the return by exercising the option and clearing the portfolio. We use OMX futures to hedge OMX index options, because it is not possible for us to replicate the whole index and to collect the data regarding the index duplication. Indeed, for European options and futures that share the same underlying asset and common expiration day, the option can be priced as if it is an option on the futures contract (Black 1976). In reality, futures and futures options are traded side by side in the same exchange, and they close at the same maturity day, this minimizes data synchronization problems.

In the second trading strategy, we try to detect the mispricing opportunity (the difference between the model calibrated prices and the market prices), and the initial portfolio will be built up by buying the option if it is underpriced (model prices > market prices) and selling the option if it is over-priced (model prices < market prices). Then a certain amount of the OMXS30 index futures according to the calculated hedging weights will be bought or sold to hedge the options. The portfolio will be properly re-balanced according to the calculated hedging ratios.

For all of the dynamic trading strategies, we use two different hedging ratios, i.e., the Delta and the mean variance hedge weights. With the stochastic volatility, and especially with a jump, the option is not a redundant asset and the market is not completed. Most derivatives cannot be completely hedged by Delta and the volatility and the jumps have to be hedged too. Therefore, we have derived and calculated the mean variance hedge weight, which minimizes the quadratic difference between the changes in contingent claims and in the underlying asset. The closed form solution for the mean variance hedge for various models will be derived and calculated by the inverse fourier transform method based on the known characteristic functions. Because the Delta weight is commonly used for the BS model, we also calculate the Delta weight in order to compare the hedging results between the BS model and other models.

Our results suggest that the average results from the boundary and the PCP tests, and from the dynamic hedging strategies fail to indicate any significant arbitrage returns. The violations from the boundary tests are much lower and the returns from the long and short hedge of the PCP test are smaller than other index option markets which have been tested and claimed to be efficient. The returns from the dynamic hedging strategies are very insignificant indicating a positive sign of the market efficiency. Turning to the performance of option price models, we find that the AJD model significantly outperforms the BS, Merton and Bates models in both of the option price forecast and the dynamic hedging strategy tests.

The main contributions of this study are at least threefold. First, this is the first study to investigate the efficiency of the OMX indexoptions market. Compared with the empirical studies in the U.S. and other European markets, studies on the Swedish market are very limited.[1] In particular, no study has been carried out to examine the Swedish index (OMX) option market efficiency, although the OMX indexoption market has been growing tremendously since it was first introduced. The OMX indexoption market is smaller than some major stock markets in the U.S. and other European countries, however, it is an essential instrument in reducing risk exposure or increasing yield over the Swedish stock market. Especially after the NASDAQ bought the Swedish OMX on February 27, 2008, the OMX index option has played a more important role in the Swedish, Nordic, and even the world financial markets, therefore, the efficiency test of the Swedish OMX indexmarket is equally important as in other large markets. Second, this study provides a comprehensive analysis of the OMX index option market. We test the market by examining the boundary and PCP conditions and carefully detecting all the possible arbitrage opportunities in different trading strategies. Third, the results have practical relevance in terms of model selections in the option pricing and the dynamic hedging strategies in the OMX option market. We have applied the most used option pricing models in the contemporary finance literature. The results from this paper provide a proxy for the application of these models in the Swedish OMX indexoption market.

The remaining part of the paper is organized as follows. First, we present the model in section 2, and the methodologies we have used will be discussed in section 3. The data are introduced in section 4. In section 5 we present the empirical results and do analysis. Section 6 concludes.

2 The models

The predictive power of the option valuation models was recognized immediately after the Black & Scholes (1973) model was introduced. More recent models have

[1] Nordén (2008) and Nordén (2009) examine the OMX indexfuture market.

included jumps,[2] stochastic volatility,[3] and state dependent diffusion terms.[4] Today, financial researchers use models combined jumps, stochastic volatility and local volatility.[5] In order to investigate the OMX index option market, we choose to calibrate a large group of the most applied option pricing models in the contemporary finance literature, i.e. the Merton (Merton 1976), Heston (Heston 1993), Bates (Bates 1996) and AJD models (Duffie et al. 2000). The BS, Merton, Heston and Bates models are all nested in the AJD model.

2.1 The AJD model

The AJD model from Duffie et al. (2000) combines stochastic volatility and jumps. It has the following specification,

$$\begin{aligned} \mathrm{d}S_t &= rS_t\mathrm{d}t + \sqrt{V_t}S_t\mathrm{d}W_t^{(S)} + S_t\mathrm{d}Z_t^{(S)}, & (1) \\ \mathrm{d}V_t &= \kappa(\xi - V_t)\mathrm{d}t + \sigma_v\sqrt{V_t}\mathrm{d}W_t^{(V)} + \mathrm{d}Z_t^{(V)}, & (2) \end{aligned}$$

where S_t is the stock prices, V_t is the volatility, r is the interest rate, W^S and W^V are standard Brownian motions with correlation ρ_w. κ and ξ are the mean reversion rate and mean reversion level, respectively. σ_v is the volatility of volatility. $Z^{(S)}$ and $Z^{(V)}$ are pure jump processes in return and variance processes, respectively. The jumps $Z^{(S)}$ and $Z^{(V)}$ can occur simultaneously or randomly. The jump processes have a constant mean jump-arrival rate λ, whose bivariate jump size distribution has the transform of Θ. The **Q**-dynamics for the log price process $\ln S_t$ is given by,

$$\mathrm{d}\begin{pmatrix} \ln S_t \\ V_t \end{pmatrix} = \begin{pmatrix} r - \lambda\bar{\mu} - V_t/2 \\ \kappa(\xi - V_t) \end{pmatrix}\mathrm{d}t + \begin{pmatrix} 1 & 0 \\ \rho_w\sigma_v & \sqrt{1-\rho_w^2}\sigma_v \end{pmatrix}\sqrt{V_t}\mathrm{d}W_t + \mathrm{d}Z_t, \quad (3)$$

where $\bar{\mu}$ is chosen as $\Theta(1,0) - 1$ such that the dynamic is risk-neutral. Following Duffie et al. (2000), we define Θ as follows,

$$\Theta(d_1, d_2) = \lambda^{-1}\big(\lambda^y\theta^y(d_1) + \lambda^v\theta^v(d_2) + \lambda^c\theta^c(d_1, d_2)\big), \quad (4)$$

[2] (see e.g., Merton 1976)

[3] (see e.g., Hull & White 1987, Heston 1993)

[4] (see e.g., Dupire 1994, Derman & Kani 1994)

[5] (see e.g., Bates 1996, Duffie et al. 2000, Eraker et al. 2003, Carr & Wu 2004, Eraker 2004, Broadie et al. 2007).

where

$$
\begin{aligned}
\lambda &= \lambda^y + \lambda^v + \lambda^c, & (5)\\
\theta^y(d_1) &= \exp\left(\mu_y d_1 + \frac{1}{2}\sigma_y^2 d_1^2\right), & (6)\\
\theta^v(d_2) &= \frac{1}{1-\mu_v d_2}, & (7)\\
\theta^c(d_1, d_2) &= \frac{\exp\left(\mu_{cy} d_1 + \frac{1}{2}\sigma_{cy}^2 d_1\right)}{1-\mu_{cv} d_2 - \rho_J \mu_{cv} d_1}. & (8)
\end{aligned}
$$

λ^y, λ^v and λ^c are the jump intensities of $Z^{(S)}$, $Z^{(V)}$ and simultaneous jumps in S and V. The jump size in S is normal distributed with $N(\mu_y, \sigma_y^2)$. The jump size in $Z^{(V)}$ is exponentially distributed with mean μ_v. For the simultaneous jumps, the marginal distribution of the jump size in V is exponential with mean μ_{cv}. The jump size in S has the distribution $N(\mu_{cy} + \rho_J z_v, \sigma_{cy}^2)$ conditional on a realization of the jumps size of z_v in V.

The AJD model nests several models. We obtain the stochastic volatility model, originally proposed by Heston (1993) by letting $\lambda = 0$. If we take $Z^V = 0$ (i.e. $\lambda^v = \lambda^c = 0$) such that jumps only present in prices we get the Bates (1996) model. If we set $\xi = V_0 = \sigma^2$, $\lambda^v = \lambda^c = 0$, and $\sigma_v = 0$, the model reduces to the pure jump diffusion model introduced by Merton (1976). Finally setting $\lambda = 0$, $\xi = V_0 = \sigma^2$ and $\sigma_v = 0$ reduce the AJD model to the Black & Scholes (1973) model.

3 Methodology

In this section, we present and motivate some methodological issues used in the analysis.

3.1 Model calibration: the Unscented Kalman Filter

The dominating calibration method is the weighted least square method, i.e. taking the parameter vector of the model that minimizes the weighted sum of the squared difference between the observed mid-price and the model price. This calibration is known to be numerically difficult to find the parameters minimizing the loss function. Another problem of this method is that it uses a small set of observations, easily overfitting data (see e.g., Bates 1996, Hull 2002, Cont & Tankov 2004).

As a alternative, the Kalman filter of Kalman (1960) is an optimal, minimum mean squared error estimator for linear systems. When system dynamics are intrinsically nonlinear, the extended Kalman filter (EKF) has customarily been used. In general, the EKF performs a truncated first order Taylor liberalization of the current state, to which the linear filter equations are applied, so the traditional Kalman filter can be applied (see e.g., Lee 1986, Maybeck 1982). Unfortunately, the EKF has two important potential drawbacks. First, it does suffer from a divergence problem, and due to the deviation of the Jacobian matrices, the linear approximations to the nonlinear functions can be complex causing implementation difficulties. Second, these linearizations can lead to filter instability if the time step intervals are not sufficiently small (see e.g., Julier et al. 1995). To address these problems, Julier & Uhlmann (1997) develop the Unscented Kalman Filter (UKF) to estimate the state of a non-linear system using the unscented transform. The UKF has been shown to be a powerful superior alternative to the EKF in the time series modeling (see e.g., Wan & Merwe 2000, Lindström et al. 2008).

We therefore calibrate all models using the unscented Kalman filter because of its nonlinear estimation high calculation speed. We implement the UKF calibration followed the method from Gove & Hollinger (2006) (see Appendix A for more details about the Unscented Kalman Filter procedure).

The calibration states in the unscented Kalman filter is given according to Lindström et al. (2008)

$$\theta_k = \theta_{k-1} + w_{k-1}, \tag{9}$$
$$C_k^{Mid} = C^{Model}(\theta_k) + v_k, \tag{10}$$

where θ_k denotes the model parameters, C_k^{Mid} is the observable option price process which is the mid price of the ask-bid spread, w_k and v_k are pairwise independent zero mean random vectors with covariance matrices Q and R respectively, and C^{Model} is the closed form of the option price with respect to the model parameter. For this specific calibration model, we apply the following equation in the measurement update equations,

$$\hat{x}_k = \hat{x}_{k|k-1} + K_k\big((C_k^{Ask} - \hat{z}_{k|k-1})1_{\{\hat{z}_{k|k-1} > C_k^{Ask}\}} + (C_k^{Bid} - \hat{z}_{k|k-1})1_{\{\hat{z}_{k|k-1} < C_k^{Bid}\}}\big), \tag{11}$$

because any estimated price in the real ask-bid spread should be considered as a correct estimation.

3.2 Theoretical option price calculation: the Inverse Fourier Transform

Since the probability density of a Lévy process is typically not known in closed form, there is no explicit formula for call option prices in stochastic volatility with jump models based on the Lévy process. However, some characteristic functions of the model can be derived, which leads to the development of the Fourier-based option pricing method. The Fourier transform based method has been frequently used in financial applications. For those who are interested in this method, Carr & Madan (1999) is a good reference to start with. A long list of references to articles using Fourier transform based methods can be found in Carr & Wu (2003).

In this paper, suppose t and T are current time and time of maturity, S_t is the stock price at time t and K is the strike level. Set $Y_T = \ln S_T$ and $k = \ln K$, then the payoff of the European vanilla call option can be written as

$$(S_T - K)^+ = (e^{Y_T} - e^k)^+ \doteq f(k).$$

Applying the Fourier transform and its inverse transform, the payoff can be derived as follows,

$$\begin{aligned}\mathbf{F}f(z) = \int_{-\infty}^{\infty} e^{zk} f(k)\mathrm{d}k &= \int_{-\infty}^{\infty} e^{zk}(e^{Y_T} - e^k)^+ \mathrm{d}k \\ &= \left[\frac{1}{z}e^{zk+Y_T} - \frac{1}{z+1}e^{(z+1)k})\right]_{-\infty}^{Y_T} \\ &= \frac{e^{(z+1)Y_T}}{z(z+1)}.\end{aligned}$$

Applying the inverse Fourier transform,

$$\mathbf{F}^{-1}\mathbf{F}f(k) = \frac{1}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} e^{-zk}\mathbf{F}f(z)\mathrm{d}z = \frac{1}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} e^{-zk}\frac{e^{(z+1)Y_T}}{z(z+1)}\mathrm{d}z = f(k).$$

In this case, $\bar{z}$ is a positive real number, and if $\bar{z} < -1$ we obtain the pay-off of a European put-option.

The European vanilla call option price can then be expressed as

$$\begin{aligned}C_t &= e^{-r(T-t)}E^{\mathbb{Q}}[(S_T - K)^+|\mathcal{F}_t] \\ &= e^{-r(T-t)}E^{\mathbb{Q}}[f(k)|\mathcal{F}_t] \\ &= e^{-r(T-t)}E^{\mathbb{Q}}[\frac{1}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} e^{-zk}\frac{e^{(z+1)Y_T}}{z(z+1)}\mathrm{d}z|\mathcal{F}_t] \\ &= e^{-r(T-t)}\frac{1}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} \frac{e^{-zk}}{z(z+1)}E^{\mathbb{Q}}[e^{(z+1)Y_T}|\mathcal{F}_t]\mathrm{d}z \\ &= e^{-r(T-t)}\frac{1}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} \frac{e^{-zk}}{z(z+1)}e^{K_{t,T}(z+1)}\mathrm{d}z, \qquad (12)\end{aligned}$$

where

$$e^{K_{t,T}(z+1)} = E^{\mathbb{Q}}[e^{(z+1)Y_T}|\mathcal{F}_t] = E^{\mathbb{Q}}[e^{(z+1)\ln(S_T)}|\mathcal{F}_t],$$

is the moment generating function of log stock-price $Y_T = \ln(S_T)$ under the risk neutral measure $\mathbb{Q}$ given S_t. For the AJD model and all related models (i.e. Black-Scholes, Merton, Heston, Bates) we have that $K_{t,T}(z)$ can be written on the form (Duffie et al. 2000)

$$K_{t,T}(z) = zY_t + \alpha(T-t,z) + \beta(T-t,z)V_t. \tag{13}$$

For this to work the real part of z has to be positive and satisfy the condition $E^{\mathbb{Q}}[S_T^{1+Re(z)}] < \infty$, i.e. $Re(z) \in A^+_{S_T} = \{x > 0 : E^{\mathbb{Q}}[S_T^{1+x}] < \infty\}$. According to the rule to choose $Re(z)$ in Lindström et al. (2008), we can use the "golden-section search" method to find an optimal value of $Re(z)$ in order to approximate the integral by a Gauss-Laguerre quadrature formula,

$$Re(z)_{\min} = \arg\min_{Re(z)\in A^+_{S_T}} g(Re(z)),$$

where

$$g(z) = \frac{e^{-zk}}{z(z+1)} e^{K_{t,T}(z+1)}.$$

In general, we have that the Gauss-Laguerre quadrature formula approximates an exponentially weighted integral from zero to infinity as,

$$\int_0^\infty e^{-x} f(x)dx \approx \sum_{j=1}^{n} \omega_j^{(n)} f(x_j^{(n)}),$$

The details of the methods can be found in Lindström et al. (2008).

3.3 The mean-variance hedging weights

As we need to hedge the stochastic volatility and the jumps besides the option itself, we use the mean-variance hedging weight in the dynamic hedging strategy test. The mean variance hedge is a type of quadratic hedging strategy which minimizes the quadratic error of replication between the contingent claim of the options and the terminal wealth of the hedging at maturity. Suppose $\{S_t\}$ is the stock price process, C_t is the option price at time t and the maturity date is denoted by T. The mean variance hedging strategy in this paper is defined by

$$\min E^{\mathbb{Q}}[(h_s\widetilde{\Delta S_t} - \widetilde{\Delta C_t})^2 | \mathcal{F}_t],$$

where

$$\begin{aligned} \widetilde{\Delta S_t} &= e^{-r\delta} S_{t+\delta} - S_t, \\ \widetilde{\Delta C_t} &= e^{-r\delta} C_{t+\delta} - C_t. \end{aligned}$$

By setting $Q(h_s) = E^{\mathbb{Q}}[(h_s\widetilde{\Delta S_t} - \widetilde{\Delta C_t})^2|\mathcal{F}_t]$ and $Q'_{h_s}(h_s) \doteq 0$, the mean variance hedging weight can be calculated as

$$h_s = \frac{E^Q[\widetilde{\Delta C_t}\widetilde{\Delta S_t}|\mathcal{F}_t]}{E^{\mathbb{Q}}[\widetilde{\Delta S_t}^2|\mathcal{F}_t]},$$

Furthermore, we argue that using Eq. (12) and (13) and the calculations in Appendix B we have the following representation,

$$h_s = \frac{e^{-r\tau}}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} e^{-z\ln\frac{K}{S_t}} \frac{e^{-2r\delta+\alpha(\tau-\delta,z_1)+\tilde{\alpha}(\delta,z_2,z_3)+\tilde{\beta}(\delta,z_2,z_3)V_t} - e^{\alpha(\tau,z_1)+\beta(\tau,z_1)V_t}}{z(z+1)(e^{-2r\delta+\alpha(\delta,2)+\beta(\delta,2)V_t}-1)}\mathrm{d}z \quad (14)$$

where $\tau = T-t,\ z_1 = z+1, z_2 = z+2, z_3 = \beta(\tau-\delta, z+1)$, $\tilde{\alpha}(u,x,y)$ and $\tilde{\beta}(u,x,y)$ are defined through

$$E^{\mathbb{Q}}[e^{xY_{t+u}+yV_{t+u}}|\mathcal{F}_t] = e^{xY_t+\tilde{\alpha}(u,x,y)+\tilde{\beta}(u,x,y)V_t},$$

for all $x, y \in \mathbb{C}$ such that $E^{\mathbb{Q}}[|e^{xY_{t+u}+yV_{t+u}}|] < \infty$, i.e. $e^{xY_t+\tilde{\alpha}(u,x,y)+\tilde{\beta}(u,x,y)V_t}$ is the simultaneous moment generating function of the log stock-price $\{Y\}$ and the stochastic volatility $\{V\}$. The relations $\tilde{\alpha}(u,x,0) = \alpha(u,x)$ and $\tilde{\beta}(u,x,0) = \beta(u,x)$ are thus immediate. The proof of the representation given by Eq. (14) can be found in Appendix B together with the exact expressions for $\tilde{\alpha}$ and $\tilde{\beta}$.

4 The OMX index market and the data

In this section, we give a brief introduction to the OMXS30 option index market and present the data used in this paper.

4.1 The OMXS30 index and index options and futures

The Swedish OMXS30 index was introduced on September 30, 1986 with the purpose of serving as an underlying security for trading in standardized European options and futures contracts. It is a market value-weighted index consisting of the 30 largest capitalized shares at NASDAQ OMX Stockholm AB.[6] The limited number of constituents guarantees that all the underlying shares of the index have excellent liquidity in order that this index is suitable as underlying for derivatives products. The composition of the index is updated every six months. The base date for the OMXS30 Index is September 30, 1986, with a base value of 500. On

[6]See the official website of the NASDAQ OMX Group, Inc. www.nasdaqomxnordic.com

April 27, 1998, the index was divided by 4. Figure 8 plots the OMXS30 index showing the big drop in 1998.

- Figure 8 about here -

The OMXS30 index option is a type of European call and put options contract with the OMXS30 index as the underlying asset, cash settlement and terms of 3, 12 and 36 months. Premium settlement day and payment of settlement occur on the first Swedish bank day following registration and the expiration day, respectively. The expiration day is the same as the last trading day which occurs on the third Friday of the expiration month of the expiration year, or, where such day is not a Swedish bank day, the preceding bank day. A new expiration month is listed four Swedish bank days prior to the expiration of the previous options series.

Besides the index options, the futures contracts on OMXS30 have also been traded with the same terms of options of 3, 12, and 36 months. The expiration day is settled on the third or fourth Friday in the expiration month of the expiration year, or the previous bank day if this Friday is not a Swedish bank day or is declared to be a half trading day. Before 2005, the OMXS30 index future was much like a forward contract because it was settled at maturity only, instead of on a marked-to-market daily basis (see e.g., Nordén 2008).[7] On February 14, 2005, the OMX introduced a daily settlement structure in a similar manner as on major index futures exchanges in the world.

4.2 The data

The sample used in this paper includes daily closing data that start on April 27 and end on August 31, 2010. The data include OMX index prices (closing bid, ask quotes), index options prices (closing bid, ask quotes), strike prices, and index futures (closing bid, ask quotes). We chose April 27 1998 as starting date due to the index base change that the index was divided by 4 on that day (see Figure 1). This change affects the model calibration process and leads to inconsistent parameter, and therefore we excluded the data before that day.

All data are provided by the NASDAQ OMX group. This includes 3094 trading days. The total number of options is 379937, of which 179460 are call options and 200477 are put options. We use the STIBOR rate as the risk free interest rate, which is collected from the national bank of Sweden. For the price calibration, if the time period is less than 30 days, the weekly rate is used; if it is between 30 and 60 days, we use the monthly rate; a 2-month rate is used for 60-90 days; a

[7] The author investigates the effects of the settlement procedure on futures market liquidity, trading activity, and futures hedging performance.

3-month rate is used for 90-182 days, etc. For the trading strategy, the constant interest rate is used during the life of the portfolio.

- Table 1 and Figure 2 about here -

Table 1 and Figure 2 offer a summary of the put and call options data divided into subgroups with respect to moneyness and time to expiration. The observations are divided into five subgroups according to moneyness and four subgroups according to the number of days left to expiration. DITM, ITM, ATM and OTM denote deep in the money, in the money, at the money, out of the money, and deep out of the money. The moneyness is defined according to the intrinsic value of the option, i.e. a call (put) option is said to be deep out of the money if $0.85 > S/K(K/S)$; out of the money (OTM) if $0.85 < S/K(K/S) < 0.98$; at the money (ATM) if $0.98 < S/K(K/S) < 1.02$; in the money (ITM) if $1.02 < S/K(K/S) < 1.15$; and deep in the money (DITM) if $S/K(K/S) > 1.15$.[8] TM denotes time to maturity. We divide the time to maturity of the options into four categories: within one week, from one week to one month, from one month to three months and longer than three months. The Spread is the average of the closing bid and ask spread. The Volume is the average of the number of contracts traded for options within the corresponding category. The Mean and Std of call and put options are the mean and standard deviation of the option prices.

We can see from Table 1, for both call and put options, that the total observations of OTM options are much larger than the ATM and ITM options. Looking at the Volume column in Table 1, we find that the most liquid options are the ATM and the OTM options with short time to maturity (less than one month) and the most illiquid options are the options with longer than 90 days time to maturity. Turning to the bid and ask spread, we find that both the option price and the bid-ask spread increase in the time to maturity and decrease in the moneyness from DITM to DOTM for both call and put options. Further, it can be seen from Figure 2, that the portion of DOTM and OTM put options over the total observations in each TM subgroup is larger than the portion of the DOTM and OTM call options over the total observations in the corresponding TM category. Turning to the time to maturity, we find that most of the put and call options are the options with a maturity time from one month to three months.

[8] This is a common way to define the moneyness (see e.g., Nordén 2001, Draper & Fung 2002, Dumas et al. 1998).

4.3 Data Synchronization, bid ask spread, and data filtering

Several problems are present in the efficiency test, i.e., the problem of the data synchronization, the problem of considering the bid and ask spread. The data synchronization is one of the most crucial requirements for the boundary and PCP tests and the dynamic hedging strategy. Ideally, we should overcome this problem with the intra-day data. However, as the intra-day data for the 30 most traded stocks are not available, we tackle the problem by hedging the OMX options with the index futures. Using future contracts to hedge options is a common choice when intra-day data is not available. In reality, futures and options are traded side by side in the same exchange, and the OMX index options and the futures expire on the same day, which minimizes data synchronization problems. Further, in order to further reduce the problems with infrequent trading and other noises in the market, we have excluded options and futures with a volume of zero, the option and future prices that are less than and equal to 0 or 0.01, and the options with a maturity of fewer than seven days. The options with a maturity of fewer than seven days usually have relatively small time premiums, and contain little information about the volatility.

Turning to the bid ask spread problem,[9] a common method is to use the midpoint of the bid and ask prices. However, according to Nordén (2009), the OMX index future is asymmetrically distributed in the bid and ask spread. This implies that representing a futures price with the bid-ask midpoint results in a bias. Another approach is to assume the bid and ask spread is constant, and then it can be estimated based on a sample of bid-ask quotations (see e.g., Phillips & Smith 1980). However, no method can fully extract all information for bid and ask spread besides a sensitivity analysis (Cavallo & Mammola 2000). Hence we consider the bid-ask spread based on two cases. First, we use the midpoint of the bid and ask price. In the second case, we fully assign the bid and ask spread by assuming that investors buy at ask prices and sell at bid prices. However, a closing call auction procedure in the OMXS30 index futures contract was introduced on June 8, 2009.[10] With this procedure, the bid-ask spread "disappears" towards the end of the trading day (see e.g., Hagström & Nordén 2012). A simple mid-point between bid and ask spread will not be the one based on the valid quote which can be collected with the intra daily data. However, as we do not have access to the

[9] For example, if we buy options, we always want to buy at the lowest price (bid price) or sell at the highest price (ask price). However, this is impossible in the market, where we usually sell at a lower price and buy at a higher price, beyond our expectation.

[10] This call auction procedure collects limit orders over a specified time interval and executes all matching orders in one trade, typically letting this last trade of the day represent the reference closing price.

intra daily data, and actually only one small part of our future data (less than one year) is affected, we ignore the effect of this closing call auction procedure when considering the case of the mid-point bid and ask spread.

5 Empirical results

In this section, we present the empirical results from the lower boundary and PCP conditions, and the results from the dynamic trading strategies.

5.1 Lower boundary conditions and the PCP test

The lower and upper boundary and the PCP must be held for an efficient market. Violations of these tests signal an inefficient market. As the upper boundary can be easily detected, we check the lower boundary condition only. As the discussion in the data section, we exclude options with a maturity of fewer than seven days and options and futures whose prices are 0 or 0.01, and the volume for both options and futures is zero. Hence, we have 95719 call options and 112891 put options. In order to perform the PCP test, we have to pair put and call options and futures having the same buying date, maturity date, and strike prices. We have 66479 paired samples for the PCP test.

We have also considered the transaction cost and the bid and ask spread. The transaction cost includes commissions fees, trading and clearing fees, costs derived from the bid and ask spread, short selling, etc. However, we only know that the trading and clearing fee for futures is 4.5 SEK/contract (before September 2006) and 3.5 SEK (after September 2006),[11] and the trading and clearing fee for options= 3.5 SEK/contract. The above mentioned other transaction costs are unknown and an accurate estimation of these transaction costs is very difficult. Not only do the transaction costs tend to vary over time, but they also depend on a particular strategy and the size of transactions. We decide to carry out the empirical study of boundary and PCP tests by a sensitivity analysis, i.e. assuming three transaction levels representing different types of investors.

As our discussions in the previous section, we consider the bid-ask spread problem by using the mid-point of the bid and ask price and by assuming that the bid and ask spread is fully assigned, i.e., by assuming that investors buy at ask prices and sell at bid prices. Therefore, the boundary and the PCP tests will be performed under two scenarios, i.e. the case with fully assigning the bid and ask spread and the case with using the mid-point of the bid ask price, which will be divided into three sub-scenarios with respect to different transaction levels.

[11] Nordén (2009) examines the effects of a change in fixed transaction costs on futures market liquidity, trading activity, volatility, futures pricing efficiency, and the futures exchange revenues.

1. TC=0. The transaction cost is equal to zero.

2. TC is at a lower level. The trading and clearing fee for options will be taken into account, but without the broker fee. Investors in this category can be members and brokers of the OMX group.[12]

3. TC is at the highest level, which represents the individual investor. The individual investor has to pay the trading and clearing cost and brokerage commissions. Following Cavallo & Mammola (2000) and Capelle-Blancard & Chaudhury (2001), we assume a brokerage commission of 0.05% of the contract value for traded options and futures. This transaction level represents individual investors.

Accounting for all transaction levels and the bid ask spread described above, the no-arbitrage lower boundary conditions for call[13] and put options[14] are, respectively,

$$
\begin{aligned}
&-C_{ask} + (F_{bid} - K)\exp^{-r(T-t)} \le TC_{bc} + TC_{sf} + TK,\\
&-P_{ask} + (K - F_{ask})\exp^{-r(T-t)} \le TC_{bp} + TC_{bf} + TK,
\end{aligned}
$$

where $_{ask}$ indicates buying at ask price, $_{bid}$ indicates selling at bid price, C, P and F denote call options, put options and future prices, respectively. TC_{bc} and TC_{bp} are, respectively, the transaction cost for buying call and put options. TC_{bf}, TC_{sf} are transaction costs for buying and short future contracts. TK is the brokerage fee.

The no-arbitrage conditions for the PCP test for long[15] and short hedge[16] are, respectively[17],

$$
\begin{aligned}
&C_{bid} - P_{ask} - F_{ask}\exp^{-r(T-t)} + K\exp^{-r(T-t)} \le TC_{sc} + TC_{bp} + TC_{bf} + TK,\\
&P_{bid} - C_{ask} + F_{bid} - K\exp^{-r(T-t)} \le TC_{sp} + TC_{bc} + TC_{sf} + TK,
\end{aligned}
$$

[12]The broker and the member of the OMX still need to pay the annual membership fee, but here we ignore this amount.

[13]If this is not satisfied, the arbitrage profit can be made by selling the underlaying at bid price and buying the call option at ask price.

[14]If this is not satisfied, the arbitrage profit can be made by buying the underlying at ask price and buying the underlying at ask price.

[15]If this is not satisfied, the arbitrage profit can be made by buying the underlying and put option at ask price and selling call at bid price.

[16]If this is not satisfied, the arbitrage profit can be made by selling the put option and the underlying at bid price and buying the call option at ask price.

[17]See Cavallo & Mammola (2000) for the PCP derivation.

where $TC_{sc}, TC_{sp}, andTC_{sf}$ are the transaction costs for writing call, put options and future contracts, respectively. TC_{bc}, TC_{bp}, and TC_{bf} are the transaction costs for buying call, put options and future contracts, respectively. TK is the brokerage fee.

- Table 2 about here -

In Table 2, we present the empirical results from the Boundary conditions and the PCP test. The results in Panel 1 correspond to using the midpoint of the bid and ask prices. The results in Panel 2 are under the assumption that the investor buys at the ask price and sells at the bid price, and the bid and ask spreads are fully assigned. For both Panel 1 and Panel 2, the table reports three columns with the results obtained under three assumptions on the level of transaction costs. In the first column, to make the results comparable, we ignore transaction costs, TC=0. In the second and third columns, we consider the level of transaction costs incurred, respectively, by an arbitrageur or a member of OMX (TC= trading and clearing cost), and a normal investor (TC = brokerage fee + trading and clearing cost).

In Panel 1, of the total sample of 95,719 observations, the lower boundary conditions for call options appear to have been violated in only 729(0.762%) samples, without taking any transaction costs into account (Panel $1, TC = 0$). After considering the trading and clearing costs and brokerage fees, the violations are, respectively, reduced to 0.404% and 0.203%. Comparatively, the violation of the lower boundary for put options is 1143 (1.012%) without transaction costs, and after taking into account transaction cost it reduces to 652(0.578%) with trading and clearing costs and 225(0.199%) with brokerage fees.

In Panel 2, with the bid and ask spread cost, both the number of violations of boundary conditions for call and put options are reduced to nearly zero. These incidences compare quite favorably with those in the U.S. and elsewhere. For example, Ackert & Tian (n.d.*b*) report the frequency of violations for the S&P 500 index options at more than 5% for call options and at more than 2% for put options from February 1992 to January 1994. In the much smaller Finnish index option market, from May 2, 1988 to December 21, 1990, Puttonen (1993) finds 7% violations for at the money call options exhibiting a significantly greater incidence. The lower boundary conditions from Capelle-Blancard & Chaudhury (2001) for the French CAC40 index option market appears to have been violated in 0.88% for call options and 0.51% for put options.

We look further on the results at the PCP test. The results from both long and short hedge of PCP for the Swedish OMX market indicate some inefficiency due to the large number of violations. However, after taking into account the bid and ask spread and other transaction costs, the arbitrage profits are nearly wiped away. This consistently matches our expectations, the number of hedges, which would have provided a profit opportunity, decreases substantially when transaction costs and bid and ask spreads are included. As can be seen in Panel 1, the results from the PCP for long hedge violations are 45.88%, 22.76%, and 6.52% and short hedge violations are 54.11%, 29.17%, and 9.82%, respectively. While the size of violations from PCP exhibits a clear tendency to decline substantially as the transaction cost increases. It can be observed that after fully imposing the bid and ask spreads in Panel 2, the PCP violations drop dramatically to 0.991% (1.515%), 0.51% (0.881%), and 0.228% (0.493%) for long (short) hedge under scenarios in which transaction costs are zero, transaction costs include only the trading cost and transaction costs include both costs and brokerage fees.

Compared with the efficiency conclusion from other studies in other index option markets, the violations for long and short hedges in OMX index option markets are very small. For the S&P 500 index (American) options, Evnine & Rudd (1985) report 52% (short hedge) and 22% (long hedge) violations.[18] For the S&P 500 index (European) options, Kamara & Miller (1995) report the PCP violation frequency at 23% (short hedge) and 10% (long hedge). When considering the cost and constraints, the violations for the short as well as the long hedge drop to 3% and 5%. For the smaller and newer Italian index option market, Cavallo & Mammola (2000) report 49% violations for the short as well as the long hedge. Considering the bid and ask spread and other transaction costs, they find the violation drops to 2% for both long and short hedges.[19]

In general, although the violations for PCP of OMX index options are not zero. According to Waggoner (2000), a quick look at the daily closing prices as reported in any newspaper reveals few violations of arbitrage conditions. The question is whether or not the violations are large enough for us to reject the market efficiency. We believe the violations from our result can be a result of the data non-synchronous. We cannot reject that the OMX index option market is efficient.

5.2 Results from the dynamic hedging strategies

In this section, we present the dynamic hedging results from the BS, Merton, Heston, Bates, and AJD models. Before we proceed to the hedging results, we

[18] In this paper, the authors use intra day data to examine the pricing of the options on the S&P 500 index options. Although the result of significant violations of the PCP suggests that the market is inefficient in some degree, authors believe that the result is highly influenced by the data non-synchronization. However, without taking into account the transaction cost can be another reason for the higher violations

[19] Berg et al. (1996) study the Oslo stock exchange equity options (American) on four four stocks from May to July 1991. Taking into account transaction costs but using daily closing data, they report 5.4% violations.

examine the performance of these models in the price forecast.

5.2.1 Results from the model calibration

The accuracy of the model parameters directly affects the theoretical price forecast and the hedging strategy. In order to examine the forecast performances of the models, we have calculated the Mean Squared Error (MSE) and the Mean Absolute Error (MAE) for both in-sample and out-of-sample forecasts as follows.

$$\text{MSE}(x) = \frac{1}{N}\sum_{t=1}^{N}(\hat{x}_t - x_t)^{=}\frac{1}{N}\sum_{t=1}^{N} e_t^2,$$

$$\text{MAE}(x) = \frac{1}{N}\sum_{i=1}^{N}|\hat{x}_t - x_t| = \frac{1}{N}\sum_{t=1}^{N}|e_t|,$$

where x_t is the mid-point of the bid and ask prices at time t and $\hat{x}$ is the model calibrated price (in-sample) and the overnight model forecasted price (out-of-sample). The performance of models has also been examined by the proportion of forecasted options price which are inside the bid-ask spread (IS):

$$\text{IS}(x) = \frac{1}{N}\sum_{i=1}^{N}\mathbf{1}_{[bid,\ ask]}(\hat{x}_t).$$

Finally, we use the DM test suggested by Diebold & Mariano (1995) to check the significance of the improved predictability of the AJD model on other models,

$$DM = \frac{E(dt)}{var(dt)} \sim N(0,1),$$

where $d_t = (e_{A,t} - e_{B,t})^2$ and $e_{A,t}$ and $e_{B,t}$ are prediction errors of two rival models, A and B, respectively. $E(d_t)$ and $var(d_t)$ are the mean and variance of the time series of d_t, respectively.

- Table 3, Table 4, Table 5, Table 6 about here -

All the model performances are evaluated by the Mean Squared Errors, the Mean Absolute Errors, the IS and the DM test statistics which are presented in Table 3, 4, 5, and 6. Table 3 and Table 4 report the performance of various models for in-sample call and put options, respectively. Tables 5, 6 report all the model performance for the out-of-sample price prediction. The burn-in time when calculating the MSE, MAE is set to 20 days. In all of these tables, the column of

"Total" reports the overall performance of the models, which is sub-categorized into moneyness and time to maturity defined according to the discussions in the data section. DITM, ITM, ATM, OTM and DOTM denote deep in the money, in the money, at the money, out of the money, and deep out of the money. TM denotes time to maturity. BS, H, M, Ba, and AJD denote the Black Scholes, Heston, Merton, Bates and Affine Jump Diffusion model, respectively. The DM test is performed under the null hypothesis that the AJD model performs significantly worse than the other models.[20]

As can be easily seen from the column "Total" of these tables, the AJD model outperforms all the other models by delivering the lowest MSE, MAE and the highest IS in both in-sample and out-of-sample forecast in general. The results from the DM test indicate that for both call and put options, the improvement of the AJD model upon other models is highly statistically significant in the in-sample and out-of-sample forecasts across moneyness and different subgroups of time to maturity besides several options with longer than 90 days time to maturity. However, the options with longer than 90 days time to maturity have very much less liquidity due to the small traded volume in the market. The superiority of the advanced model in the option price forecast cannot be reflected in the calibration. Therefore, the model performance cannot be evaluated by the options with longer than 90 days time to maturity. Due to the illiquidity, the options with longer than 90 days to maturity will not be able to affect the hedging strategy results.

Further, both MSE and MAE for the put and call options from the in-sample and out-of-sample forecasts increase in the time to maturity. Interestingly, it is evident from these four tables that the MSE and MAE of the Heston model are much lower than those of the Bates models implying that the Heston model outperforms the Bates model in the OMX index option market. For example, it can be seen from Tables 3 and 5, that the MSE (MAE) are 13.43 (1.71) and 15.54 (1.95) for the call option of the Heston model, respectively, while those of the Bates model are 15.99 (1.9) and 20.4 (2.19), respectively. This may also indicate that jumps in the Swedish OMX index option market in most cases happen in the volatility process. Therefore, adding jumps to the price process of the Bates model does not improve the goodness of fit of this model compared to the Heston model. On the other hand, the Bates model is still a better model in describing the price dynamics in the OMX index option market compared to the Black Scholes and the Merton models by delivering lower forecast errors.

- Figure 3 about here -

In Figure 3, we plot the estimated volatility from various models. The line

[20] The reported DM test statistics are for the MSE. We have also performed the DM test for the MAE and the IS; however, the test statistics from them are similar to the one we reported for the MSE. The DM test results for the MAE and the IS are available upon request.

in the red color is the estimated volatility from the AJD model. The blue line in each plot in the background is the estimated volatility from the BS, Merton, Heston and Bates model, respectively. We can see from the first two plots in Figure 3, the BS and the Merton model underestimate the volatility during the financial crisis period from 2008 to 2010. These two models cannot catch up the up and down movements in the volatility compared with the performance of the AJD model. The third and the fourth plot in this figure indicate that the Bates and the Heston models perform better than the BS and the Merton models in the financial crisis period. However, these two models over-estimate the volatility in the normal period from 2001 to 2003 compared with the estimated volatility from the AJD model.

5.2.2 Results from trading strategy 1

In the first trading strategy, we assume that an investor sells one call/put option when it is first issued to the market. Then a portfolio is built up to neutralize the risk by buying/selling a certain amount of future contracts according to the calculated hedging weights. The rest of the money is put into a bank account. The portfolio will be re-balanced according to the Delta and MV hedging weights. Table 7 presents the hedging results from this strategy with the MV and the Delta weights considering the clearing fees and the brokerage fees assumed as in the boundary and put-call-parity section. We categorize the results into different option types (call and put options) and different moneyness, which is defined in the same way as in the previous section. We report the mean, standard deviation and the MSE of all portfolio returns. As the portfolios we have created are risk neutral, if the market is efficient, the return of the portfolio should be close or equal to zero. Therefore, we calculate and report the MSE between the obtained return of the portfolio and zero, and if the return obtained from the model is very close to zero, this implies that the model performs well in the hedging strategy. Therefore, the lower MSE, the better the model performance.

- Table 7 here -

We first consider the hedging strategy results with the MV weights in Table 7. We find that the MSE of the AJD model is much lower than in any other model. This phenomenon can be observed consistently across the moneyness (the only exception is the ATM put option). This is an indication that the AJD model performs much better in the MV hedging than the BS, Merton, Heston, Bates models. Again, as we have observed in the price forecast section, the Bates model does not outperform the Heston model. Adding jumps to the price process does not improve the hedging results due to the large errors in the price forecast. Surprisingly, the BS model provides a not too bad hedging result compared with the Merton, Heston and

Bates models. It provides even better results than the Heston model for e.g. the ATM and ITM call and OTM put options.

The superior performance of the AJD model over other models can be double confirmed when looking at the standard deviations of the portfolio returns. The standard deviation of the returns from the portfolio with the MV weight shows that the AJD model is less volatile (more stable) than other models by delivering the lowest standard deviation for both call and put options and across the moneyness from DITM to DOTM. Again, the BS model offers a more stable hedging performance than the Merton, Bates and Heston models for the ITM and ATM call and ATM and OTM put options.

However, turning to the Delta hedge results, we have a different situation. The AJD model loses its superiority. Instead, the BS model offers the lowest MSE and the lowest standard deviation in most of the cases (besides the DOTM and DITM call and put options). For DOTM and DITM call and put options, the AJD still performs better than the BS model. The reason why the AJD model lost its superiority can be that due to the presentation of the jumps and the stochastic feature in the volatility, the market is not completed. With a market incompleteness, the Delta hedge cannot eliminate the volatility risk and the jump risk. If we still use the Delta weight, the improvement of the performance of the more advanced models will be limited. This emphasizes the importance of the MV hedge when the market presents jumps and stochastic volatility.

By looking at the mean of the portfolio returns from all models in Table 7, we find that the mean of the return of hedged portfolios with both Delta and MV weights is either negative for call options and close to zero for put options (without considering the bid and ask spread). Although the returns from put options are higher than the returns obtained from the call options, we believe that the returns will be zero or negative if we consider the bid and ask spread and other administrative costs in reality. Most importantly, by comparing the return with the standard deviation of the returns from all models, we find that the returns are not significant. This indicates that there is no arbitrage opportunity of realizing significant profits in the market.

5.2.3 Results from trading strategy 2

In the second strategy, we detect the mispricing opportunity (the difference between the model calibrated prices and the market prices), and the initial portfolio will be built up by buying the option if it is underpriced (model prices > market prices) and selling the option if it is over-priced (model prices < market prices). Then a certain amount of the OMXS30 index futures according to the calculated hedging weights will be bought or sold to hedge the options. The portfolio will be properly re-balanced according to the calculated hedging ratios. Because the prices of OTM and ITM options are quite different, we define the mispricing by percentage (the model price is 15% less (more) than the market price as over (under) price) and in absolute value (the model price is two index points less (more) than the market price as over (under) price).

- Figure 6 about here -

We first look at the detected mispricing options. We plot all the mispriced options in Figure 4. In Figure 4 (a), we plot the mispriced options when the mispricing is defined by percentage. We observe that the OTM and the DOTM options are more likely to be mispriced. This is consistent with the finding from other studies. For example, Cavallo & Mammola (2000) report that for both call and put options of the Italian index (MIBO30) market, the highest arbitrage opportunities are discovered for options out of the money and the lowest are found for options in the money. However, when we try to analyze the reason behind this phenomenon, we find that the OTM and DOTM prices are very low, and if we define the mispricing by percentage, a small difference between the model price and the market price will be identified as mispricing of the OTM and DOTM options and it will not be considered as miss pricing of the ITM and DITM options. Therefore, we change the way of defining mispricing by using the absolute value. The detected number of the mispriced options are plotted in Figure 4 (b). We observe that a large amount of mispriced options are the ITM and DITM options. It is difficult to find a measure for evaluating the performance of different models in this strategy on the same scale as in the first trading strategy, because the identified mispriced options are different and the portfolios created based on the mispricing identifications are also different. However, we do observe that the number of detected mispriced options from all models decreases from the AJD to the BS models.

Can the detected mispriced options result in arbitrage profits in the trading strategies? Table 8 and Table 9 report the average returns and the standard deviations from trading strategy 2 with the MV weight when the mispricing is defined in absolute values (considering the clearing fees and assumed brokerage fees as in trading strategy 1, but without considering the bid and ask spread) for call and put options, respectively. In these tables, OP and UP denote overpricing and underpricing. The number after OP and UP is the classification of different identified mispricing groups. We have six mispricing groups in terms of OP 2-5, OP 5-10, OP >10, UP 2-5, UP 5-10, and UP >10. E.g., OP 2-5 indicates that the option is overpriced between 2 to 5 index points. We can see from Table 8 and Table 9 that most mispriced options are within the 10 index points. The returns from the OP >10 (UP >10) group are larger than other over(under)priced groups such as OP 2-5 and OP 5-10 (UP 2-5 and UP 5-10). However, the standard deviations are also increasing dramatically when the mispricing limits increase. The returns from the put options obtained from the trading strategy are larger than from the call options, in particular when the options are overpriced. However, none of the returns are significant when looking at the standard errors. We also believe that the obtained returns will be reduced largely when we take into account the bid and ask spread. The market efficiency then cannot be rejected based on the results we have obtained.

6 Conclusion

In this paper, we have empirically tested the market efficiency of the Swedish OMXS30 index option market. The market efficiency definition is the absence of arbitrage opportunity in the market. We first checked the boundary conditions and the Put-Call Parity, and then performed a variance based efficiency test, which was carried out by establishing a risk neutral portfolio and re-balancing the initial portfolio with the Delta and the MV hedge weights and with different hedging strategies. The portfolio returns have been obtained according to different levels of the transaction costs and under different assumptions of bid and ask spread.

In order to perform the various hedging strategies, we have calibrated several most applied variance based models, i.e. the BS, Merton, Heston, Bates and Affine Jump Diffusion models with the unscented Kalman filter. The Fourier transform method has been employed to calculate the closed form solution of option price based on the calibrated parameters from these models. We find that the AJD model significantly outperforms the BS, Merton, Bates models in both option price forecast and trading strategies.

The boundary and the PCP tests and the dynamic hedging strategy results provide evidence that no significant abnormal returns can be obtained in the OMXS30 option market, thereby supporting the hypothesis of no arbitrage opportunity and market efficiency.

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7 Tables

Table 1: Summary Statistics for Options Data

		Call					Put				
Money	TM	Number	Mean	Std.	Volume	Spread	Number	Mean	Std.	Volume	Spread
DITM	<7	246	183.42	97.56	3	6.04	500	164.62	60.34	13	6.29
	7-30	2 343	222.35	107.51	2	6.46	3 972	184.02	76.35	6	6.30
	31-90	3 056	262.63	146.92	2	6.73	4 604	204.31	116.05	4	6.18
	>90	3 832	291.83	143.12	4	9.13	7 050	278.81	150.65	7	6.09
ITM	<7	2 491	55.42	32.09	235	4.57	2 475	58.37	34.83	183	5.03
	7-30	11 509	64.46	33.23	128	4.44	10 818	64.31	33.90	124	4.48
	31-90	22 563	72.76	31.46	29	4.40	21 753	71.98	29.95	28	4.34
	>90	14 454	108.64	39.10	14	5.00	12 200	103.01	31.82	16	4.51
ATM	<7	1 674	9.13	6.96	3262	1.42	1 646	9.28	6.53	2848	1.52
	7-30	6 794	19.02	10.13	1405	1.96	6 789	18.89	9.50	1137	2.01
	31-90	15 680	34.56	15.66	221	3.19	15 261	34.22	14.47	212	3.11
	>90	7 814	61.52	22.98	55	4.16	7 141	63.32	22.27	47	3.96
OTM	<7	2 711	0.83	1.50	1280	0.43	3 627	0.96	1.59	982	0.55
	7-30	14 777	4.27	5.12	1117	0.97	19 082	4.64	5.09	835	1.07
	31-90	31 336	14.67	11.39	361	2.49	38 941	14.65	10.84	280	2.30
	>90	16 490	37.77	19.81	91	3.95	19 872	38.36	21.13	77	3.75
DOTM	<7	1 132	0.42	0.20	37	0.82	933	0.42	0.40	58	0.81
	7-30	5 220	0.72	0.85	111	0.94	6 417	0.76	1.05	208	0.82
	31-90	6 721	2.99	3.22	79	1.82	8 867	2.74	3.24	148	1.35
	>90	8 617	12.74	12.63	53	3.4	8 529	13.21	13.22	96	2.98

Note: This table provide the summary statistics for the data used in this paper. The option data are from 27 April 1998 to 31 August 2010. The total options are 379937, of which 179460 are call options and 200477 are put options. Base on Time to Maturity, data are grouped into "less than or equal to 7 days", "between 8 and 30 days", "between 31 to 90 days" and "over 90 days". DITM, ITM, ATM and OTM denotes deep in the money, in the money, at the money, out of the money, and deep out of the money. TM denotes time to maturity. The moneyness is defined according to the intrinsic value of the option, i.e. a call (put) option is said to be deep out of the money, if $0.85 < S/K(K/S)$; out of the money (OTM) if $0.85 < S/K(K/S) < 0.98$; at the money (ATM), if $0.98 < S/K(K/S) < 1.02$; in the money (ITM) if $1.02 < S/K(K/S) < 1.15$; deep in the money (DITM) if $S/K(K/S) > 1.15$ where K is the strike price and S is the current index price. The "Spread" is the average of the closing bid and ask spread. The "Volume" is the average of the number of contracts traded for options within the corresponding category. The mean and Std of call and put options are the mean and standard deviation of the option prices.

Table 2: Results for the lower Boundary and PCP tests

	Profitable hedges	Panel 1 –Transaction Prices			Panel 2 – Bid Ask Spread		
		TC=0	TC=TC1	TC=TC2	TC=0	TC=TC1	TC=TC2
Lower Boundary Call	Total sample: 95719						
	Number of violations	729	387	194	31	24	15
	% of violations	0.762%	0.404%	0.203%	0.032%	0.028%	0.016%
	Average value	-1.633	-2.503	-3.760	-2.106	-1.981	-2.156
Lower Boundary Put	Total sample: 112891						
	Number of violations	1143	652	225	52	38	24
	% of violations	1.012%	0.578%	0.199%	0.046%	0.034%	0.021%
	Average value	-1.507	-2.094	-4.575	-2.437	-2.850	-3.455
Long Hedge	Total sample: 66479						
	Number of violations	30501	15131	4340	659	339	158
	% of violations	45.881%	22.761%	6.528%	0.991%	0.510%	0.228%
	Average value	1.020	1.184	2.103	1.397	1.913	2.581
Short hedge	Total sample: 66479						
	Number of violations	35972	19395	6529	1031	586	328
	% of violations	54.110%	29.175%	9.821%	1.551%	0.881%	0.493%
	Average value	1.141	1.280	1.950	1.728	2.245	2.636

Note: This table reports the results for the lower boundary and PCP tests. the In Panel 1, Lower boundary conditions for put and call from this panel using the transaction prices of midpoint between bid and ask prices. In Panel 2, the investors are assumed to buy asset at ask prices and sell at bid prices. TC is the total transaction cost. In the first column, $TC = 0$. $TC1$ represents the transaction cost level for the brokers of OMX, only trading cost is paid. $TC2$ is the transaction cost incurred by an individual investor who pays both trading and clearing costs and the brokerage fee.

Table 3: Price Calibration In-sample Performance - Call Options

Money	TM	MSE					MAE					IS					DMTest(vs.AJD)			
		BS	M	H	Ba	AJD	BS	M	H	Ba	AJD	BS	M	H	Ba	AJD	BS	M	H	Ba
Total		24.28	20.12	13.43	15.99	12.57	2.87	2.34	1.71	1.90	1.67	0.41	0.50	0.61	0.58	0.62	67.4	49.7	7.6	15.3
DITM		88.96	84.91	86.83	88.64	80.40	4.59	4.10	3.96	3.94	3.74	0.62	0.71	0.72	0.74	0.75	6.3	4.4	7.2	3.7
	8−30	15.44	13.82	14.46	23.34	13.55	3.05	2.79	2.88	2.93	2.71	0.64	0.82	0.76	0.83	0.85	20.9	4.4	17.2	7.0
	31−90	14.05	11.56	11.22	15.22	10.81	2.86	2.27	2.18	2.24	2.07	0.78	0.88	0.89	0.88	0.91	17.2	10.6	4.8	7.5
	>90	199.3	192.3	196.9	188.5	181.9	7.04	6.44	6.11	5.88	5.75	0.49	0.50	0.56	0.56	0.57	5.2	4.1	6.8	3.3
ITM		31.89	21.05	15.30	18.64	14.45	3.80	2.77	2.12	2.30	2.05	0.45	0.59	0.69	0.67	0.71	48.1	23.6	3.5	10.0
	8−30	9.06	6.35	4.32	6.60	4.20	2.23	1.66	1.36	1.40	1.29	0.59	0.76	0.84	0.84	0.86	32.6	17.2	1.8	2.4
	31−90	18.74	11.30	8.03	10.83	7.54	3.13	2.19	1.74	1.89	1.67	0.47	0.63	0.73	0.70	0.74	49.0	28.2	9.4	9.4
	>90	74.72	50.44	36.99	41.71	34.92	6.42	4.71	3.42	3.77	3.32	0.26	0.36	0.51	0.47	0.53	33.3	16.3	2.5	7.7
ATM		12.61	11.90	7.95	10.59	7.58	2.18	2.02	1.62	1.81	1.58	0.47	0.52	0.58	0.56	0.59	18.7	18.1	2.3	7.7
	8−30	5.05	5.32	2.37	3.30	2.32	1.47	1.44	1.00	1.09	0.98	0.40	0.43	0.51	0.52	0.53	18.1	19.6	1.1	2.8
	31−90	8.45	7.11	6.09	7.38	5.39	1.89	1.65	1.51	1.61	1.43	0.53	0.61	0.64	0.63	0.65	16.6	14.3	7.4	5.5
	>90	29.09	28.18	17.67	24.76	17.31	3.52	3.29	2.49	2.95	2.49	0.43	0.48	0.56	0.52	0.57	11.5	11.7	0.6	5.9
OTM		15.56	13.79	7.15	9.36	6.46	2.41	1.96	1.32	1.56	1.30	0.32	0.43	0.56	0.52	0.56	35.6	30.2	3.5	8.9
	8−30	2.28	2.12	0.94	1.50	0.92	0.95	0.81	0.55	0.65	0.53	0.33	0.39	0.51	0.50	0.53	22.0	18.3	0.7	2.6
	31−90	8.43	5.29	4.50	5.67	4.14	2.05	1.37	1.19	1.35	1.14	0.36	0.53	0.61	0.57	0.61	45.3	16.8	6.7	8.3
	>90	43.33	42.36	18.80	24.80	16.79	4.71	4.33	2.42	2.93	2.42	0.26	0.31	0.55	0.48	0.53	27.1	27.4	2.6	7.0
DOTM		21.57	20.80	3.97	5.97	4.27	2.34	2.19	1.06	1.26	1.11	0.38	0.37	0.57	0.55	0.56	35.9	30.9	-3.0	1.7
	8−30	0.49	0.52	0.38	0.60	0.37	0.50	0.52	0.44	0.50	0.44	0.30	0.30	0.48	0.50	0.50	5.6	7.3	0.9	3.3
	31−90	2.30	1.88	1.48	1.85	1.41	1.01	0.90	0.76	0.86	0.75	0.54	0.50	0.67	0.65	0.69	12.3	7.8	1.4	2.1
	>90	52.17	50.54	8.57	13.16	9.38	4.73	4.43	1.76	2.12	1.88	0.36	0.35	0.62	0.57	0.57	36.9	31.7	-3.2	1.5

Note: This table provides the price Calibration in-sample results for call options. The column of Total indicates the general results of each model. The MSE and MAE are the Mean Squared Errors and the Mean Absolute Errors between the market price and the calibrated prices from various models, respectively. The IS is the measure of the proportion of options price inside the bid-ask spread. BS, H, M, Ba, and AJD denote the Black Scholes, Heston, Merton, Bates and Affine Jump Diffusion models, respectively The reported values of the DM test are the test statistics under the null hypothesis that the AJD model significantly worse than the other models (the reported results are for the MSE). TM denotes time to maturity. DITM, ITM, ATM, OTM and DOTM denote deep in the money, in the money, at the money, out of the money, and deep out of the money, respectively.

Table 4: Price Calibration In-sample Performance - Put Options

Money	TM	MSE					MAE					IS					DMTest(vs.AJD)			
		BS	M	H	Ba	AJD	BS	M	H	Ba	AJD	BS	M	H	Ba	AJD	BS	M	H	Ba
Total		22.27	16.16	10.30	11.51	9.29	2.97	2.25	1.64	1.75	1.57	0.32	0.46	0.58	0.58	0.62	106.5	62.9	15.6	19.7
DITM		42.80	42.93	30.10	30.42	29.49	4.05	4.07	3.47	3.52	3.44	0.52	0.47	0.57	0.58	0.56	19.7	18.1	3.0	2.6
	8−30	16.25	16.53	15.85	15.95	15.85	3.12	3.15	3.05	3.06	3.05	0.49	0.46	0.64	0.66	0.60	11.5	21.1	0.2	1.5
	31−90	14.50	14.39	14.46	14.03	13.82	3.04	3.19	3.17	3.13	3.11	0.65	0.50	0.63	0.65	0.63	2.5	5.5	7.0	1.3
	>90	79.30	78.86	49.83	50.79	48.88	5.32	5.24	3.95	4.07	3.92	0.45	0.44	0.49	0.48	0.49	20.1	17.9	2.1	3.4
ITM		22.49	21.76	14.05	16.22	12.26	3.04	2.83	2.23	2.41	2.15	0.51	0.56	0.65	0.63	0.66	38.7	34.2	9.7	12.7
	8−30	7.37	8.32	5.32	5.82	5.54	2.00	2.07	1.64	1.71	1.65	0.65	0.62	0.78	0.76	0.78	16.0	22.4	0.8	2.3
	31−90	12.30	10.13	9.53	10.02	8.34	2.55	2.17	2.09	2.11	1.95	0.53	0.63	0.65	0.65	0.68	24.1	15.5	13.9	10.0
	>90	57.08	57.19	31.31	38.32	26.28	5.05	4.84	3.07	3.67	3.01	0.30	0.34	0.51	0.44	0.50	32.5	30.2	7.2	10.1
ATM		11.90	10.54	6.56	8.52	5.77	2.13	1.93	1.50	1.65	1.44	0.46	0.52	0.59	0.59	0.61	24.5	20.4	5.5	9.7
	8−30	5.56	5.92	2.27	2.60	2.27	1.45	1.46	0.90	0.94	0.89	0.42	0.46	0.58	0.59	0.58	18.3	20.7	0.1	3.3
	31−90	9.37	7.28	5.64	6.21	4.73	1.94	1.64	1.45	1.48	1.34	0.52	0.61	0.64	0.65	0.67	20.9	18.4	7.4	7.4
	>90	24.94	22.71	13.50	20.23	11.81	3.34	3.02	2.29	2.78	2.22	0.42	0.48	0.55	0.50	0.58	13.9	11.5	3.0	7.4
OTM		21.53	10.35	5.59	6.52	4.83	3.15	1.86	1.17	1.27	1.10	0.18	0.40	0.56	0.55	0.61	94.8	47.2	9.1	13.1
	8−30	5.12	2.90	1.05	1.12	0.93	1.52	0.94	0.56	0.56	0.49	0.14	0.37	0.49	0.52	0.59	46.6	27.8	9.7	6.1
	31−90	14.45	5.89	2.90	3.39	2.49	2.87	1.56	1.09	1.14	1.01	0.21	0.46	0.61	0.60	0.64	86.5	43.0	11.6	11.2
	>90	54.65	27.83	16.05	18.86	13.84	5.69	3.55	2.03	2.36	1.97	0.18	0.32	0.59	0.54	0.60	64.7	31.6	6.5	9.9
DOTM		23.77	14.04	10.35	10.32	9.53	2.63	1.64	1.03	1.01	0.93	0.16	0.39	0.50	0.56	0.62	48.5	27.5	5.6	0.4
	8−30	1.25	0.71	0.39	0.33	0.29	0.69	0.49	0.40	0.35	0.32	0.08	0.48	0.43	0.56	0.68	16.5	10.3	15.8	6.4
	31−90	6.06	2.11	1.38	1.41	1.15	1.68	0.88	0.70	0.68	0.61	0.24	0.46	0.55	0.58	0.64	41.3	19.1	10.3	2.0
	>90	62.03	38.20	28.44	28.36	26.35	5.35	3.44	1.93	1.91	1.78	0.17	0.24	0.56	0.56	0.57	44.8	25.5	4.9	0.7

Note: This table provides the price Calibration in-sample results for put options. The column of Total indicates the general results of each model. The MSE and MAE are the Mean Squared Errors and the Mean Absolute Errors between the market price and the calibrated prices from various models, respectively. The IS is the measure of the proportion of options price inside the bid-ask spread. BS, H, M, Ba, and AJD denote the Black Scholes, Heston, Merton, Bates and Affine Jump Diffusion models, respectively The reported values of the DM test are the test statistics under the null hypothesis that the AJD model significantly worse than the other models (the reported results are for the MSE). TM denotes time to maturity. DITM, ITM, ATM, OTM and DOTM denote deep in the money, in the money, at the money, out of the money, and deep out of the money, respectively.

Table 5: Price Calibration Out-of-sample Performance - Call Options

Money	TM	MSE					MAE					IS					DMTest(vs.AJD)			
		BS	M	H	Ba	AJD	BS	M	H	Ba	AJD	BS	M	H	Ba	AJD	BS	M	H	Ba
Total		26.42	23.23	15.54	20.40	15.47	2.98	2.53	1.95	2.19	1.96	0.40	0.47	0.55	0.53	0.55	62.22	47.84	0.63	22.27
DITM		86.41	83.74	84.87	88.27	80.40	4.57	4.16	4.00	4.00	3.86	0.62	0.70	0.70	0.72	0.73	4.78	3.57	5.26	6.79
	8−30	15.89	14.29	14.97	23.01	14.10	3.07	2.82	2.90	2.92	2.75	0.64	0.80	0.74	0.82	0.82	19.50	3.04	16.54	3.70
	31−90	14.60	12.29	11.87	15.85	11.61	2.88	2.33	2.22	2.28	2.13	0.79	0.88	0.89	0.88	0.90	15.45	9.08	3.03	2.77
	>90	206.91	202.8	205.6	200.6	194.5	7.24	6.76	6.37	6.19	6.17	0.48	0.50	0.53	0.55	0.55	3.69	3.30	4.90	3.95
ITM		33.52	23.81	17.27	21.55	17.25	3.84	2.91	2.33	2.52	2.32	0.45	0.58	0.64	0.63	0.65	42.97	22.21	0.08	11.08
	7−30	9.52	6.80	5.00	7.46	4.75	2.26	1.72	1.50	1.53	1.43	0.59	0.74	0.79	0.79	0.81	35.87	19.78	8.73	4.53
	31−90	19.99	12.94	9.68	13.46	9.53	3.18	2.32	1.96	2.13	1.93	0.48	0.61	0.67	0.65	0.67	43.64	23.30	1.97	7.49
	>90	78.98	57.78	41.16	47.16	41.72	6.54	5.02	3.73	4.06	3.79	0.26	0.35	0.47	0.45	0.47	29.47	15.94	-0.63	6.21
ATM		15.20	15.21	10.36	14.61	10.79	2.42	2.32	1.94	2.15	1.97	0.43	0.46	0.50	0.48	0.50	14.32	14.94	-1.95	7.81
	7−30	6.16	6.52	3.82	6.16	3.84	1.66	1.63	1.33	1.45	1.32	0.36	0.38	0.41	0.41	0.43	14.98	16.39	-0.28	3.50
	31−90	10.85	9.57	8.38	11.77	8.06	2.16	1.96	1.83	1.97	1.81	0.48	0.53	0.55	0.54	0.55	12.37	10.03	2.50	5.77
	>90	34.03	35.91	21.63	29.47	23.87	3.86	3.76	2.88	3.30	3.01	0.40	0.42	0.50	0.47	0.49	8.71	10.39	-2.56	4.10
OTM		18.48	17.67	9.57	14.80	9.56	2.57	2.20	1.61	1.90	1.64	0.31	0.40	0.48	0.45	0.47	36.36	30.84	0.09	13.71
	7−30	2.78	2.71	1.73	4.30	1.71	1.04	0.92	0.76	0.93	0.75	0.32	0.37	0.42	0.41	0.43	16.99	14.61	0.82	5.91
	31−90	10.45	7.48	6.50	11.23	6.40	2.23	1.64	1.48	1.71	1.48	0.35	0.48	0.53	0.50	0.52	34.18	12.22	1.48	9.76
	>90	50.89	53.35	24.03	32.30	24.17	4.96	4.70	2.84	3.35	2.96	0.26	0.30	0.48	0.43	0.46	28.64	28.82	-0.20	7.65
DOTM		22.09	22.03	4.94	11.55	4.91	2.31	2.20	1.16	1.53	1.23	0.37	0.35	0.52	0.50	0.51	33.46	28.97	-0.25	8.93
	7−30	0.52	0.59	0.46	4.28	0.46	0.52	0.54	0.48	0.74	0.48	0.28	0.29	0.44	0.44	0.47	3.03	5.98	0.26	4.08
	31−90	3.15	2.72	2.29	13.05	2.29	1.09	1.00	0.89	1.32	0.89	0.52	0.47	0.61	0.59	0.62	10.42	5.51	0.05	6.75
	>90	54.85	55.00	10.75	16.14	10.71	4.77	4.52	1.94	2.34	2.14	0.36	0.35	0.58	0.54	0.53	34.26	29.58	0.74	3.87

Note: This table provides the price Calibration out-of-sample sample results for call options. The column of Total indicates the general results of each model. The MSE and MAE are the Mean Squared Errors and the Mean Absolute Errors between the market price and the calibrated prices from various models, respectively. The IS is the measure of the proportion of options price inside the bid-ask spread. BS, H, M, Ba, and AJD denote the Black Scholes, Heston, Merton, Bates and Affine Jump Diffusion models, respectively The reported values of the DM test are the test statistics under the null hypothesis that the AJD model significantly worse than the other models (the reported results are for the MSE). TM denotes time to maturity. DITM, ITM, ATM, OTM and DOTM denote deep in the money, in the money, at the money, out of the money, and deep out of the money, respectively.

Table 6: Price Calibration Out-of-sample Performance - Put Options

Money	TM	MSE					MAE					IS					DMTest(vs.AJD)			
		BS	M	H	Ba	AJD	BS	M	H	Ba	AJD	BS	M	H	Ba	AJD	BS	M	H	Ba
Total		24.48	19.38	12.44	14.01	12.37	3.08	2.43	1.87	1.98	1.85	0.32	0.44	0.53	0.53	0.55	89.83	53.56	0.35	17.11
DITM		42.41	43.55	30.05	30.97	30.04	4.05	4.07	3.50	3.55	3.49	0.52	0.48	0.57	0.58	0.57	18.84	17.46	0.06	2.65
	7−30	15.85	16.10	15.47	15.53	15.47	3.11	3.13	3.04	3.04	3.04	0.51	0.48	0.65	0.66	0.62	10.25	20.94	0.08	2.94
	31−90	14.51	15.53	15.25	15.06	14.94	3.10	3.25	3.20	3.18	3.17	0.63	0.51	0.62	0.64	0.62	-3.32	6.21	5.37	2.22
	>90	82.22	84.19	51.87	54.19	51.85	5.41	5.35	4.05	4.19	4.04	0.44	0.43	0.49	0.47	0.49	19.19	17.26	0.34	2.44
ITM		26.54	27.06	17.07	19.93	16.85	3.23	3.06	2.50	2.68	2.47	0.50	0.53	0.60	0.58	0.61	31.74	28.66	0.85	10.53
	7−30	7.77	8.70	5.97	6.43	5.97	2.04	2.11	1.74	1.80	1.73	0.64	0.61	0.74	0.73	0.74	18.82	26.83	0.03	10.77
	31−90	15.20	12.95	12.72	13.41	11.91	2.75	2.41	2.39	2.42	2.30	0.51	0.59	0.59	0.60	0.62	17.19	8.11	7.53	10.75
	>90	67.79	72.89	37.07	46.47	37.63	5.43	5.32	3.53	4.14	3.60	0.30	0.33	0.46	0.40	0.44	27.05	26.36	-0.57	7.99
ATM		15.34	14.65	9.50	11.61	9.62	2.40	2.24	1.85	2.00	1.86	0.42	0.47	0.51	0.50	0.51	19.18	18.52	-0.54	7.23
	7−30	6.43	6.90	3.58	3.85	3.53	1.60	1.59	1.19	1.22	1.18	0.39	0.42	0.49	0.49	0.49	15.25	18.34	1.02	5.05
	31−90	12.23	10.17	8.69	9.41	8.13	2.21	1.96	1.82	1.86	1.78	0.47	0.53	0.55	0.55	0.55	14.62	12.24	3.41	7.23
	>90	33.13	33.65	18.57	25.71	20.14	3.83	3.60	2.77	3.23	2.85	0.38	0.42	0.48	0.44	0.48	11.54	12.28	-1.69	4.95
OTM		23.11	12.90	7.32	8.69	7.22	3.20	2.01	1.40	1.51	1.38	0.18	0.39	0.50	0.50	0.53	82.14	38.51	0.94	10.93
	7−30	5.39	3.24	1.56	1.59	1.43	1.53	0.99	0.70	0.69	0.64	0.14	0.36	0.44	0.46	0.50	43.40	25.52	8.82	8.00
	31−90	15.56	7.42	4.55	5.31	4.48	2.91	1.71	1.34	1.40	1.31	0.21	0.45	0.54	0.53	0.55	76.31	35.56	1.44	12.42
	>90	58.79	34.98	19.43	23.55	19.30	5.81	3.84	2.36	2.68	2.41	0.19	0.32	0.55	0.50	0.54	55.45	27.03	1.59	7.54
DOTM		23.93	15.11	10.99	11.40	10.97	2.62	1.69	1.11	1.10	1.04	0.16	0.39	0.47	0.53	0.58	36.02	16.66	0.13	4.07
	7−30	1.22	0.73	0.42	0.36	0.34	0.69	0.49	0.42	0.37	0.35	0.08	0.49	0.43	0.56	0.66	15.82	10.14	12.47	4.42
	31−90	6.08	2.38	1.70	1.94	1.55	1.67	0.92	0.77	0.77	0.69	0.23	0.47	0.52	0.54	0.59	37.80	16.77	3.68	9.11
	<90	63.67	41.77	30.69	31.49	30.66	5.42	3.61	2.11	2.12	2.03	0.16	0.24	0.52	0.52	0.54	32.19	15.23	0.40	2.71

Note: This table provides the price Calibration out-of-sample sample results for put options. The column of Total indicates the general results of each model. The MSE and MAE are the Mean Squared Errors and the Mean Absolute Errors between the market price and the calibrated prices from various models, respectively. The IS is the measure of the proportion of options price inside the bid-ask spread. BS, H, M, Ba, and AJD denote the Black Scholes, Heston, Merton, Bates and Affine Jump Diffusion models, respectively The reported values of the DM test are the test statistics under the null hypothesis that the AJD model significantly worse than the other models (the reported results are for the MSE). TM denotes time to maturity. DITM, ITM, ATM, OTM and DOTM denote deep in the money, in the money, at the money, out of the money, and deep out of the money, respectively.

Table 7: Results from dynamic Hedging Strategy 1 with transaction cost

		Mean Variance Hedge														
Type	Moneyness	Mean					Standard Deviation					MSE				
		BS	M	H	Ba	AJD	BS	M	H	Ba	AJD	BS	M	H	Ba	AJD
Call		-1.20	-0.52	-0.87	-1.18	-1.05	10.22	10.92	10.15	10.78	9.87	105.98	119.63	103.68	117.64	98.47
	DITM	-4.65	-3.67	-4.58	-5.18	-4.61	14.14	13.18	13.89	13.77	13.37	221.48	187.20	213.96	216.53	200.10
	ITM	-2.28	-0.55	-1.49	-1.74	-2.18	11.70	13.00	11.75	12.32	11.45	142.03	169.42	140.37	154.68	135.80
	ATM	-1.32	-1.02	-1.16	-1.45	-1.21	10.06	11.27	10.09	11.00	10.00	102.95	128.10	103.09	123.04	101.53
	OTM	-0.23	-0.19	-0.10	-0.51	0.00	8.61	9.43	8.65	9.61	8.19	74.26	89.02	74.75	92.53	67.07
	DOTM	0.75	1.09	0.95	0.89	0.89	6.89	6.36	6.02	6.21	6.02	48.06	41.58	37.11	39.40	37.02
Put		1.64	1.84	1.81	1.54	1.66	8.98	10.65	9.08	9.63	8.82	83.39	116.81	85.71	95.09	80.52
	DITM	3.31	3.64	3.67	3.76	3.52	9.04	8.72	8.45	8.57	8.57	92.64	89.36	84.97	87.57	85.80
	ITM	2.92	3.30	3.38	3.01	3.13	9.67	10.75	9.53	10.34	9.38	102.13	126.36	102.28	115.95	97.72
	ATM	1.64	1.50	1.63	1.34	1.66	9.24	11.67	9.62	10.50	9.35	88.07	138.36	95.29	112.08	90.22
	OTM	1.15	1.32	1.25	0.99	1.08	8.86	11.30	9.18	9.66	8.76	79.90	129.39	85.87	94.25	77.89
	DOTM	-0.29	-0.02	-0.38	-0.74	-0.46	7.08	7.70	6.87	7.01	6.68	50.20	59.29	47.28	49.75	44.80

		Delta Hedge														
Type	Moneyness	Mean					Standard Deviation					MSE				
		BS	M	H	Ba	AJD	BS	M	H	Ba	AJD	BS	M	H	Ba	AJD
Call		-1.20	-1.24	-1.27	-1.24	-1.24	10.22	10.66	10.60	10.65	10.60	105.86	115.13	113.94	114.85	113.91
	DITM	-4.65	-4.65	-4.51	-4.70	-4.58	14.14	13.76	13.52	13.66	13.51	221.54	210.98	203.14	208.59	203.42
	ITM	-2.28	-2.47	-2.71	-2.48	-2.67	11.69	12.33	12.42	12.33	12.36	141.89	158.13	161.63	158.29	159.85
	ATM	-1.31	-1.31	-1.33	-1.23	-1.32	10.05	10.78	10.89	10.85	10.92	102.77	117.98	120.28	119.15	120.97
	OTM	-0.22	-0.24	-0.15	-0.26	-0.09	8.61	9.02	8.92	9.12	8.94	74.14	81.42	79.60	83.22	79.93
	DOTM	0.75	0.86	0.90	0.88	0.89	6.89	7.08	6.30	6.55	6.43	47.98	50.84	40.47	43.70	42.18
Put		1.65	1.62	1.56	1.66	1.58	8.98	9.22	9.16	9.13	9.07	83.31	87.68	86.33	86.08	84.74
	DITM	3.31	3.44	3.50	3.60	3.53	9.03	9.09	8.77	8.81	8.81	92.57	94.33	89.10	90.61	89.97
	ITM	2.93	2.83	2.86	2.82	2.91	9.67	9.82	9.89	9.90	9.87	102.03	104.46	106.06	106.04	105.89
	ATM	1.64	1.67	1.60	1.76	1.61	9.23	9.80	9.96	9.92	9.88	87.95	98.85	101.81	101.41	100.21
	OTM	1.15	1.12	0.96	1.16	0.97	8.86	9.23	9.00	8.93	8.84	79.81	86.52	81.94	81.07	79.07
	DOTM	-0.30	-0.32	-0.33	-0.38	-0.36	7.08	6.72	6.88	6.88	6.74	50.20	45.32	47.47	47.50	45.51

Note: This table reports the results from the dynamic hedging strategy with MV and Delta weights. BS, H, M, Ba, AJD denote the Black Scholes, Heston, Merton, Bates and Affine Jump Diffusion models. The MSE is the mean squared error between the return of the portfolio and zero. DITM, ITM, ATM, OTM and DOTM denotes deep in the money, in the money, at the money, out of the money, and deep out of the money.

Table 8: Hedging Strategy 2 with transaction cost - Call

Category	Moneyness	Mean of NPV					STD of NPV					Number				
		BS	M	H	Ba	AJD	BS	M	H	Ba	AJD	BS	M	H	Ba	AJD
OP Over 10	DITM	-1.54	12.31	10.21	7.73	10.37	34.66	38.55	41.99	40.52	44.56	403	245	198	217	179
	ITM	1.42	10.11	6.02	7.22	10.50	15.53	18.05	18.44	17.11	17.78	1333	489	359	382	289
	ATM	7.93	5.38	5.39	5.60	6.92	14.08	17.35	16.40	16.21	17.10	151	120	144	129	110
	OTM	14.67	5.03	2.84	9.15	18.40	18.16	21.84	22.85	22.70	19.37	206	177	344	202	206
	DOTM	13.00	7.19	7.06	23.10	24.55	15.55	20.72	30.67	16.16	14.28	15	11	46	41	34
OP 5-10	DITM	-3.92	0.31	-4.93	-3.69	-4.18	14.87	12.18	11.44	12.66	13.32	578	342	268	312	264
	ITM	-2.14	3.22	-1.89	-2.51	-3.47	12.75	15.41	14.22	15.65	14.98	3620	1621	863	978	825
	ATM	-0.03	3.33	1.09	0.36	-0.52	12.70	13.14	14.33	14.21	14.67	567	479	344	371	279
	OTM	3.51	2.33	0.03	3.01	3.42	12.85	13.85	16.34	13.39	11.31	392	436	570	620	438
	DOTM	6.42	3.40	9.38	10.20	9.74	13.32	13.15	15.49	9.71	10.70	44	52	163	204	196
OP 2-5	DITM	-0.78	-0.38	-0.74	-1.96	-0.86	6.53	7.29	6.43	7.73	7.01	5257	3958	3806	3319	3466
	ITM	-0.82	1.84	-0.16	0.11	-0.89	9.73	12.85	10.94	12.28	11.23	11447	6795	5342	4480	4289
	ATM	-1.12	-0.16	-1.13	-0.92	-1.16	11.15	12.76	12.43	13.22	12.26	1917	1854	1934	1650	1430
	OTM	0.06	1.08	1.35	1.23	1.94	10.46	10.44	10.51	10.67	9.62	1764	2474	3194	2901	2747
	DOTM	2.11	2.71	4.99	4.32	4.68	5.15	5.66	7.75	7.13	6.21	342	467	1141	966	933
UP 2-5	DITM	6.69	3.32	4.14	4.90	5.70	9.58	10.35	10.19	10.98	9.67	435	598	689	685	642
	ITM	1.87	0.27	0.45	0.18	0.73	13.38	15.19	12.41	13.36	11.67	1337	2136	2401	3161	2731
	ATM	-1.55	-3.81	-1.01	-1.91	-2.15	10.70	14.23	11.96	13.17	11.38	2548	1654	1595	1913	1768
	OTM	-3.13	-2.62	-2.82	-1.94	-3.37	8.93	12.24	11.77	12.46	10.75	8793	4751	2685	3841	3011
	DOTM	-2.80	-3.25	-5.96	-2.51	-4.66	10.34	9.71	12.19	11.94	12.51	1612	1325	540	1016	680
UP 5-10	DITM	9.66	6.04	9.40	9.23	9.50	10.87	12.19	9.90	11.74	9.48	400	507	453	490	443
	ITM	5.54	2.38	7.36	5.25	6.30	14.70	19.41	17.49	18.03	14.61	746	1055	930	1041	1025
	ATM	3.18	-1.97	4.75	0.83	2.65	15.23	17.69	19.05	18.58	17.29	656	536	298	412	405
	OTM	-0.99	-4.82	0.04	-0.58	-2.03	13.22	14.60	16.28	17.61	17.50	2414	1528	478	936	748
	DOTM	-7.32	-9.03	-2.87	-0.73	-6.95	14.39	12.27	17.01	17.41	18.17	981	878	161	312	258
UP Over 10	DITM	26.89	19.91	27.67	24.68	22.20	23.02	23.99	23.99	23.00	21.38	360	452	459	411	438
	ITM	20.66	12.31	20.71	13.22	15.28	25.94	27.72	30.44	29.62	25.55	487	600	445	521	474
	ATM	9.37	2.38	9.76	3.39	5.46	25.38	25.59	33.26	28.79	26.21	211	252	84	128	114
	OTM	1.88	-3.74	5.39	2.93	6.59	24.31	22.30	34.39	24.94	27.09	807	775	148	280	221
	DOTM	-16.63	-15.89	-17.54	-14.39	-12.63	20.22	18.58	24.69	22.23	25.28	564	584	70	98	105

Note: This table reports the mean and the standard deviation from the dynamic hedging strategy 2 with MV weights for call options and the miss pricing is defined in the absolute value. BS, H, M, Ba, AJD denote the Black Scholes, Heston, Merton, Bates and Affine Jump Diffusion models. OP denotes over price and UP denotes under price. DITM, ITM, ATM, OTM and DOTM denotes deep in the money, in the money, at the money, out of the money, and deep out of the money. Number indicates the number of the miss priced options.

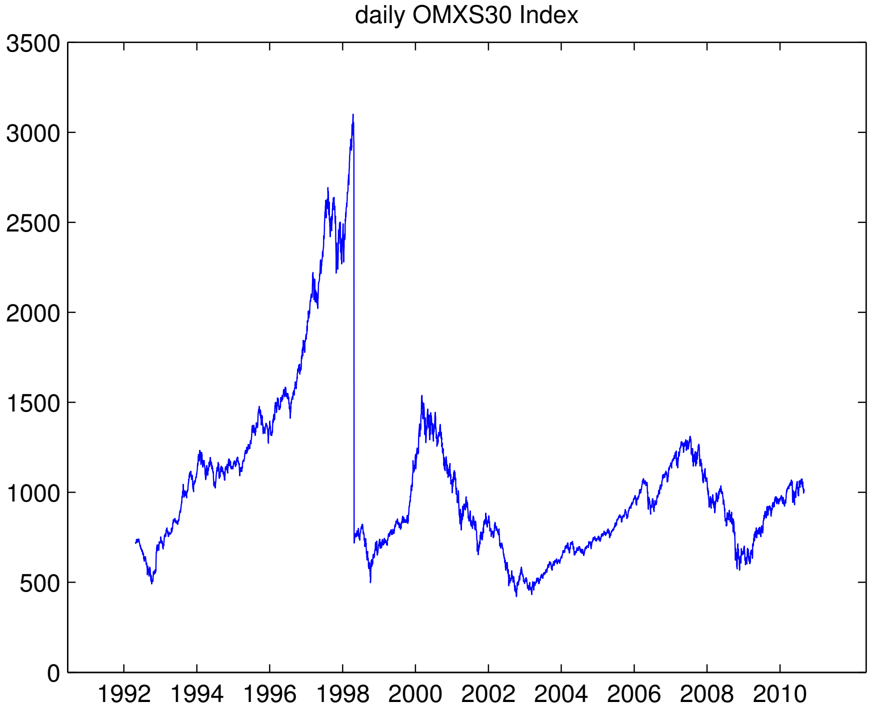
Table 9: Hedging Strategy 2 with transaction cost - Put

Category	Moneyness	Mean of NPV					STD of NPV					Number				
		BS	M	H	Ba	AJD	BS	M	H	Ba	AJD	BS	M	H	Ba	AJD
OP Over 10	DITM	11.43	12.00	11.19	12.70	11.80	11.72	11.85	11.80	11.94	12.01	301	300	419	403	379
	ITM	15.24	9.59	7.73	10.49	12.92	12.74	13.14	15.37	14.28	13.77	267	281	473	385	357
	ATM	15.05	10.02	7.84	8.96	11.58	15.31	18.47	23.91	23.24	20.77	184	124	158	128	101
	OTM	15.61	18.09	6.60	6.77	9.18	17.32	27.16	18.94	20.54	22.25	1481	331	307	272	203
	DOTM	8.38	28.55	20.78	21.42	22.88	17.86	31.65	28.81	32.03	31.55	477	107	108	90	86
OP 5-10	DITM	3.38	3.48	4.76	5.43	5.41	7.22	7.12	8.85	8.99	9.08	548	612	841	826	856
	ITM	7.01	6.04	5.82	5.88	7.30	10.93	10.95	13.08	12.96	11.64	866	1130	1168	1195	1045
	ATM	5.73	7.79	5.90	5.36	5.94	13.84	15.70	16.04	15.81	15.33	587	553	425	443	369
	OTM	4.17	12.10	6.56	5.66	5.77	12.05	20.33	15.73	16.64	13.93	4972	1892	854	1038	782
	DOTM	2.37	7.77	1.64	1.02	3.63	8.79	14.71	8.68	11.13	10.52	1441	523	234	226	214
OP 2-5	DITM	2.69	3.02	3.58	3.61	3.55	4.96	5.05	6.47	6.40	6.34	8170	8378	9529	9337	9322
	ITM	3.88	3.70	4.44	4.04	4.40	6.86	8.75	7.86	8.44	7.71	6290	7579	8602	8459	8161
	ATM	2.15	3.12	3.82	3.20	4.08	11.98	12.42	11.46	12.80	12.02	1967	1924	1928	1851	1469
	OTM	1.29	4.67	4.56	3.58	4.38	8.07	14.12	12.87	12.95	11.88	15187	7728	5196	5050	4022
	DOTM	1.03	3.61	1.86	0.11	2.54	6.57	11.10	7.62	9.38	7.70	2711	1951	905	1106	931
UP 2-5	DITM	-9.69	-9.38	-6.18	-6.30	-5.87	15.78	14.82	13.50	13.09	13.36	856	816	422	435	459
	ITM	-5.95	-7.20	-6.83	-5.73	-6.25	11.34	14.96	13.28	14.22	12.15	5258	3151	1979	2146	2193
	ATM	-5.58	-6.71	-4.73	-5.56	-5.67	9.72	14.88	12.46	14.50	10.31	2248	1464	1297	1116	1372
	OTM	-2.97	-4.87	-4.17	-6.80	-3.92	11.86	14.86	14.27	17.22	12.76	1199	2052	2420	2376	2599
	DOTM	3.82	0.94	1.85	1.63	2.24	10.13	9.06	8.24	11.38	9.42	275	609	967	786	788
UP 5-10	DITM	-10.22	-9.92	-5.65	-6.13	-4.43	15.14	14.81	14.40	15.03	15.38	667	614	257	309	293
	ITM	-6.85	-10.32	-6.51	-7.36	-6.60	14.09	14.66	14.26	16.79	14.30	1904	1284	521	779	632
	ATM	-5.57	-14.10	-6.96	-12.84	-7.43	11.58	14.41	12.75	16.81	12.66	492	393	195	246	238
	OTM	-2.91	-10.18	0.67	-10.50	-3.47	12.36	18.61	13.51	24.93	16.71	441	589	296	438	402
	DOTM	7.22	-2.17	7.47	6.80	7.62	12.14	14.17	9.22	12.50	11.20	151	226	212	144	150
UP Over 10	DITM	-9.60	-11.15	8.32	4.70	6.39	20.89	20.07	24.26	25.54	24.66	424	428	56	72	66
	ITM	-8.40	-12.20	-2.76	-5.72	-2.95	14.01	14.58	15.30	15.35	15.25	680	653	144	282	193
	ATM	-6.78	-13.55	-4.74	-10.92	-6.35	16.01	17.55	14.88	15.15	11.14	111	148	32	74	57
	OTM	5.32	-3.10	15.00	-0.69	7.16	21.49	23.38	16.37	24.85	17.20	173	224	101	138	114
	DOTM	11.79	2.17	17.43	9.99	12.43	11.21	11.93	8.40	9.72	5.20	94	91	48	31	30

Note: This table reports the mean and the standard deviation from the dynamic hedging strategy 2 with MV weights for put options and the miss pricing is defined in the absolute value. BS, H, M, Ba, AJD denote the Black Scholes, Heston, Merton, Bates and Affine Jump Diffusion models. OP denotes over price and UP denotes under price. DITM, ITM, ATM OTM, and DOTM denotes deep in the money, in the money, at the money, out of the money, and deep out of the money. Number indicates the number of the miss priced options.

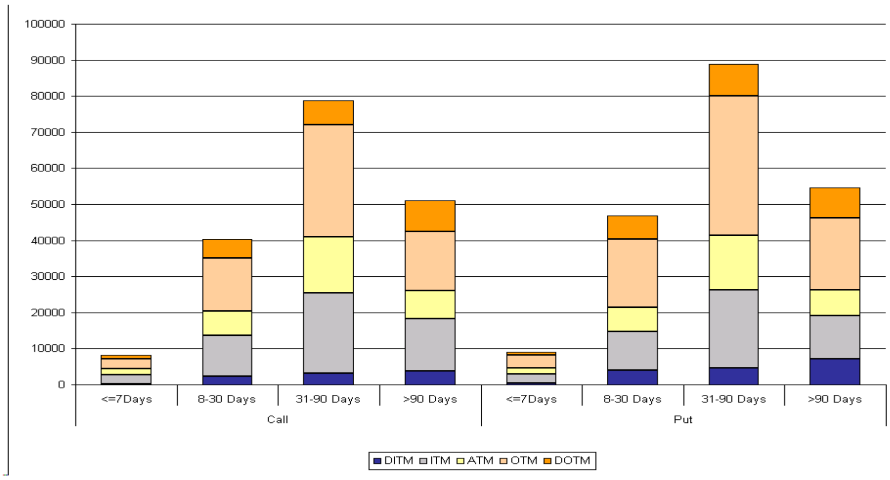
8 Figures

Figure 1: The OMXS30 Index prices



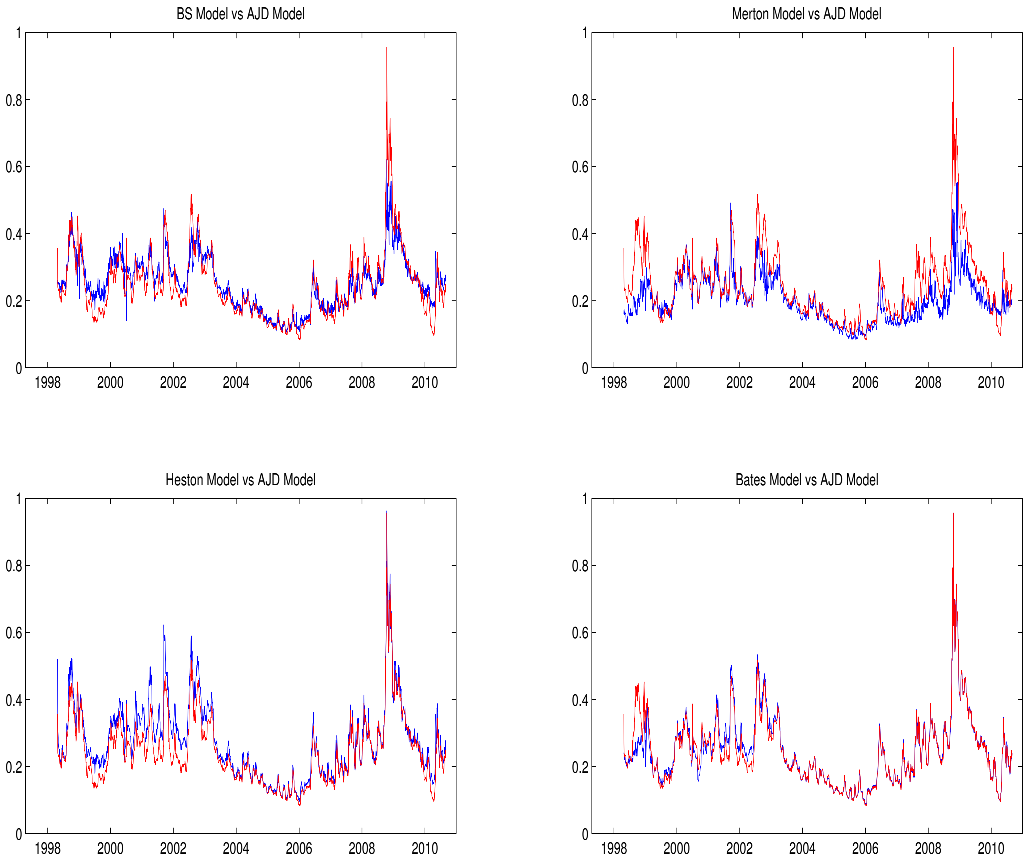
Note: This figure plots the OMX index prices from 1992 to 2010. The index was divided by 4 on April 27, 1998.

Figure 2: Number of Options



Note: This figure plots the number of call and put options in different time to maturity and different moneyness. DITM, ITM, ATM OTM, and DOTM denotes deep in the money, in the money, at the money, out of the money, and deep out of the money.

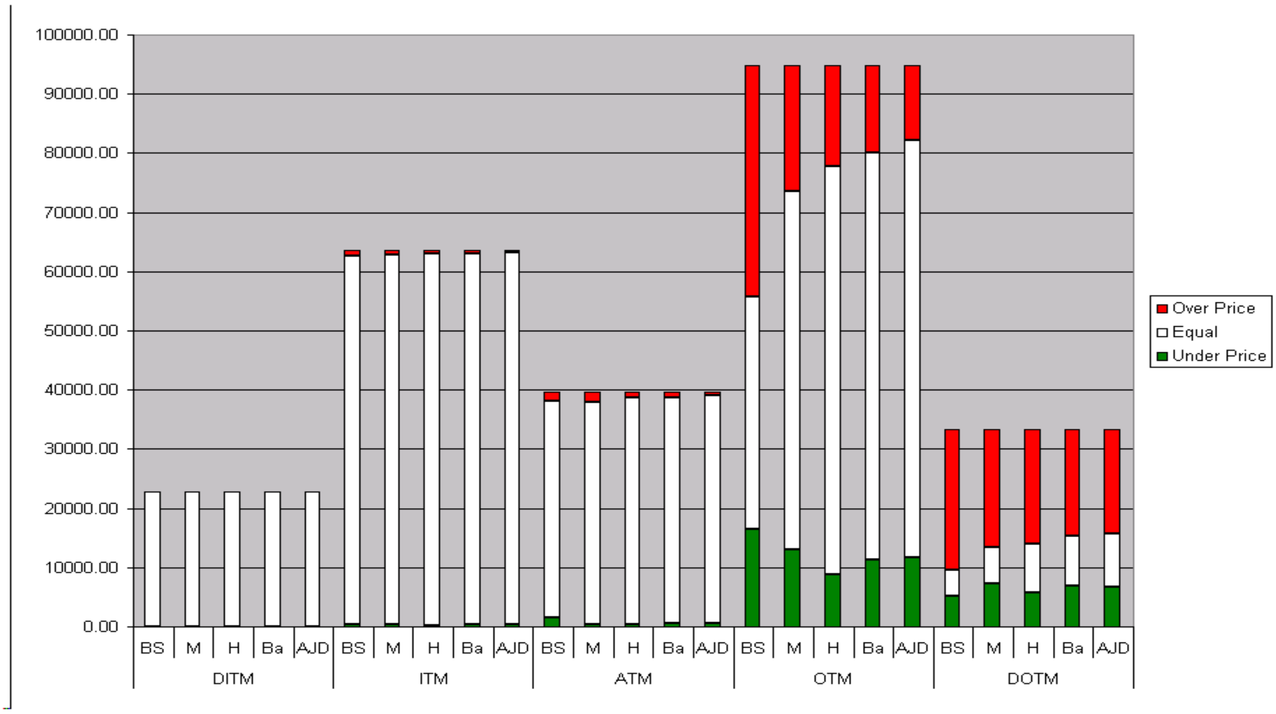
Figure 3: Estimated Volatility



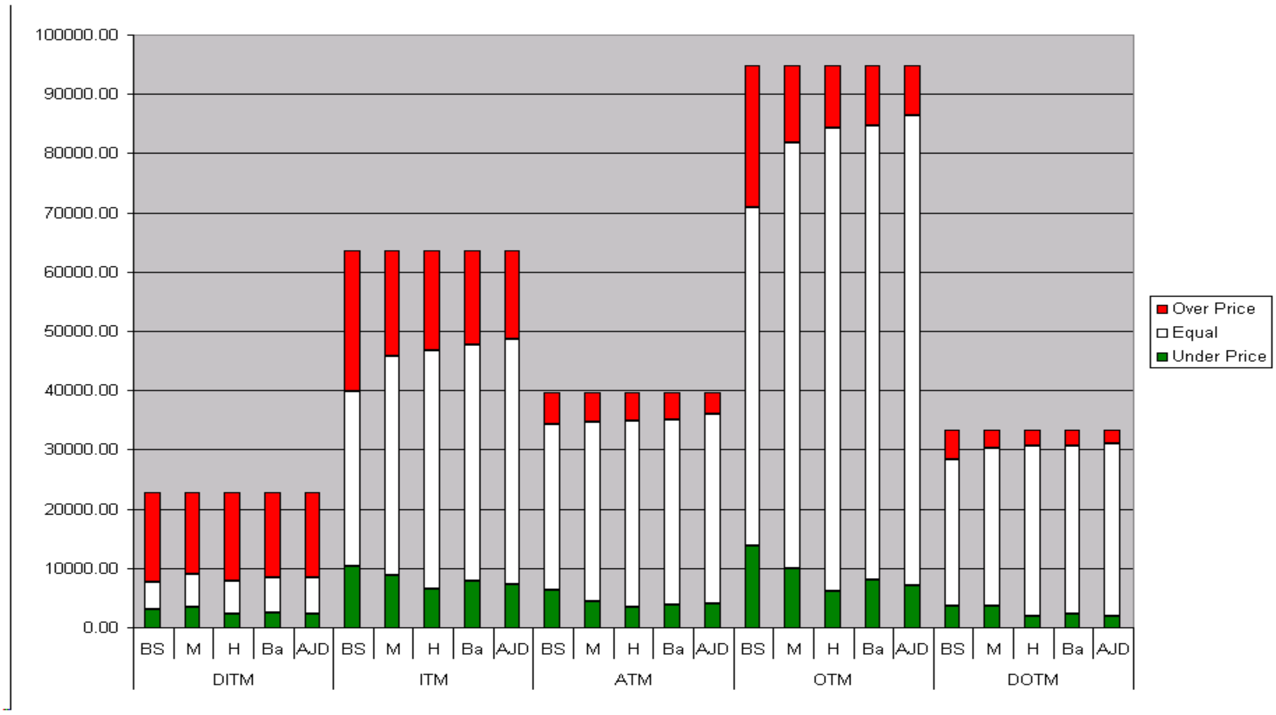
Note: This figure plots the estimated volatility from various models.

Figure 4: Number of Detected Misprice

(a) Misprice by Percentage (15%)



(b) Misprice by Value (2)



Note: This figure plots the number of detected mispriced options when the mispricing is defined in the percentage and in the absolute value.

9 Appendix

Appendix A. The implementations of the unscented Kalman Filter (UKF)

Suppose that we have the non-linear system as follows,

$$\begin{aligned} x_k &= f(x_{k-1}, u_{k-1}) + w_{k-1}, &(15)\\ z_k &= h(x_k) + v_k. &(16)\end{aligned}$$

The basic idea of the UKF is as follows: the state x_t is approximated by a set of weighted points called *sigma points* with the same mean $\bar{x}$ and covariance $\mathbf{P}_{xx}$. Applying the non-linear function f to each sigma point to yield the set of transformed sigma points, we can then obtain the mean and covariance of the transformed state $f(x_t)$ following the method from Gove & Hollinger (2006).

The sigma points and the weight for the ith points of their respective mean (m) and covariance (c) have the form,

$$\begin{aligned} \mathcal{X}^0 &= \bar{x} & W_0^{(m)} &= \lambda/(n+\lambda)\\ \mathcal{X}^i &= \bar{x} + (\sqrt{(n+\lambda)\mathbf{P}_{xx}})_i & W_0^{(c)} &= \lambda/(n+\lambda) + (1-\alpha^2+\beta)\\ \mathcal{X}^{i+n} &= \bar{x} - (\sqrt{(n+\lambda)\mathbf{P}_{xx}})_i & W_i^{(m)} &= W_i^{(c)} = 1/2(n+\lambda)\end{aligned}$$

where the parameters $0 \leq \alpha \leq 1$ and $\beta \geq 0$ control the spread of the sigma points and weighting for higher-order moments, parameter $\kappa \geq 0$ is often set to zero, $\lambda = \alpha^2(n+\kappa) - n$, $(\sqrt{(n+\lambda)\mathbf{P}_{xx}})_i$ is the ith row of the matrix square root of $(n+\kappa)\mathbf{P}_{xx}$. Then the transformed state $\mathcal{Z}_i = f(\mathcal{X}_i)$, with the mean and covariance as follows,

$$\begin{aligned} \bar{z} &= \sum_{i=0}^{2n} W_i^{(m)} \mathcal{Z}_i,\\ \mathbf{P}_{zz} &= \sum_{i=0}^{2n} W_i^{(c)} \{\mathcal{Z}_i - \bar{z}\}\{\mathcal{Z}_i - \bar{z}\}^T,\\ \mathbf{P}_{xz} &= \sum_{i=0}^{2n} W_i^{(c)} \{\mathcal{X}_i - \bar{x}\}\{\mathcal{Z}_i - \bar{z}\}^T.\end{aligned}$$

Second, using the unscented transform above, we can derive the UKF procedure. The sigma points of the state x_{k-1}

$$\mathcal{X}_{k-1} = \left[\hat{x}_{k-1} \quad \hat{x}_{k-1} + \sqrt{(n+\kappa)\mathbf{P}_{xx,k-1}} \quad \hat{x}_{k-1} - \sqrt{(n+\kappa)\mathbf{P}_{xx,k-1}}\right],$$

and the transformed state

$$\begin{aligned} \mathcal{X}_{k|k-1} &= f(\mathcal{X}_{k-1}, u_{k-1}),\\ \mathcal{Z}_{k|k-1} &= h(\mathcal{Z}_{k-1}).\end{aligned}$$

The prediction equations are just the mean and covariance of the transformed state.
Prediction equations:

$$
\begin{aligned}
\hat{x}_{k|k-1} &= \sum_{i=0}^{2n} W_i^{(m)} \mathcal{X}_{k|k-1}^i, \\
\hat{z}_{k|k-1} &= \sum_{i=0}^{2n} W_i^{(m)} \mathcal{Z}_{k|k-1}^i, \\
\mathbf{P}_{xx,k|k-1} &= \sum_{i=0}^{2n} W_i^{(c)} [\mathcal{X}_{k|k-1}^i - \hat{x}_{k|k-1}][\mathcal{X}_{k|k-1}^i - \hat{x}_{k|k-1}]^T + Q_k.
\end{aligned}
$$

Then we update the prediction equation in the UKF about the measurement state $\hat{z}_{k|k-1}$,
Update equations:

$$
\begin{aligned}
K_k &= \mathbf{P}_{xz}\mathbf{P}_{zz}^{-1}, \\
\hat{x}_k &= \hat{x}_{k|k-1} + K_k(z_k - \hat{z}_{k|k-1}), \\
\mathbf{P}_{xx,k} &= \mathbf{P}_{xx,k|k-1} - K_k \mathbf{P}_{zz} K_k^T,
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{P}_{zz} &= \sum_{i=0}^{2n} W_i^{(c)} [\mathcal{Z}_{k|k-1}^i - \hat{z}_{k|k-1}][\mathcal{Z}_{k|k-1}^i - \hat{z}_{k|k-1}]^T + R_k, \\
\mathbf{P}_{xz} &= \sum_{i=0}^{2n} W_i^{(c)} [\mathcal{X}_{k|k-1}^i - \hat{x}_{k|k-1}][\mathcal{Z}_{k|k-1}^i - \hat{z}_{k|k-1}]^T.
\end{aligned}
$$

Appendix B. The derivation of the mean variance hedge for the Affine Jump Diffusion model

Suppose $\{S_t\}$ is the stock price process; C_t is the option price at time t and the maturity date is denoted by T. The mean variance hedging weight can be calculated as

$$h_s = \frac{E^Q[\widetilde{\Delta C_t}\widetilde{\Delta S_t}|\mathcal{F}_t]}{E^{\mathbb{Q}}[\widetilde{\Delta S_t}^2|\mathcal{F}_t]}$$

Furthermore, we argue that using Eq. (12) that

$$h_s = \frac{e^{-r\tau}}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} e^{-z\ln K}\frac{e^{-2r\delta}E^{\mathbb{Q}}[e^{\ln S_{t+\delta}+K_{t+\delta,T}(z+1)}|\mathcal{F}_t] - e^{\ln S_t+K_{t,T}(z+1)}}{z(z+1)(e^{-2r\delta}E^{\mathbb{Q}}[e^{2\ln S_{t+\delta}}|\mathcal{F}_t] - e^{2\ln S_t})}\mathrm{d}z.$$

We now want to show that this expression can be written as

$$\frac{e^{-r\tau}}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} e^{-z\ln\frac{K}{S_t}}\frac{e^{-2r\delta+\alpha(\tau-\delta,z_1)+\tilde{\alpha}(\delta,z_2,z_3)+\tilde{\beta}(\delta,z_2,z_3)V_t} - e^{\alpha(\tau,z_1)+\beta(\tau,z_1)V_t}}{z(z+1)(e^{-2r\delta+\alpha(\delta,2)+\beta(\delta,2)V_t}-1)}\mathrm{d}z \tag{17}$$

where $\tau = T-t,\ z_1 = z+1, z_2 = z+2, z_3 = \beta(\tau-\delta, z+1)$, $\tilde{\alpha}(u,x,y)$ and $\tilde{\beta}(u,x,y)$ are defined through

$$E^{\mathbb{Q}}[e^{xY_{t+u}+yV_{t+u}}|\mathcal{F}_t] = e^{xY_t+\tilde{\alpha}(u,x,y)+\tilde{\beta}(u,x,y)V_t},$$

for all $x, y \in \mathbb{C}$ such that $E^{\mathbb{Q}}[|e^{xY_{t+u}+yV_{t+u}}|] < \infty$, i.e. $e^{xY_t+\tilde{\alpha}(u,x,y)+\tilde{\beta}(u,x,y)V_t}$ is the simultaneous moment generating function of the log-stockprice $\{Y\}$ and the stochastic volatility $\{V\}$. We prove this in two steps. Equation (17) will be derived in Part I and the closed form of the AJD model will be calculated in Part II.

Part I Calculation of $h_s = \frac{E^Q[\widetilde{\Delta C_t}\widetilde{\Delta S_t}|\mathcal{F}_t]}{E^{\mathbb{Q}}[\widetilde{\Delta S_t}^2|\mathcal{F}_t]}$.
The denominator of h_s is derived by

$$\begin{aligned}
E^{\mathbb{Q}}[\widetilde{\Delta S_t}^2|\mathcal{F}_t] &= E^{\mathbb{Q}}[(e^{-r\delta}S_{t+\delta}-S_t)(e^{-r\delta}S_{t+\delta}-S_t)|\mathcal{F}_t] \\
&= E^{\mathbb{Q}}[e^{-2r\delta}S_{t+\delta}^2 - 2e^{-r\delta}S_{t+\delta}S_t + S_t^2|\mathcal{F}_t] \\
&= E^{\mathbb{Q}}[e^{-2r\delta}S_{t+\delta}^2|\mathcal{F}_t] - S_t^2 \\
&= e^{-2r\delta}E^{\mathbb{Q}}[e^{2\ln S_{t+\delta}}|\mathcal{F}_t] - S_t^2 \\
&= e^{-2r\delta+2\ln S_t+\alpha(\delta,2)+\beta(\delta,2)V_t} - e^{2\ln(S_t)} \\
&= e^{2\ln(S_t)}(e^{-2r\delta+\alpha(\delta,2)+\beta(\delta,2)V_t} - 1).
\end{aligned}$$

The numerator of h_s is separated into two parts I_1 and I_2,

$$\begin{aligned}
E^Q[\widetilde{\Delta C_t}\widetilde{\Delta S_t}|\mathcal{F}_t] &= E^Q[(e^{-r\delta}C_{t+\delta}-C_t)(e^{-r\delta}S_{t+\delta}-S_t)|\mathcal{F}_t] \\
&= E^Q[e^{-2r\delta}C_{t+\delta}S_{t+\delta} - e^{-r\delta}C_{t+\delta}S_t - e^{-r\delta}C_tS_{t+\delta} + C_tS_t|\mathcal{F}_t] \\
&= \underbrace{E^Q[e^{-2r\delta}C_{t+\delta}S_{t+\delta}|\mathcal{F}_t]}_{I_2} - \underbrace{C_tS_t}_{I_1}
\end{aligned}$$

where using Eq. (12) and (13) we obtain

$$
\begin{aligned}
I_1 &= \frac{e^{-r\tau}}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} e^{-z\ln K}\frac{e^{(z+2)Y_t+\alpha(\tau,z+1)+\beta(\tau,z+1)V_t}}{z(z+1)}\mathrm{d}z \\
I_2 &= \frac{e^{-r\tau}}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} e^{-z\ln K}\frac{e^{-2r\delta+\alpha(\tau-\delta,z+1)}E^{\mathbb{Q}}[e^{(z+2)Y_{t+\delta}+\beta(\tau-\delta,z+1)V_{t+\delta}}|\mathcal{F}_t]}{z(z+1)}\mathrm{d}z
\end{aligned}
$$

Now using the definition of $\tilde{\alpha}$ and $\tilde{\beta}$ as the representation of the simultaneous moment generating function of Y and V we can rewrite I_2 as

$$
\begin{aligned}
I_2 &= \frac{e^{-r\tau}}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} e^{-z\ln K}\frac{e^{-2r\delta+\alpha(\tau-\delta,z+1)+(z+2)Y_t+\tilde{\alpha}(\delta,z+2,\beta(\tau-\delta,z+1))+\tilde{\beta}(\delta,z+2,\beta(\tau-\delta,z+1))V_t}}{z(z+1)}\mathrm{d}z \\
&= \frac{e^{-r\tau}}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} e^{-z\ln K}\frac{e^{-2r\delta+\alpha(\tau-\delta,z_1)+(z+2)Y_t+\tilde{\alpha}(\delta,z_2,z_3)+\tilde{\beta}(\delta,z_2,z_3)V_t}}{z(z+1)}\mathrm{d}z
\end{aligned}
$$

where $\tau = T-t,\ z_1 = z+1, z_2 = z+2, z_3 = \beta(\tau-\delta, z+1)$. Now putting everything together and canceling common factors we obtain

$$
\begin{aligned}
h_s &= \frac{E^{\mathbb{Q}}[\widetilde{\Delta C_t}\widetilde{\Delta S_t}|\mathcal{F}_t]}{E^{\mathbb{Q}}[\widetilde{\Delta S_t}^2|\mathcal{F}_t]} \\
&= \frac{I_1 - I_2}{E^{\mathbb{Q}}[\widetilde{\Delta S_t}^2|\mathcal{F}_t]} \\
&= \frac{e^{-r\tau}}{2\pi i}\int_{-i\infty+\bar{z}}^{i\infty+\bar{z}} e^{-z\ln\frac{K}{S_t}}\frac{e^{-2r\delta+\tilde{\alpha}(\delta,z_2,z_3)+\tilde{\beta}(\delta,z_2,z_3)V_t+\alpha(\tau-\delta,z_1)} - e^{\alpha(\tau,z_1)+\beta(\tau,z_1)V_t}}{z(z+1)(e^{-2r\delta+\alpha(\delta,2)+\beta(\delta,2)V_t}-1)}\mathrm{d}z,
\end{aligned}
$$

which is exactly the equation(17).

Part II Calculation of of the conditional simultaneous moment generating function for the log stock-price Y and the volatility V for the AJD model and its corresponding sub models.

$$E^{\mathbb{Q}}[e^{z_1 Y_u + z_2 V_u}|\mathcal{F}_t] = e^{z_1 Y_t + \tilde{\alpha}(u-t,z_1,z_2)+\tilde{\beta}(u-t,z_1,z_2)V_t},$$

here we assume that $z_1, z_2 \in \mathbb{C}$ are such that $E^{\mathbb{Q}}[|e^{z_1 Y_u + z_2 V_u}|] < \infty$. In the calculations below we assume that logarithms and square roots of complex numbers are taken as the principal branch.

We view t as a fixed number and also suppress the dependence on z_1 and z_2 for notational convenience and write $g(u) = \tilde{\alpha}(u-t, z_1, z_2)$ and $h(u) = \tilde{\beta}(u-t, z_1, z_2)$ as functions with respect to u ($t < u < T$). Since

$$E^{\mathbb{Q}}[e^{z_1 Y_t + z_2 V_t}|\mathcal{F}_t] = e^{z_1 Y_t + z_2 V_t}.$$

we get $g(t) = 0$ and $h(t) = z_2$. Let $\mathcal{A}$ denote the generator of the bivariate Markov process (Y_t, V_t). We have that, assuming f being $C^2(\mathbb{R} \times \mathbb{R}^+)$ and sufficiently integrable,

$$\begin{aligned}
\mathcal{A}(f)(y,v) &= (r - \frac{v}{2} - \lambda\bar{\mu})\frac{\partial}{\partial y}f(y,v) + \kappa(\xi - v)\frac{\partial}{\partial v}f(y,v) \\
&+\frac{1}{2}\left(v\frac{\partial^2}{\partial y^2}f(y,v) + \sigma_v^2 v\frac{\partial^2}{\partial v^2}f(y,v) + 2\rho_w\sigma_v v\frac{\partial^2}{\partial x\partial v}f(y,v)\right) \\
&+\lambda\int_{\mathbb{R}^2}\left(f(y+y_1, v+v_1) - f(y,v)\right)J(\mathrm{d}y_1, \mathrm{d}v_1),
\end{aligned}$$

with

$$\bar{\mu} = \int_{\mathbb{R}^2}\left(e^{y_1} - 1\right)J(\mathrm{d}y_1, \mathrm{d}v_1),$$

where J is the simultaneous density for the jumps of the processes Y and V. The process

$$M_t = e^{z_1 Y_t + \tilde{\alpha}(u-t, z_1, z_2) + \tilde{\beta}(u-t, z_1, z_2)V_t}$$

is a $\mathbb{Q}$-martingale by the tower property of conditional expectation. The martingale property is now equivalent to,

$$\frac{\partial}{\partial t}e^{z_1 Y_t + \tilde{\alpha}(u-t,z_1,z_2) + \tilde{\beta}(u-t,z_1,z_2)V_t} + \mathcal{A}(e^{z_1 Y_t + \tilde{\alpha}(u-t,z_1,z_2) + \tilde{\beta}(u-t,z_1,z_2)V_t}) = 0,$$

together with the integrability condition stated above. Using the specific form of $\mathcal{A}$ and that M is a martingale as well as that

$$\begin{aligned}
\frac{\partial}{\partial u}g(u) &= -\frac{\partial}{\partial t}\tilde{\alpha}(u-t, z_1, z_2) \\
\frac{\partial}{\partial u}h(u) &= -\frac{\partial}{\partial t}\tilde{\beta}(u-t, z_1, z_2)
\end{aligned}$$

we obtain a system of ODEs with boundary conditions

$$\begin{aligned}
\dot{h}(u) &= \frac{z_1^2 - z_1}{2} - (\kappa - \rho_w\sigma_v z_1)h(u) + \frac{\sigma_v^2}{2}h(u)^2, \\
\dot{g}(u) &= (r - \lambda\bar{\mu})z_1 + \kappa\xi h(u) + \lambda(\Theta(z_1, h(u)) - 1), \\
h(t) &= z_2, \\
g(t) &= 0,
\end{aligned}$$

where Θ is simultaneous moment generating function for the jumps in Y and V, i.e.

$$\begin{aligned}
\Theta(d_1, d_2) &= \int_{\mathbb{R}^2} e^{d_1 y + d_2 v} J(\mathrm{d}x, \mathrm{d}v), \\
&= \frac{\lambda^y}{\lambda}e^{\mu_y d_1 + \frac{1}{2}\sigma_y^2 d_1^2} + \frac{\lambda^v}{\lambda}\frac{1}{1 - \mu_v d_2} + \frac{\lambda^c}{\lambda}\frac{e^{\mu_{cy}d_1 + \frac{1}{2}\sigma_{cy}^2 d_1^2}}{1 - \mu_{cv}d_2 - \rho_J\mu_{cv}d_1},
\end{aligned}$$

where $\lambda = \lambda^y + \lambda^v + \lambda^c$. For $h(u)$, we rewrite the ODE above as

$$\dot{h}(u) = a_0 + a_1 h(u) + a_2 h(u)^2, \tag{18}$$

where

$$a_0 = \frac{z_1^2 - z_1}{2}, a_1 = -(\kappa - \rho_w \sigma_v z_1), a_2 = \frac{\sigma_v^2}{2}.$$

Set $\gamma = \sqrt{-4a_0 a_2 + a_1^2}$, $l(u) = \frac{-2a_2}{\gamma}(h(u) + \frac{a_1}{2a_2})$ and substitute it into (18) gives

$$\dot{l}(u) = \frac{\gamma}{2}(1 - l(u)^2).$$

This is a separable ODE which we solve as follows:

$$\begin{aligned}
\frac{\mathrm{d}l}{1-y^2} = \frac{\gamma}{2}\mathrm{d}u \quad &\Rightarrow \quad \frac{1}{2}\Big(\frac{1}{1-l} + \frac{1}{1+l}\Big)\mathrm{d}l = \frac{\gamma}{2}\mathrm{d}u, \\
&\Rightarrow \quad \ln\Big(\frac{1+l(u)}{1-l(u)}\Big) = u\gamma + b'', \\
&\Rightarrow \quad l(u) = \frac{b' e^{u\gamma} - 1}{b' e^{u\gamma} + 1}.
\end{aligned}$$

Since $h(u) = \frac{-a_1}{2a_2} - \frac{\gamma}{2a_2} l(u)$ and using the boundary condition $h(t) = z_2$, we obtain

$$\begin{aligned}
h(u) &= \frac{-a_1}{2a_2} - \frac{\gamma}{2a_2}\frac{e^{(u-t)\gamma} - b}{e^{(u-t)\gamma} + b} \\
&= \frac{-1}{2a_2}\left(\gamma + a_1 - 2\gamma\frac{e^{-(u-t)\gamma} b}{1 + be^{-(u-t)\gamma}}\right) \\
b &= \frac{\gamma + a_1 + 2a_2 z_2}{\gamma - a_1 - 2a_2 z_2}
\end{aligned}$$

and because $\dot{g}(u) = (r - \lambda\bar{\mu})h(u) + \kappa\xi h(u) + \lambda(\Theta(z_1, h(u)) - 1)$ and $g(t) = 0$ we have the expression of $g(u)$ as follows

$$g(u) = (r - \lambda\bar{\mu})z_1(u-t) + \kappa\xi\int_t^u h(s)\mathrm{d}s + \lambda(\int_t^u \Theta(z_1, h(s))\mathrm{d}s - (u-t)),$$

where

$$\begin{aligned}
\kappa\xi\int_t^u h(s)\mathrm{d}s &= \frac{-\kappa\xi}{2a_2}\left((\gamma + a_1)(u-t) - 2\ln\left(\frac{1+b}{1+be^{-(u-t)\gamma}}\right)\right), \\
\lambda\int_t^u \Theta(z_1, h(s))\mathrm{d}s &= f_1(u) + f_2(u) + f_3(u),
\end{aligned}$$

and where finally

$$
\begin{aligned}
f_1(u) &= \lambda^y (u-t) e^{\mu_y z_1 + \frac{1}{2}\sigma_y^2 z_1^2}, \\
f_2(u) &= \frac{2a_2\lambda^v}{c_1}\left((u-t) + \frac{1}{c_2}\ln\left(\frac{c_1 + c_2 b}{c_1 + e^{-(u-t)\gamma} c_2 b}\right)\right), \\
f_3(u) &= \frac{2a_2\lambda^c}{c_3}\left((u-t) + \frac{1}{c_4}\ln\left(\frac{c_3 + c_4 b}{c_3 + e^{-(u-t)\gamma} c_4 b}\right)\right) e^{\mu_{cy} z_1 + \frac{1}{2}\sigma_{cy}^2 z_1^2}, \\
a_0 &= \frac{z_1^2 - z_1}{2}, \\
a_1 &= -(\kappa - \rho_w \sigma_v z_1), \\
a_2 &= \frac{\sigma_v^2}{2}, \\
b &= \frac{\gamma + a_1 + 2a_2 z_2}{\gamma - a_1 - 2a_2 z_2}, \\
c_1 &= 2a_2 + \mu_v(a_1 + \gamma), \\
c_2 &= 2a_2 + \mu_v(a_1 - \gamma), \\
c_3 &= 2a_2(1 - \rho_J \mu_{cv} z_1) + \mu_{cv}(a_1 + \gamma), \\
c_4 &= 2a_2(1 - \rho_J \mu_{cv} z_1) + \mu_{cv}(a_1 - \gamma), \\
\gamma &= \sqrt{-4a_0 a_2 + a_1^2}.
\end{aligned}
$$

In conclusion, the conditional simultaneous moment generating for the AJD model is

$$
E^{\mathbb{Q}}[e^{z_1 Y_u + z_2 V_u} | \mathcal{F}_t] = e^{z_1 Y_t + \tilde{\alpha}(u-t, z_1, z_2) + \tilde{\beta}(u-t, z_1, z_2) V_t},
$$

where $\tilde{\alpha}(u-t, z_1, z_2) = g(u)$, $\tilde{\beta}(u-t, z_1, z_2) = h(u)$ with g and h defined as above.